Dynamical tides in binary neutron stars:
effects of spin, tidal spin, and nonlinear hydrodynamics

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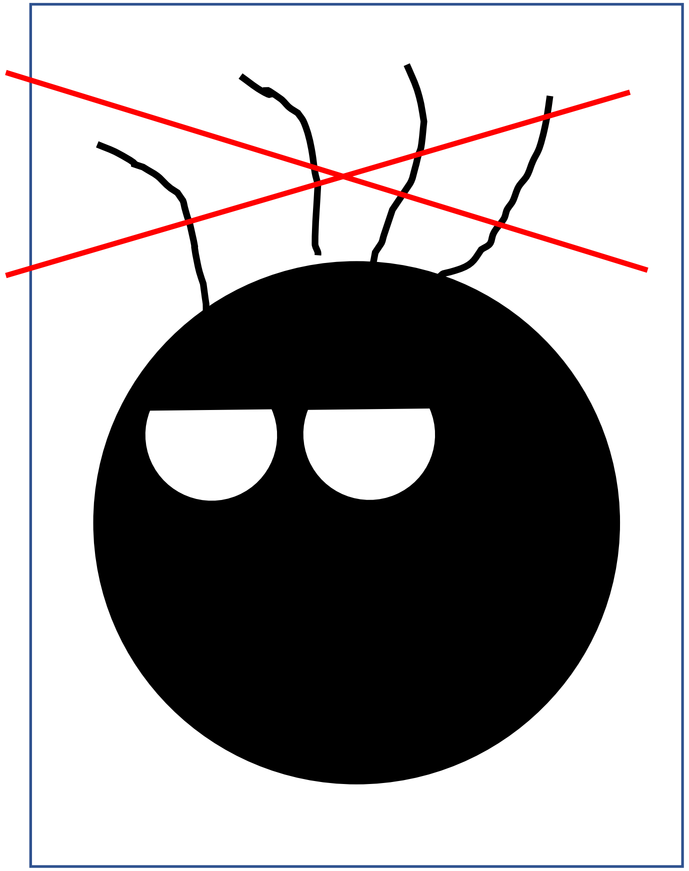
N3AS Seminar
10/07/25

MONTANA STATE UNIVERSITY

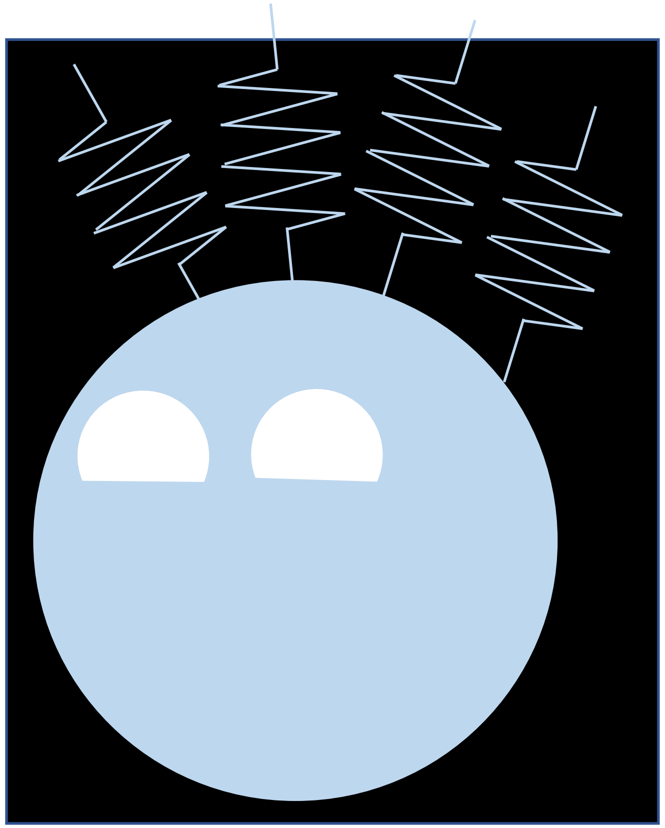
extreme Gravity Institute

Black holes are boring (not really)
No hair; only mass and spin



Artist's impression of a black hole (BH)

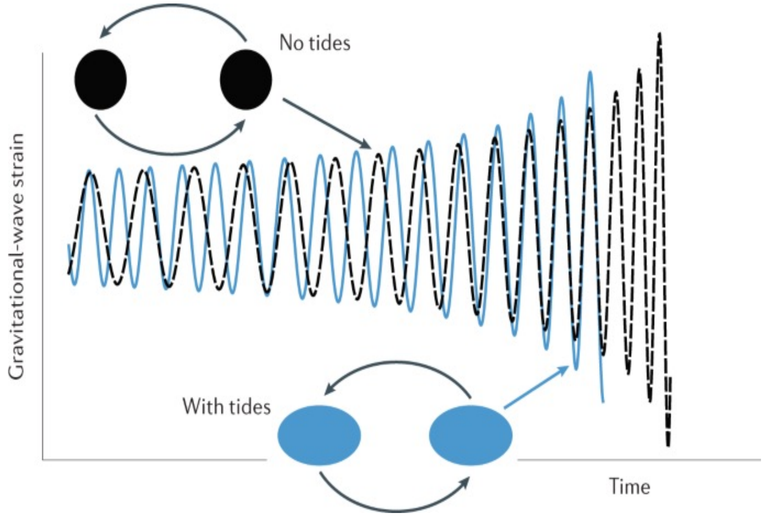
Neutron stars are exciting (really)!
Mass, spin, matter distributions!
Probes neutron star equation of state



And a neutron star (NS)

Tides in BNSs

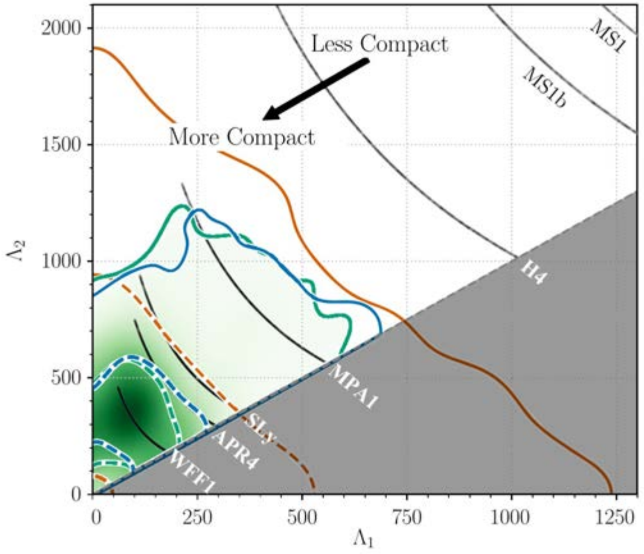
- Extra sink of energy
- Accelerating orbital decay



Yunes+ (2022)

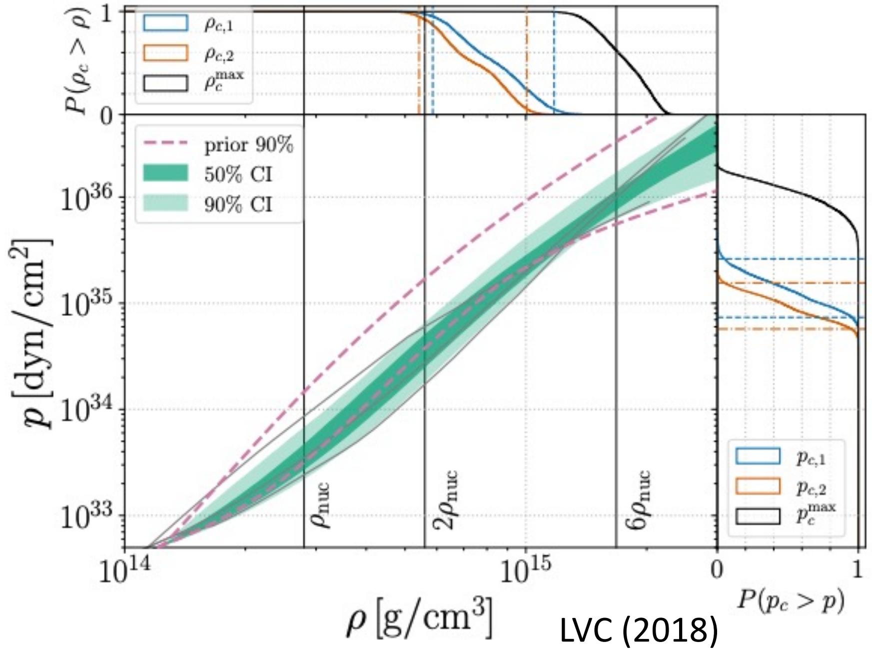
Need accurate templates!

This talk



LVC (2018)

Equation of state from tidal deformability

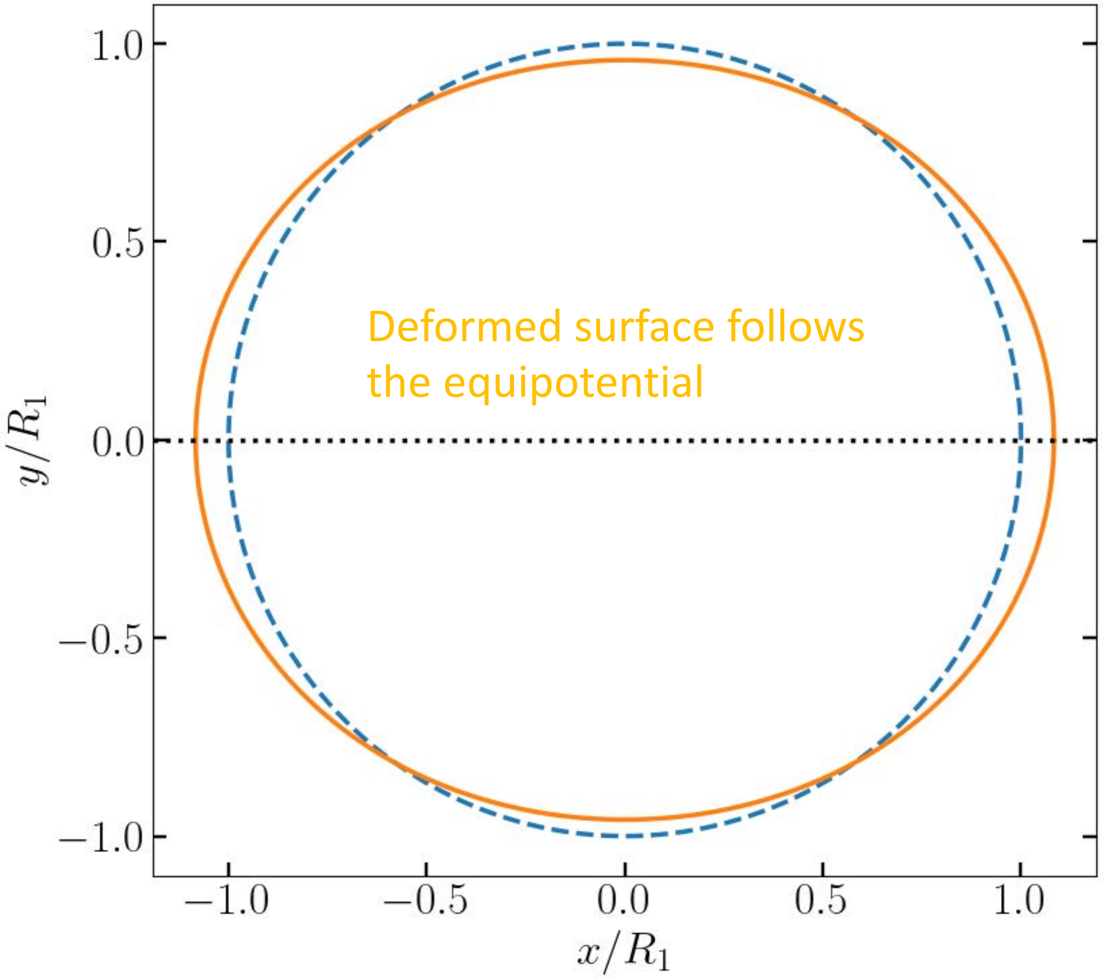


LVC (2018)

Part I: a new effective-one-body model including tidal spin

(arXiv:2501.13064,
HY & S. Lau, 2025)

Love number (tidal deformability in the adiabatic limit)



NS surface in the equatorial plane in a frame co-rotating w/ the orbit.
The companion is always on the +x axis.

Tidal deformability

$$Q^{ij} = -\lambda \mathcal{E}^{ij}.$$

$$k_2 = \frac{3}{2}\frac{\lambda}{R^5}.$$

$$g_r^{(\text{tide})} = -9\frac{M_2}{M_1}\left(\frac{M}{r^2}\right)\left(\frac{\lambda}{r^5}\right).$$

Tidal back-reaction in the radial direction

(Lai+93, 94a, 94b;
Flanagan & Hinderer 08; Hinderer+10; …)

Tide as forced springs

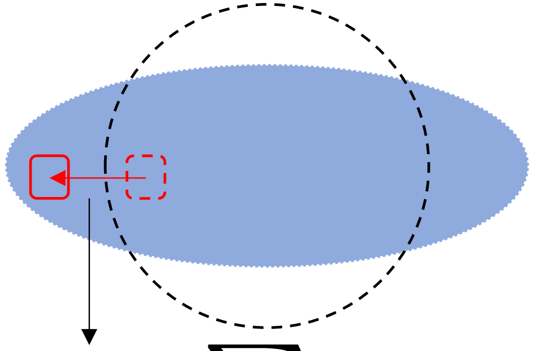
$$\ddot{q}_a + \omega_a^2 q_a = \omega_a^2 U_a$$

Natural/eigen frequency

$$\omega_a \simeq \sqrt{M_1/R_1^3} \simeq 2\pi \times 2000\,\mathrm{Hz}$$

Tidal forcing

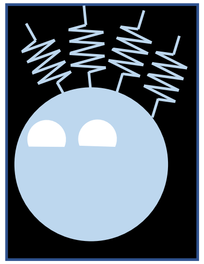
$$U_a \sim e^{-i2\omega t}$$



$$\boldsymbol{\xi} = \sum_a q_a \boldsymbol{\xi}_a$$

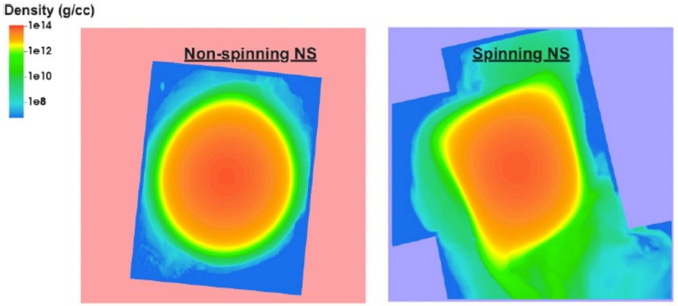
Mode amplitude

Spring-like hair

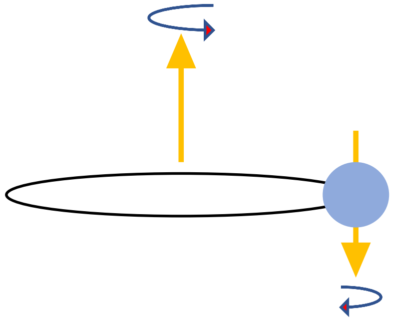


- Equilibrium solution: $q_a \simeq \dfrac{\omega_a^2}{\omega_a^2 - 4\omega^2} U_a$
- Early inspiral: adiabatic tide
- Near merger: $2\omega \simeq 2\sqrt{\dfrac{(M_1+M_1)}{(2R_1)^3}} \simeq \omega_a!$
- Including spin: Doppler (↓↓): $\omega_a \rightarrow \omega_a + m_a\Omega$

  & Coriolis (↑)

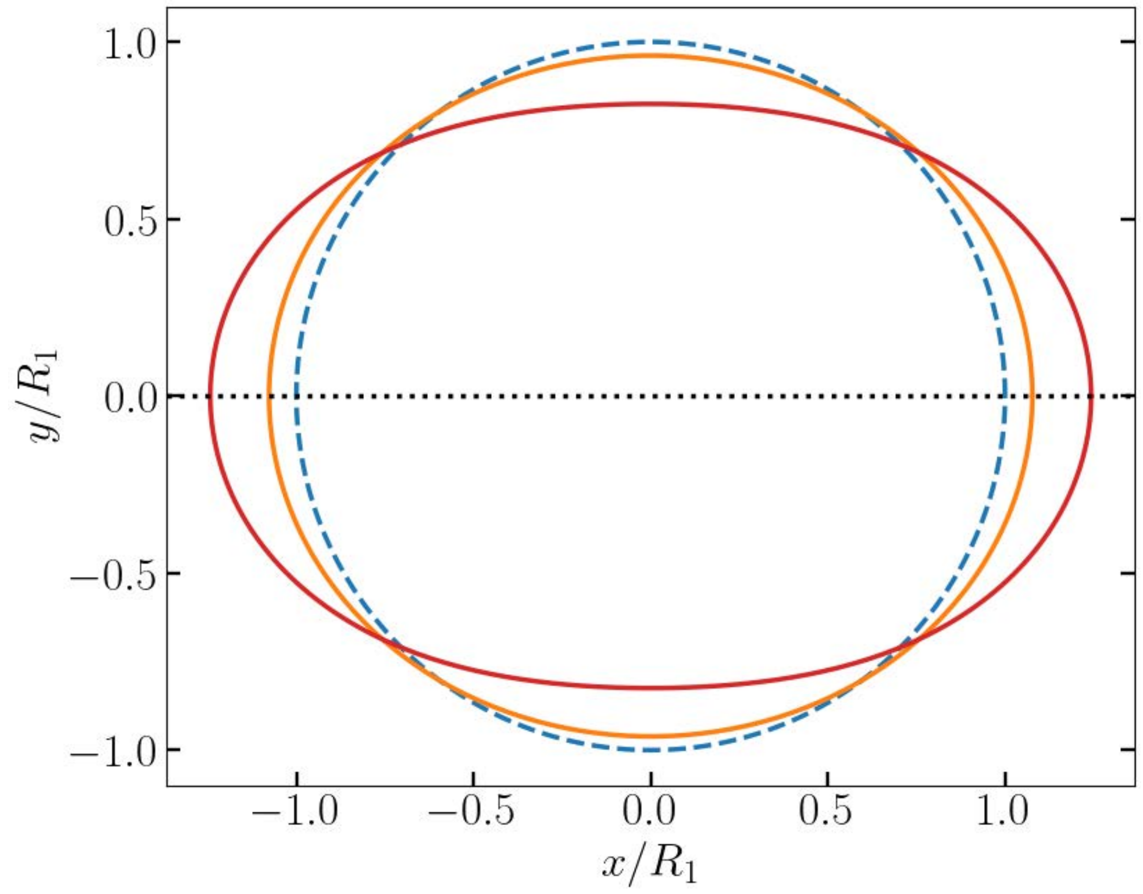


Steinhoff+ (2021)



Effective Love number

Amplified radial interaction by finite-frequency corrections

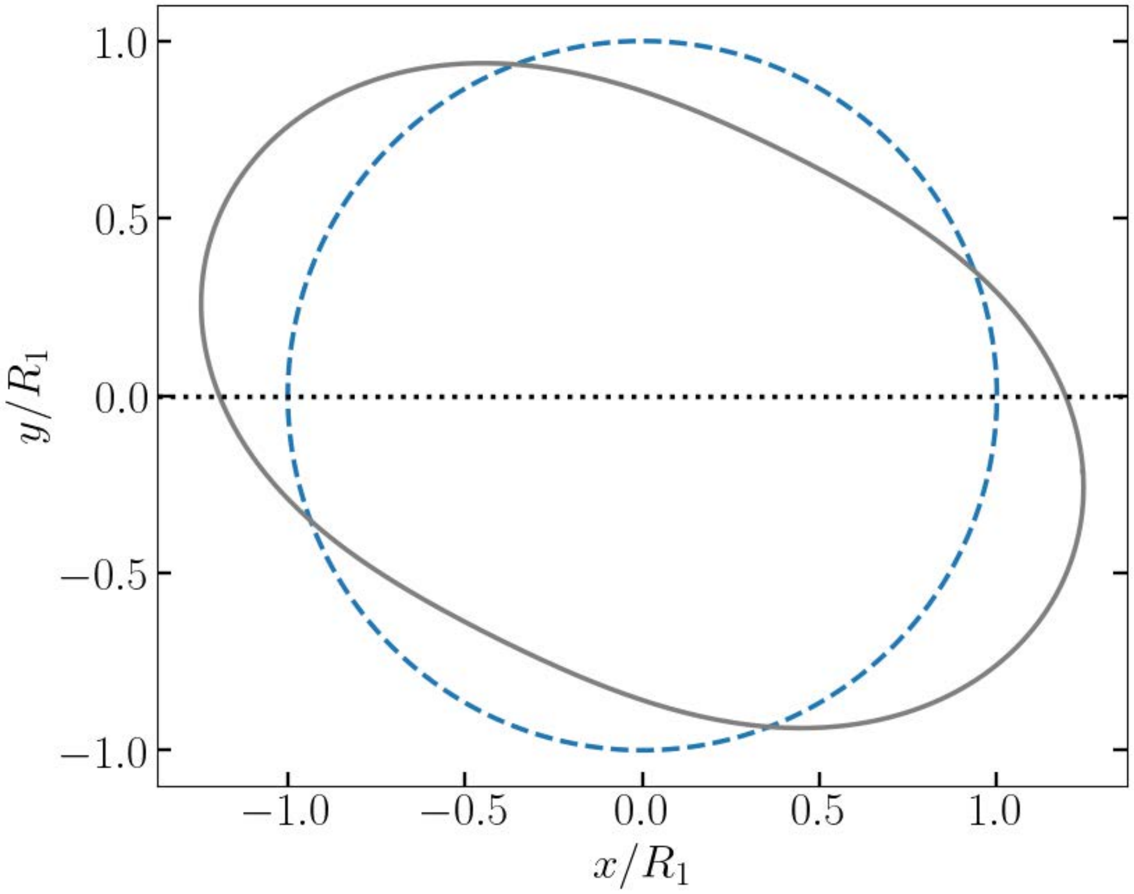


$$\lambda \to \hat{\kappa}^{\rm eff}\lambda$$ Steinhoff+ (2021)

$$\hat{k}_\ell^{\rm eff} = a_\ell + b_\ell\left(1 - \frac{(\Delta\omega_{0\ell})^2}{\omega_{0\ell}^2}\right)\left\{\frac{\omega_{0\ell}^2}{\omega_{0\ell}^2 - (\ell\omega_{\rm orb} - \Delta\omega_{0\ell})^2}\right.$$

+ (other terms)}

The actual bulge

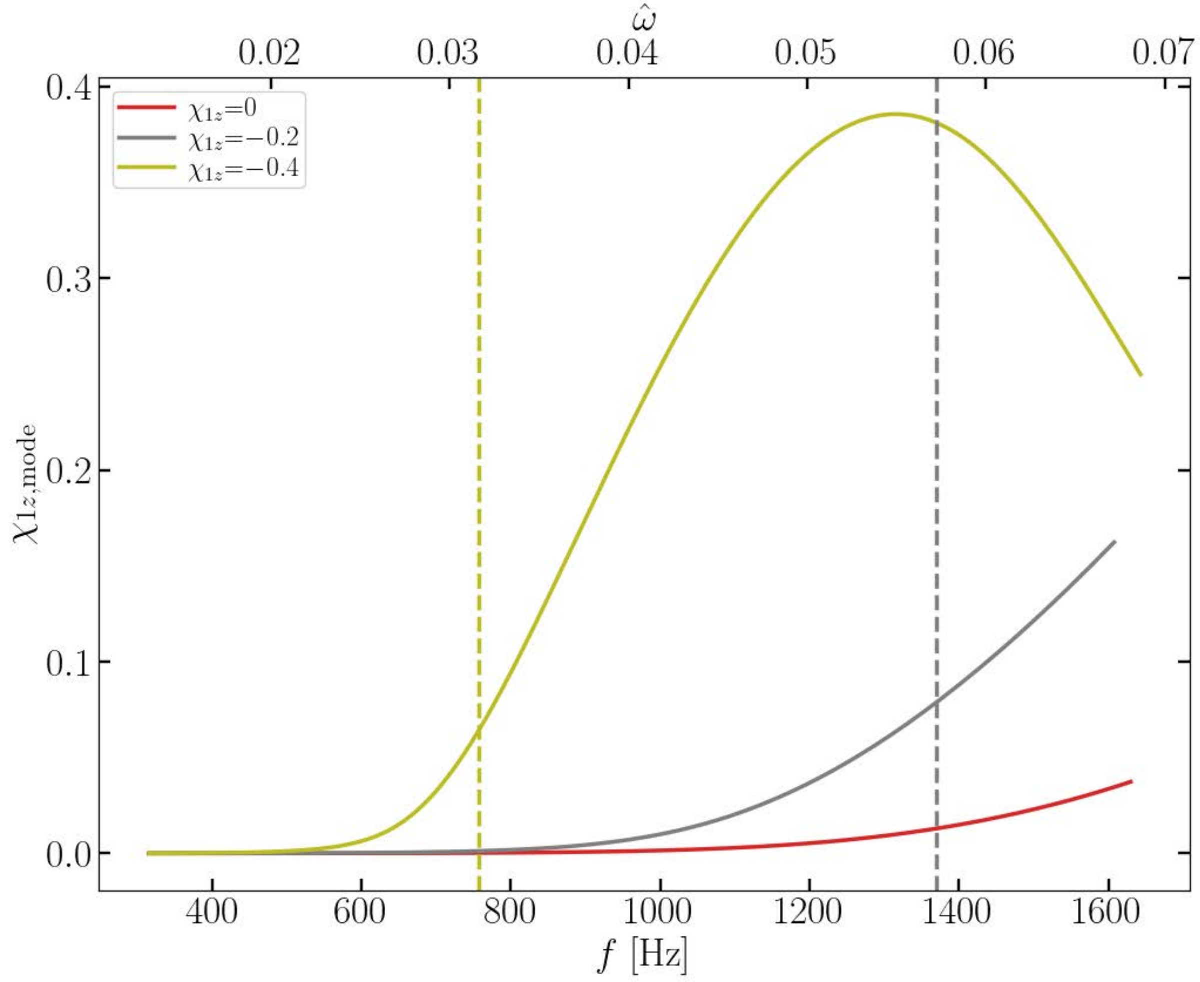


GW-induced orbital decay creates a lag in the tidal bulge!

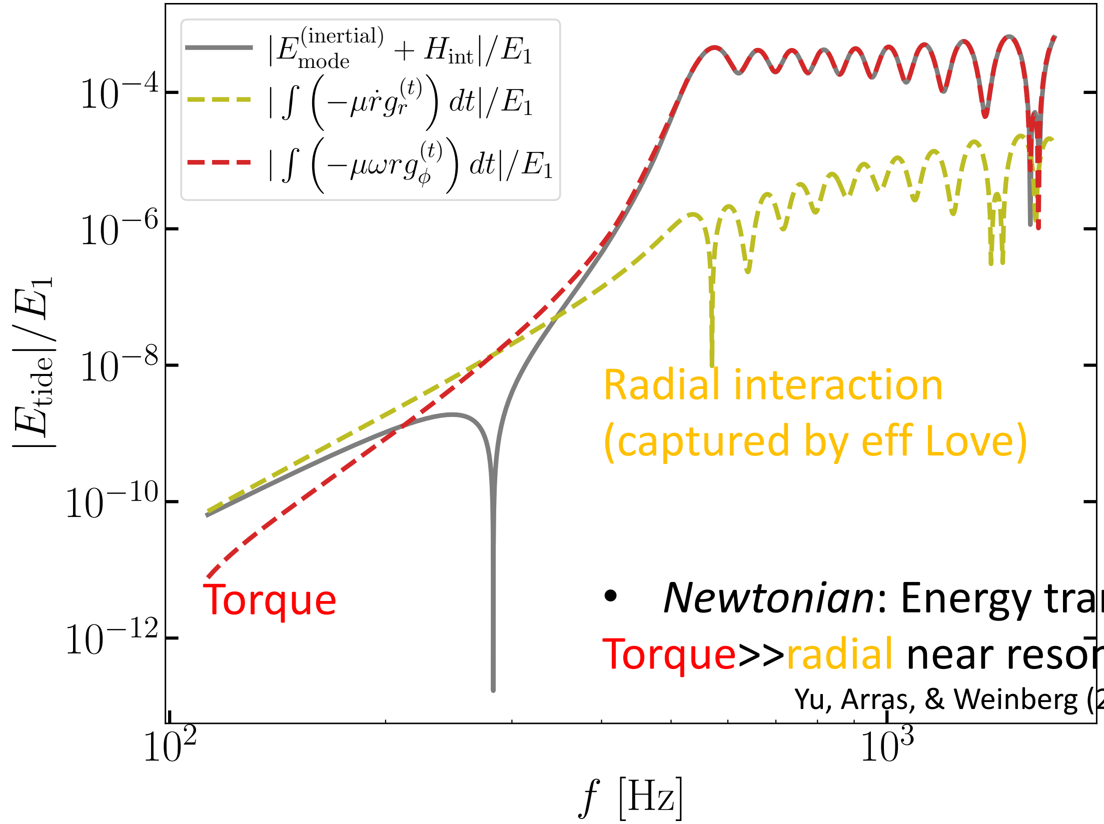
"Effective" damping. (Lai 94; S. Ma, HY, & Y. Chen 2020, HY, Arras, & Weinberg 24)

Both radial (eff. Love) and tangential (torque) components!

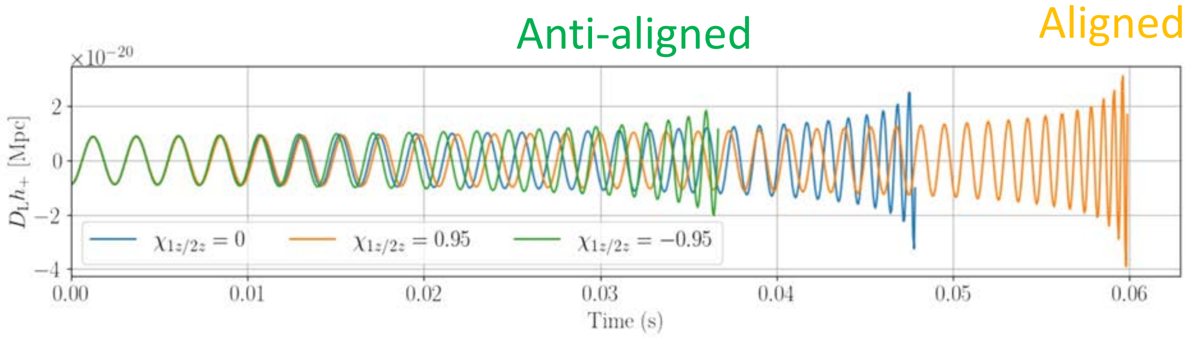
Tidal spin



- Tidal spin can reach significant sizes (~0.4) near resonance (vertical lines)
- Total NS spin (background + tidal) not constant!
- Ignored by all LVK models



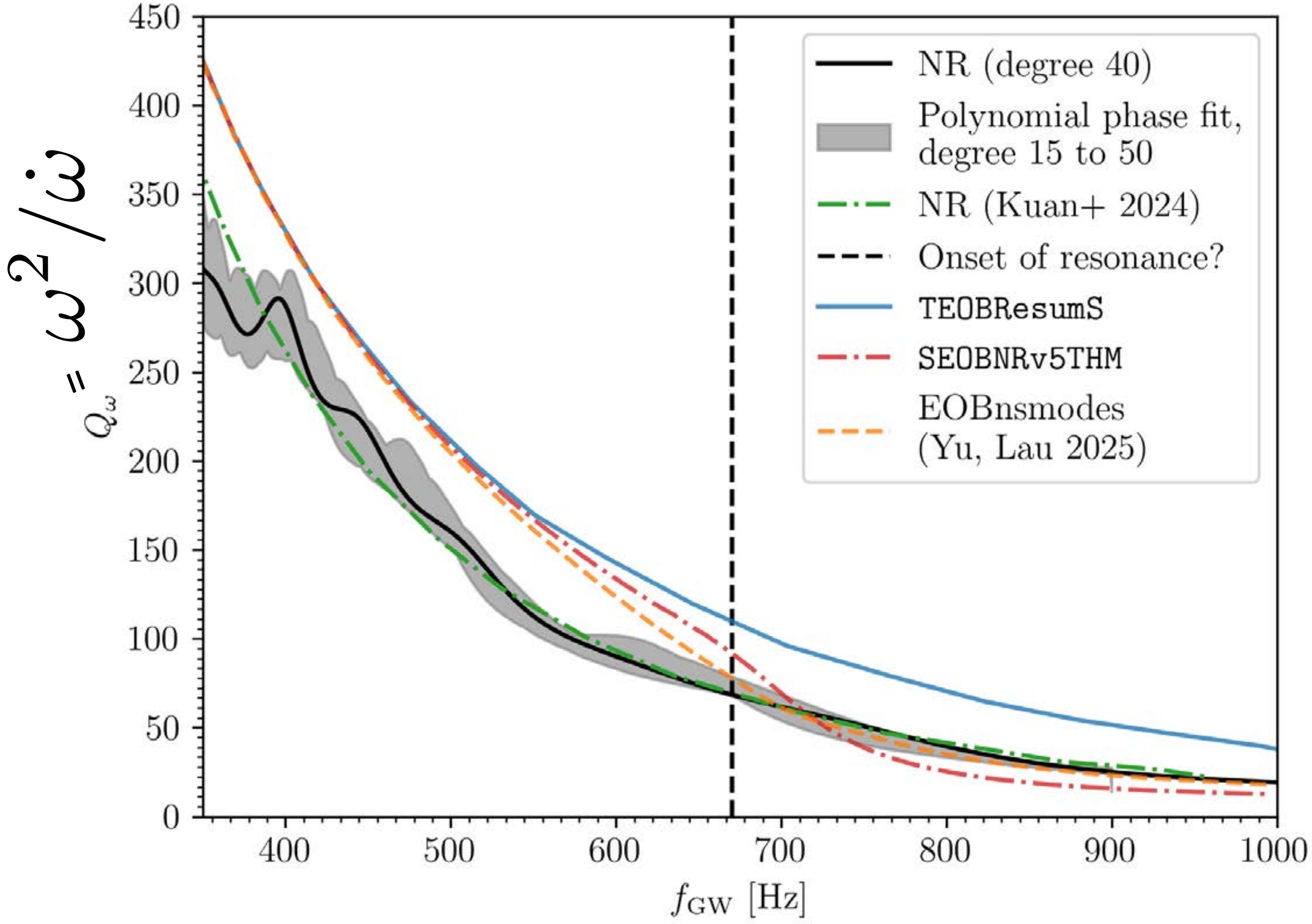
- *Newtonian*: Energy transfer Torque>>radial near resonance



- *Post-Newtonian*: orbital hang-up

Wang & Nitz (2023)

Theoretical models vs numerical relativity



NR simulations by Kuan et al. (2024) on rapidly spinning BNSs $\chi_z = -0.48$

Adiabatic Love Number

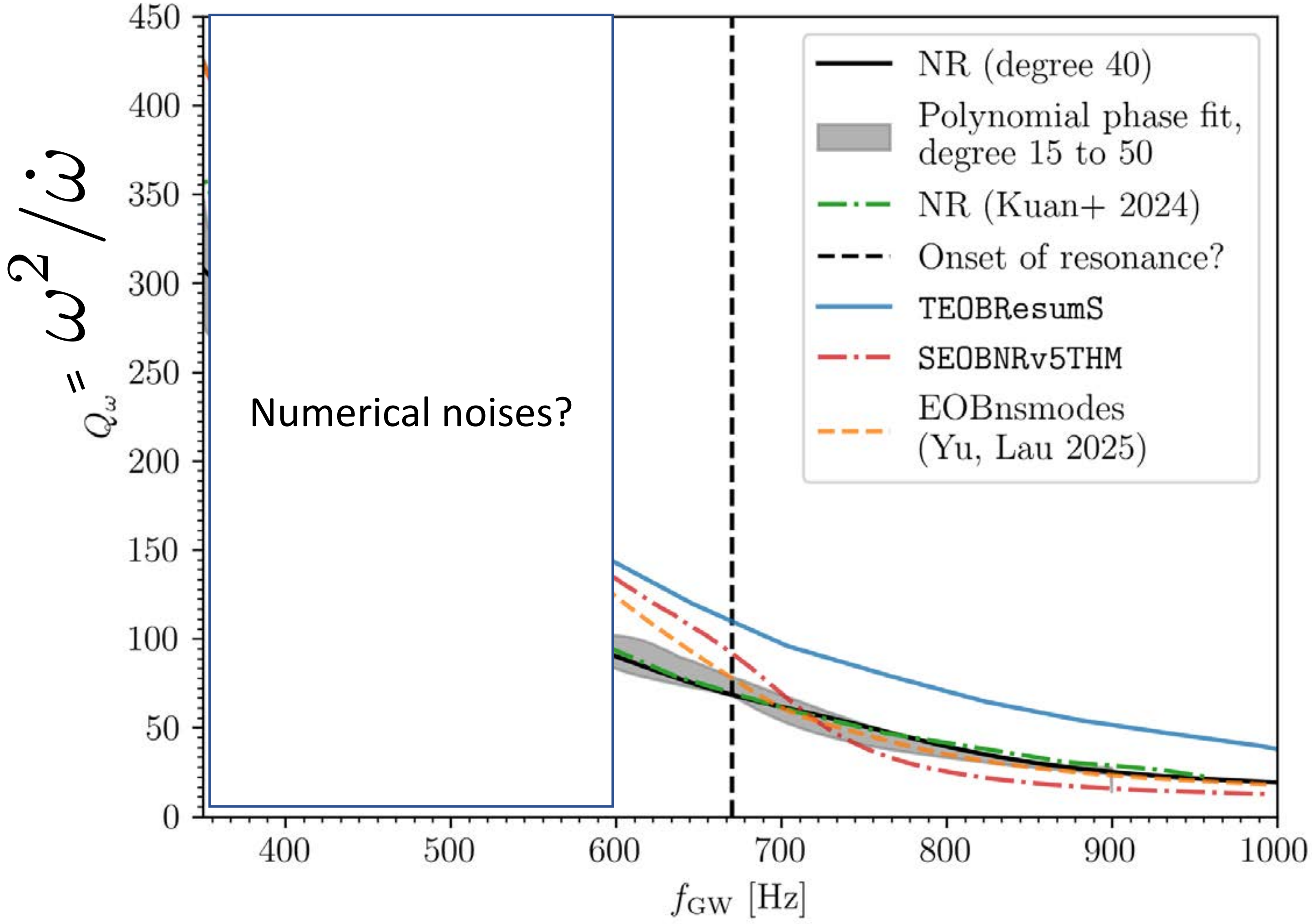
Effective Love Number

New EOB (this work)

The new EOB shows the best match to NR! (without any calibrations)

Figure Credit: Marcus Haberland (AEI)

Theoretical models vs numerical relativity



NR simulations by Kuan et al. (2024) on rapidly spinning BNSs $\chi_z = -0.48$

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Figure Credit: Marcus Haberland (AEI)

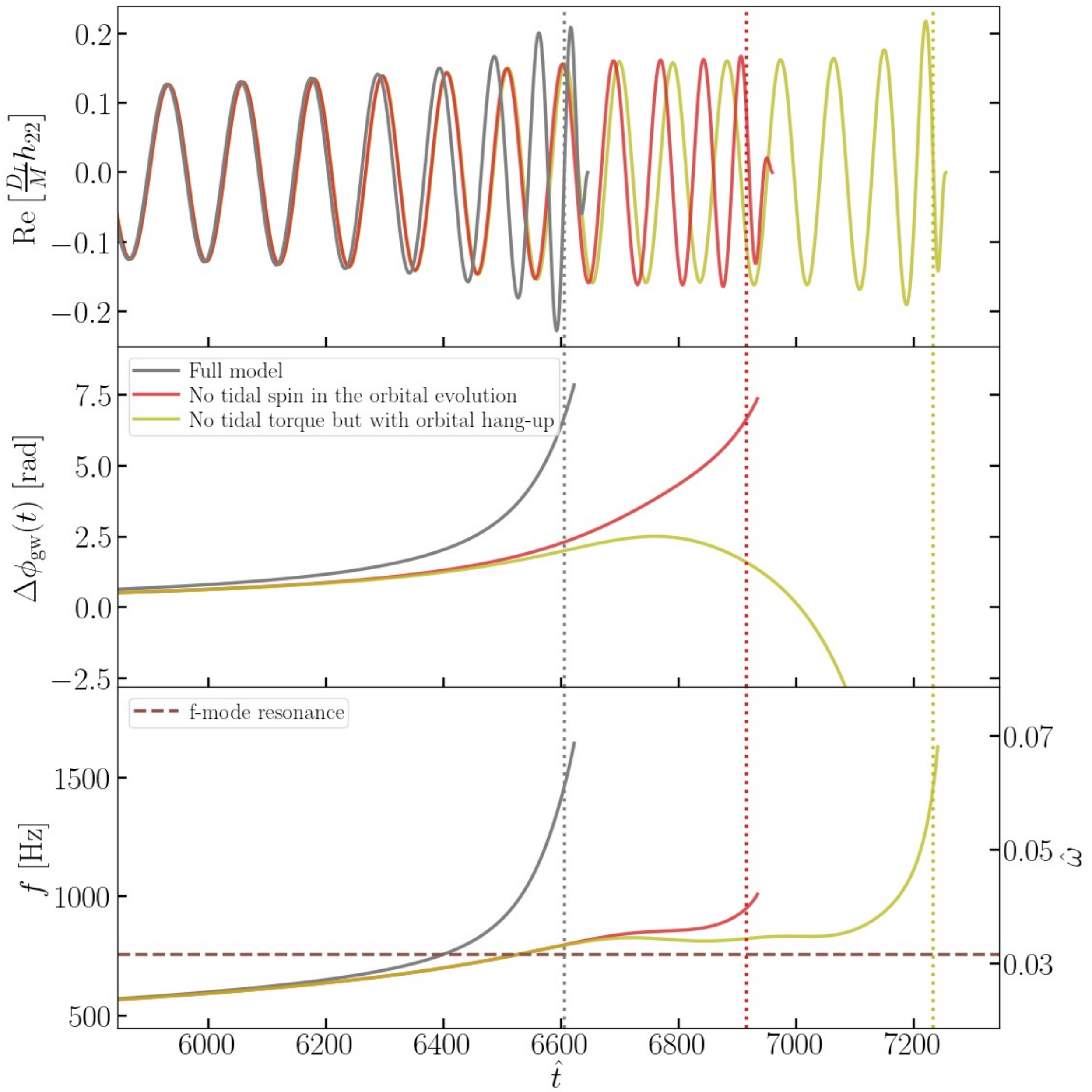
A closer look

Background spin: $\chi_{1z}{=}{-}0.4 \quad \left(\frac{\Omega_{\rm spin}}{2\pi}{=}{-}780\,{\rm Hz}\right)$

Black: Full EOB
Red: No tidal spin (effective Love number)
Yellow: no Newtonian torque but with PN hang-up.

Black vs Yellow:
Newtonian torque

Red vs Yellow:
PN hang-up



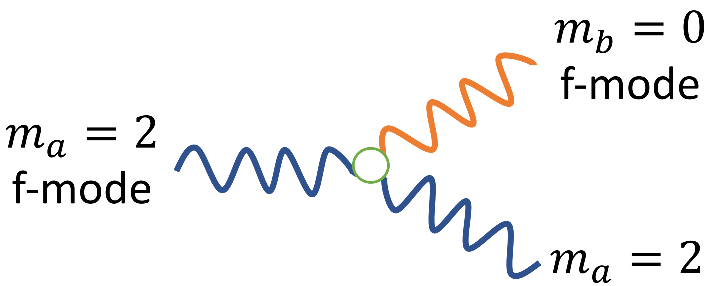
(Vertical lines: Peaks of the 22 GW mode)

Part II: impact of nonlinear hydrodynamics

Beyond the linear tide

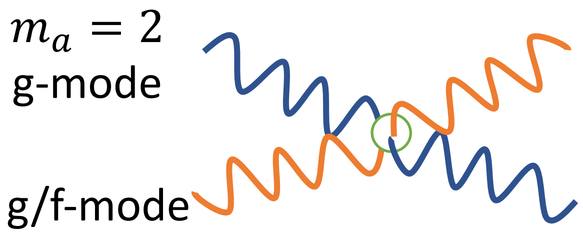
(arXiv:2211.07002,
HY, N.N. Weinberg, P. Arras, et al., 2023)

- Artificial calibrations in S/TEOB/NRtidal models for non-spinning BNSs
- Solution: nonlinear hydrodynamics!



$$\ddot{q}_a + \omega_a^2 q_a = \omega_a^2 \kappa q_a q_b,$$
$$\ddot{q}_a + \underbrace{\omega_a^2 \left[1 - \kappa q_b\right]}_{\omega_{a,\mathrm{eff}}^2} q_a = 0.$$

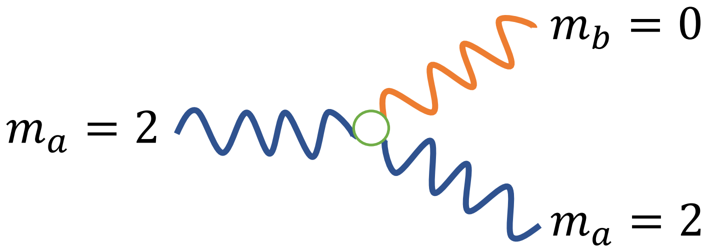
- Frequency shift produced by perturbed eq.
- $\kappa q_b > 0,\ \omega_{a,\mathrm{eff}}^2 \downarrow$
- Enhanced f-mode resonance!



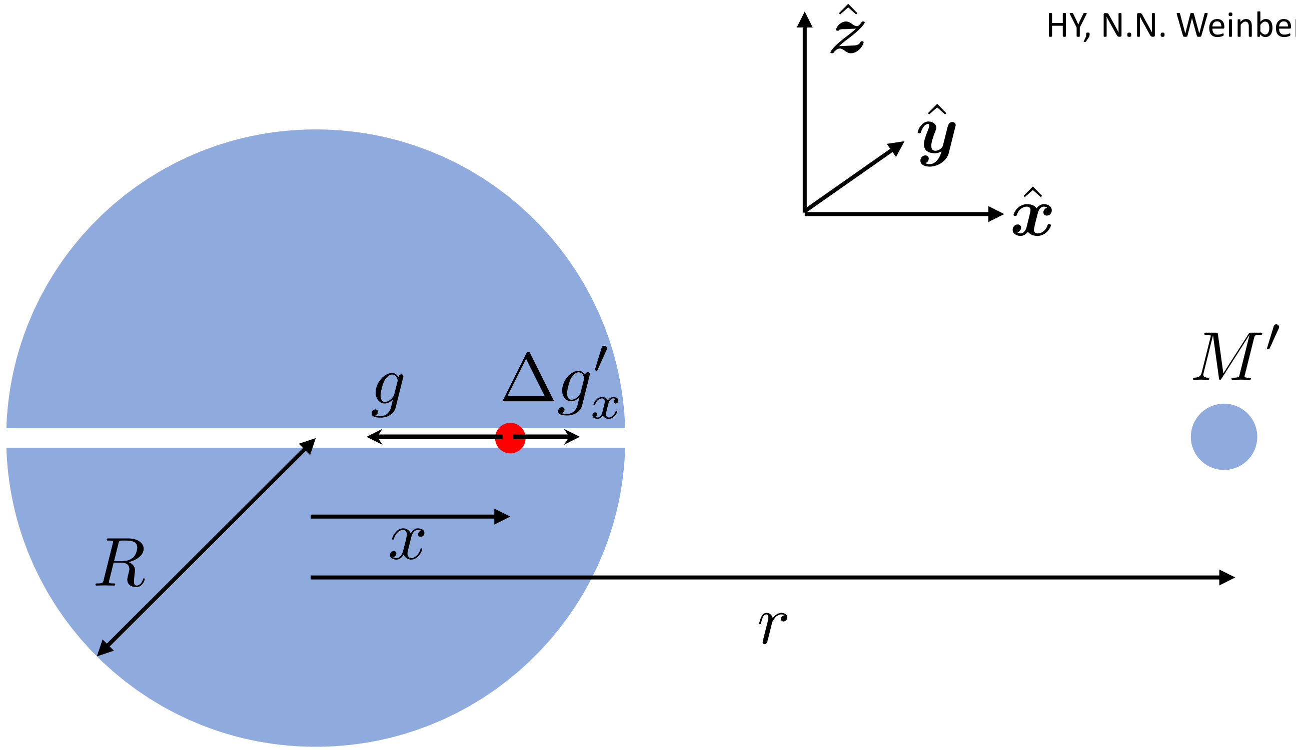
$$\ddot{q}_a + \omega_a^2 q_a = \omega_a^2 \zeta(q_b q_b^*) q_a,$$
$$\ddot{q}_a + \underbrace{\omega_a^2 \left[1 - \zeta(q_b q_b^*)\right]}_{\omega_{a,\mathrm{eff}}^2} q_a = 0.$$

- Frequency shift produced by anharmonicity.
- Important for the g-mode!
- $\omega_{a,\mathrm{eff}}^2 \uparrow$ —resonance locking (low-order g-modes)
- $\omega_{a,\mathrm{eff}}^2 < 0$ —p-g instability (high-order g-modes)

Origin of the f-mode frequency shift



(arXiv:2211.07002,
HY, N.N. Weinberg, P. Arras, et al., 2023)

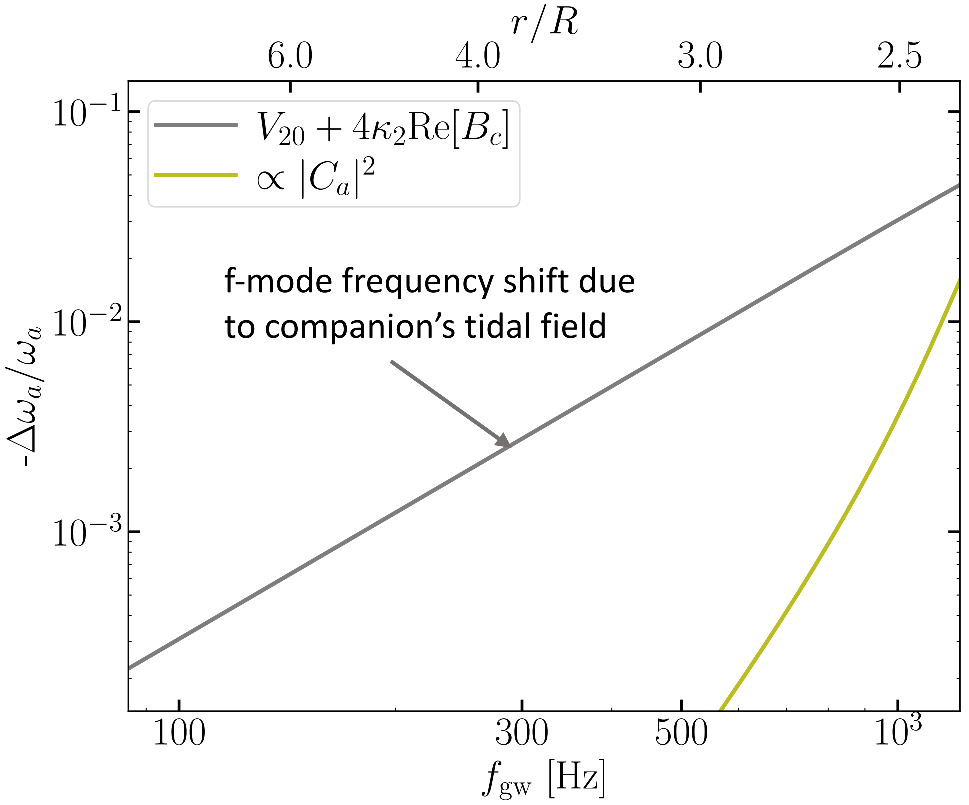


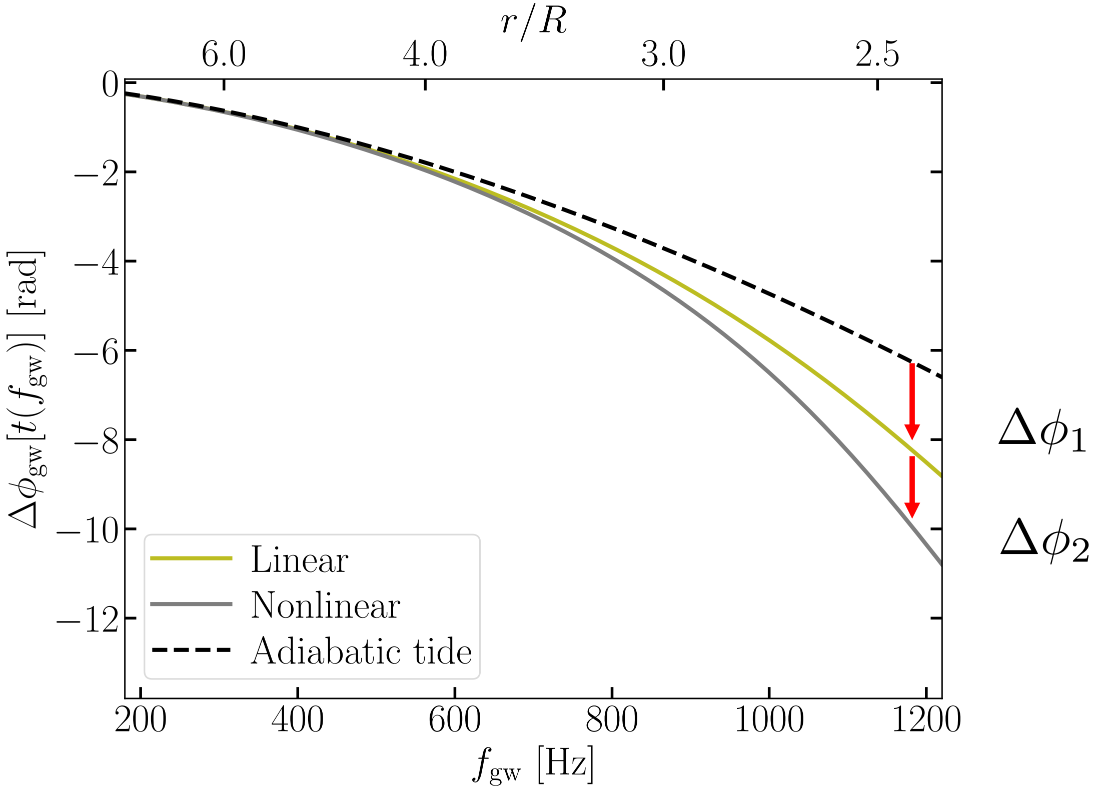
- Test particle in a small hole through the star:
- $\ddot{x} = -g \simeq -\rho x = -\omega^2 x$
- ~ eigenfrequency of the f-mode

- Companion's tidal gravity reduces inward acc. along x
- Fractional frequency decrease ~ (R/r)^3~ω^2
- ($\Delta\omega_y = -\Delta\omega_x/2,\ (\Delta\omega_x + \Delta\omega_y)/2 = \Delta\omega_x/4 < 0.$)

Enhanced mode resonance

$$q_a \simeq \frac{\omega_a^2}{\omega_a^2 - 4\omega^2} U_a$$

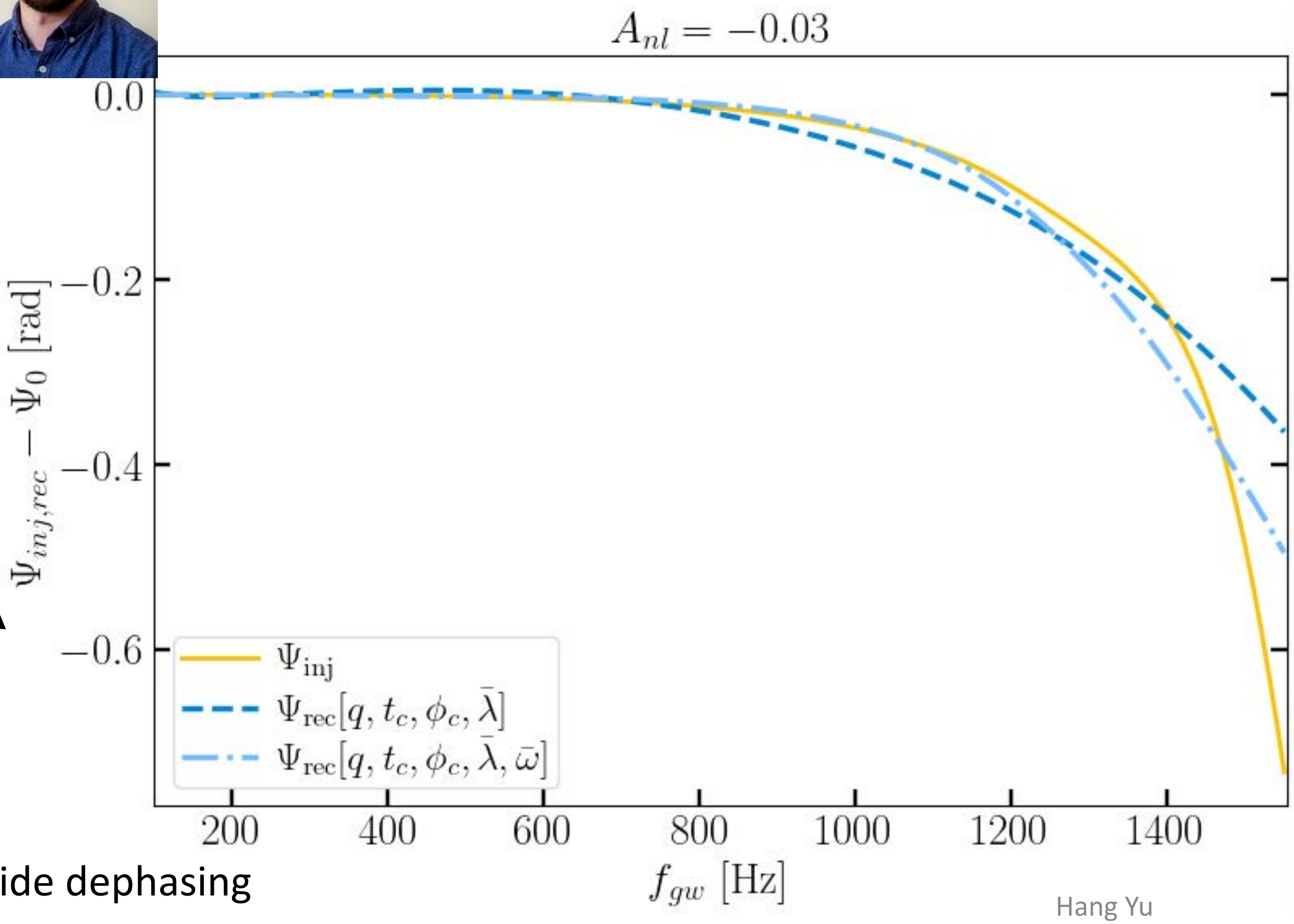




- The nonlinear f-mode frequency shift can be ~ 5% @ 1 kHz
- Confirmed by Pitre & Poisson 25 in GR (w/ low-freq approximation)
- Impact amplified by the Lorentzian!

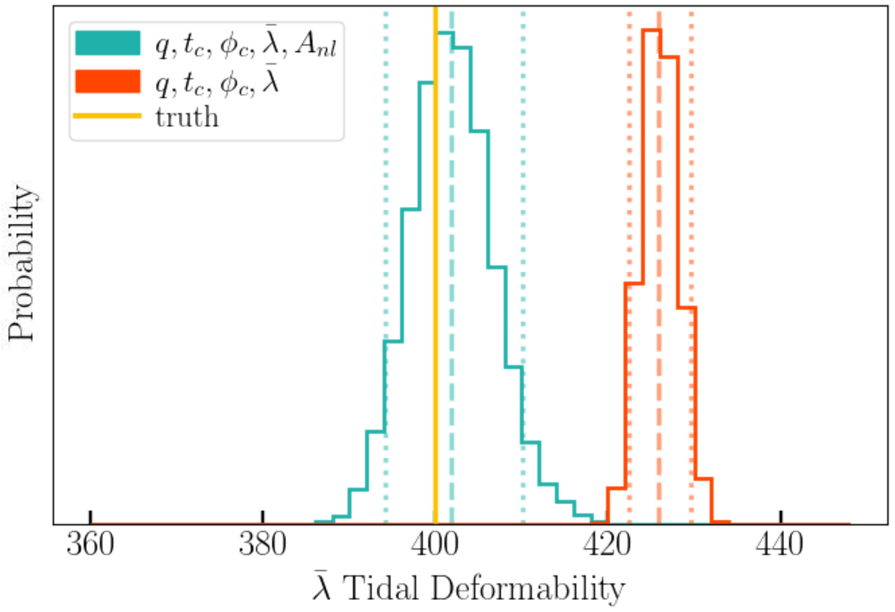
$\Delta\phi_1$~ linear dyn. – ad. tide
$\Delta\phi_2$~ nonlinear – lin. dyn. Tide
$\Delta\phi_2 \simeq \Delta\phi_1$!
Affecting every BNS!

Biased EOS inference (J. Bretz & HY, in prep)

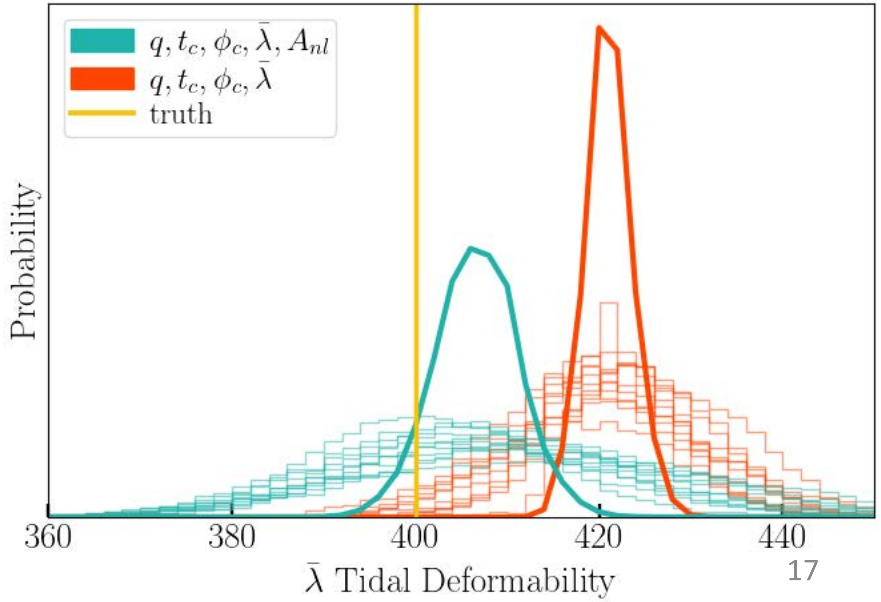


NL tide dephasing

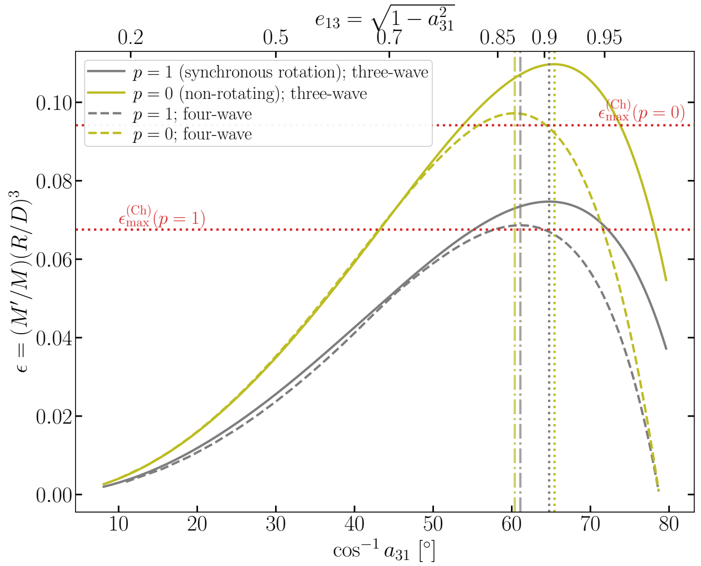
Single loud event



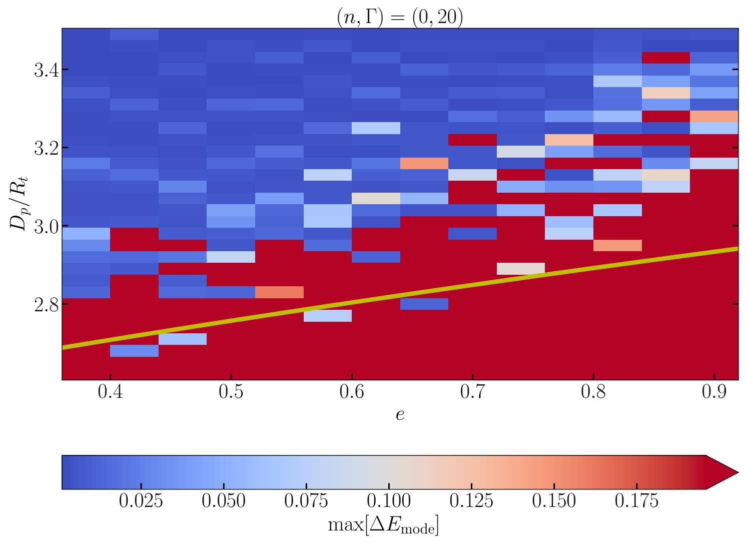
Stacking multiple events

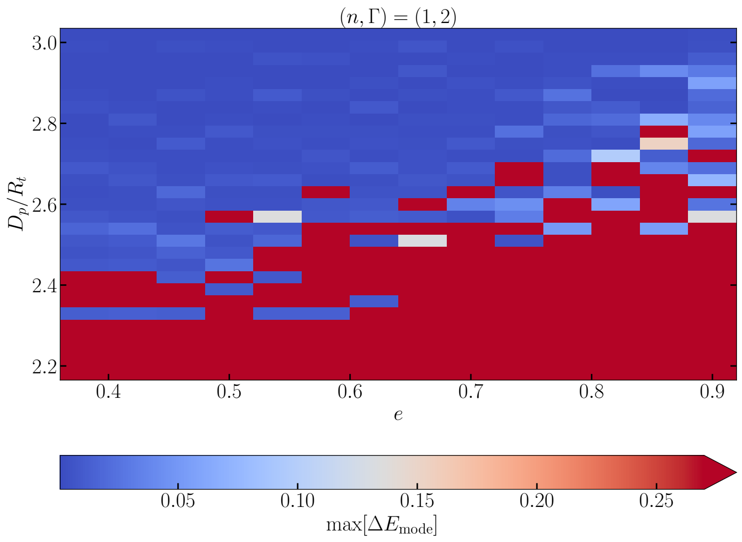


Aside: Roche limit (Yu+ 25, arXiv:2508.20183)

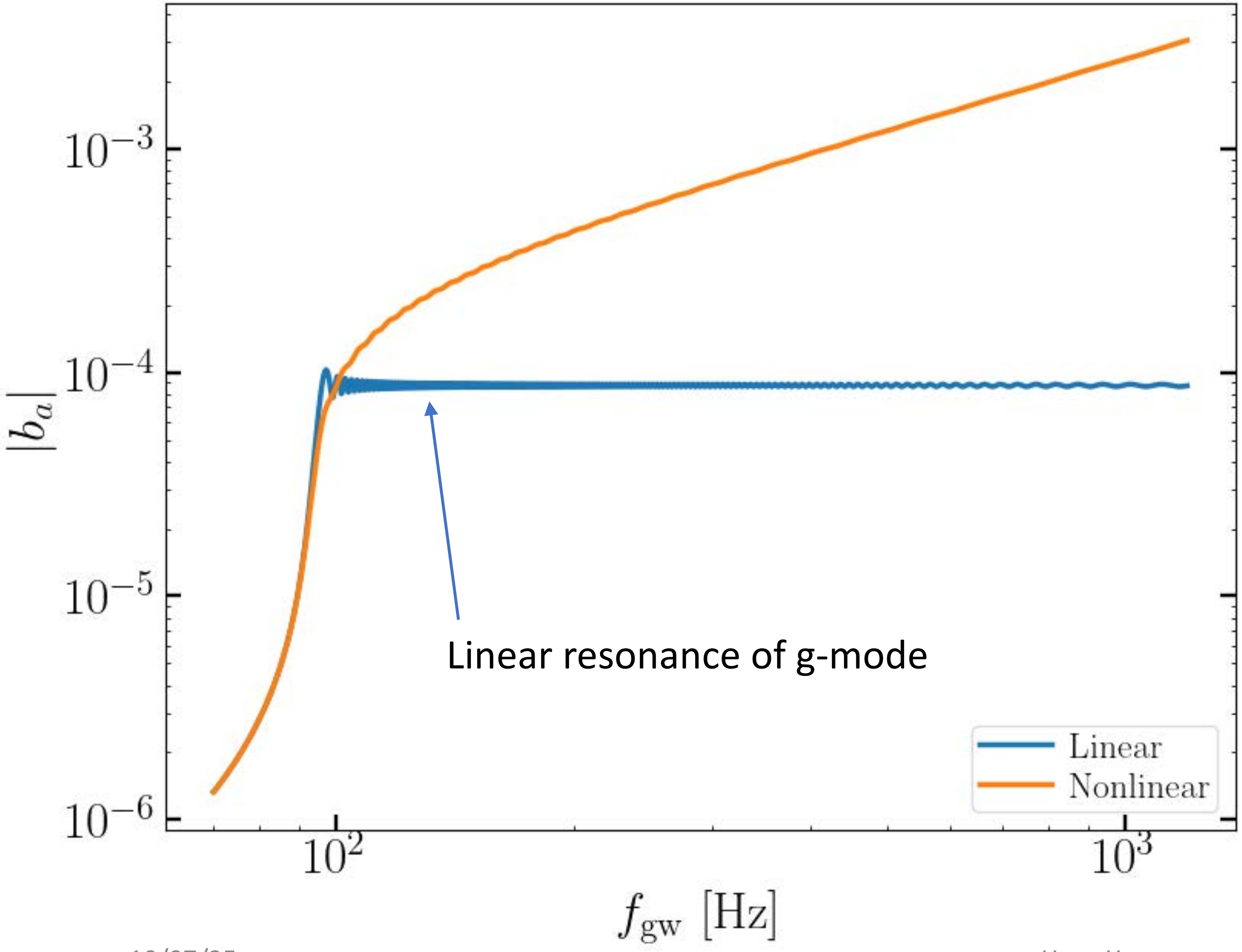


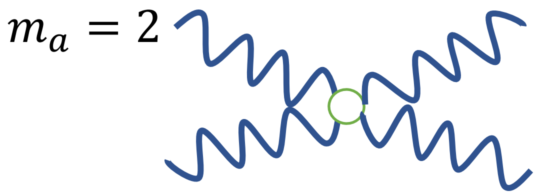
- When the nonlinear tide destabilizes the f-mode, Roche limit!
- Reproducing Chandrasekhar's hydrostatic result (synchronous rotation, circular orbit, & homogenous ellipsoids)
- Applicable to real stars w/ arbitrary eccentricities/rotation
- Dynamical tide significantly modifies the Roche limit!
- Terminal frequency of BNS inspirals? (Lai 93)
- Repeated tidal disruption events? Migration of hot Jupiters?





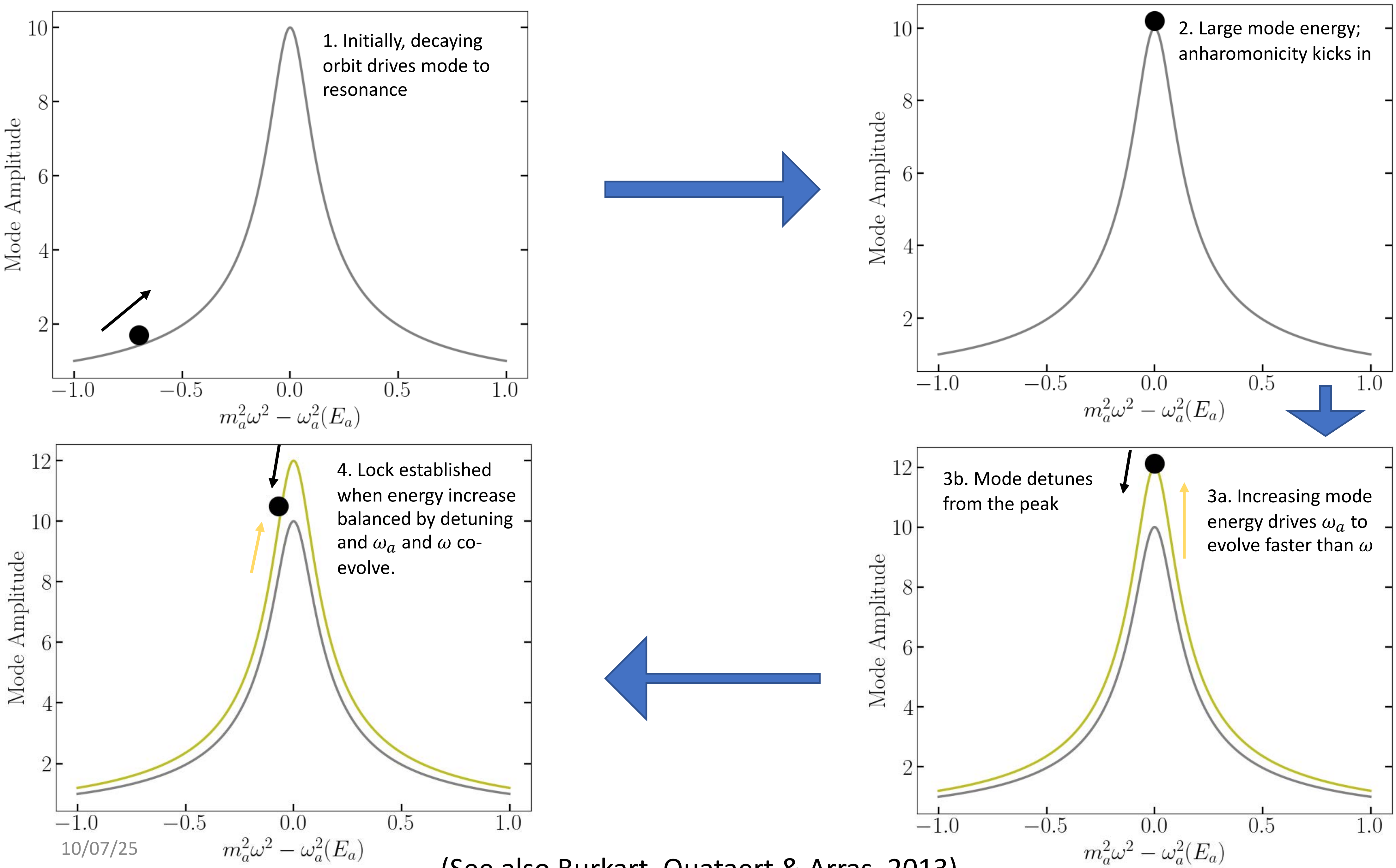
Resonance locking of g-modes (arXiv:2410.03831, 2503.11837, K. J. Kwon, HY, & T. Venumadhav, 2025)



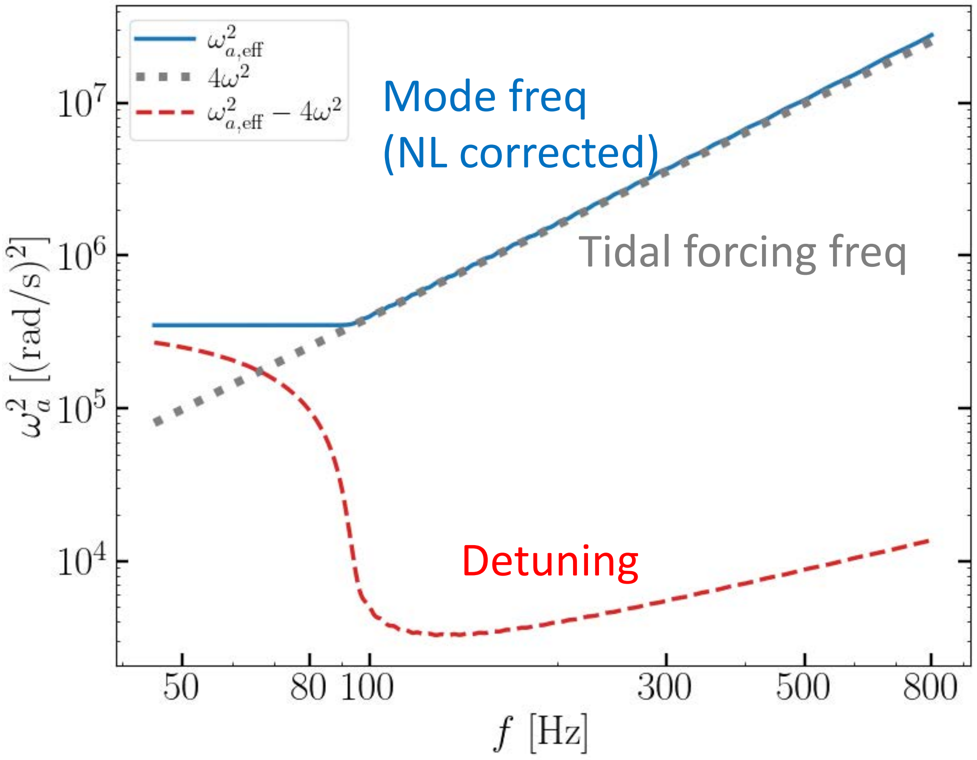


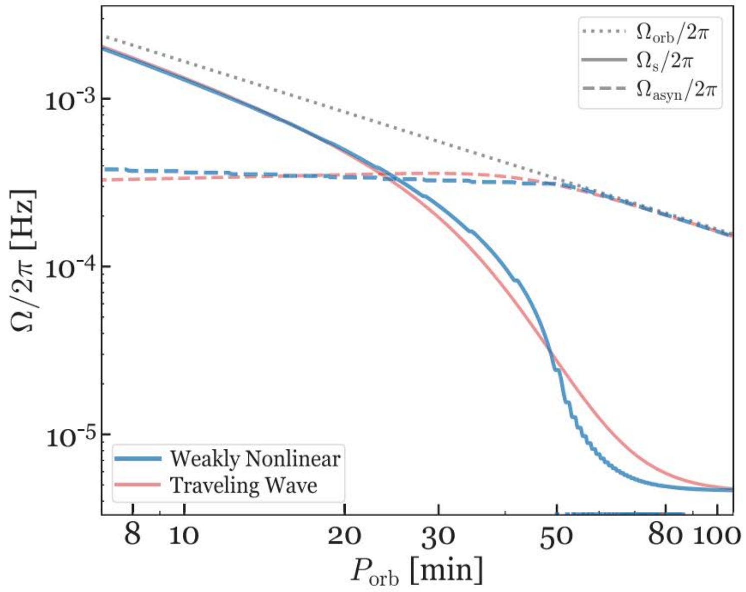
Linear case:
Decoupled with the orbit afterward.
Small phase shift < 0.01 rad (Lai 94; HY & Weinberg 17a, b)

Nonlinear case:
Continuously amplified by resonance and coupled with the orbit.
Significant phase correction ~ 3 rad!



(See also Burkart, Quataert & Arras, 2013)





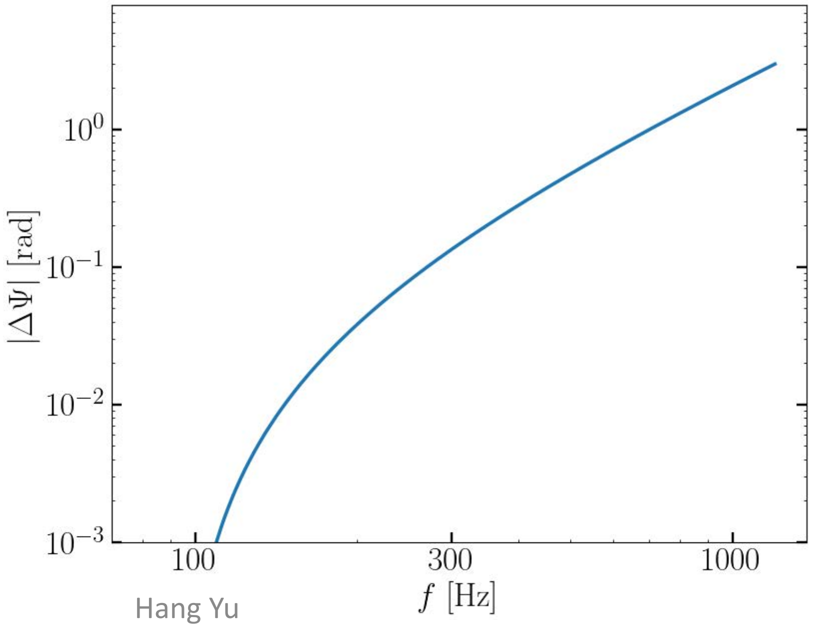
Cf. Resonance locking in double white dwarfs

(HY, N. N. Weinberg, J. Fuller 2021)

WD: freq shift due to spin (Doppler)

NS: freq shift due to anharmonicity

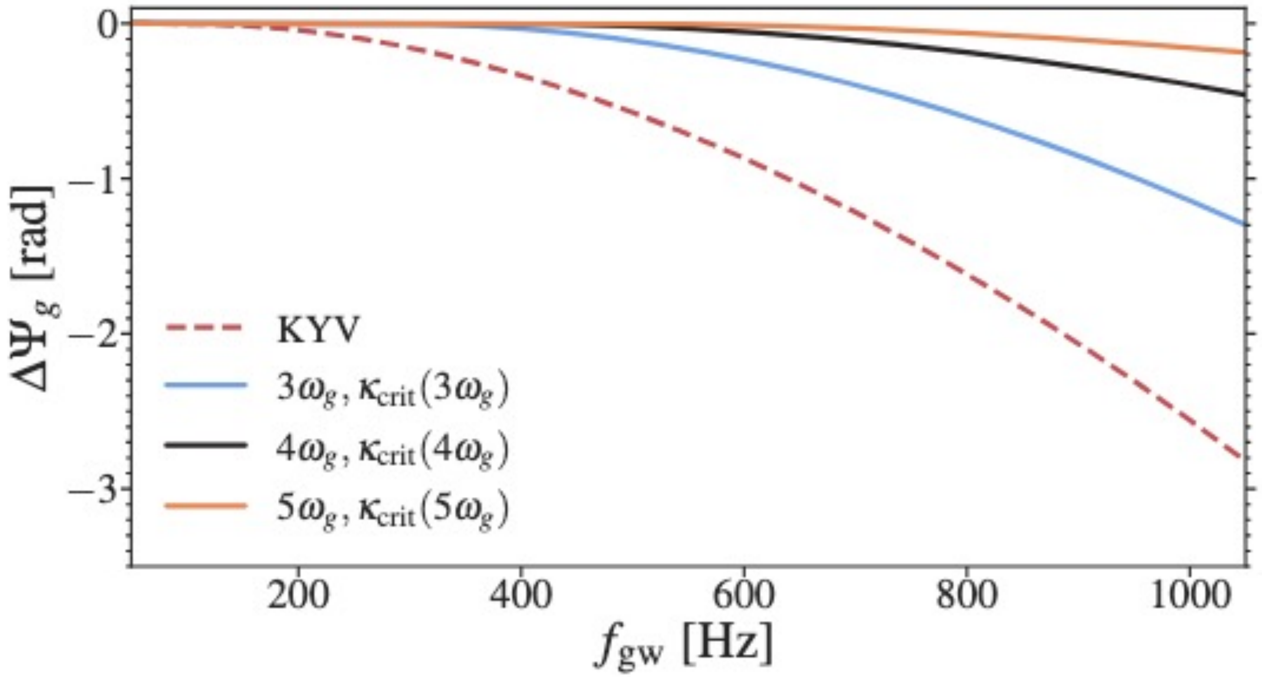
(Also Burkart et al 13; Fuller & Lai 13)



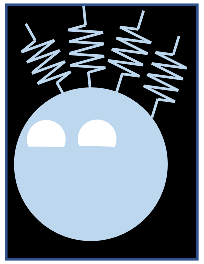
Phase shift ~ 3 rad from the g-mode!

Caveat/Opportunity—superfluid g-modes?

- Core of cold NSs is a superfluid
  - Very different g-mode spectra (x3-x5 higher frequencies) (Kantor & Gusakov 14, Passamonti, Andersson, & Ho 16, Yu & Weinberg 17)
- Assuming optimal coupling strength, effect suppressed by increasing frequency
- Test NS superfluidity?



Conclusion



- Dynamical tides in BNSs have rich phenomenology beyond just (effective) Love number
- Need accurate theoretical models to infer NS EOS

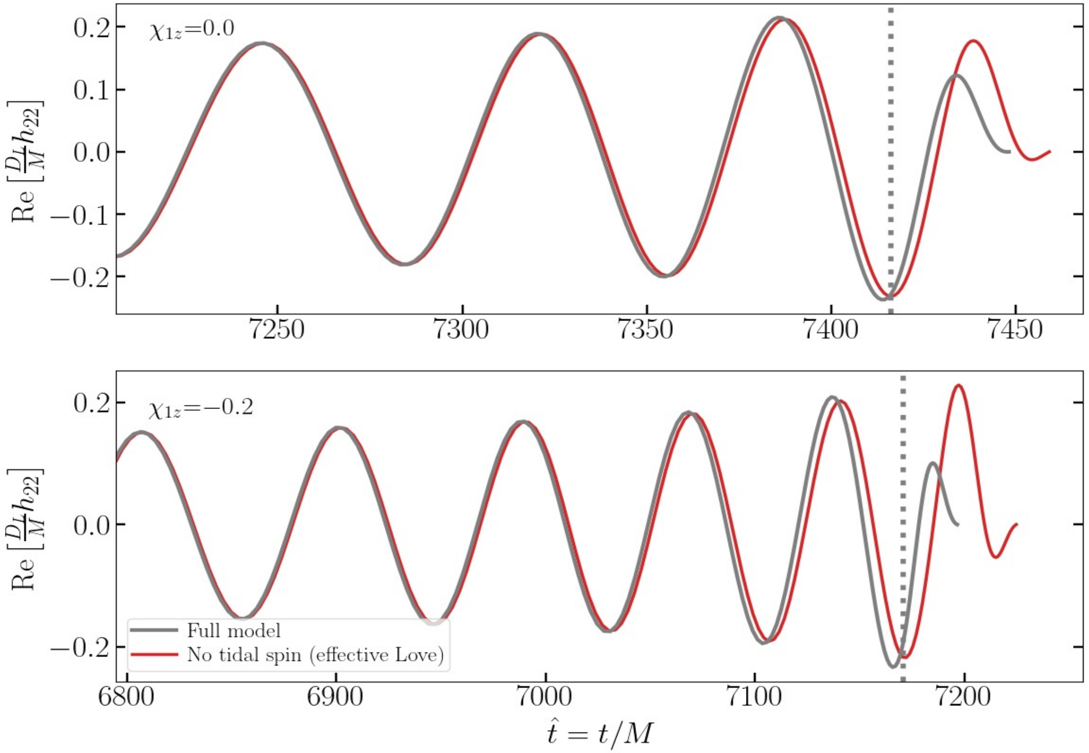
- New EOB model for BNSs w/ aligned spins
  - Tidal spin: ~0.5 rad (no spin) – >5 rad (anti-aligned spin)
  - (Mode resonance in GW flux at 1 PN.)

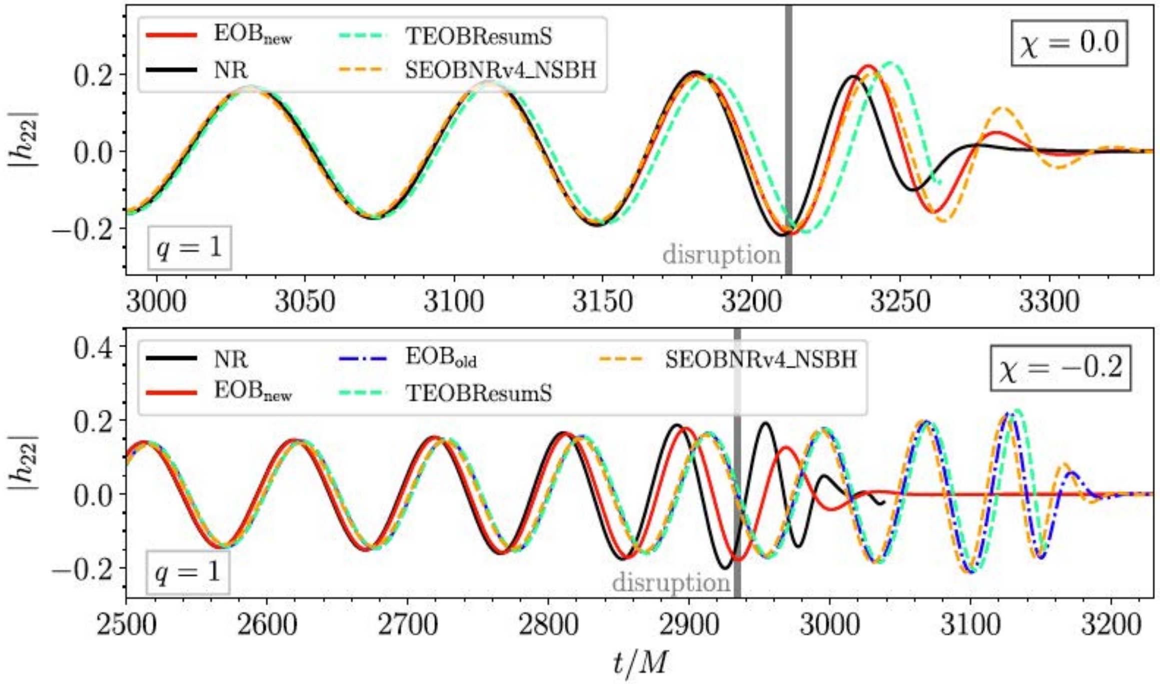
- Nonlinear hydrodynamics (shifts of mode frequencies)
  - f-mode: enhancing f-mode resonance; ~1 rad
  - g-mode: resonance locking from anharmonicity; < 1 rad (superfluid) -3 rad (normal fluid)

- Questions? Reach me at hang.yu2@montana.edu

New EOB model (with aligned/anti-aligned background spins)

Peak of the 22 GW mode



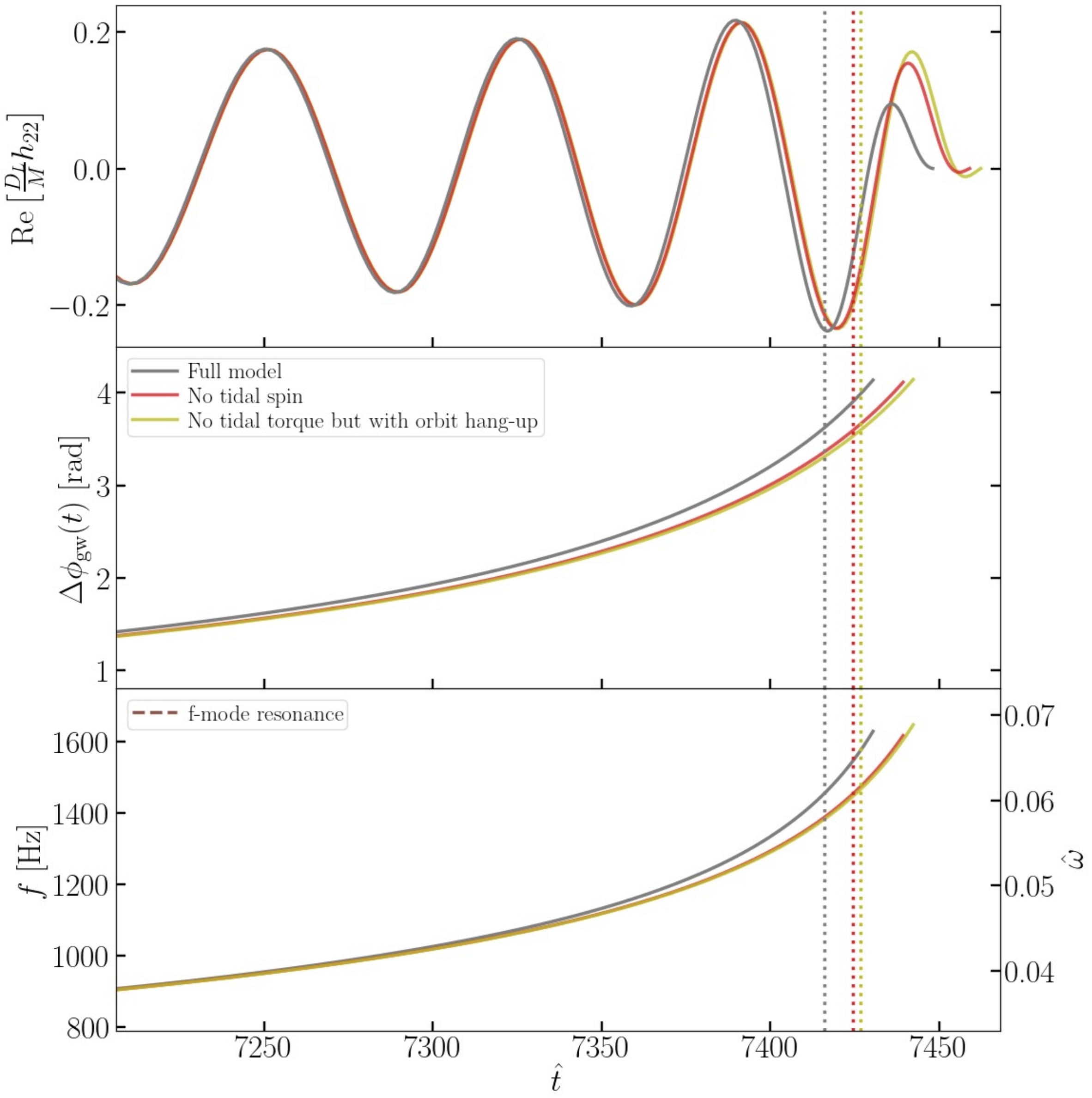


Steinhoff+ (2021)

Black: new EOB w/ tidal spin
Red: no tidal spin (as in eff. Love)

Black: Numerical relativity
Red: best EOB using eff. Love

Non-spinning



$\chi = -0.2$