

How Weak Interactions Suppress g-modes in Hot Neutron Stars (NS)

arXiv:2504.12230

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Aug 6, 2025



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Oscillations of NS

Fluid perturbations

- Radial oscillation ($l=0$): $\varepsilon^r = R_n^r(r)e^{i\omega t}$
don't couple to gravitational waves

$$\omega = 2\pi\nu + \frac{i}{\tau}$$

- Non-radial oscillation ($l \geq 2$):

$$\xi_{\text{even}}^{r, \theta, \phi} = \nabla (R_n(r) Y_m^l(\theta, \phi) e^{i\omega t})$$

$$\xi_{\text{odd}}^{\theta, \phi} = \hat{r} \times \nabla (R_n(r) Y_m^l(\theta, \phi) e^{i\omega t})$$

f-mode (fundamental $n=0$) (even),

p-modes (pressure $n=1, 2, \dots$) (even)

g-modes (gravity $n=1, 2, \dots$) (even)

r-modes (rotation $m=\pm 1, \pm 2, \dots$) (odd)

- Spacetime perturbations:**

Family I w-modes (even)

Family II w-modes (odd)

important for BBH ring-down

even-parity
(polar mode)

odd-parity

(axial modes)

	ν (kHz)	τ (s)
f-mode	1.3-2.8	0.1-1
g-mode	<0.8	>100
p-mode	>2.7	1-1000
r-mode	~ spin	<0
w-mode	~10	~1E-5

$$h_{\mu\nu}^{\text{even}} = \begin{pmatrix} H_0 & H_1 & 0 & 0 \\ H_1 & H_2 & 0 & 0 \\ 0 & 0 & r^2 K & 0 \\ 0 & 0 & 0 & r^2 \sin^2 \theta K \end{pmatrix} Y_{lm}(\theta, \phi)$$

$$h_{\mu\nu}^{\text{odd}} = \begin{pmatrix} 0 & 0 & \frac{h_0 \partial_\phi}{\sin \theta} & -h_0 \sin \theta \partial_\theta \\ 0 & 0 & \frac{h_1 \partial_\phi}{\sin \theta} & -h_1 \sin \theta \partial_\theta \\ \dots & \dots & 0 & 0 \\ \dots & \dots & 0 & 0 \end{pmatrix} Y_{lm}(\theta, \phi)$$

gravity mode (g-mode)

Adiabatic (de)compression:

$$\varepsilon - \delta\varepsilon, p - \delta p$$

$$\varepsilon, p$$

Ambient environment:

$$\varepsilon - d\varepsilon, p - dp$$

$$\varepsilon, p$$

Gravity field $\vec{g}(r)$



- Mechanical equilibrium: $p - \delta p = p - dp$
- Gravity as recovering: $\varepsilon - \delta\varepsilon > \varepsilon - d\varepsilon$
- Non-convecting(stable): $c_{ad}^2 = \frac{\delta p}{\delta\varepsilon} > \frac{dp}{d\varepsilon} = c_{eq}^2$

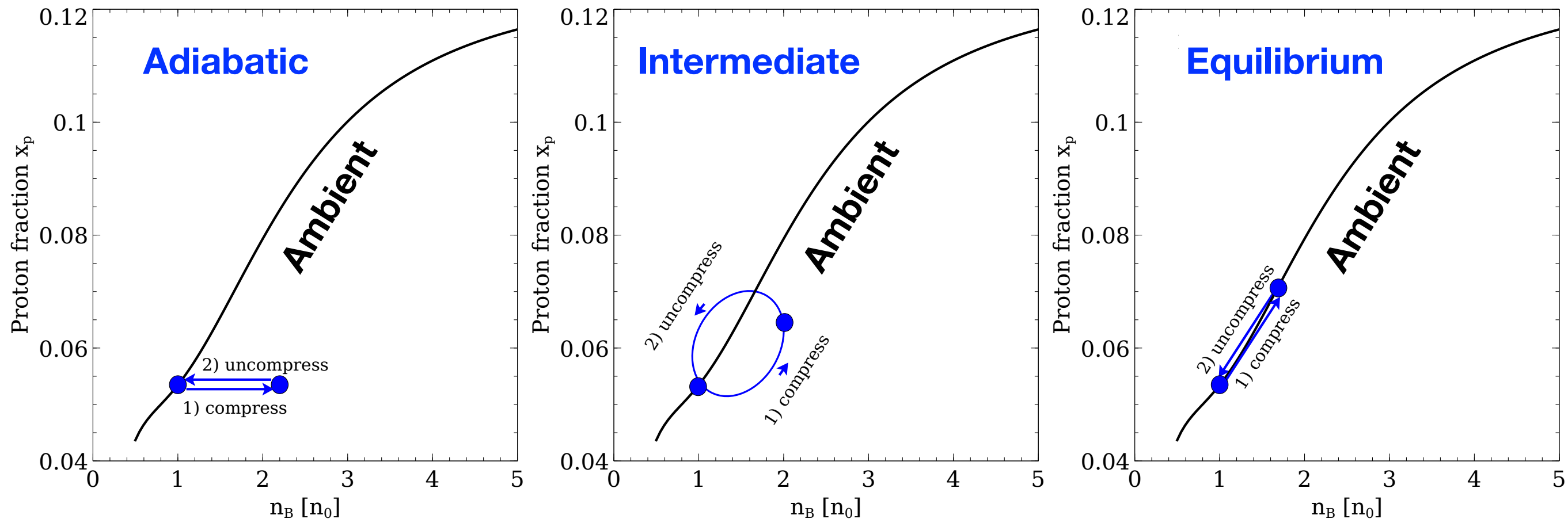
$$c_{ad}^2 = \left[\frac{\partial p}{\partial n_B} \right]_{\{x\}} \left[\frac{\partial \varepsilon}{\partial n_B} \right]_{\{x\}}^{-1}$$

$$c_{eq}^2 = \frac{dp}{dr} \left[\frac{d\varepsilon}{dr} \right]^{-1}$$

fix composition & entropy, e . g . $\{x\} = \left\{ \frac{n_p}{n_B}, \frac{n_n}{n_B}, S, \dots \right\}$

Dissipation of npe matter

- Path of a fluid element as it is compressed and uncompressed.



- Path in $\{x_p, n_B\}$ plane maps to path in $\{P, V\}$ plane.

Traversing the path leads to dissipative work, $\oint P \cdot dV$

Dissipation from β -reactions

- Nucleon weak β -equilibrium at low neutrino opacity,

$$(N+) \quad n \rightarrow p + e^- + \bar{\nu}$$

$$(N+) \quad p + e^- \rightarrow n + \nu$$

leads to $\delta_\mu = \mu_n - \mu_p - \mu_e \rightarrow 0$, and $x = \frac{n_p}{n_B} \rightarrow x_{eq}$.

- off β -equilibrium, when $\delta_x = x - x_{eq}$,

$$\partial_t \delta_x = \frac{1}{n_B} (\Gamma_{n \rightarrow p} - \Gamma_{p \rightarrow n})$$

$$\Gamma_{n \rightarrow p}^{\text{dUrca}} \sim \int \left(\prod_{n,p,e,\bar{\nu}} \frac{d^3 p_i}{2E_i} \right) \delta^4 \left(\sum_{n,p,e,\bar{\nu}} p_i \right) \sum_{\text{spins}} |\mathcal{M}|^2 f_n (1 - f_p) (1 - f_e) \propto T^5$$

$$\Gamma_{p \rightarrow n}^{\text{dUrca}} \sim \int \left(\prod_{n,p,e,\nu} \frac{d^3 p_i}{2E_i} \right) \delta^4 \left(\sum_{n,p,e,\nu} p_i \right) \sum_{\text{spins}} |\mathcal{M}|^2 (1 - f_n) f_p f_e \propto T^5$$

$\Gamma_{n \rightarrow p}^{\text{mUrca}}$ and $\Gamma_{p \rightarrow n}^{\text{mUrca}}$ have additional factors of $d^3 p_N^{\text{in}} d^3 p_N^{\text{out}} f_N (1 - f_N) \propto T^2$

- Treat dUrca+mUrca jointly: Nucleon Width Appro. (Alford, Haber, & Zhang 2406.13717)

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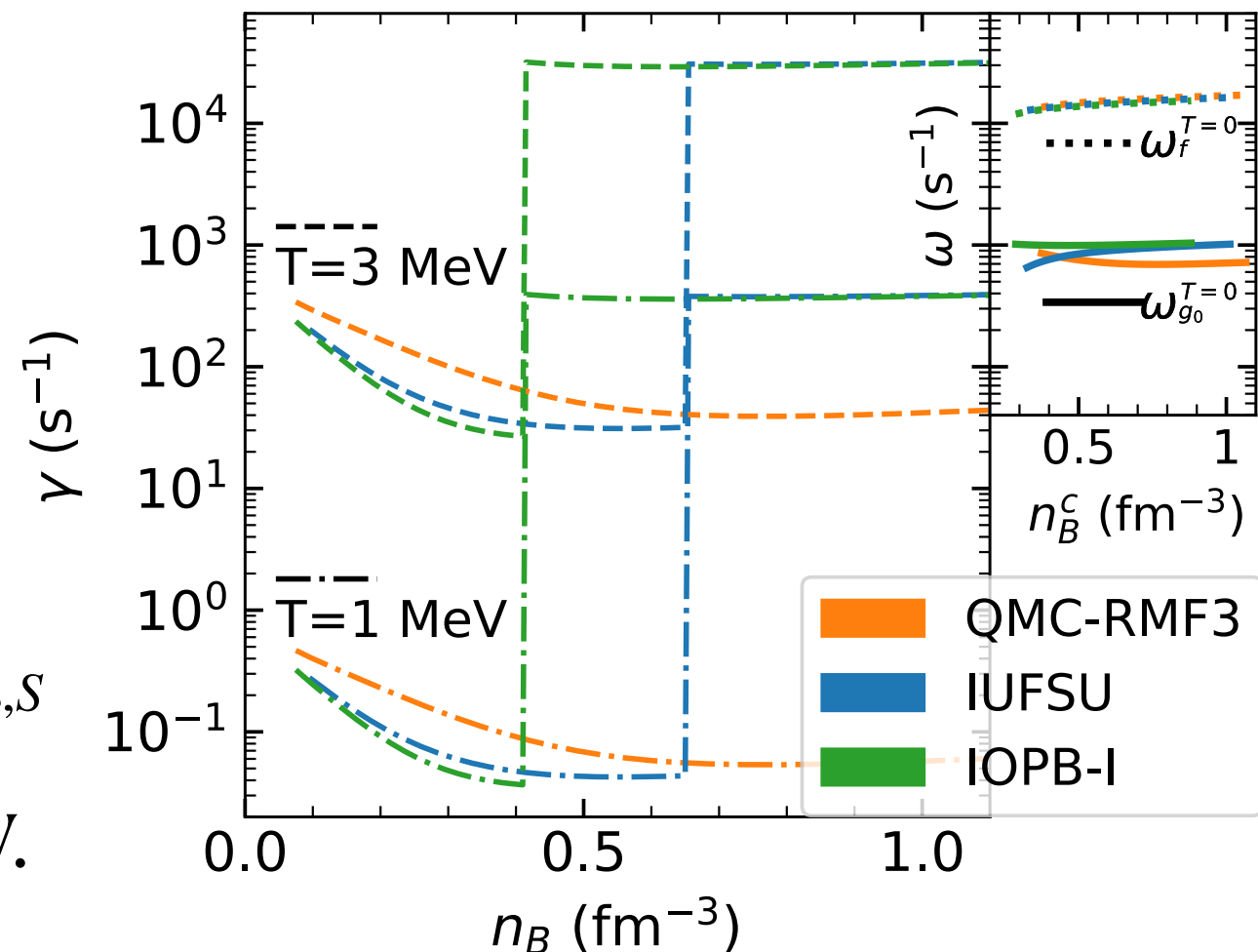
$$\partial_t \delta_x = \frac{1}{n_B} (\Gamma_{n \rightarrow p} - \Gamma_{p \rightarrow n}) \approx -\gamma \delta_x,$$

(linear order)

where γ is the weak relaxation rate,

$$\gamma = - \frac{1}{n_B} \frac{\partial(\Gamma_{n \rightarrow p} - \Gamma_{p \rightarrow n})}{\partial \delta_\mu} \bigg|_{n_{B,S}} \frac{\partial \delta_\mu}{\partial \delta_x} \bigg|_{n_{B,S}}$$

- γ reaches kHz frequency at $T \sim 3$ MeV.



Dynamic sound speed

$$c_{dy}^2 = c_{eq}^2 + \frac{c_{ad}^2 - c_{eq}^2}{1 - i\frac{\gamma}{\omega}}$$

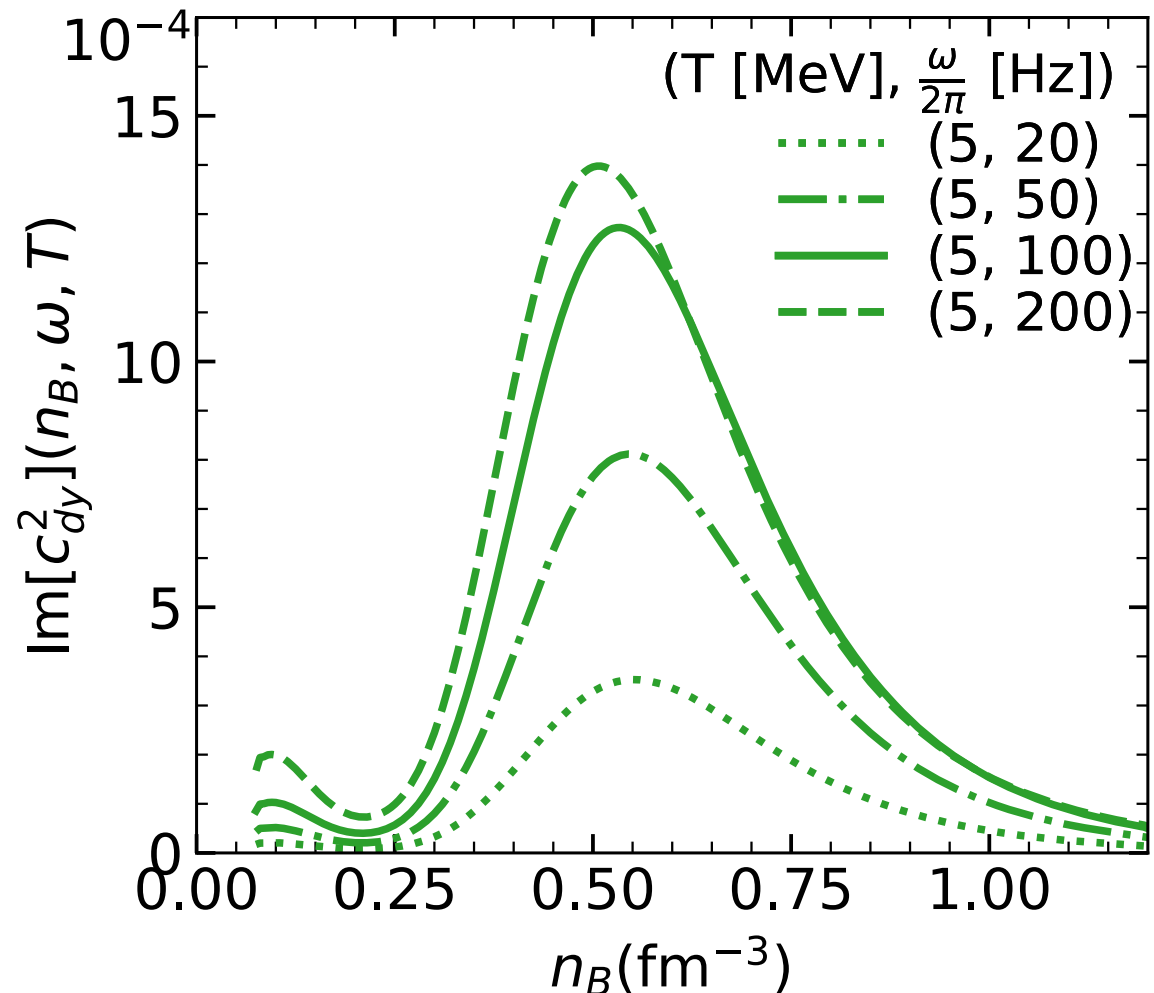
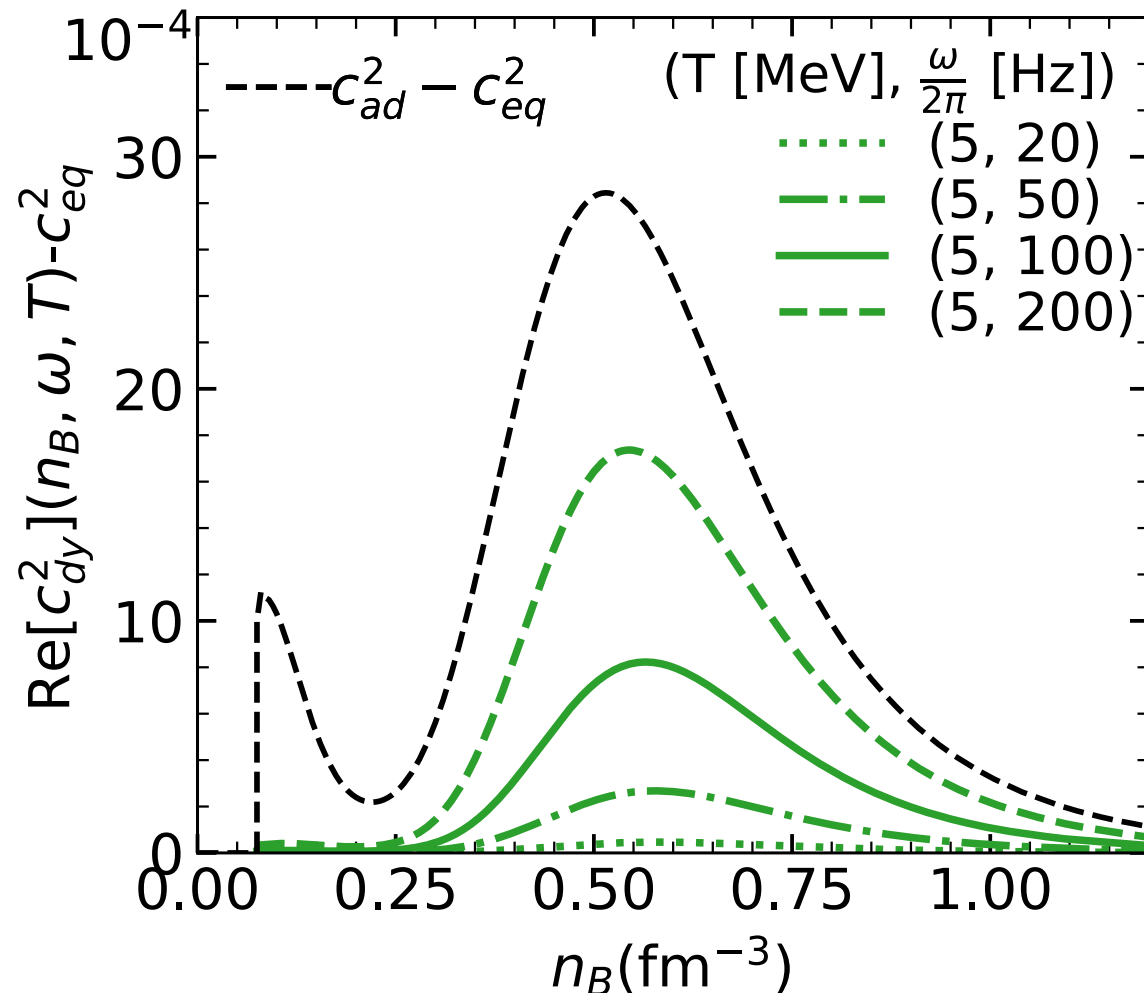
- Complex, frequency-dependent quantity derived from the **period ansatz** that captures both **restoring force** and **damping**. $n_B(t) = n_B^0 + \delta n_B \cos(\omega t)$

1. Restoring force: $c_{eq}^2 < \text{Re}[c_{dy}^2] < c_{ad}^2$

$$\text{Re}[c_{dy}^2] - c_{eq}^2 = (c_{ad}^2 - c_{eq}^2) \frac{\omega^2}{\omega^2 + \gamma^2}$$

2. Dissipative damping: resonance at $\gamma \approx \omega$

$$\text{Im}[c_{dy}^2] = (c_{ad}^2 - c_{eq}^2) \frac{\omega\gamma}{\omega^2 + \gamma^2}$$



Dynamic sound speed

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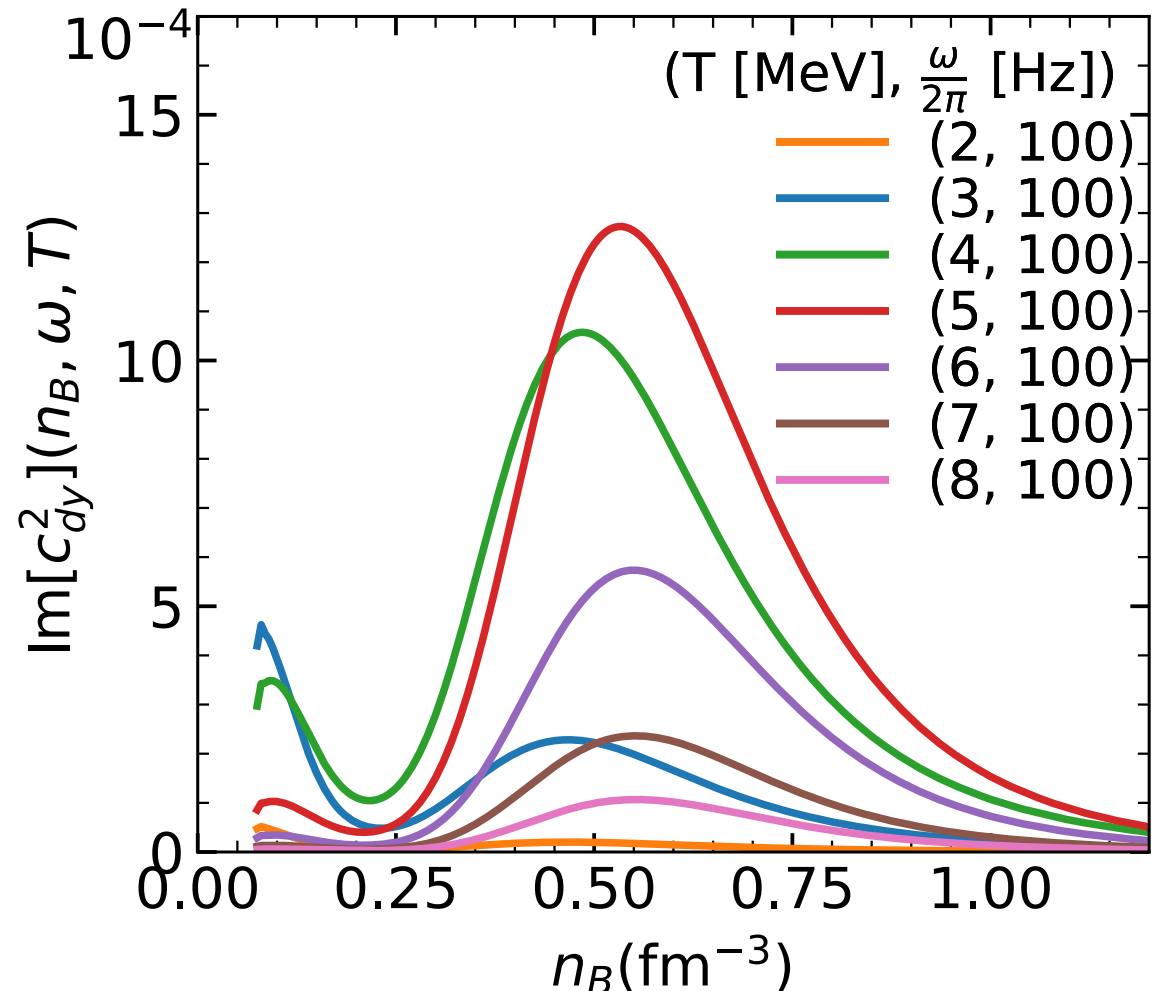
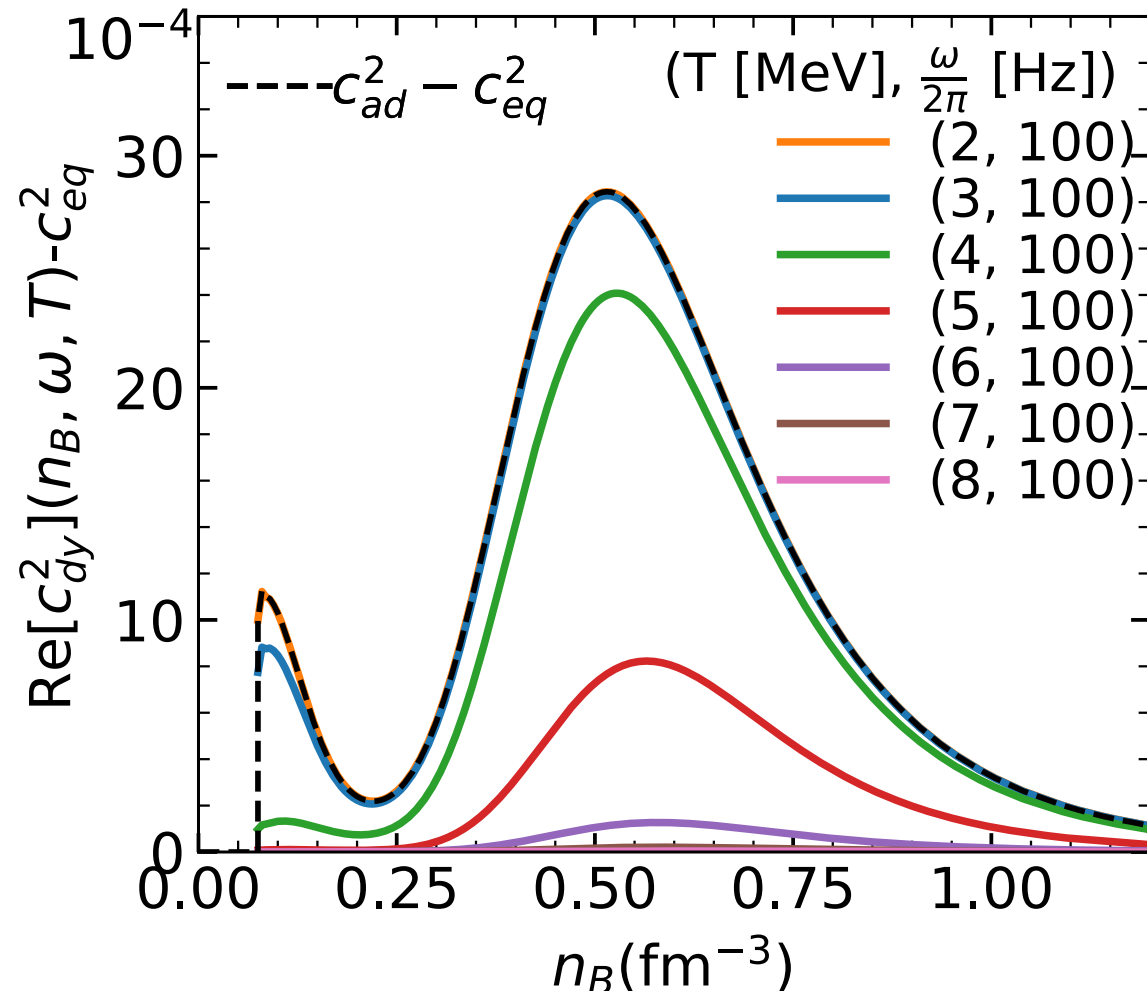
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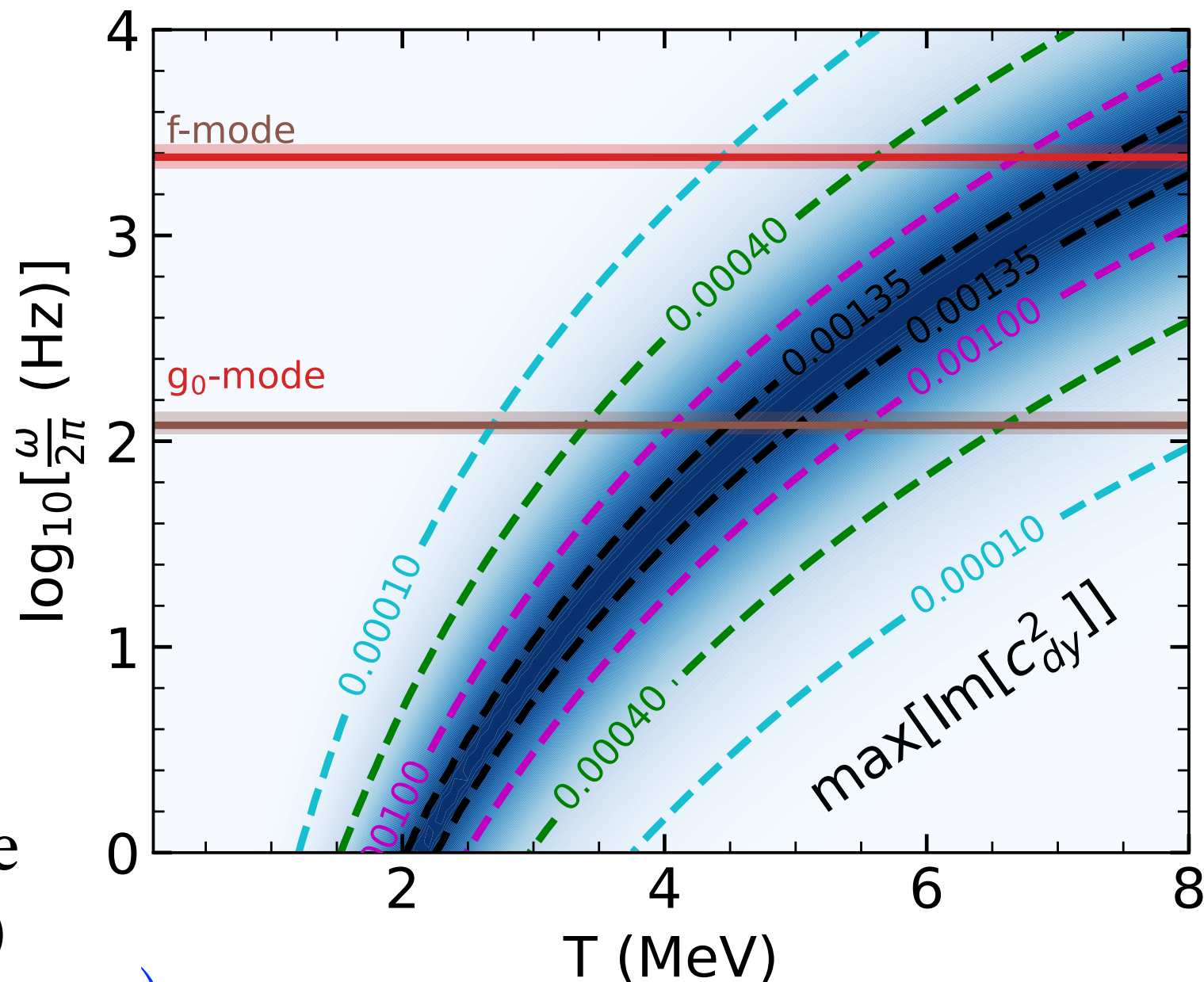
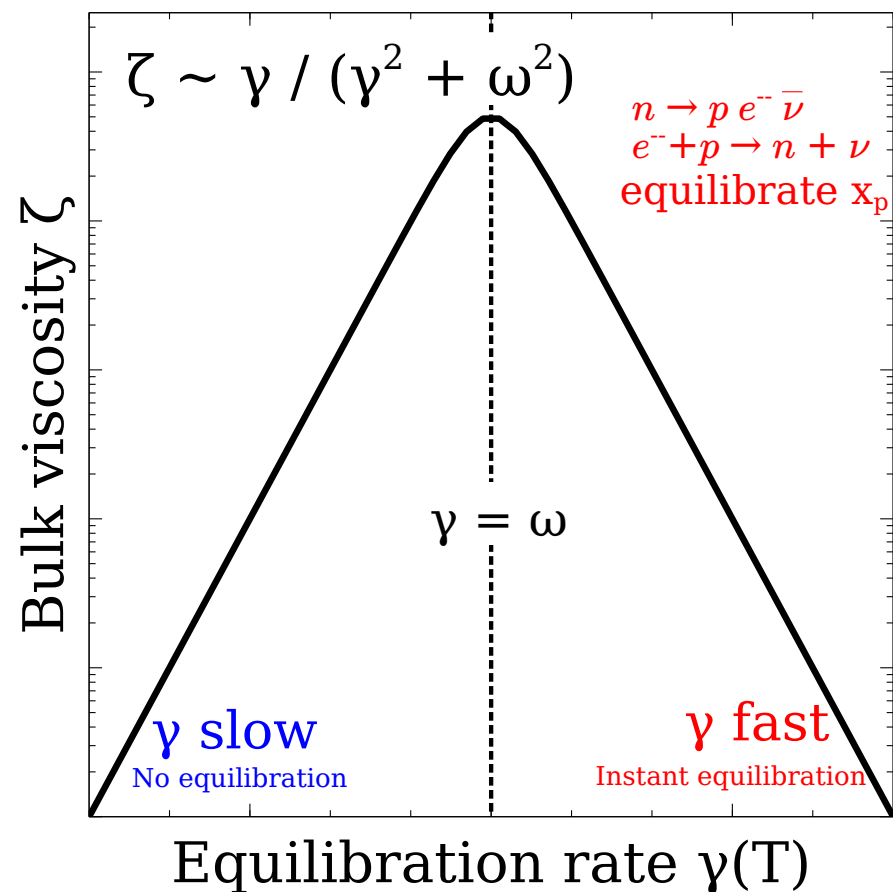


Bulk Viscosity

- Bulk viscosity $\zeta = \frac{\varepsilon + p}{\omega} \text{Im} [c_{dy}^2] = \frac{\gamma}{\omega^2 + \gamma^2} (c_{ad}^2 - c_{eq}^2) (\varepsilon + p)$

quantifies energy dissipation from compressional flow,

$$\frac{d\varepsilon}{dt} = -\zeta (\nabla \cdot \mathbf{v})^2.$$



- Even modes have divergence (f-mode, g-modes, p-modes)

$$\nabla \cdot \xi_{\text{even}}^{r, \theta, \phi} = \nabla \cdot \left(\nabla (R_n(r) Y_m^l(\theta, \phi) e^{i\omega t}) \right)$$

ODEs of Non-radial Oscillation

Eigen value problem of even quasi-normal modes

- Solve Einstein's equations:

$$8\pi\delta\left(T_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T\right) = \delta R_{\mu\nu}$$

with fluid perturbations, and metric perturbations:

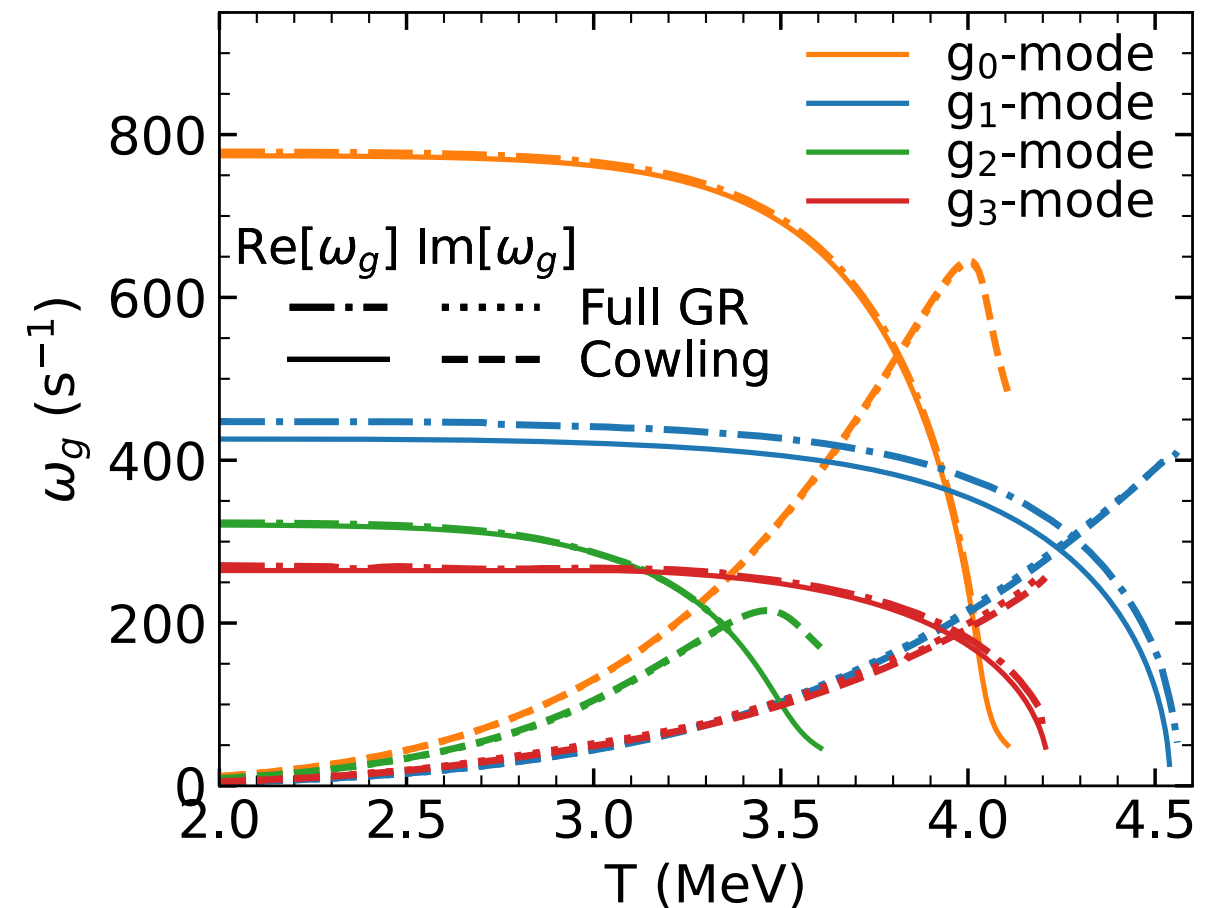
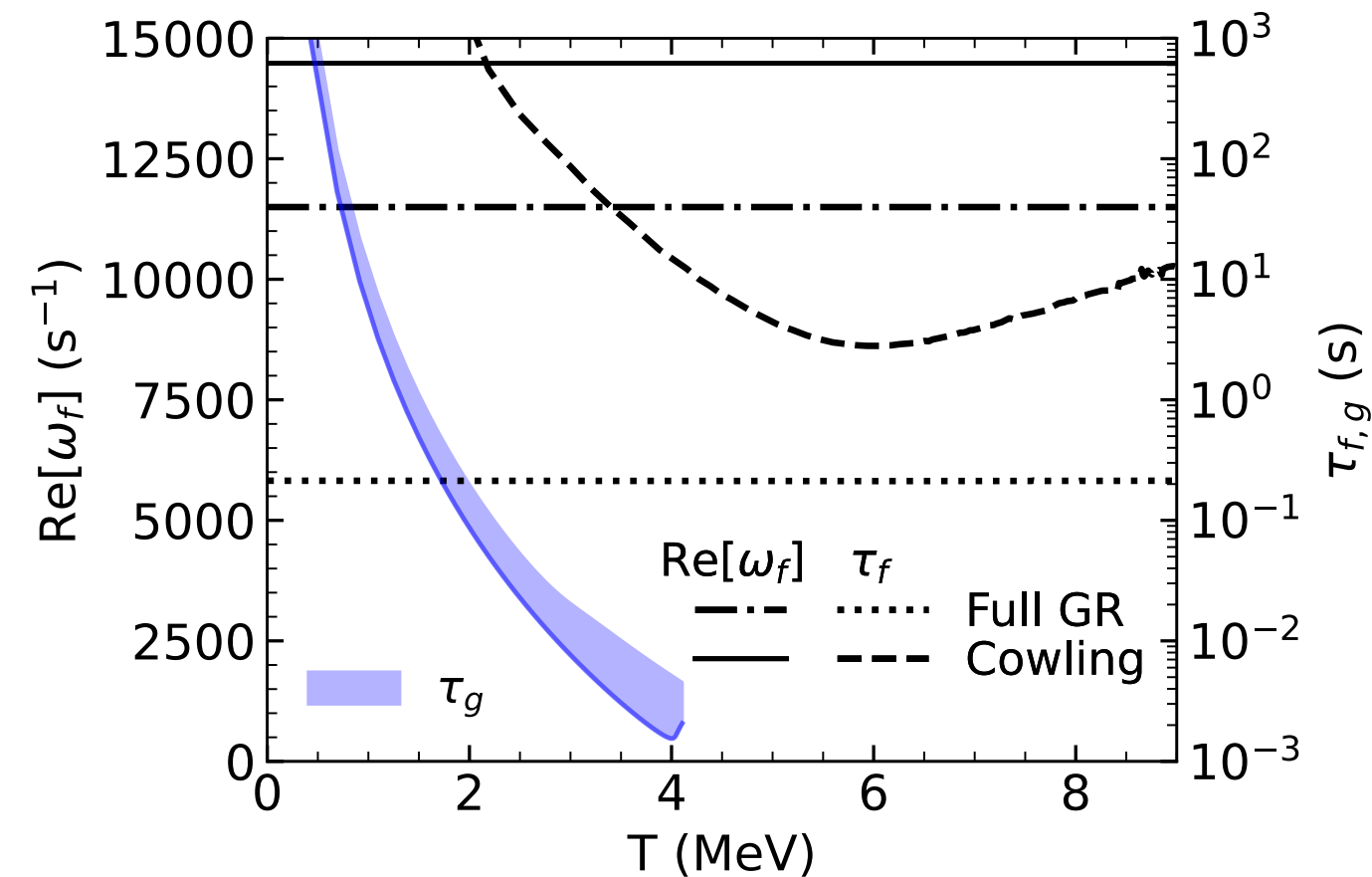
$$\xi_{\text{even}}^{r, \theta, \phi} = \nabla\left(R_n(r)Y_m^l(\theta, \phi)e^{i\omega t}\right)$$

$$\delta T_{\mu\nu} \text{ from } \xi^{r, \theta, \phi} \text{ and } C_{dy}^2 \quad \delta g_{\mu\nu}^{\text{even}} = \begin{pmatrix} H_0 e^\nu & H_1 & 0 & 0 \\ H_1 & H_2 e^\lambda & 0 & 0 \\ 0 & 0 & r^2 K & 0 \\ 0 & 0 & 0 & r^2 \sin^2 \theta K \end{pmatrix} Y_{lm}(\theta, \phi) e^{i\omega t}$$

- To get eigenvalue $\omega = 2\pi\nu + \frac{\mathbf{i}}{\tau}$,

and eigenfunction $\xi_{\text{even}}^{r, \theta, \phi}$ and H_0, H_1, H_2, K

Impact of bulk viscosity on $\omega = \text{Re}[\omega] + \frac{\mathbf{i}}{\tau}$

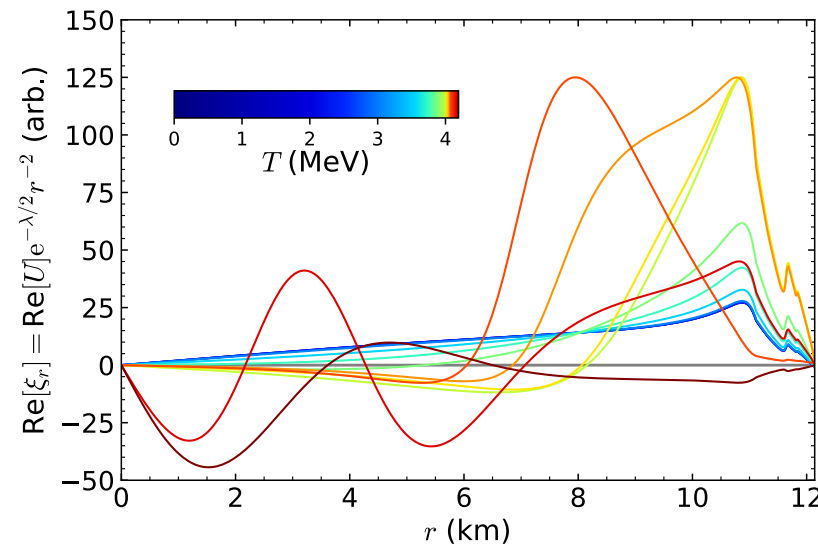


- No impact to f-mode.
f-mode is semi divergence-free.
- g-mode gets suppressed at larger T .
Avoided crossing between g-modes.

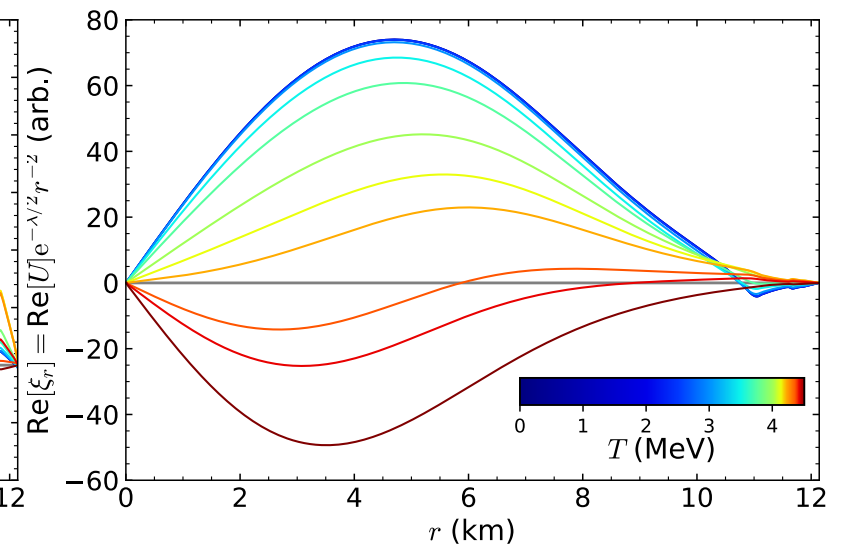
Fluid perturbation

- QMC-RMF3

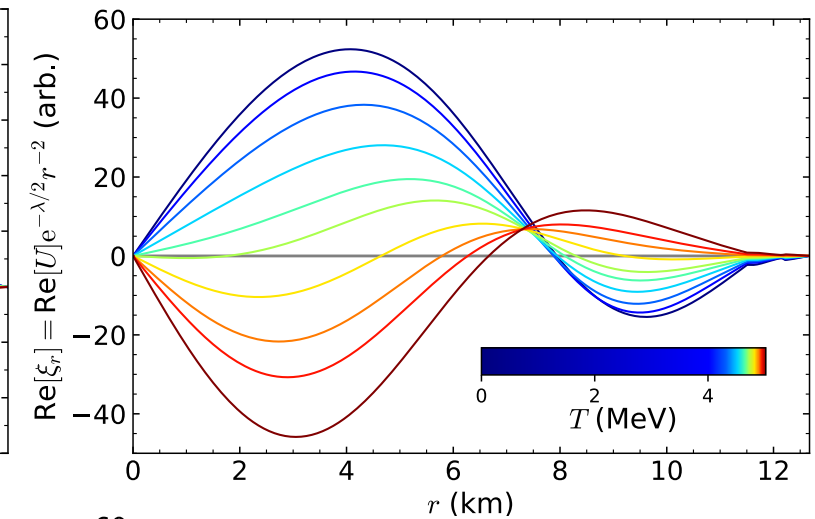
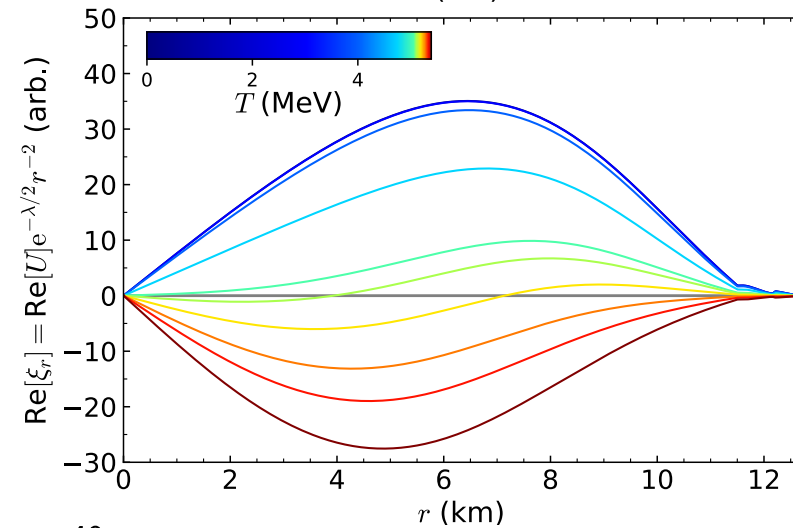
g_1 – mode



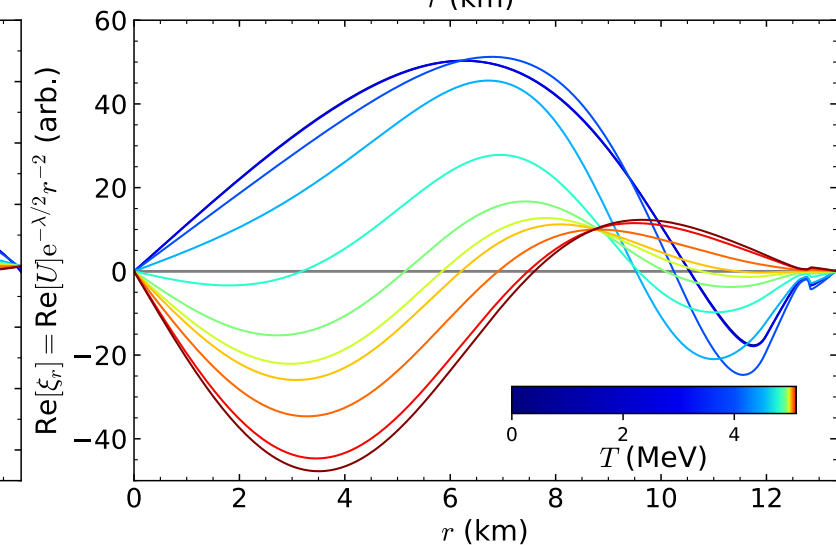
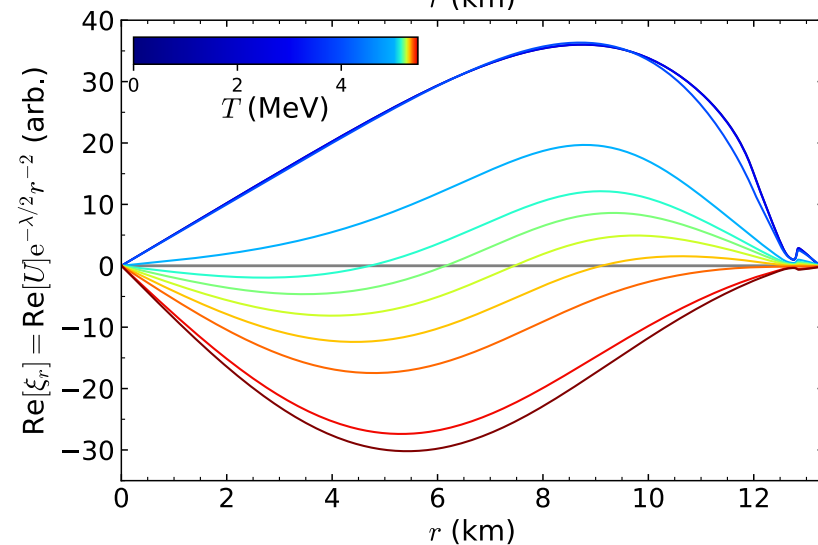
g_2 – mode



- IUFSU



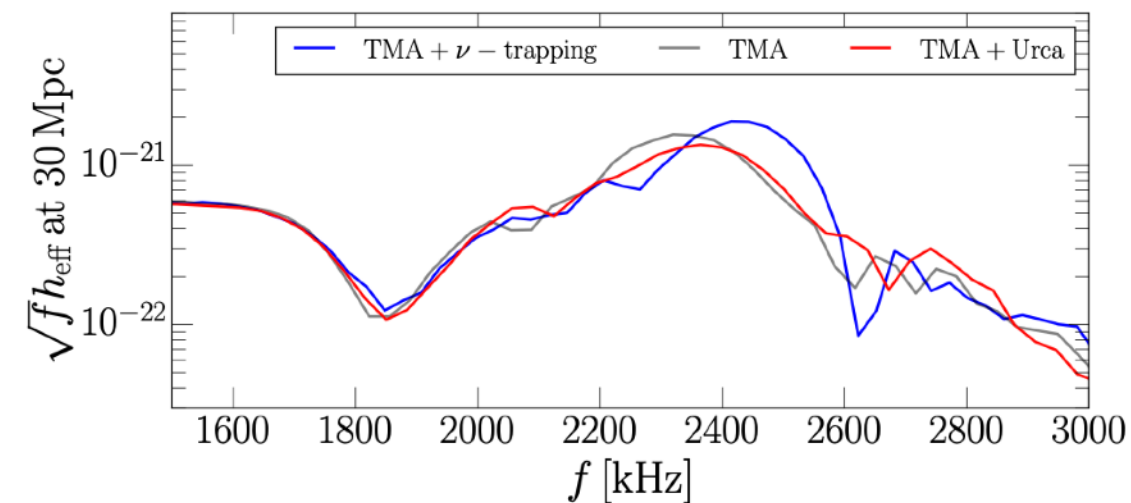
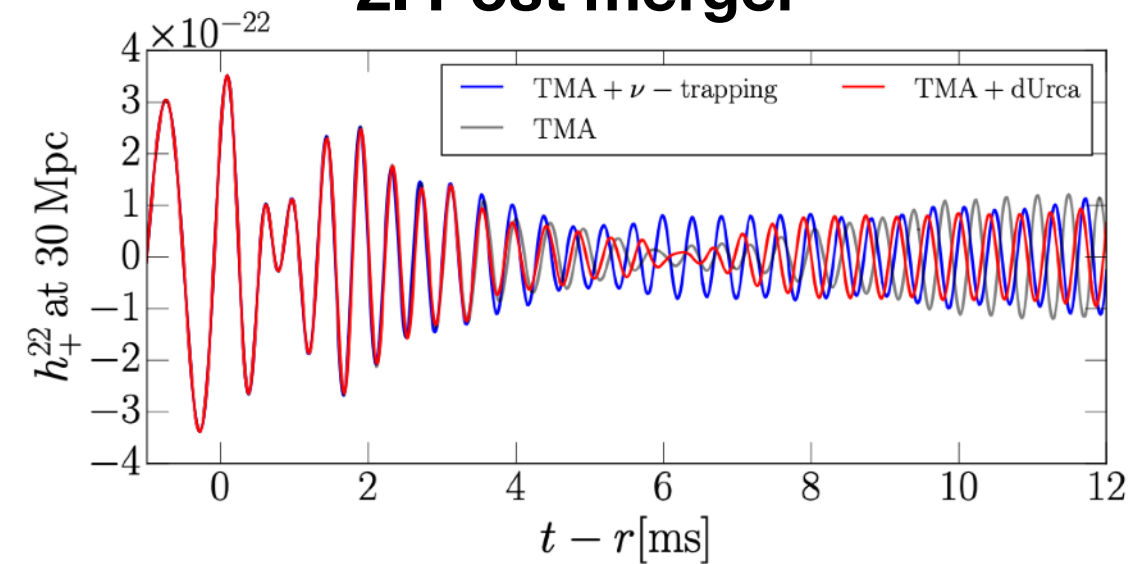
- IOPB-I



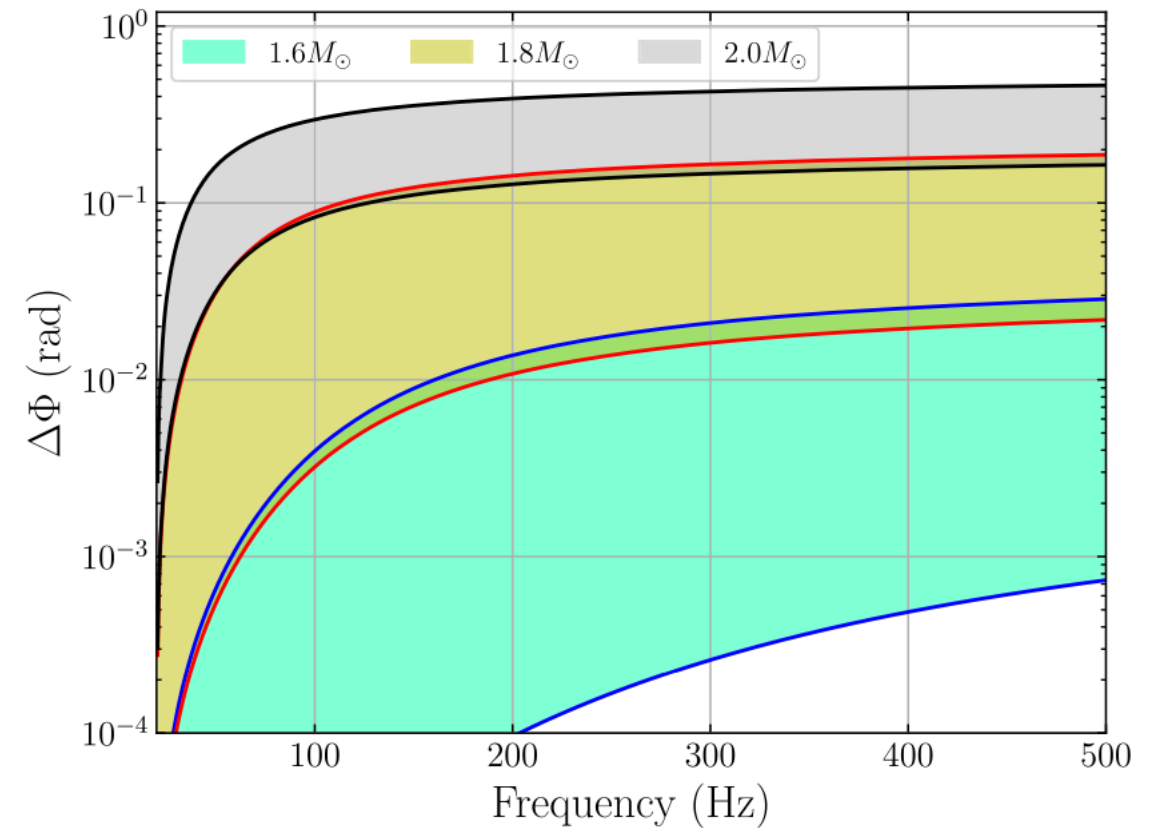
Where this matters:

- $$\zeta = \frac{\varepsilon + p}{\omega} \text{Im} \left[c_{dy}^2 \right]$$

2. Post merger

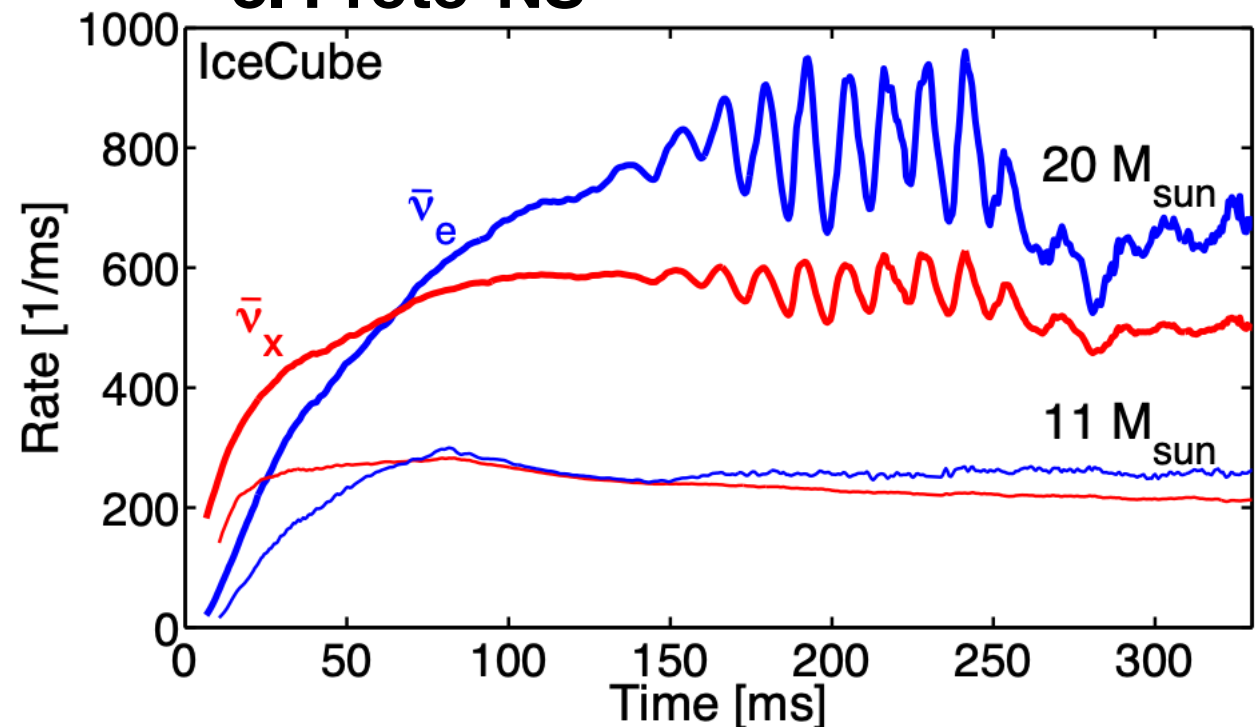


**Most, Haber, Harris, Zhang,
Alford, and Noronha 2207.00442**



Ghosh, Pradhan and Chatterjee 2306.14737

3. Proto-NS



Tamborra, Hanke, Muller, Janka and Raffelt 1307.7936

To-do Next

- Beyond the Fermi surface approximation
- More realistic composition & entropy profile.
- Consider the Neutrino-trapped scenario.
- Gravitational waveform with tidal excitation in NS spiraling.
- Viscosity due to other degrees of freedom: muons, pions, hyperons, deconfined quarks, and perhaps dark matter ...

Between Maxwell & Gibbs

Partially local & partially global

- Locally neutral lepton densities:

$$n_{e,N} = n_p, \quad n_{e,Q} = \frac{2}{3}n_u - \frac{1}{3}n_d - \frac{1}{3}n_s$$

- Global lepton density, $n_{e,G}$

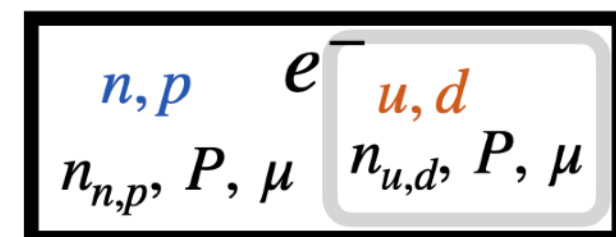
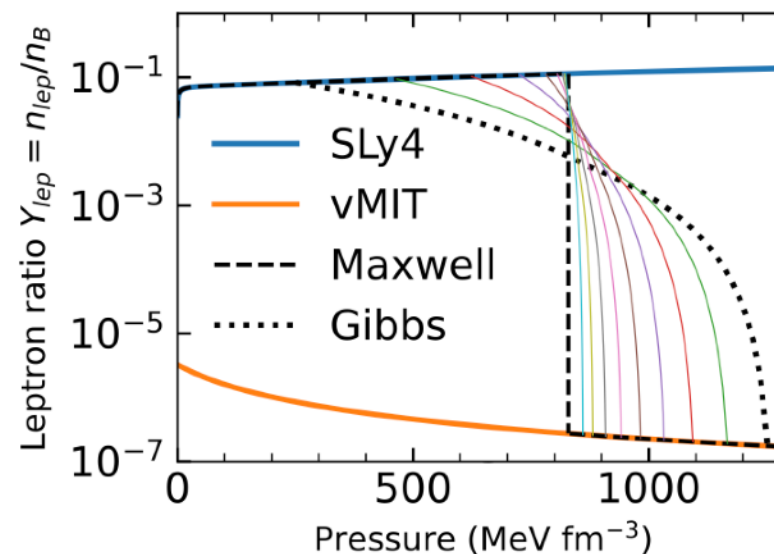
- Total lepton density:

$$n_e = g(fn_{e,N} + (1-f)n_{e,Q}) + (1-g)n_{e,G}$$

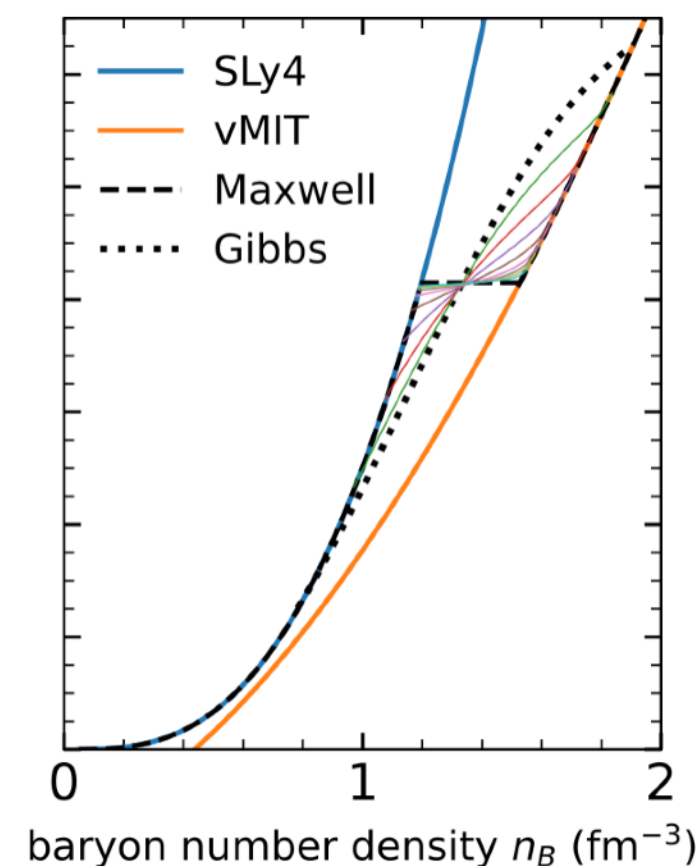
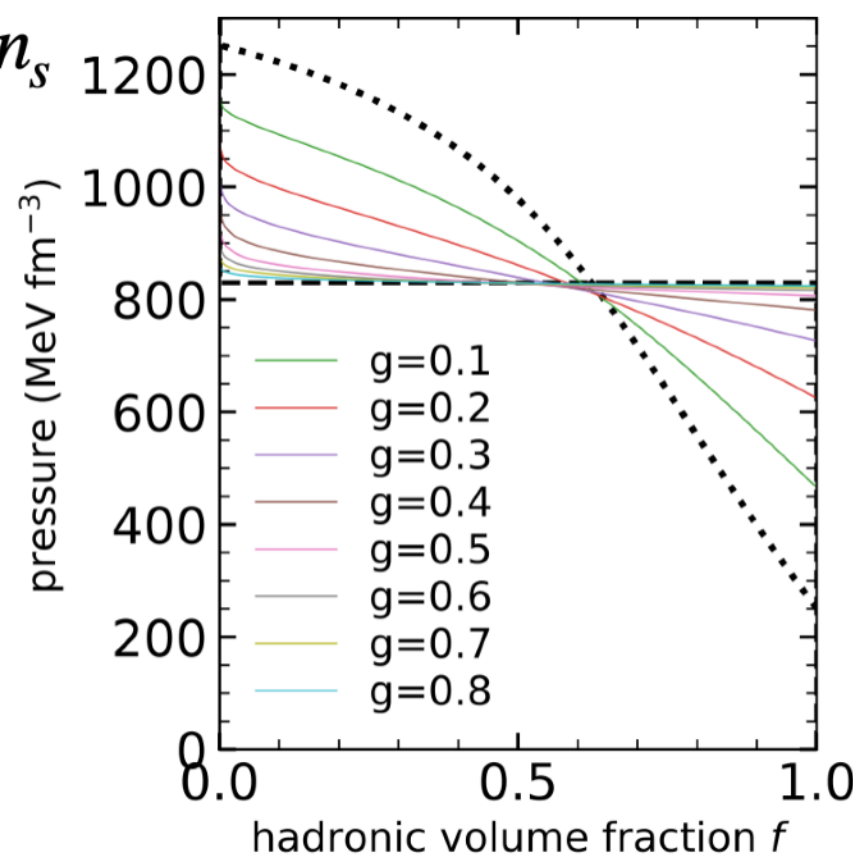
- $g = 0 \rightarrow$ Gibbs transition

- $g = 1 \rightarrow$ Maxwell transition

- g could be determined by Surface & Coulomb energy.



Global lepton fraction
 $1 - g$



[arXiv: 2302.04289 \(cold\)](https://arxiv.org/abs/2302.04289)
[arXiv: 2506.20418 \(hot\)](https://arxiv.org/abs/2506.20418)

Between Maxwell & Gibbs

Extend to finite temperature:

- Introduce anti-particles as,

$$\mu_{e^-} = -\mu_{e^+}$$

$$\mu_{\mu^-} = -\mu_{\mu^+}$$

$$\mu_{u^-} = -\mu_{u^+}$$

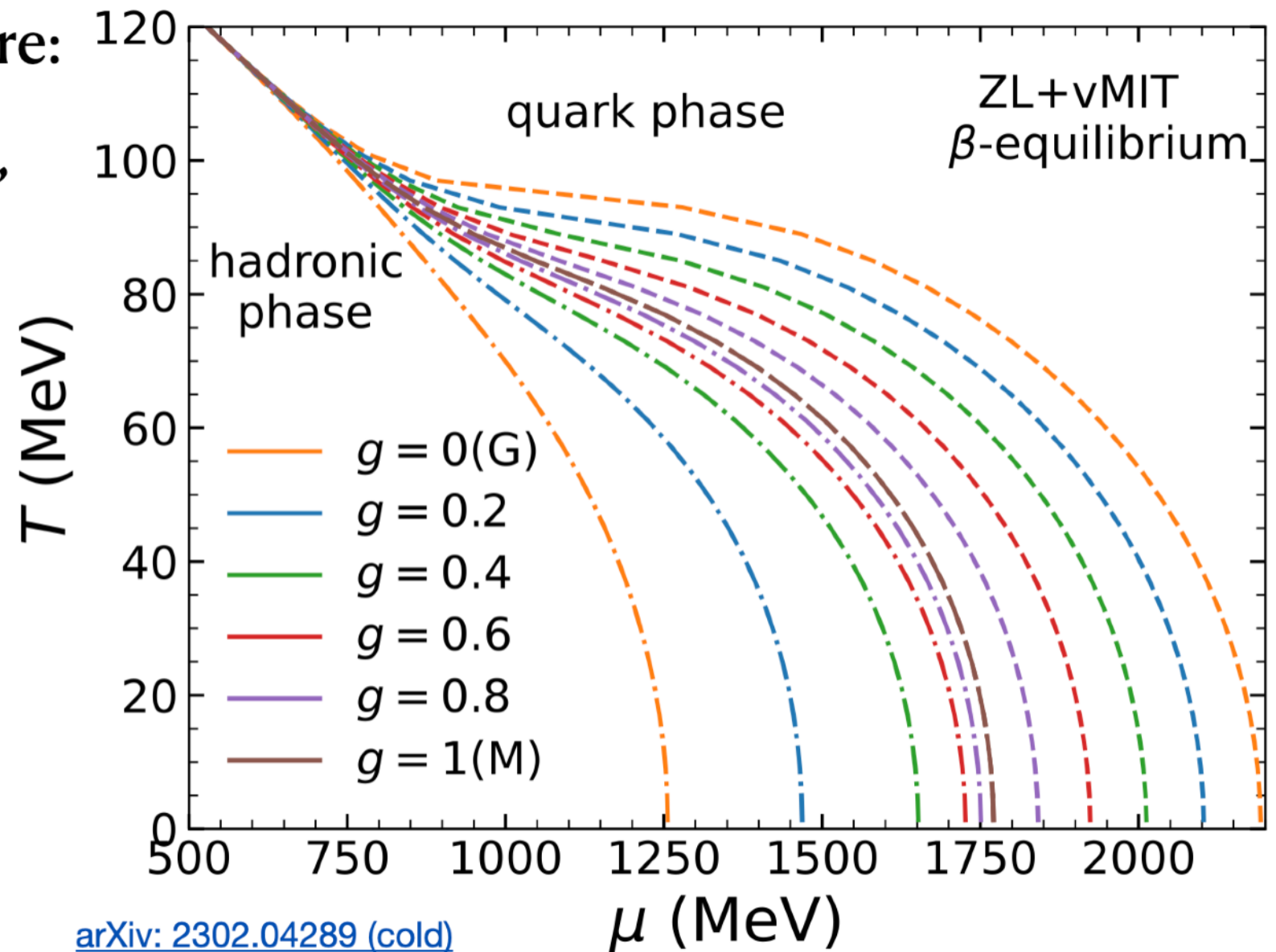
$$\mu_{d^-} = -\mu_{d^+}$$

$$\mu_{s^-} = -\mu_{s^+}$$

- Add photon contribution,

$$\varepsilon_{\text{photon}} \propto T^4$$

- neutrino contribution
(equilibrium)



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[arXiv: 2506.20418](https://arxiv.org/abs/2506.20418) (hot)

Reference

- Framework for phase transitions between the Maxwell and Gibbs constructions
C Constantinou, T Zhao, S Han, M Prakash
Physical Review D 107 (7), 074013 [2023]
- Framework for phase transitions between the Maxwell and Gibbs constructions at finite temperature [2025]
C Constantinou, M Guerrini, T Zhao, S Han, M Prakash
arXiv:2506.20418
- Suppression of composition g-modes in chemically-equilibrating warm neutron stars. T Zhao, P Rau, A Haber, S Harris, C Constantinou, S Han [2025]
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