CP Violation and Baryon Spin-Flavor Symmetry

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arXiv:2504.01105 (to appear in PRC)

arXiv:2508.xxxxx

Where is the anti-matter?

- Big bang cosmology determines the baryon asymmetry

$$\frac{n_B - n_{\bar{B}}}{n_\gamma} \sim O(10^{-10})$$

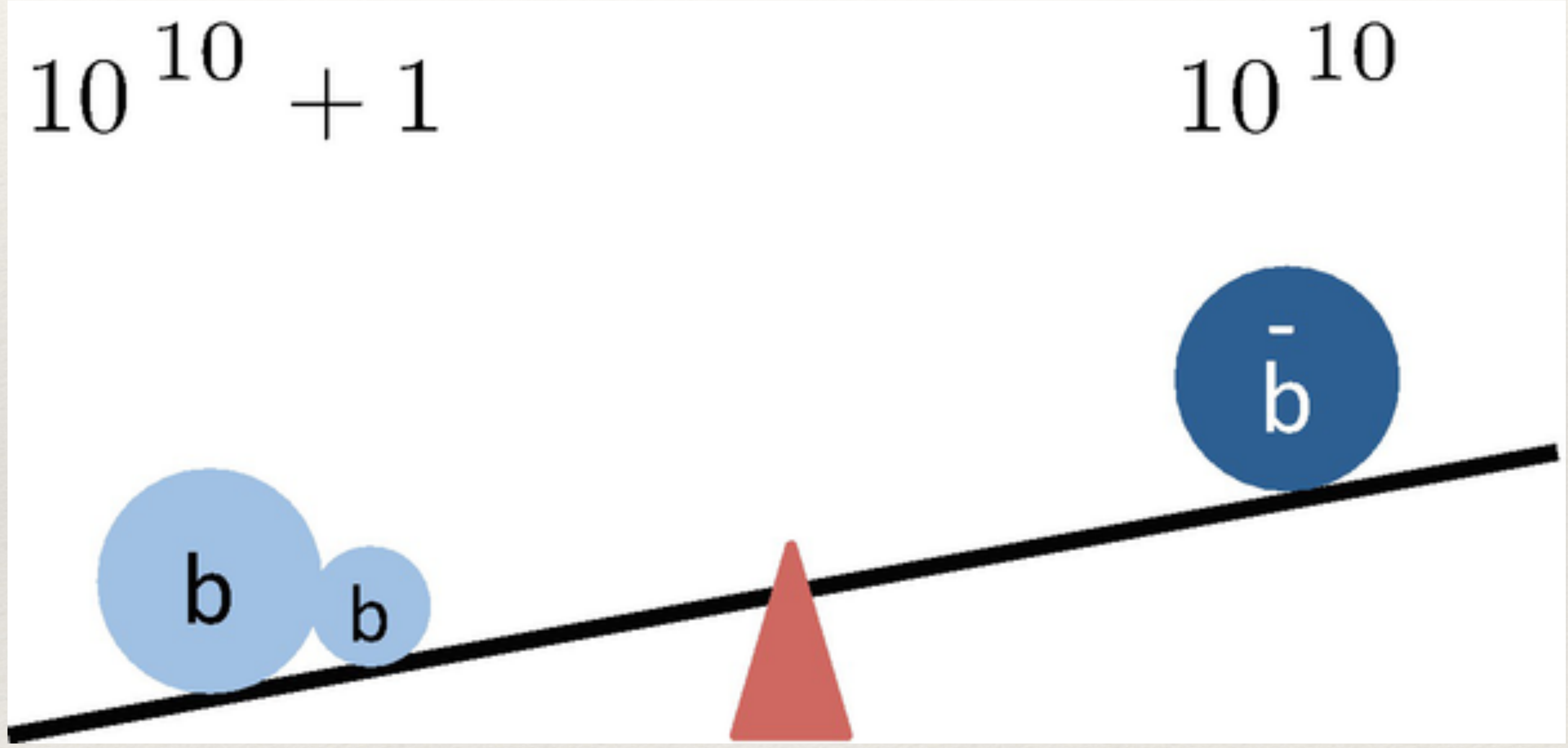


Image credit: K. Fuyuto

- Why???
- Three deal-breakers (Sakharov):
  1. Baryon number violation
  2. Out-of-equilibrium interactions
  3. $\not{C}$ and $\boxed{\not{CP}}$

CP Violation in the quark sector

- Many options—quark EDM, θ term, chromo-EDM, higher-dim operators…

$$\mathcal{L} = \bar{q}\,(i\not{D} - M)\,q - \frac{1}{4}G_{\mu\nu}G^{\mu\nu} + m_*(\bar{\theta} - \bar{\theta}_{\mathrm{ind}})\bar{q}i\gamma^5 q$$
$$+ r\bar{q}i\gamma^5\tilde{d}_{\mathrm{cE}}q - \frac{g}{2}\bar{q}\sigma^{\mu\nu}G_{\mu\nu}\left(\tilde{d}_{\mathrm{cM}} + \tilde{d}_{\mathrm{cE}}i\gamma^5\right)q$$

$$r = \frac{g}{2}\frac{\langle 0|\bar{q}\sigma_{\mu\nu}G^{\mu\nu}q|0\rangle}{\langle 0|\bar{q}q|0\rangle} \qquad m_* = \frac{m_u m_d}{m_u + m_d} \qquad \bar{\theta}_{\mathrm{ind}} = r\mathrm{Tr}(M^{-1}\tilde{d}_{\mathrm{cE}})$$

- Chromo-EDM is related to chromo-MDM by chiral symmetry

de Vries, Hockings, Mereghetti, Walker-Loud, Ottnad, Meißner, Guo, Borasoy, Pich, de Rafael, Crewther, Witten, di Vecchia, Veneziano…

$\not{C}\!\not{P}$ Pion-Nucleon Couplings from cMDM

$$\mathcal{L}_{\pi N} \supset -\frac{\bar{g}_0}{2F_0} N^\dagger \left(\pi^A \tau^A\right) N - \frac{\bar{g}_1}{2F_0} \pi^0 N^\dagger N$$

$$\boxed{\begin{aligned} \bar{g}_0 &= \frac{1}{2}\left(\tilde{d}_u + \tilde{d}_d\right)\left(\sigma_C^3 + \frac{r\sigma^3}{\bar{m}\varepsilon}\right) \\ \sigma^3 &= \frac{\bar{m}\epsilon}{M_N} \langle P| \bar{q}\tau^3 q |P\rangle \\ \sigma_C^3 &= \frac{1}{2M_N} \langle P| g_s \bar{q}\,(\sigma\cdot G)\,\tau^3 q |P\rangle \end{aligned}}$$

$$\boxed{\begin{aligned} \bar{g}_1 &= \left(\tilde{d}_d - \tilde{d}_u\right)\left(\sigma_C^0 - \frac{r\sigma^0}{\bar{m}}\right) \\ \sigma^0 &= \frac{\bar{m}}{2M_N} \langle P| \bar{q} q |P\rangle \\ \sigma_C^0 &= \frac{1}{4M_N} \langle P| g_s \bar{q}\,(\sigma\cdot G)\, q |P\rangle \end{aligned}}$$

Need to fix LECs: Lattice, sum rules, hadron structure, large-N_c

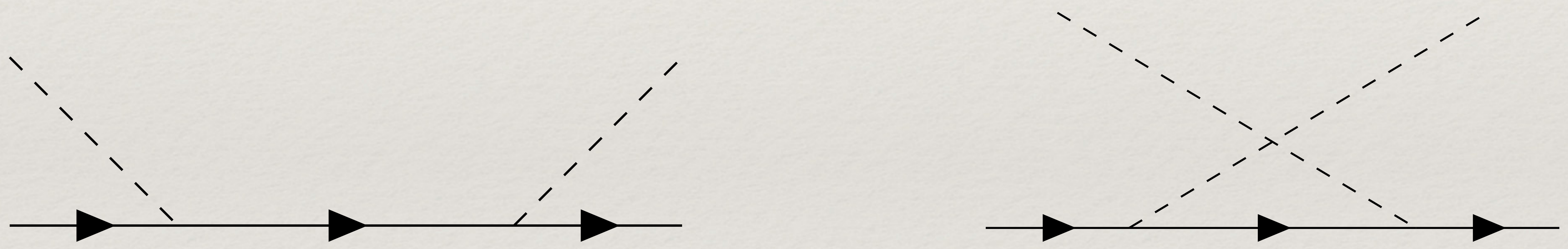
de Vries et al. PRC 92, PLB 766

Consistency Conditions

- Axial current matrix elements

$$\langle B' | \, \bar{q}\gamma^i \gamma^5 \tau^a q \, | B \rangle = \hat{g}_A N_c \, \langle B' | \, X^{ia} \, | B \rangle$$

- Baryon-meson scattering amplitude *should be* $O(N_c^0)$



$$\frac{\hat{g}_A^2}{F^2} N_c^2 \left[X^{ia}, X^{jb} \right] \Rightarrow \left[X^{ia}, X^{jb} \right] \lesssim O(1/N_c^2)$$

- Baryons transform under contracted $\mathrm{SU}(2N_F)$

Gervais and Sakita PRL 52, PRD 30; Dashen and Manohar PLB 315

Spin-Flavor Symmetry

- Infinite tower of degenerate baryons transforming under SU($2N_f$)

$$S^i = q^\dagger \frac{\sigma^i}{2} q \qquad I^a = q^\dagger \frac{\tau^a}{2} q \qquad G^{ia} = q^\dagger \frac{\sigma^i \tau^a}{4} q$$

$$\langle B' | \frac{\mathcal{O}^{(n)}}{N_c^n} | B \rangle \sim N_c^{-|I-S|}$$

Dashen PRD 49, 51;
Carone PLB 322;

Luty and March-Russell NPB 426

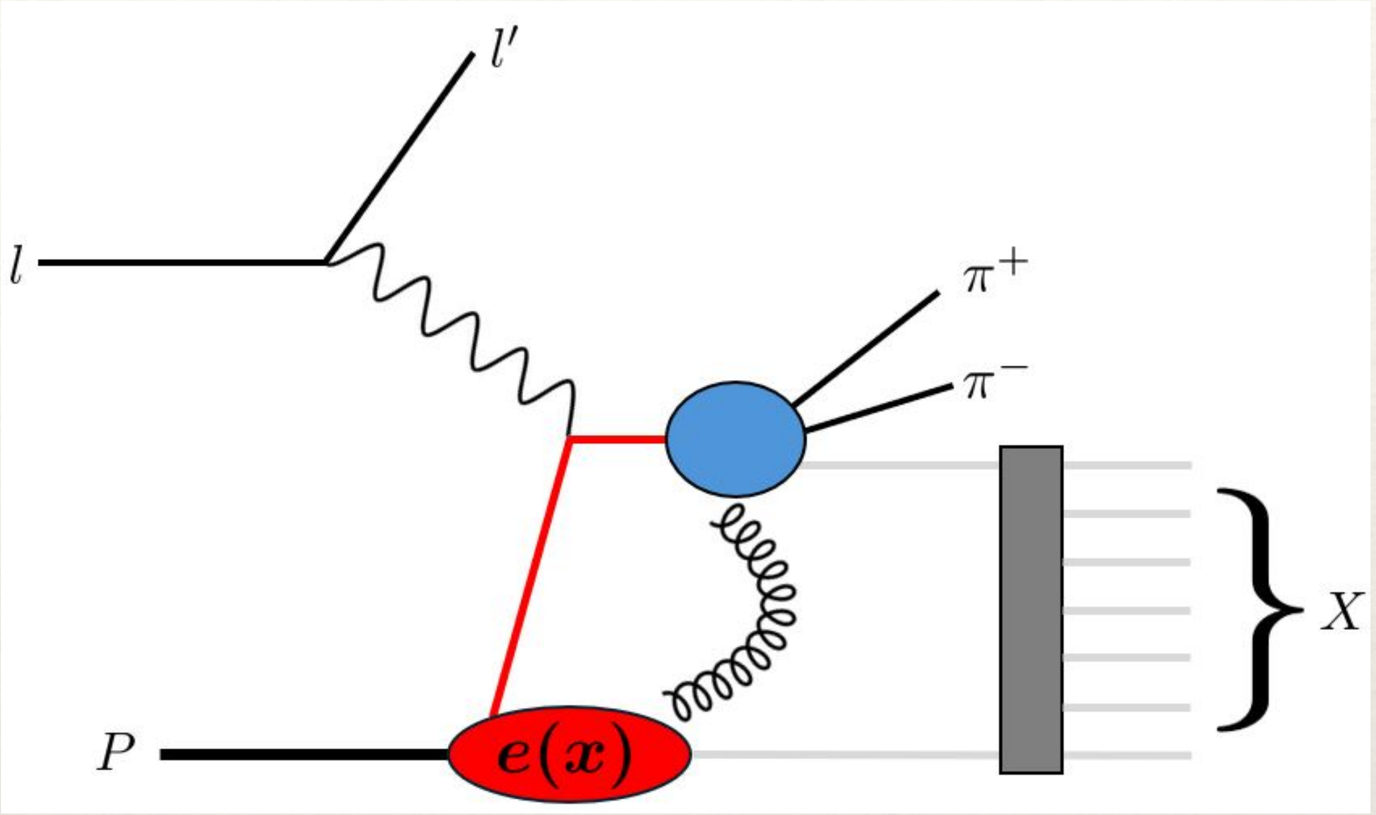
- Expand QCD operators and take baryon matrix elements

$$\mathcal{O}^{(m)}_{\mathrm{QCD}} = N_c^m \sum_{n,s,t} c_n \left(\frac{S^i}{N_c}\right)^s \left(\frac{I^a}{N_c}\right)^t \left(\frac{G^{ia}}{N_c}\right)^{n-s-t}$$

- Map scalings onto EFT operators: electroweak currents, neutrinoless double beta decay, dark matter direct detection

TRR et al. ARNPS 73, PRC 106, PRC 103, PRC 101

Twist-3 Chiral Odd PDF and the cMDM



- Nucleon matrix element of cMDM parameterized by 2 form factors

$$\langle P|\bar{\psi}^q(0)g_s G^{\alpha\mu}(0)\sigma_\alpha{}^\nu\psi^q(0)|P\rangle = A^q m_N\left(m_N^2 g^{\mu\nu} - P^\mu P^\nu\right) + B^q m_N P^\mu P^\nu$$

$$\sigma_C^0 = \frac{m_N^2}{4}\left[3A^{(u+d)} + B^{(u+d)}\right] \qquad \sigma_C^3 = -\frac{m_N^2}{2}\left[3A^{(u-d)} + B^{(u-d)}\right]$$

- Third (Mellin) moment of twist-3 distribution

$$\mathcal{M}_3\left[e^q_{\mathrm{tw3}}\right] = \frac{1}{4m_N p_+^2}\sum_{i=1}^{2}\langle P|\bar{\psi}_q(0)gG^{+i}(0)\sigma^{+i}\psi_q(0)|P\rangle$$
$$= \frac{1}{4}\left(A^q - B^q\right)$$

This is the central result of the Letter: the CMDM sigma terms $\sigma_C^{0,3}$ are related to the third moment of $e^{u,d}(x)$, barring the two unknown constants $B^{u,d}$ that vanish in the non-relativistic (NR) limit. The latter can be seen by working in the nucleon's rest frame [i.e., $P^\mu = (m_N, \vec{0})$] and realizing that in the NR limit the quark bilinear $\bar{q}\sigma_\alpha^\nu q$ is nonzero only when $\alpha, \nu \neq 0$. That is, the right side of Eq. (16) must be zero when $\mu = \nu = 0$, and this can be achieved only if $B_q = 0$. It is well known that the symmetry relations obtained from NR quark models are identical to those implied by spin-flavor symmetry, which is a direct consequence of the large N_c expansion [49]. Therefore, the terms B^q are subdominant and we shall neglect them in our numerical analysis henceforth. Under this assumption one

Seng PRL 122

Spin-flavor constraints for CMDM

Bhattacharya, Fuyuto, Mereghetti, TRR
arXiv:2504.01105

- Both isoscalar form factors are $O(N_c)$!

$$A^{(u+d)} m_N^3 g^{ij} = N_c t_A^{(u+d)} \left\{ \frac{G^{ia}}{N_c}, \frac{G^{ja}}{N_c} \right\}$$

$$A^{(u-d)} m_N^3 g^{ij} = N_c t_A^{(u-d)} \left\{ \frac{G^{i3}}{N_c}, \frac{J^j}{N_c} \right\}$$

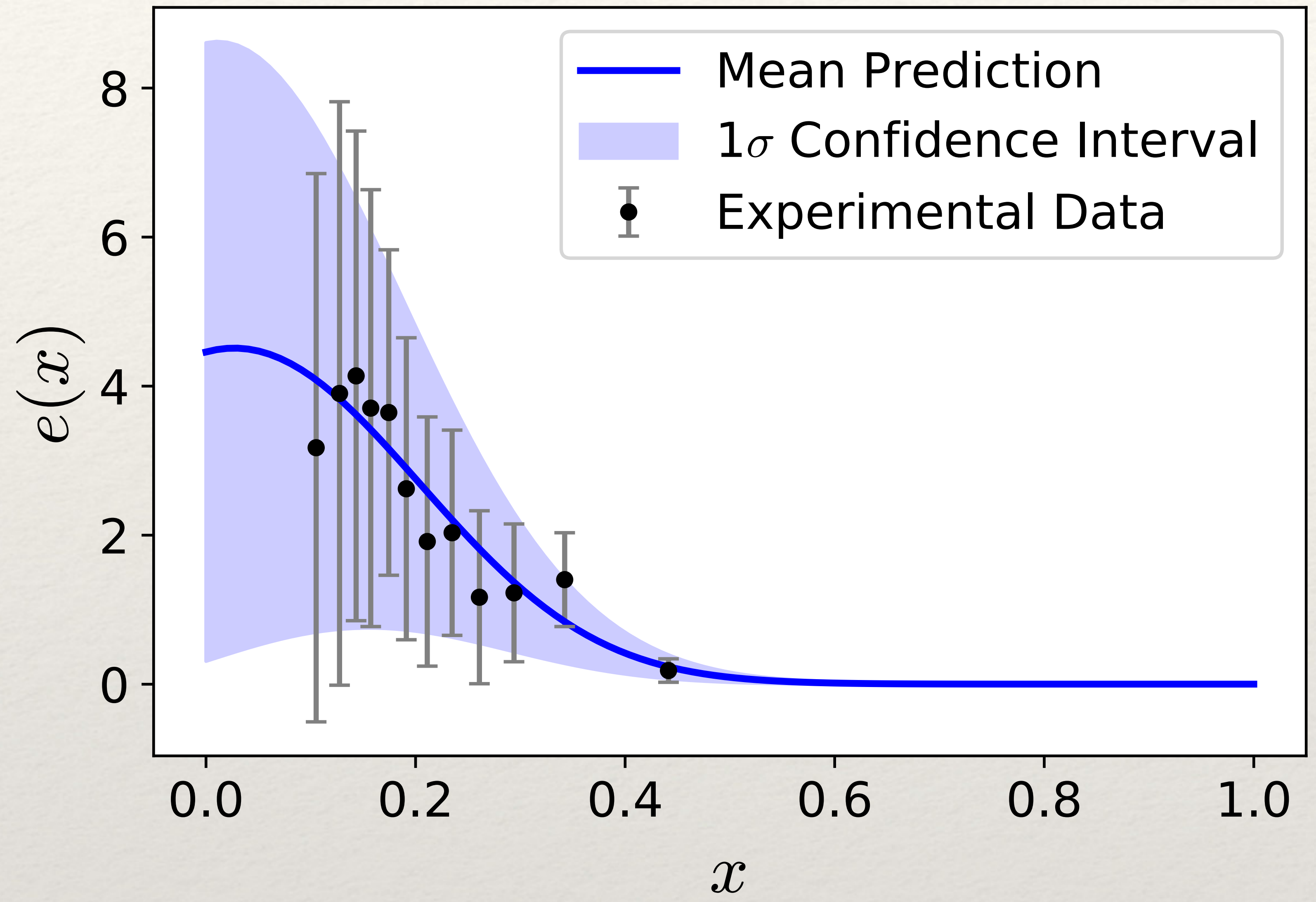
$$B^{(u+d)} m_N^3 = t_{B,0}^{(u+d)} N_c + t_{B,2}^{(u+d)} \frac{J^2}{N_c}$$

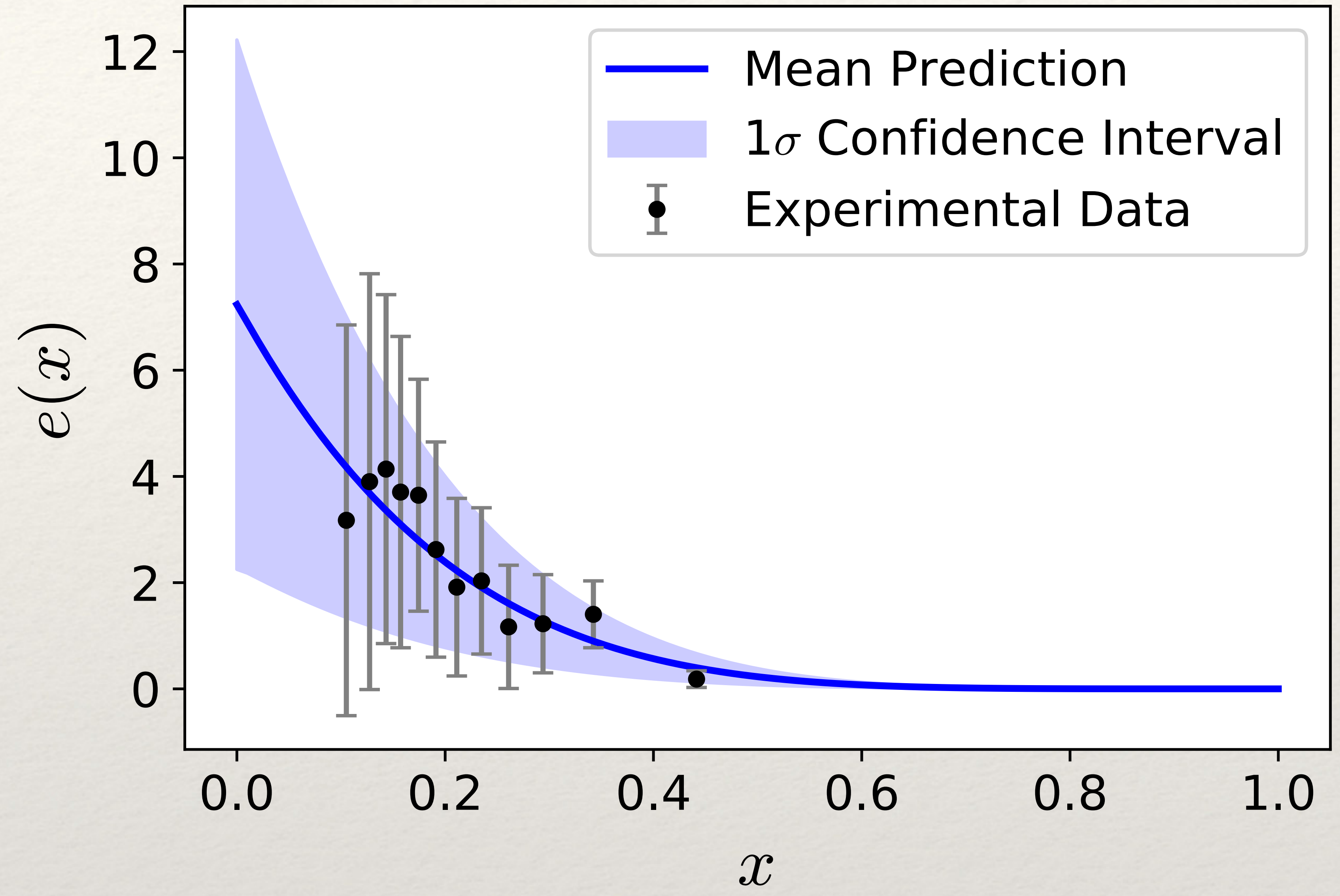
$$B^{(u-d)} m_N^3 = t_{B,1}^{(u-d)} I^3$$

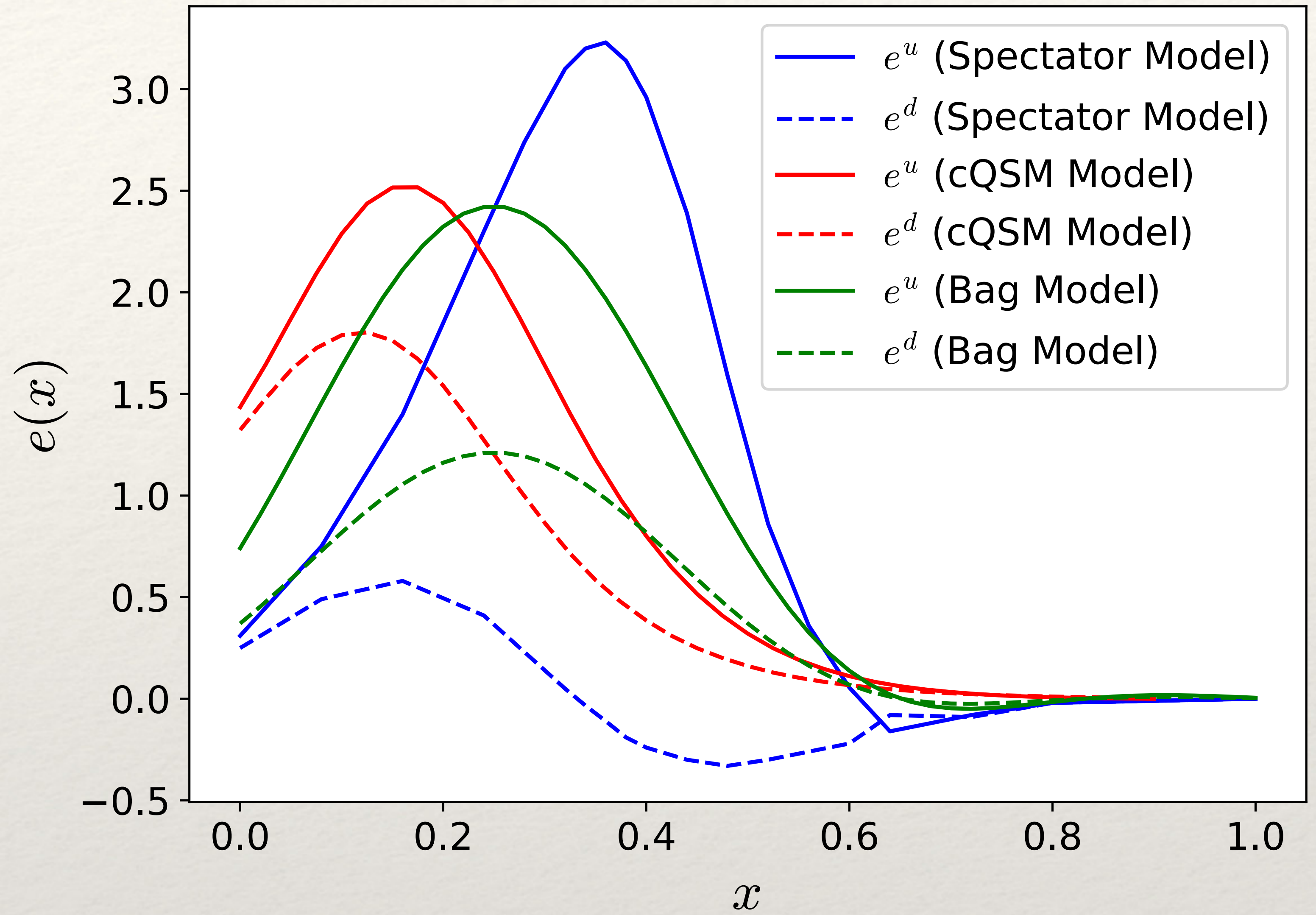
- Relative scaling of isoscalar and isovector PDFs in agreement with model calculations

$$\frac{\mathcal{M}_3[e^u] - \mathcal{M}_3[e^d]}{\mathcal{M}_3[e^u] + \mathcal{M}_3[e^d]} \sim O(1/N_c)$$

Extraction from CLAS12 and Models







$$\sigma_C^0 = \begin{cases} +4.4 \\ -3.6 \end{cases} \text{GeV}^2, \qquad \frac{r\sigma^0}{\bar{m}} = 5.9\ \text{GeV}^2$$

$$\sigma_C^3 = \begin{cases} +2.5 \\ -2.9 \end{cases} \text{GeV}^2, \qquad \frac{r\sigma^3}{\bar{m}\varepsilon} = 0.87\ \text{GeV}^2$$

$$\bar{g}_1 = -\left(\tilde{d}_u - \tilde{d}_d\right) \times \begin{cases} -1.5 \\ -9.6 \end{cases} \text{GeV}^2$$

$$\bar{g}_0 = \left(\tilde{d}_u + \tilde{d}_d\right) \times \begin{cases} +1.7 \\ -1.0 \end{cases} \text{GeV}^2$$

Bhattacharya, Fuyuto, Mereghetti, TRR
arXiv:2504.01105

Strong CP from Spin-Flavor Symmetry

- $\bar{\theta}$ arises from instantons in QCD vacuum and induces neutron EDM

$$d_n \vec{\sigma} \cdot \vec{E}$$

… but it's tiny

$$d_n^{(\mathrm{expt})} \leq 1.8 \times 10^{-13}\, e\,\mathrm{fm} \quad \Longrightarrow \quad |\bar{\theta}| \lesssim 10^{-10}$$

Abel et al. PRL 124

- Intimate connection to $U(1)_A$ anomaly

- Rich history of neutron EDM estimates

  $\rightarrow$ with large-N_c

  $\Longrightarrow$ but no spin-flavor symmetry!

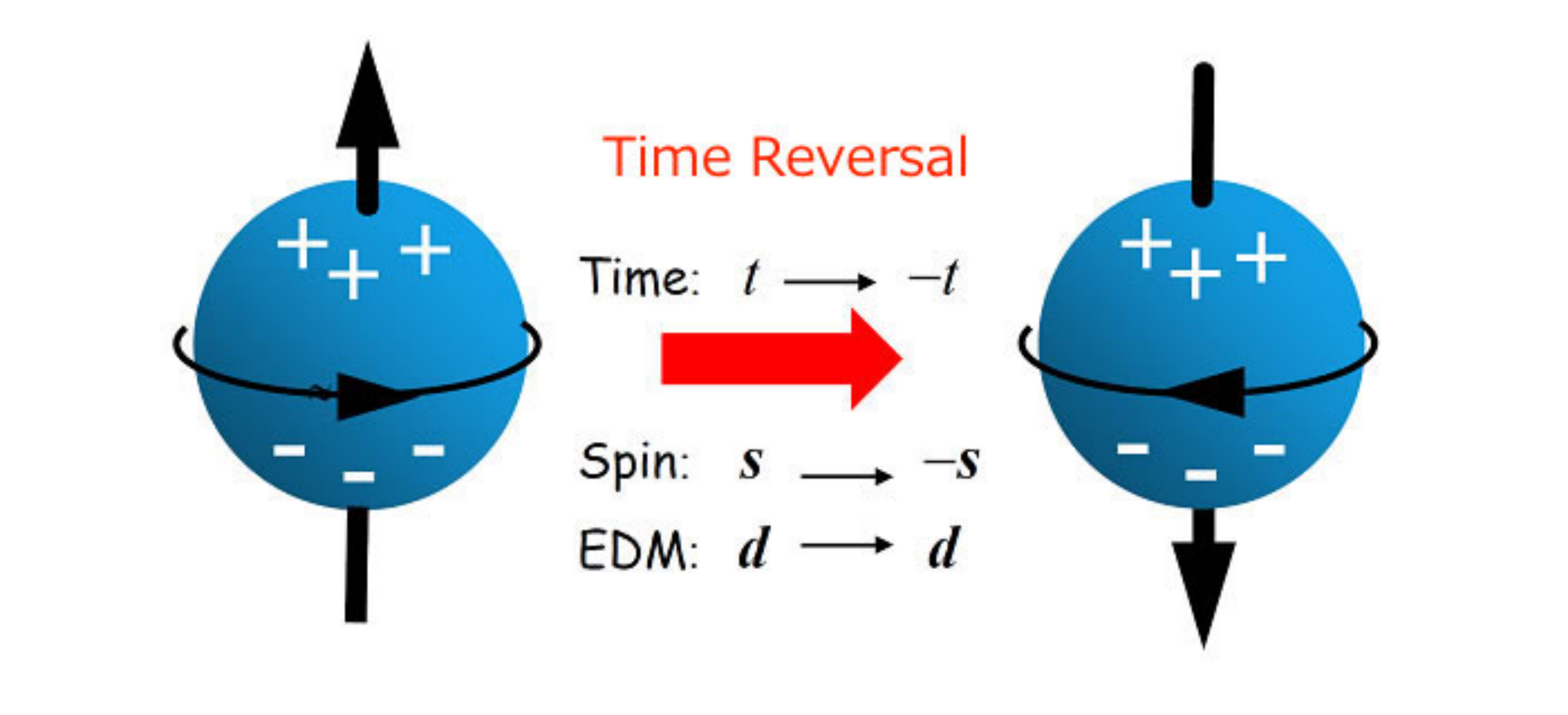


Image: Riken Nishina Center

Chiral Perturbation Theory at Large-N_c^*

- Axial anomaly is suppressed→additional Goldstone mode

$$\partial^\mu J^5_\mu = \frac{2N_f}{N_c}\left(\frac{g}{4\pi}\right)^2 \mathrm{Tr}(G\cdot\tilde{G}) \quad\Longrightarrow\quad U(N_f)_L\times U(N_f)_R \to U(N_f)_V$$

$$\begin{aligned}\mathcal{L}_\pi = {} & \frac{F_0^2}{4}\mathrm{Tr}\left(D_\mu U(D^\mu U)^\dagger\right) + \frac{1}{2}B_0F_0^2\mathrm{Tr}\left(\widetilde{M}U + U^\dagger\widetilde{M}\right)\\ & + \frac{F_0^2}{4}\frac{ia\tilde{\theta}}{N_c}\mathrm{Tr}\left[\left(U - U^\dagger\right) - \log U + \log U^\dagger\right]\\ & - \frac{F_0^2}{4}\frac{a}{N_c}\left[\frac{i}{2}\left(\log\det U - \log\det U^\dagger\right)\right]^2\end{aligned}$$

di Vecchia & Veneziano, Witten, Rosenzweig & Tahern

$^*N_c = 3 \gg 1$

Chiral Perturbation Theory at Large-N_c^*

Jenkins PRD 53

- Larger spin-flavor symmetry in baryon sector

$$\mathcal{L}_B = iD_0 + \frac{1}{2}\mathrm{Tr}\left(u_i\tau^a\right)A^{ia} + \cdots$$

$$A^{ia}_{B'B} = \langle B'|\bar{q}\gamma^i\gamma^5\frac{\tau^a}{2}|B\rangle$$

$$= \langle B'|a_1^{(v)}G^{ia} + \frac{a_2^{(v)}}{N_c}J^iI^a + \frac{a_3^{(v)}}{N_c^2}\left\{J^2, G^{ia}\right\}|B\rangle$$

- $\not{C}\not{P}$ baryon-pion couplings are proportional to SU(4) generators

$$\mathcal{L} = -\frac{4a\tilde{\theta}}{F_0 N_c}\phi_0\mathcal{H}^0 - \frac{4a\tilde{\theta}}{F_0 N_c}\phi_a\mathcal{H}^a$$

$$\frac{\bar{g}_0^{(\pi NN)}}{2F_0} = \frac{m_\pi^2}{F_0}c_5\bar{\theta}$$

$$\frac{\bar{g}_1^{(\pi NN)}}{2F_0} = \frac{F_0}{2\chi_{\mathrm{YM}}}(2B_0\bar{m})^2\varepsilon(1-\varepsilon^2)c_1\bar{\theta}$$

$^*N_c = 3 \gg 1$

Electric Dipole Moments

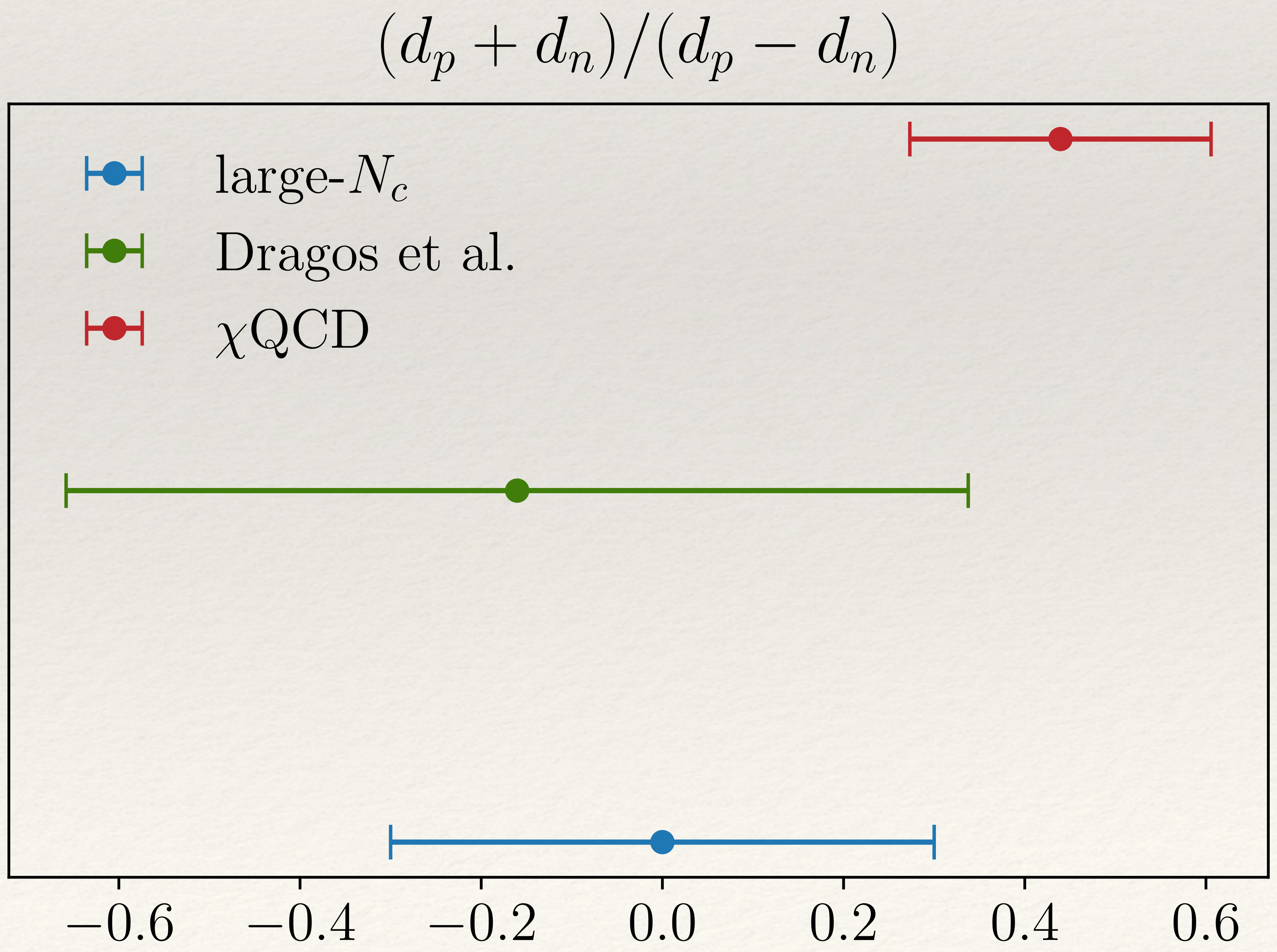
- Diagonal nucleon and Δ EDMs given by "universal" formula to one-loop

$$d_B = \left[d_1^{(v)} + \left(\frac{1}{4\pi F_0}\right)^2 \frac{8a\tilde{\theta}}{N_c} a_1^{(v)} h_1^{(v)} \log \frac{\mu^2}{m_\pi^2} \right] G^{i3}_{BB} + d_1^{(0)} J^i_{BB} + \frac{1}{N_c} d_2^{(v)} (J^i I^3)_{BB}$$

- Conventional wisdom: tree ~ one-loop
- Large-N_c wisdom: loops are $1/N_c^l$ suppressed
- Isospin relations can be tested on the lattice

$$d_{\Delta^{++}} = \frac{9}{10}(d_p - d_n) + \frac{3}{2}(d_p + d_n)$$

$$d_{\Delta^{++}} - d_{\Delta^-} = \frac{9}{5}(d_p - d_n)$$



Electric Dipole Moments

- $\Delta \to N$ transition moments are easier to handle in this framework

$$d_{\Delta N} = a_1^{(v)} h_1^{(v)} G_{N\Delta}^{i3} \frac{8a\tilde{\theta}}{N_c} \frac{1}{(4\pi F_0)^2} \left[\log\left(\frac{\mu^2}{m_\pi^2}\right) - \frac{2\Delta^2}{m_\pi^2} - \frac{\Delta}{\sqrt{\Delta^2 - m_\pi^2}} \log\left(\frac{2(\Delta^2 - \Delta\sqrt{\Delta^2 - m_\pi^2}) - m_\pi^2}{m_\pi^2} \right) \right]$$

- Identical to diagonal moments when $\Delta = M_\Delta - M_N \to 0$

- More isospin relations

$$d_{\Delta^+ p} = d_{\Delta^0 n}$$

$$d_{\Delta^+ p} + d_{\Delta^0 n} = \frac{4\sqrt{2}}{5}(d_p - d_n) + O(1/N_c^2)$$

Summary

- The large-N_c limit leads to a spin-flavor symmetry that can constrain baryon matrix elements
- Combined effort with hadron structure can provide new constraints for cMDM/cEDM
- New relations between N and Δ EDMs from $\bar{\theta}$—could be tested on lattice
- Next: role of Δ in $\cancel{CP}$ nuclear forces, higher-dimensional sources of CP violation