

Two Lectures on
The Standard Cosmology
Rocky Kolb—University of Chicago

4th Santa Cruz School on



**Multi-Messenger
Astrophysics**

The observed universe

The cosmic symphony

Where to start?

The beginning and work forward?

Today and work backwards?

Historical developments?

Classical issues with the Standard Model

Age

Flatness

Unwanted relics

Why homo/iso

Inflation

Density perturbations and gravitational waves

Reheating

Dark Matter

Evidence

Particle Dark Matter

Freeze-in/Freeze-out

Axions, Axion-like particles

Dark Energy

Evidence

If Λ , why so small?

BBN

Limits on neutrino properties

CMB

Temperature

Sound Horizon

Relativistic Cosmology

Lightning Review of GR

The Cosmological Principle

Einstein's Static Universe

Friedmann Blows Up the Universe

Friedmann Equation

Lemaitre rediscovers Friedmann

Einstein Becomes a Big Banger

Thermal History

A primer on Thermodynamics

The Program

Rocky I

The Universe Observed

Energy and pressure

The Friedmann equation

Primordial origin of Dark Matter[#]

Rocky II

Primordial Inflation

Baryo/Leptogenesis

CMB^{*}

Dark Energy[†]

[#] Matt Pyle

^{*} Adrian Lee

[†] Dave Schlegel

Some Reference Texts

Baumann, *Cosmology*

Dodelson, *Modern Cosmology*

Hooper, *Particle Cosmology & Astrophysics*

Kolb & Turner, *The Early Universe*

Lyth and Liddle, *The Primordial Density Perturbation*

Mukhanov, *Physical Foundations of Cosmology*

Weinberg, *Cosmology*

Standard Cosmology

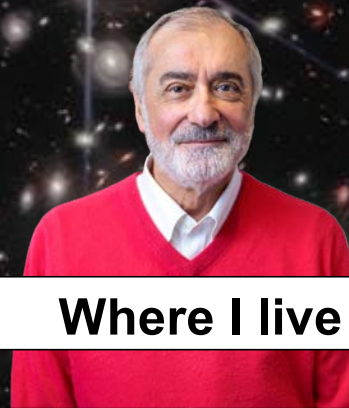
Astronomical Observations

General Relativity

Particle & Nuclear Physics

Where I live

Pillars of Standard Cosmology



The Standard Cosmology (Λ CDM)

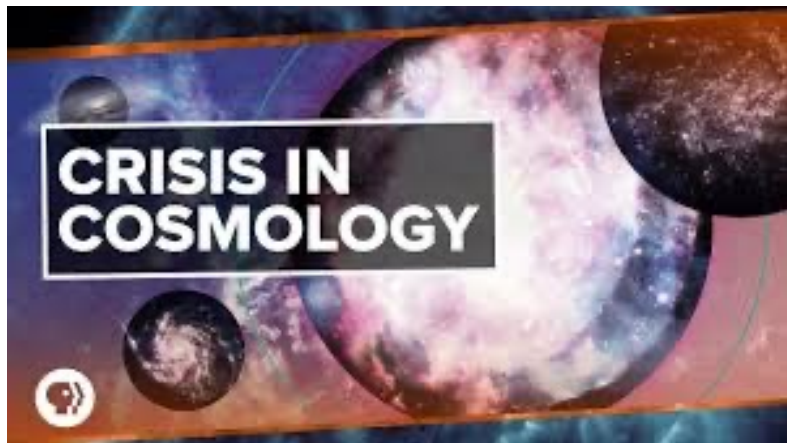
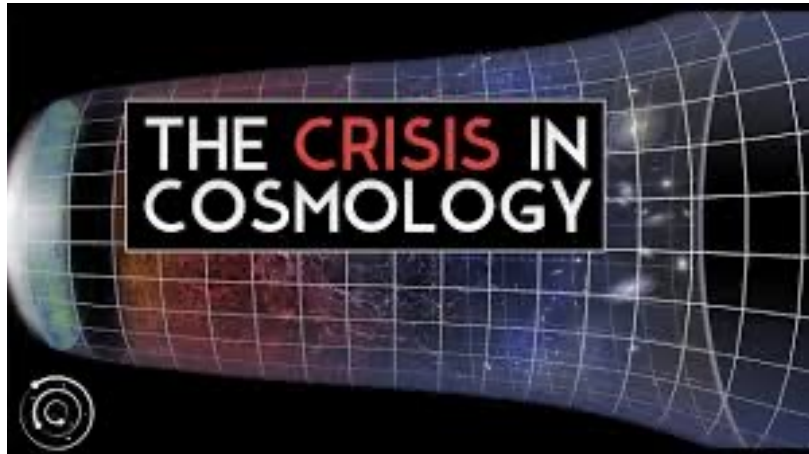
- Gravity described by General Relativity.
- Big Bang: Universe emerged from a state of high density, energy, and temperature a finite time in the past, expanding according to Einstein's equations.
- Λ : Einstein's Cosmological Constant (dark energy).
- CDM: Cold Dark Matter.
- An inflationary era very early in the expansion history that set the primordial temperature and density perturbations.
- The standard model of particle and nuclear physics (+ DM + DE + inflation).
- Primordial density perturbations grew via gravitational instability (aided by dark matter) to form large-scale structure.

Cosmic Symphony (Harmonice Mundi)

expansion tempo	movement	epoch	relic
pizzicato	string	$t_{\text{Planck}} = 10^{-43}\text{s}$???
presto	inflation	$t_{\text{inflation}} = 10^{-35}\text{s}$	CMB fluctuations gravitational waves seeds of structure
allegro	radiation	earlier than 60,000 years	abundances of the light elements
andante	matter	later than 60,000 years	growth of structure: galaxies, clusters,...
largo	inflation	day before yesterday	acceleration of the universe

The Standard Cosmology (Λ CDM)

- Most predictive cosmological model, in agreement with (almost) all observational data.
- But not without issues and questions:
 - 95% of the present universe is dark energy and dark matter, which we don't understand. (Beyond the Standard Model of particle physics.)
 - Cosmic inflation is phenomenological success, but without grounding in a fundamental theory. (Beyond the Standard Model of particle physics.)
 - Why is there a universe, why are there only 3 spatial dimensions, why did structure form so early, ...?
- But premature to say, "Cosmology is in Crisis."



The New York Times

OPINION
GUEST ESSAY

The Story of Our Universe May Be Starting to Unravel

Sept. 2, 2023



Virginia Gabrielli

By Adam Frank and Marcelo Gleiser

Dr. Frank is an astrophysicist at the University of Rochester.


Dr. Gleiser is a theoretical physicist at Dartmouth College.

Cosmology Crisis: What Is It? Why Is It Happening?



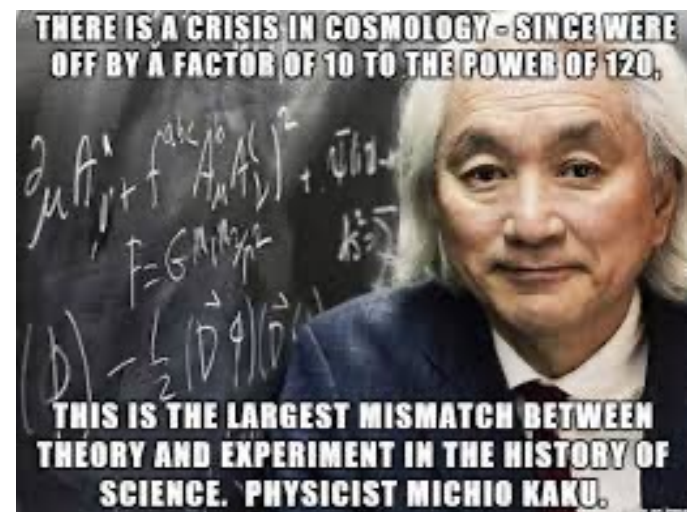
20th Anniversary

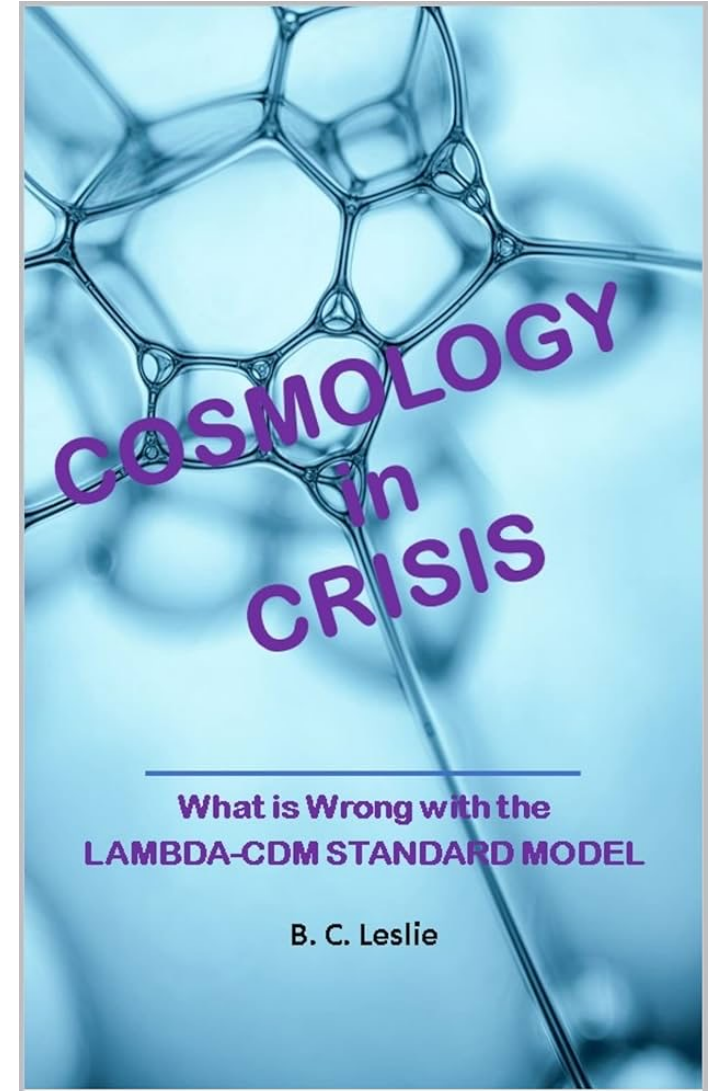
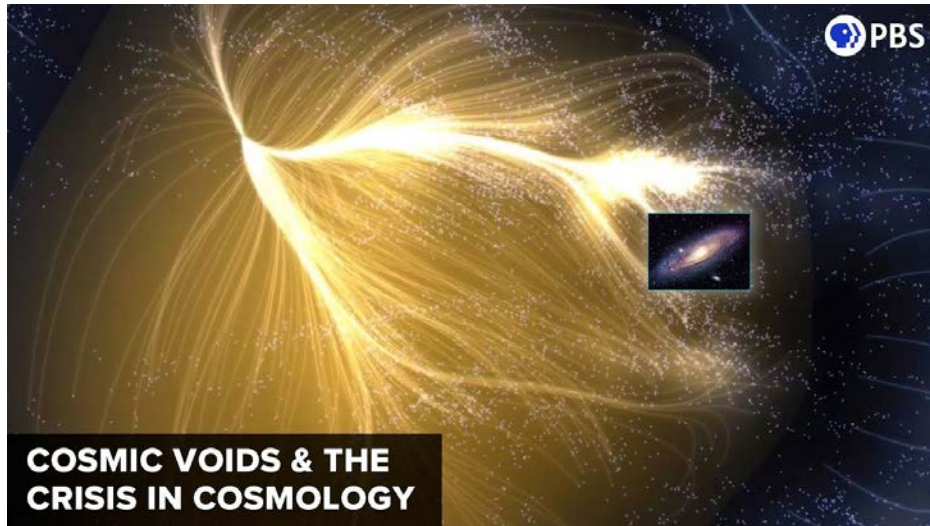
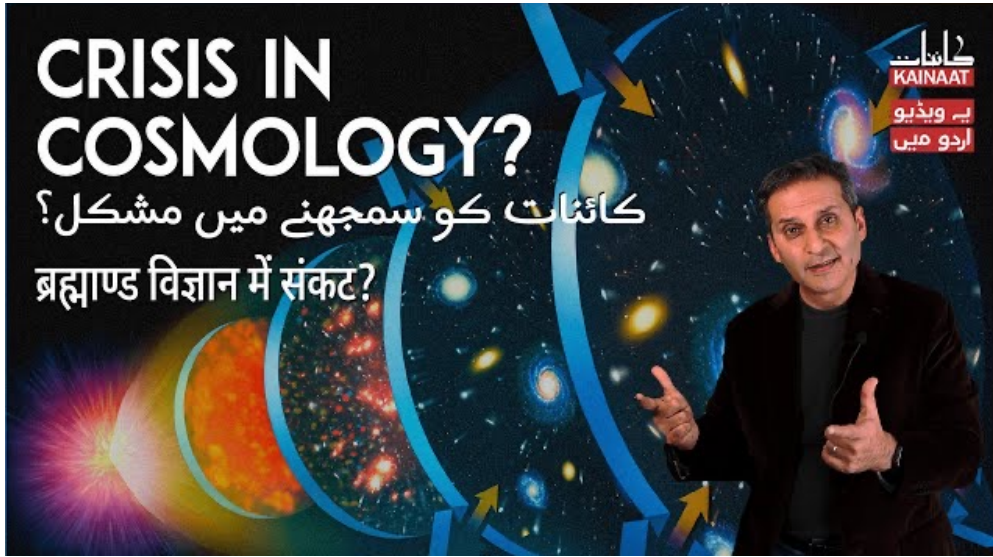
CARNEGIE SCIENCE
The Carnegie Observatories

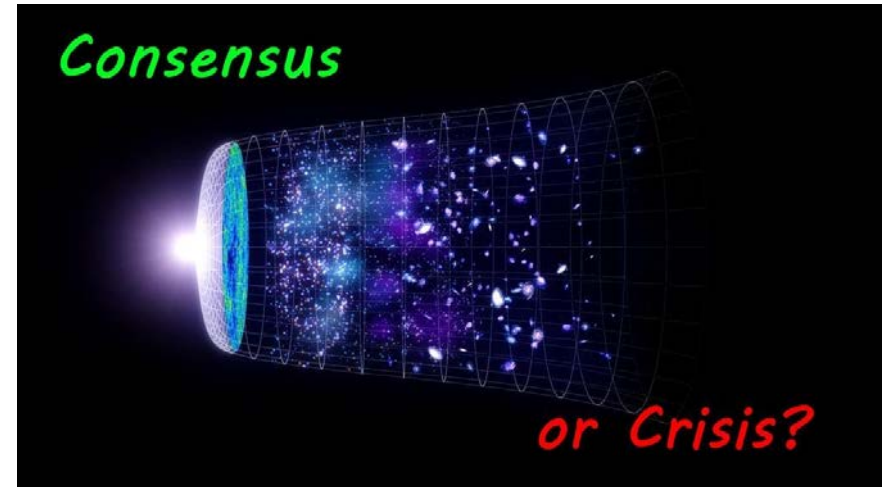


Is There a Crisis in Cosmology?

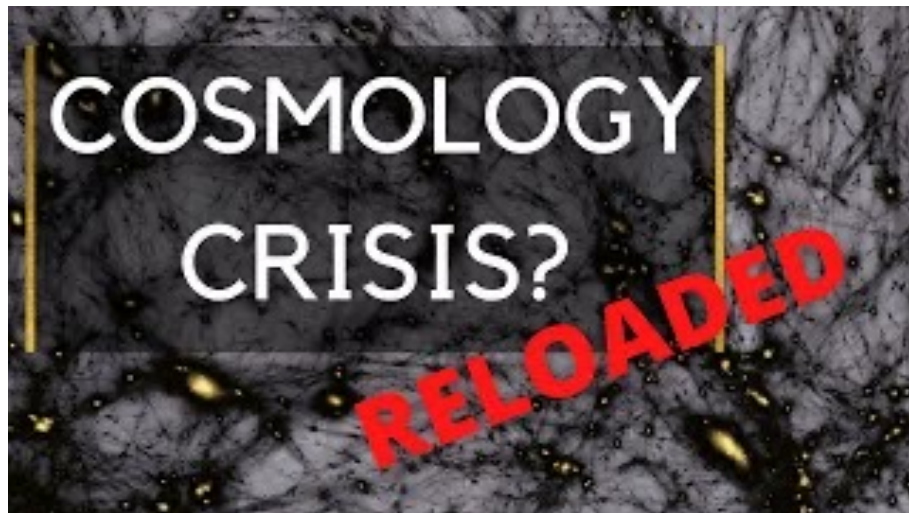
Dr. Wendy L. Freedman







The "crisis in cosmology" is
pure exaggeration Ethan Siegel



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How a Dispute over a Single Number Became a Cosmological Crisis

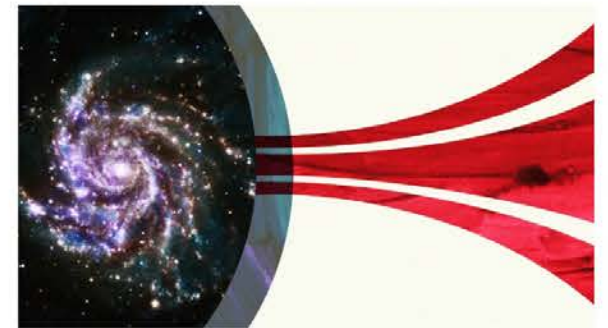
Two divergent measurements of how fast the universe is expanding cannot both be right. Something must give—but what?

By Richard Panek on March 1, 2020

13.8 — MAY 4, 2023

Some scientists speak of a “crisis in cosmology.” They have a good reason

The standard model of cosmology has a big new problem: Some galaxies seem to be too old.



Credit: Annelisa Leinbach / Big Think; NASA

KEY TAKEAWAYS

● Just like atoms are the building blocks of chemistry, galaxies are the building blocks of cosmology. ● The standard model of cosmology gives astronomers a way to link observed distances to objects with their age. ● However, new images from the James Webb Space Telescope found galaxies far more distant (and, hence, far older) than what the standard model of cosmology predicts. The new data provide compelling evidence that the model might need to be updated.

[News](#) > [Science & Astronomy](#)

Why is there a 'crisis' in cosmology?

By [Paul Sutter](#) published October 30, 2021



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Paul M. Sutter is an astrophysicist at SUNY Stony Brook and the Flatiron Institute, host of "Ask a Spaceman" and "Space Radio," and author of "How to Die in Space." Sutter contributed this article to [Space.com's Expert Voices: Op-Ed & Insights](#).

Is there really a 'crisis' in cosmology?

By [Paul Sutter](#) last updated February 21, 2022

Maybe we just don't understand some cosmic phenomena as well as we think we do.

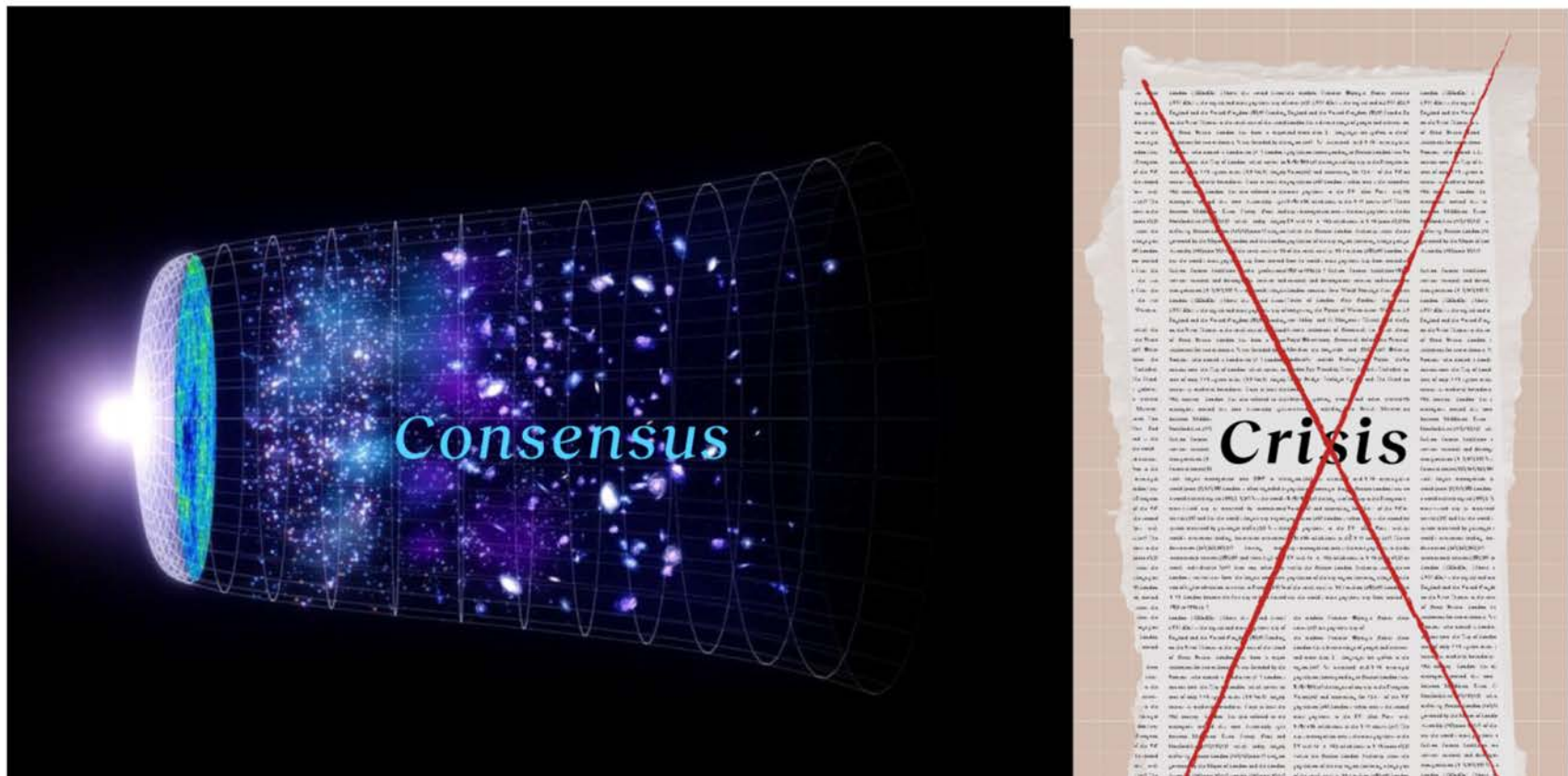


The Crab Nebula as seen by the Hubble Space Telescope and ground-based telescopes in a composite view. The nebula is the aftermath of a brilliant supernova spotted in 1054. (Image credit: NASA, ESA, NRAO/AUI/NSF and G. Dubner (University of Buenos Aires))

STARTS WITH A BANG — SEPTEMBER 6, 2023

The “crisis in cosmology” is pure exaggeration

There are a few clues that the Universe isn't completely adding up. Even so, the standard model of cosmology holds up stronger than ever.

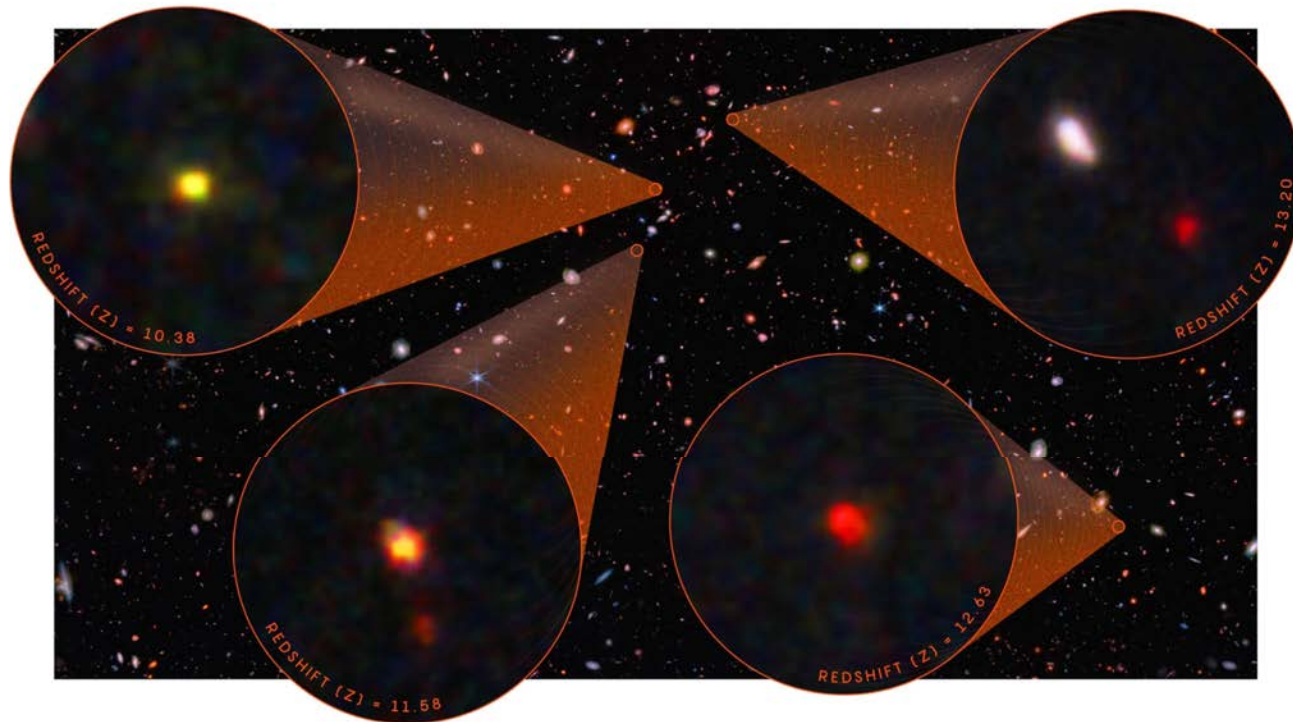


ASTROPHYSICS

Standard Model of Cosmology Survives a Telescope's Surprising Finds

📖 📱 📧

Reports that the James Webb Space Telescope killed the reigning cosmological model turn out to have been exaggerated. But astronomers still have much to learn from distant galaxies glimpsed by Webb.



The Webb telescope has spotted galaxies surprisingly far away in space and deep in the past. These four, studied by a team called JADES, are all seen as they appeared less than 500 million years after the Big Bang.

Samuel Velazco/Quantum Magazine, source: NASA



Rebecca Boyle

The cracks in cosmology were supposed to take a while to appear. But when the James Webb Space Telescope (JWST) opened its lens last spring, extremely distant yet very bright galaxies immediately shone into the telescope's field of view. "They were just so stupidly bright, and they just stood out," said Rohan Naidu, an

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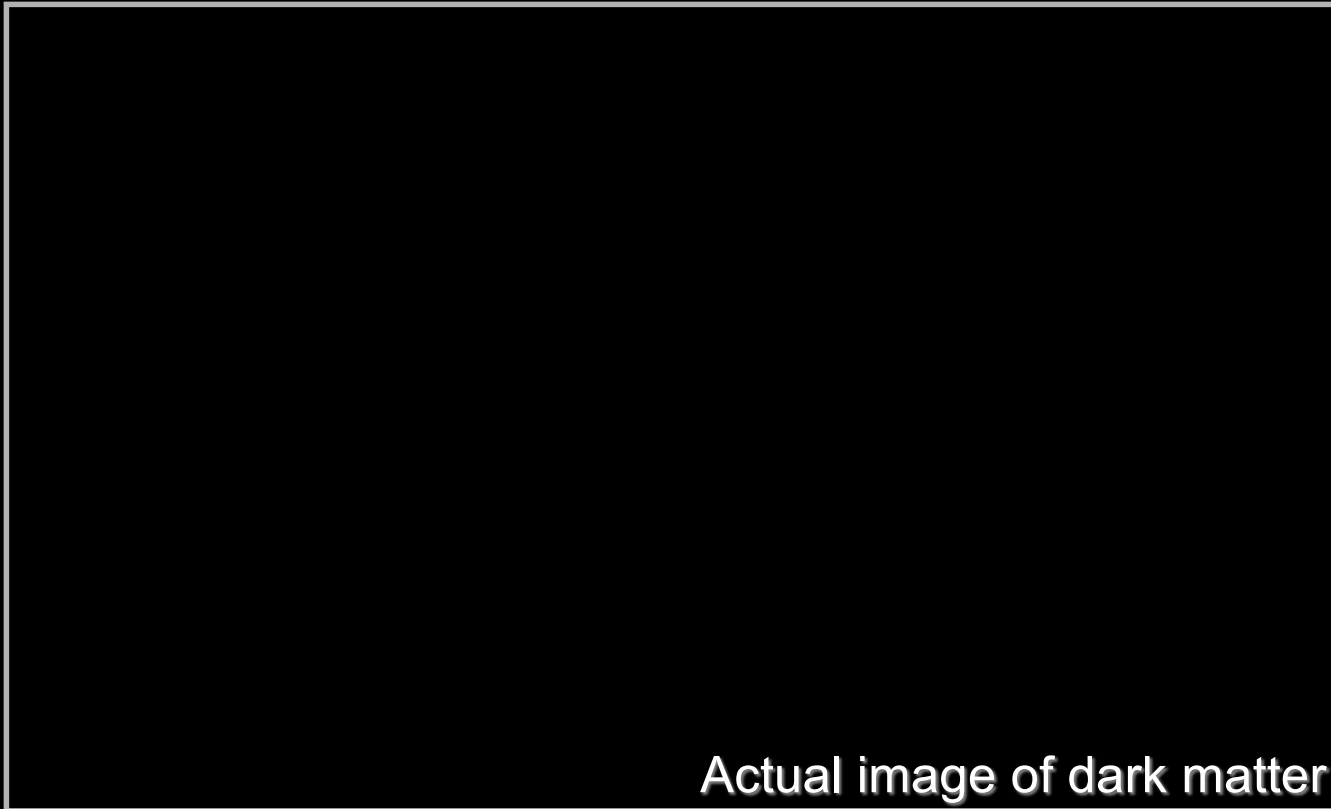
January 20, 2023

Observer's View of the Universe



**Inhomogeneous, rich structure,
full of stars, black holes, galaxies, clusters,**

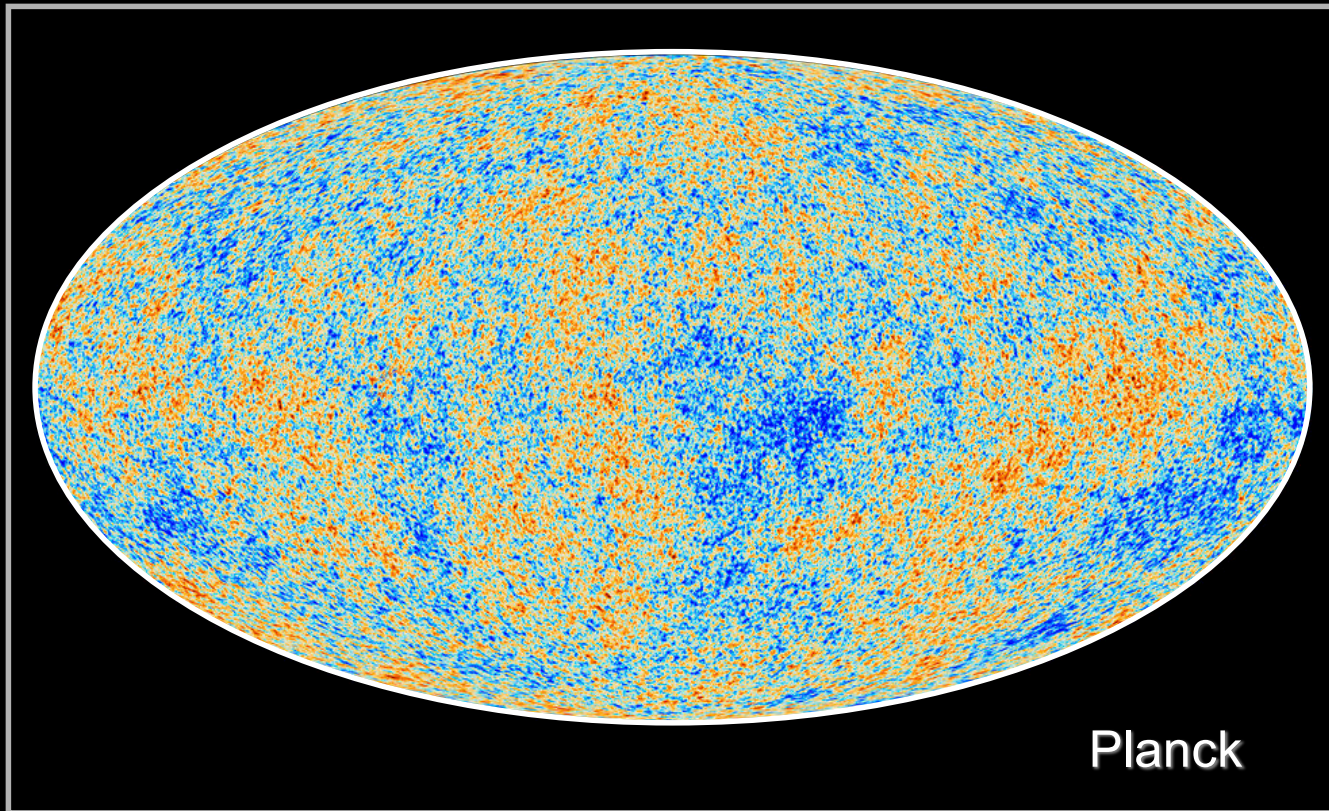
Theorist's View of the Universe



Actual image of dark matter

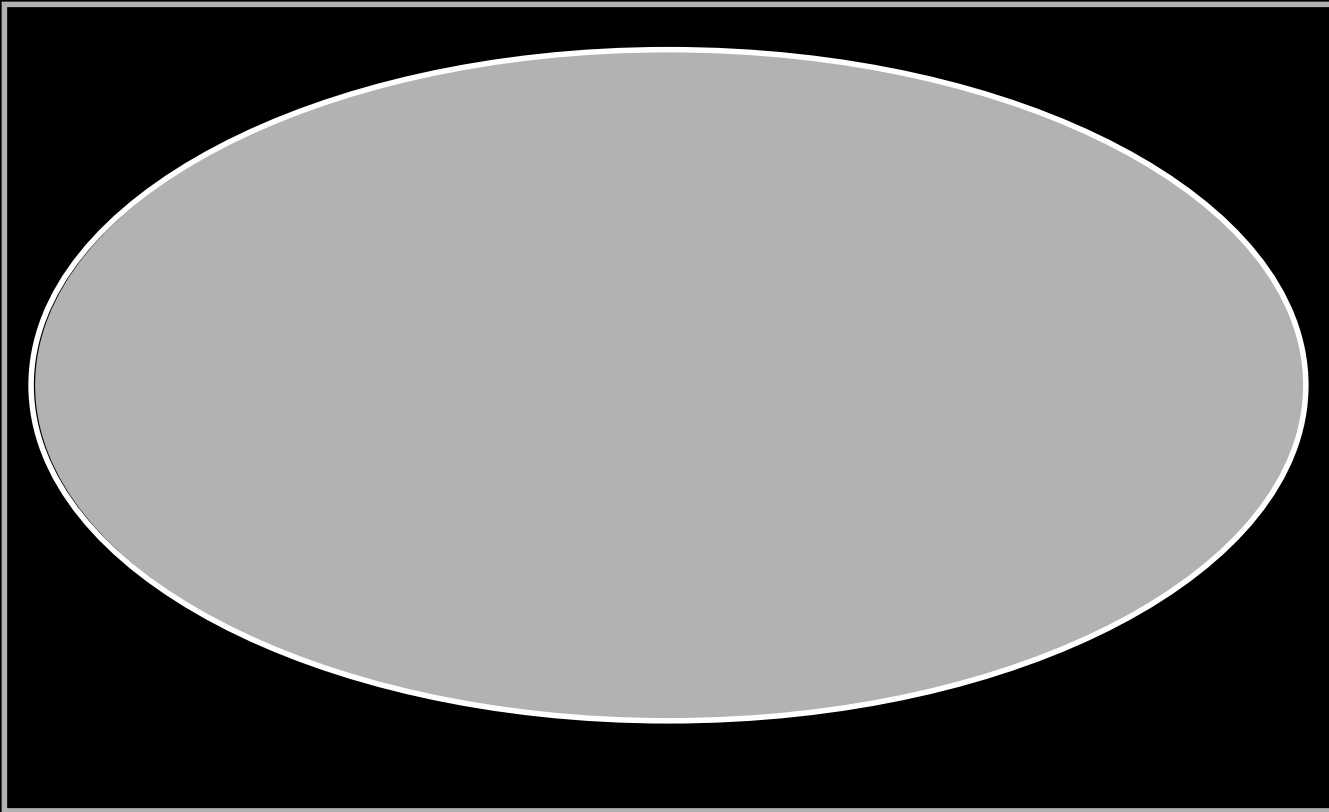
**smooth (homogeneous and isotropic)
full of dark matter and dark energy**

Observer's View of the Universe



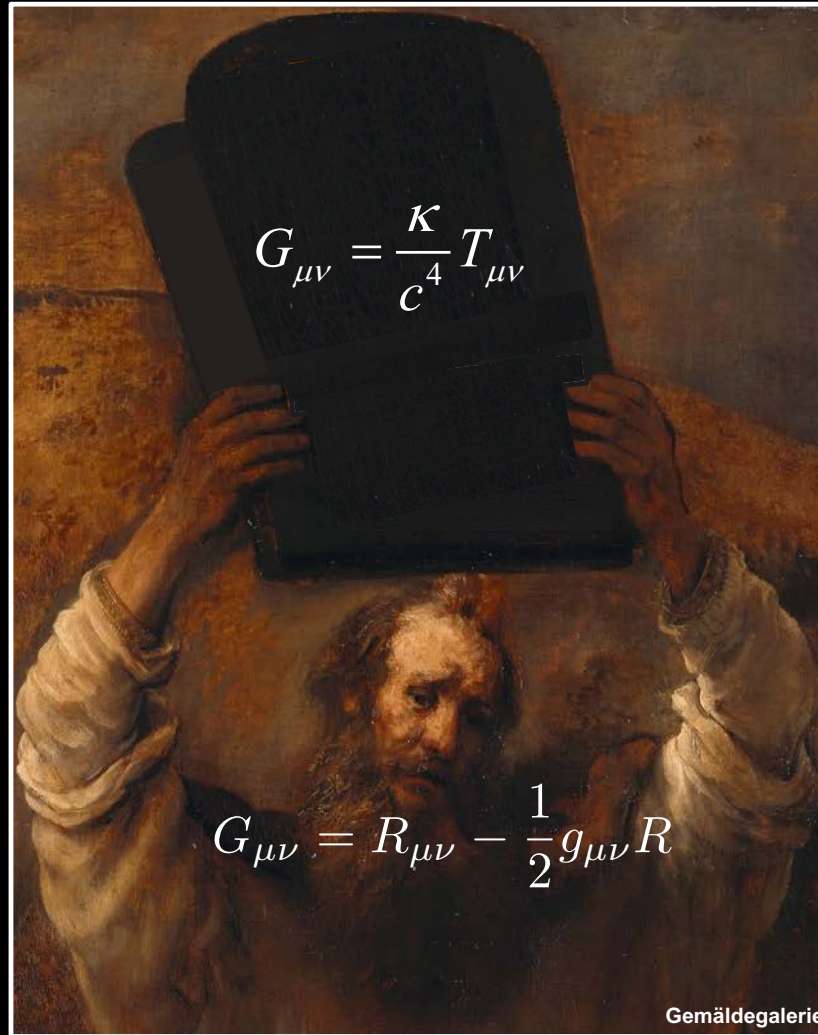
Temperature fluctuations $\mathcal{O}(10^{-5})$ in the CMB
What is their origin? Inflation!

Theorist's View of the Universe



Temperature remarkably uniform
Why so smooth? Inflation!

Ten Commandments of Modern Genesis





Einstein's 1917 Static World Model

1. Assumed the cosmological principle: “All locations in the universe are equivalent; in particular the locally averaged density of stellar matter should therefore be the same everywhere.”
2. Assumed the universe was spherical (finite volume), space curved. Boundary condition on space: space has no boundary.
3. Assumed the universe is “static” (no beginning, no end): Boundary condition on time: time has no boundary.

Einstein's 1917 Static World Model

1. "All locations in the universe are equivalent; in particular the locally averaged density of stellar matter should therefore be the same everywhere."
2. Space is finite, hence spherical.
3. "Spatial structure and density should be constant over time."
4. Dynamics determined by Einstein field equations of general relativity.
 1. Implies there is one dynamical variable, constant in space: the scale factor " $P(t)$ "
 2. Implies that P is the "radius of the universe."
 3. Implies that P is constant in time
 4. Implies that P satisfies the system of equations

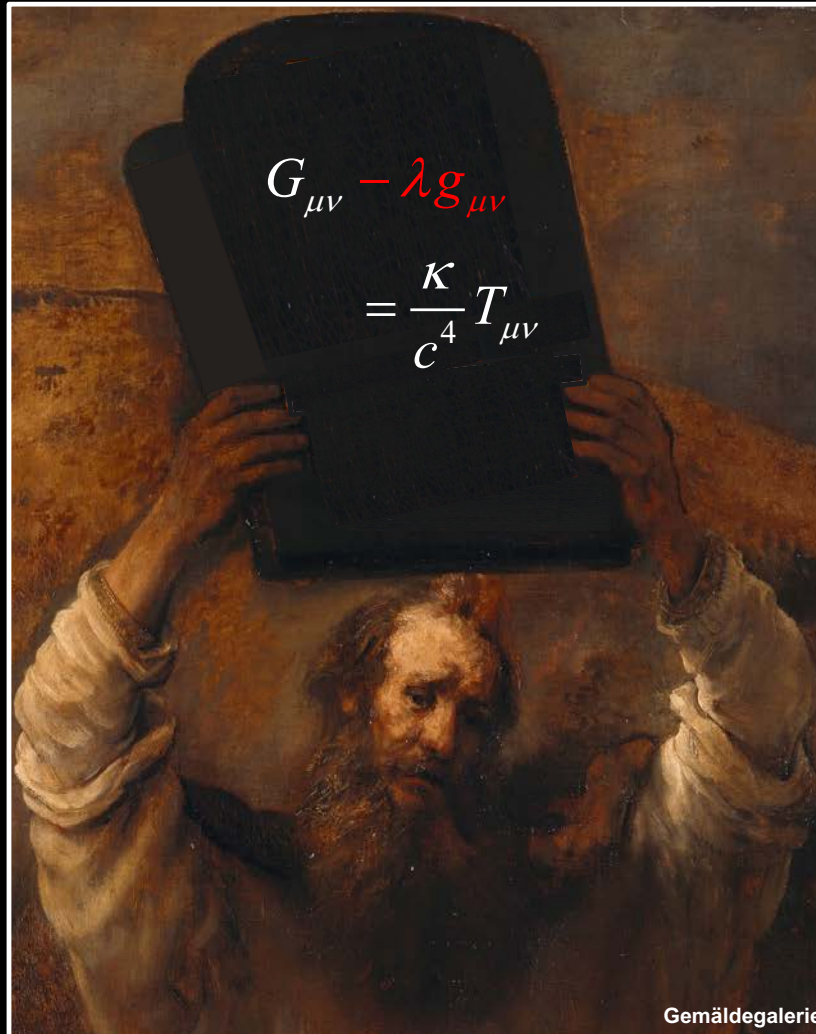
$$\begin{aligned}
 2 \frac{P''}{P} + \frac{P'^2}{P^2} + \frac{c^2}{P^2} &= 0 & P &= \text{scale factor (radius of universe)} \\
 \frac{3P'^2}{P^2} + \frac{3c^2}{P^2} &= \kappa c^2 \rho & P'' &= \text{acceleration} \\
 & & P' &= \text{velocity} \\
 & & \kappa &= 8\pi G_N \\
 & & \rho &= \text{mass density}
 \end{aligned}$$

Einstein's 1915-1916 field equations do not admit a static solution.

Ten Commandments of Modern Genesis

Left-hand side

cosmological
constant λ



Right-hand side

Einstein's 1917 Static World Model

Starting with a universe full of stars

Assumed

1. the cosmological principle: "All locations in the universe are equivalent; in particular the locally averaged density of stellar matter should therefore be the same everywhere."
2. the universe was spherical (finite volume), space curved.
3. the universe is "static."

Original field equations do not admit a static solution.

Added a cosmological constant.

Could calculate the volume of the universe in terms of λ and the stellar density.

Einstein's 1917 Static World Model

1. "All locations in the universe are equivalent; in particular the locally averaged density of stellar matter should therefore be the same everywhere." [Cosmological Principle]
2. "Spatial structure and density should be constant over time."
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4. Implies that P satisfies the system of equations

$$2\frac{P''}{P} + \frac{P'^2}{P^2} + \frac{c^2}{P^2} - \lambda = 0$$

$$\frac{3P'^2}{P^2} + \frac{3c^2}{P^2} - \lambda = \kappa c^2 \rho$$

P = scale factor (radius of universe)

P' = velocity

P'' = acceleration

$\kappa = 8\pi G$

ρ = mass density

Einstein's 1915-1916 field equations do not admit a static solution.

They do with the introduction of λ . $\lambda = \frac{c^2}{P^2}$ $\frac{2c^2}{P^2} = \kappa c^2 \rho$

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$$\begin{array}{ll} \frac{c^2}{P^2} - \lambda = 0 & P = \text{scale factor (radius of universe)} \\ & P' = \text{velocity} \\ & P'' = \text{acceleration} \\ \frac{3c^2}{P^2} - \lambda = \kappa c^2 \rho & \kappa = 8\pi G \\ & \rho = \text{mass density} \end{array}$$

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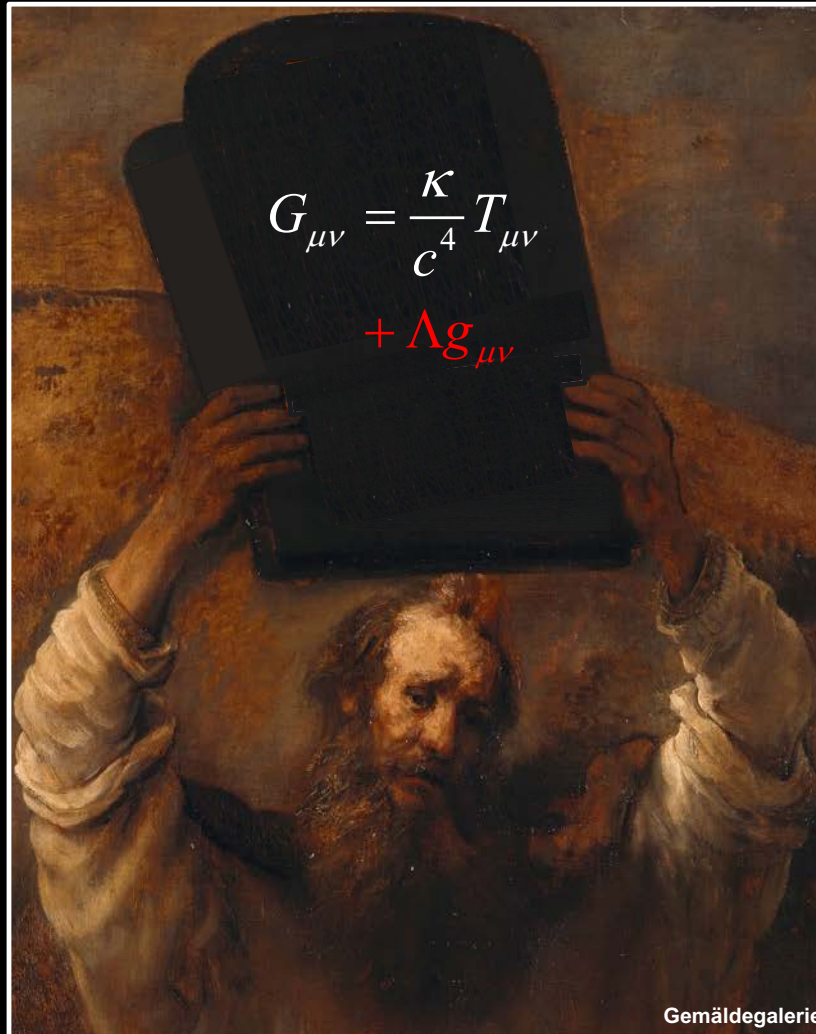
Ten Commandments of Modern Genesis

Left-hand side

$$G_{\mu\nu} = \frac{\kappa}{c^4} T_{\mu\nu} + \Lambda g_{\mu\nu}$$

Right-hand side

dark energy Λ





Hubble



University of Chicago 1909 National Champions

Hubble Recommendation Letters From the UChicago Archives



“... physically, he is a splendid specimen...”

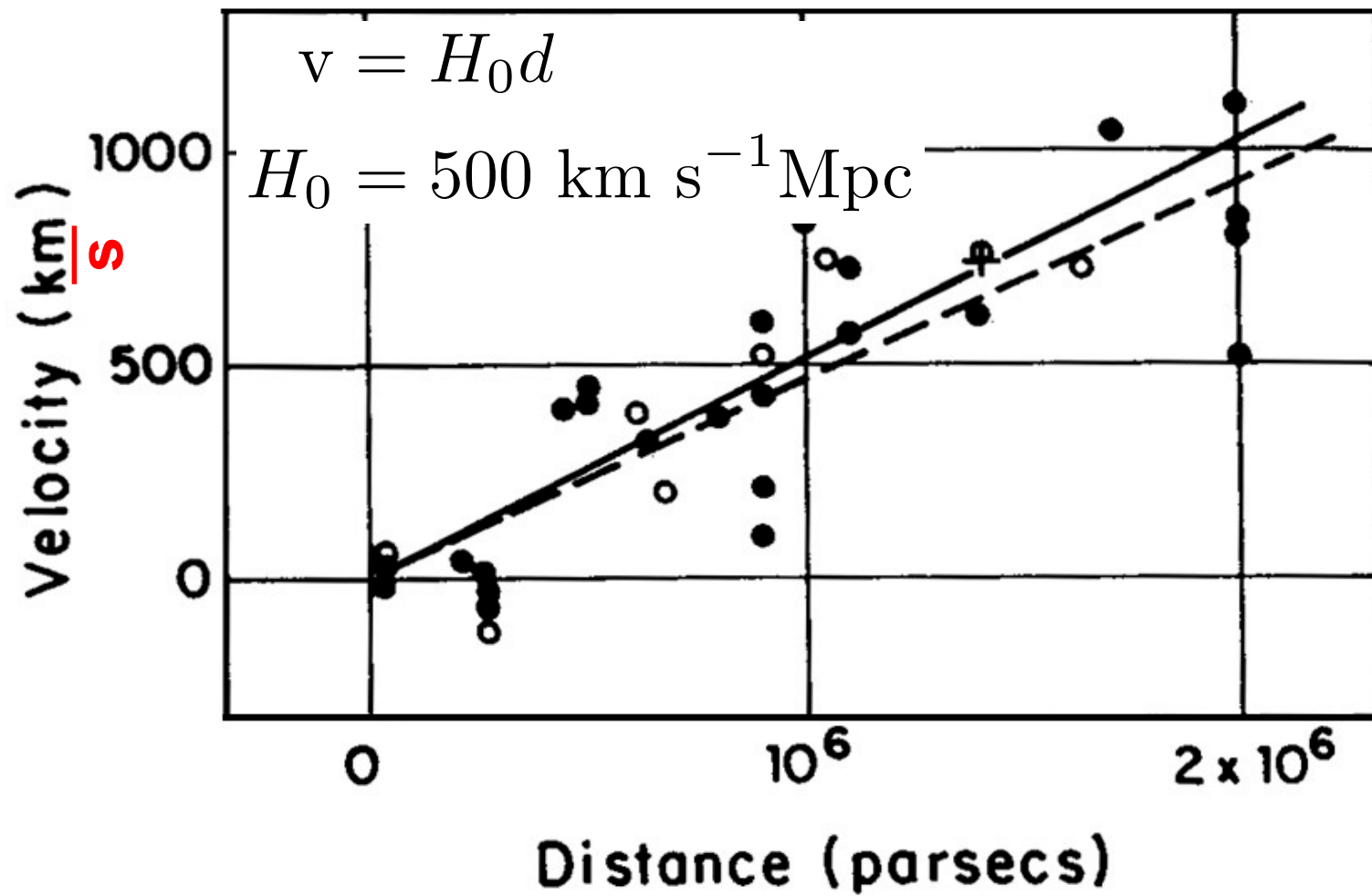
“... magnificent physique”

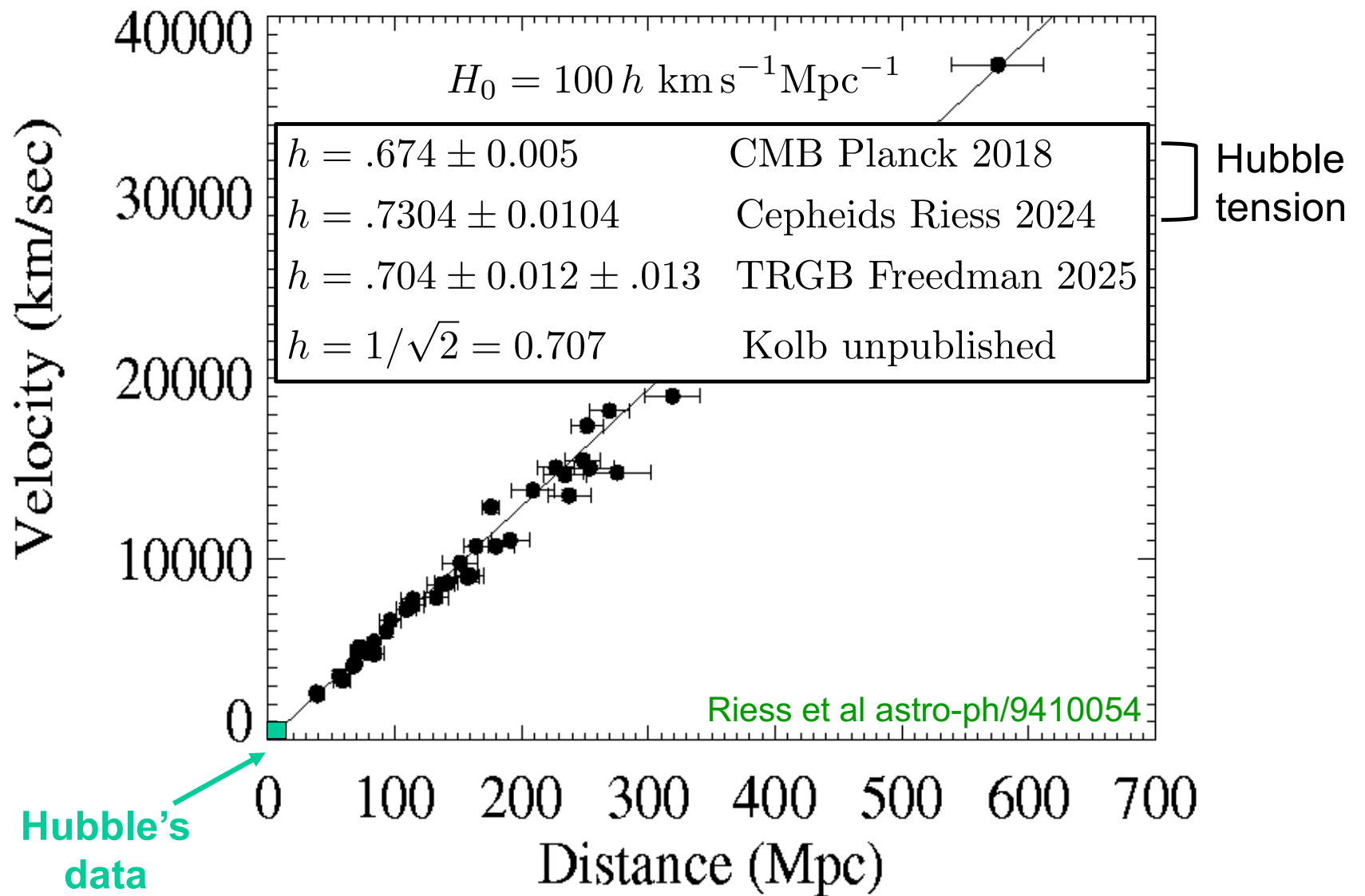
“... manly ...”

“... loveable character ...”

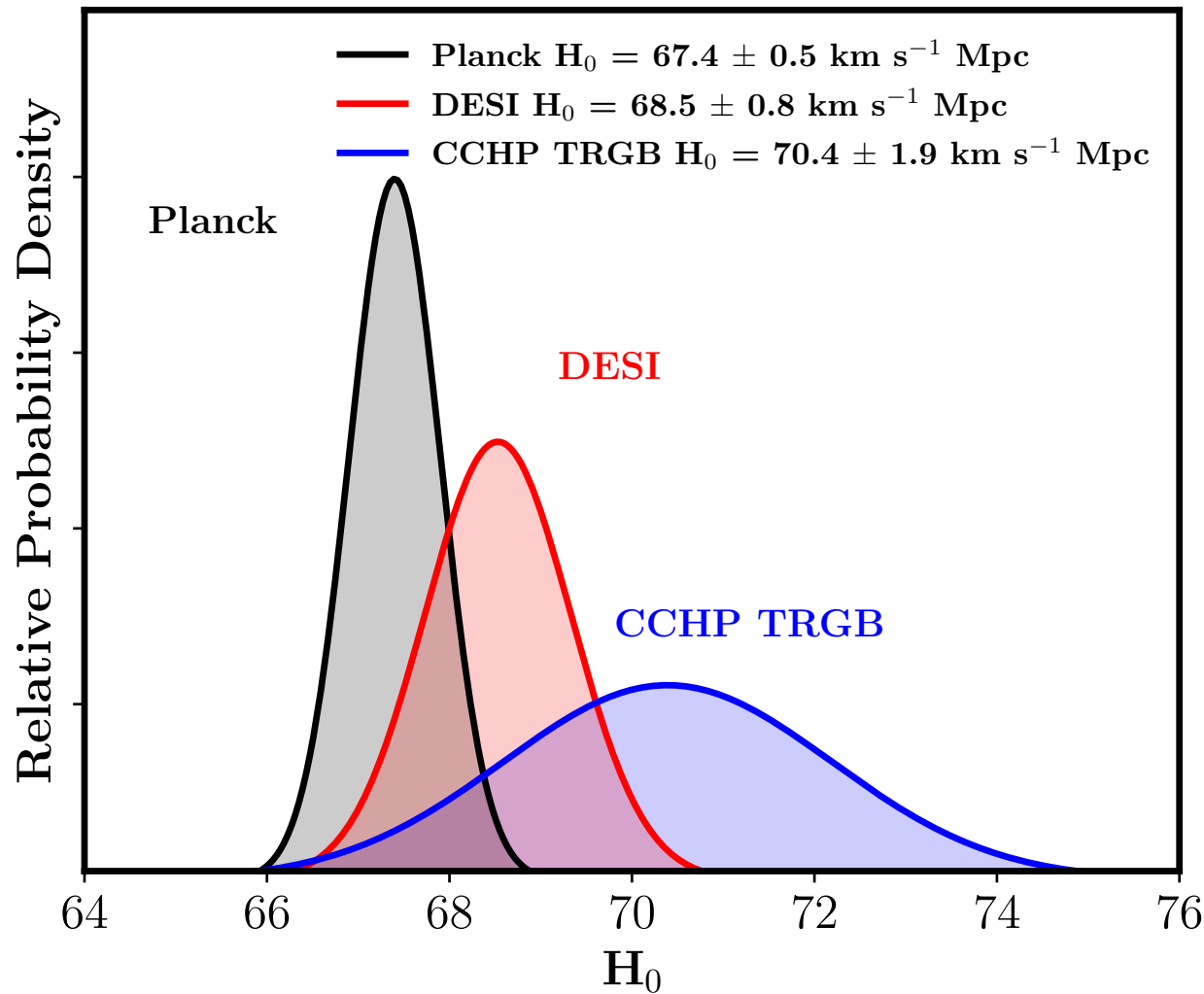


Hubble's 1929 Discovery Paper





Freedman et al. (2025) 2408.06153v2



Using only JWST + SNe

$$H_0 = 68.81 \pm 1.79 \pm 1.32 \text{ TRGB}$$

$$H_0 = 67.80 \pm 2.17 \pm 1.64 \text{ JAGB}$$

Using HST + JWST + SNe

$$H_0 = 70.4 \pm 1.2 \pm 1.3 \text{ TRGB}$$

“Our results are consistent with the current standard LCDM model without the need for the inclusion of new physics.”

Big-Bang Theory

Robertson-Walker metric $\longleftrightarrow G_{\mu\nu} = 8\pi G_N T_{\mu\nu}$ \longrightarrow Perfect-fluid stress tensor

$a(t)$ = cosmic scale factor

$$ds^2 = dt^2 - a^2(t) \left(\frac{dr^2}{1 - kr^2} + r^2 d\Omega^2 \right)$$

$k = +1, -1, 0$ for spherical,
hyperbolic, or flat 3-spaces.

Often convenient to use
conformal time η

$$dt = a d\eta$$

$$ds^2 = a^2(\eta)(d\eta^2 + d\vec{x}^2)$$

ρ = energy density

p = pressure

$$T^\mu_\nu = \text{diag}(\rho, p, p, p)$$

Robertson-Walker Metric

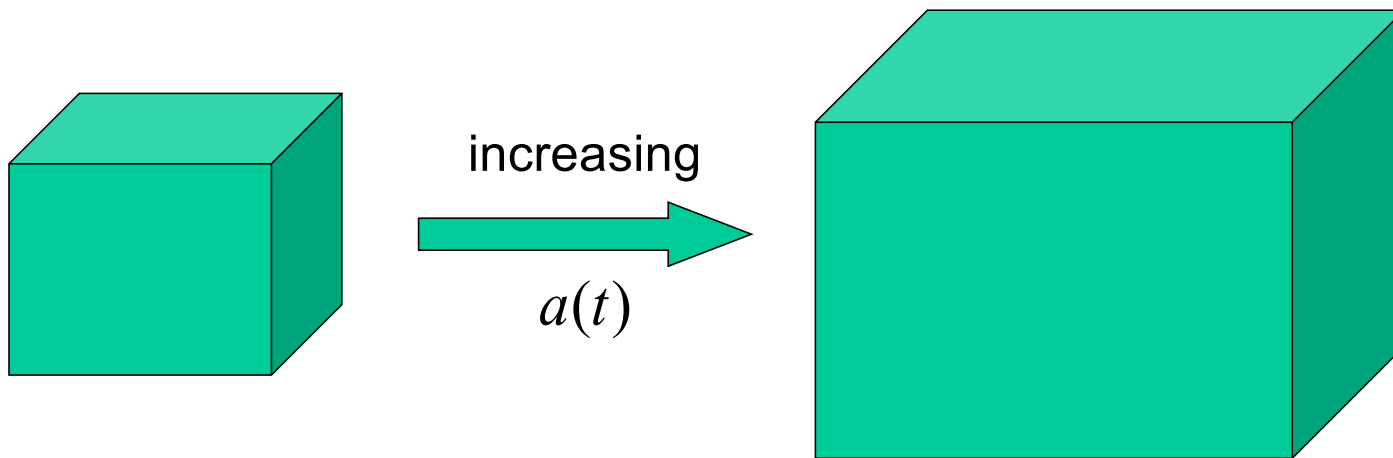
$$ds^2 = dt^2 - a^2(t) \left(\frac{dr^2}{1 - kr^2} + r^2 d\Omega^2 \right)$$

$k = 0$ (spatially flat)

$$ds^2 = dt^2 - a^2(t) (dr^2 + r^2 d\Omega^2) = dt^2 - a^2(t) (d\vec{x}^2 + d\vec{y}^2 + d\vec{z}^2)$$

(comoving coordinates: $\vec{x}, \vec{y}, \vec{z}$)

(physical distance: $d\vec{l}^2 = a^2(t) (d\vec{x}^2 + d\vec{y}^2 + d\vec{z}^2)$)



Perfect-Fluid Stress-Energy Tensor

- Must specify energy and pressure content of the Universe
- Assume pressure is related to energy density: $p_i = w_i \rho_i$
- Conservation of stress-energy tensor: $T^{\mu\nu}_{;\nu} = 0 \rightarrow \rho_i \propto a^{-3(1+w_i)}$

$T^{\mu\nu}$: fluids with different w

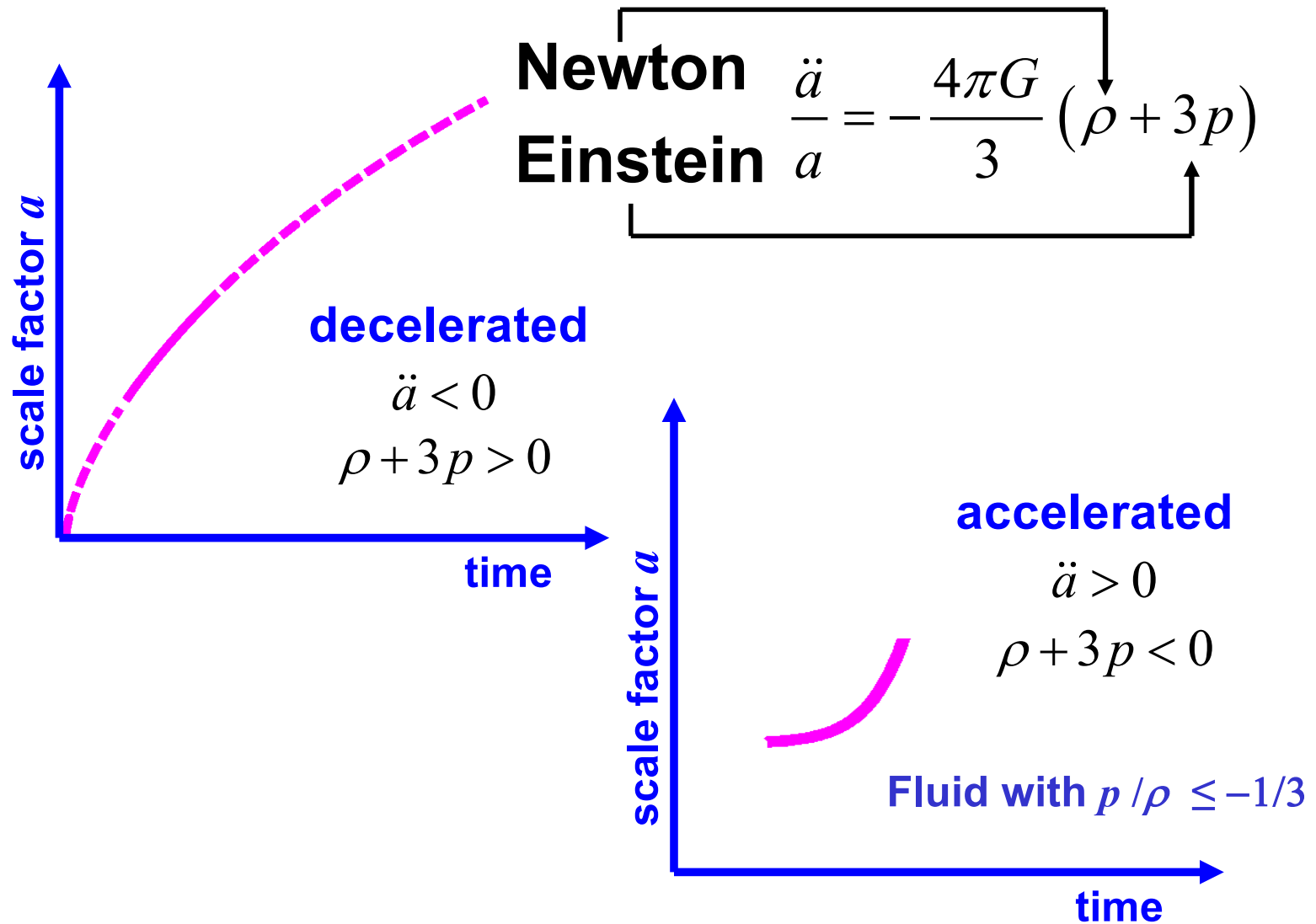
Kination:	$p_K = \rho_K$	$w = 1$	$\rho_K \propto a^{-6}$
Radiation:	$p_R = \rho_R/3$	$w = 1/3$	$\rho_R \propto a^{-4}$
Matter:	$p_M = 0$	$w = 0$	$\rho_M \propto a^{-3}$
Curvature:	$p_k = -\rho_k/3$	$w = -1/3$	$\rho_k \propto a^{-2}$
Vacuum (Λ):	$p_V = -\rho_V$	$w = -1$	$\rho_V \propto a^0$

Dynamics From Field Equations

$$(00) \quad \left(\frac{\dot{a}}{a} \right)^2 + \frac{k}{a^2} = \frac{8\pi G}{3} \rho \quad \text{Friedmann Equation}$$

$$(00) - (ii) \quad \frac{\ddot{a}}{a} = -\frac{4\pi G}{3} (\rho + 3p) \quad \text{Deceleration equation}$$

$$H \equiv \frac{\dot{a}}{a} = \text{expansion rate}$$



Expansion Rate Is A Key Quantity

Friedmann equation ($G_{00} = 8\pi G T_{00}$): Expansion rate $H(z)$

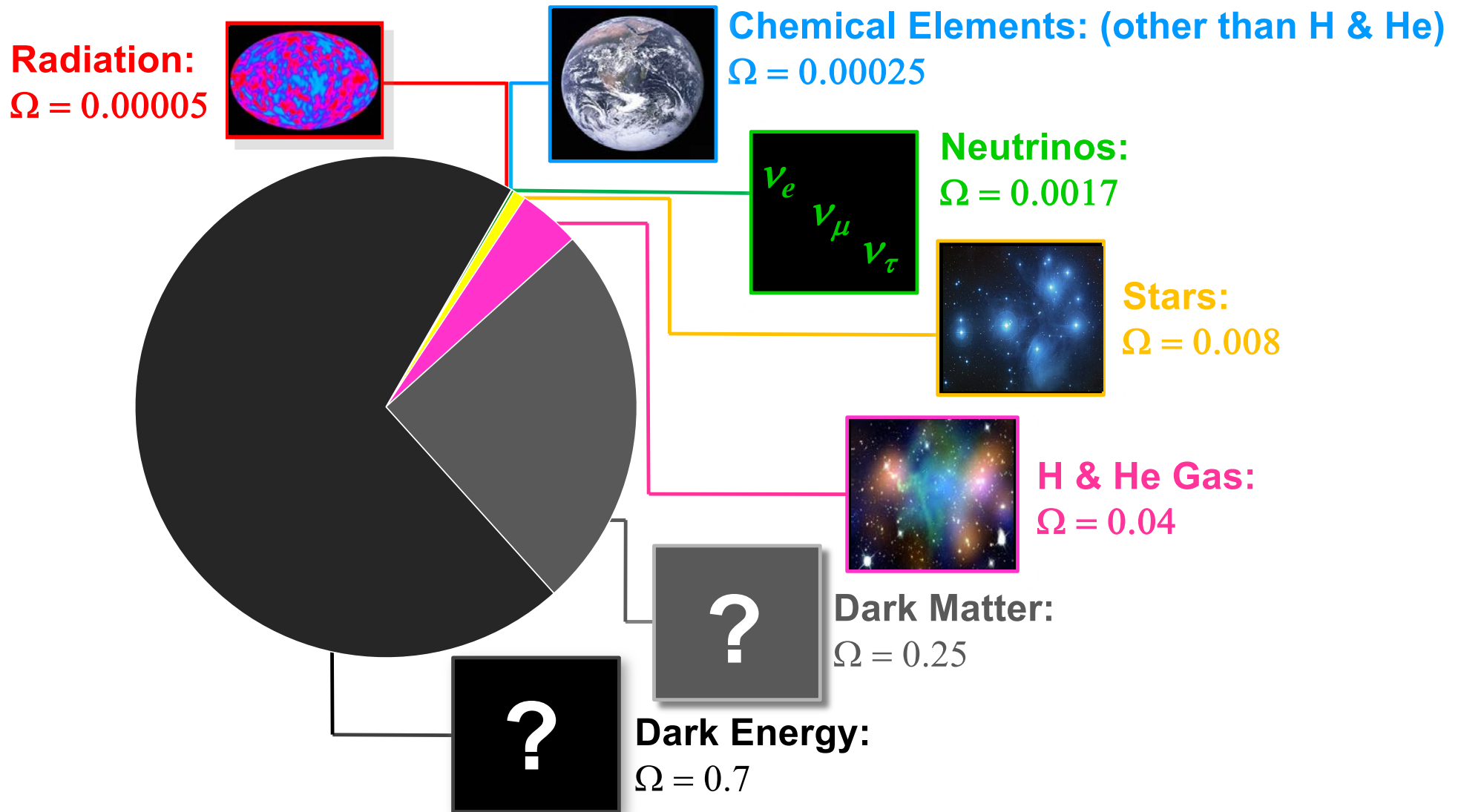
$$\Omega_i \equiv \rho_i / \rho_C \quad \rho_C \equiv 3H_0^2 / 8\pi G_N \quad 1+z = a_0/a$$

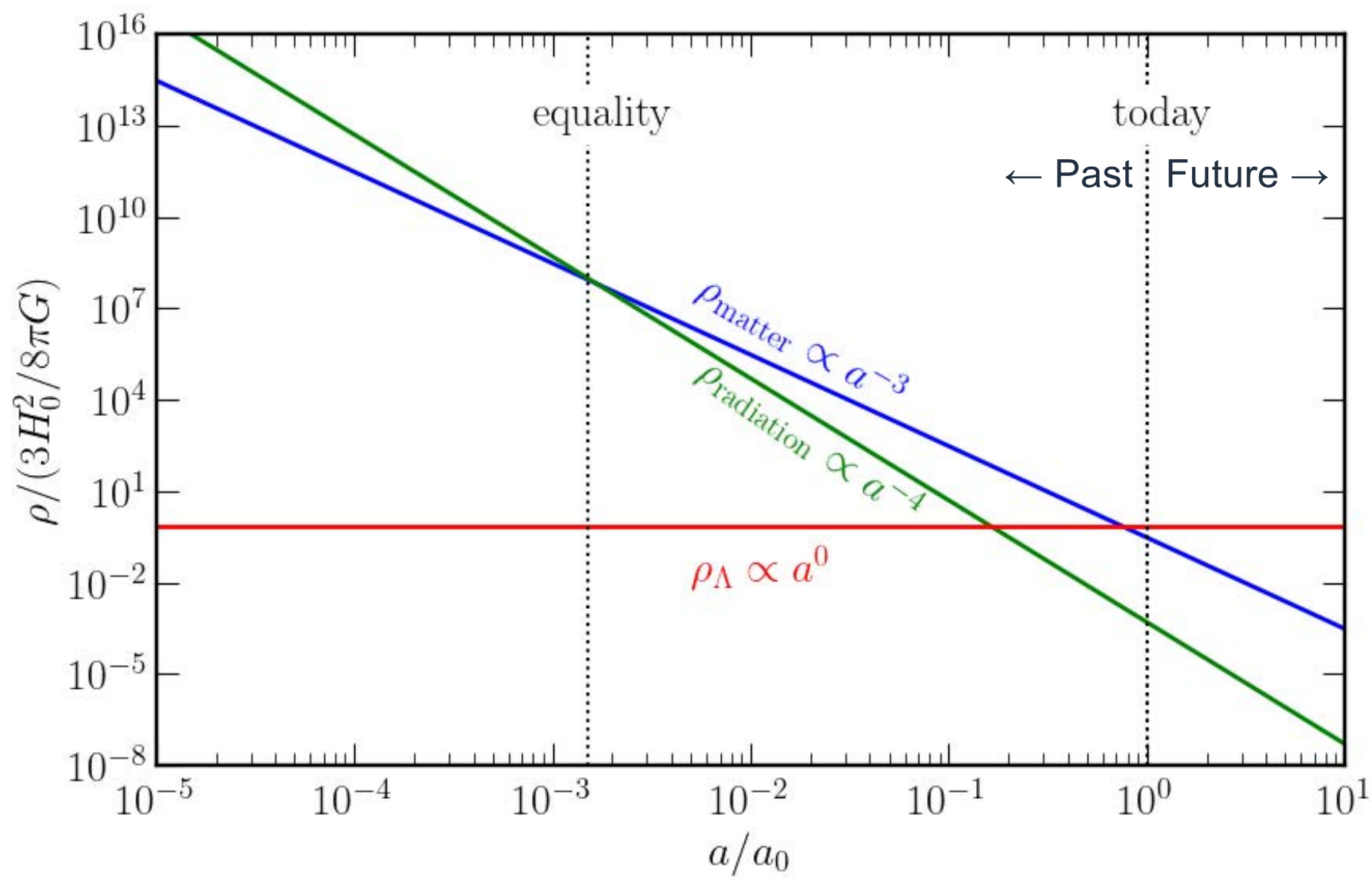
Hubble const. radiation matter curvature dark energy

$$H^2(z) = H_0^2 \left[\underbrace{\Omega_R (1+z)^4}_{\text{CMB}} + \underbrace{\Omega_M (1+z)^3}_{\text{LSS}} + \underbrace{(1-\Omega_{\text{TOTAL}})(1+z)^2}_{\text{CMB}} + \underbrace{\Omega_w (1+z)^{3(1+w)}}_{H(z)} \right]$$

Equation of state parameter: $w = P / \rho, \rho_i \propto a^{-3(1+w_i)}$

{	1	kination	ρ
	1/3	radiation	a^{-6}
	0	matter	a^{-4}
	-1/3	curvature	a^{-3}
	-1	Λ	a^{-2}
			a^0





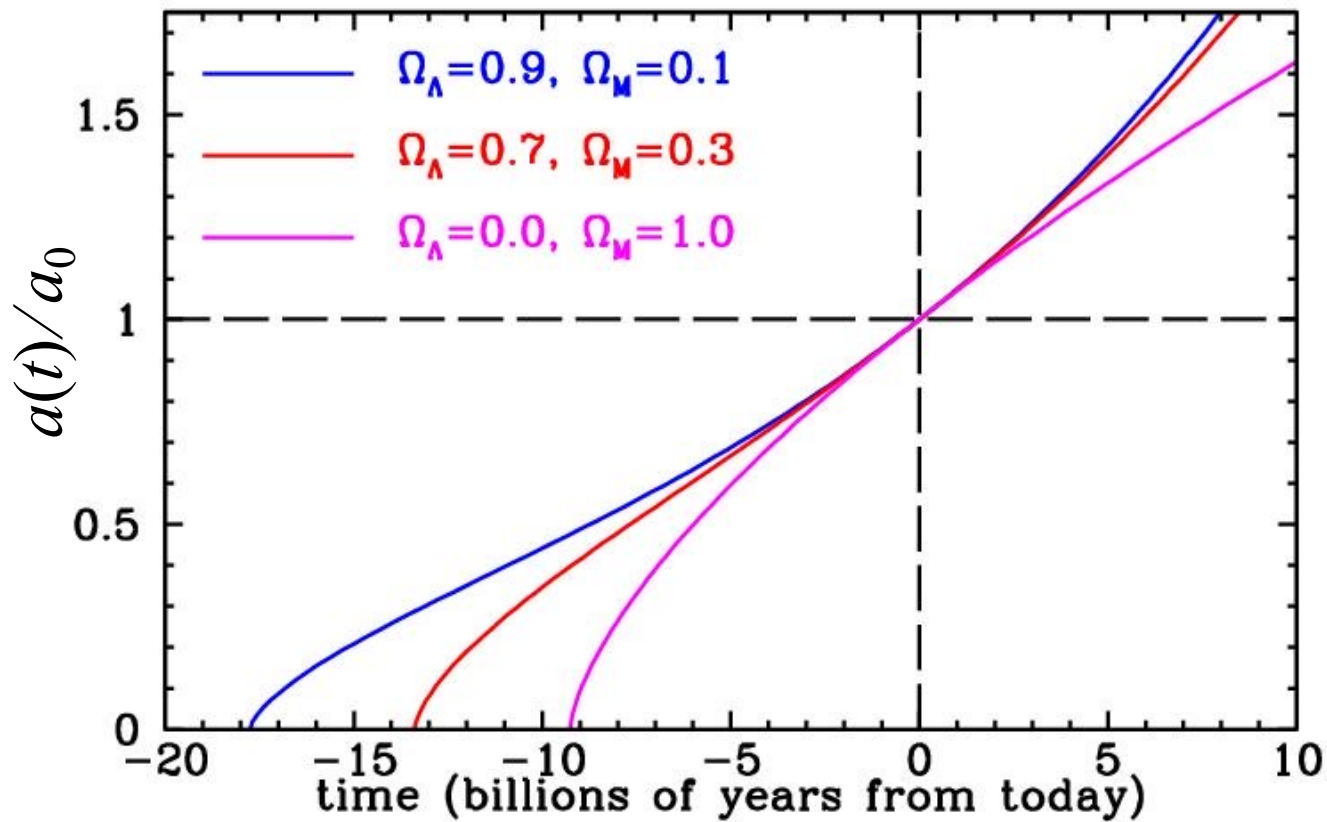
Dynamics \Rightarrow Evolution

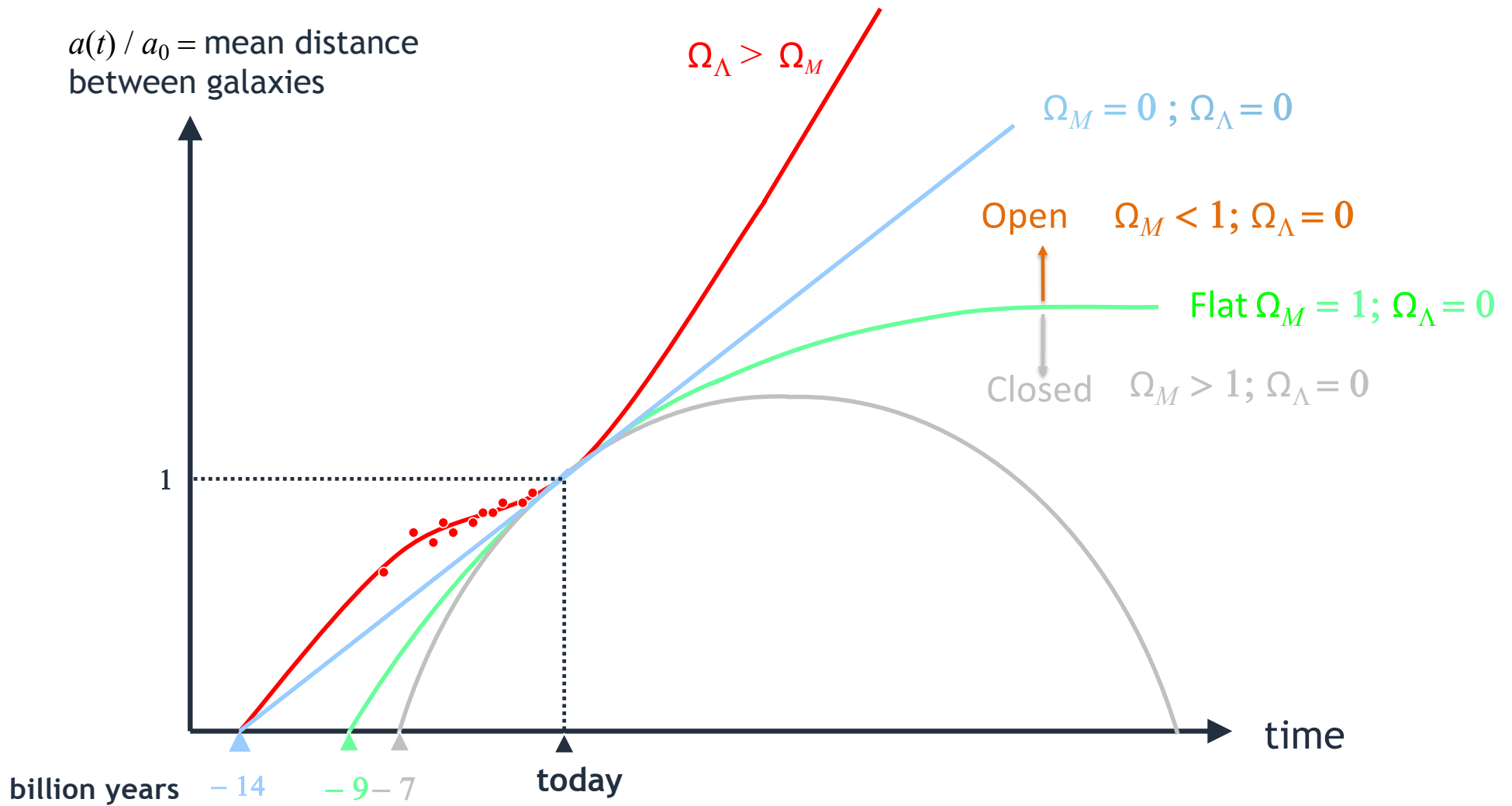
$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} = \frac{8\pi G}{3} \rho \quad \rho = \rho_M(a) + \rho_R(a) + \rho_\Lambda + \dots$$

- $a(t)$ & $H(t)$ depend on matter/energy content
- $a(t)$ measurable via redshift
- Redshift z is a proxy for time or scale factor: $1 + z = a_0/a(t)$

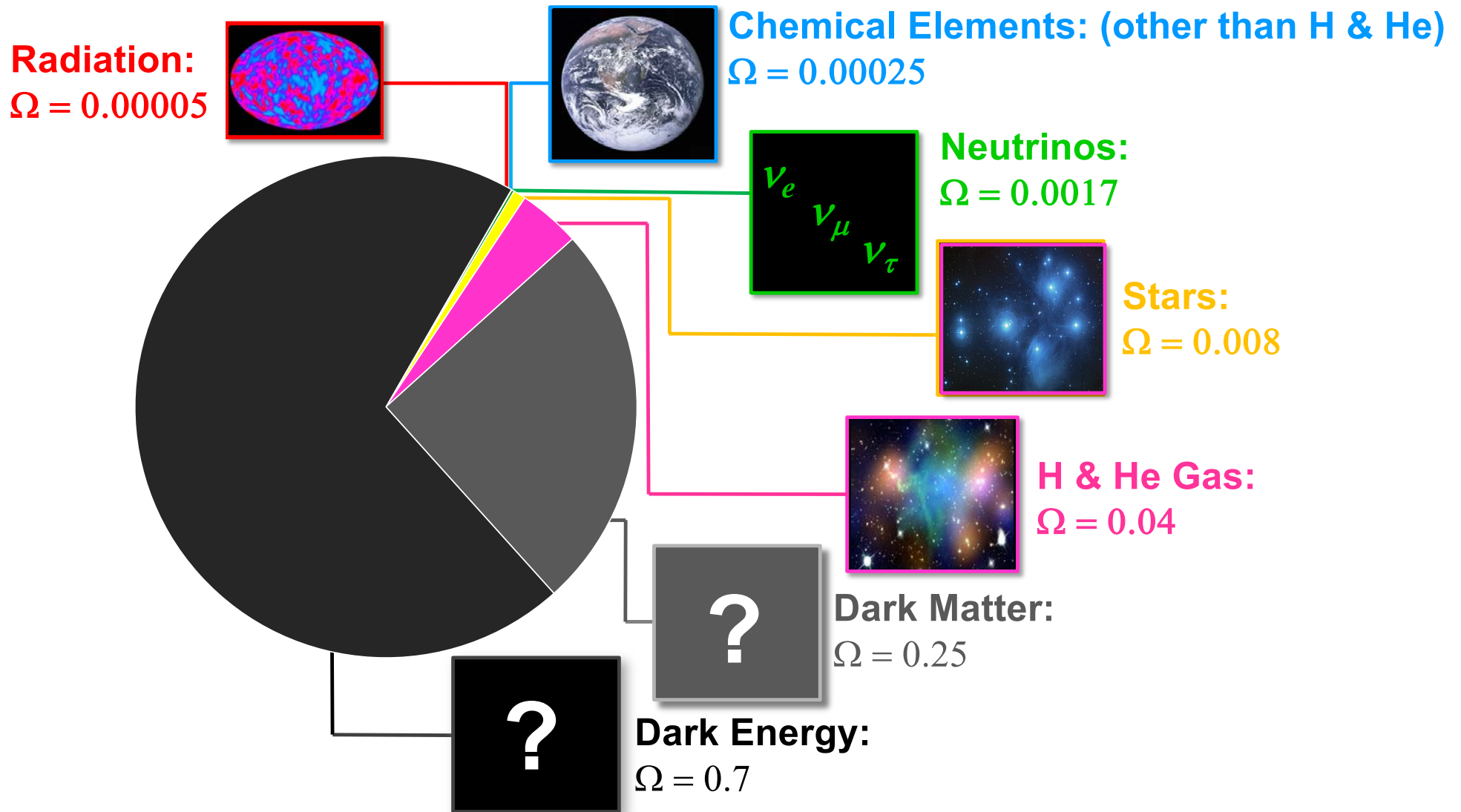
Evolution of $a(t)$ for Matter + Λ Flat Models

$$\left(\frac{\dot{a}}{a}\right)^2 = H_0^2 \left[\Omega_M \left(\frac{a_0}{a}\right)^3 + \Omega_\Lambda \right]$$





From B. Leibundgut



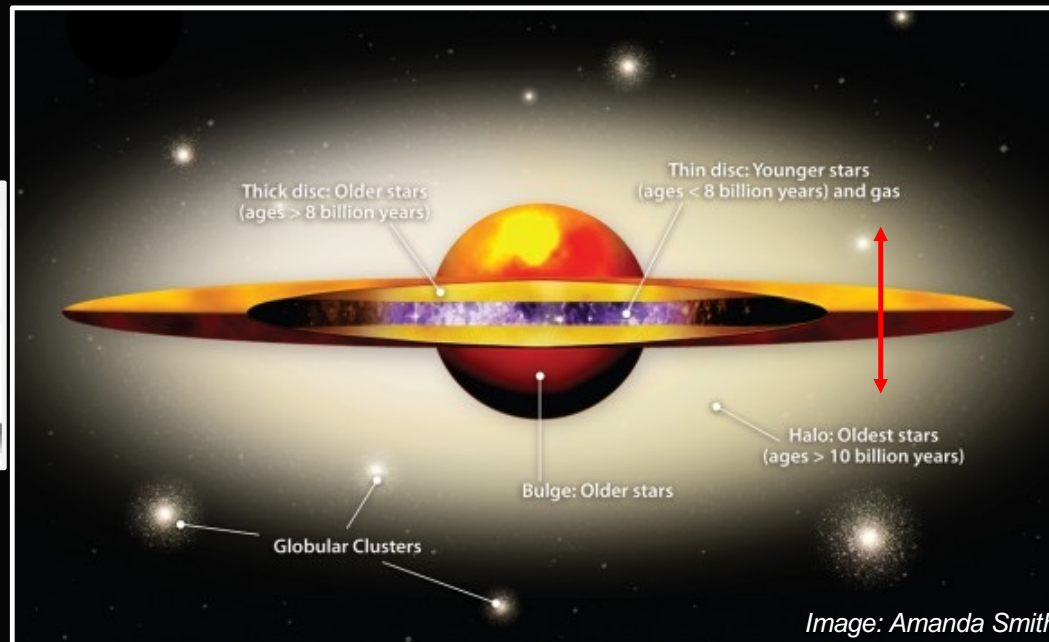
Ninety-three Years of Dark Matter

Sun: 2×10^{27} tons; 4×10^{26} watts \rightarrow Mass/Luminosity = 5 tons/watt

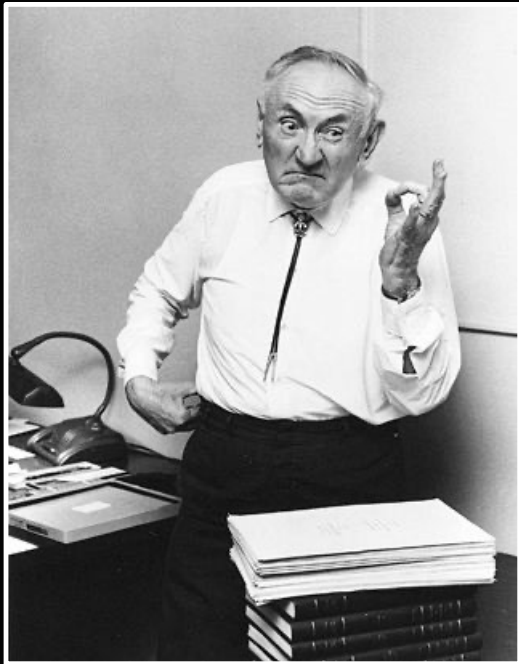
Oort 1932 Local Neighborhood Dim 10–15 tons/watt



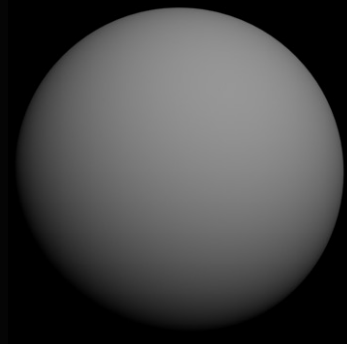
Jan Oort



Fritz Zwicky



Described his colleagues as
“spherical bastards”



Ninety-three Years of Dark Matter

Sun: 2×10^{27} tons; 4×10^{26} watts \rightarrow Mass/Luminosity = 5 tons/watt

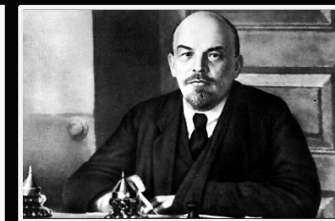
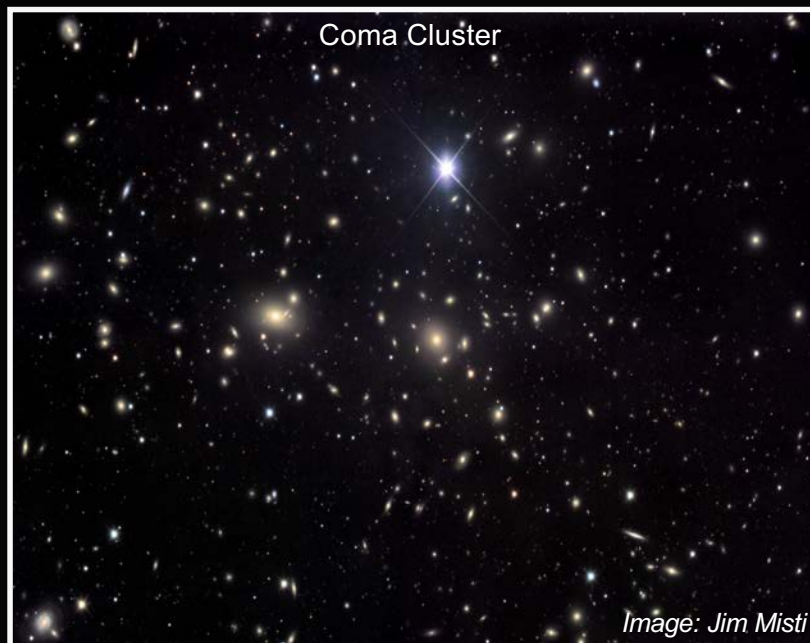
Oort	1932	Local Neighborhood Dim	10–15 tons/watt
Zwicky	1937	Galaxy Clusters Really Dark	2,500 tons/watt



Fritz Zwicky

Varna, Bulgaria

IN THIS HOME
WAS BORN FRITZ ZWICKY -
THE ASTRONOMER
WHO DISCOVERED
NEUTRON STARS
AND THE DARK MATTER
IN THE UNIVERSE.

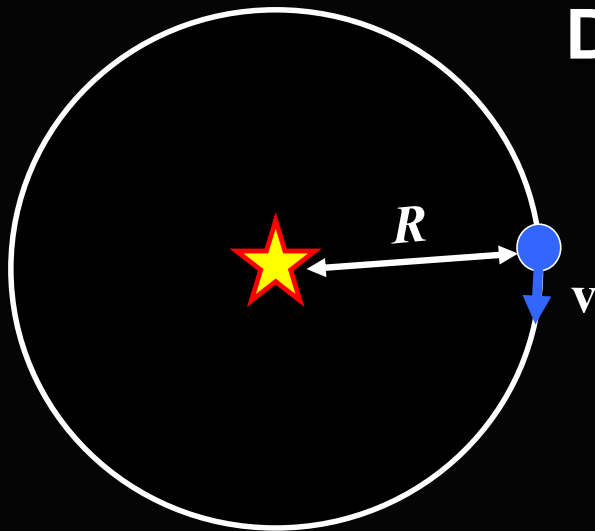


Vladimir Lenin 1916

Zurich, Switzerland
(Spiegelgasse 17)

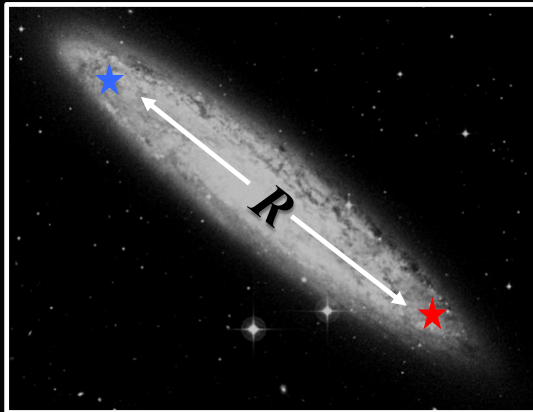
HIER WOHNTE
V. 21. FEBR. 1916 BIS 2. APRIL 1917
LENIN
DER FÜHRER DER RUSSISCHEN
REVOLUTION

Dark Matter



$$\frac{G_N m M_\odot}{R^2} = m \frac{v^2}{R} \quad (F = ma)$$

measure v & $R \rightarrow M_\odot$



$$v^2 = \frac{G_N M_{<R}}{R}$$

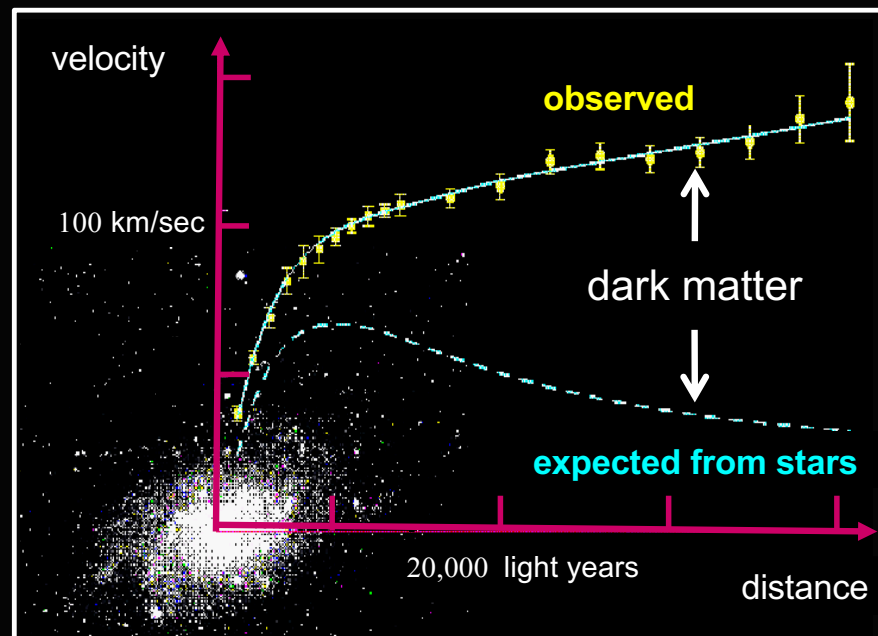
measure v & $R \rightarrow M_{<R}$

“outside” of galaxy, measure v & $R \rightarrow M_{\text{GALAXY}}$

Ninety-three Years of Dark Matter

Sun: 2×10^{27} tons; 4×10^{26} watts \rightarrow Mass/Luminosity = 5 tons/watt

Oort	1932	Local Neighborhood Dim	10–15 tons/watt
Zwicky	1937	Galaxy Clusters Really Dark	2,500 tons/watt
Rubin, Ford & others	1970s	Individual Galaxies Also Dark	300 tons/watt

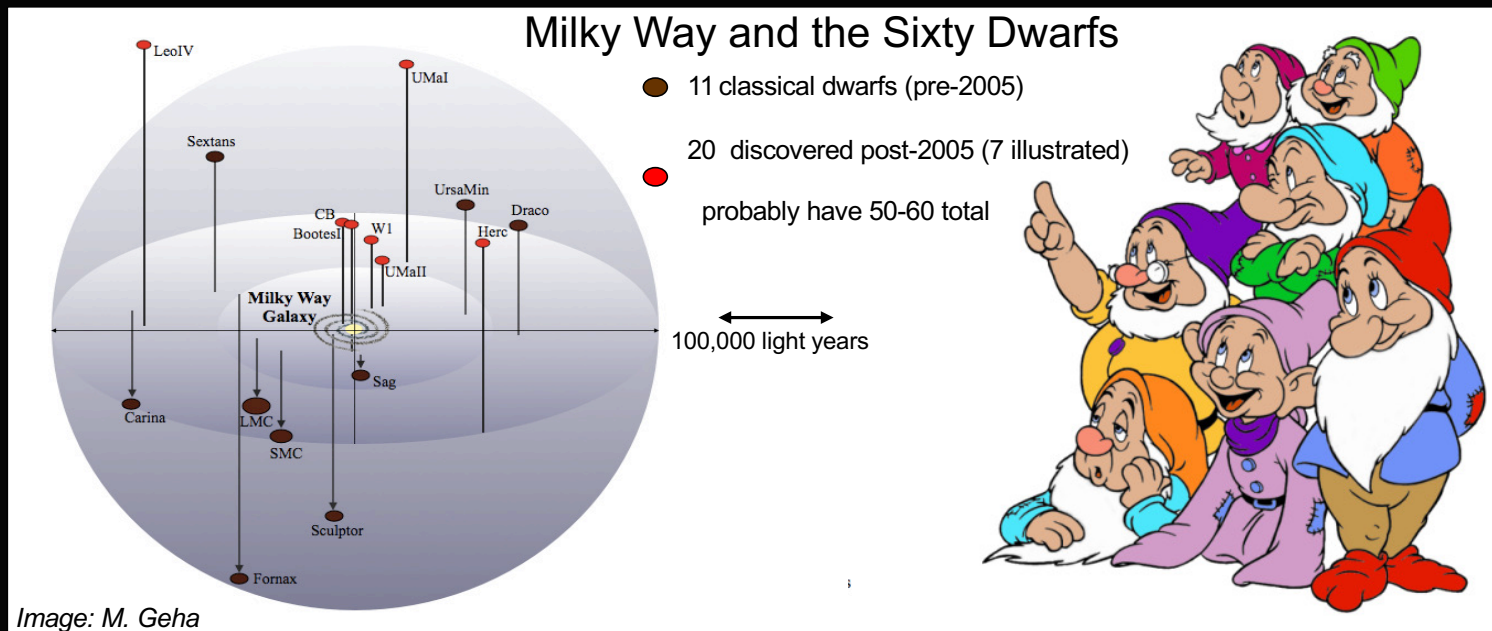


Vera Rubin

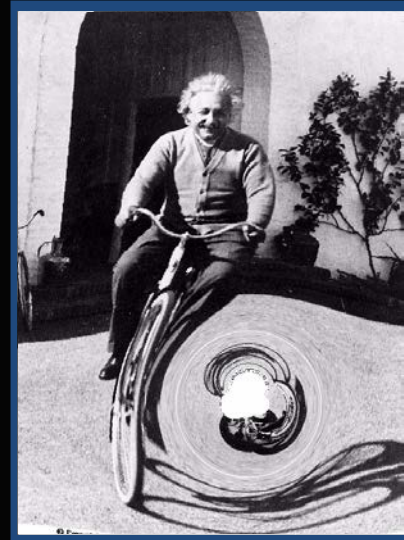
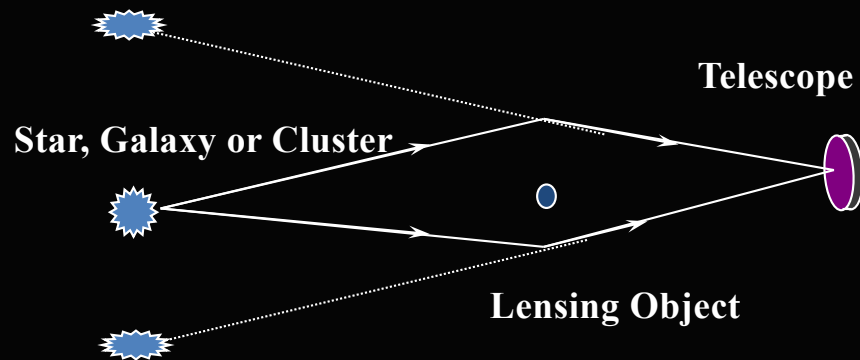
Ninety-three Years of Dark Matter

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Dwarf Observers	1990s	Dwarf Galaxies Darker Still	15,000 tons/watt

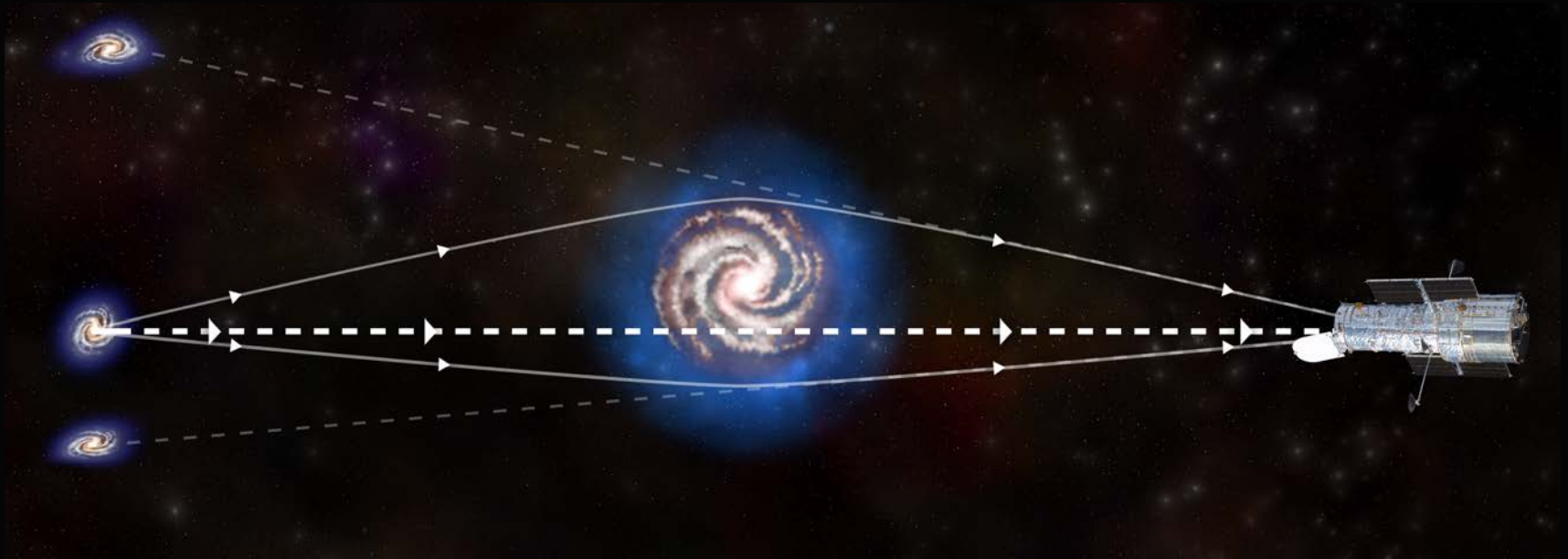


Gravitational Lensing



Gravitational Lensing

The bending of light by gravity makes it possible to measure the mass of a galaxy (scale of 10 kiloparsec) or cluster of galaxies (scales of dozens of megaparsec)



Gravitational Lensing



**Hubble Space
Telescope**

**Galaxy Cluster
Abell 2218**

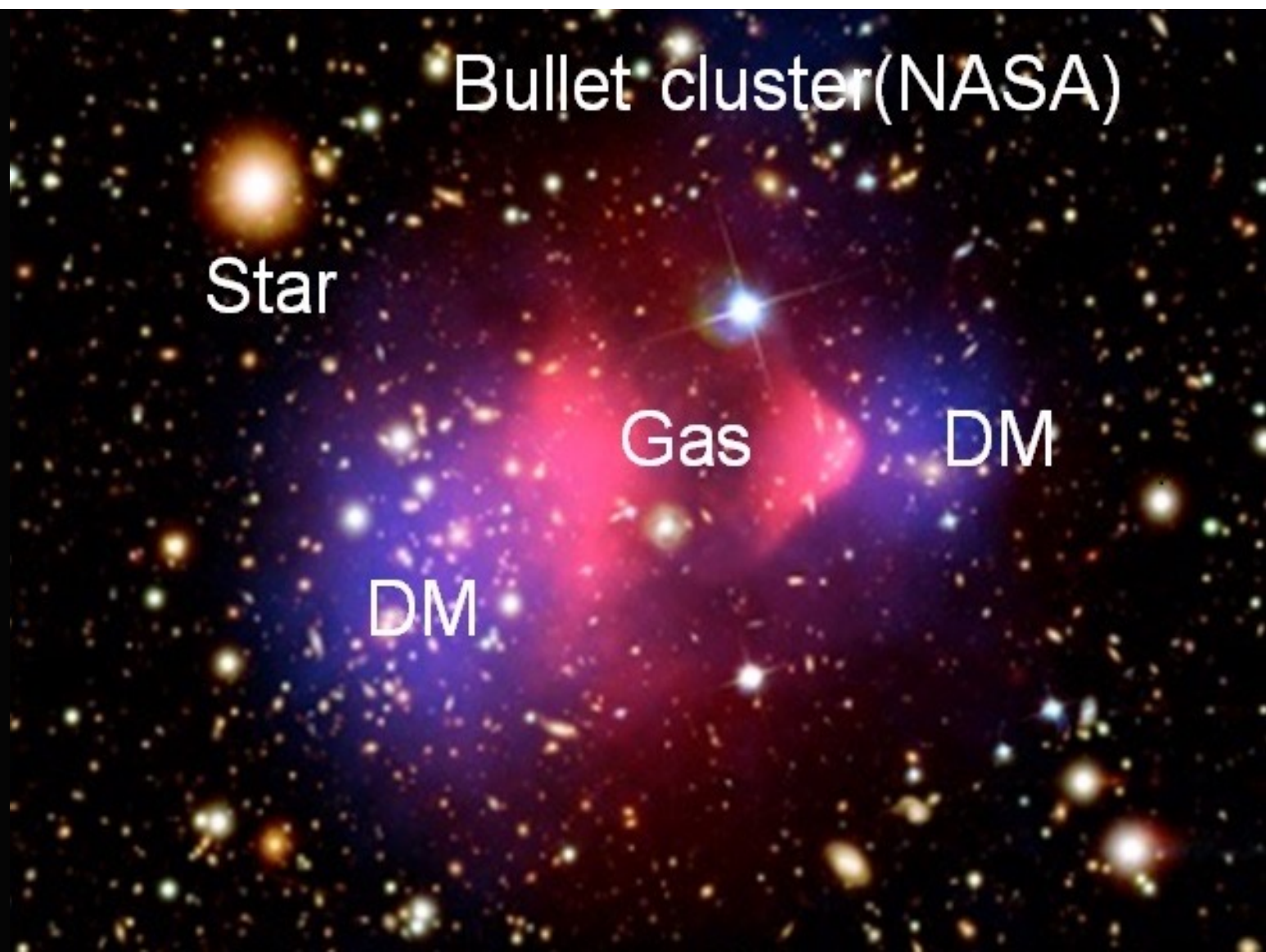
Bullet cluster(NASA)

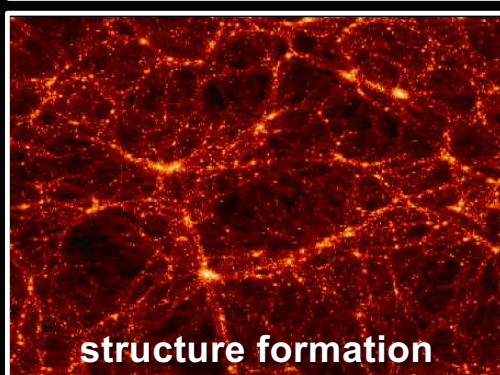
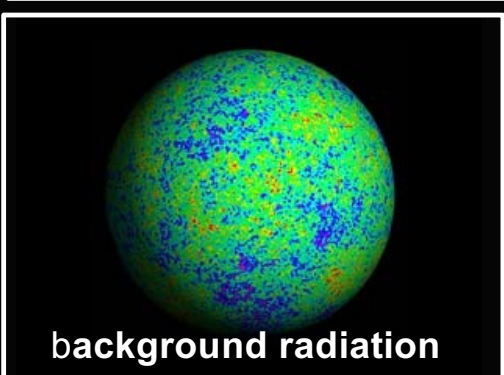
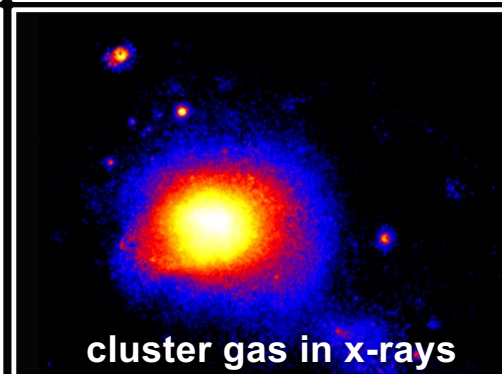
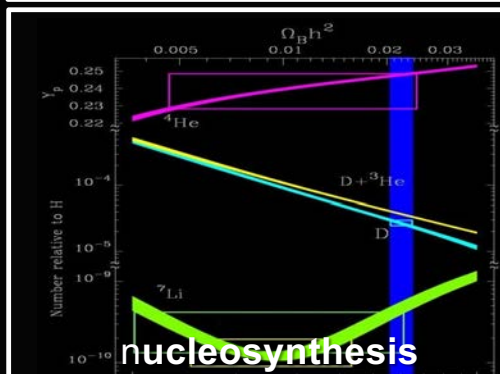
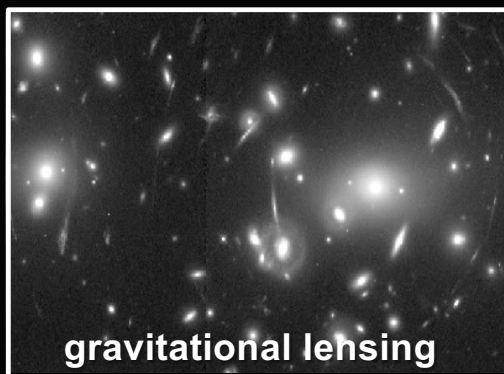
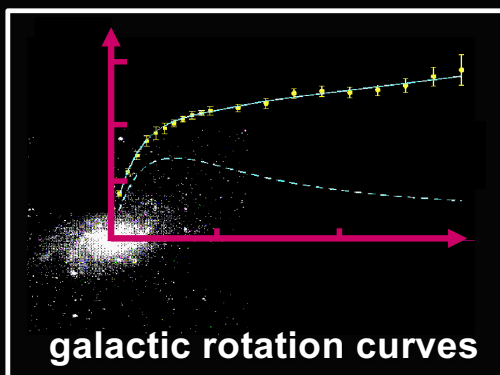
Star

Gas

DM

DM







What is a Simulation?

Oxford English Dictionary

Lost for
Words?



simulation



Find
Word

simulation

SECOND EDITION 1989

Pronunciation

Spellings

Etymology

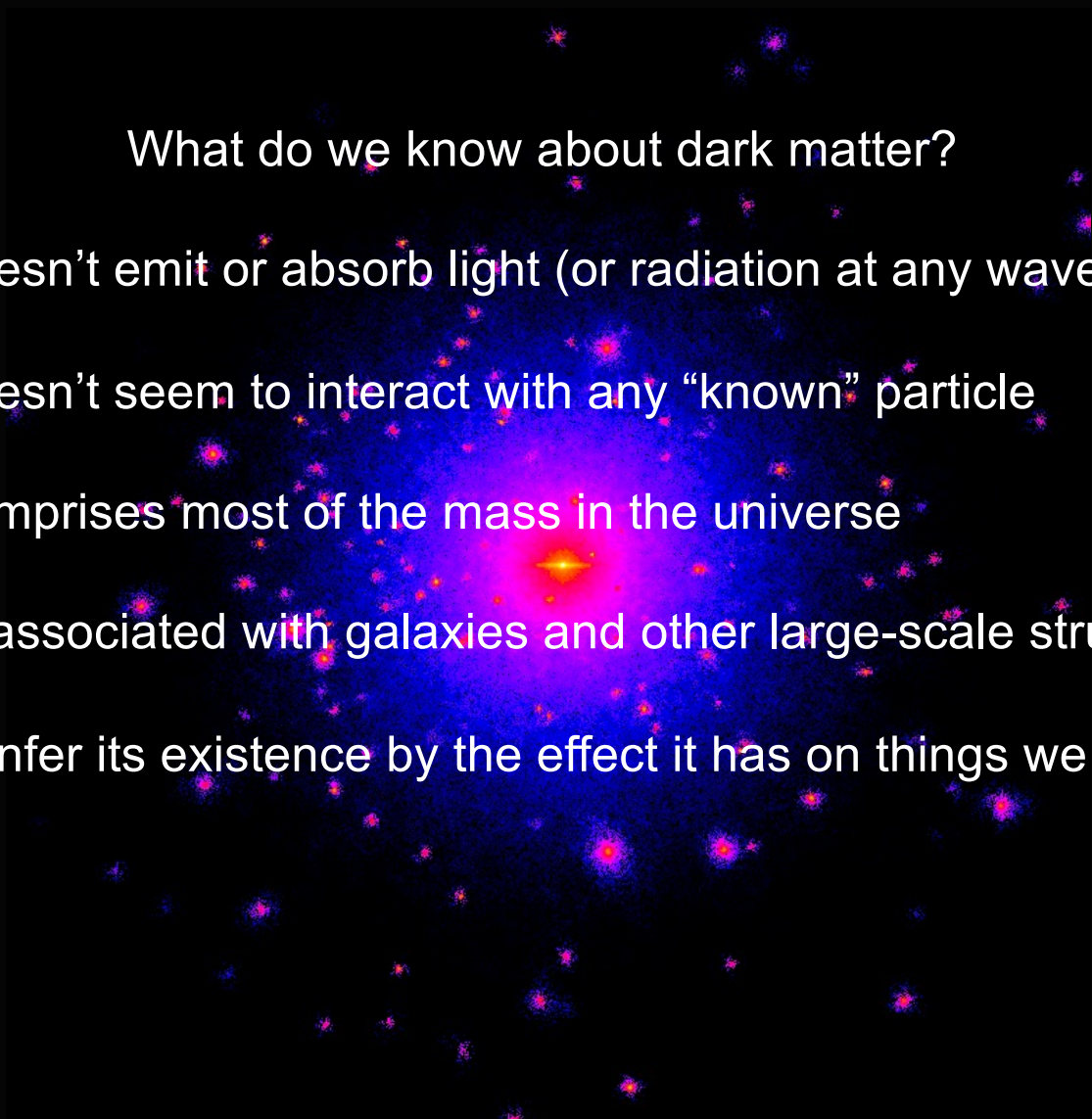
Quotations

Date chart

1. a. The action or practice of simulating, with intent to deceive; false pretence, deceitful profession.

We See Only the Tip of the Iceberg

Most of the matter is dark and it's not even "normal" stuff!



What do we know about dark matter?

1. It doesn't emit or absorb light (or radiation at any wavelength)
2. It doesn't seem to interact with any "known" particle
3. It comprises most of the mass in the universe
4. It is associated with galaxies and other large-scale structures
5. We infer its existence by the effect it has on things we do see

Dark Matter

- Einstein or Newton didn't have the last word
Modified Gravity
MOND (Modified Newtonian Dynamics, *i.e.*, $F \neq m a$)

- Rocky Rogue Planets

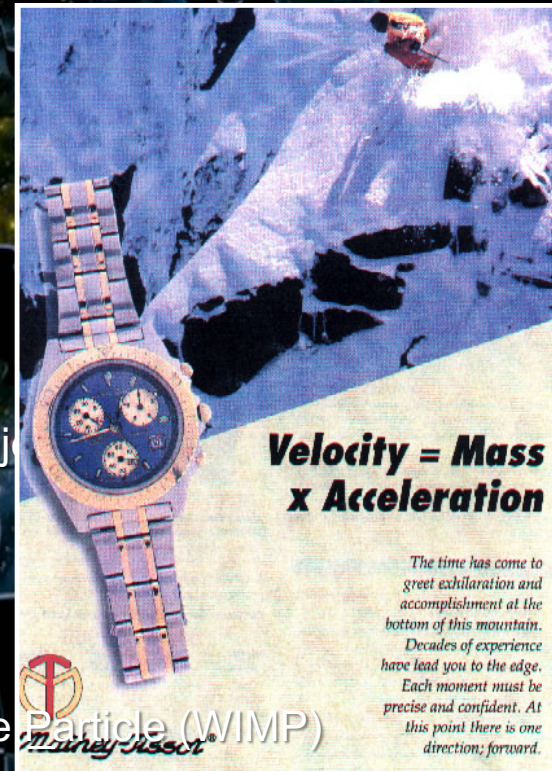
- Mass Challenged Stars

- Black Holes

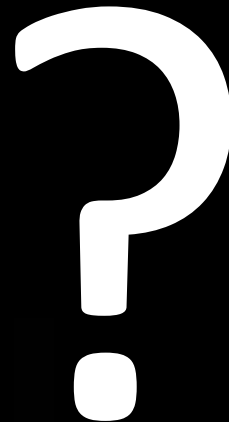
- Unknown Particle Species

Massive Compact Halo Obj

Weakly Interacting Massive Particle (WIMP)

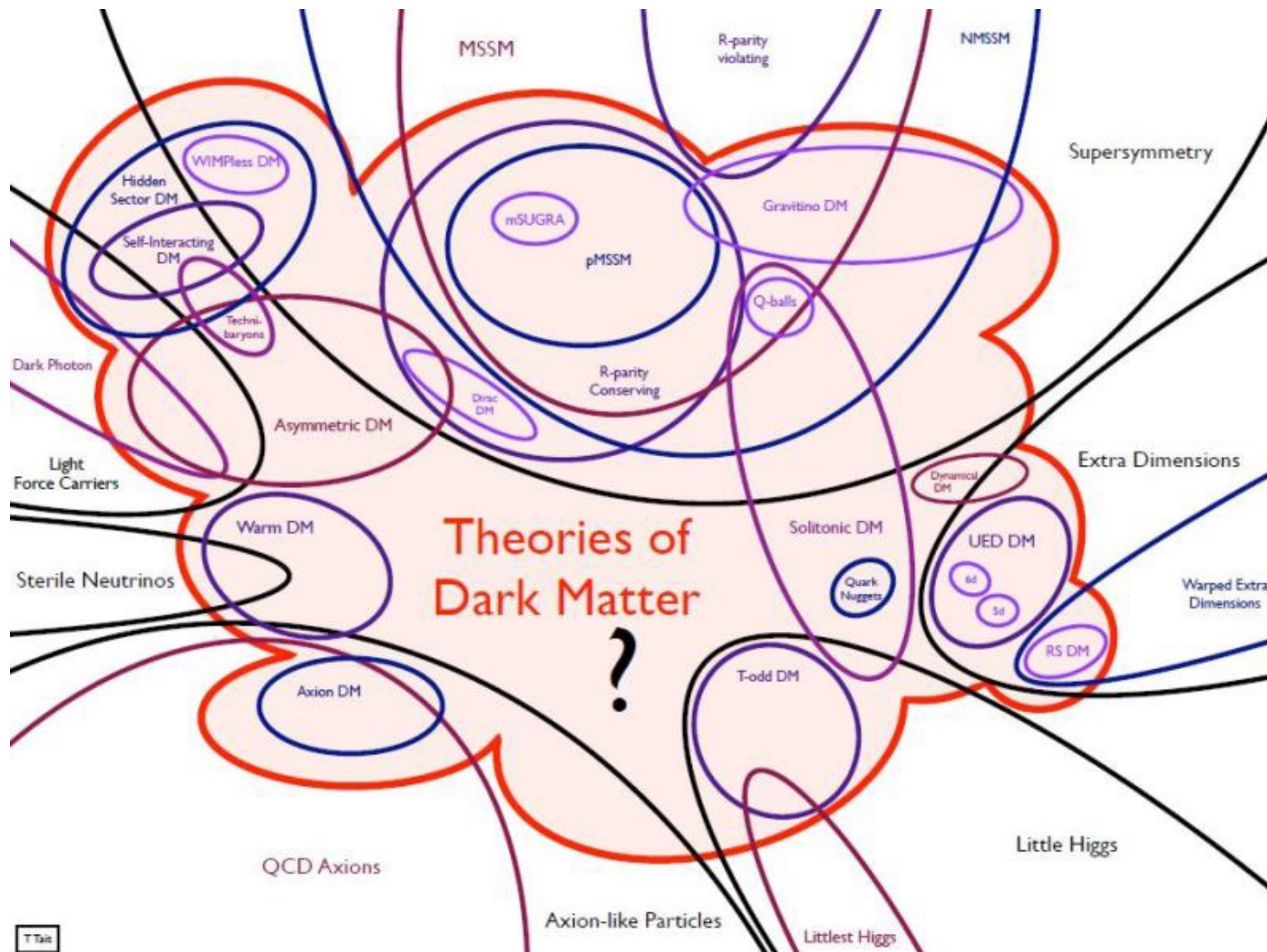


Known Particle Species



Dark particle must be stable and massive and interact weakly
Dark particle must be “Beyond the Standard Model” (BSM)

Let 10^3 flowers bloom?



Don't look now, but ...

... invisible things are passing through you!

A mysterious, invisible particle species is all around us,
a relic of the first fraction of a second of the Universe,
about a hundred million are in this room at any instant
flying around at about a half million miles per hour,
about 1 million-million will pass through you during this class,
but you can't see them, feel them, or smell them, and yet ...
... they shape the large-scale structure of the Universe.

A Fantastical Story!

Image: Navarro et al.

Particle Dark Matter Bestiary

- sub-eV mass neutrinos (WIMPs exist!) (hot)
 - sterile neutrinos, gravitini (warm)
 - lightest supersymmetric particle (cold)
 - lightest Kaluza-Klein particle (cold)
 - Bose-Einstein condensates
 - axions, axion clusters
 - solitons (Q-balls, B-balls, ...)
 - supermassive wimpzillas
- thermal relics
or decay of or
oscillation from
thermal relics
- from phase
transitions
- nonthermal
relics
- from inflation

Mass

10^{-22} eV (10^{-56} g) Bose-Einstein
 $10^{-8} M_{\odot}$ (10^{+25} g) axion clusters

Interaction Strength

only gravitational: wimpzillas
 strongly interacting: B balls

Freeze Out

Details in classic paper of Gondolo and Gelmini NPB360 (1991) 145

Consider a massive particle species χ , with mass m .

Assuming M-B statistics with phase-space density $f(p_\chi)$
(no stimulated emission or Pauli blocking)

Creation and annihilation of χ through interaction with a light field ψ
 $\psi_1 + \psi_2 \leftrightarrow \chi_1 + \chi_2$

Boltzmann equation for evolution of n_ψ (in absence of scattering, $n_\chi \propto a^{-3}$)

$$\dot{n}_\chi + 3Hn_\chi = \frac{1}{(2\pi)^8} \int \frac{d^3p_{\psi_1}}{2E_{\psi_1}} \int \frac{d^3p_{\psi_2}}{2E_{\psi_2}} \int \frac{d^3p_{\chi_1}}{2E_{\chi_1}} \int \frac{d^3p_{\chi_2}}{2E_{\chi_2}} [f(p_{\psi_1})f(p_{\psi_2}) - f(p_{\chi_1})f(p_{\chi_2})] \\ \times |\mathcal{M}_{\chi_1\chi_2 \rightarrow \psi_1\psi_2}|^2 \delta^4(p_{\psi_1} + p_{\psi_2} - p_{\chi_1} - p_{\chi_2})$$

Have assumed $|\mathcal{M}_{\psi_1\psi_2 \rightarrow \chi_1\chi_2}|^2 = |\mathcal{M}_{\chi_1\chi_2 \rightarrow \psi_1\psi_2}|^2$

Freeze Out

Kinetic Equilibrium and Chemical Equilibrium of a particle X :

Kinetic equilibrium if rate of reactions that change the energy of a particle is $\gg H$, e.g.,

$$\Gamma(\gamma + X \leftrightarrow \gamma + X) \gg H$$

$$f_X(p_X) = e^{-(E_X - \mu)/T} = e^{\mu/T} f_X^{\text{EQ}}(p_X)$$

Chemical equilibrium if rate of reactions that change the number of particles is $\gg H$, e.g.,

$$\Gamma(X + X \leftrightarrow \gamma + \gamma) \gg H \Rightarrow \mu_X = \mu_\gamma = 0$$

$$f_X(p_X) = e^{-E_X/T} = f_X^{\text{EQ}}(p_X)$$

In almost all cases, rate for kinetic equilibrium \gg rate for chemical equilibrium

Freeze Out

$$\begin{aligned} \dot{n}_\chi + 3Hn_\chi = & \frac{1}{(2\pi)^8} \int \frac{d^3p_{\psi_1}}{2E_{\psi_1}} \int \frac{d^3p_{\psi_2}}{2E_{\psi_2}} \int \frac{d^3p_{\chi_1}}{2E_{\chi_1}} \int \frac{d^3p_{\chi_2}}{2E_{\chi_2}} [f(p_{\psi_1})f(p_{\psi_2}) - f(p_{\chi_1})f(p_{\chi_2})] \\ & \times |\mathcal{M}_{\chi_1\chi_2 \rightarrow \psi_1\psi_2}|^2 \delta^4(p_{\psi_1} + p_{\psi_2} - p_{\chi_1} - p_{\chi_2}) \end{aligned}$$

Assume light particle in chemical equilibrium: $f_\psi(p) = e^{-E_\psi/T}$

$$f_\psi(p_{\psi_1})f_\psi(p_{\psi_2}) = e^{-E_{\psi_1}/T}e^{-E_{\psi_2}/T}, \quad \delta^4 \Rightarrow e^{-E_{\psi_1}/T}e^{-E_{\psi_2}/T} = e^{-E_{\chi_1}/T}e^{-E_{\chi_2}/T} = f_\chi^{\text{EQ}}(p_{\chi_1})f_\chi^{\text{EQ}}(p_{\chi_2})$$

$$\begin{aligned} \dot{n}_\chi + 3Hn_\chi = & \frac{1}{(2\pi)^8} \int \frac{d^3p_{\psi_1}}{2E_{\psi_1}} \int \frac{d^3p_{\psi_2}}{2E_{\psi_2}} \int \frac{d^3p_{\chi_1}}{2E_{\chi_1}} \int \frac{d^3p_{\chi_2}}{2E_{\chi_2}} [f^{\text{EQ}}(p_{\chi_1})f^{\text{EQ}}(p_{\chi_2}) - f(p_{\chi_1})f(p_{\chi_2})] \\ & \times |\mathcal{M}_{\chi_1\chi_2 \rightarrow \psi_1\psi_2}|^2 \delta^4(p_{\psi_1} + p_{\psi_2} - p_{\chi_1} - p_{\chi_2}) \end{aligned}$$

Rearrange:

$$\begin{aligned} \dot{n}_\chi + 3Hn_\chi = & \frac{1}{(2\pi)^8} \int d^3p_{\chi_1} \int d^3p_{\chi_2} [f^{\text{EQ}}(p_{\chi_1})f^{\text{EQ}}(p_{\chi_2}) - f(p_{\chi_1})f(p_{\chi_2})] \\ & \times \frac{1}{2E_{\chi_1}} \frac{1}{2E_{\chi_2}} \int \frac{d^3p_{\psi_1}}{2E_{\psi_1}} \frac{d^3p_{\psi_2}}{2E_{\psi_2}} |\mathcal{M}_{\chi_1\chi_2 \rightarrow \psi_1\psi_2}|^2 \delta^4(p_{\psi_1} + p_{\psi_2} - p_{\chi_1} - p_{\chi_2}) \end{aligned}$$

Freeze Out

$$\begin{aligned}\dot{n}_\chi + 3Hn_\chi &= \frac{1}{(2\pi)^8} \int d^3p_{\chi_1} \int d^3p_{\chi_2} [f^{\text{EQ}}(p_{\chi_1})f^{\text{EQ}}(p_{\chi_2}) - f(p_{\chi_1})f(p_{\chi_2})] \\ &\quad \times \frac{1}{2E_{\chi_1}} \frac{1}{2E_{\chi_2}} \int \frac{d^3p_{\psi_1}}{2E_{\psi_1}} \frac{d^3p_{\psi_2}}{2E_{\psi_2}} |\mathcal{M}_{\chi_1\chi_2 \rightarrow \psi_1\psi_2}|^2 \delta^4(p_{\psi_1} + p_{\psi_2} - p_{\chi_1} - p_{\chi_2})\end{aligned}$$

Annihilation cross section for $\chi_1 + \chi_2 \rightarrow \psi_1 + \psi_2$:

$$\sigma_A = \frac{1}{(2\pi)^2} \frac{1}{\sqrt{(p_{\chi_1} \cdot p_{\chi_2})^2 - m_\chi^4}} \int \frac{d^3p_{\psi_1}}{2E_{\psi_1}} \frac{d^3p_{\psi_2}}{2E_{\psi_2}} |\mathcal{M}_{\chi_1\chi_2 \rightarrow \psi_1\psi_2}|^2 \delta^4(p_{\psi_1} + p_{\psi_2} - p_{\chi_1} - p_{\chi_2})$$

$$\dot{n}_\chi + 3Hn_\chi = \frac{1}{(2\pi)^8} \int d^3p_{\chi_1} \int d^3p_{\chi_2} [f^{\text{EQ}}(p_{\chi_1})f^{\text{EQ}}(p_{\chi_2}) - f(p_{\chi_1})f(p_{\chi_2})] \frac{\sqrt{(p_{\chi_1} \cdot p_{\chi_2})^2 - m_\chi^4}}{2E_{\chi_1} 2E_{\chi_2}} \sigma_A$$

Freeze Out

$$\dot{n}_\chi + 3Hn_\chi = \frac{1}{(2\pi)^8} \int d^3p_{\chi_1} \int d^3p_{\chi_2} [f^{\text{EQ}}(p_{\chi_1})f^{\text{EQ}}(p_{\chi_2}) - f(p_{\chi_1})f(p_{\chi_2})] \frac{\sqrt{(p_{\chi_1} \cdot p_{\chi_2})^2 - m_\chi^4}}{2E_{\chi_1}2E_{\chi_2}} \sigma_A$$

Now assume χ in Kinetic Equilibrium: $f_\chi(p) = e^{\mu/T} e^{-E/T}$

$$\dot{n}_\chi + 3Hn_\chi = \frac{[1 - e^{2\mu/T}]}{(2\pi)^8} \int d^3p_{\chi_1} \int d^3p_{\chi_2} e^{-E_{x_1}/T} e^{-E_{x_2}/T} \frac{\sqrt{(p_{\chi_1} \cdot p_{\chi_2})^2 - m_\chi^4}}{2E_{\chi_1}2E_{\chi_2}} \sigma_A$$

$$\frac{\sqrt{(p_{\chi_1} \cdot p_{\chi_2})^2 - m_\chi^4}}{2E_{\chi_1}2E_{\chi_2}} \equiv |v_{\text{Moller}}| = \text{Moller velocity}$$

$$\dot{n}_\chi + 3Hn_\chi = [1 - e^{2\mu/T}] \int d^3p_{\chi_1} \int d^3p_{\chi_2} e^{-E_{x_1}/T} e^{-E_{x_2}/T} |v_{\text{Moller}}| \sigma_A$$

Freeze Out

$$\dot{n}_\chi + 3Hn_\chi = \left[1 - e^{2\mu/T}\right] \int d^3p_{\chi_1} \int d^3p_{\chi_2} e^{-E_{x_1}/T} e^{-E_{x_2}/T} |v_{\text{Moller}}| \sigma_A$$

$$\dot{n}_\chi + 3Hn_\chi = \underbrace{\int d^3p'_{\chi_1} \int d^3p'_{\chi_2} f^{\text{EQ}}(p'_{\chi_1}) f^{\text{EQ}}(p'_{\chi_2}) \left[1 - e^{2\mu/T}\right]}_{[(n_\chi^{\text{EQ}})^2 - n_\chi^2]} \underbrace{\frac{\int d^3p_{\chi_1} \int d^3p_{\chi_2} e^{-E_{x_1}/T} e^{-E_{x_2}/T} |v_{\text{Moller}}| \sigma_A}{\int d^3p'_{\chi_1} \int d^3p'_{\chi_2} f^{\text{EQ}}(p'_{\chi_1}) f^{\text{EQ}}(p'_{\chi_2})}}_{\langle |v_{\text{Moller}}| \sigma_A \rangle}$$

$$\dot{n}_\chi + 3Hn_\chi = [(n_\chi^{\text{EQ}})^2 - n_\chi^2] \langle |v_{\text{Moller}}| \sigma_A \rangle$$

Freeze Out

$$\dot{n}_\chi + 3Hn_\chi = \left[(n_\chi^{\text{EQ}})^2 - n_\chi^2 \right] \langle |v_{\text{Moller}}| \sigma_A \rangle$$

- I lost factors of 2π along the way, but the result is right.
- Have to account for identical particles in initial and final states.
- Take account of spin states.
- Need initial conditions
 - If at high temperatures ($T \gg m$) $n_\chi = n_\chi^{\text{EQ}}$ then “Freeze OUT”
 - If at high temperatures ($T \gg m$) $n_\chi \ll n_\chi^{\text{EQ}}$ then “Freeze IN”
- When $\text{Max} [n_\chi, n_\chi^{\text{EQ}}] \langle |v_{\text{Moller}}| \sigma_A \rangle \ll H$, $n_\chi \propto a^{-3}$ and $n_\chi a^{-3}$ frozen.
- More details in Goldolo and Gelmini.

Freeze Out

$$\dot{n}_\chi + 3Hn_\chi = \left[(n_\chi^{\text{EQ}})^2 - n_\chi^2 \right] \langle |v_{\text{Moller}}| \sigma_A \rangle$$

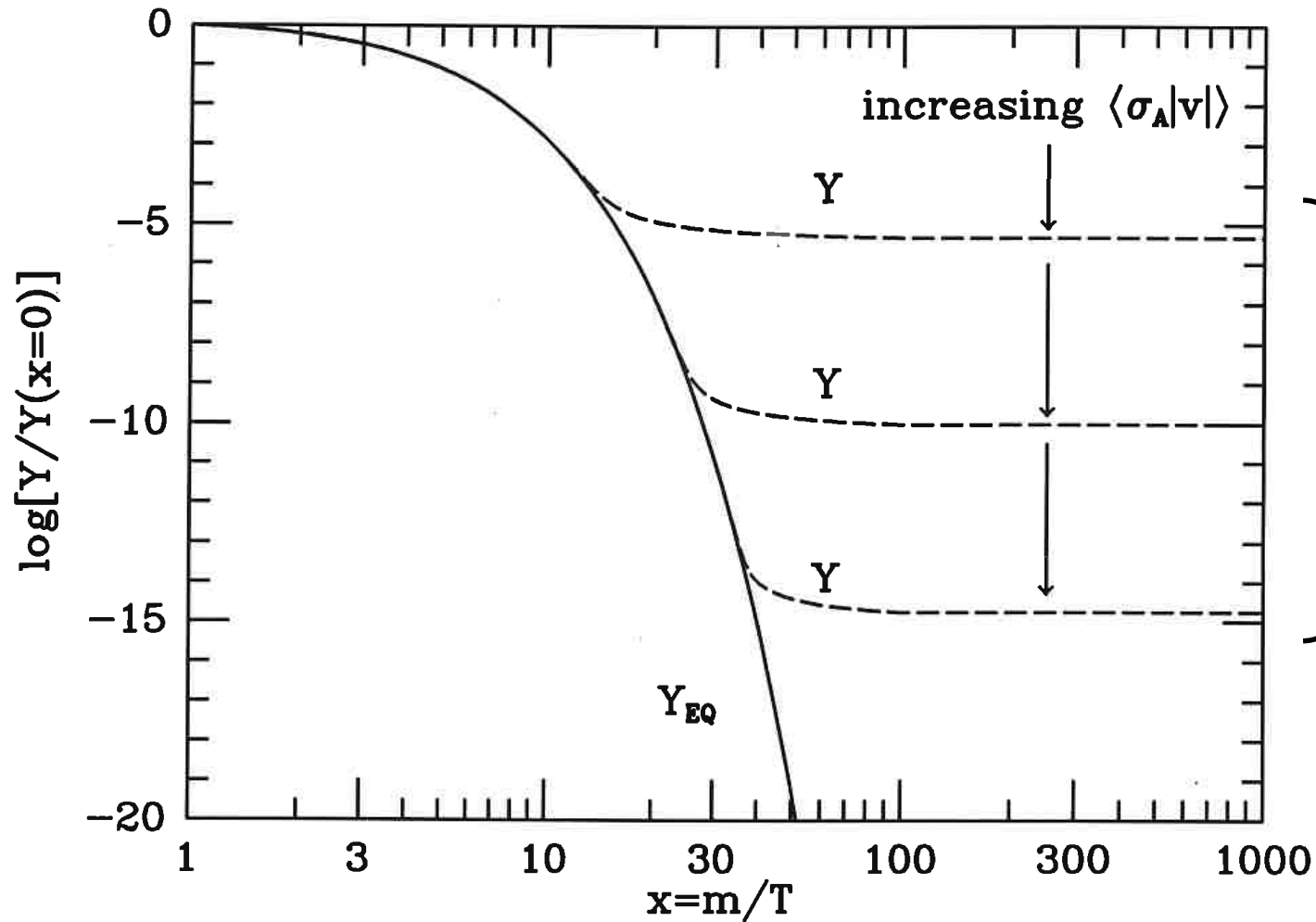
- Convenient to track $Y_\chi = n_\chi/s$, s is the entropy density

$$s = \frac{\rho + P}{T} = \frac{2\pi^2}{45} g_{*s} T^3 \quad \text{where} \quad g_{*s} = \sum_{i=\text{bosons}} \left(\frac{T_i}{T} \right)^3 + \frac{7}{8} \sum_{i=\text{fermions}} \left(\frac{T_i}{T} \right)^3$$

For most of history of the universe all species at same temperature and $g_{*s} = g_*$

- And track evolution in $x = m/T$ rather than time

Freeze Out of a Cold Thermal Relic



$$Y_{\infty} \propto \frac{1}{mm_{Pl}\langle\sigma_A|v_{Moller}|\rangle}$$

Freeze-out value of Y
determines present
mass density

$$\Omega \propto mY_{\infty}$$

$$\propto \frac{1}{\langle\sigma_A|v_{Moller}|\rangle}$$



Fermi National Accelerator Laboratory

FERMILAB-Pub-77/41-THY
May 1977

Cosmological Lower Bound on

Heavy Neutrino Masses

BENJAMIN W. LEE *

Fermi National Accelerator Laboratory, Batavia, Illinois 60510

AND

STEVEN WEINBERG **

Stanford University, Physics Department, Stanford, California 94305

$$\frac{dn}{dt} = - \frac{3R}{R} n - \langle \sigma v \rangle n^2 + \langle \sigma v \rangle n_0^2 \quad (2)$$

Here n is the actual number density of heavy neutrinos at time t ; R is the cosmic scale factor; $\langle \sigma v \rangle$ is the average value of the $\bar{\nu} \nu$ annihilation cross-section times the relative velocity and n_0 is the number density of heavy neutrinos in thermal (and chemical) equilibrium⁶:

$$n(T) = \frac{1}{2} \int_0^\infty \frac{4\pi p^2 dp}{(2\pi)^3} \left[\exp \left(\frac{m^2 + p^2}{2T} \right) + 1 \right]^{-1} \quad (3)$$

$$\frac{dn}{dt} = - \frac{3\dot{R}}{R} n - \langle \sigma v \rangle n^2 + \langle \sigma v \rangle n_0^2$$

NR annihilation
cross section
 $\langle \sigma_{AV} \rangle =$ \times Møller flux
(thermal avg.)

$$\Omega h^2 \approx 0.11 \times \frac{10^{-36} \text{ cm}^2}{\langle \sigma_{AV} \rangle}$$

$$10^{-36} \text{ cm}^2 = \frac{\alpha^2}{(150 \text{ GeV})^2}$$

weak scale!

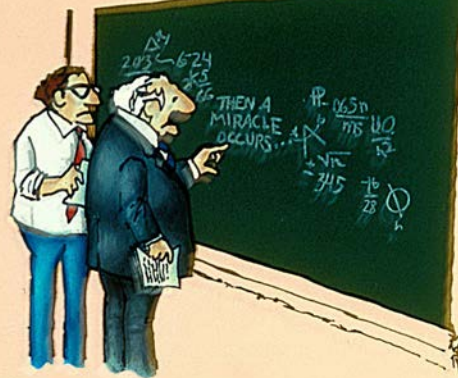
Not quite so clean:

- velocity dependence
- resonances
- co-annihilation
- log dependence on M
- decay production
- spin-dependence
- asymmetries
- ...

The WIMP “Miracle”



1 : an extraordinary
divine interventio



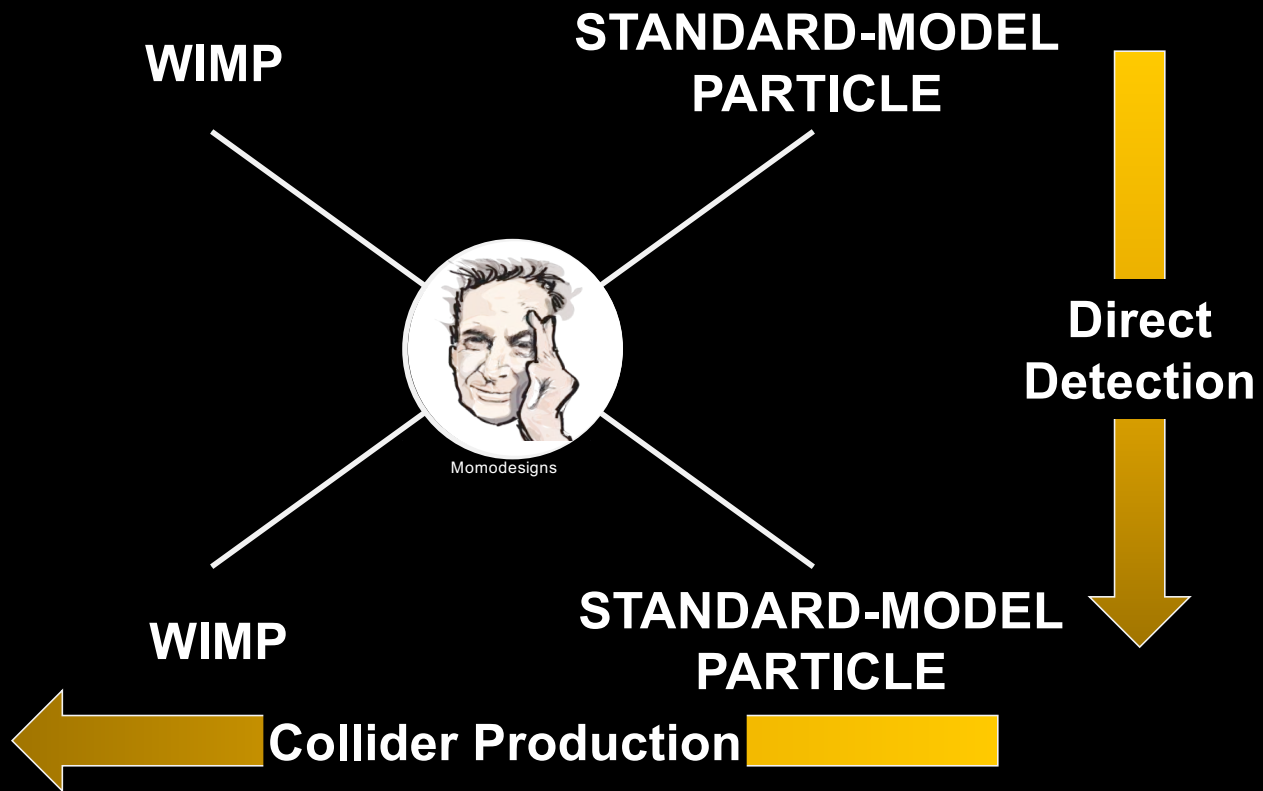
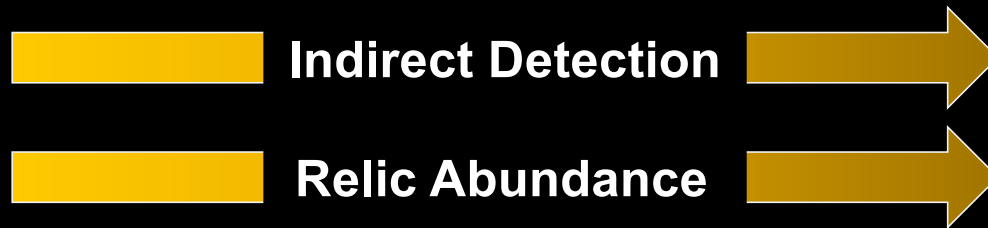
I think you should be more
explicit here in step two

encyclopedia

. often used to give an
impression of great and
unusual value in a trivial
context ...

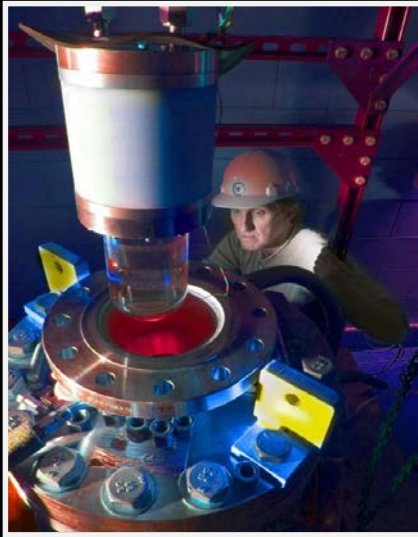
WIMPs: BSM (but not far BSM)
Interact with Standard Model particles (but “weakly”)

WIMPs



Direct Detection (Matt Pyle)

COUPP



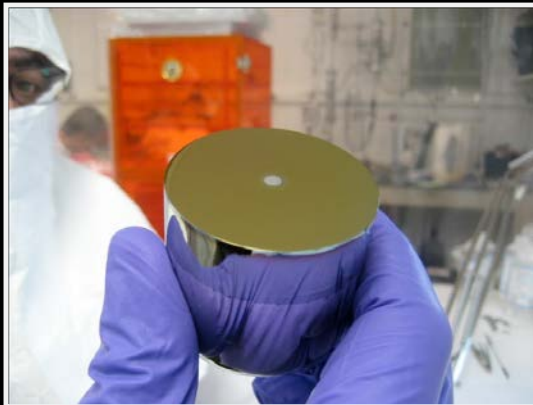
CDMS



CRESST



CoGeNT

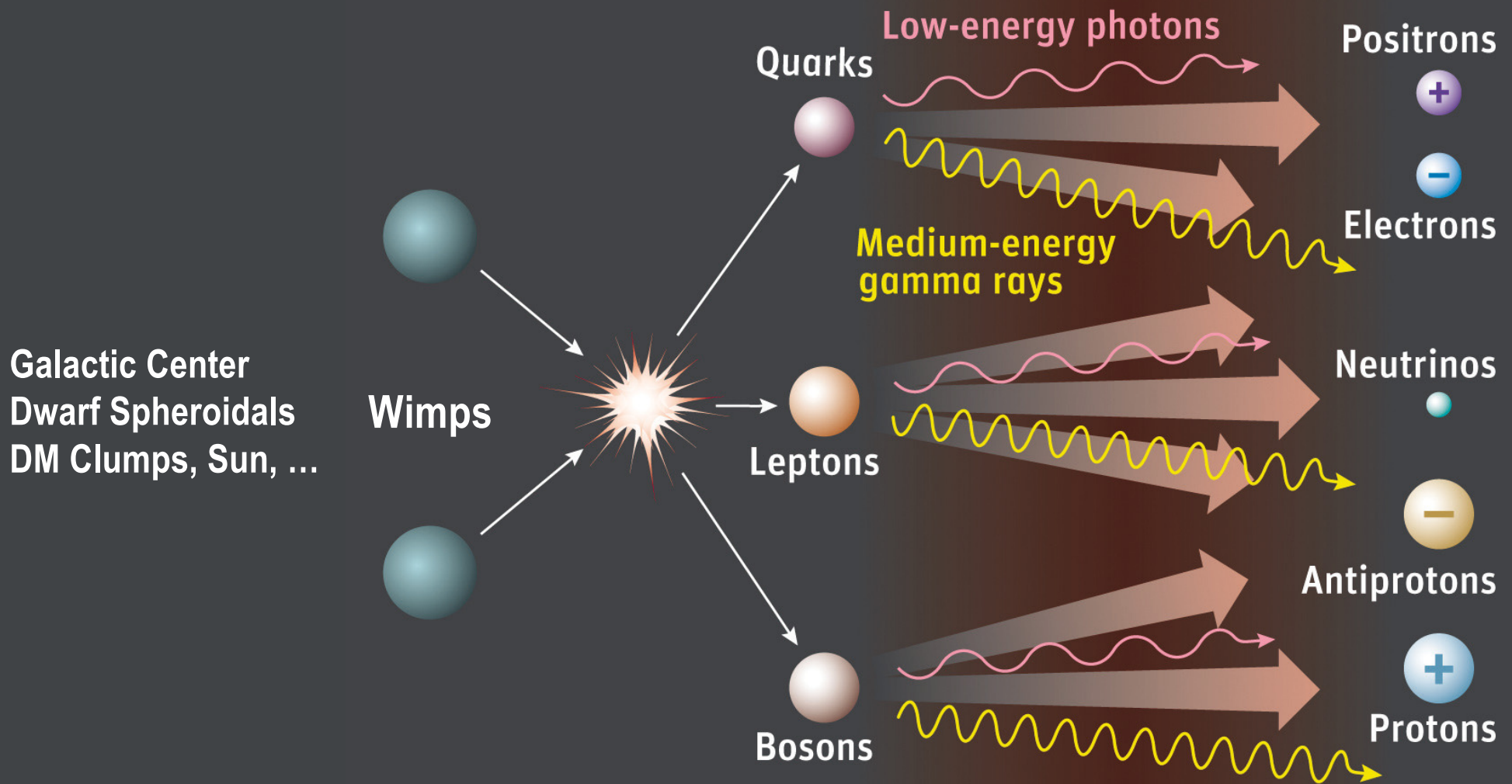


Xenon



(+ EDELWEISS,
DAMA, EURECA,
ZEPLIN, DEAP, ArDM,
WARP, LUX, SIMPLE,
PICASSO, DMTPC,
DRIFT, KIMS, LUX,
ARDM, ANAIS, CDEX
PandaX, DarkSide,
DAMA/LIBRA ...)

Indirect Detection



Indirect Detection

$$\Phi_{\gamma, e^+, n, \dots}(E, \hat{n}) = \frac{1}{4\pi} \left[\frac{\langle \sigma v \rangle}{2m_\chi^2} \frac{dN_{\gamma, e^+, n, \dots}}{dE} \right] \left[\int_{\text{line of sight}} \rho_\chi^2 dl \right]$$

$$\Phi_{\gamma, e^+, n, \dots}(E, \hat{n}) = \quad \left[\text{particle physics} \right] \quad \left[J \text{ factor (astronomy)} \right]$$

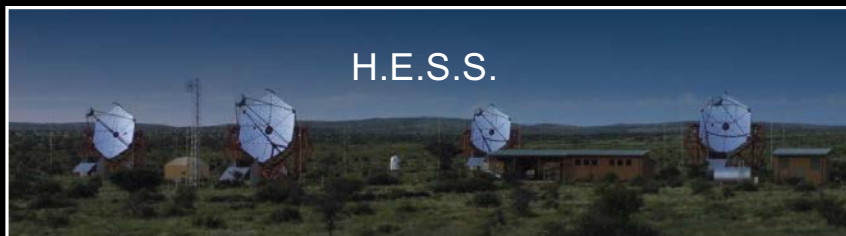
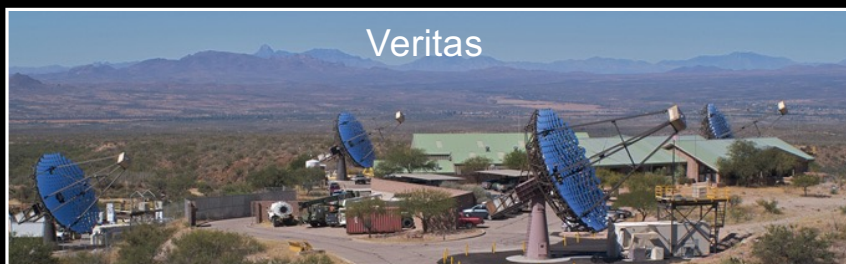
What to look for

- Charged particles: \bar{p} , high-energy e^-e^+
astronomical backgrounds
easy to detect
bent by magnetic field
- Continuum photons, neutrinos
astronomical backgrounds
 γ easy to detect
 ν hard to detect/often not dominant
- Monoenergetic photon line ($\bar{\chi}\chi \rightarrow \gamma\gamma$)
low background
(probably) low signal
“golden” detection channel

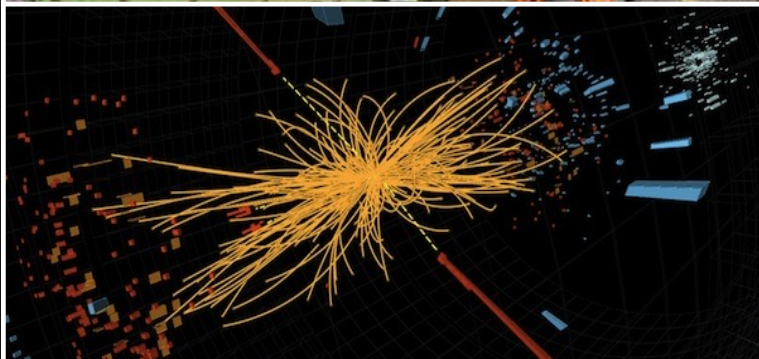
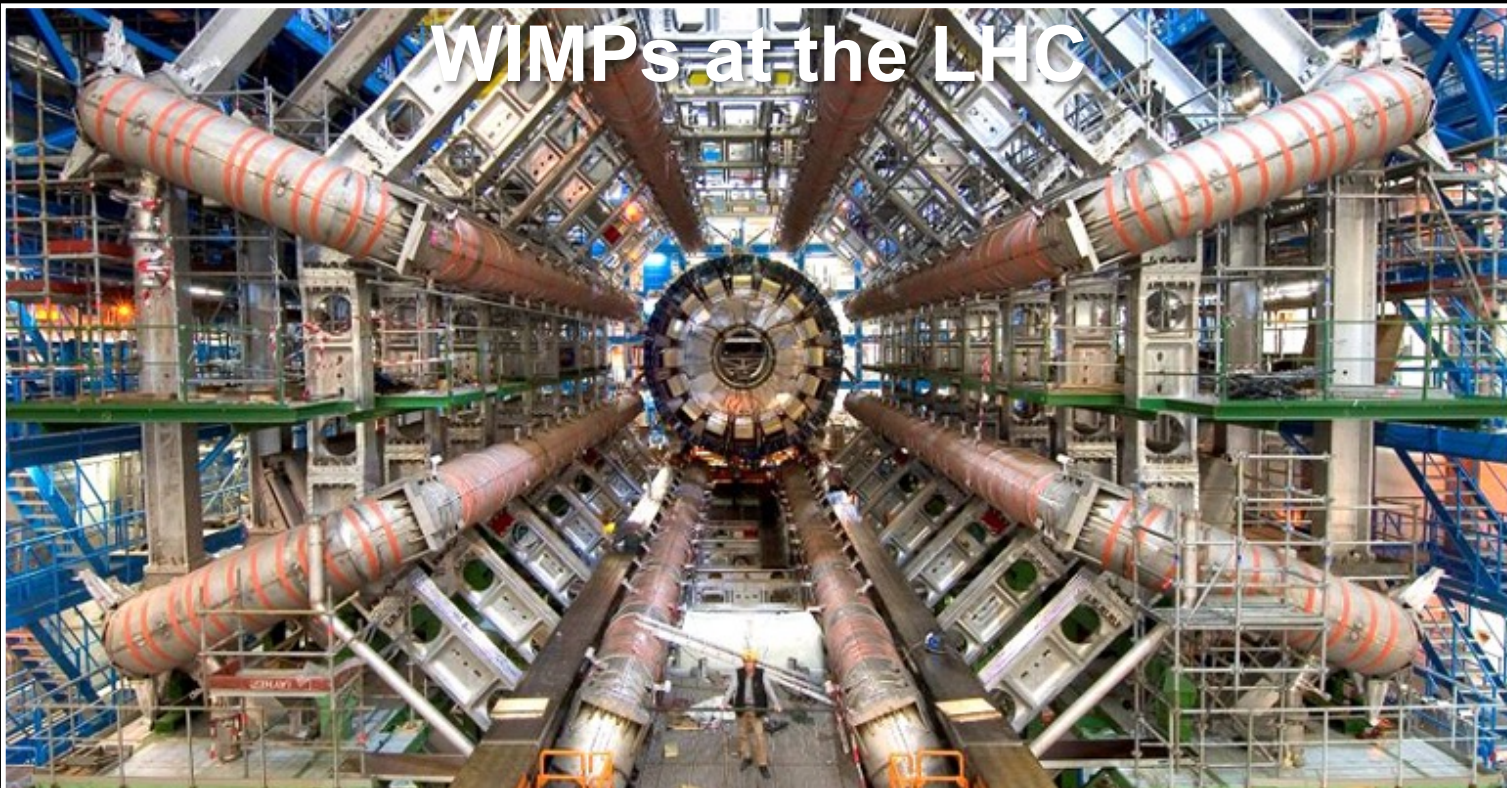
Where to look for it

- Galactic Center
know where to look
largest signal
largest backgrounds
- Nearby subclumps
don't know where to look
signal down 10^{-3}
clean: no baryons
- Dwarf spheroidals ($\mathcal{M}/\mathcal{L} > 3000$)
know where to look (about 20)
signal down another 10^{-3}
clean: very few baryons

Indirect Detection



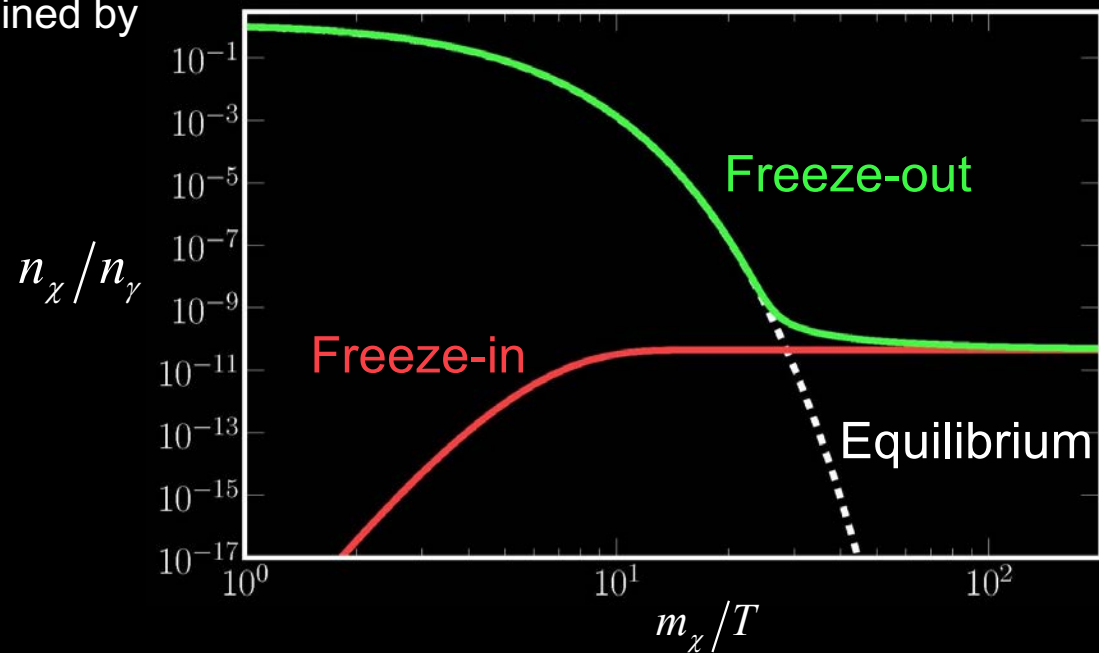
WIMPs at the LHC



Looking for an
invisible
needle in a haystack

Most popular paradigm: cold thermal relic*

- DM abundance set by creation/annihilation with standard-model particles
- Equilibrium abundance of DM determined by M / T (no asymmetry)
- DM species final abundance determined by “freeze-out” of equilibrium, or “freeze-in” to final abundance



- Freeze-out/in: interplay between particle physics (DM—SM interactions) and cosmology (expansion rate)

* An object of particular veneration.

Direct, Indirect, Accelerator

Where is the WIMP?

No signal in direct (DAMA?), indirect (galactic center γ -ray excess?), or accelerator searches.

Even more troubling, no sign of BSM physics at LHC.

This doesn't seem to be the decade of the WIMP!!!

(Perhaps) DM is NOT a WIMP (cold thermal relic), time to focus elsewhere



go lighter
dark sector



go ultralight
axion, dark photon

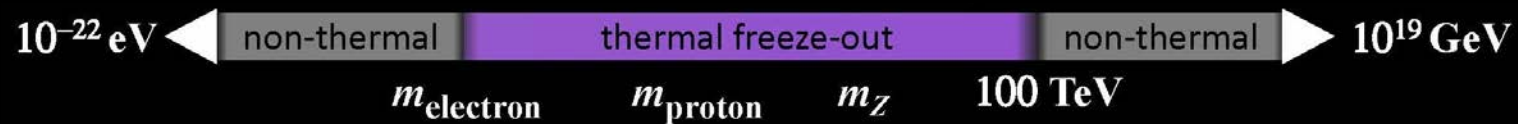


go ultraheavy
WIMPzilla



boldly go where no
one has gone before

Dark Matter particle mass range



Mass Range for Particle Dark Matter

$m \geq 10^{-22}$ eV: de Broglie wavelength smaller than dark-matter dominated objects

$m \leq 10^{19}$ GeV: mass less than Planck mass

$m \geq \text{few eV}$: if fermion (exclusion principle)

If thermal freeze-out

$m \geq m_e$: annihilation to SM particles

$m \leq 100$ TeV: annihilation cross section too small for larger masses

Is Dark Matter a Particle Or a Wave?

$m \leq 1$ eV: occupation number in de Broglie-wavelength volume $\geq 1 \rightarrow$ WAVE

$m \geq 1$ eV: occupation number in de Broglie-wavelength volume $\leq 1 \rightarrow$ PARTICLE

Dark Matter particle mass range (mere 41 orders of magnitude)



Plancktons:

$$m \sim m_{\text{Planck}} = 10^{19} \text{ GeV}$$

WIMPzillas:

$$m \sim m_{\text{inflaton}} = 10^{10} - 10^{13} \text{ GeV}$$

Supermassive:

$$m > 100 \text{ TeV}$$

WIMP range (e.g., neutralino):

$$m_{\text{proton}} < m < 1 \text{ TeV}$$

Light dark matter (e.g., dark photon):

$$m_{\text{electron}} < m < m_{\text{proton}}$$

Ultralight dark matter (e.g., axion):

$$m < 1 \text{ eV}$$

Fuzzy dark matter:

$$m \sim 10^{-22} \text{ eV}$$

Is Dark Matter Really a WIMP?

- Observation/experiment will tell!
- So far, after 30 years of effort nothing definite seen.

**KEEP
CALM
AND
CARRY ON SEARCHING
FOR
DARK MATTER**

- or, ...
- Pursue other ideas.
- Hope for a disruptive discovery.

The Universe Observed

- Cosmological parameters (just numbers)

H_0 → the present expansion rate (Hubble's constant)

Ω_i → the present cosmic food chain
(Ω_{TOTAL} , Ω_M , Ω_B , Ω_Λ , Ω_γ , Ω_ν , ...)

T_0 → the present temperature of the Universe

t_0 → the present age of the Universe

- Power spectra—characterization of perturbations:

Galaxies: $P(k)$ Radiation: C_ℓ