Neutron Stars as Laboratories

- Equation of State of Neutron-Rich Matter
- Speed of Sound at High Density
- Heating and Cooling in Accreting Neutron Stars
- Dark Matter in Neutron Stars

General Constraints on the Equation of State

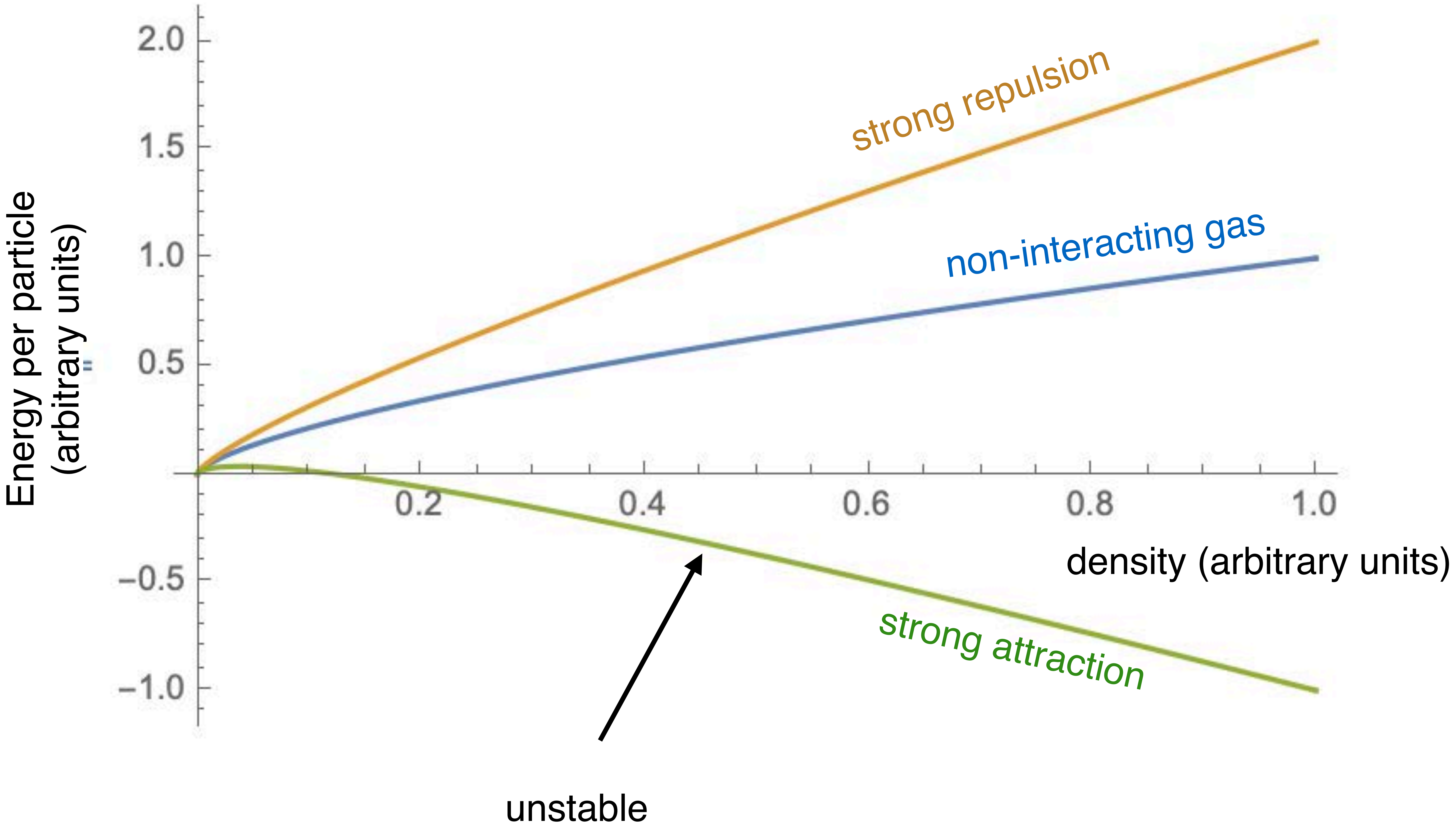
Preliminaries: Uniform Fermi Liquids

$$\frac{\epsilon}{n} = \frac{3}{5}\frac{k_F^2}{2M} + V(n)$$

$$n = \frac{N}{V} = \frac{k_F^3}{3\pi^2}$$

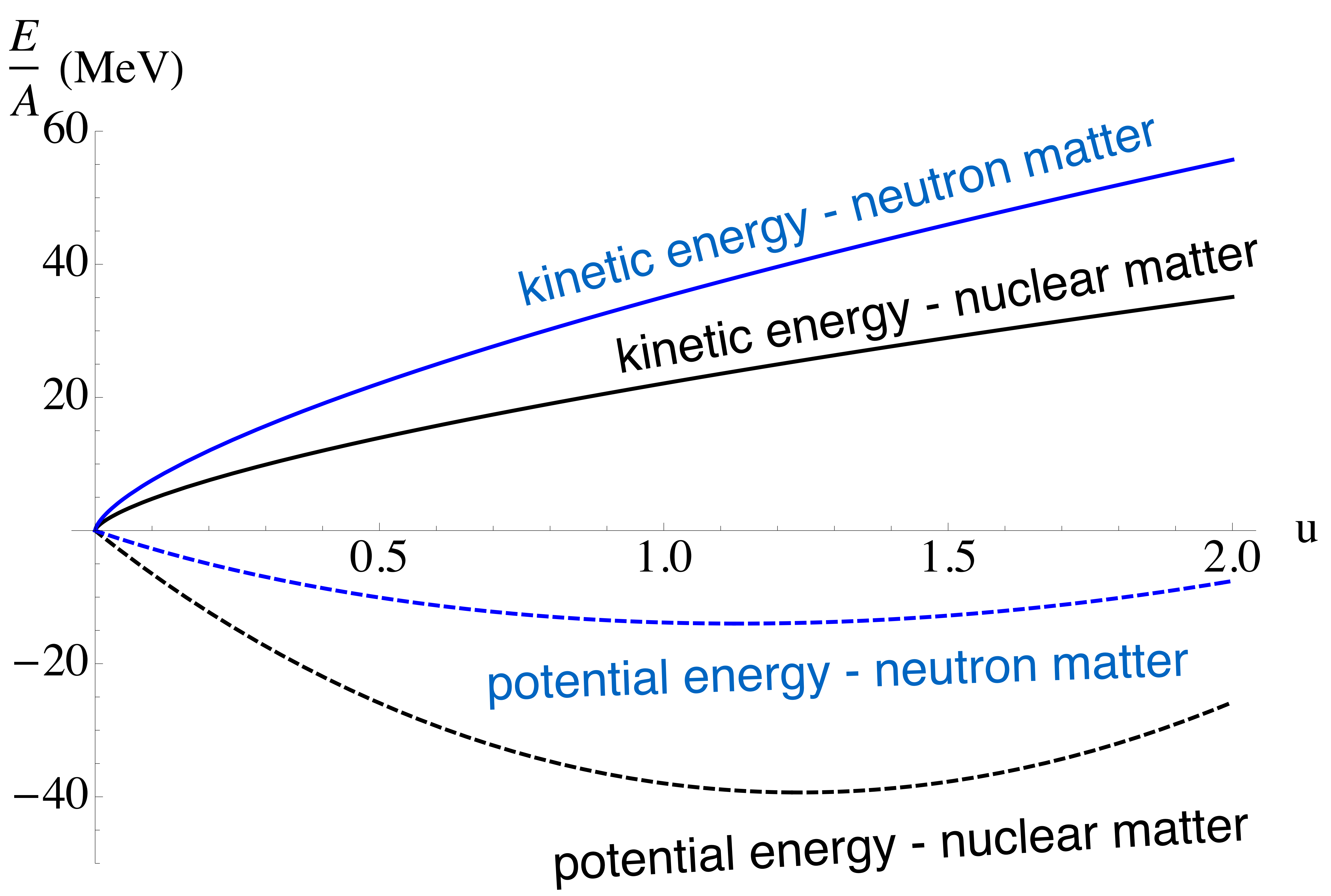
$$P(n) = n^2 \left(\frac{\partial(\epsilon/n)}{\partial n}\right)$$
$$= -\epsilon + \mu n$$

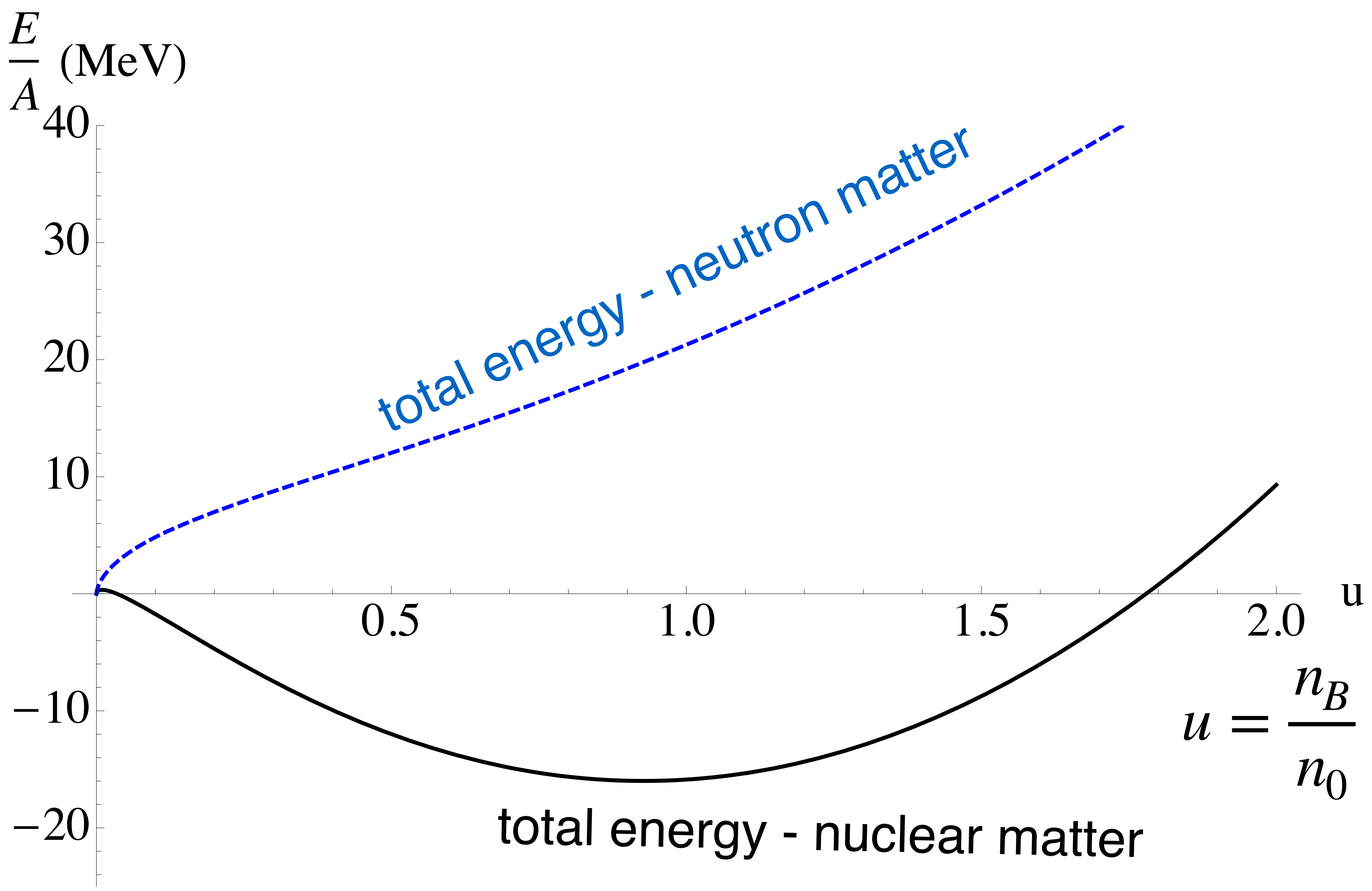
$$\mu = \left(\frac{\partial \epsilon}{\partial n}\right)$$



Neutron Matter and Nuclear Matter

$$\frac{\epsilon(n_n, n_p)}{n_B} = \frac{3}{5}\ \frac{p_{Fn}^2}{2\ m_n}\ \frac{n_n}{n_B} + \frac{3}{5}\ \frac{p_{Fp}^2}{2\ m_p}\ \frac{n_p}{n_B} + \frac{V_{np}(n_n, n_p)}{n_B}$$





Empirical Properties of Nuclear and Neutron Matter at Saturation

$$n_0 = 0.16 \pm 0.01 \ \mathrm{fm}^{-3}$$

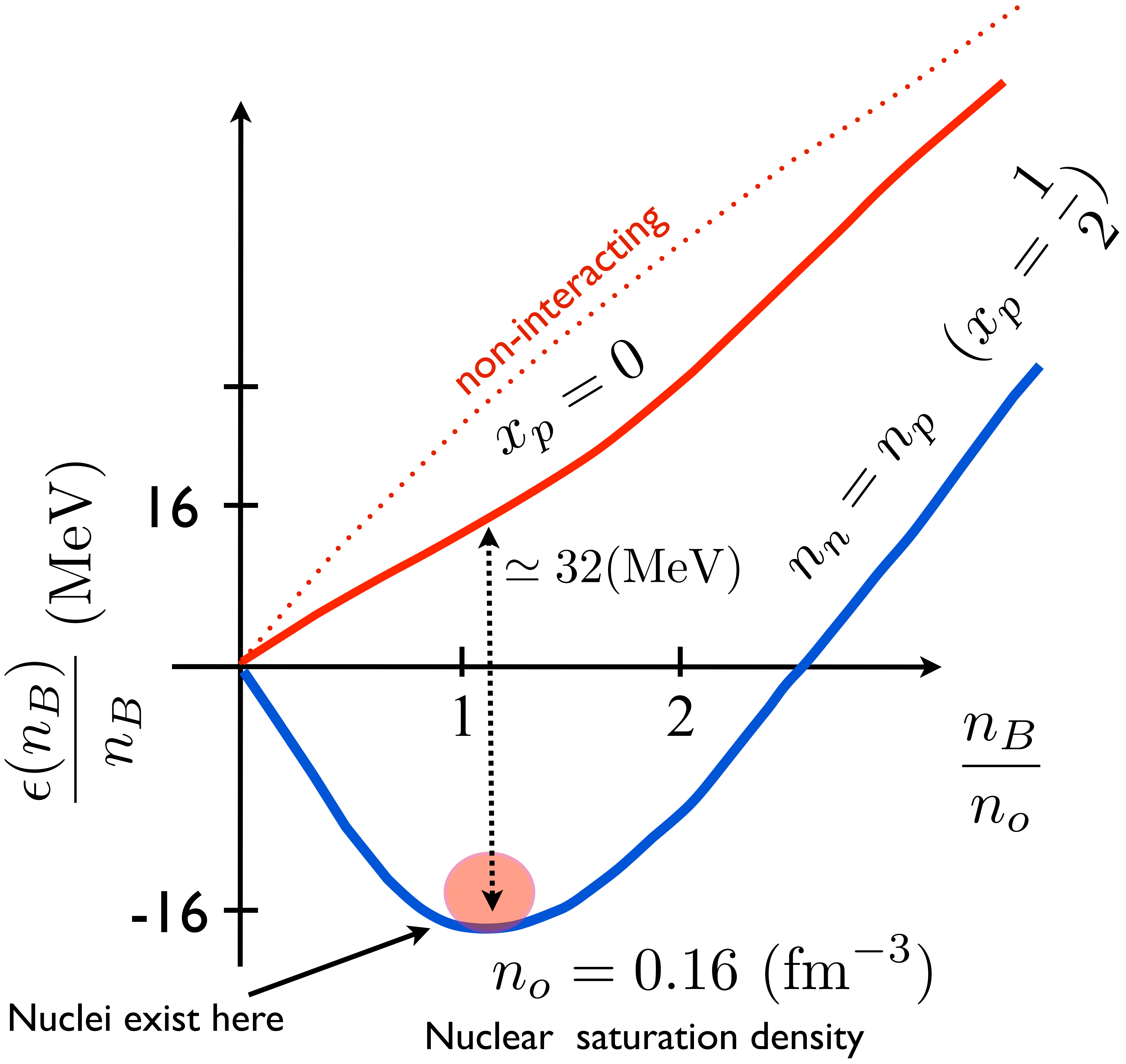
$$\frac{\epsilon(n_n = n_p = \frac{n_0}{2})}{n_0} = -16 \ \mathrm{MeV}$$

$$K = 9n_0^2 \left(\frac{\partial^2(\epsilon(n_B)/n_B)}{\partial n_B^2} \right)_{n_B=n_0} = 240 \pm 20 \ \mathrm{MeV}$$

$$S = \frac{\epsilon(n_B = n_0, n_p = 0) - \epsilon(n_n = n_p = \frac{n_B}{2})}{n_B}$$

$$= 32 \pm 2 \mathrm{MeV}$$

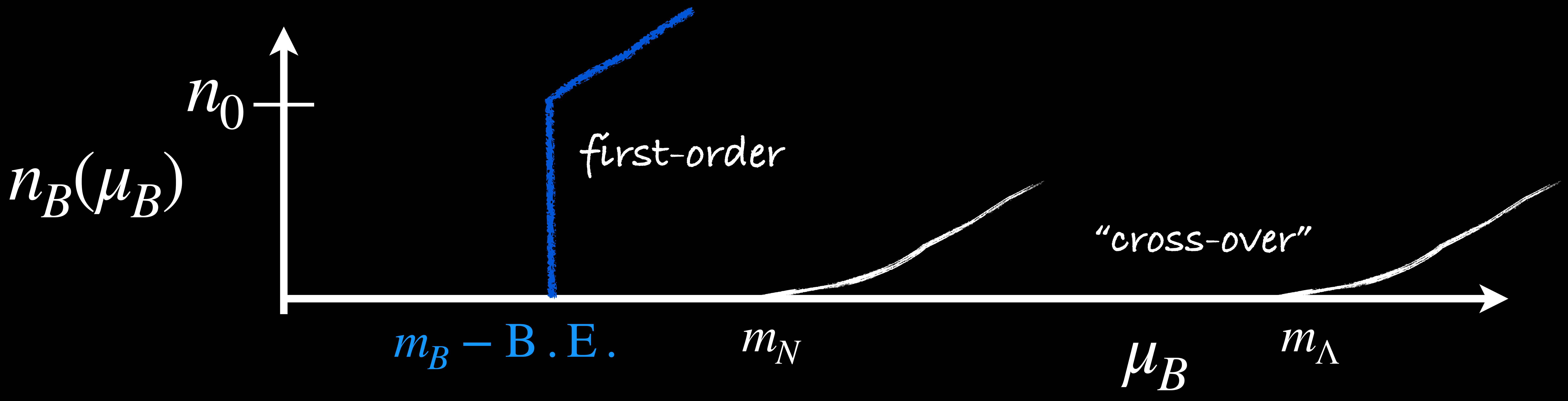
$$L = 3n_0 \left(\frac{\partial S(n_B)}{\partial n_B} \right)_{n_B=n_0} \simeq 65 \pm 25 \ \mathrm{MeV}$$



First-order Phase Transition Due to Attractive Nuclear Interactions

The vacuum responds to a chemical potential by producing a finite density of particles with the lowest free energy. Density is discountinuos at first-order phase transitions.

At T=0 there is a first-order phase transition from the QCD vacuum to a state with a finite density of neutrons and protons (n_B=n_0).



Charge Neutrality and Beta-Equilibrium

- Stable matter is electrically neutral. $N_p = N_e$

- All allowed reactions are in equilibrium. $\mu_e + \mu_p = \mu_n$

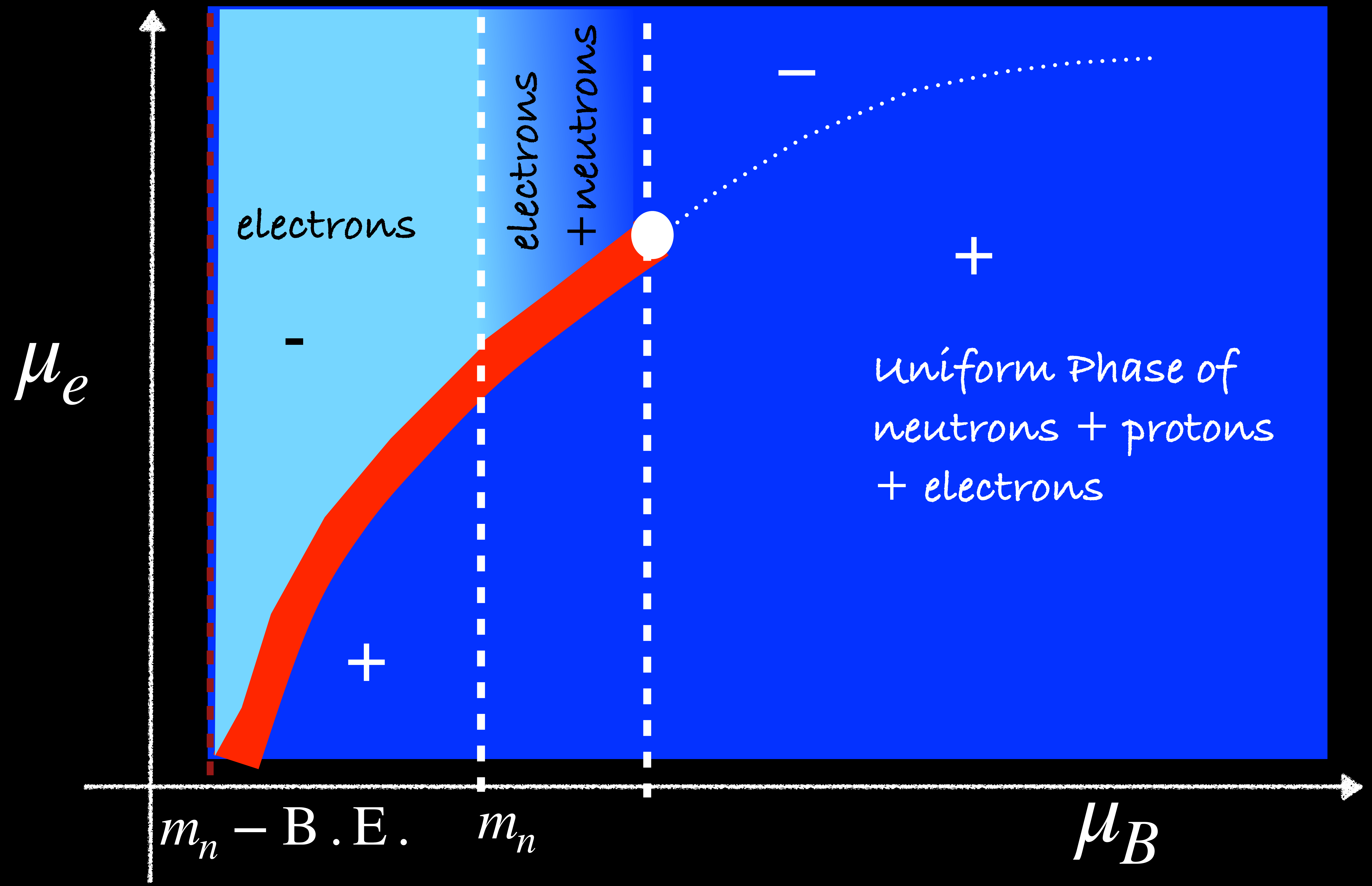
$$e^- + p \leftrightarrow n + \nu_e$$

In equilibrium chemical potentials are determined by the conserved charges.

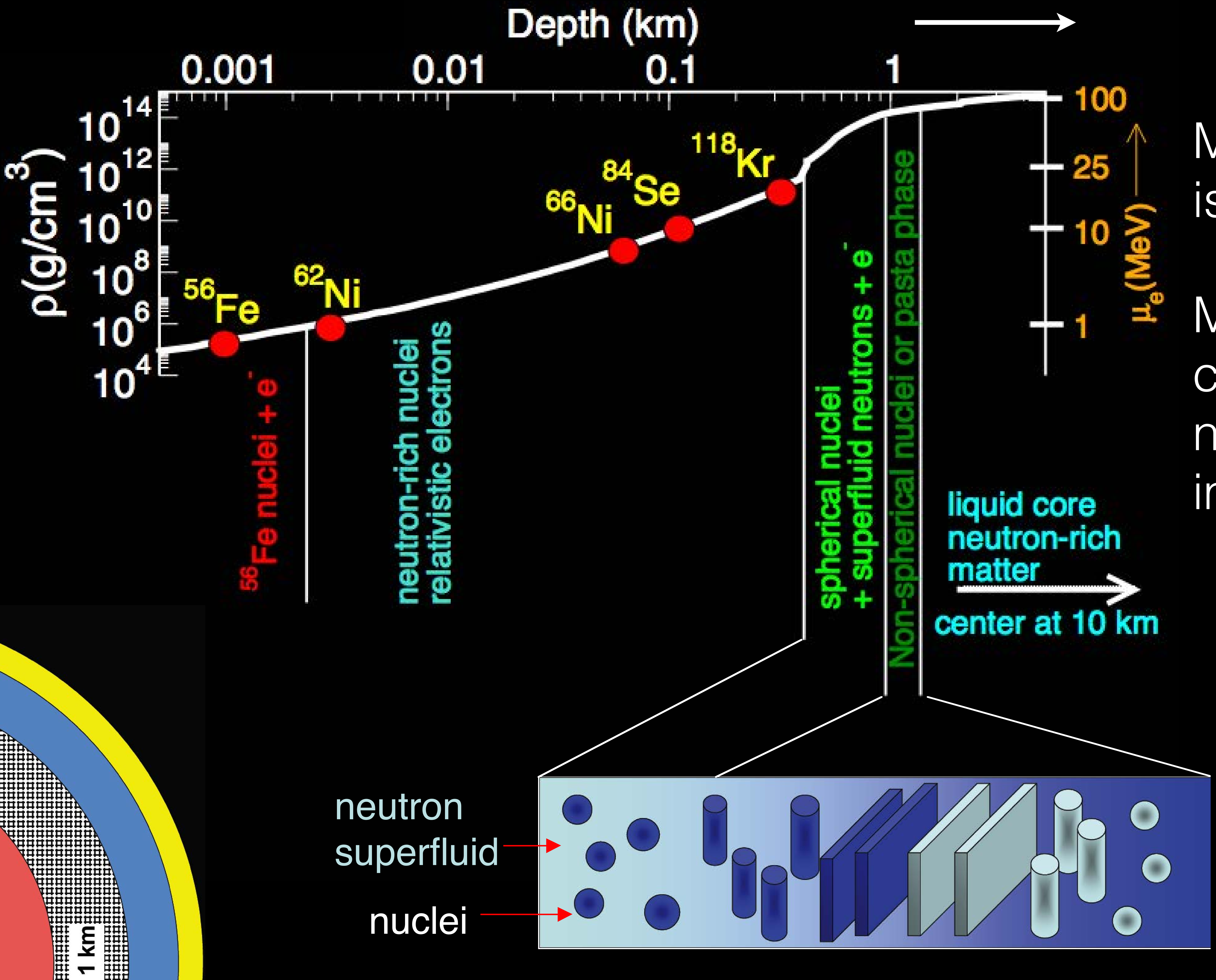
Conserved charges: Baryon number and electric charge.
The associated chemical potentials are μ_B and μ_Q

$$\mu_i = b_i \mu_B + q_i \mu_Q$$

Matter with 2 Conserved Charges




Global charge neutrality
Local charge neutrality
n,p
e⁻
n,p
e⁻
n
n,p
e⁻
Energy cost due to Coulomb
and surface energies.



Mass contained in the crust is small ~ few percent.

Most of it is in the inner-crust as either spherical or non-spherical nuclei immersed in a neutron fluid.

Equation of State of the Outer Core

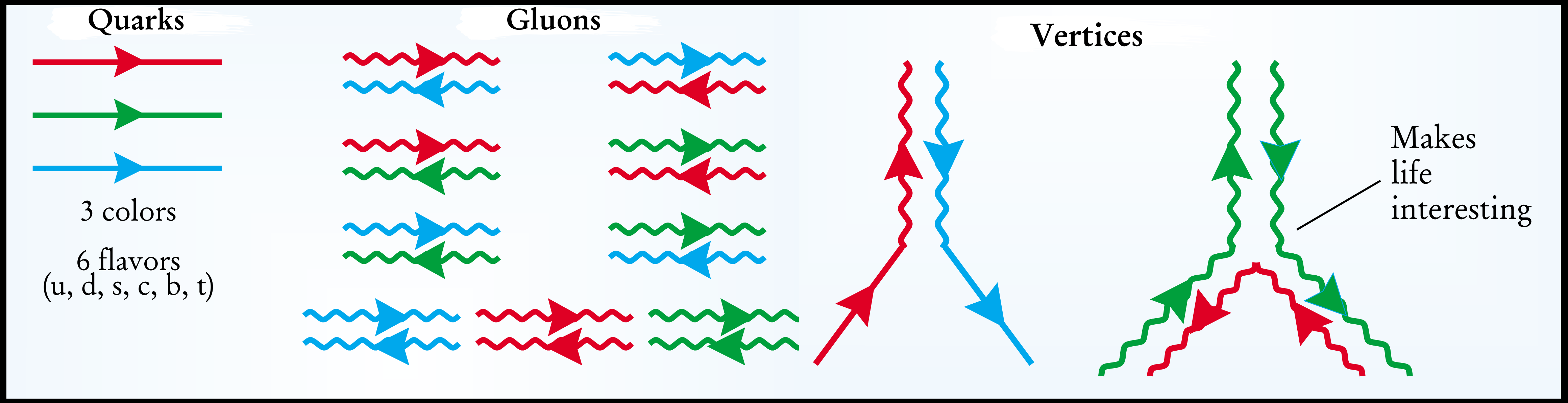
Nuclear Interactions

QCD (Lagrangian) is simple is write down

$$D_\mu \equiv \partial_\mu + i t^a A_\mu^a$$

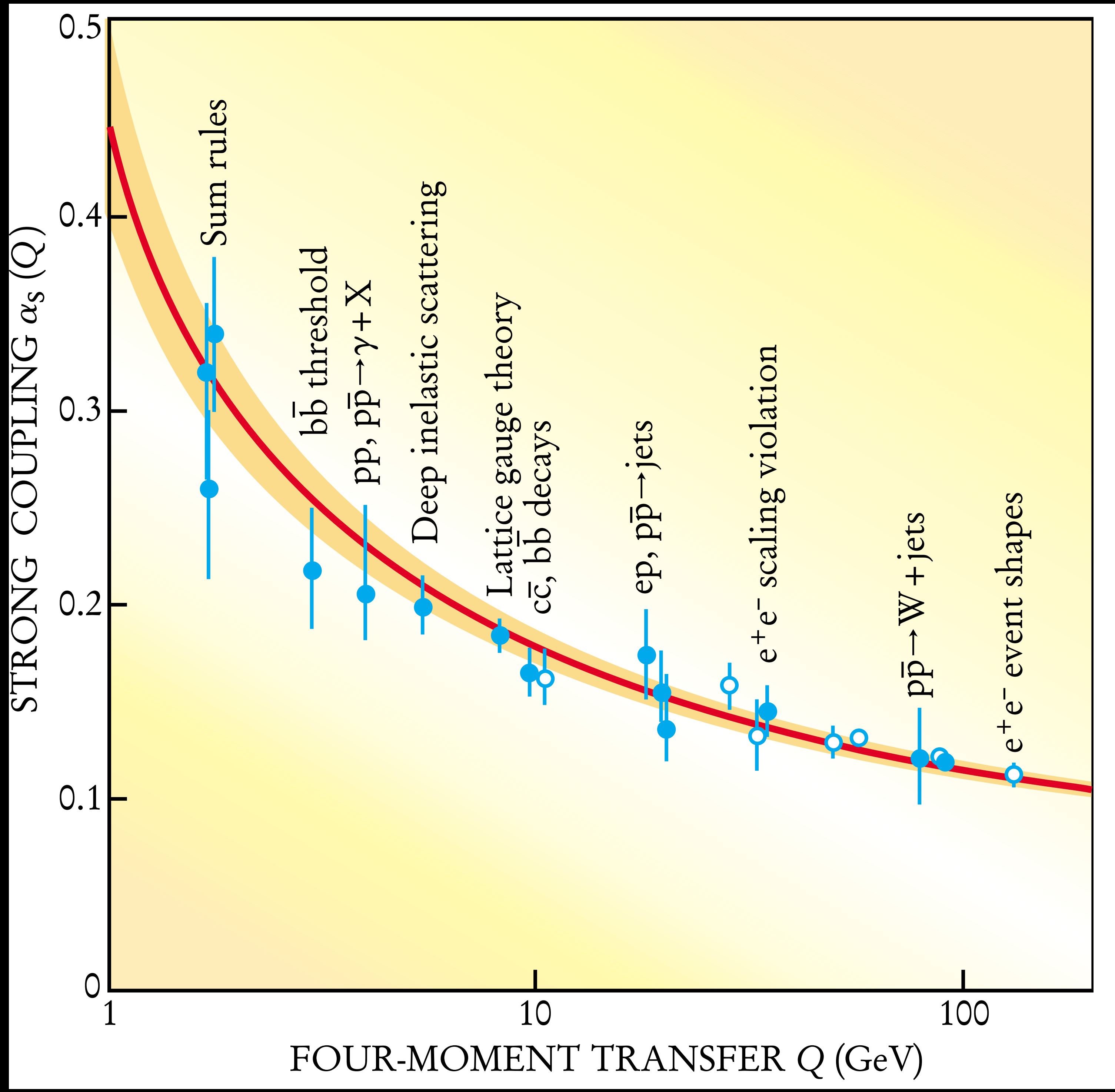
$$\mathcal{L} = \frac{1}{4g^2} G^a_{\mu\nu} G^a_{\mu\nu} + \sum_j \bar{q}_j (i\gamma^\mu D_\mu + m_j) q_j$$

$$G^a_{\mu\nu} \equiv \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + i f^a_{bc} A^b_\mu A^c_\nu$$



but is difficult to solve at low energy.

F. Wilczek, Physics Today (2000)



It gets simpler at high energy (asymptotic freedom).

The low energy QCD vacuum is non-perturbative:

- It confines quarks to color singlet states.
- Spontaneously breaks chiral symmetry.

Nuclear Interactions

- Baryons and mesons are the relevant low energy degrees of freedom at low energy. Interactions between them are strong, complex, and short-range.

- Pions are special. They are the Goldstone bosons associated with chiral symmetry breaking and provide the longest range force between nucleons.

- Other mesons are significantly heavier. It is not very useful to single them out as mediators of the strong interaction between composite color singlet states.

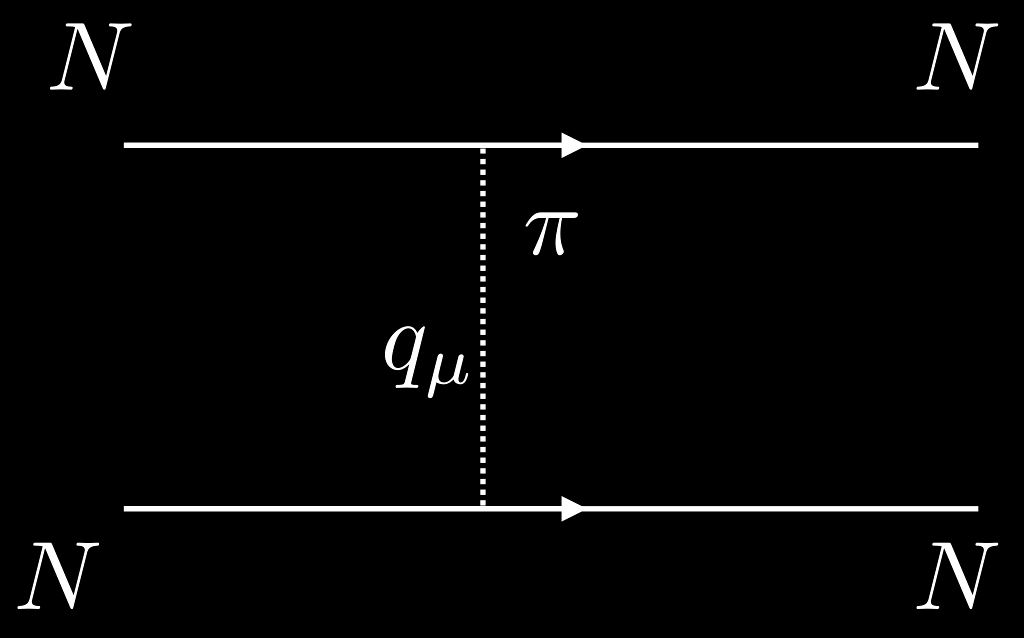
- How then can we write down a theory of strong interactions between nucleons at low energy ?

Potential Models   Effective Field Theories (EFT)

Nucleon-Nucleon Potentials

One-pion exchange:

$$\mathcal{L}_{NN\pi} = \psi_N^\dagger \left(i\partial_t - \frac{\nabla^2}{2M_N} \right) \psi_N \;\; - \frac{g_a}{f_\pi} \; \psi_N^\dagger \tau^a \sigma \cdot \nabla \pi^a \psi_N$$



$$V_\pi(q) = -\left(\frac{g_A}{\sqrt{2} f_\pi} \right)^2 \; \tau_1 \cdot \tau_2 \; \frac{\sigma_1 \cdot \mathbf{q} \; \sigma_2 \cdot \mathbf{q}}{q^2 + m_\pi^2}$$

In coordinate space the potential is

$$V_\pi(q) = -\left(\frac{g_A}{\sqrt{2} f_\pi} \right)^2 \; \tau_1 \cdot \tau_2 \; \left[S_{12} \; \left(1 + \frac{3}{m_\pi r} + \frac{3}{m_\pi^2 r^2} \right) \frac{e^{-m_\pi r}}{r} - \frac{4\pi}{3} \; \sigma_1 \cdot \sigma_2 \; \delta^3(r) \right]$$

Potential depends on spin and iso-spin.

It has a tensor component: $S_{12} = 3(\sigma_1 \cdot \hat{r}_1) \; (\sigma_2 \cdot \hat{r}_2) - \sigma_1 \cdot \sigma_2$

It is singular: $V(r \to 0) \approx \dfrac{1}{r^3}$

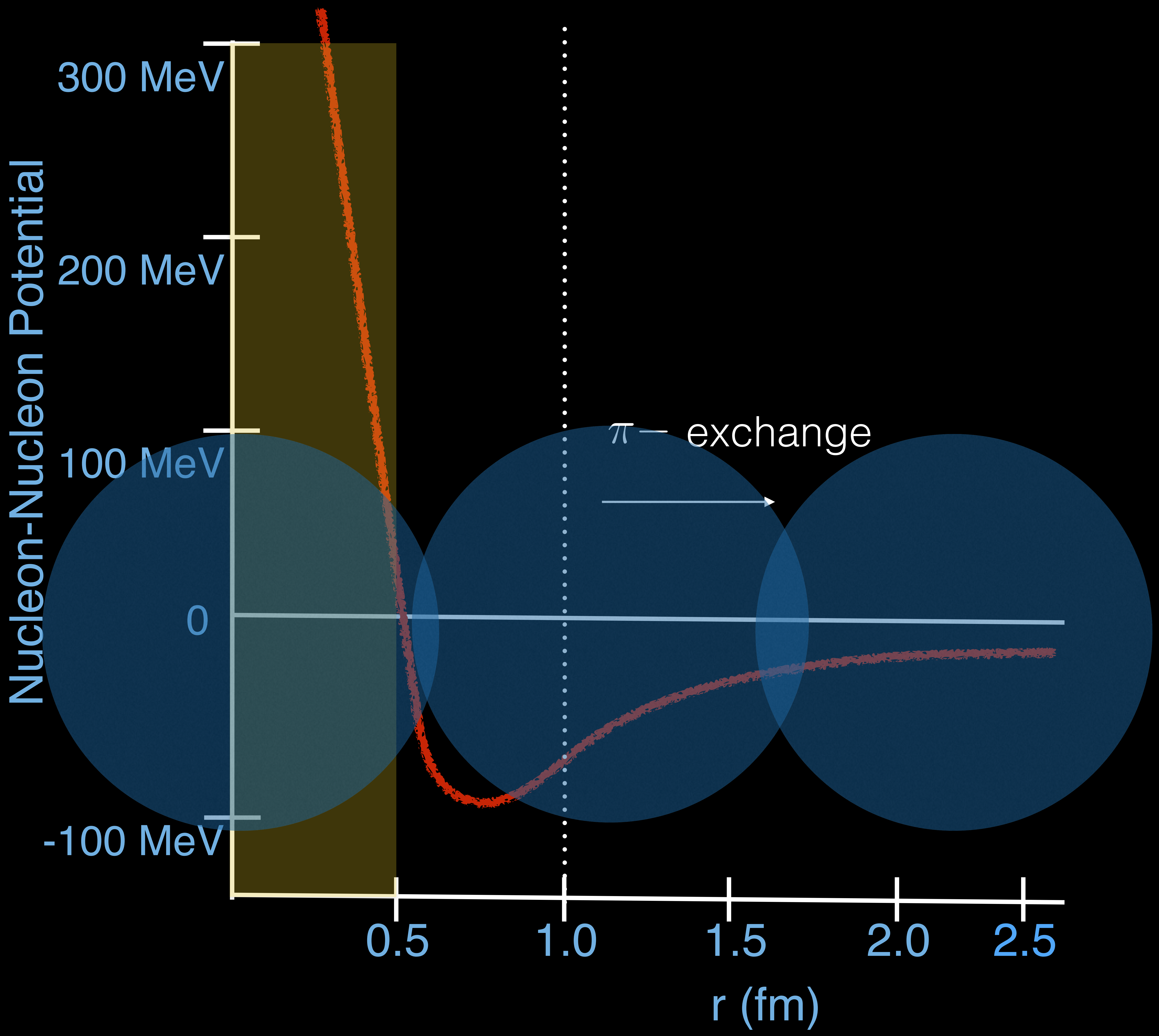
Nuclear Forces at Short Distances

They are essential even at low energy.

Are constrained by nucleon-nucleon scattering data (phase shifts).

Models favor strong repulsion. (hard-core)

Range of these forces is comparable to the intrinsic size of the nucleon.



Potential Models

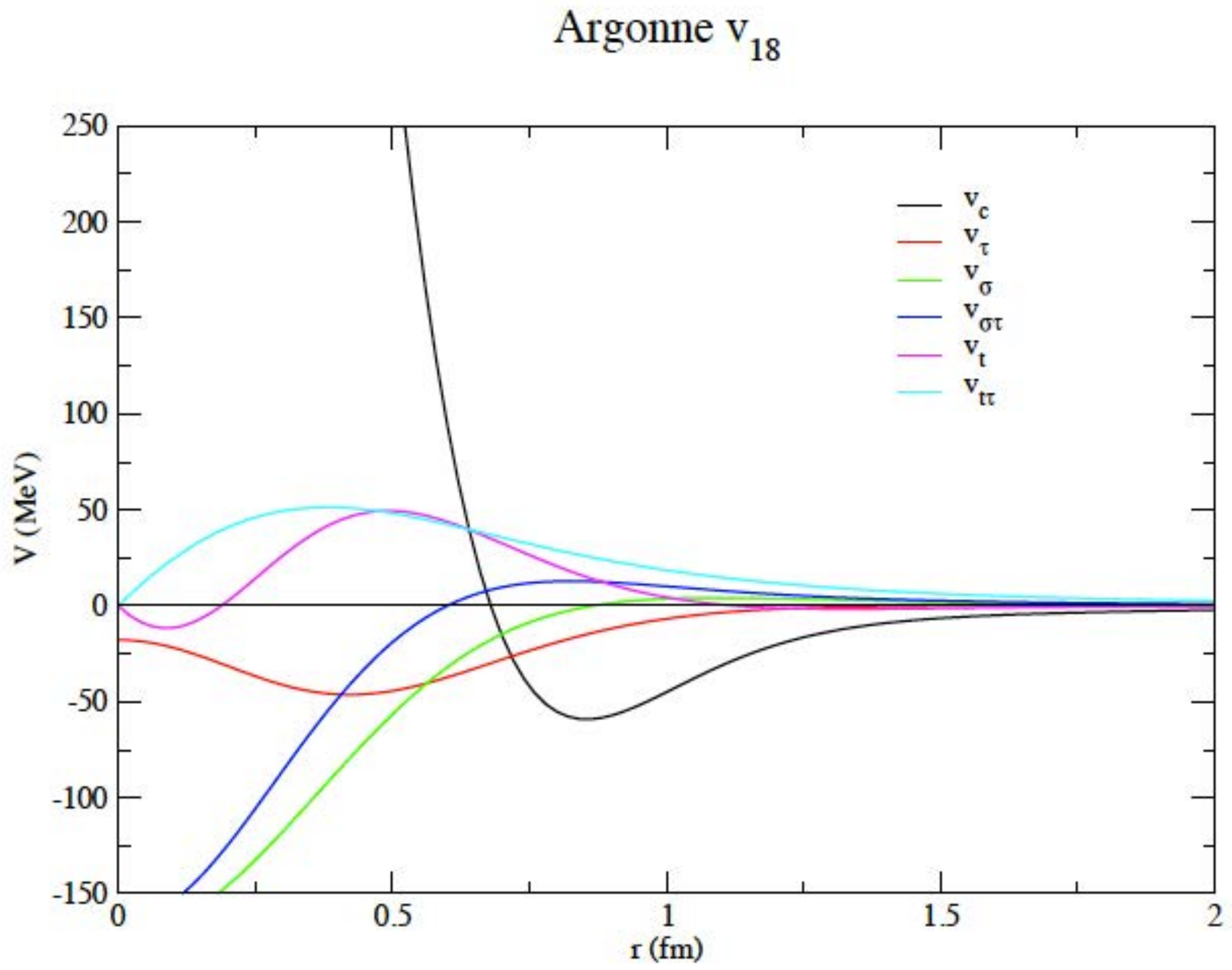
Insert a model potential in Schrödinger equation and use scattering data to constrain the parameters:

$$V_{ij} = \sum_p v_p(r_{ij})\ O^p_{ij}$$

Intricate spin, isospin and tensor structure.

$$\begin{aligned} O^p_{ij} = & [1, \sigma_i \cdot \sigma_j, S_{ij}, \mathbf{L}\cdot\mathbf{S}, \mathbf{L}^2, \mathbf{L}^2(\sigma_i\cdot\sigma_j), (\mathbf{L}\cdot\mathbf{S})^2] \\ & + [1, \sigma_i \cdot \sigma_j, S_{ij}, \mathbf{L}\cdot\mathbf{S}, \mathbf{L}^2, \mathbf{L}^2(\sigma_i\cdot\sigma_j), (\mathbf{L}\cdot\mathbf{S})^2] \otimes \tau_i\cdot\tau_j \\ & + [1, \sigma_i \cdot \sigma_j, S_{ij}, \mathbf{L}\cdot\mathbf{S}] \otimes T_{ij} \\ & + [1, \sigma_i \cdot \sigma_j, S_{ij}, \mathbf{L}\cdot\mathbf{S}] \otimes (\tau_i + \tau_j)_z \end{aligned}$$

$$S_{ij} = 3\sigma_i\cdot\hat{r}_{ij}\sigma_j\cdot\hat{r}_{ij} - \sigma_i\cdot\sigma_j \qquad T_{ij} = 3\tau_{iz}\tau_{jz} - \tau_i\cdot\tau_j$$



Potential is Neither Unique Nor Observable (in QM)

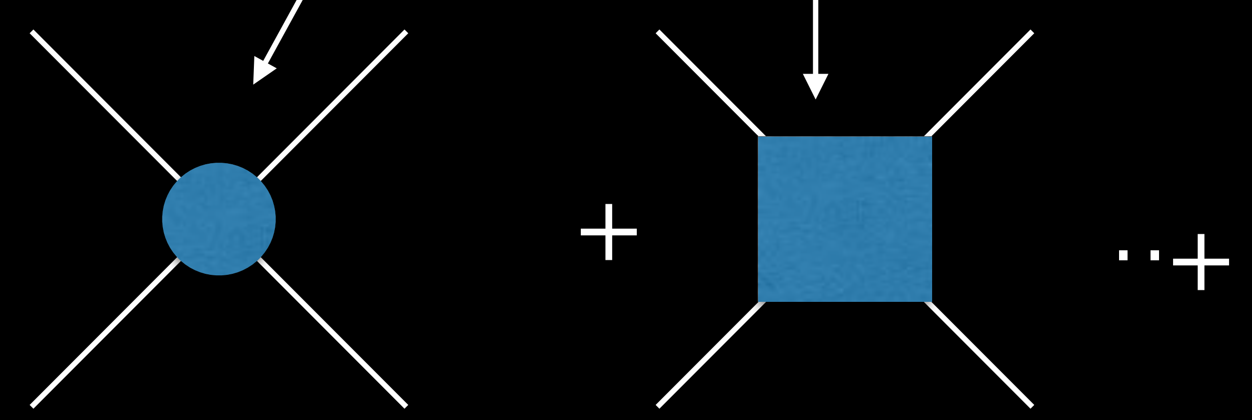
Potential Models: Relies on a set of (reasonable) assumptions about the short distance behavior to solve the Schrödinger equation and fit observables.

Effective Field Theory: Relies on a separation of scales to Taylor expand potential in powers of momenta or inverse radial separation. Coefficients of the expansion are determined by fitting to observables.

A simple (heuristic) EFT example:

$$V_\omega(q) = \frac{g_\omega^2}{q^2 + m_\omega^2} \approx \frac{g_\omega^2}{m_\omega^2} - \frac{g_\omega^2}{m_\omega^2}\frac{q^2}{m_\omega^2} + \ldots$$

Exchange of heavy bosons at low energy cannot be resolved.



When several heavy particles may be exchanged, or when the underlying mechanism is unknown, the general expansion is

$$V_{\text{short}}(q) = C_0 + C_2\,\frac{q^2}{\Lambda^2} + \ldots$$

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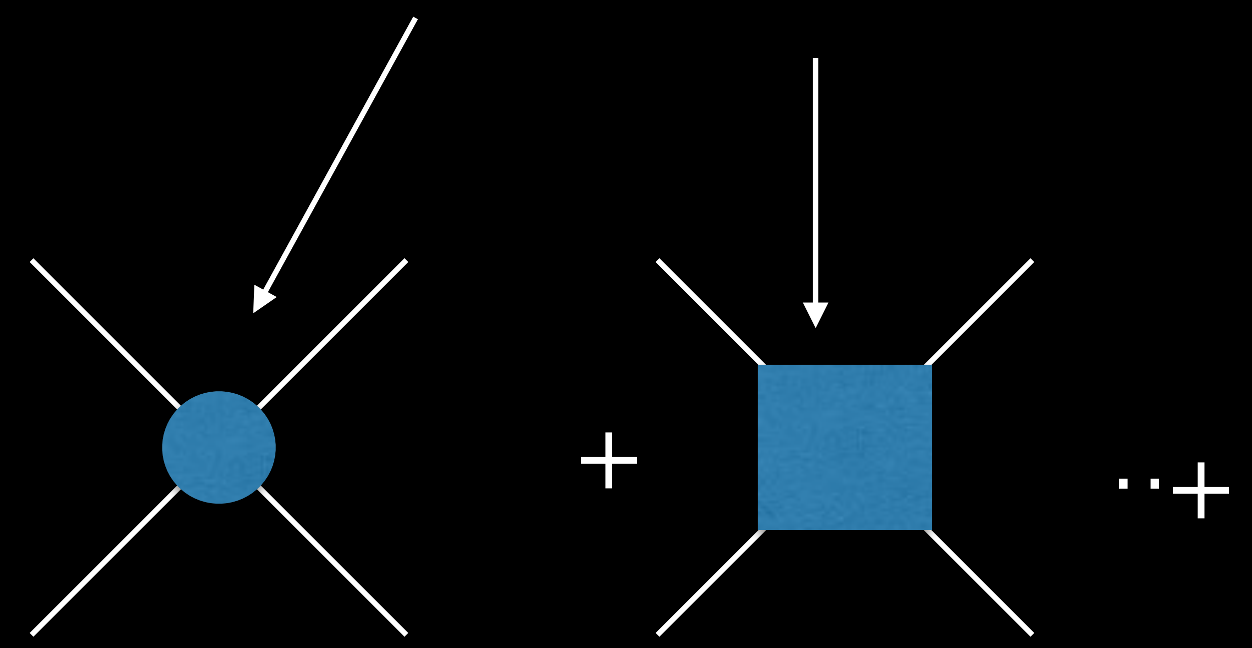

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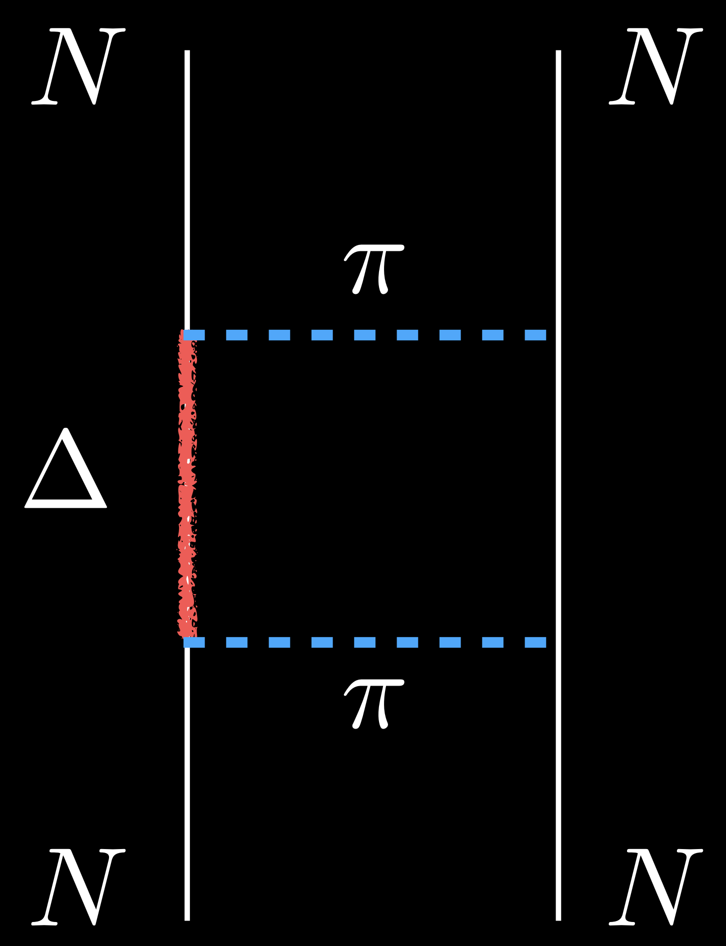
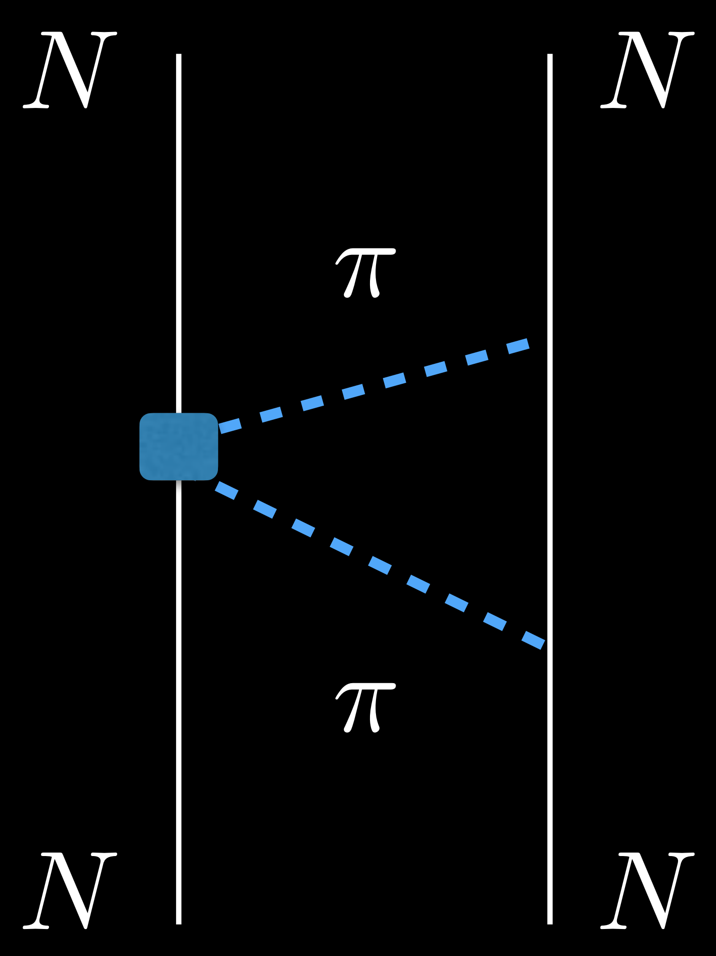
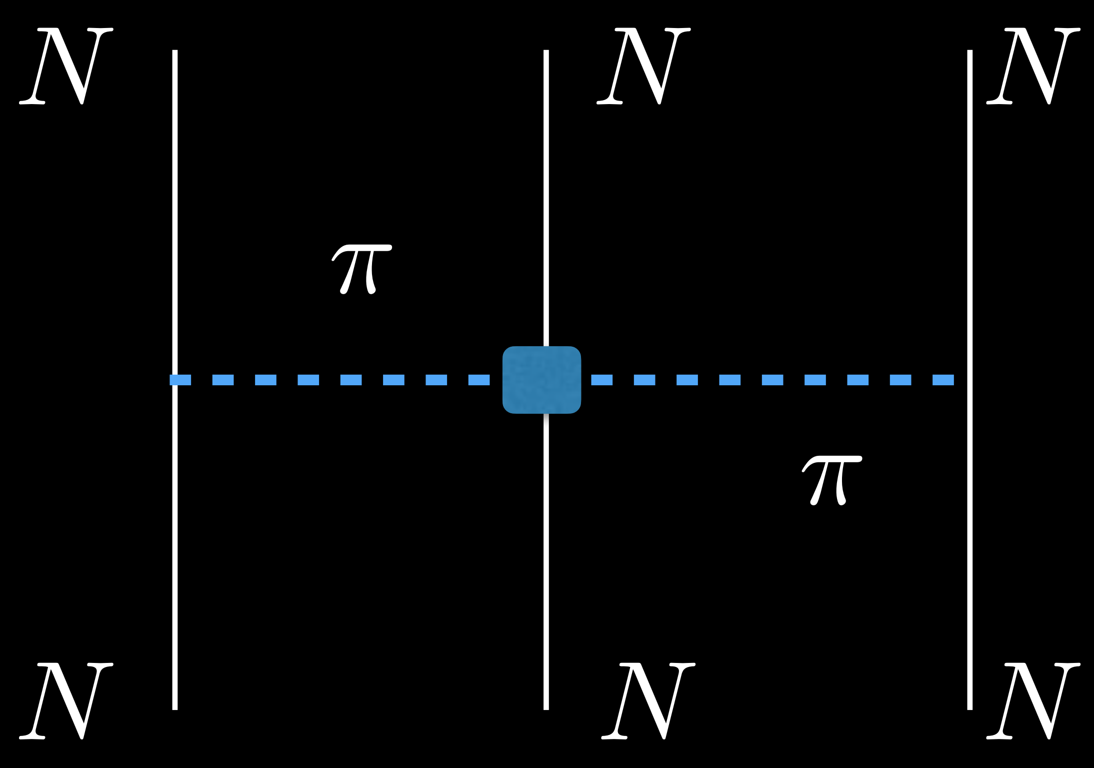
Nucleons are composite with internal excitations



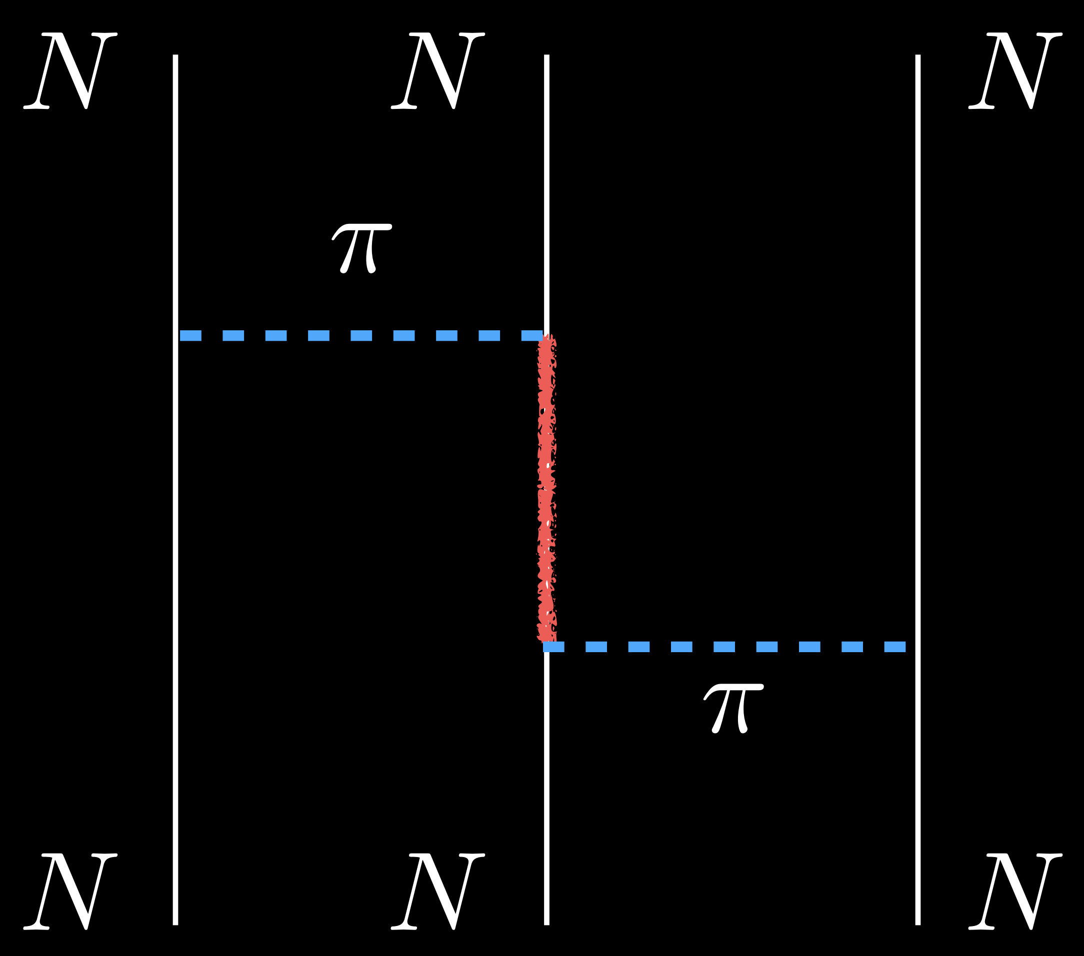
At low energy



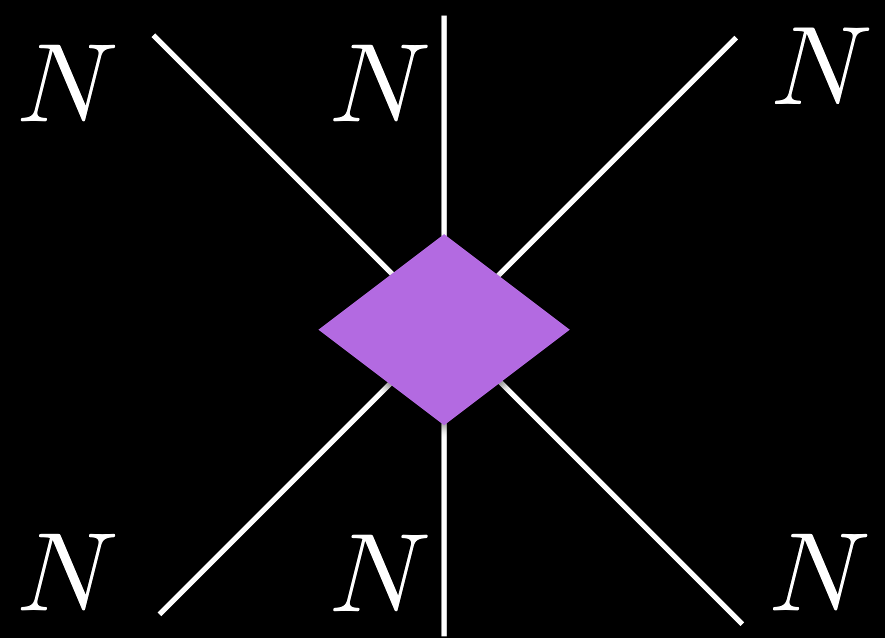
$$m_\Delta - m_N \simeq 1232 - 939 \text{ MeV} \approx 300 \text{ MeV}$$



There are three and many-body forces:

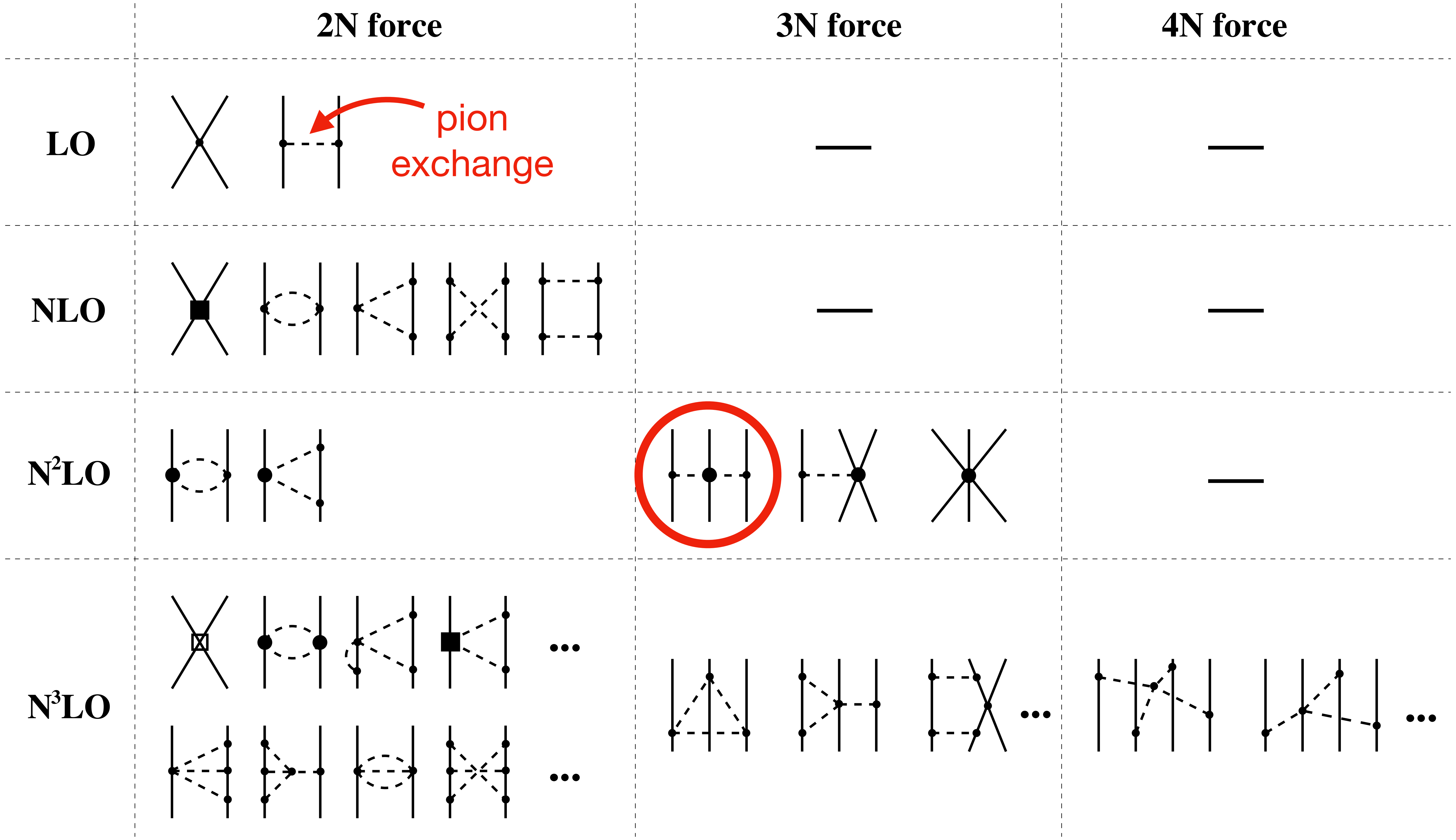


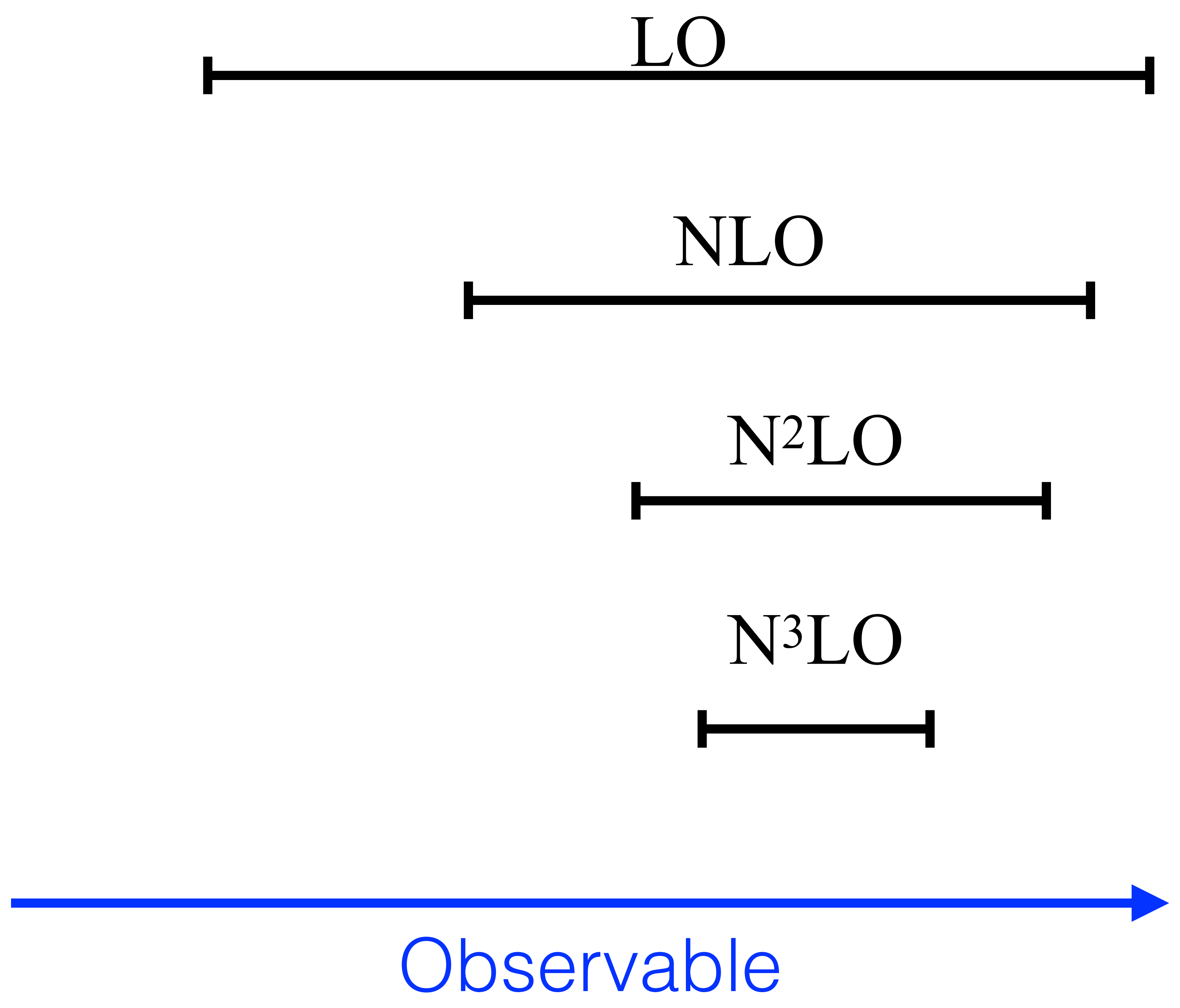
At low energy



Nuclear Forces from Effective Field Theory (EFT)

EFT Hamiltonians organizes operators in powers of the momentum: $\frac{Q}{\Lambda_B}$



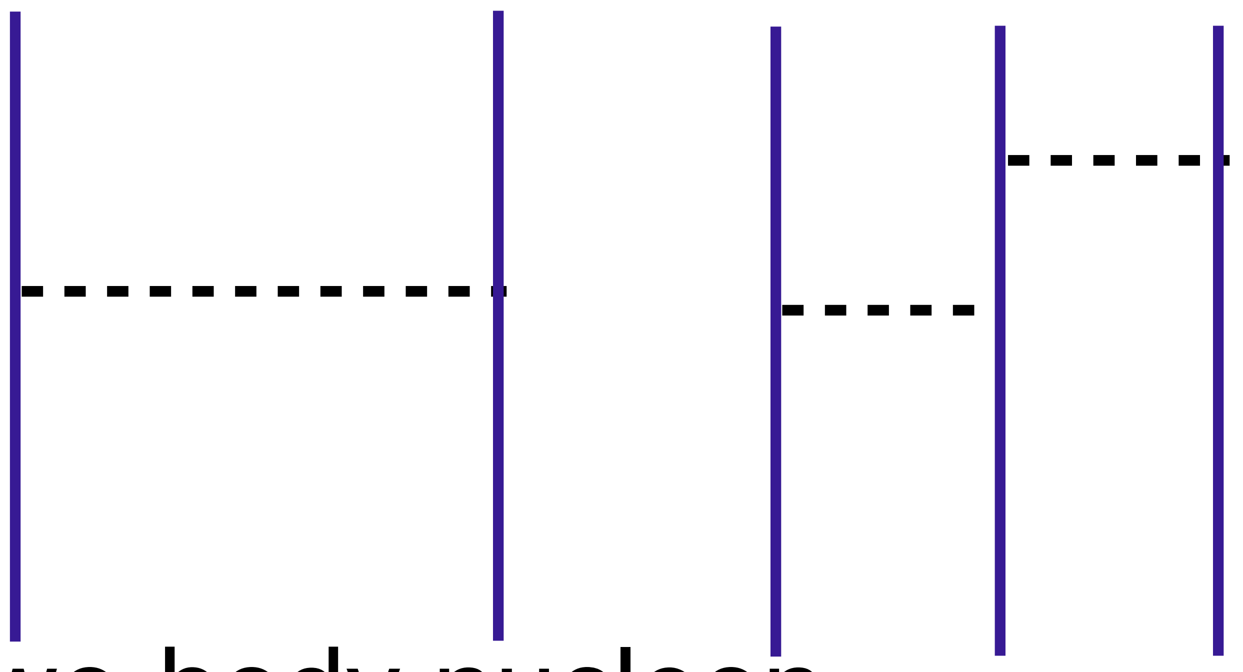


Allows for error estimation*. Provides guidance for the structure of three and many-body forces.

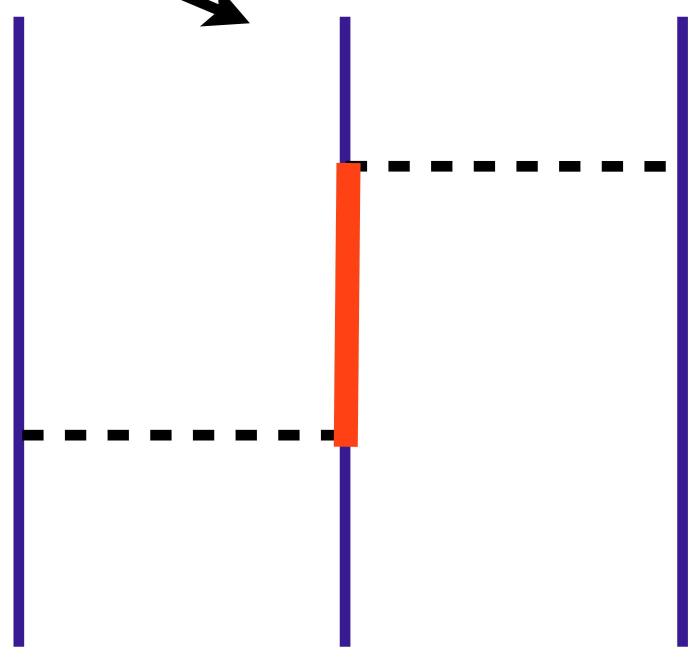
Beane, Bedaque, Epelbaum, Kaplan, Machliedt, Meisner, Phillips, Savage, **van Klock**, **Weinberg**, Wise ..

Ground State Energy

$$H_{\text{nuclear}} = \frac{\nabla^2}{2M} + V_{\text{NN}} + V_{\text{NNN}} + \cdots$$



two-body nucleon-nucleon potential is well constrained by scattering data.



three-neutron potential is constrained by light nuclei.

Quantum Many-Body Theory:
Quantum Monte Carlo
Diagrammatic Methods
(perturbation theory)

$E(\rho_n, \rho_p)$: Energy per particle

Equation of State of Dense Nuclear Matter

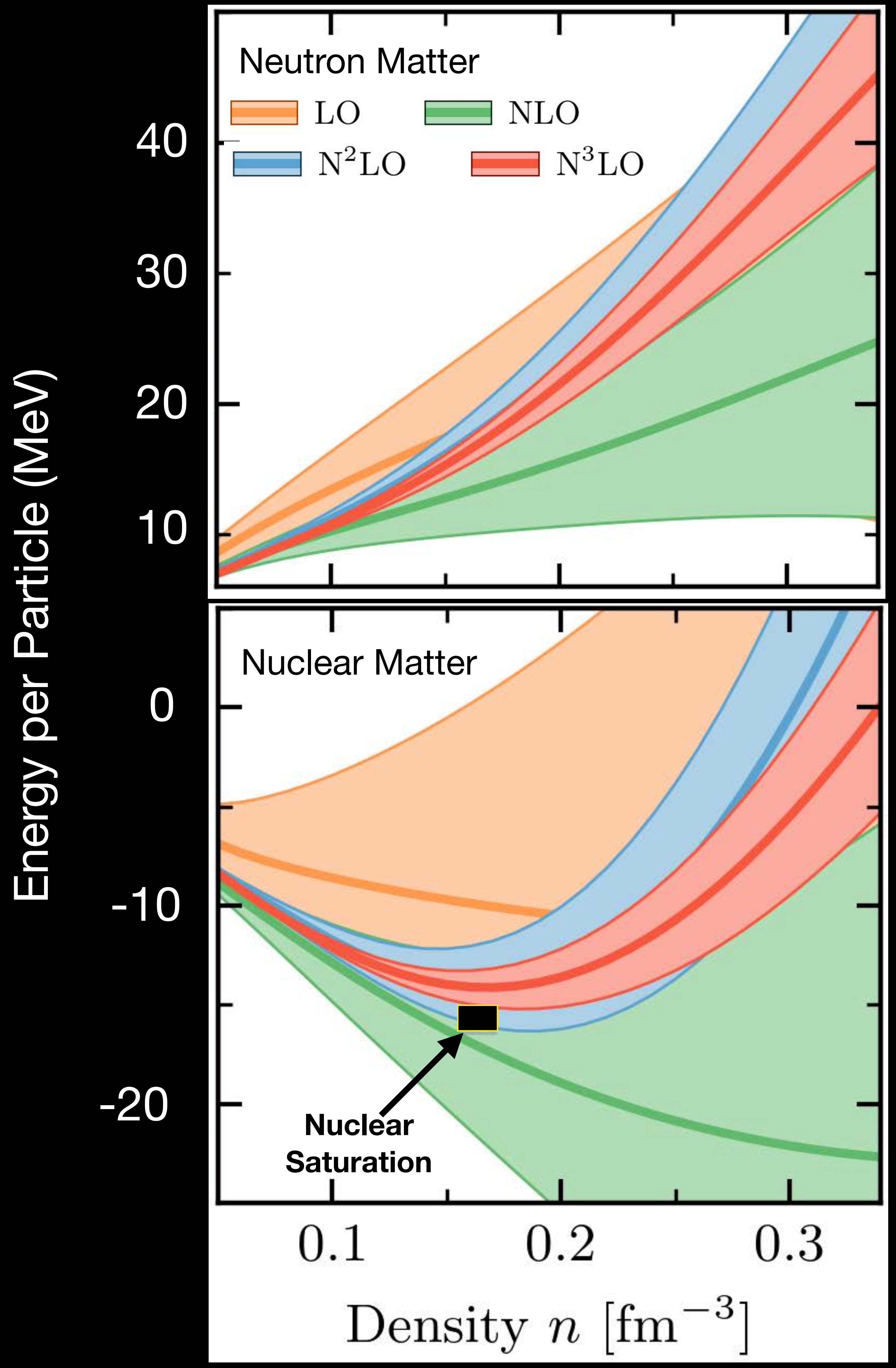
Quantum many-body calculations of neutron matter and nuclear matter using EFT potentials show convergence up to about twice nuclear saturation density.

Hebeler and Schwenk (2009), Gandolfi, Carlson, Reddy (2010), Gezerlis et al. (2013), Tews, Kruger, Hebeler, Schwenk (2013), Holt Kaiser, Weise (2013), Hagen et al. (2013), Roggero, Mukherjee, Pederiva (2014), Wlazlowski, Holt, Moroz, Bulgac, Roche (2014), Tews et al. (2018), Drischler et al., (2020).

Three-nucleon forces at N²LO play a key role. They provide the repuslion needed for saturation the pressure needed to hold up neutron stars.

Drischler et al. used Bayesian methods to systematically estimate the EFT truncation errors in neutron and nuclear matter.

Drischler, Furnstahl, Melendez, Phillips, (2020).



Drischler et al., (2020).

Equation of State of Neutron Star Matter

In neutron stars, matter is in equilibrium with respect to weak interactions and contains a small fraction (about 5-10%) of protons, electrons and muons:

Many-body perturbation theory and Bayesian estimates of the EFT truncation errors predict:

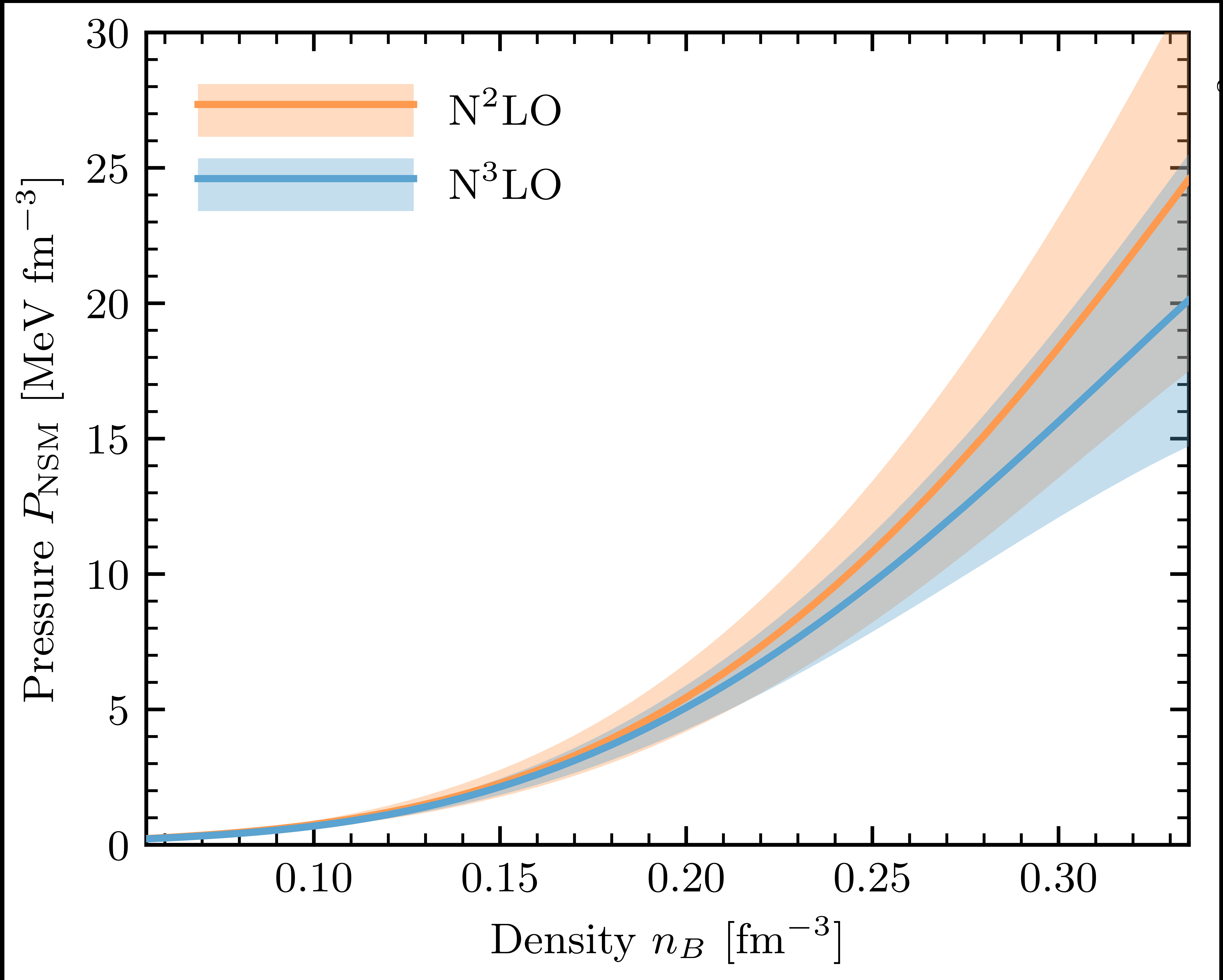
$$P_{\mathrm{NSM}}(n_B = 0.16\ \mathrm{fm}^{-3}) = 3.0 \pm 0.4\ \mathrm{MeV/fm}^3$$

$$P_{\mathrm{NSM}}(n_B = 0.34\ \mathrm{fm}^{-3}) = 20.0 \pm 5\ \mathrm{MeV/fm}^3$$

Christian Drischler

Sophia Han

Tianqi Zhao



Drischler, Han, Lattimer, Prakash, Reddy, Zhao (2020)

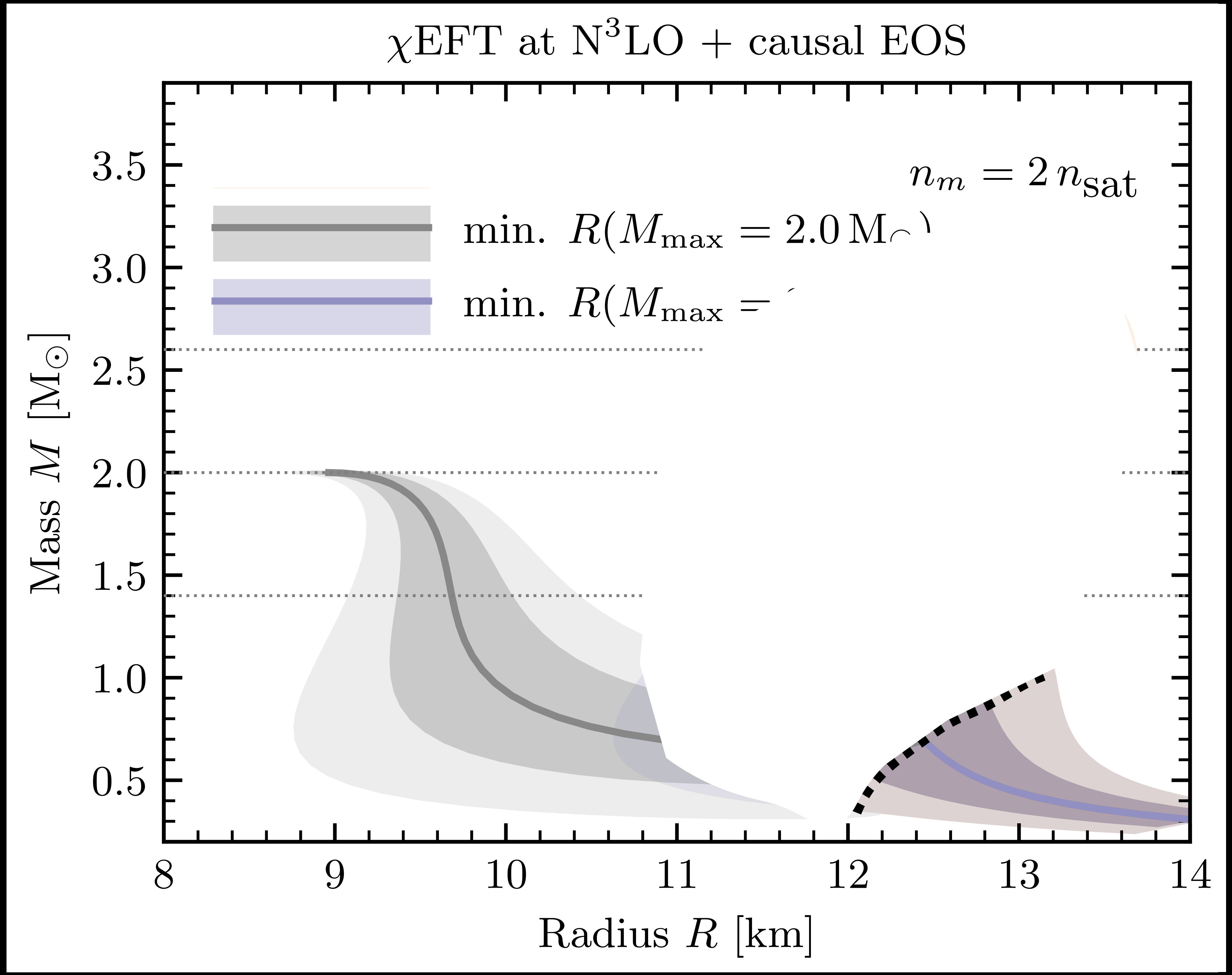
Bounds on Neutron Star Radii

EFT predictions for the EOS can be combined with extremal high-density EOS (with $c_s^2 = 1$) to derive robust bounds on the radius of a NS of any mass.

The lower limit on the NS maximum mass obtained from observations strengthen these bounds:

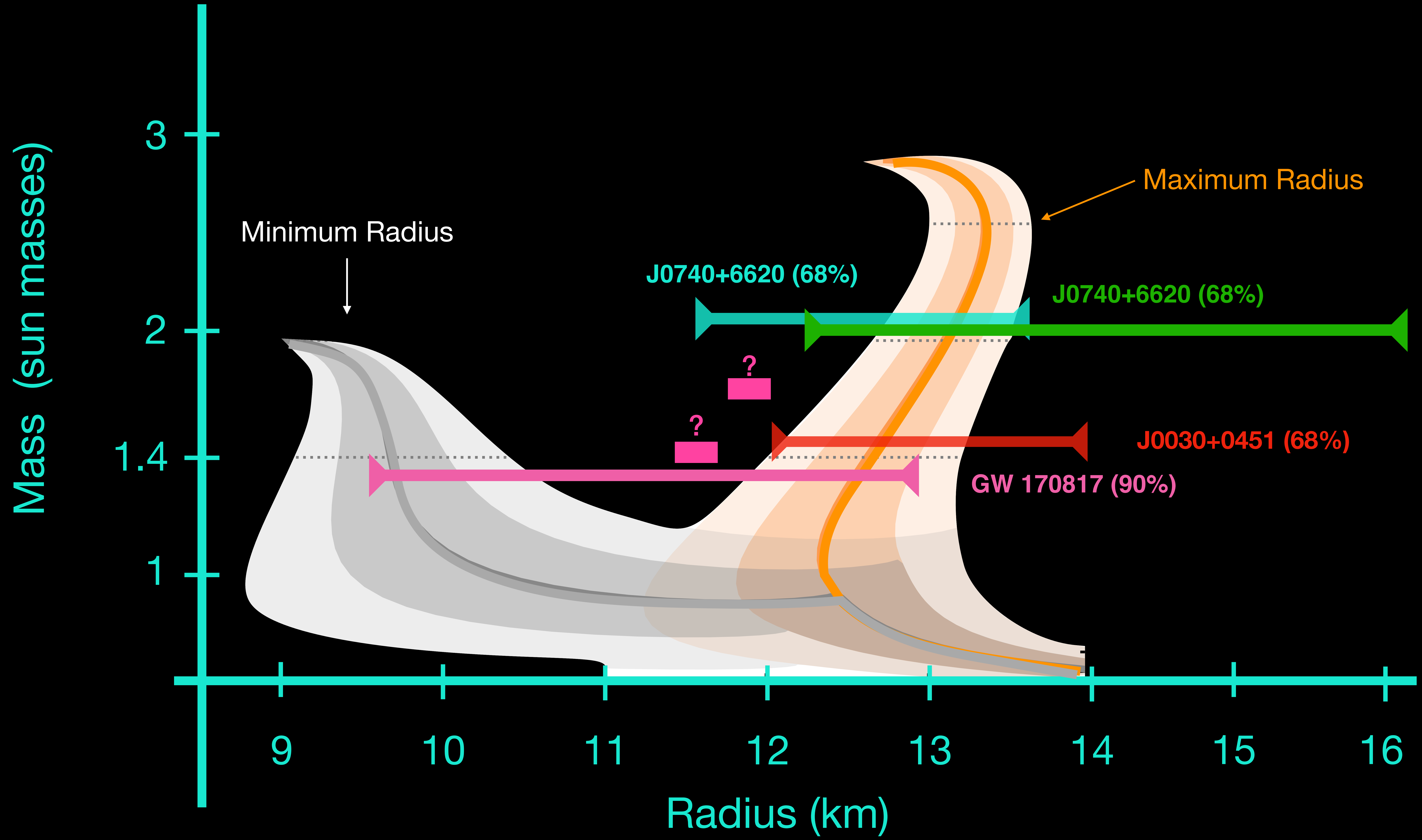
- $M_{\max} > 2.0\ M_{\odot}$, 9.2 km < $R_{1.4}$ < 13.2 km
- $M_{\max} > 2.6\ M_{\odot}$, 11.2 km < $R_{1.4}$ < 13.2 km

If $R_{1.4}$ is small (<11.5 km) or large (>12.5 km), it would imply a very large speed of sound in the cores of massive neutron stars.



Drischler, Han, Lattimer, Prakash, Reddy, Zhao (2020)

Observational Constraints



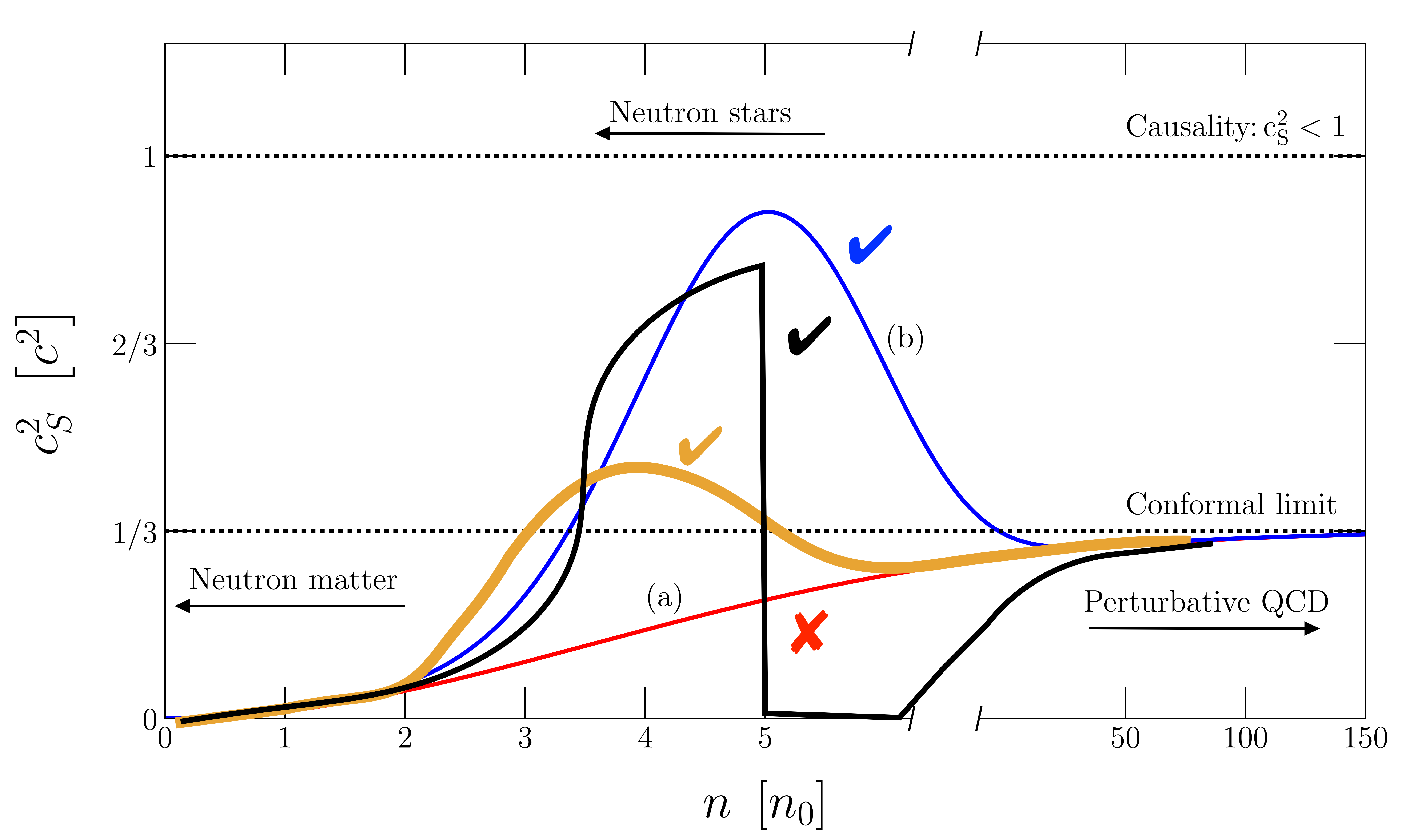
Speed of Sound in Dense Matter

$$c_s^2 = \frac{\partial P}{\partial \epsilon}$$

Large maximum mass and observed radii, combined with neutron matter calculations suggests a rapid increase in pressure in the neutron star core.

This implies a large and non-monotonic sound speed in dense QCD matter.

Suggests the existence of a strongly interacting phase of relativistic matter.

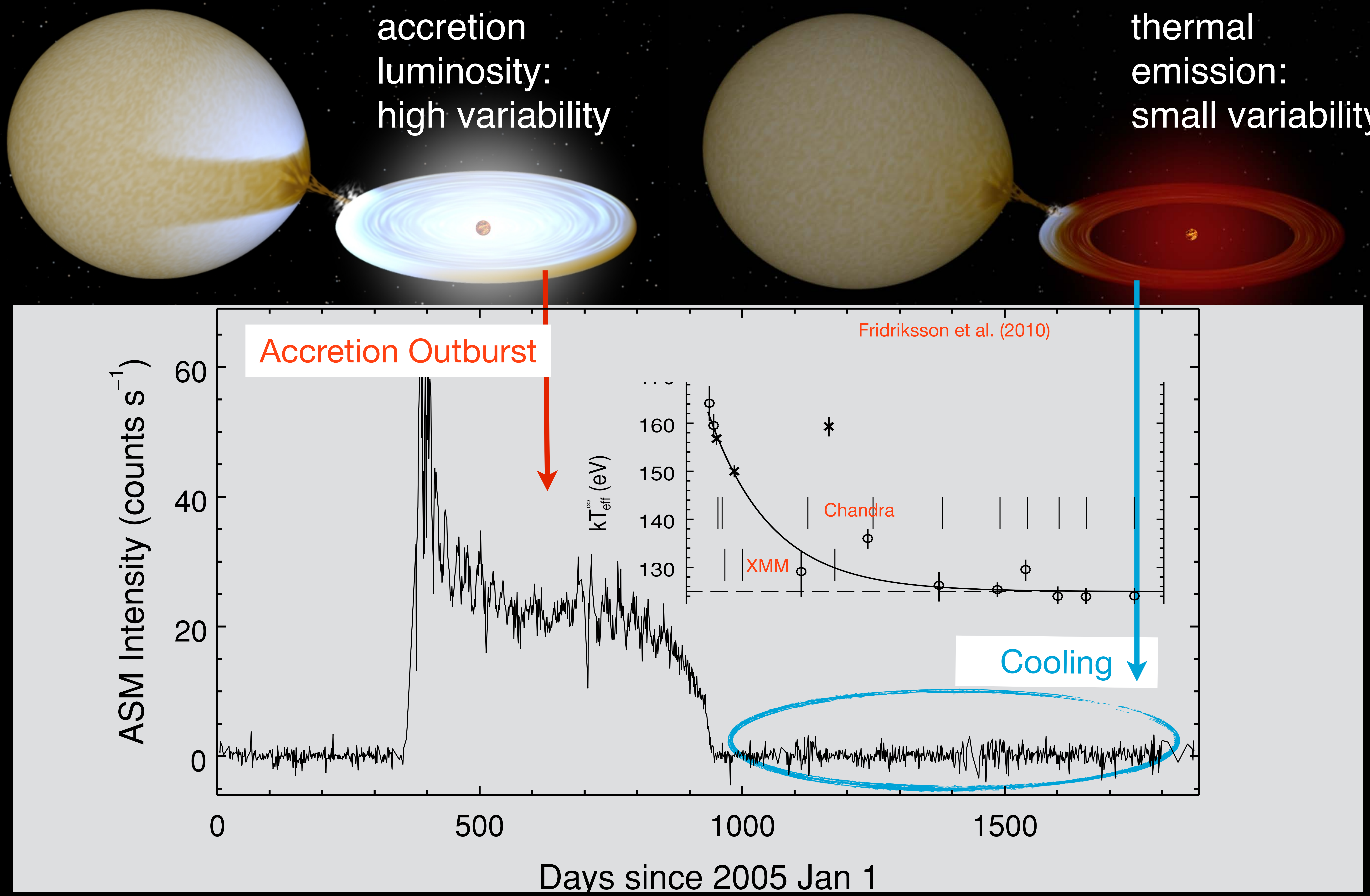


Tews, Carlson, Gandolfi and Reddy (2018)
Steiner & Bedaque (2016)

Thermal Evolution of Accreting Neutron Stars

Evidence for a solid and superfluid state of matter in the crust.

Transiently Accreting Neutron Stars



Cooling Post Accretion

- This relaxation was first discovered in 2001 and 6 sources have been studied to date.

- All known Quasi-persistent sources show cooling after accretion

- Cools on a time scale of ~1000 days.

- The thermal and transport properties of the solid and superfluid inner crust plays a key role.

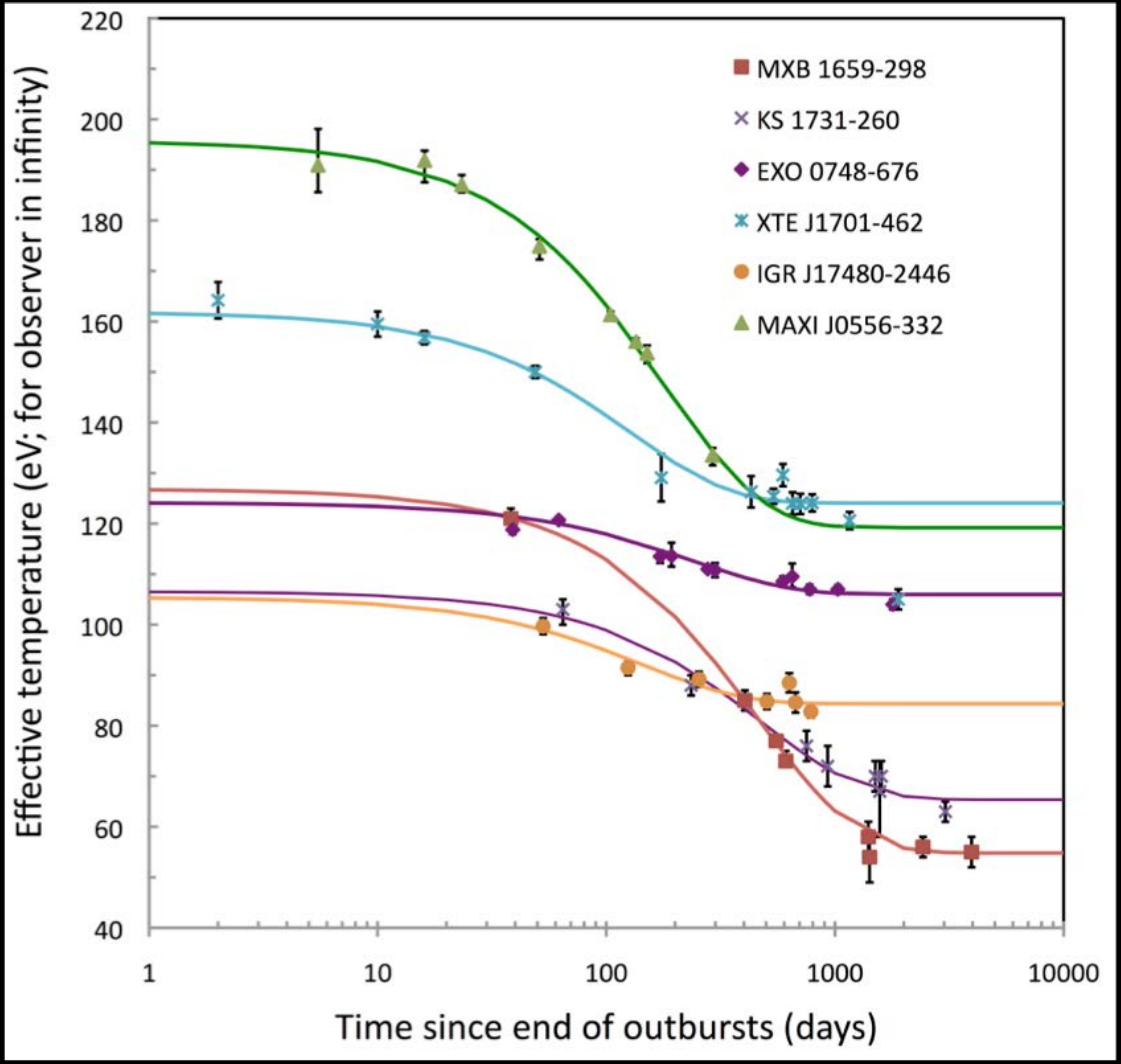
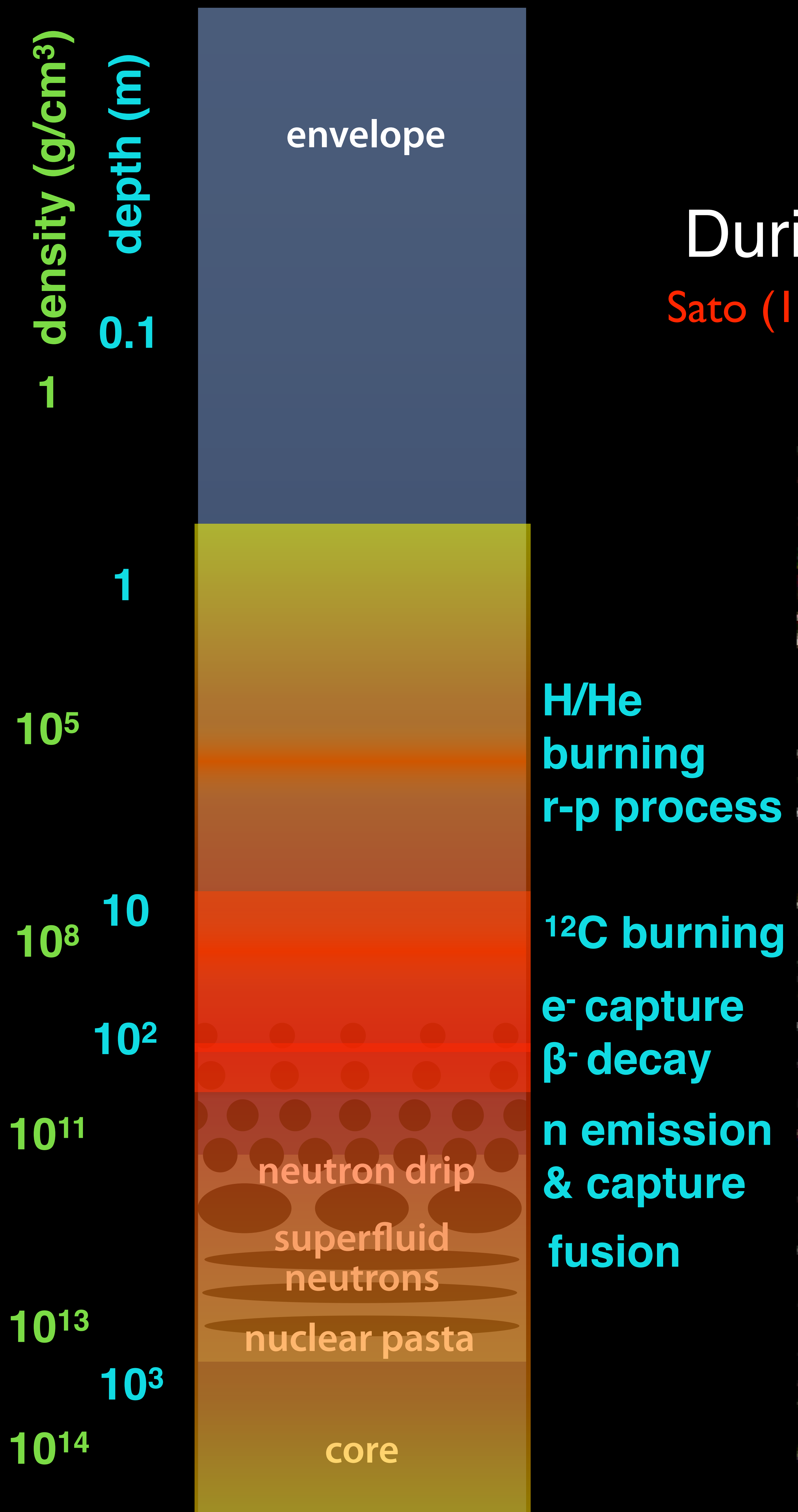


Figure from Rudy Wijnands (2013)

Deep Crustal Heating

During accretion nuclear reactions release: ~ 2-4 MeV / nucleon

Sato (1974), Haensel & Zdunik (1990), Brown, Bildsten Rutledge (1998) Gupta et al (2007,2011).

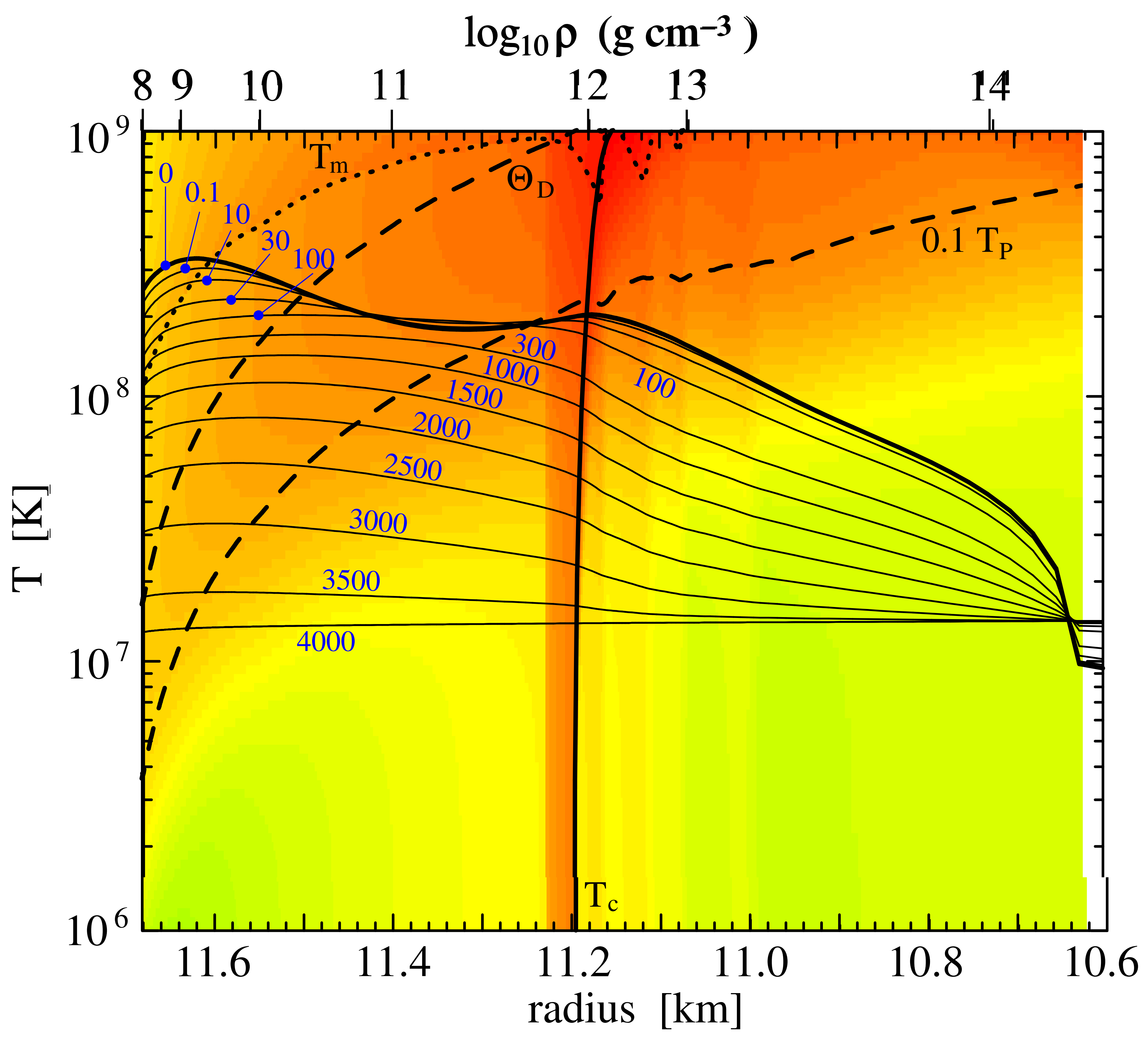


Thermal Evolution of the Crust

Temperature profile in the crust depends on the duration of the accretion phase.

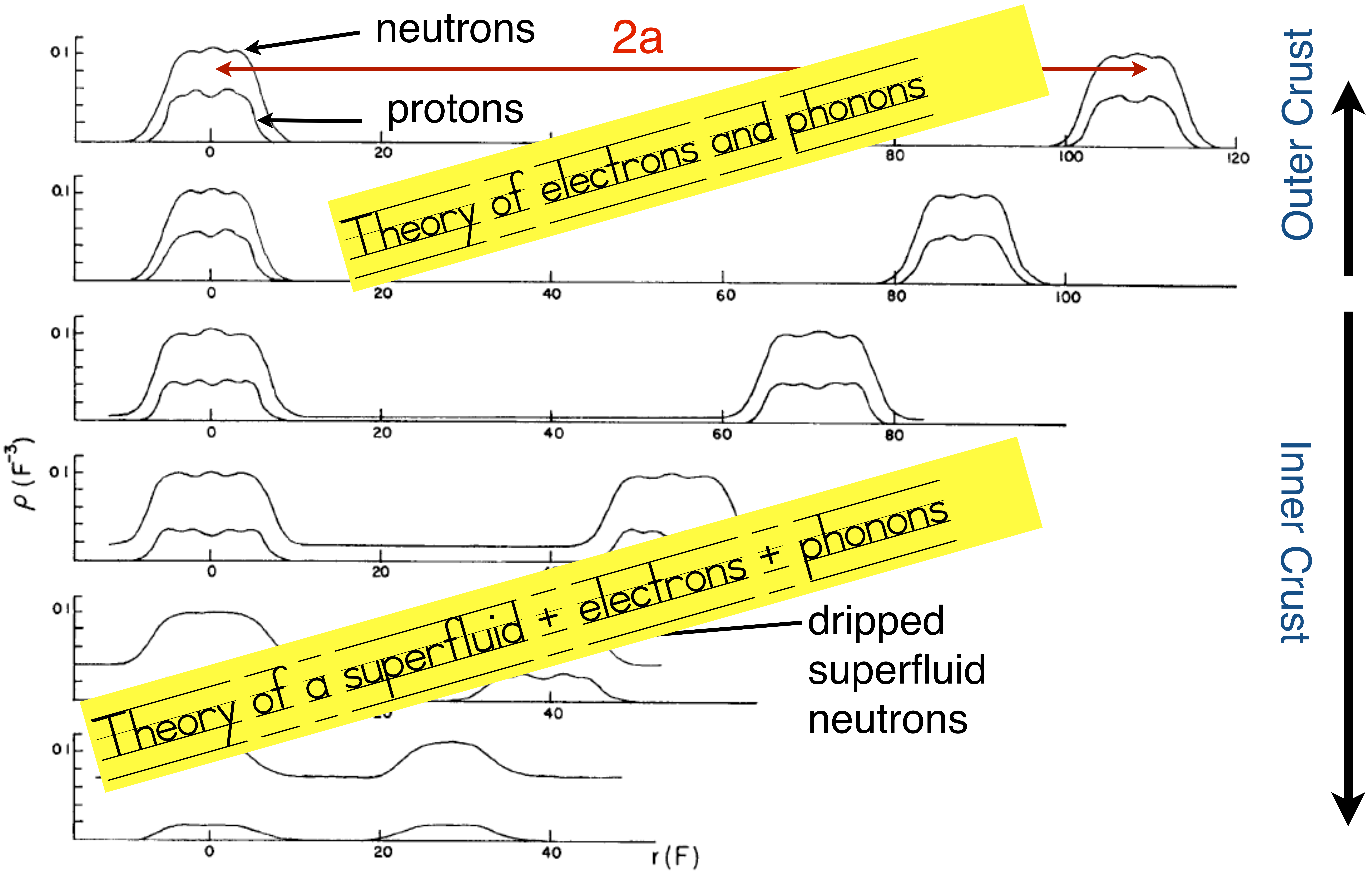
When accretion ends heat flows into the core and is radiated away as neutrinos.

Timescale for cooling is set by the heat diffusion time.

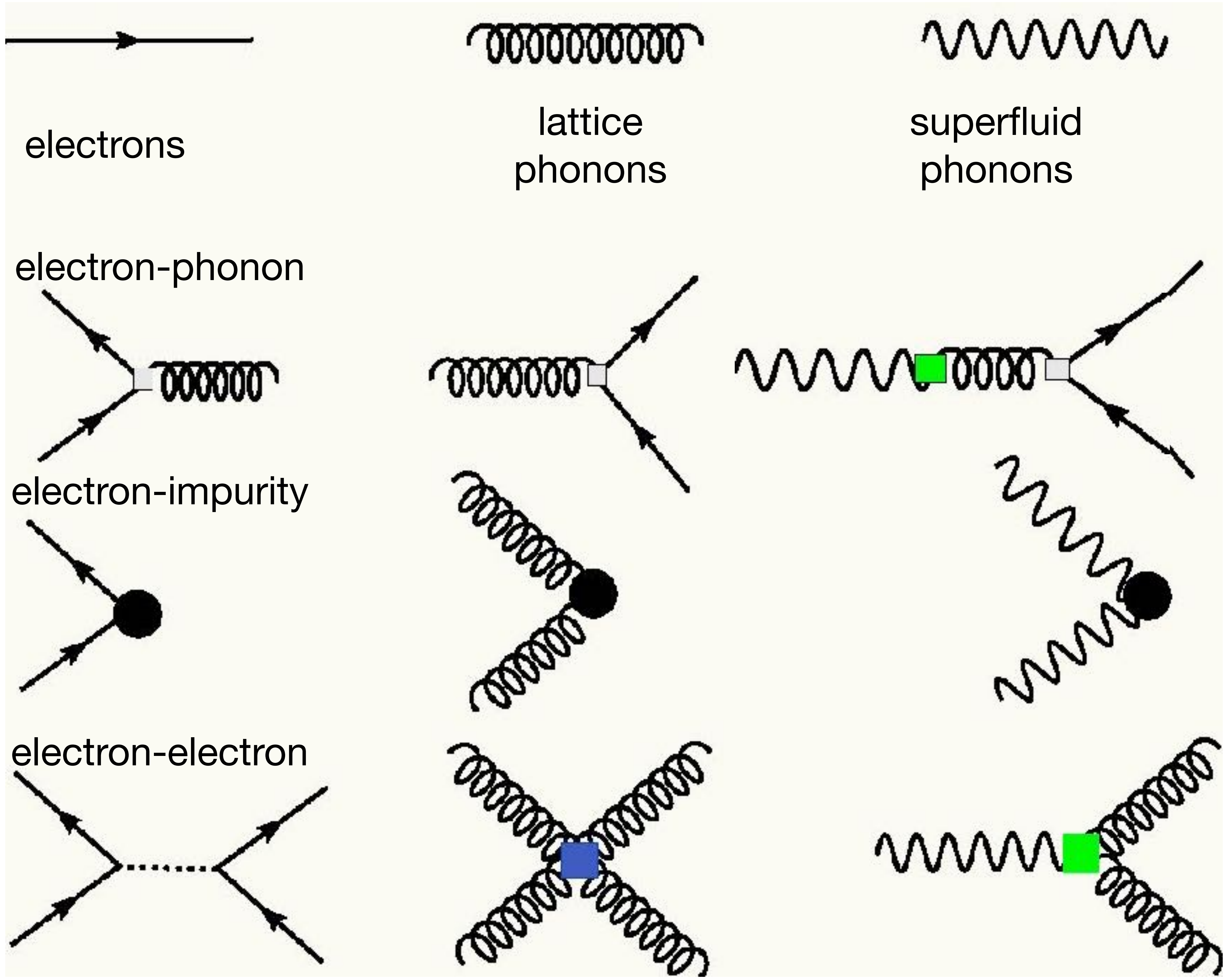


Shternin & Yakovlev (2007) Cumming & Brown (2009) Page & Reddy (2011)

Low Energy Excitations in the Crust



Baym Pethick & Sutherland (1971)   Negele & Vautherin (1973)



Cirigliano, Reddy & Sharma (2011), Page & Reddy (2012), Chamel, Page, & Reddy (2013)

Connecting to Crust Microphysics

Crustal Specific Heat

Crust Thickness

$$\tau_{\rm cool} \simeq \frac{C_V}{\kappa} \Delta R^2$$

Thermal Conductivity

- Observed timescales are short.
- Requires small specific heat and large thermal conductivity.

Observations suggest inner curst is solid and superfluid!

Shternin & Yakovlev (2007) Cumming & Brown (2009) Page & Reddy (2011)

The Dark Side of Neuton stars

Neutron stars are great places to look for dark matter

- They accrete and trap dark matter.

$$M_\chi < 10^{-14} M_\odot \left(\frac{\rho_\chi}{1\ \mathrm{GeV/cm^3}}\right) \frac{t}{\mathrm{Gyr}}$$

- Produce dark matter due to its high density.

$$M_\chi \lesssim M_\odot \text{ for } m_\chi < 2\ \mathrm{GeV}$$

- Produce dark matter due to high temperatures at birth or during mergers.

$$M_\chi \lesssim 10^{-1}\ M_\odot \text{ for } m_\chi < 500\ \mathrm{MeV}$$

Black-Holes in the Neutron Star Mass-Range

Idea: Accretion of asymmetric bosonic dark matter can induce the collapse of an NS to a BH.

Goldman & Nussinov (1989)

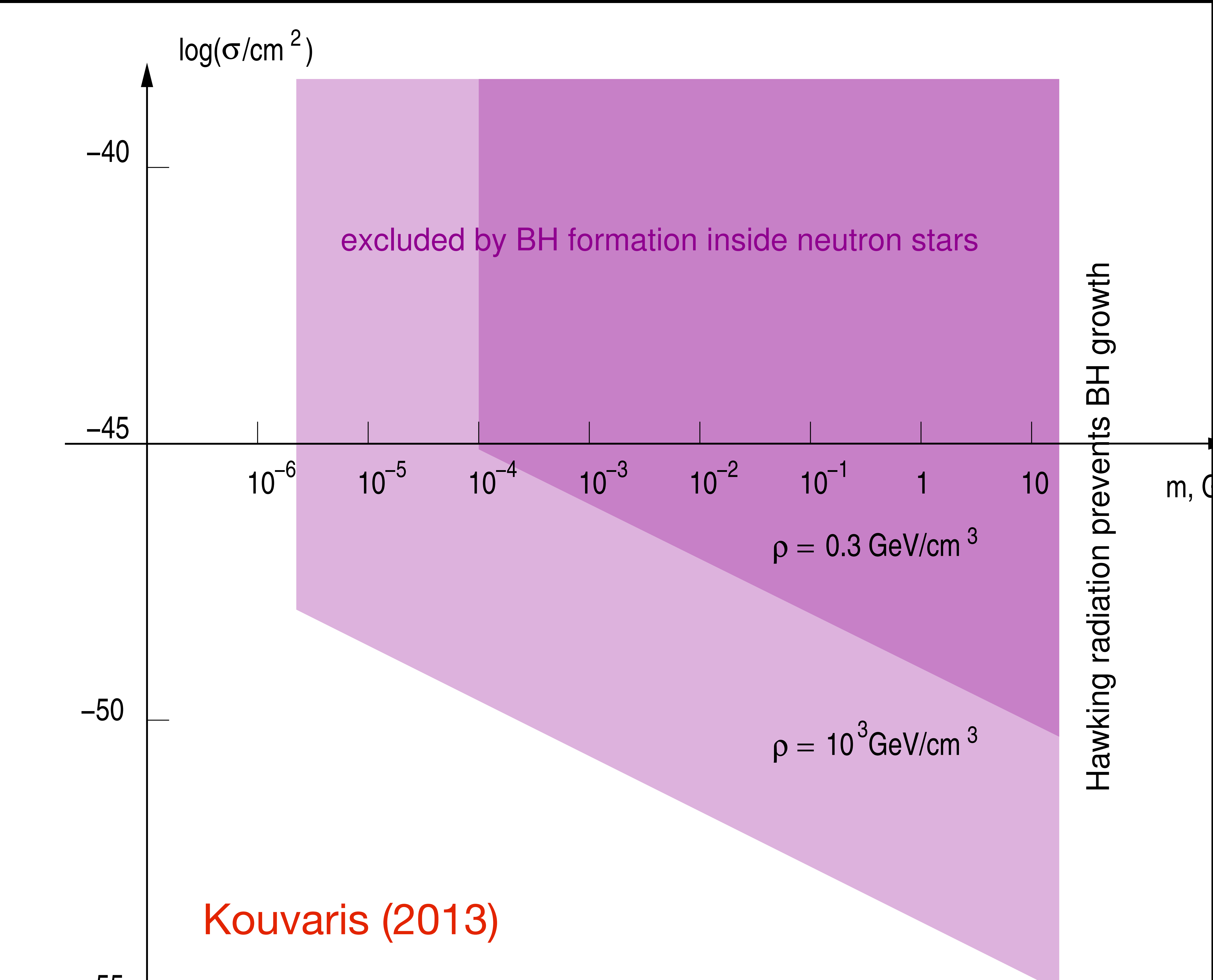
$$M_\chi \approx 10^{-14} M_\odot \ \mathrm{Min}\left[\frac{\sigma}{2\times 10^{-45}\mathrm{cm}^2}, 1\right] \left(\frac{\rho_\chi}{1\ \mathrm{GeV/cm}^3}\right) \frac{t}{\mathrm{Gyr}}$$

The maximum mass of weakly Interacting bosons is negligible:

$$M_{\mathrm{Bosons}} \approx 10^{-18}\ M_\odot \left(\frac{\mathrm{GeV}}{m_\chi}\right)$$

The existence of old neutron stars in the Milkyway with estimated ages ~ 10^{10} years provides strong constraints on asymmetric DM.

For a concise reviews see Kouvaris (2013) and Zurek (2013)



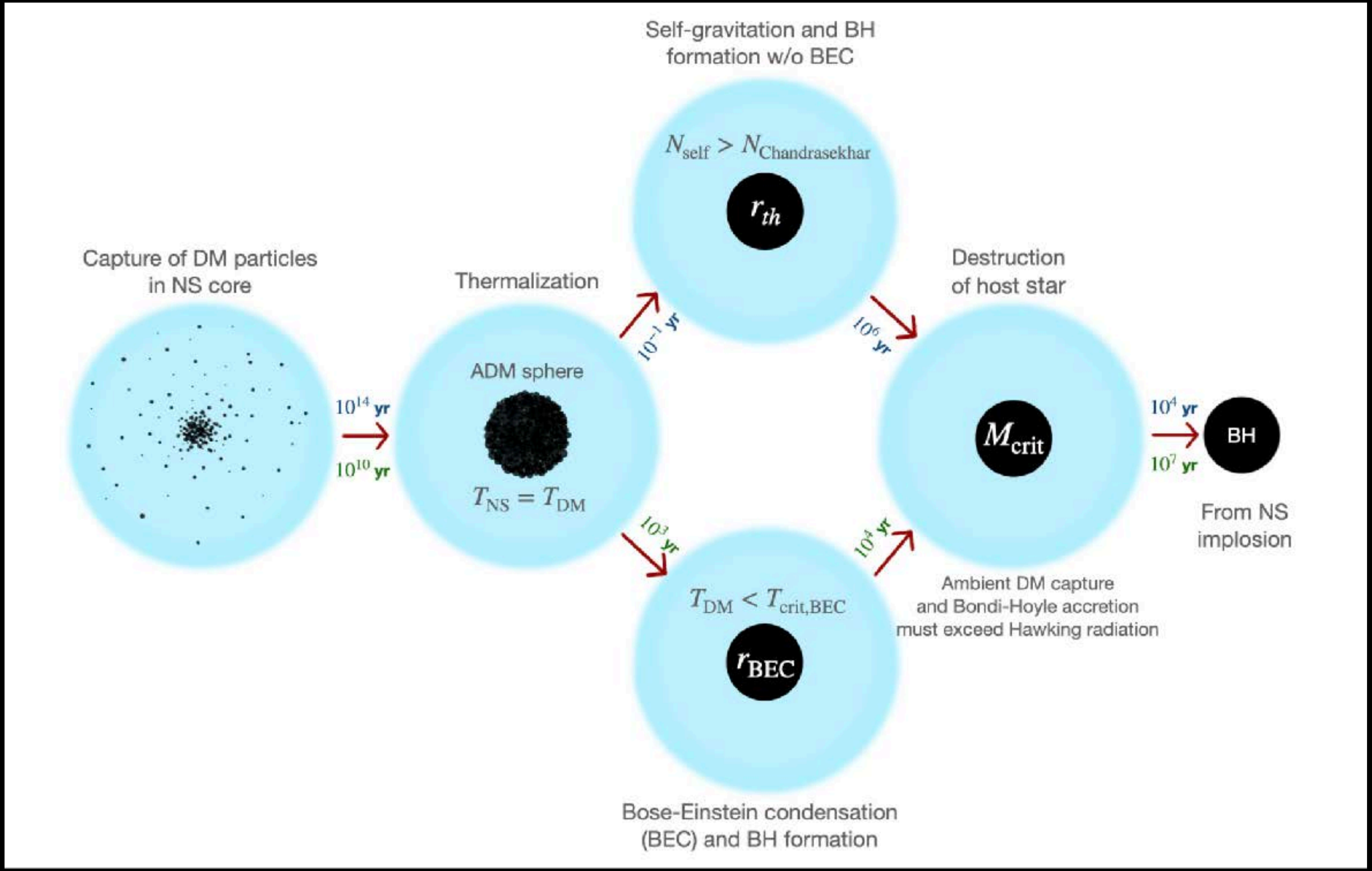
Kouvaris (2013)

Converting NSs into BHs

For dark matter in the 1-10^6 GeV mass range, black hole formation is complex and involves several timescales.

Capture time is typically the limiting step. But, thermalization can be slow in exotic superfluid phases and depends on processes in the inner core!

C. Kouvaris and P. Tinyakov (2011)
S. D. McDermott, H.-B. Yu, and K. M. Zurek, (2012)
B. Bertoni, A. E. Nelson, and S. Reddy (2013)
+ many more more refined recent analyses.



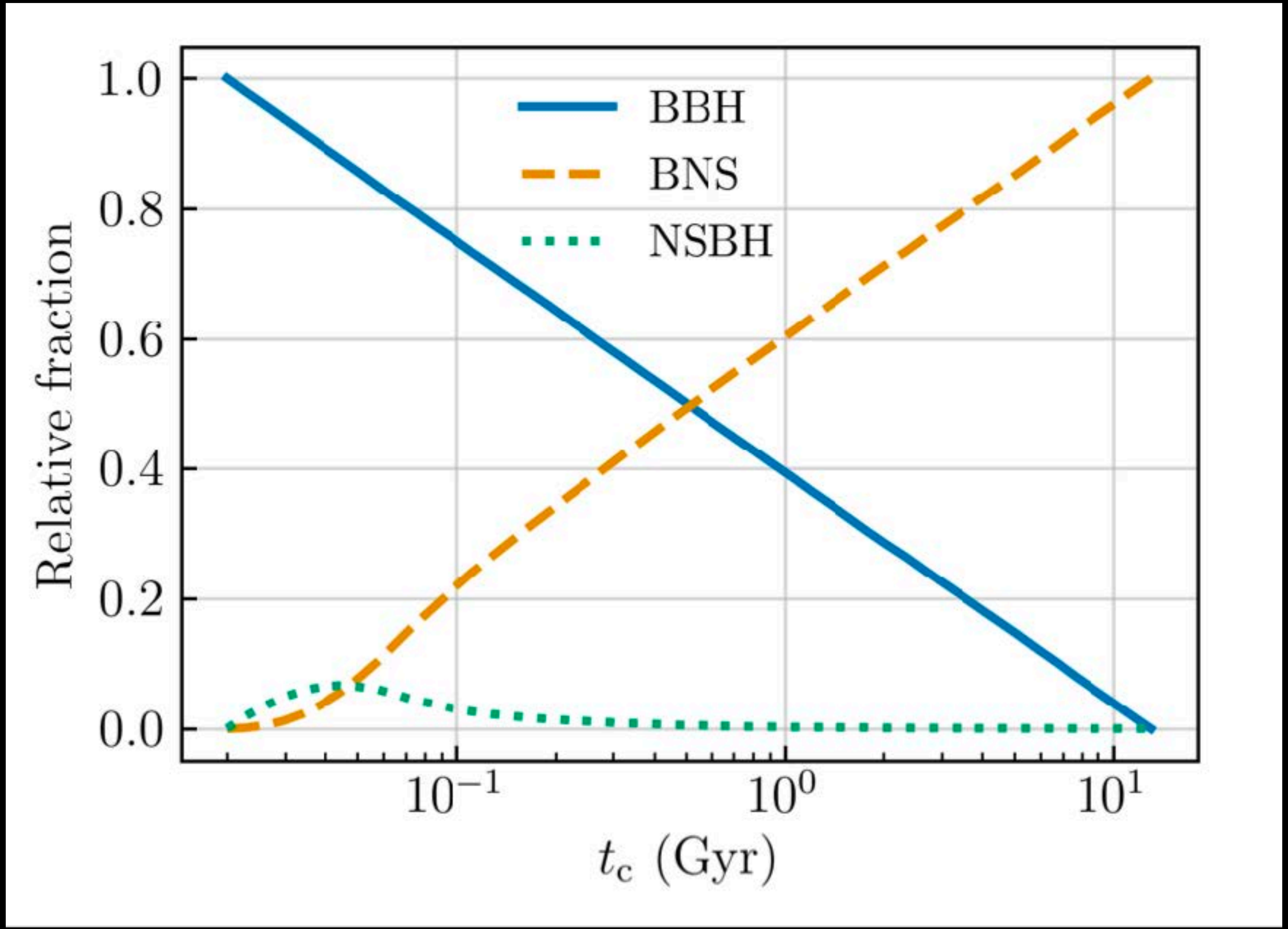
Divya Singh, Gupta, Berti, Reddy, Sathyaprakash (2023)

Inferring Conversion Timescales from Future GW Observations

- Measuring many binary masses and tidal deformability presents unique opportunities beyond discovering BHs in the NS mass range.

- The conversion timescale can be inferred if it is comparable to the binary coalescence time scale (delay timescale) from the fraction of BBH in the NS mass-range.

- In simple scenarios, the conversion timescale can be inferred quite accurately with next-generation detectors.



BBH and BNS distributions for a hypothetical conversion timescale ~ 1 Gyr.

Divya Singh, Gupta, Berti, Reddy, Sathyaprakash (2023)

Baryon Number Violation in Neutron Stars

Particles in the MeV-GeV mass range that mix with baryons very weakly are natural dark matter candidates.

There was speculation that a dark baryon with mass m_χ between 937.76 - 938.78 MeV might explain the neutron life-time discrepancy:

$$n \to \chi + \ldots$$

Fornal & Grinstein (2018)

$$\tau_n^{\text{bottle}} = 879.6 \pm 0.6 \text{ s}$$ -- counts neutrons

$$\tau_n^{\text{beam}} = 888.0 \pm 2.0 \text{ s}$$ -- counts protons

$$\text{Br}_{n\to\chi} = 1 - \frac{\tau_n^{\text{bottle}}}{\tau_n^{\text{beam}}} = (0.9 \pm 0.2) \times 10^{-2}$$

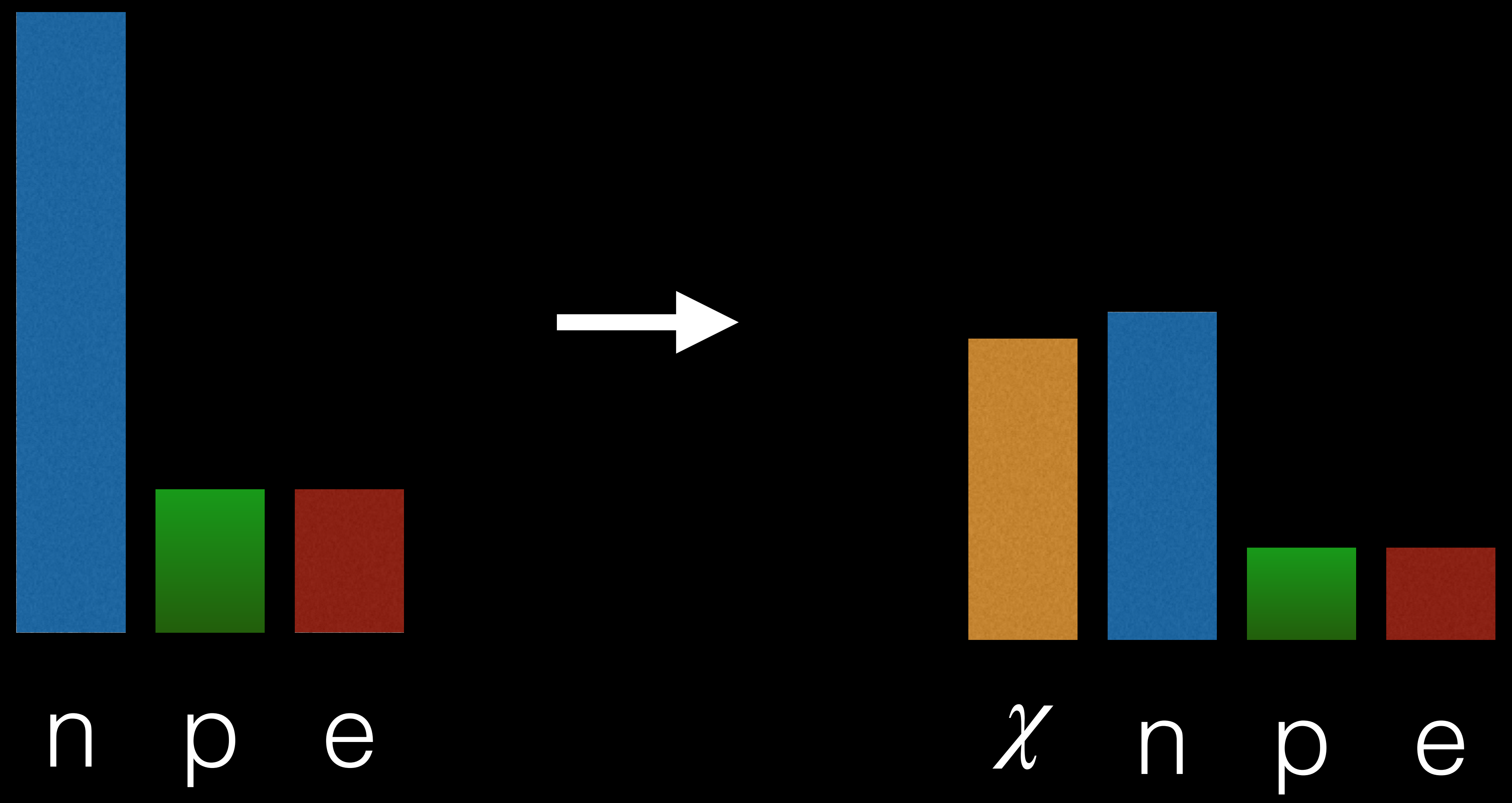
A model for hidden baryons which mix with the neutron:

$$\mathcal{L}_{\text{eff}} = \bar{n}\,(i\partial\!\!\!/ - m_n)\,n + \bar{\chi}\,(i\partial\!\!\!/ - m_\chi)\,\chi - \delta\,(\bar{\chi}n + \bar{n}\chi)$$

Mixing angle: $\theta = \dfrac{\delta}{\Delta M}$ $\Rightarrow$ An explanation of the anomaly requires $\theta \simeq 10^{-9}$

Neutron stars can probe smaller mixing angles $\theta \simeq 10^{-18}$ and masses up to 2 GeV.

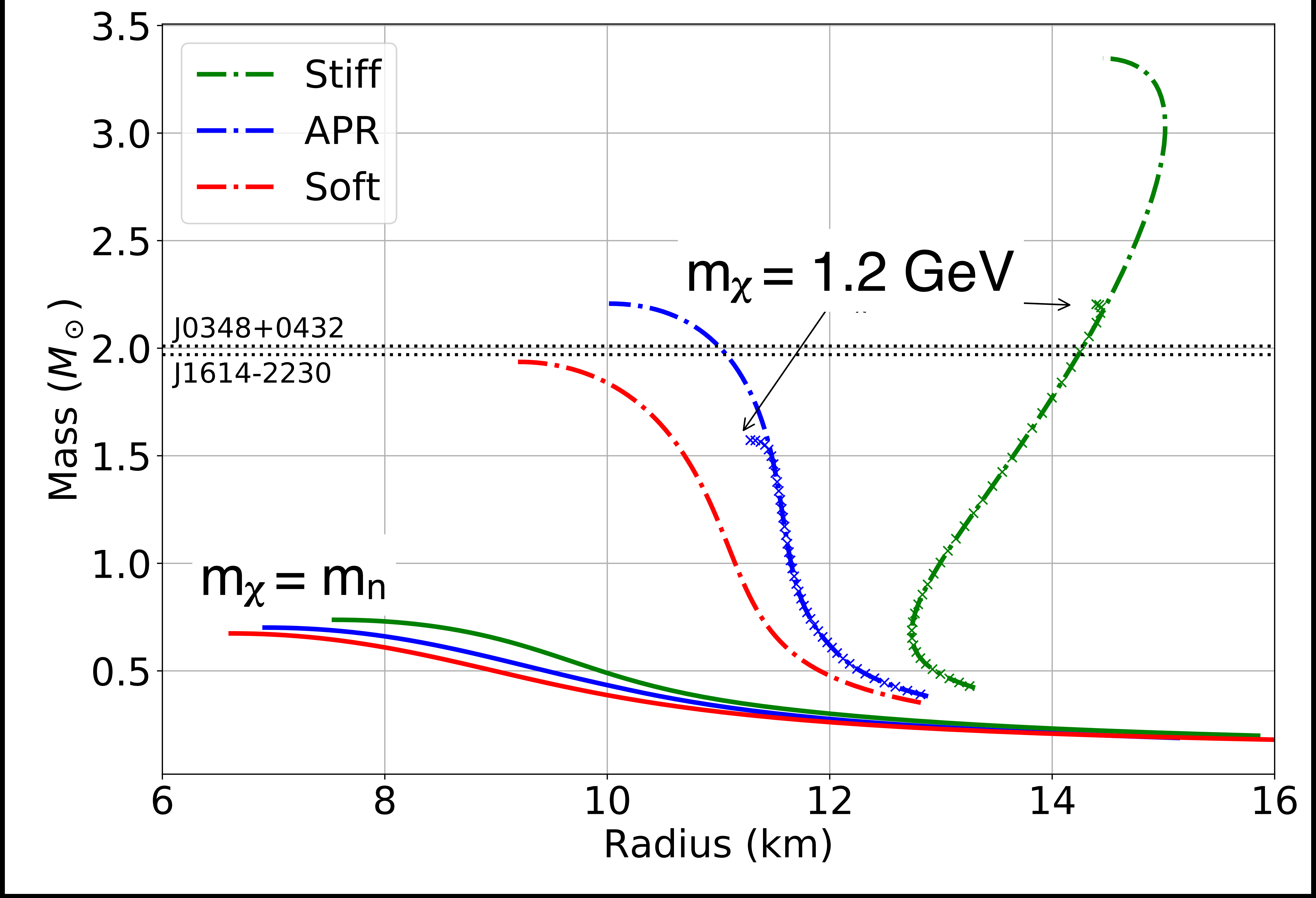
Weakly Interacting Dark Baryons Destabilize Neutron Stars



Neutron decay lowers the nucleon density at a given energy density.

When dark baryons are weakly interacting the maximum mass of neutron stars is greatly reduced.

Observed neutron stars exclude dark baryons with mass < 1.2 GeV.



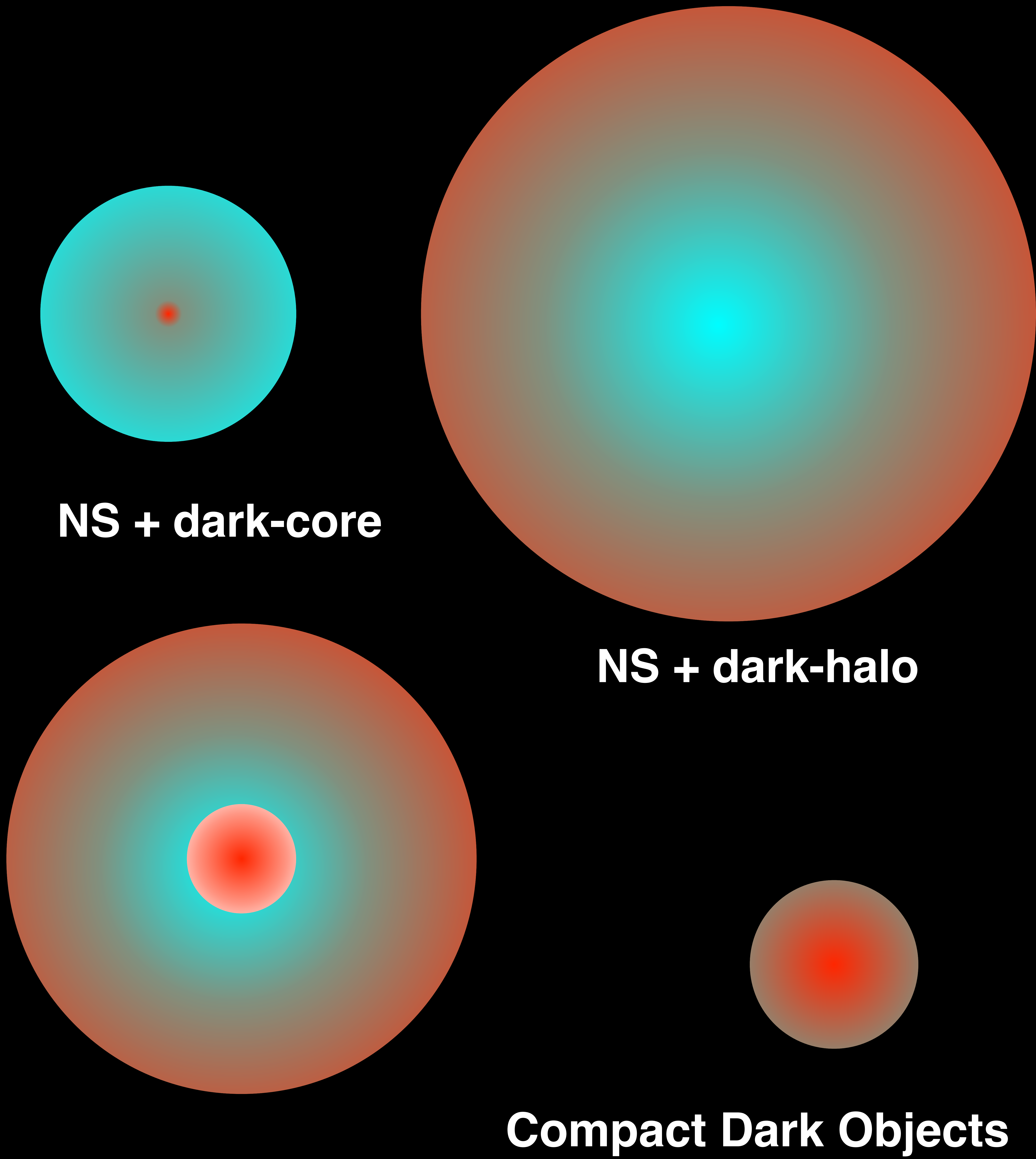
Mckeen, Nelson, Reddy, Zhou (2018) Baym, Beck, Geltenbort, Shelton (2018) Motto, Guichon, Thomas (2018)

Self-interacting Dark Matter

Self-interacting dark matter could form hybrid neutron stars and compact dark objects.

Gravitational wave observations of binary compact objects whose masses and tidal deformability's differ from those expected from neutron stars and stellar black holes would provide conclusive evidence for a strongly self-interacting dark sector:

Mass $< 0.1\ M_{solar}$
Tidal Deformability > 600



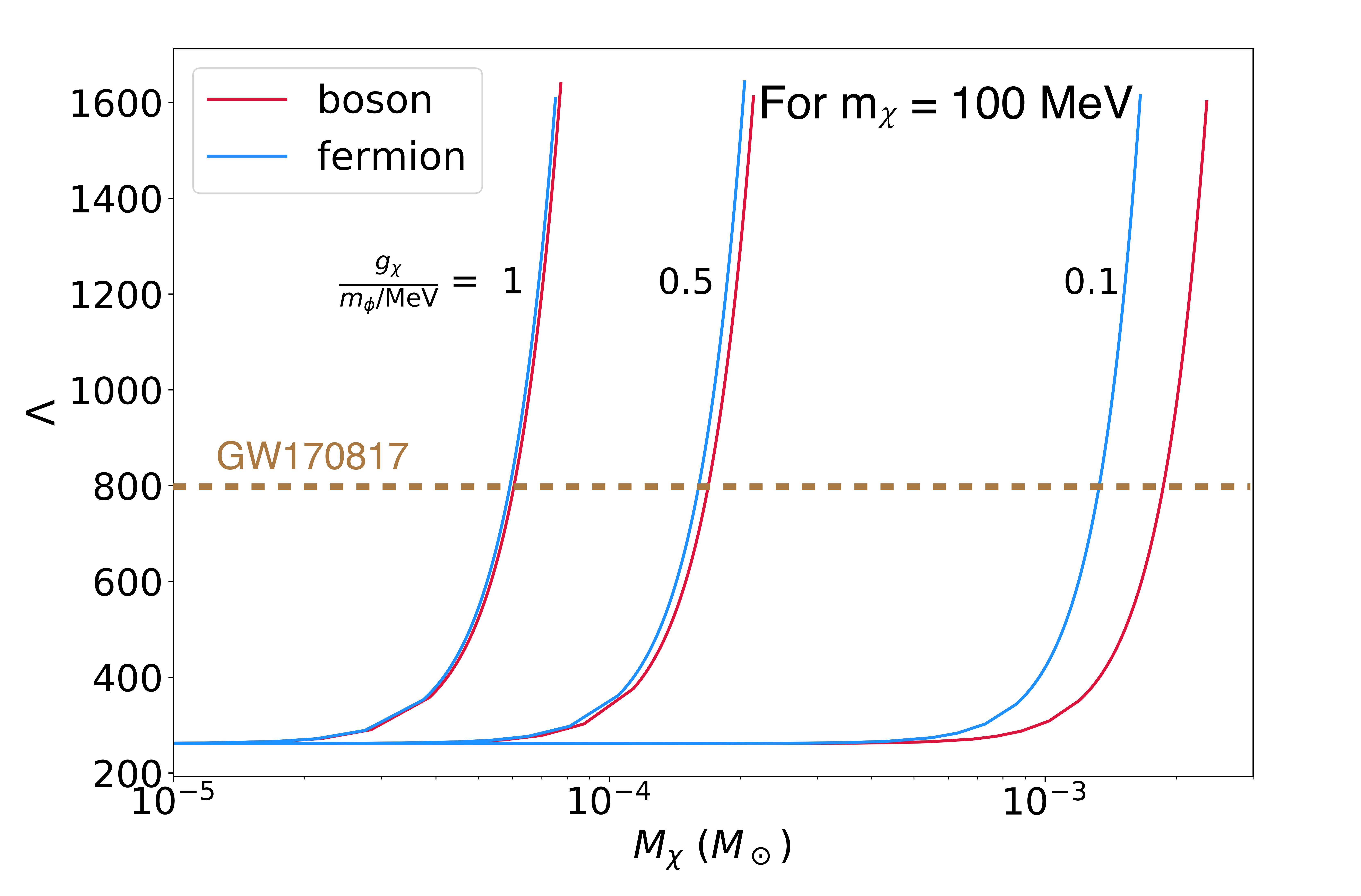
Nelson, Reddy, & Zhou (2018)   Horowitz & Reddy (2018)

Dark Halos Alter Tidal Interactions

Trace amount of light dark matter ~ 10^{-4}-10^{-2} M_{solar} is adequate to enhance the tidal deformability $\Lambda > 800$!

Self-Interactions of "natural" size provides adequate repulsion.

g_χ/m_Φ = (0.1/MeV) or (10^{-6}/eV)



Nelson, Reddy, Zhou (2018)