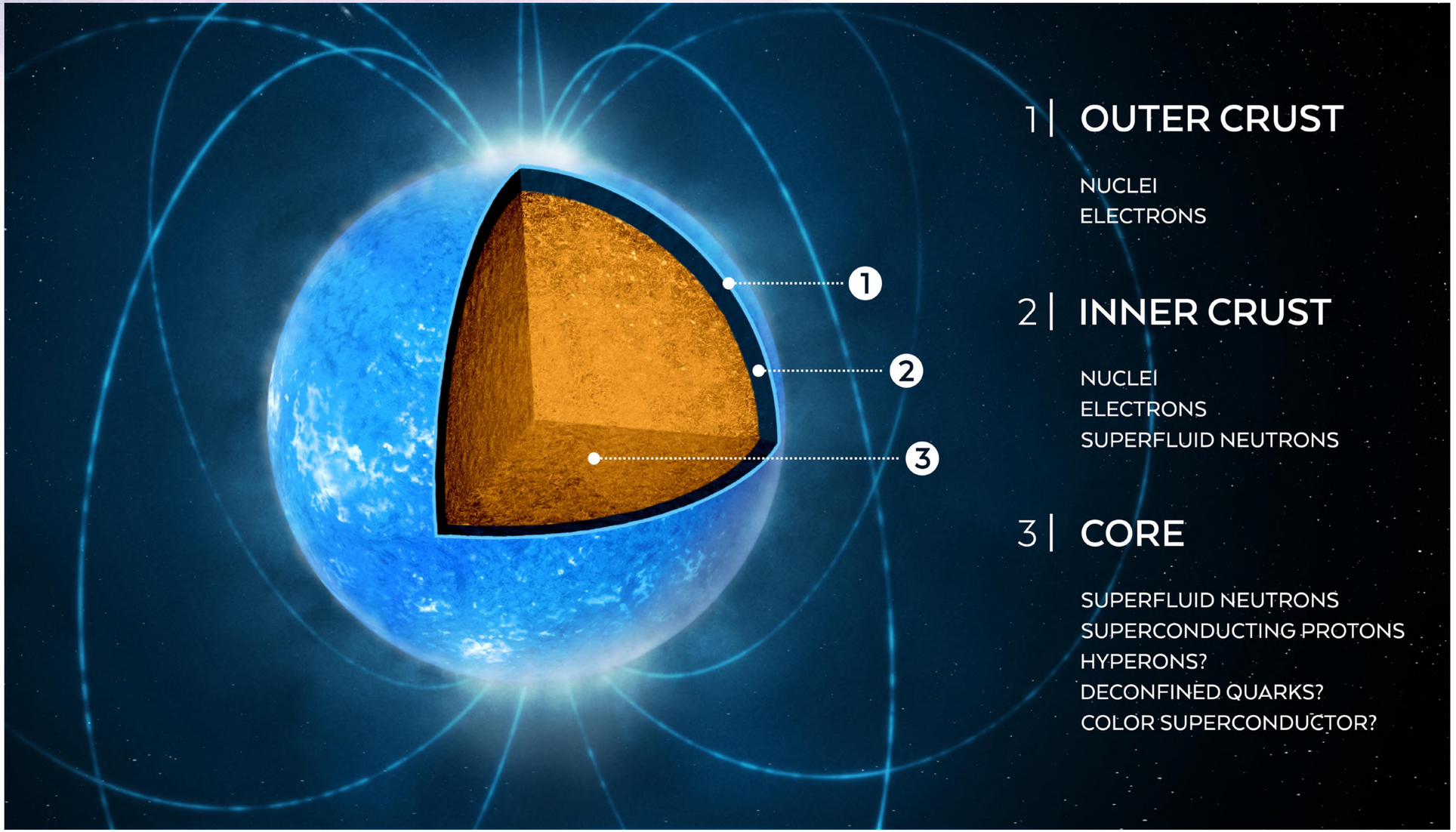
# Constrained Brownian-bridge prior for neutron-star equation-of-state inference

*TG, O. Komoltsev, A. Kurkela, E. Sunde 2512.XXXXX*

Tyler Gorda
Ohio State University

*N3AS Seminar*
16 December 2025

# Neutron stars – probes of the highest densities




Watts, Andersson, Chakrabarty+ Rev. Mod. Phys. 88 (2016)

Collapsed remnant of dead stars, held from collapse by repulsive strong nuclear force

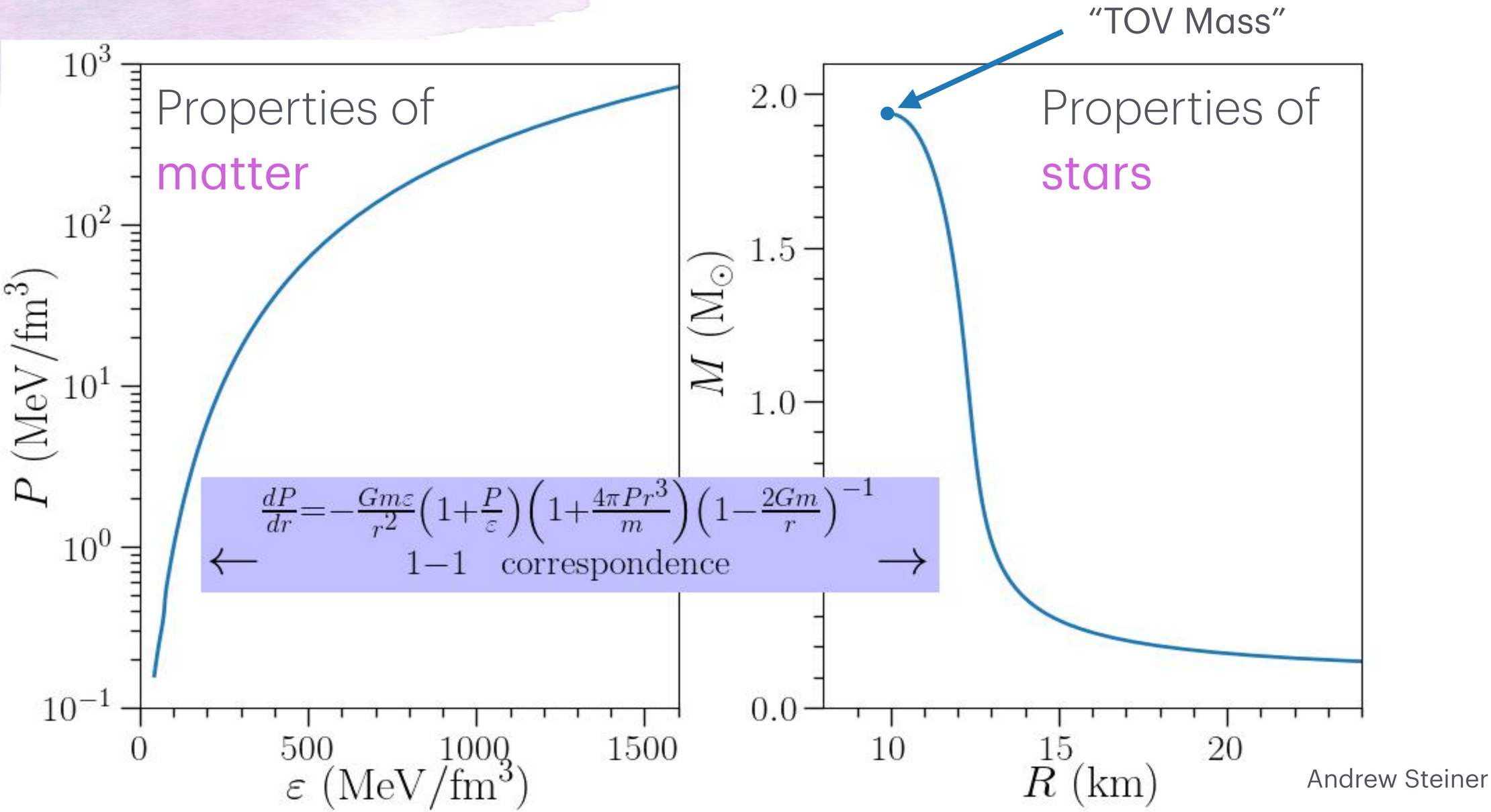
$$\text{Mass} \lesssim 2M_\odot,$$

$$11\,\text{km} \lesssim R \lesssim 13\,\text{km}$$

$$T \lesssim \text{keV} \sim 10^7\,\text{K}$$

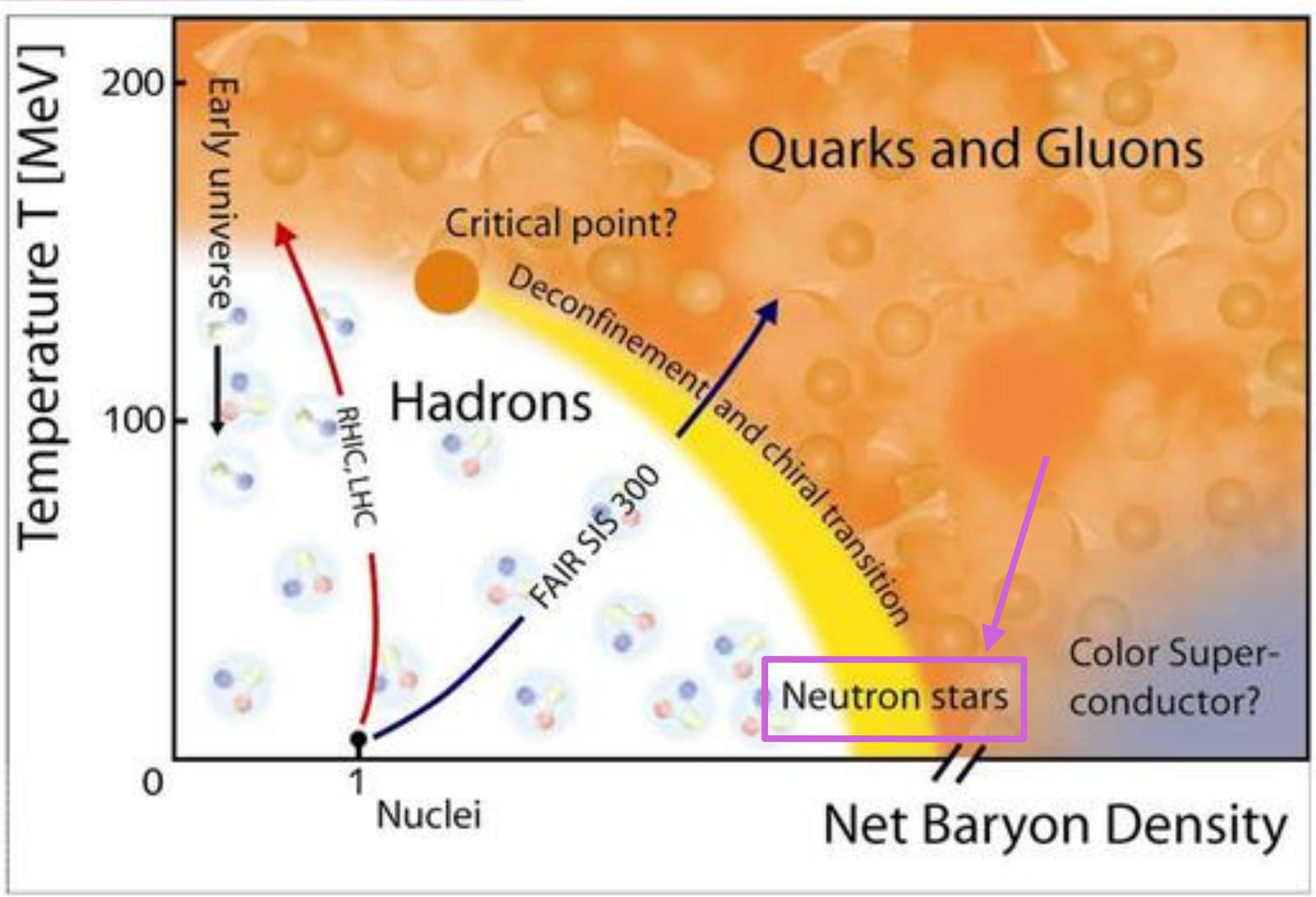
Cold means simple

# (Micro)physics from (macro) observations



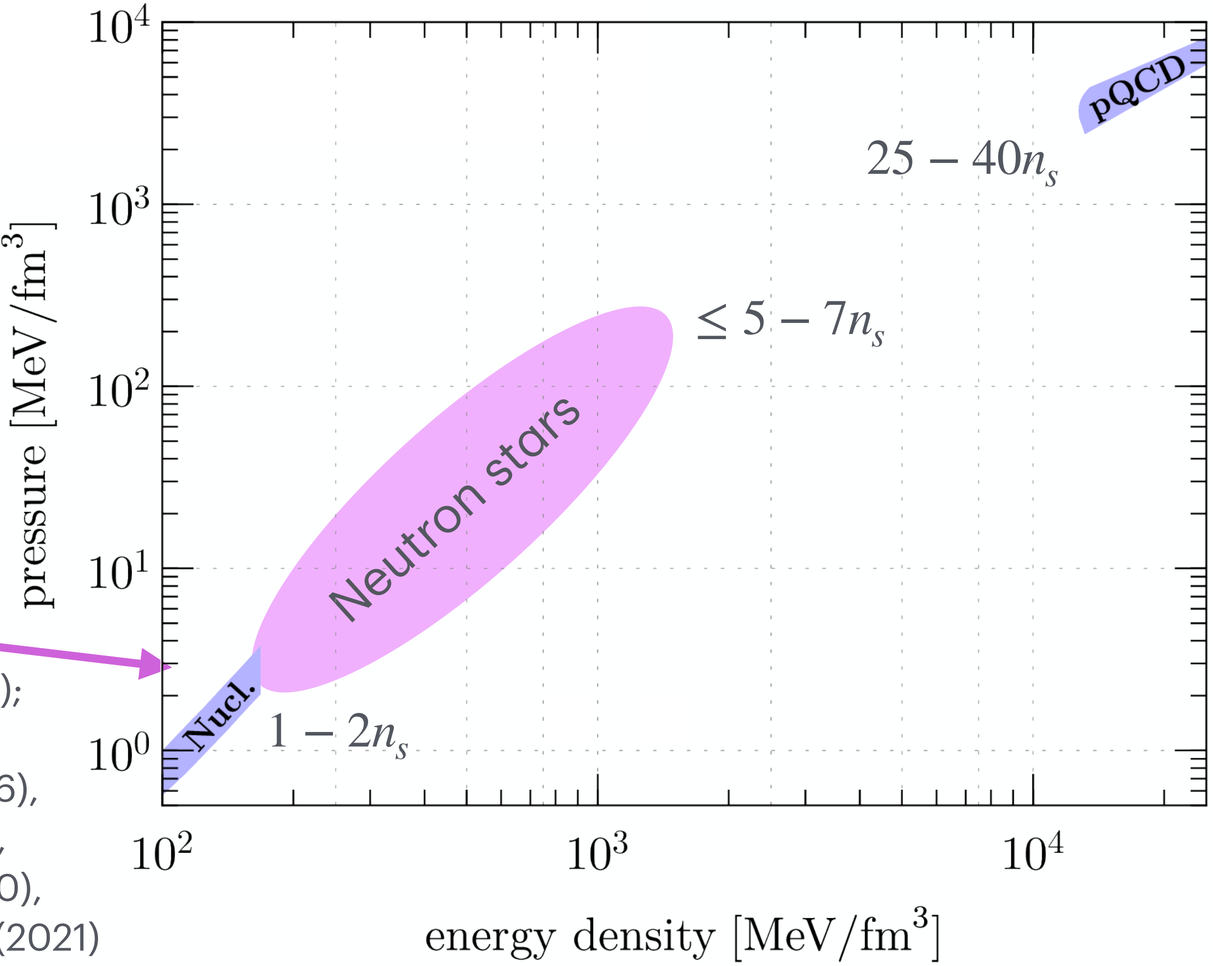

Andrew Steiner

# Neutron stars in the context of the strong nuclear force




Compressed Baryonic Matter (CBM) experiment

# Different (theory) inputs constrain different parts of the EoS




Perturbative QCD

Freedman & McLerran PRD 16 (1977)
Kurkela, Romatschke+ PRD 81 (2010),
TG, Kurkela+ PRL 121 (2018),
TG, Kurkela+ PRL 127 (2021)
& PRD 104 (2021),
TG, Paatelainen+ PRL 131 (2023)

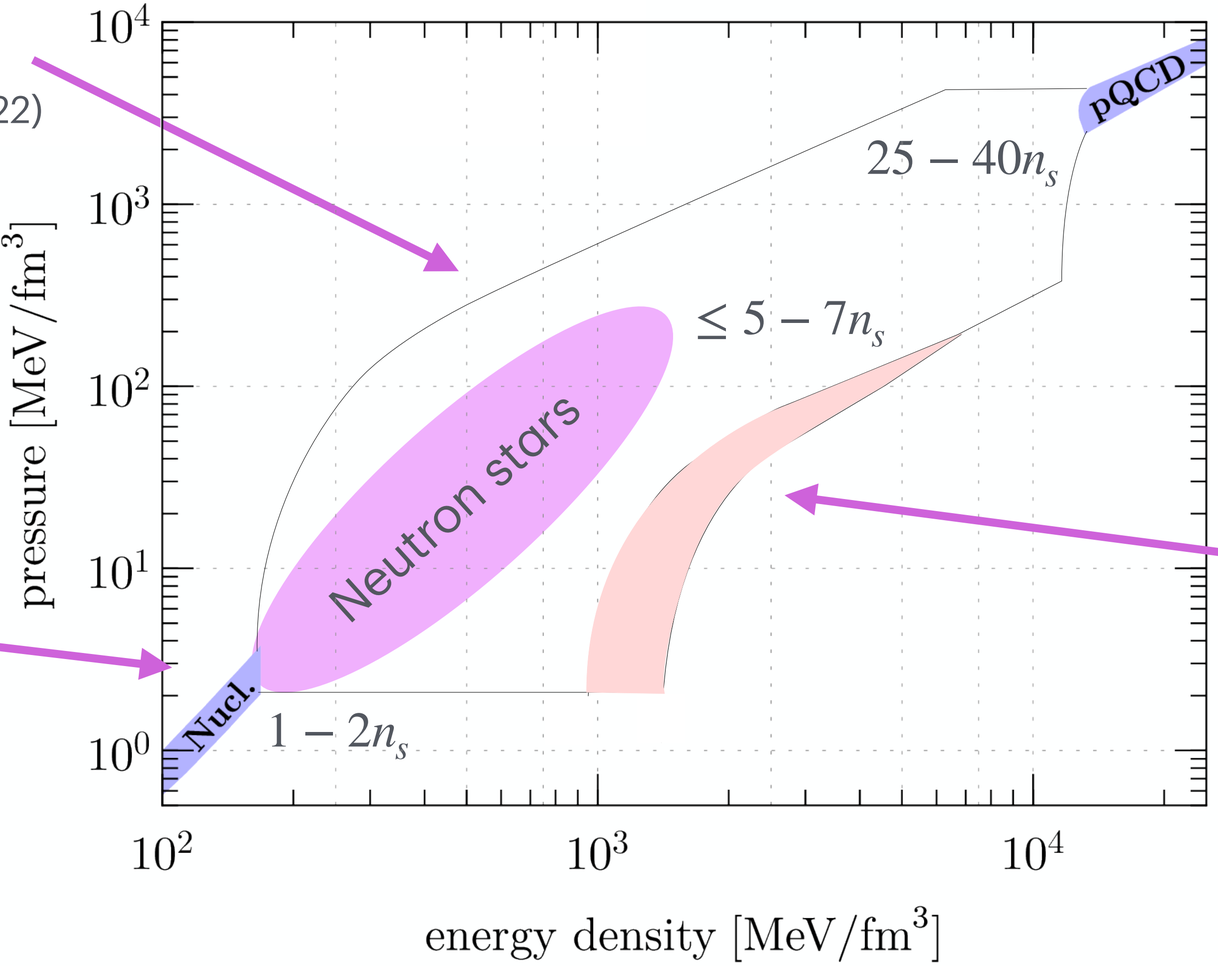
Chiral EFT

Hebeler, Lattimer+. ApJ. 773 (2013);
Tews, Krüger+, PRL 110 (2013)
Lynn, Tews, Carlson+ PRL 116 (2016),
Drischler, Hebeler+ PRL 122 (2019),
Drischler, Furnstahl+ PRL 125 (2020),
Drischler, Han, Lattimer, PRC 103 (2021)
Keller, Hebeler+ PRL 130 (2023).

# Intermediate constraints on the EoS

## Thermodynamics

Komoltsev, Kurkela PRL 128 (2022)




## Perturbative QCD

## Chiral EFT

## Lattice (Isospin QCD)

Abbot, Detmold+ PRD 108 (2023),

Brandt, Cuterib+, JHEP 07 (2023),
Fujimoto, Reddy, PRD 109 (2024),

(see also
Moore, TG, JHEP 12 (2023),
Navarrete+, PRD 110, (2024),
TG, Navarrete+ 2511.09627 )

# How the global thermodynamic constraints work

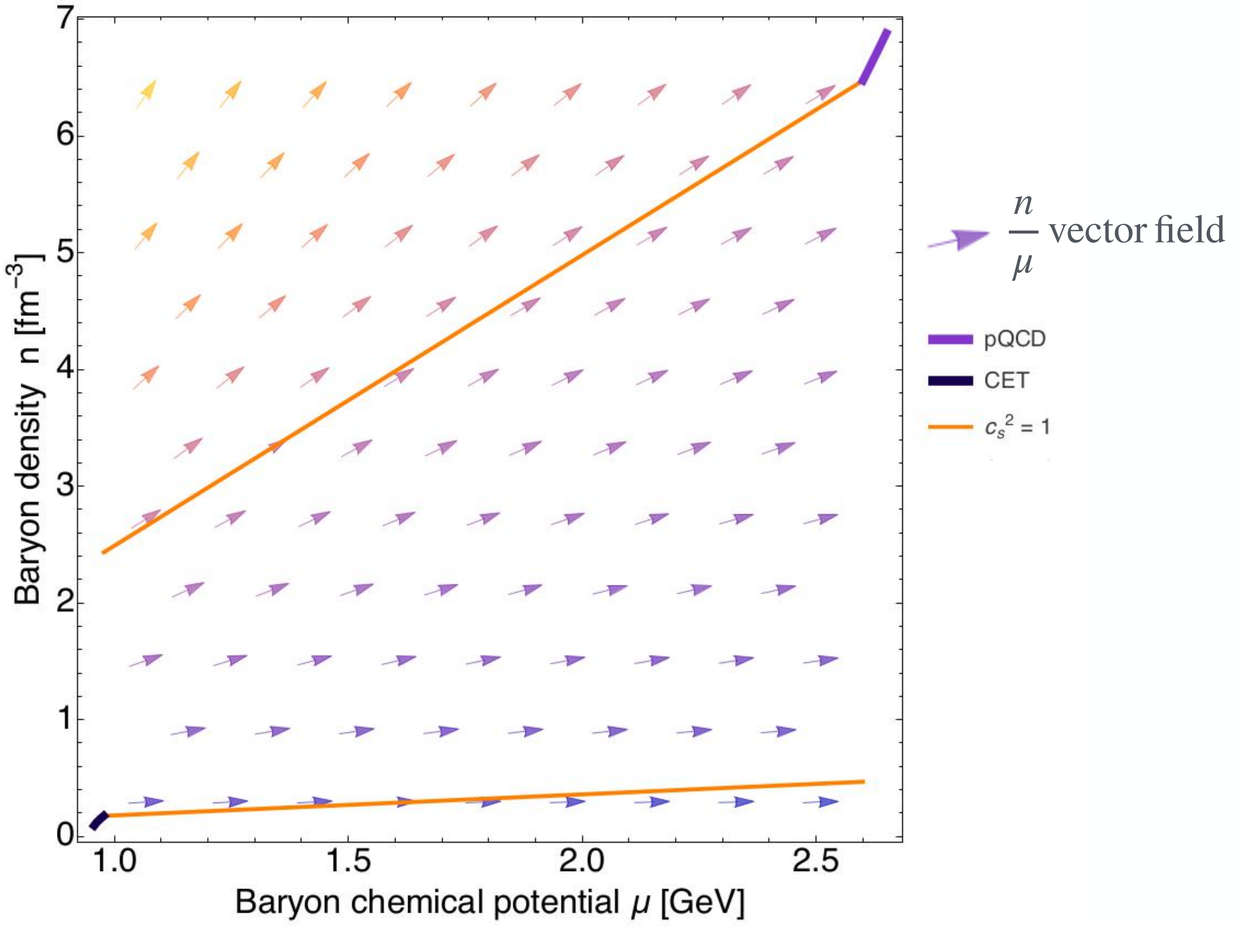
Komoltsev, Kurkela PRL 128, 202701 (2022)

1. Stability

$$\partial_\mu^2 \Omega(\mu) \leq 0 \implies \partial_\mu n(\mu) \geq 0$$

2. Causality

3. Consistency



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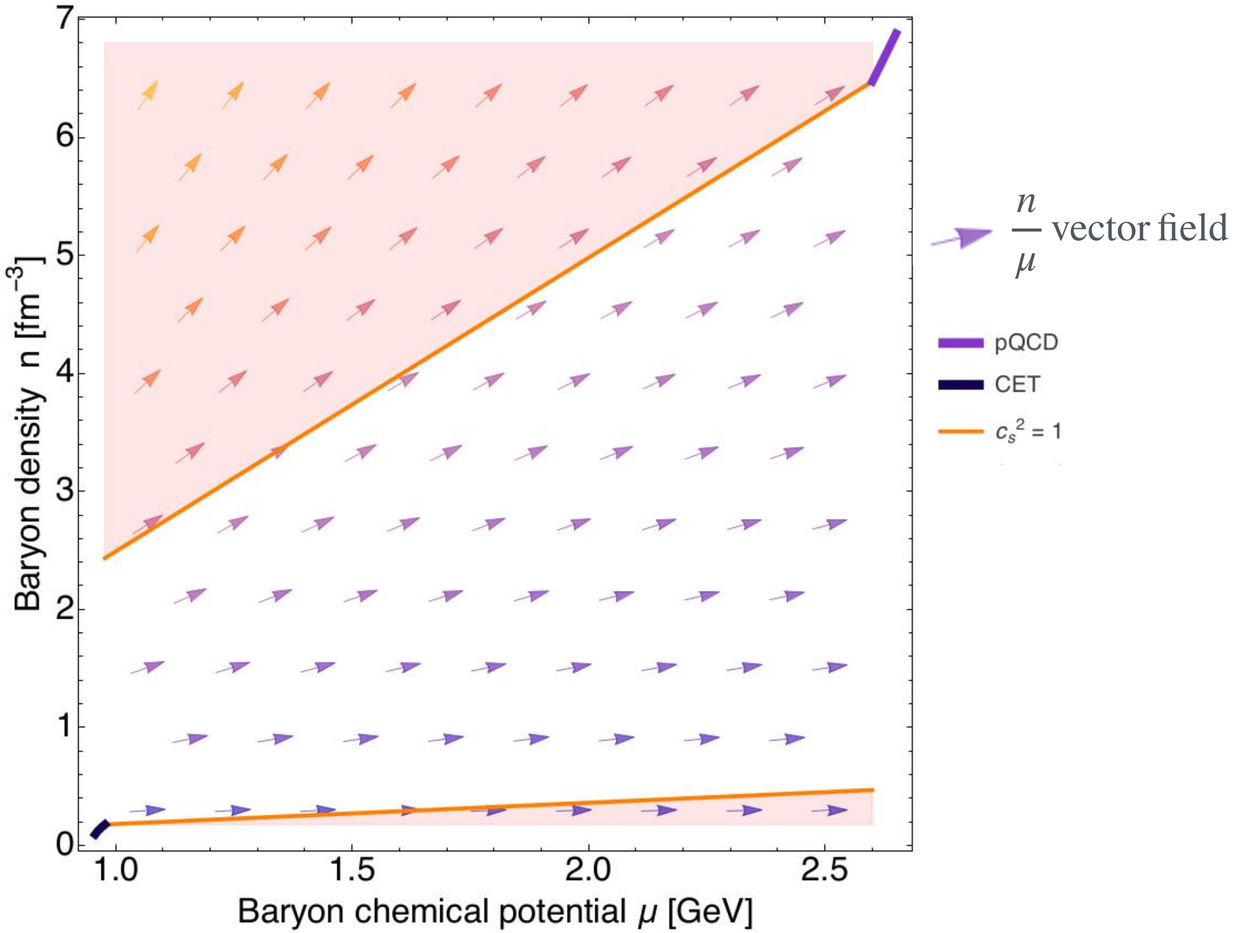

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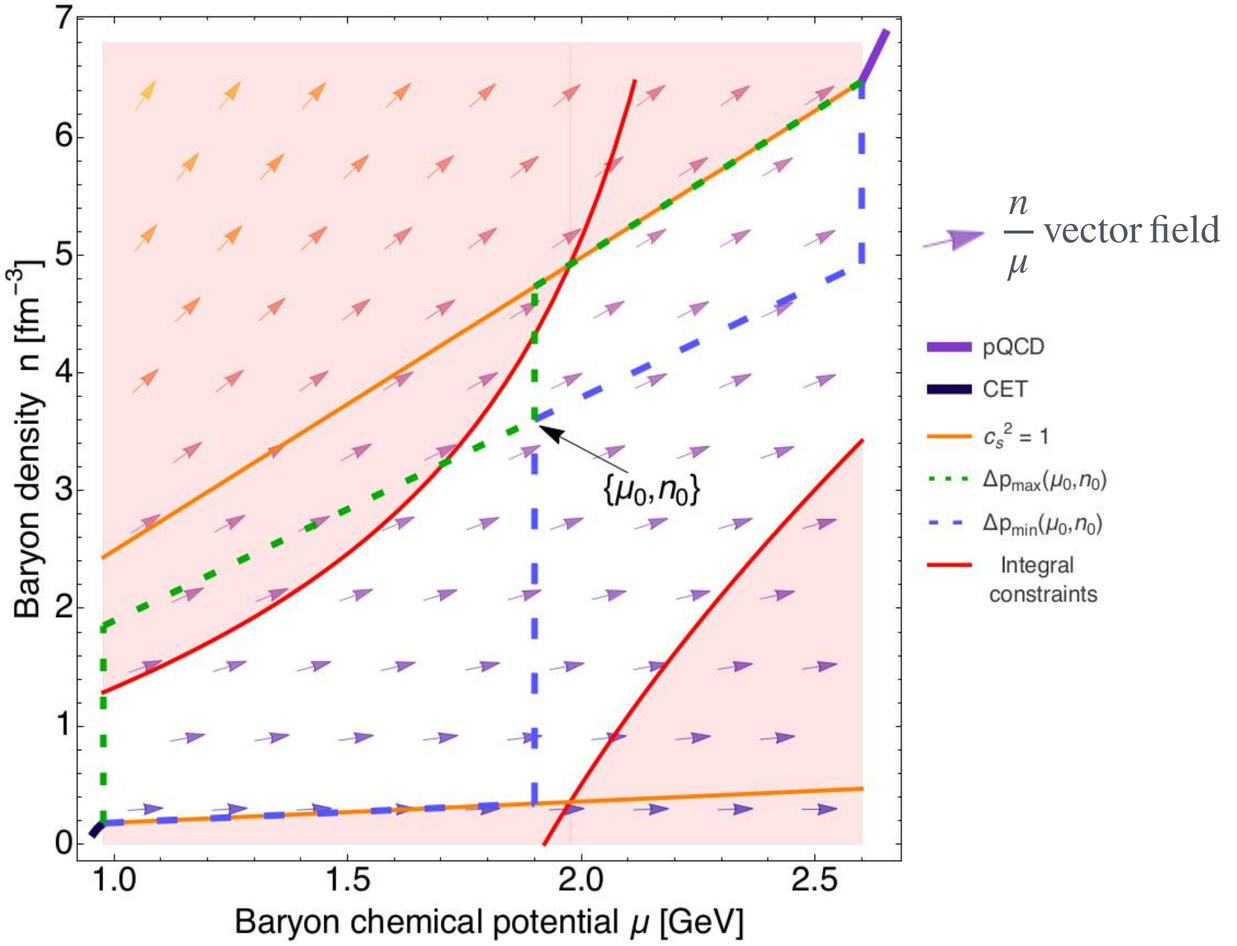
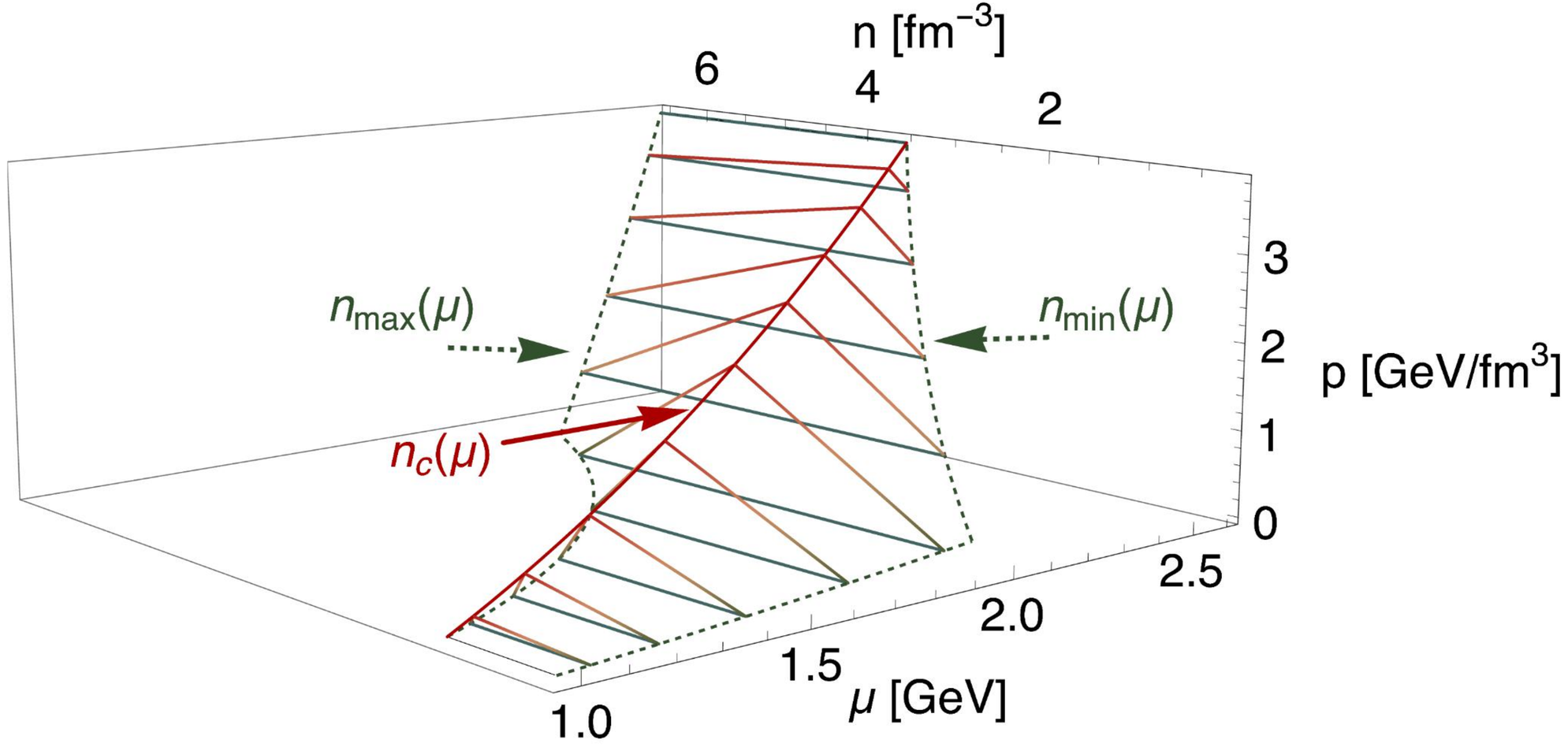
$$\int_{\mu_{\rm CEFT}}^{\mu_{\rm pQCD}} \mathrm{d}\mu\, n(\mu) = p_{\rm pQCD} - p_{\rm CEFT}$$ Fixed!



# Thermodynamic constraints are 3 dimensional

Komoltsev, Kurkela PRL 128, 202701 (2022)



# Desired minimal assumptions between CEFT and pQCD

- Satisfy global causal + consistency constraints
- Minimal local assumptions about the EOS

Legred, Chatziioannou, Essick+ PRD 105 (2022)

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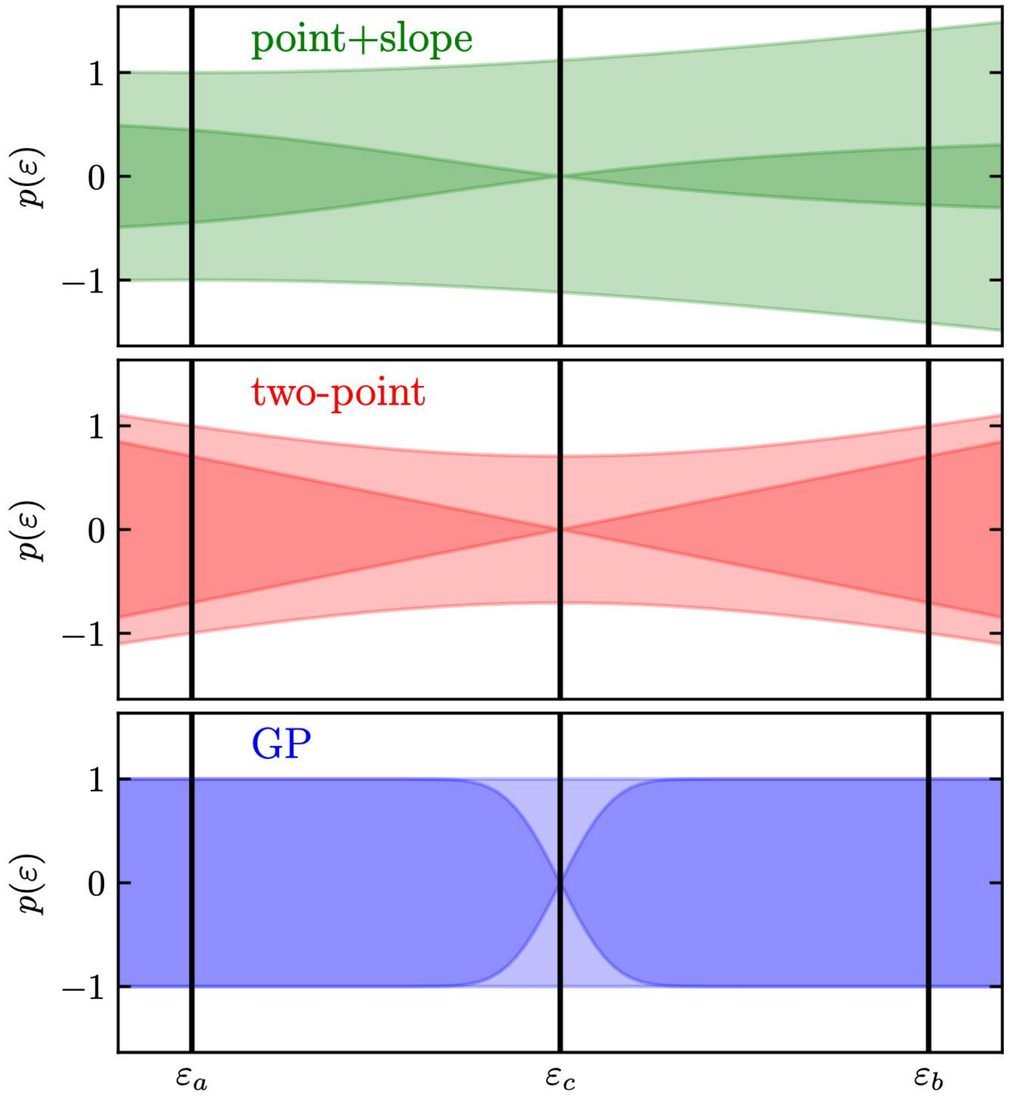
- Satisfy global causal + consistency constraints
- Minimal local assumptions about the EOS
  - Assuming parametric form of EOS introduces additional **correlations between densities**

    Legred, Chatziioannou, Essick+ PRD 105 (2022)

  - *"Only the GP process fills the prior volume rapidly as one moves away from the observation point $\varepsilon_c$"*
- Desire **local Gaussian Correlations,** but **global thermodynamic constraints**

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- Parametric ensemble?

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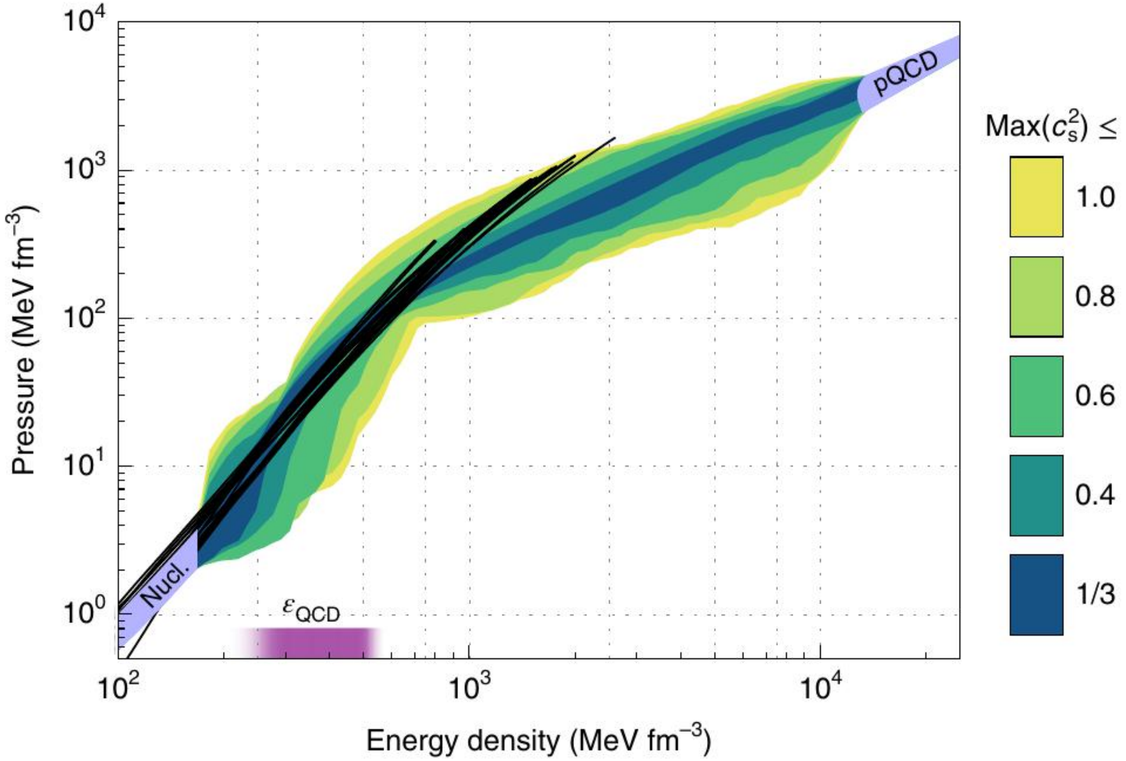

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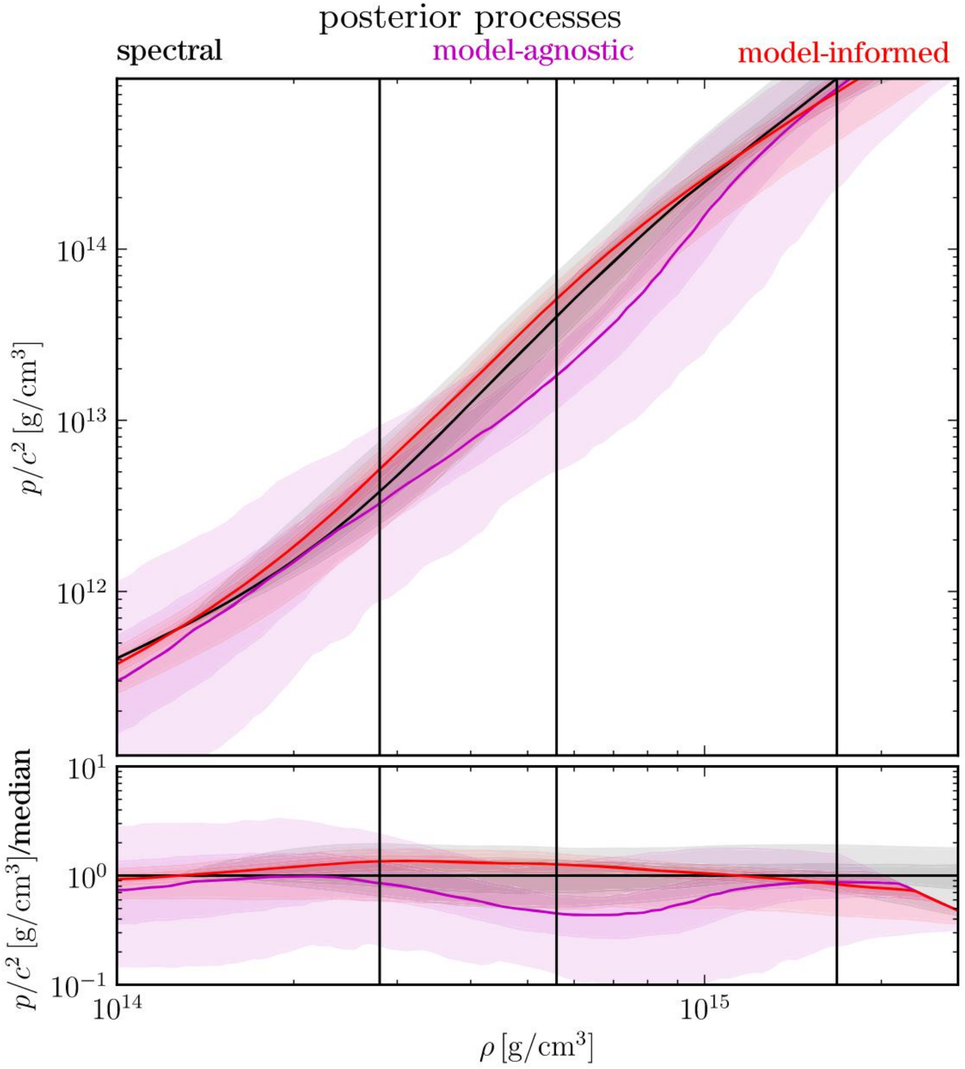

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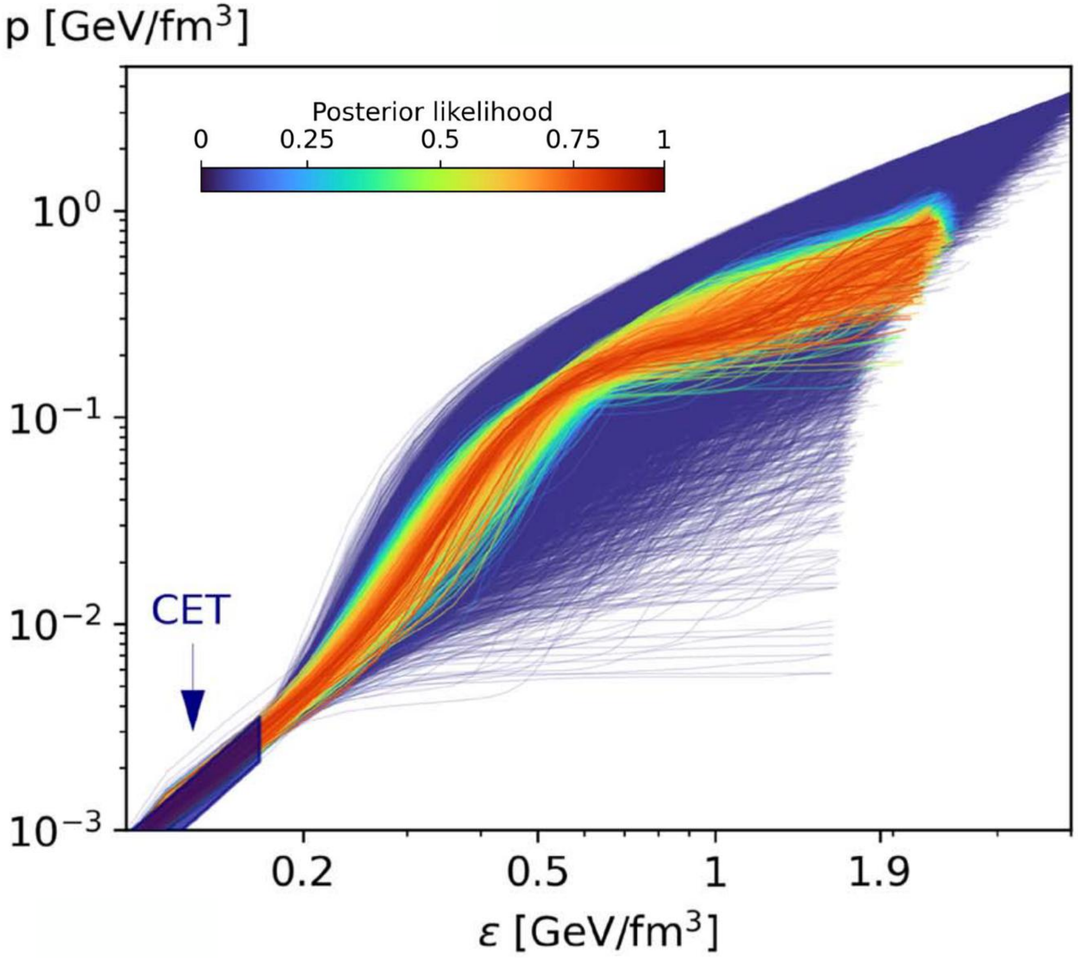
  - + Weighting by some consistency likelihood at fixed density?

    TG, Komoltsev, Kurkela, ApJ 950 (2023)



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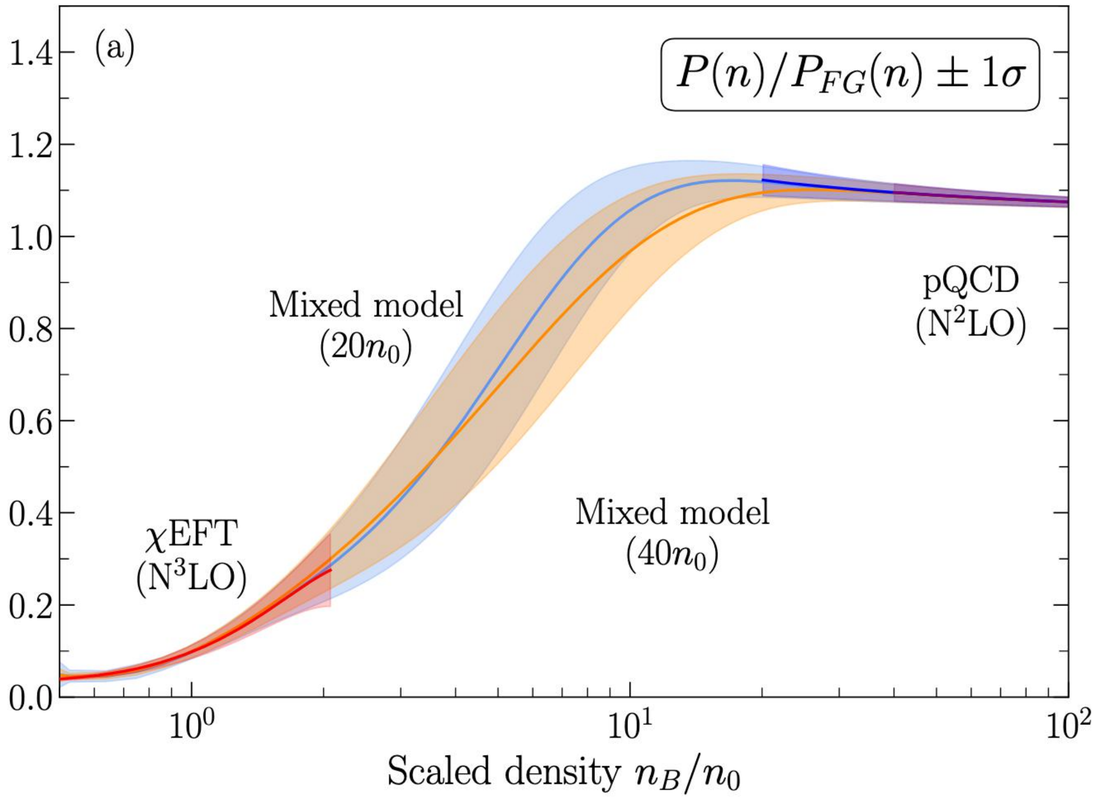
  Semposki, Drischler, Furnstahl+ 2505.18921 (Bayesian model mixing)



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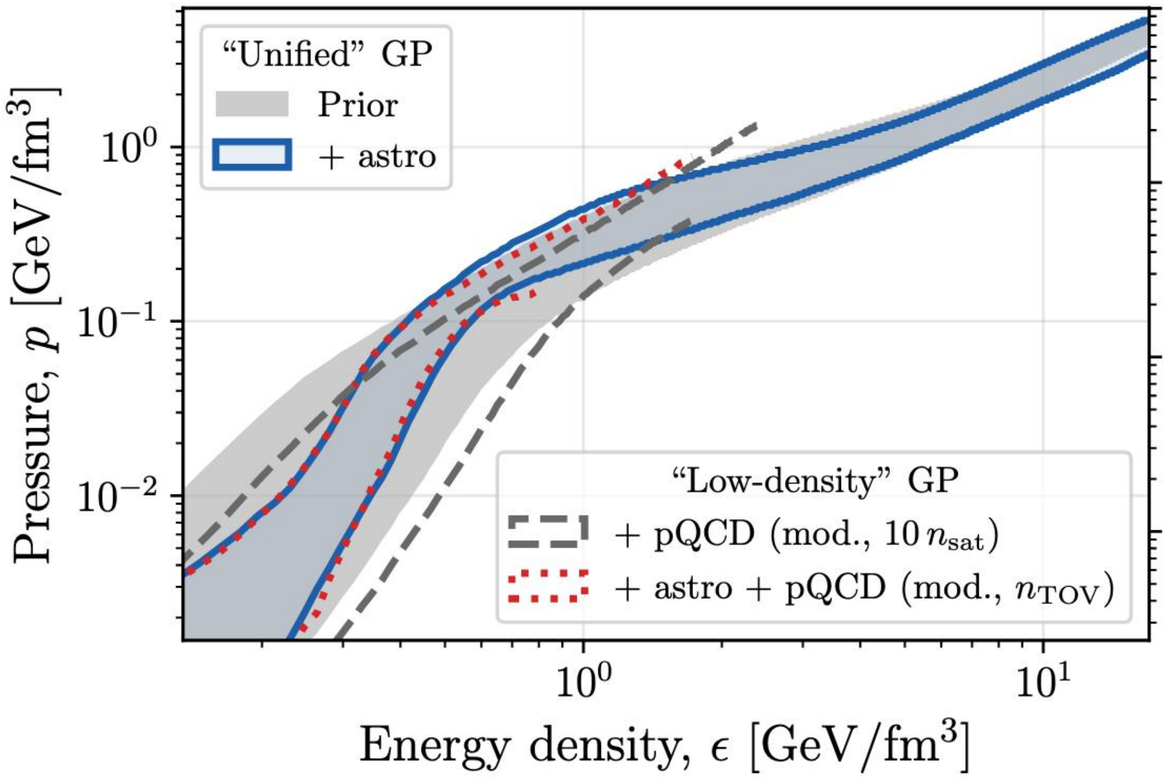
- Check consistency *a posteriori*? (Shooting)

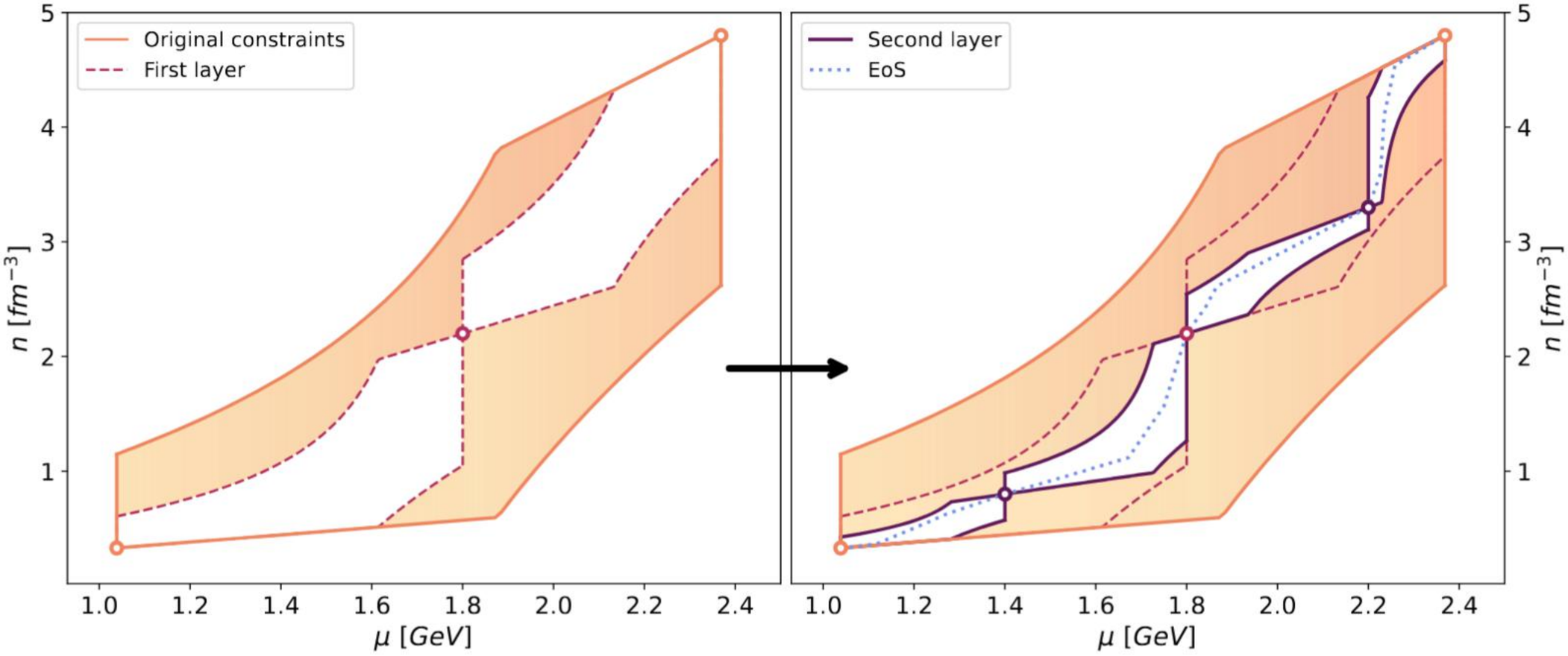
  Finch, Legred, Chatziioannou+ 2505.13691



# New approach: constrained Brownian bridge

TG, Komoltsev, Kurkela, Sunde 2512.XXXXX

1. **Bridge** by sampling points within the allowed volume with self-similar refinement



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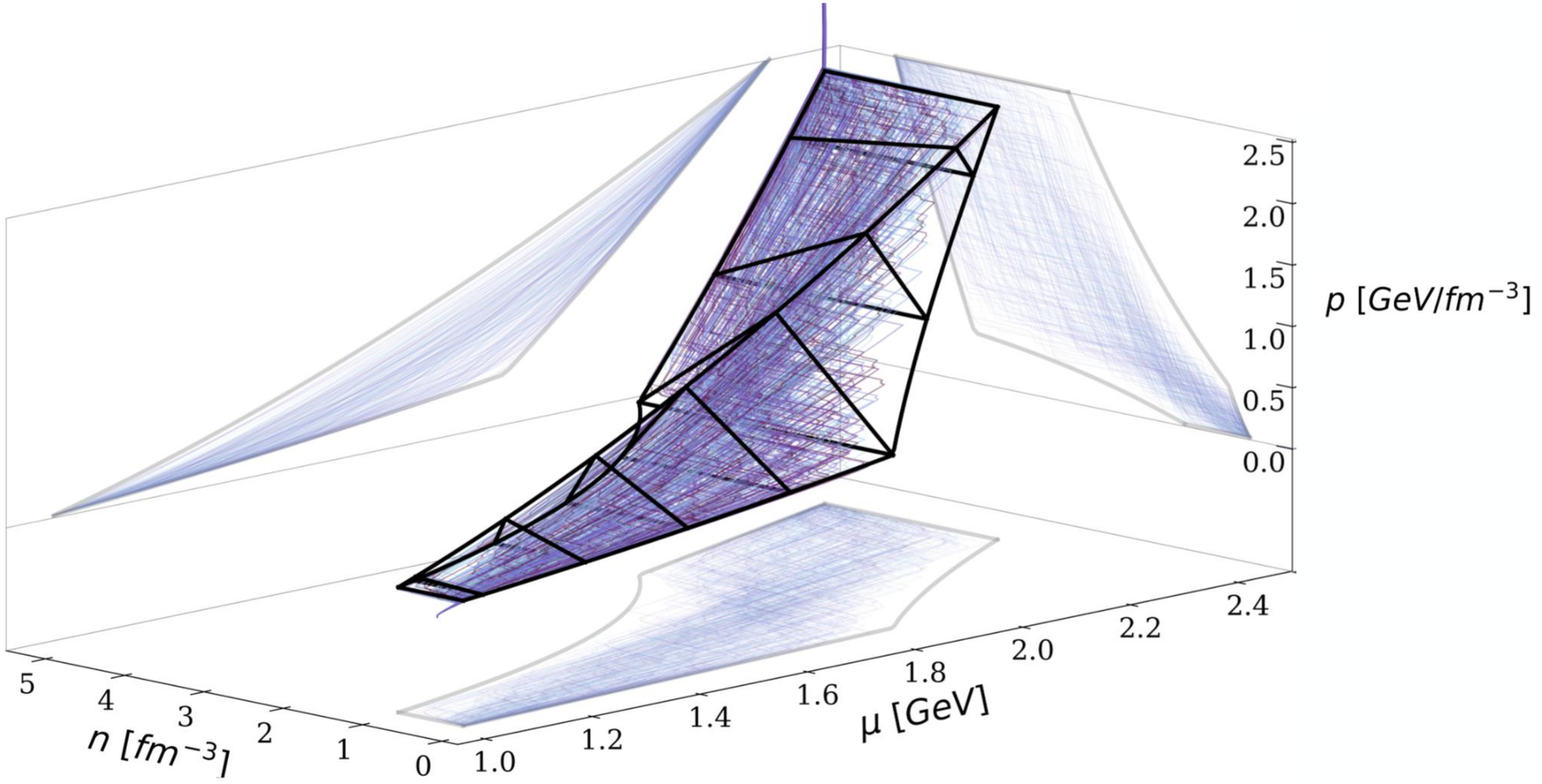

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2. Introduce local Gaussian correlations by diffusing $\mu(n) \to \mu(n,\tau)$ for flow time $\tau$

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$$\left\langle \delta c_s^2(n,\tau)\, \delta c_s^2(n',\tau) \right\rangle = \int d\xi\, d\xi'\, G(n,\xi;\tau)\, G(n',\xi';\tau)\, \overbrace{\left\langle \delta c_s^2(\xi,0)\, \delta c_s^2(\xi',0) \right\rangle}^{\approx \sigma_s^2 \delta(\xi-\xi') \text{ (initially white)}},$$

$$\propto \frac{1}{\sqrt{8\pi D(n)\tau}} \exp\left[ -\frac{(n-n')^2}{8D(n)\tau} \right] \quad \text{(using local Taylor expansions)}$$

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$\Longrightarrow$ characteristic correlation length $\sigma = \sqrt{4D(n)\tau}$

Use correlation length that is **logarithmically constant** in $n$ in this work

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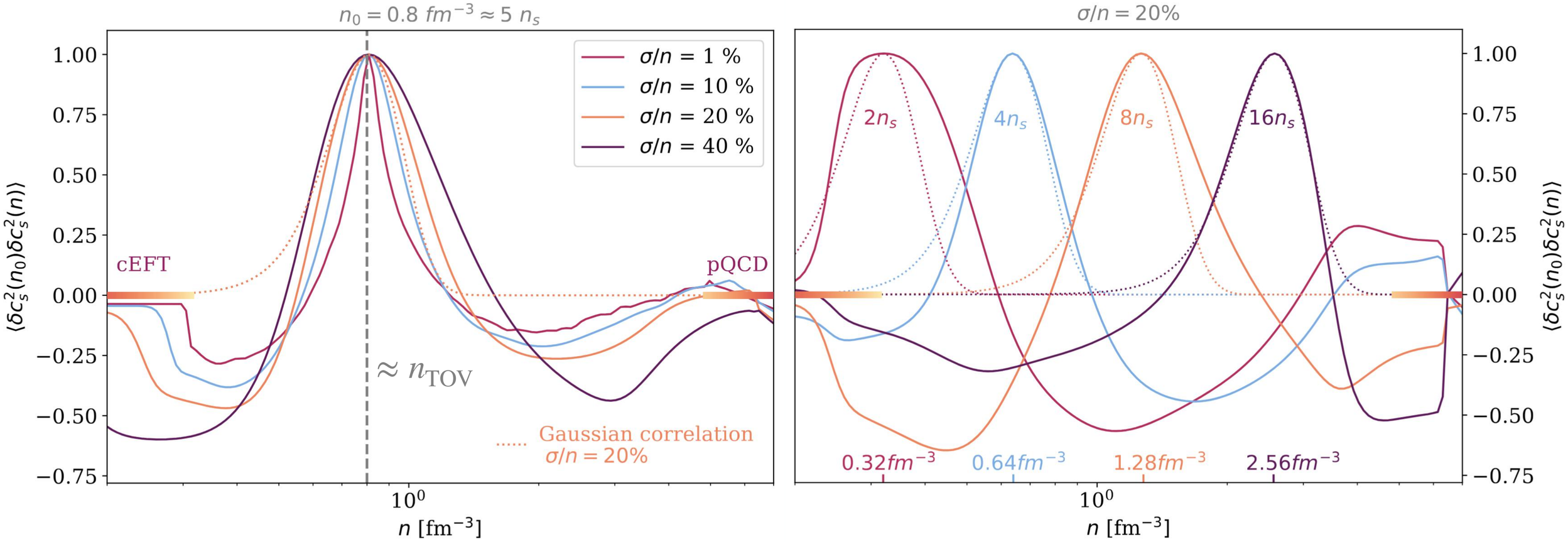
**Important**: Diffusion locally conserves the integral $\epsilon(n,\tau) = \int \mathrm{d}n \, \mu(n,\tau)$, so thermodynamic consistency preserved in the flow! (At least to % level for non-vanishing boundary $D(n)$ — well within errors of CEFT and pQCD )

# Global anticorrelation in speed of sound + local Gaussian behavior

TG, Komoltsev, Kurkela, Sunde 2512.XXXXX

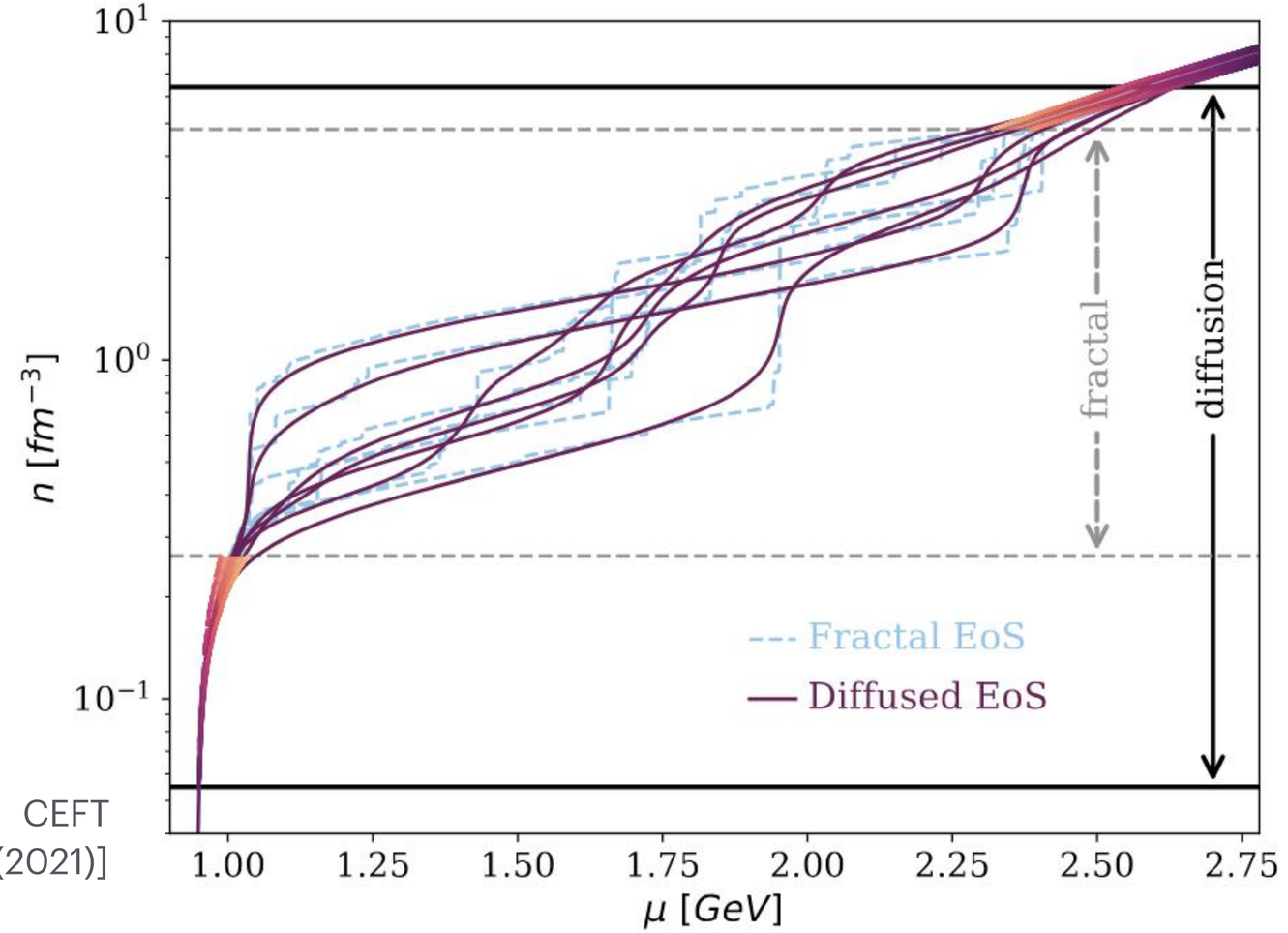



Consistent with previous works showing softening above $n_{\rm TOV}$

Komoltsev & Kurkela PRL 128 (2022); Komoltsev, Somasundaram, TG+ PRD 109 (2024); ...

# Diffuse over wider range of density, to correlate with theory

TG, Komoltsev, Kurkela, Sunde 2512.XXXXX

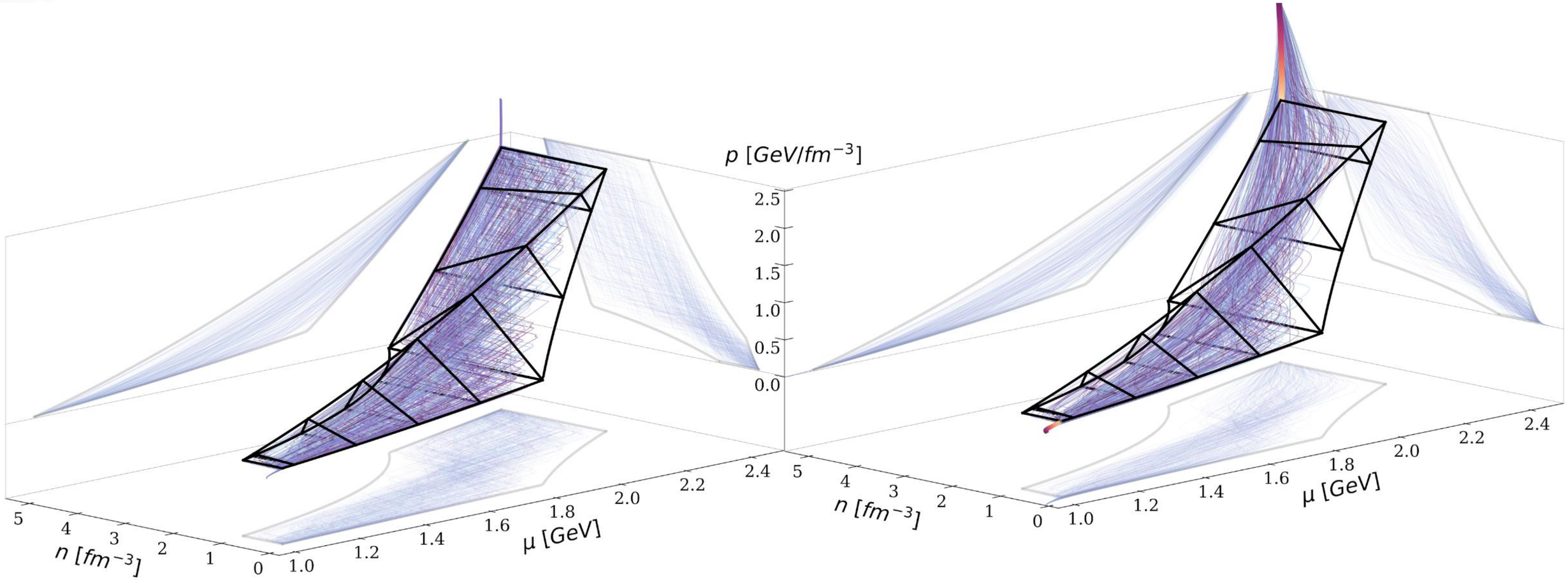



pQCD
[TG, Kurkela+ PRL 127 (2021)]

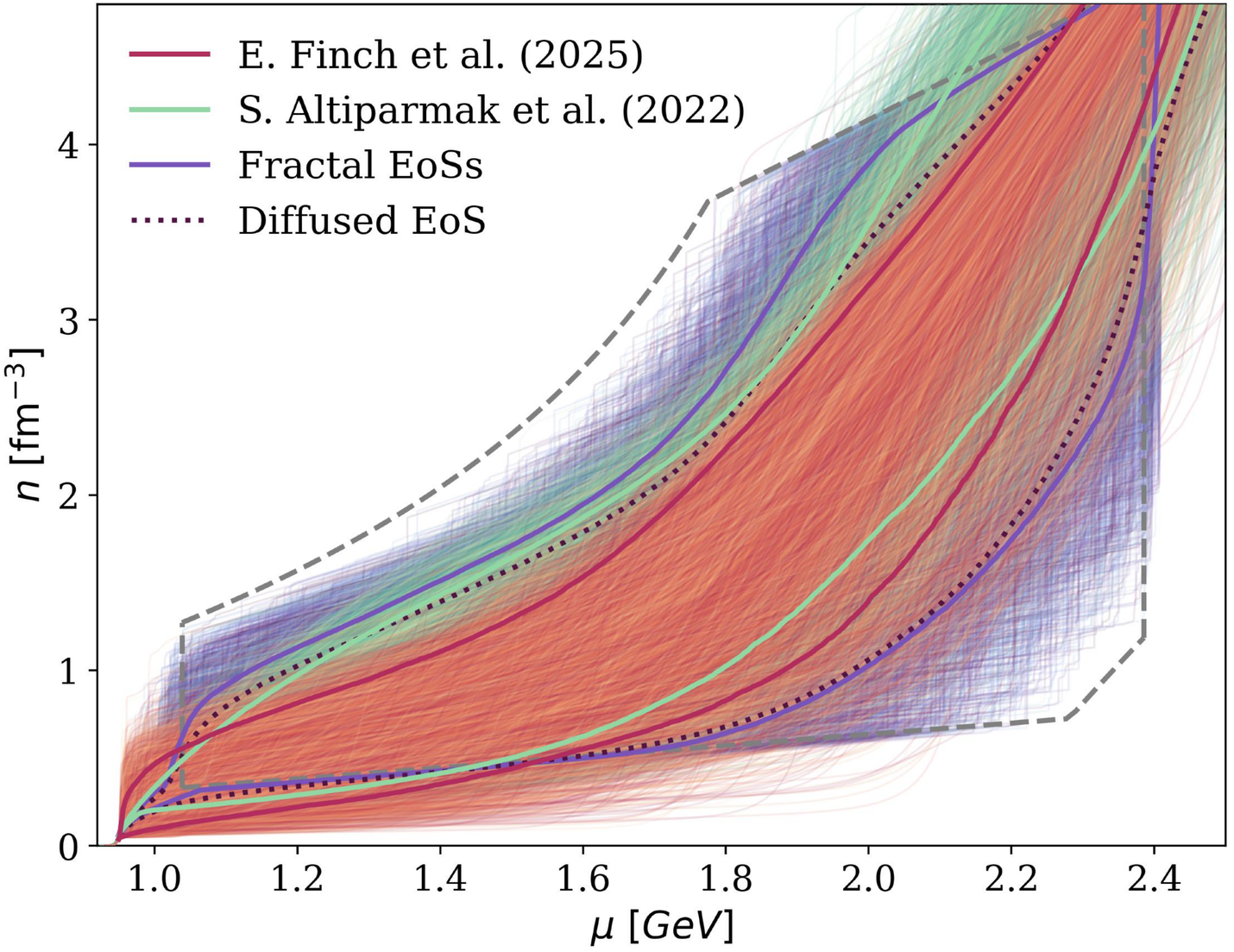
CEFT
[Drischler, Han, Lattimer PRC 103 (2021)]

# Diffused EoSs still fill much of the full allowed volume

TG, Komoltsev, Kurkela, Sunde 2512.XXXXX



# Brownian Bridge as/more general than other priors



# Perform EoS inference. In a nutshell:

$$P(\text{EoS} \mid \text{data}) = \frac{P(\text{EoS})P(\text{data} \mid \text{EoS})}{P(\text{data})}$$

Posterior

Prior

Likelihood

Prior: our new Brownian bridge

Likelihood: Astrophysics from high-mass pulsars, mass-radius measurements, and gravitational waves

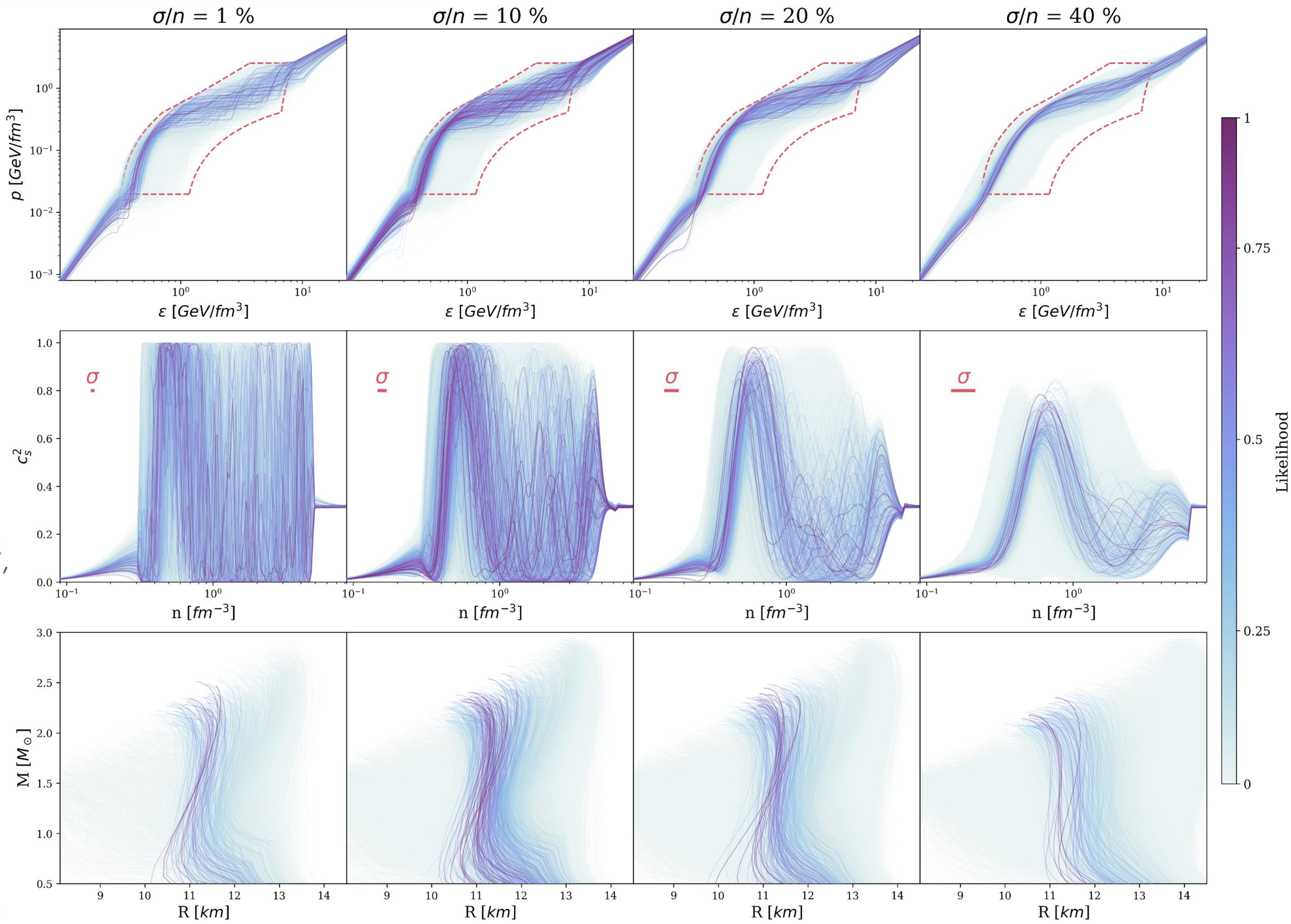
Posterior: EoSs most consistent with the data

# Posteriors

- Stiff at low densities + extended softening above $n_{\mathrm{TOV}}$

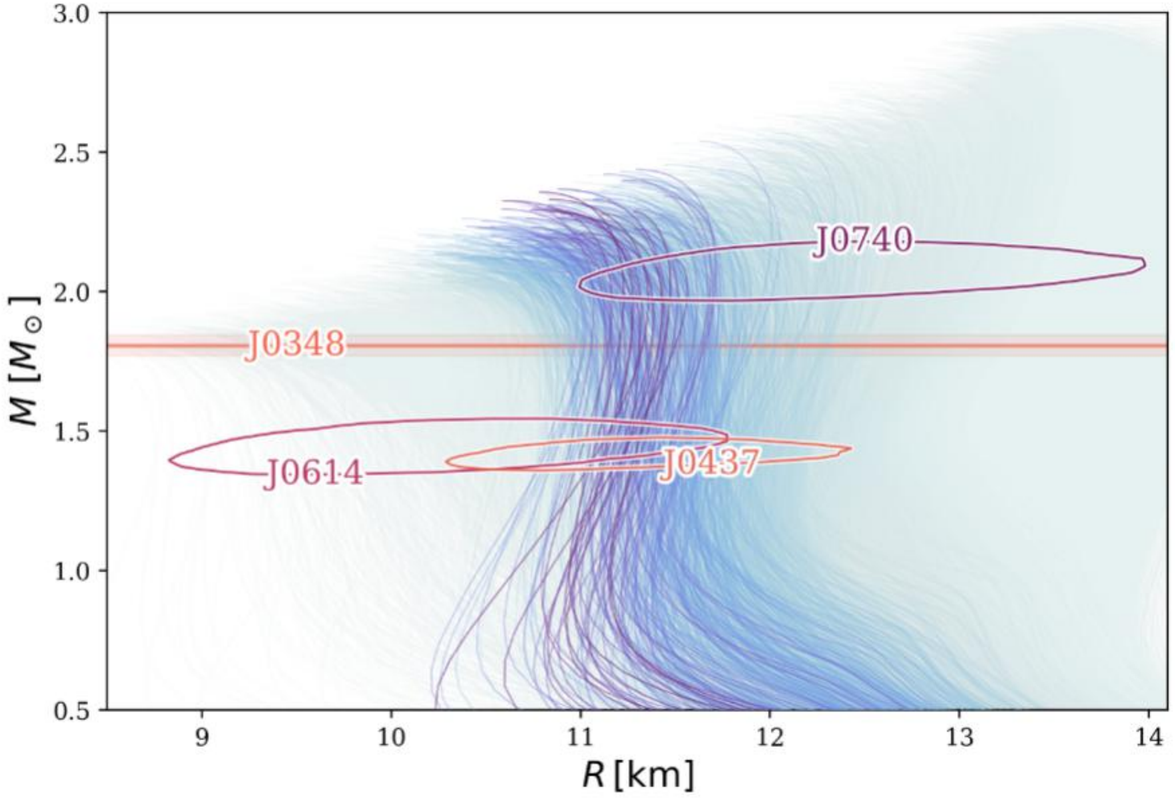
  Komoltsev+ PRD 109 (2024)

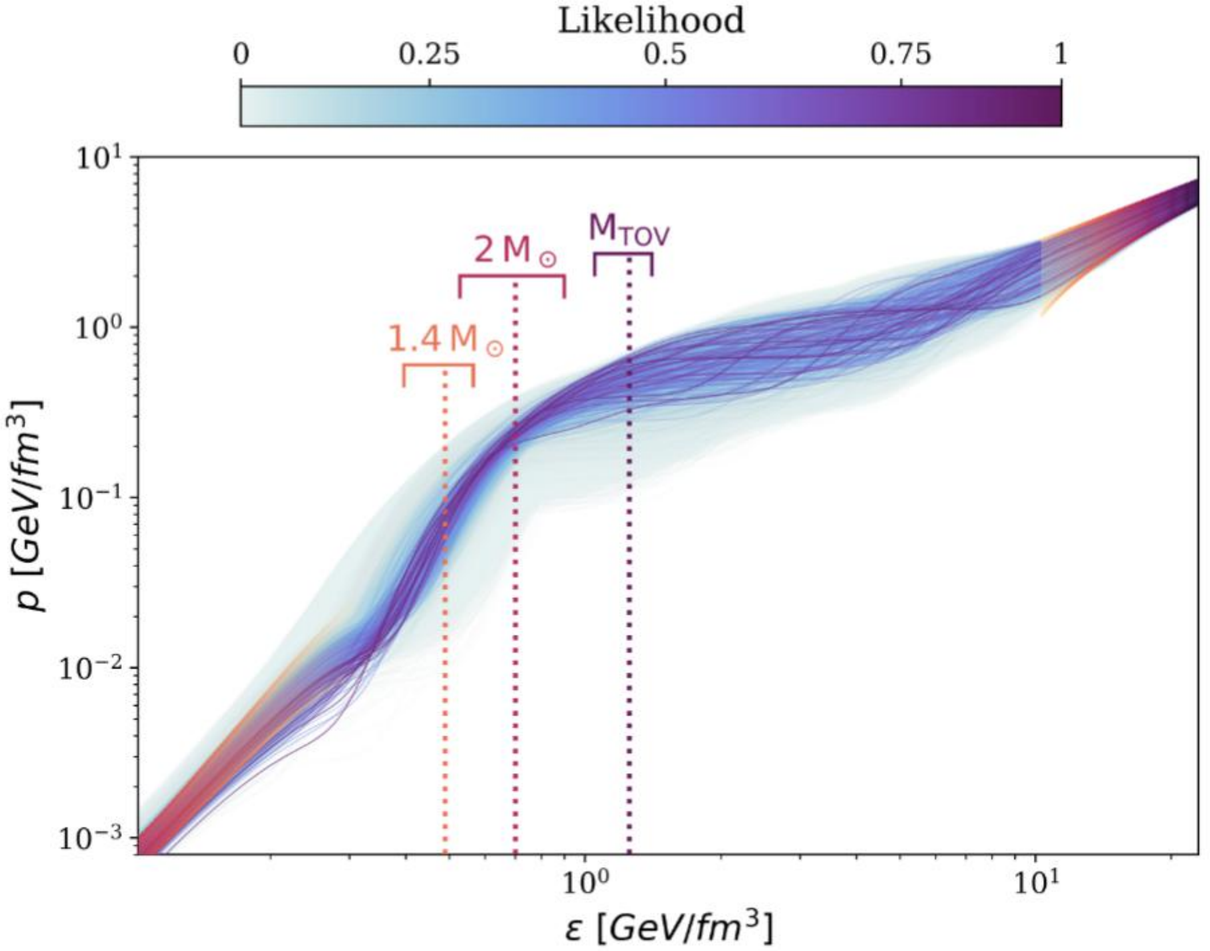
- Larger correlations reach less extreme $c_s^2$, more extreme radii



# Hierarchical ensemble shows stiff+soft behavior

TG, Komoltsev, Kurkela, Sunde 2512.XXXXX




Uniform $\sigma/n \in [0.2,0.4]$

Softening starts between $2M_\odot$ and $M_{\rm TOV}$ — Quark matter?

See Annala, TG, Kurkela+ Nat. Phys. 16 (2020) , Annala, TG+ Nat. Comm. 14 (2023)...

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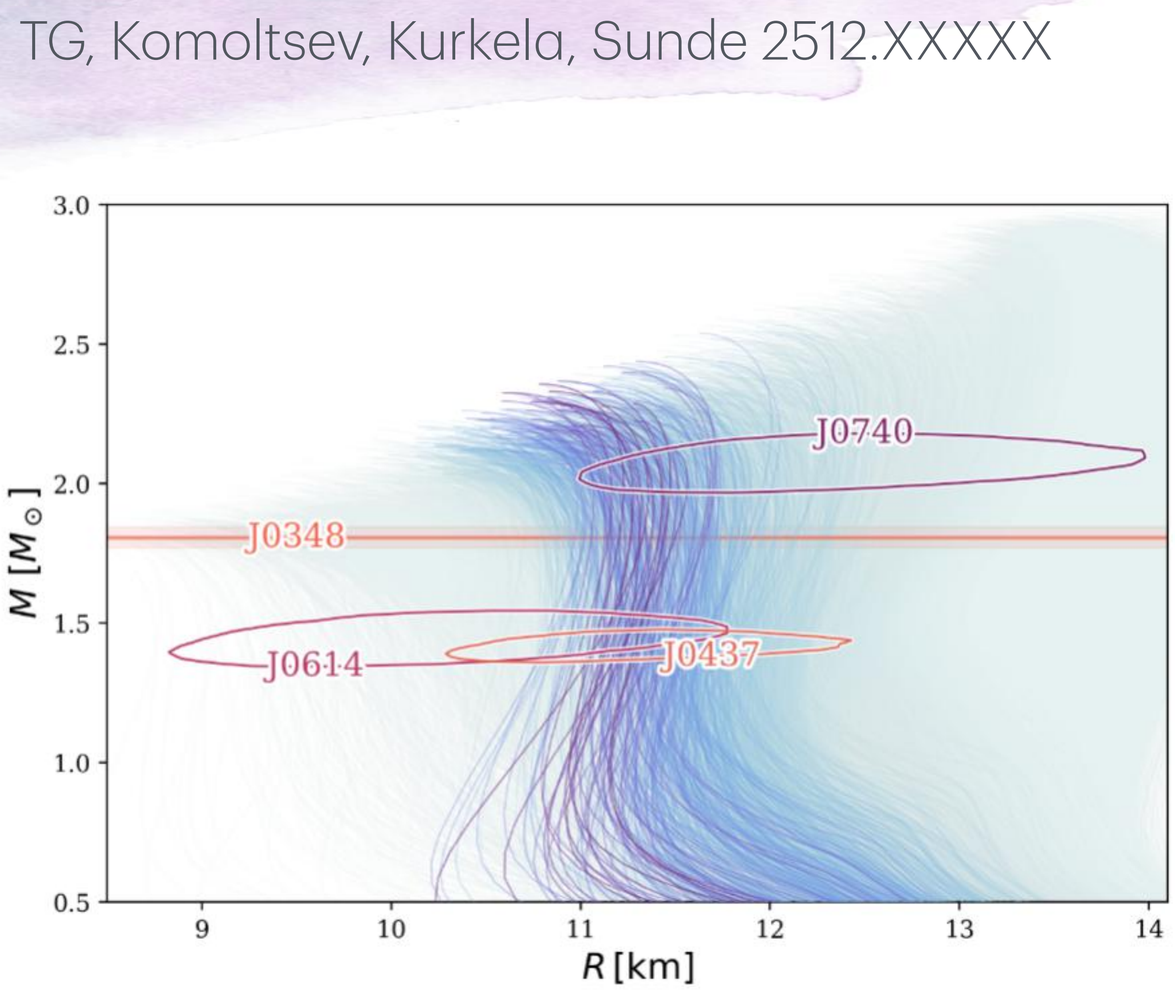


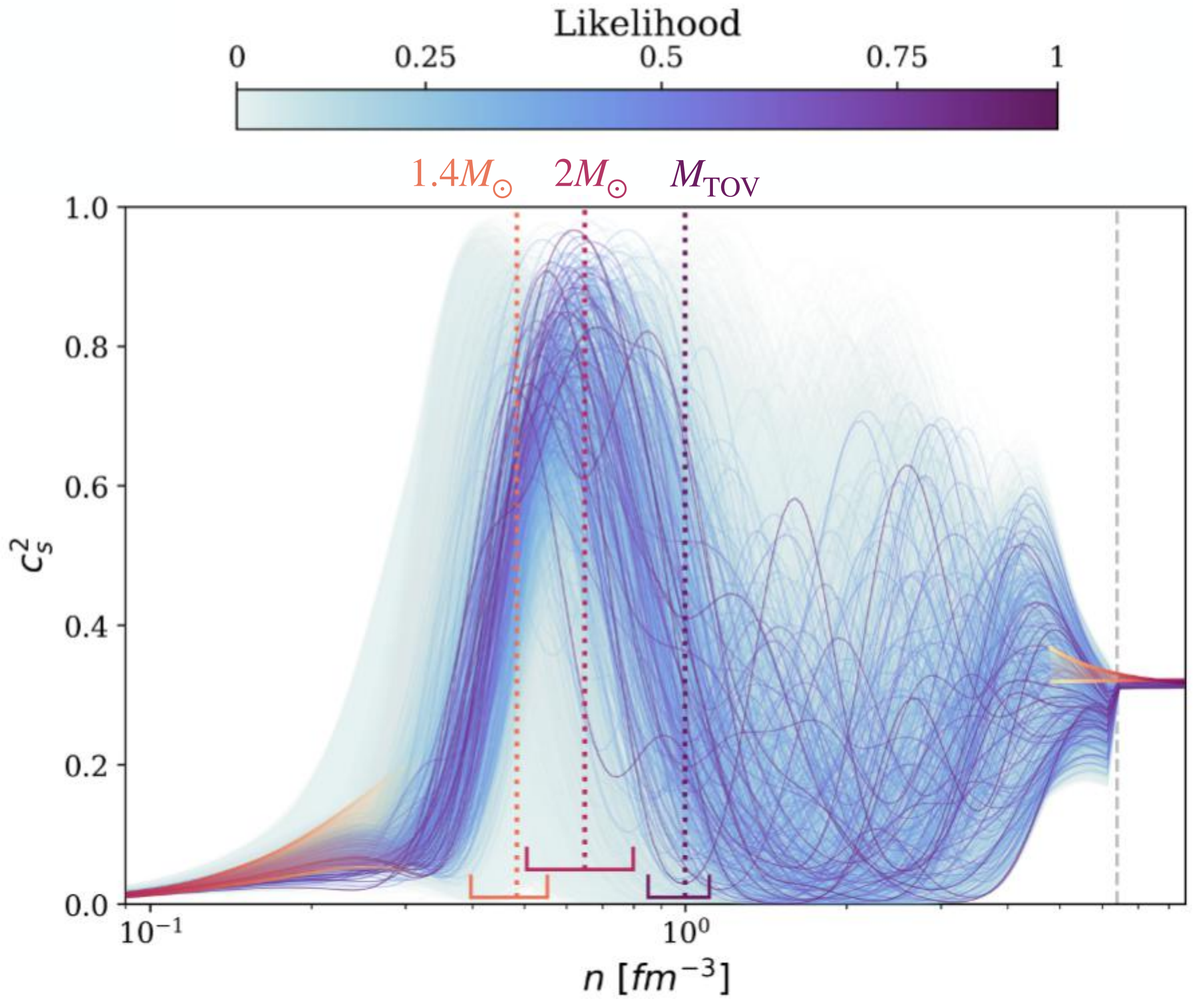


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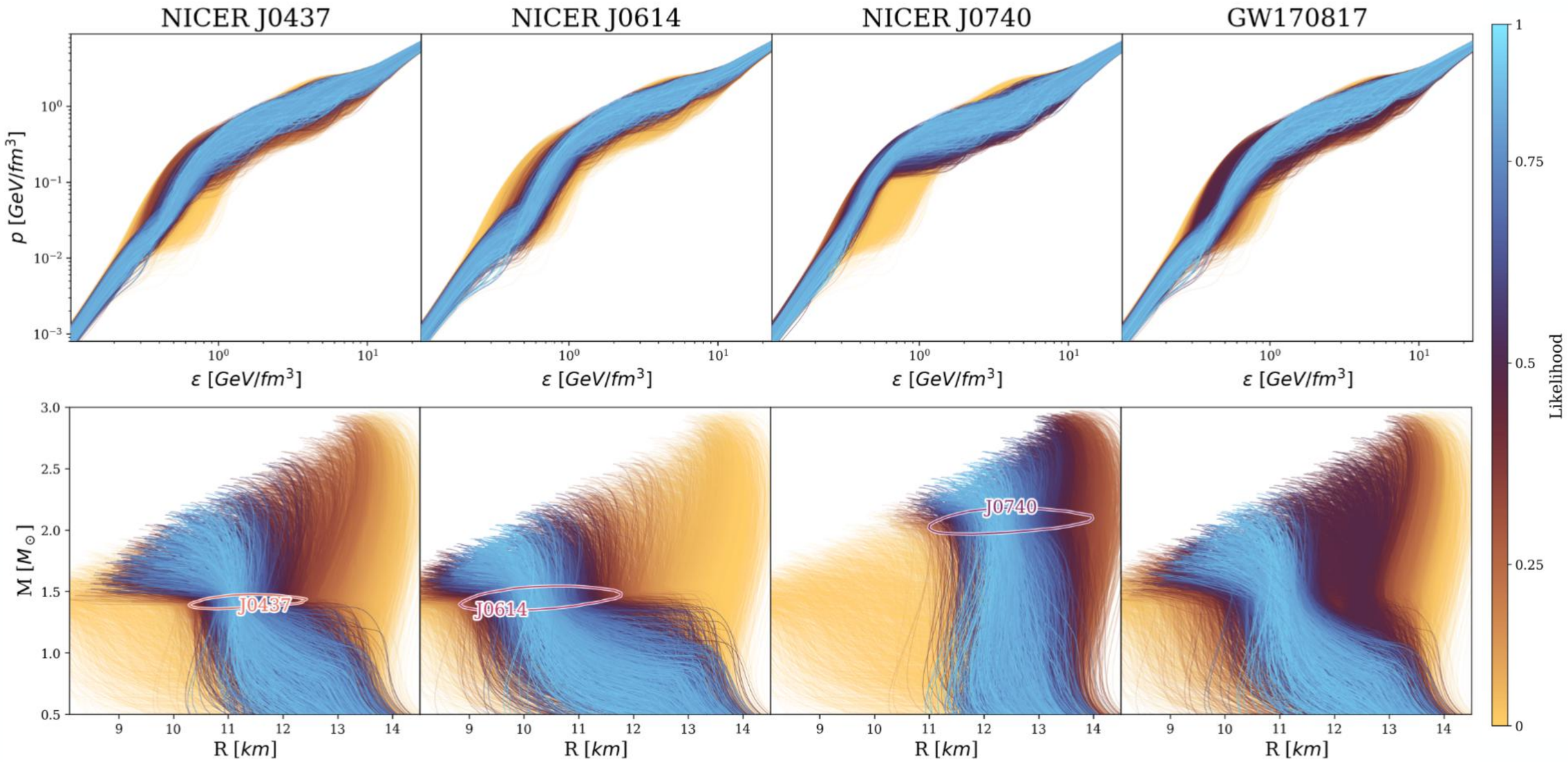
# Effects of different measurements



# Summary & Conclusions

- Presented **Brownian Bridge approach** to combine global thermodynamic constraints + minimal local assumptions (nonparametric) **efficiently**
- Add correlations by **diffusing EoS in auxiliary flow time — preserves thermodynamic consistency** within the errors of CEFT and pQCD. (**"Correct" way to add correlations**)
- Fills causal + thermodynamic volume well both before and after correlations are added; **as or more general than other priors in literature —** "exactly as general as minimal assumptions allow"
- Approach can handle arbitrary density dependent correlations
  (causal as long as $D''(n) - D'(n)/n \leq 0$; saturated for logarithmic correlation, stable as long as $D(n) \geq 0$)
- Continue to verify that stiff NS branch followed by prolonged softening beginning from masses in the range of $2M_\odot - M_{\rm TOV}$ stars is favored — **Quark matter?**
- Soon will appear on arXiv, and public code will be released

# Summary & Conclusions

- Presented **Brownian Bridge approach** to combine global thermodynamic constraints + minimal local assumptions (nonparametric) **efficiently**
- Add correlations by **diffusing EoS in auxiliary flow time — preserves thermodynamic consistency** within the errors of CEFT and pQCD. (**"Correct" way to add correlations**)
- Fills causal + thermodynamic volume well both before and after correlations are added; **as or more general than other priors in literature** — "exactly as general as minimal assumptions allow"
- Approach can handle arbitrary density dependent correlations
  (causal as long as $D''(n) - D'(n)/n \leq 0$; saturated for logarithmic correlation, stable as long as $D(n) \geq 0$)
- Continue to verify that stiff NS branch followed by prolonged softening beginning from masses in the range of $2M_\odot - M_{\rm TOV}$ stars is favored — **Quark matter?**
- Soon will appear on arXiv, and public code will be released

*Thanks for your attention!*

# Backup slides