The Low-Frequency Bremsstrahlung/Memory Tail of Gravitational Radiation

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Berkeley UNIVERSITY OF CALIFORNIA

N3AS Network for Neutrinos, Nuclear Astrophysics, and Symmetries PHYSICS FRONTIER CENTER

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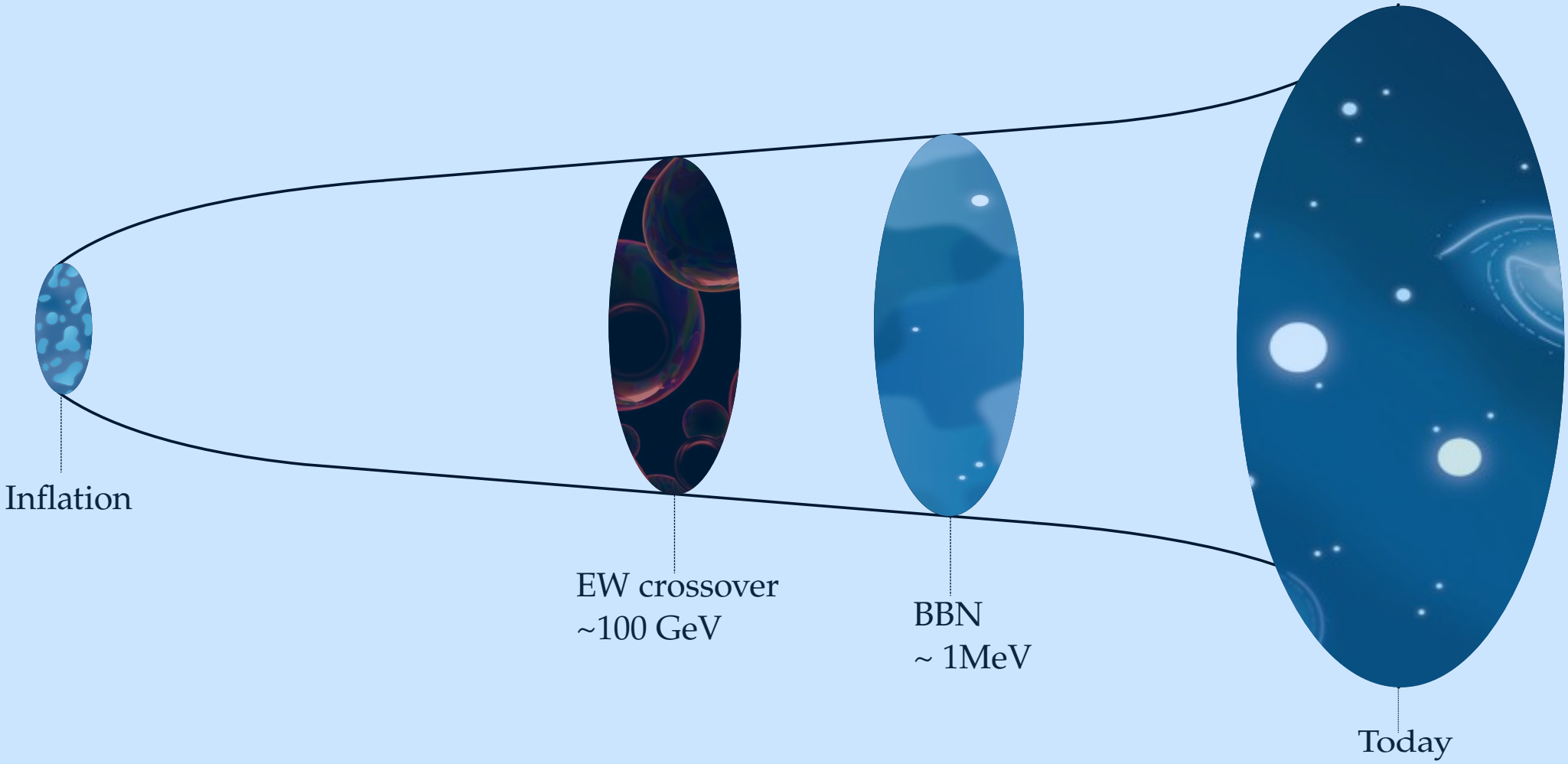
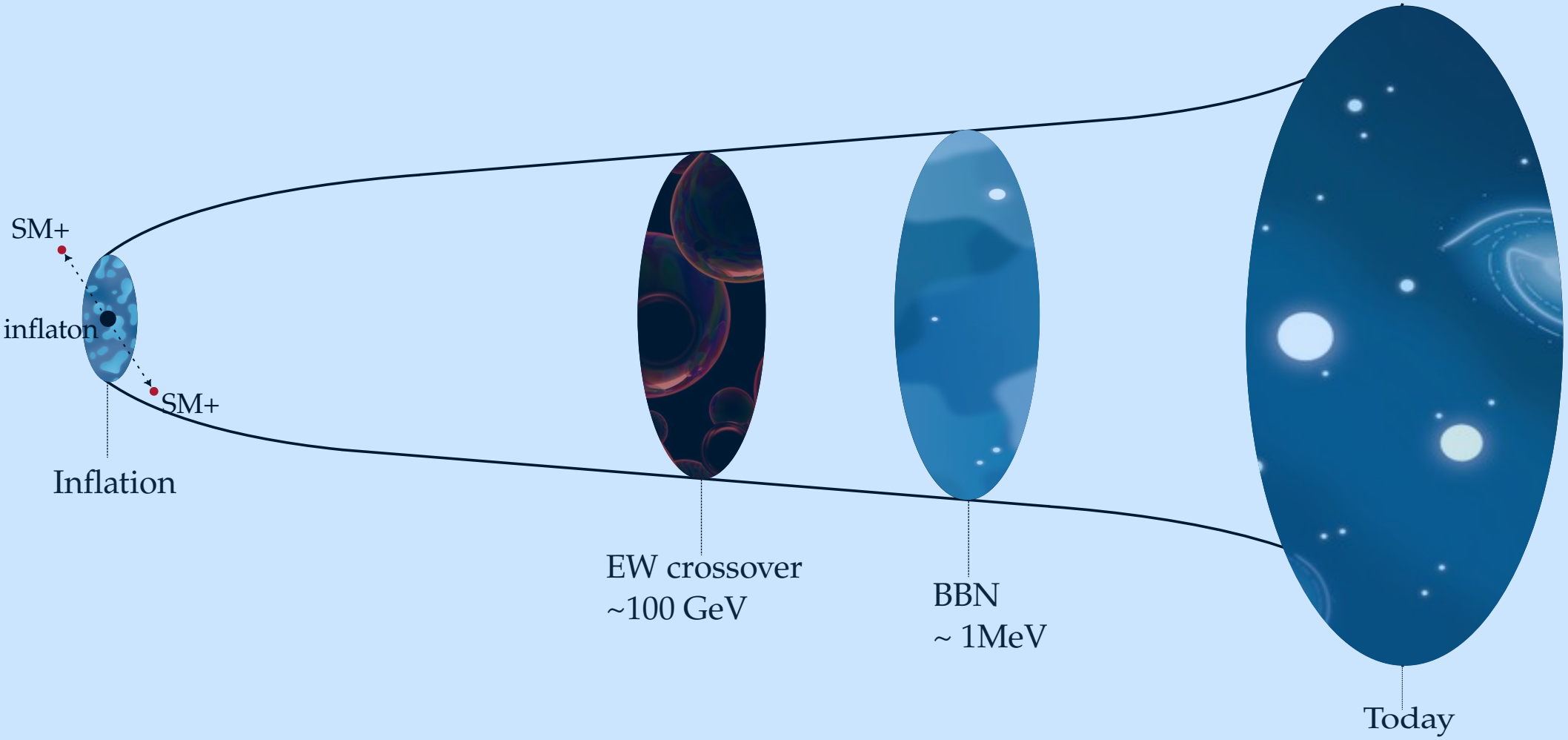

Inflation
EW crossover
~100 GeV
BBN
~ 1MeV
Today


SM+
inflaton
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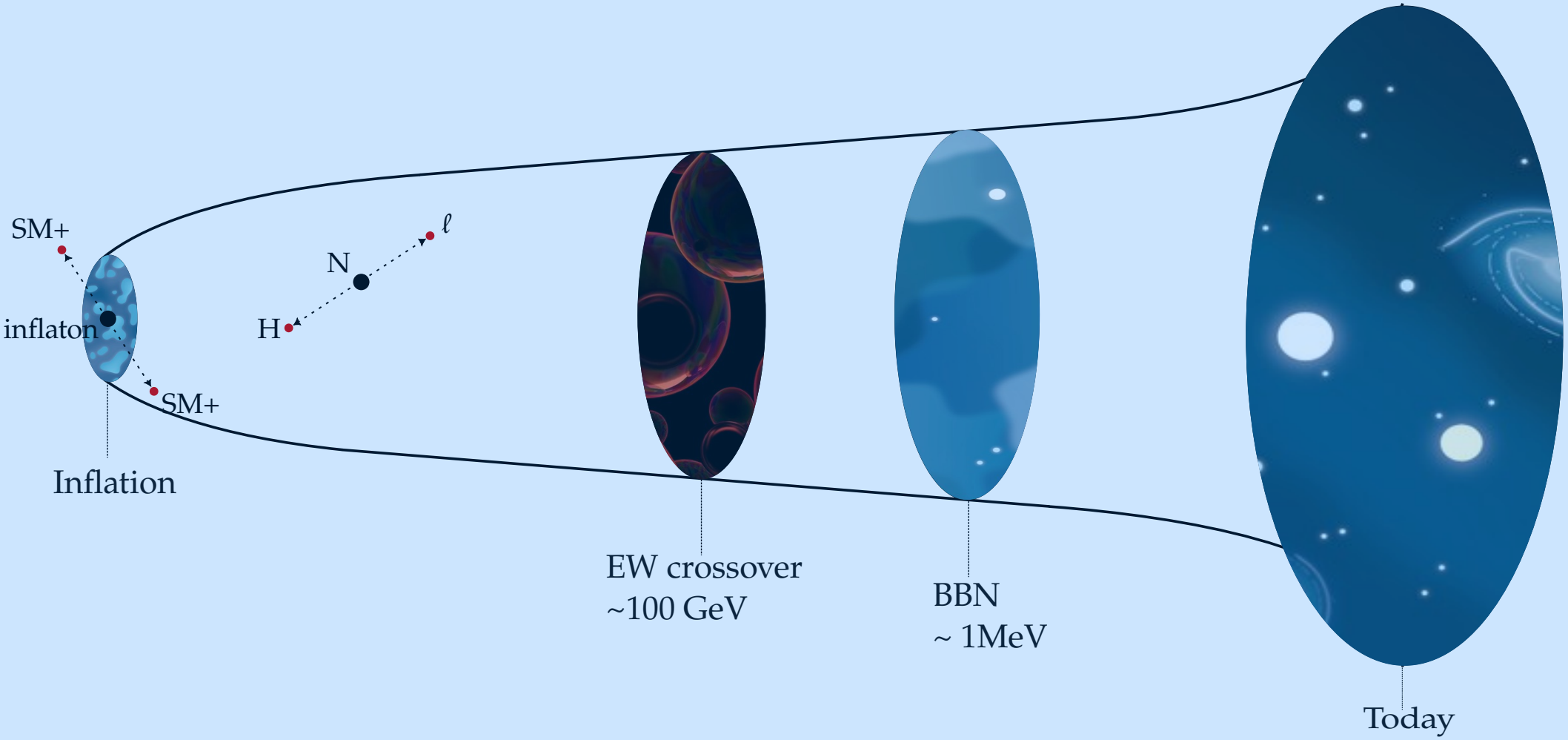
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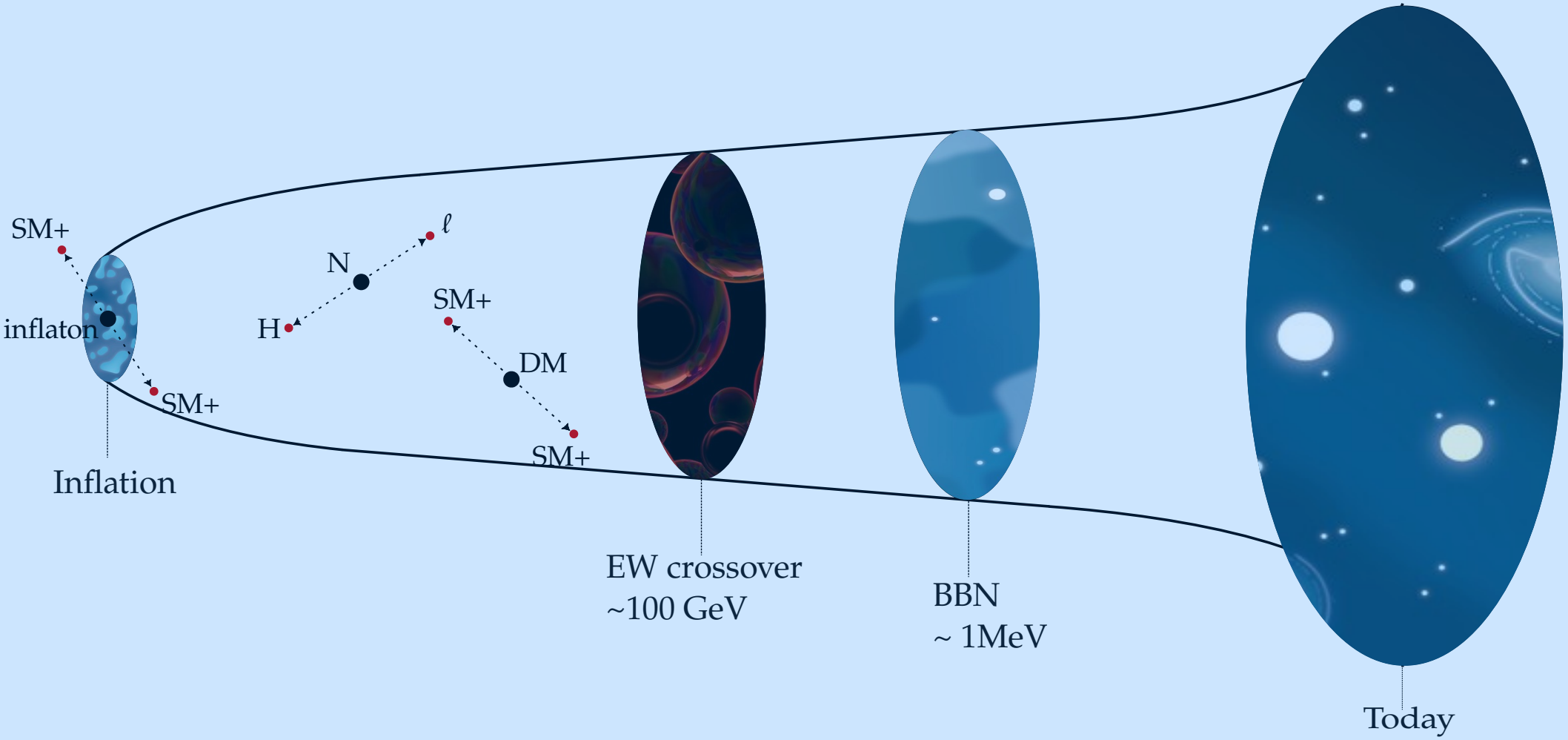
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SM+
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~100 GeV
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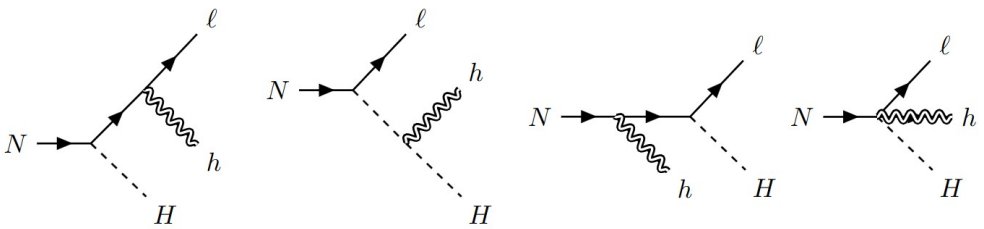
Today

Example: Leptogenesis

$$f_{\rm gw}^{\rm peak} = \frac{M}{2}\left(\frac{g_*^0}{g_*^D}\right)^{1/3}\frac{T_0}{T_D} = 7.9\times 10^{12}\,{\rm Hz}\left(\frac{M}{10^{15}\,{\rm GeV}}\right)^{1/2}\left(\frac{10^{-4}}{Y}\right)$$

$$(\partial_t + 4H)\rho_{\rm h} = \int \frac{d^3k}{(2\pi)^3}\frac{1}{2k}\Pi_{p,q,r}\, k\,|\mathcal{M}|^2\,({\rm dist.\ funct.})\delta^{(4)}(\cdots)$$

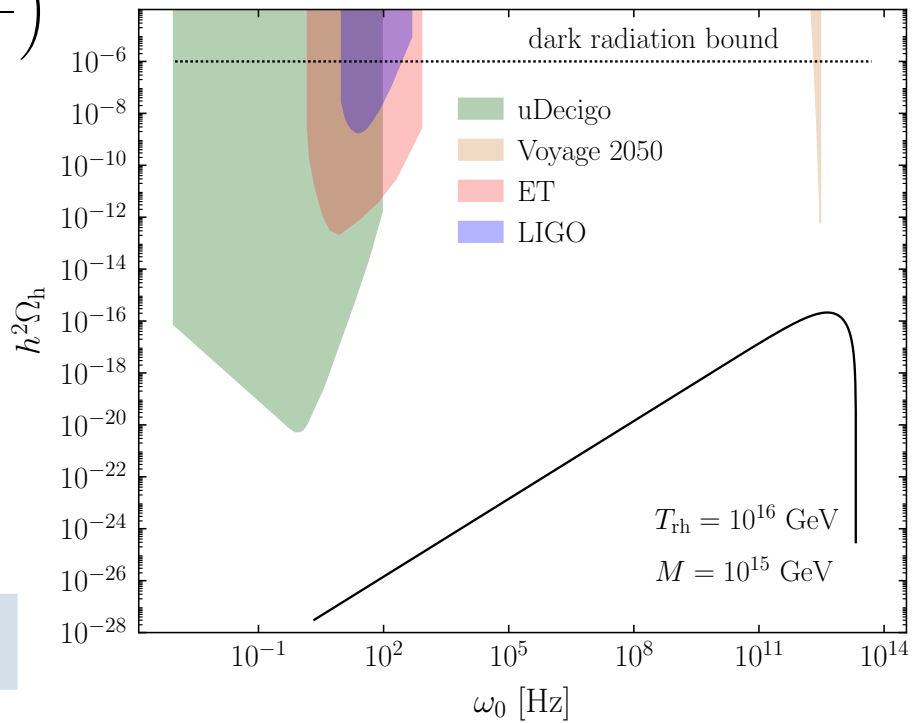


Weinberg's soft graviton theorem:

$$A = A_0\left(\frac{\kappa}{2}\right)\left[\frac{q_\mu q_\nu}{q\cdot k} + \frac{r_\mu r_\nu}{r\cdot k} + \frac{p_\mu p_\nu}{p\cdot k}\right]\epsilon^{\lambda,\mu\nu *}(k) \quad \Rightarrow |\mathcal{M}|^2 \sim \frac{1}{k^2}$$

$$d\rho_{\rm h} \sim dk\,k^2\,k\,\frac{1}{k^2} \sim dk$$

$$\Omega_h \sim \frac{d\rho_h}{d\ln k} \sim \frac{d\rho_h}{dk}\,k \sim k$$

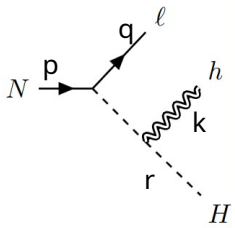


[Murayama, Noether, **JSE**, 25']

Q1: Is the low frequency tail a robust prediction?
Maybe better target than high-frequency peak

Q2: Can gravitational memory enhance the
low frequency signal?

Example: Leptogenesis



$$A = A_0 \left(\frac{\kappa}{2}\right) V_{\mu\nu}(k,r) \frac{1}{(r+k)^2} \epsilon^{\lambda,\mu\nu *} \sim A_0 \left(\frac{\kappa}{2}\right) \frac{r_\mu r_\nu}{r \cdot k} \epsilon^{\lambda,\mu\nu *}(k)$$

Medium effects: Propagator gets finite width

$$A = A_0 \left(\frac{\kappa}{2}\right) \left[\frac{q_\mu q_\nu}{q \cdot k + \Sigma_q^2} + \frac{r_\mu r_\nu}{r \cdot k + \Sigma_r^2} + \frac{p_\mu p_\nu}{p \cdot k + \Sigma_p^2}\right] \epsilon^{\lambda,\mu\nu *}(k)$$

$|\mathcal{M}|^2 \sim \frac{1}{k^2}$

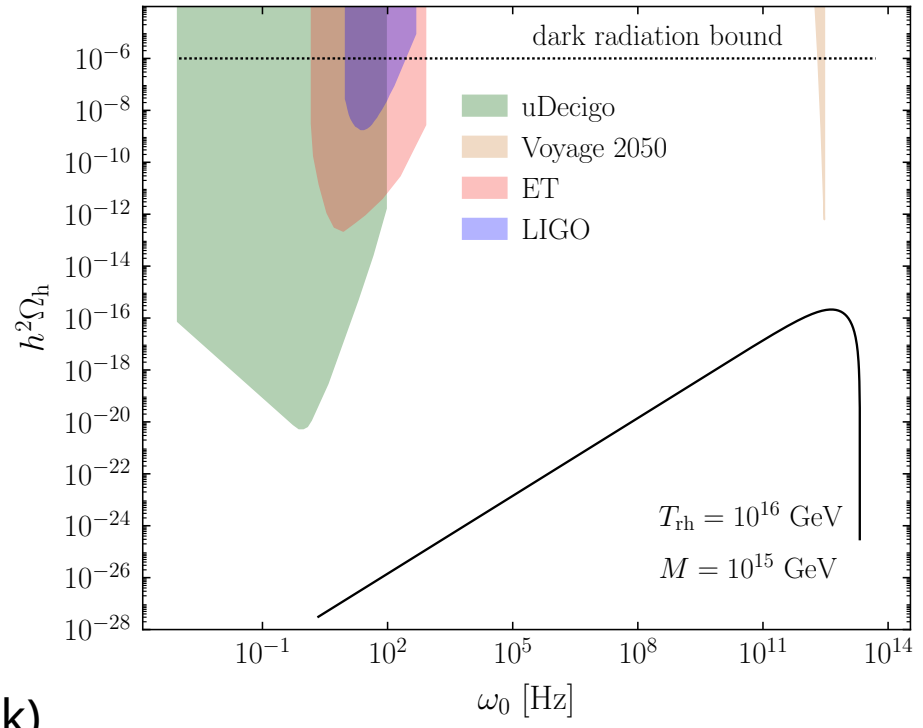
$\Omega_h \sim k$

(small k)

$|\mathcal{M}|^2 \sim \text{const}$

$\Omega_h \sim k^3$

(very small k)



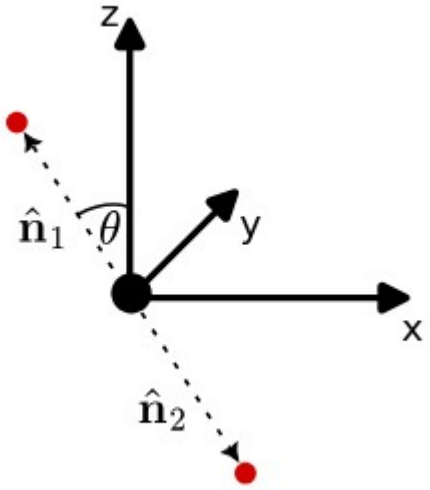
[Murayama, Noether, **JSE**, 25']

Physical interpretation: Interaction of final states with medium

Classical model to understand physical effect of finite width

$$T^{\mu\nu}(x,t) = \frac{p^\mu p^\nu}{p^0}\delta(x)\,\Theta(-t)\,e^{t/\tau} + \sum_a \frac{p_a^\mu p_a^\nu}{p_a^0}\,\delta(x-\hat{n}_a t)\,\Theta(t) e^{-t/t_\star}$$

$$t_\star = \frac{1}{\omega_\star} = \frac{1}{\sigma n}$$



$$T^{\mu\nu}(k,\omega) = \frac{p^\mu p^\nu}{p^0}\,\frac{1}{i\omega + 1/\tau} - \sum_a \frac{p_a^\mu p_a^\nu}{p_a^0}\,\frac{1}{i\omega - ik\cdot\hat{n}_a - \omega_\star}$$

$$\frac{dE}{d\omega} = \frac{G}{2\pi^2}\int_S d\hat{\Omega}\;\omega^2 T_{ij}^{\mathrm{TT}}(k=\omega\hat{x},\omega)T^{\mathrm{TT}\;ij*}(k=\omega\hat{x},\omega),$$

$$\frac{dE}{d\omega} = \left(\frac{M^2}{m_{\mathrm{pl}}^2}\right) f(\omega,\omega_\star), \qquad f(\omega,\omega_\star) = \begin{cases} \frac{1}{\pi} & \omega \gg \omega_\star \\ \frac{8}{15\pi}(\omega/\omega_\star)^2 & \omega \ll \omega_\star \end{cases}$$

Same result with modified soft graviton theorem

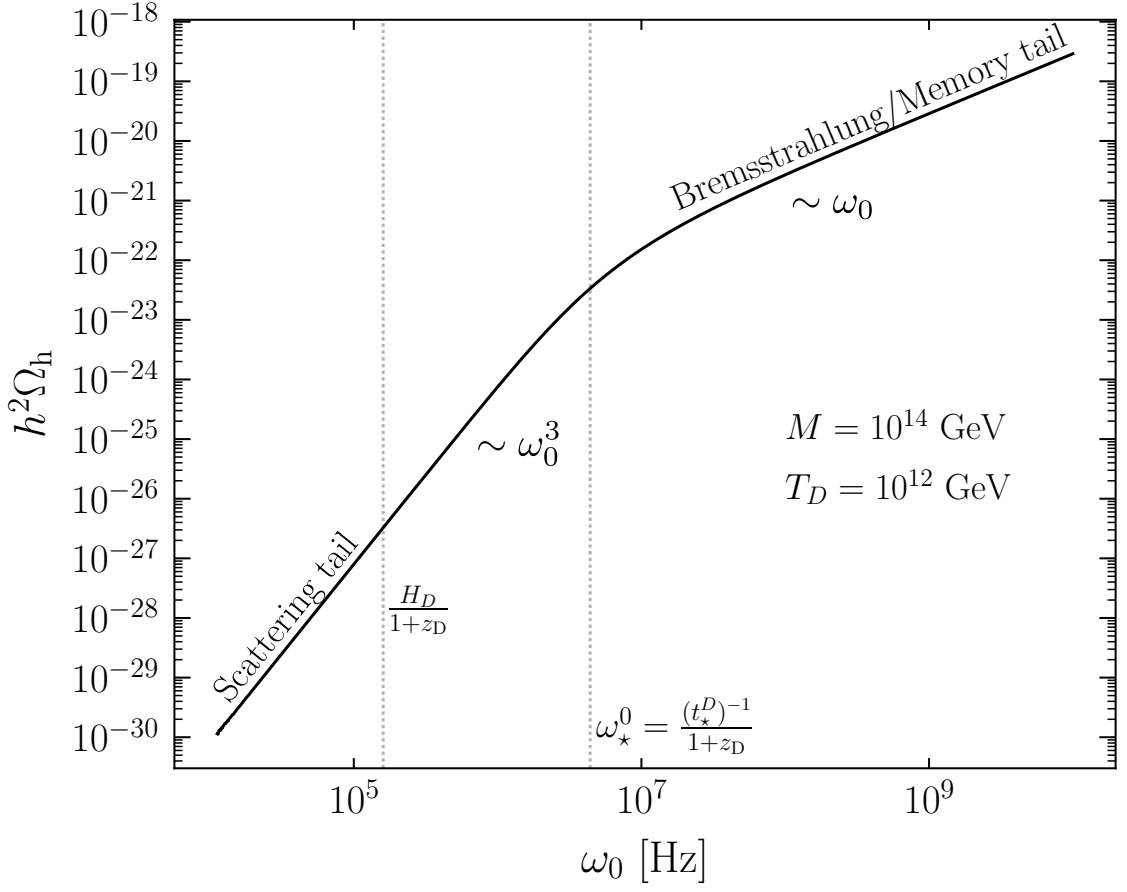
$$\frac{dE}{d\omega} \equiv \omega \frac{d}{d\omega}\,\frac{\Gamma}{\Gamma_0},$$

$$|\Sigma|^2 = \frac{3}{2}M^2\omega_\star^2$$

Cosmological stochastic energy spectrum

$$h^2\Omega_{\rm h} = \frac{1}{\rho_c^0}\int dz\ \frac{\omega}{(1+z)}\frac{dE}{d\omega}\frac{dN}{dV\,dz},$$

$$\frac{dE}{d\omega} = \left(\frac{M^2}{m_{\rm pl}^2}\right)\begin{cases}\frac{1}{\pi} & \omega \gg \omega_\star \\ \frac{8}{15\pi}(\omega/\omega_\star)^2 & \omega \ll \omega_\star\end{cases}$$

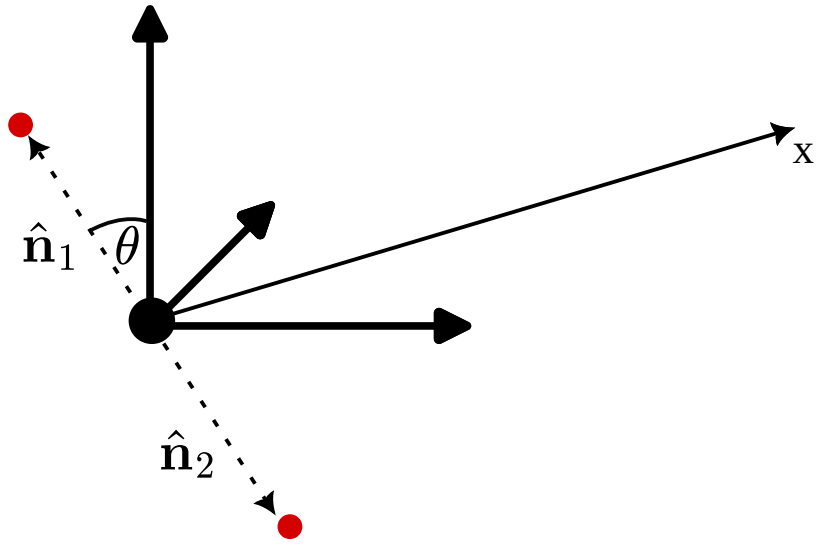


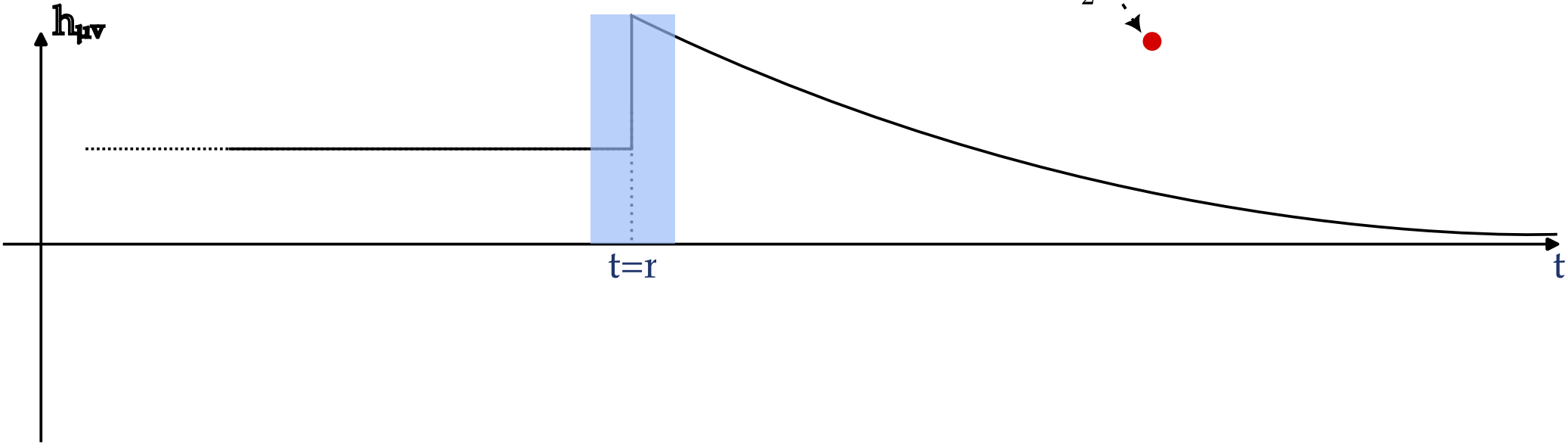
Q1: Is the low frequency tail a robust prediction?
Maybe better target than high-frequency peak

A: Low frequency tail is suppressed. Seems not detectable with laser interferometers

Gravitational Memory

$$\bar{h}_{\mu\nu}(x,t) = \frac{4G}{r}\left[\frac{p_\mu p_\nu}{p^0}\,\Theta(r-t) + \sum_{a=1,2}\frac{1}{t/r - \hat{n}_a\cdot\hat{x}}\,\frac{p_{a\mu}p_{a\nu}}{p_a^0}\Theta(t-r)\right]$$





Gravitational Memory

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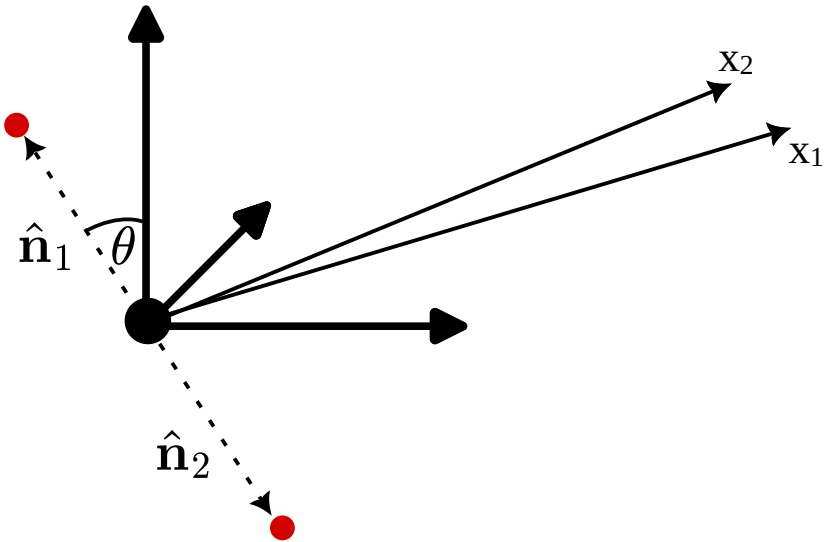
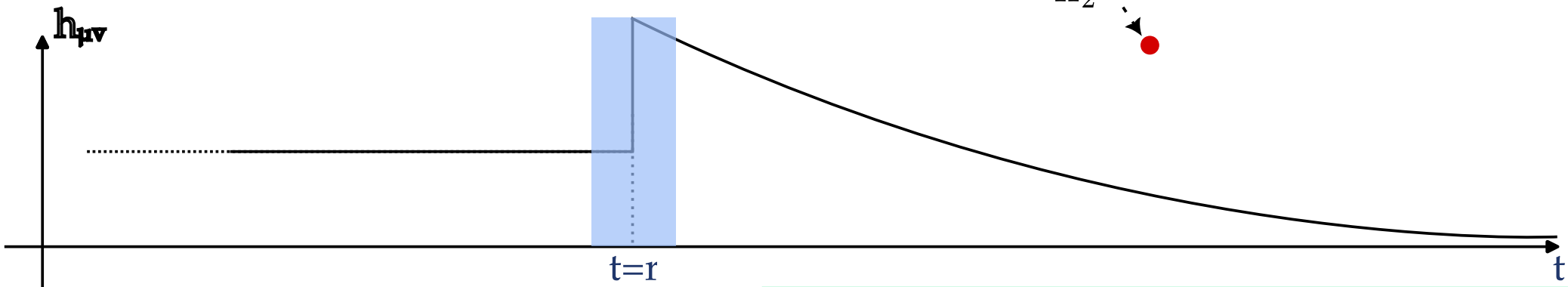




$$\Delta s \equiv s(t>r) - s(t<r) = -\frac{1}{2}\frac{1}{|\Delta x|}\Delta h_{ij}^{\mathrm{TT}}\Delta x^i \Delta x^j$$

$$\Delta h_{ij}^{TT} \sim \left[\sum_a \frac{p_{ai}p_{aj}}{p_a\cdot\tilde{k}} - \frac{p_i p_j}{p\cdot\tilde{k}}\right]^{\mathrm{TT}}$$

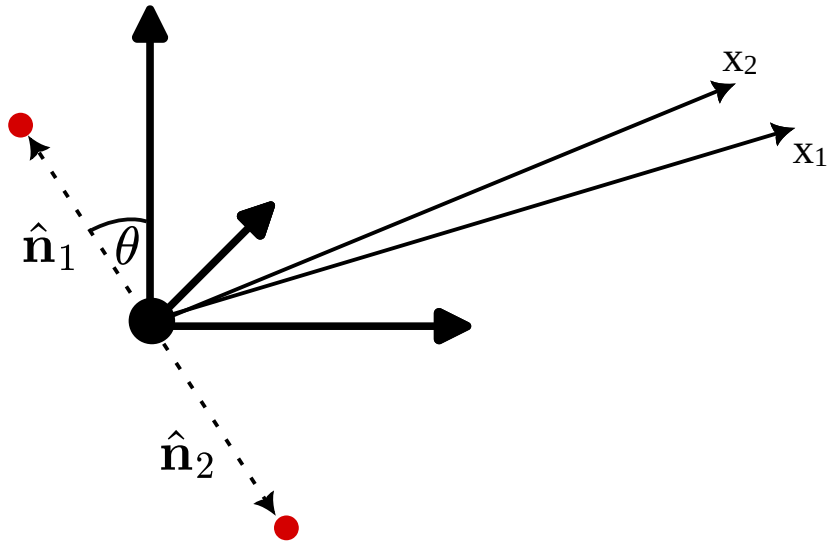
$\Delta x = x_2 - x_1$ take limit to null infinity

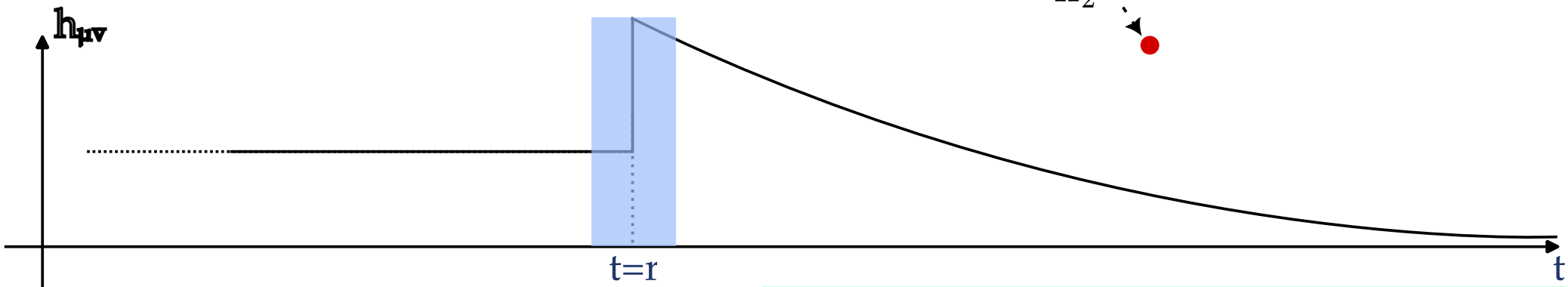
$$\tilde{k}^\mu = 1/\sqrt{32\pi G}\,(1,\hat{x})$$

[Zel'dovich, Polnarev 69; Braginsky, Grishchuk 85; Braginsky, Thorne 87]

Gravitational Memory

$$\bar{h}_{\mu\nu}(x,t) = \frac{4G}{r}\left[\frac{p_\mu p_\nu}{p^0}\,\Theta(r-t) + \sum_{a=1,2}\frac{1}{t/r - \hat{n}_a\cdot\hat{x}}\,\frac{p_{a\mu}p_{a\nu}}{p_a^0}\Theta(t-r)\right]$$





$$\Delta s \equiv s(t>r) - s(t<r) = -\frac{1}{2}\frac{1}{|\Delta x|}\Delta h_{ij}^{\mathrm{TT}}\Delta x^i \Delta x^j$$

$$\Delta h_{ij}^{TT} \sim \left[\sum_a \frac{p_{ai}p_{aj}}{p_a\cdot\tilde{k}}\,\mathrm{Exp}\left[-\frac{u_+}{t_\star(1-\hat{x}\cdot\hat{n}_a)}\right] - \frac{p_i p_j}{p\cdot\tilde{k}}\,\mathrm{Exp}\left[-\frac{u_-}{\tau}\right]\right]^{\mathrm{TT}}$$

$\Delta x = x_2 - x_1$ take limit to null infinity

$\tilde{k}^\mu = 1/\sqrt{32\pi G}\,(1,\hat{x})$ $\quad u_+ = t - r \geq 0, \quad u_- = r - t \geq 0$

[Zel'dovich, Polnarev 69; Braginsky, Grishchuk 85; Braginsky, Thorne 87]

Gravitational Memory

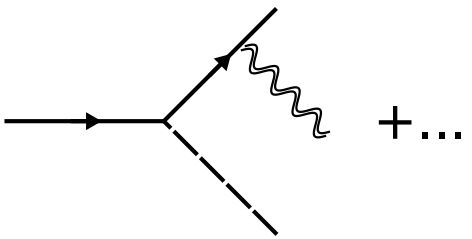
Linear memory

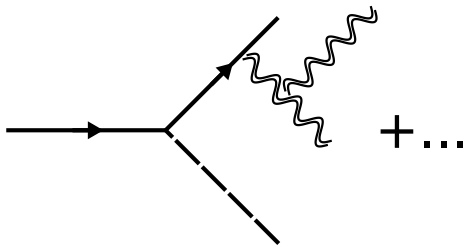
Non-linear memory

Boltzmann equations

$$(\partial_t + 4H(t))\rho_{\rm h}(t) = \int d\Omega\, E_h |\mathcal{M}|^2$$

$$h^2\Omega_{\rm h} = \frac{1}{\rho_c^0}\frac{d\rho_{\rm h}^0}{d\ln\omega_0}$$

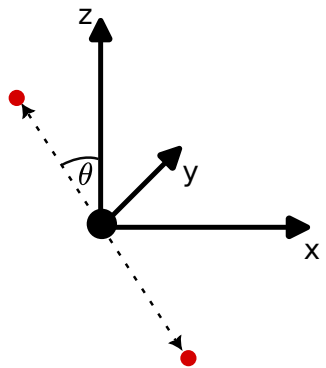


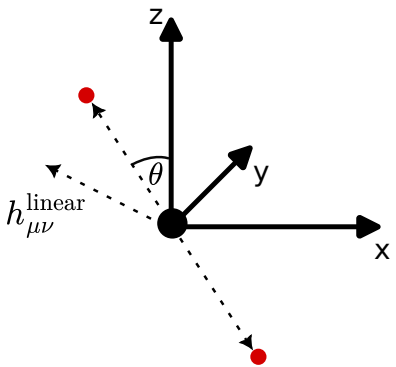


Equations of motions

$$T_{\mu\nu} \Rightarrow h_{\mu\nu},\ \frac{dE}{d\omega}$$

$$h^2\Omega_{\rm h} = \frac{1}{\rho_c^0}\int dz\,\frac{\omega}{1+z}\,\frac{dE}{d\omega}\,\frac{dN}{dVdz}$$



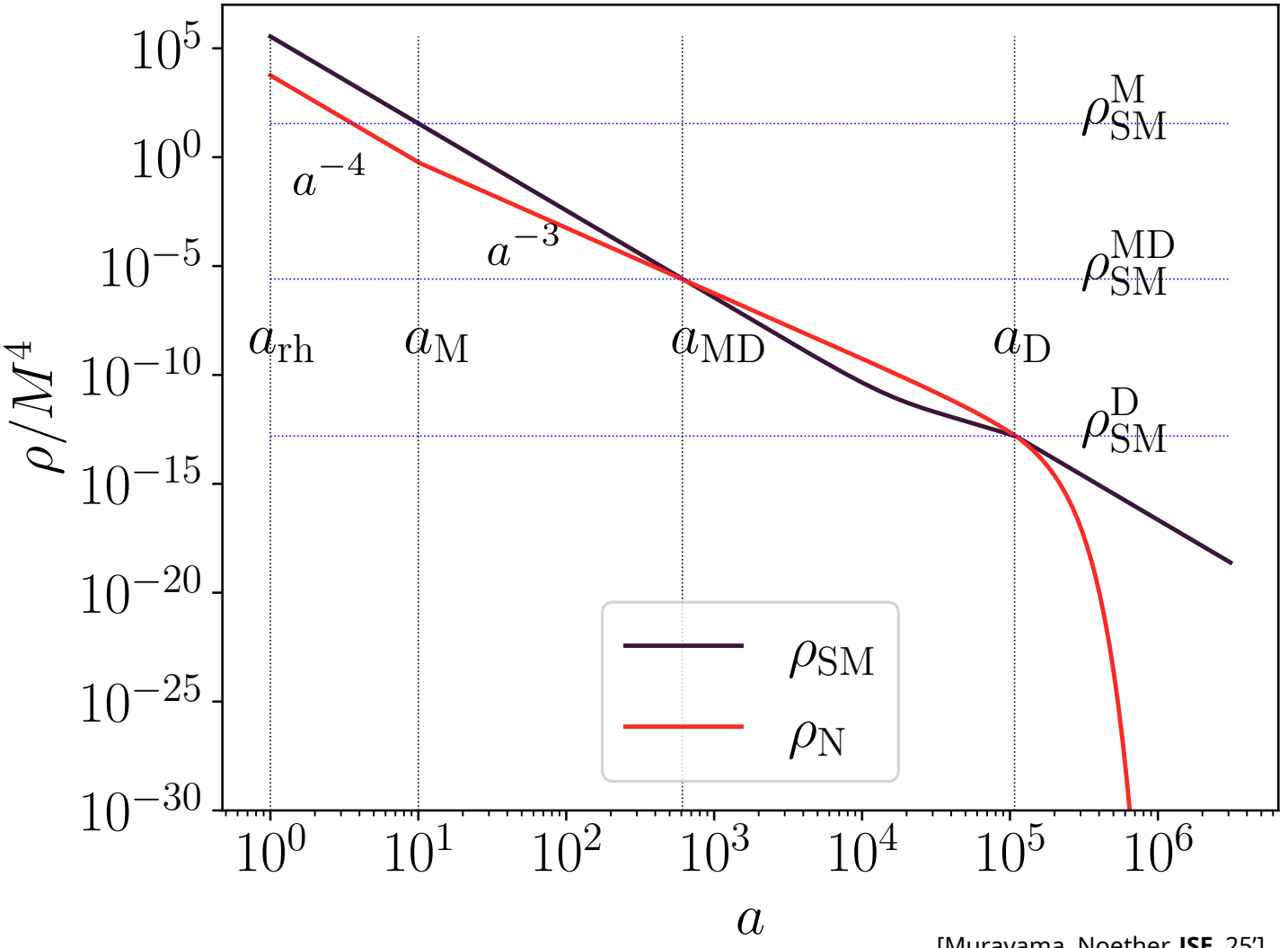


Q2: Can gravitational memory enhance the low frequency signal?

A: No. Bremsstrahlung = Linear memory. Maybe non-linear memory

Conclusions

- GW spectrum from decaying particles can be suppressed due to the interaction with the surrounding medium
- Calculating GW spectrum with classical methods (GW memory) and Weinberg's soft graviton theorem gives same answer.
- Bremsstrahlung = Memory
- Memory formula is modified if final states are not asymptotic.



[Murayama, Noether, **JSE**, 25']

Early matter domination

$$t_{\rm MD} < \Gamma^{-1}$$

$$\Rightarrow Y^2 < 0.23\,\frac{M}{m_{\rm Pl}}$$

$$T_{\rm rh} = 10^{16}{\rm GeV}$$

$$M = 10^{15}\,{\rm GeV}$$

$$Y = 10^{-4}$$