Efficient Propagation of Neutrino Flavors in Matter

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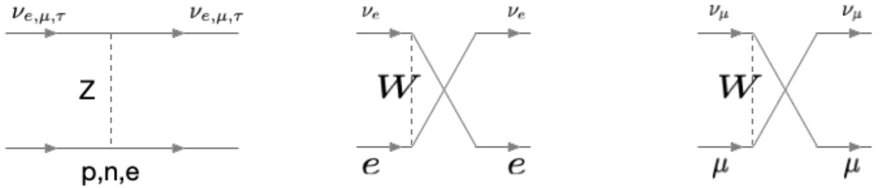
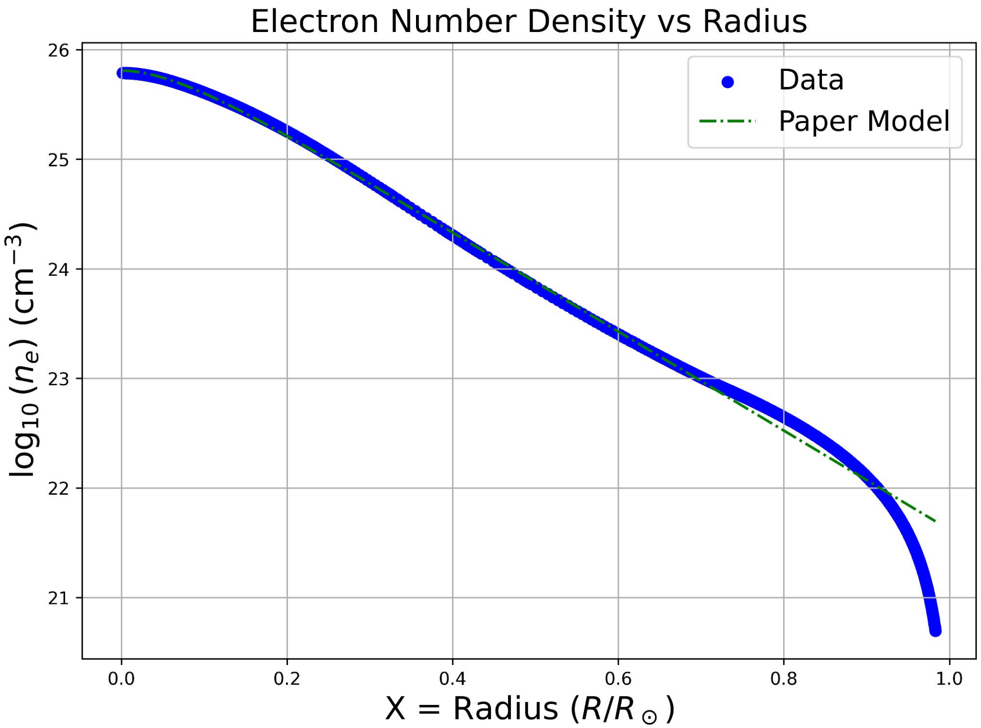
Motivation – Solar Neutrino Problem

- Neutrinos come in 3 flavors: electron, muon, and tau.
- As they travel, they **oscillate** — changing flavor due to quantum mixing.
- In dense environments like the Sun, interactions with electrons modify oscillation behavior (MSW effect).
    - In a supernova's proto-neutron star the muon density can be much higher, with a bigger effect
- This effect is central to the **solar neutrino problem**, where fewer electron neutrinos were detected on Earth than expected.
- The Sun provides a well-understood test case to develop and validate models before extending to more complex settings



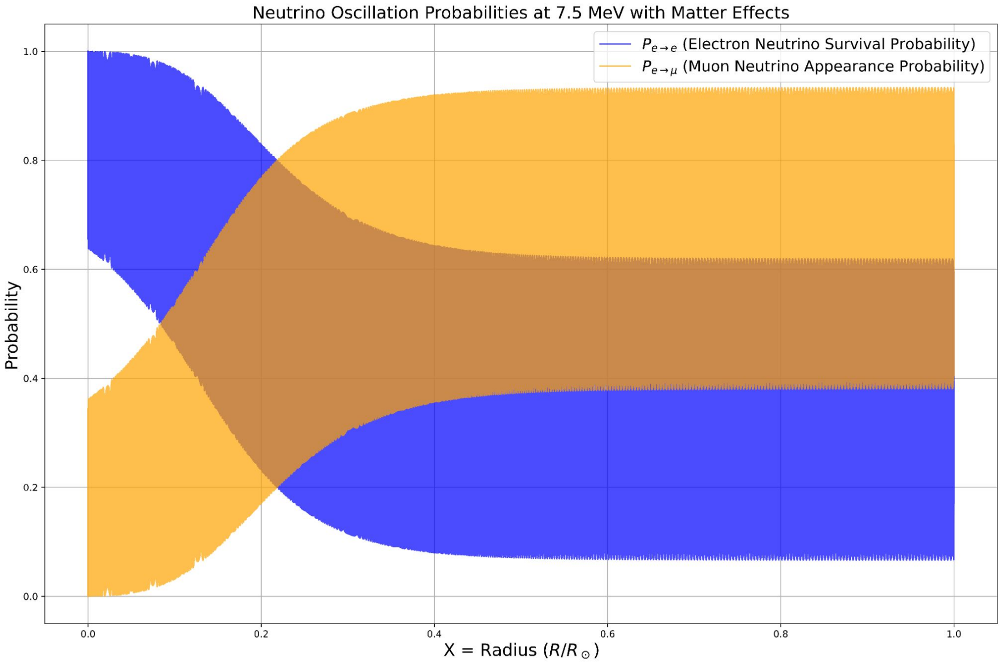
neutral current interaction, all flavors — charged current interaction, only electron neutrino — charged current interaction, only muon neutrino

Solar Electron Density



- Electron number density decreases exponentially with solar radius
- Data closely follows the analytical model from Bahcall [3], especially in the core and mid-radius
- Deviations appear near the surface, likely due to increased uncertainties in solar structure models
- This density profile is critical for modeling **neutrino flavor transformation** in the Sun (e.g. MSW effect)

Rapid Neutrino Oscillation in Flavor Basis



The first term below is an effective mass adjustment due to the interaction between electron neutrinos and the density of electrons.

$$X(t) = \frac{2\sqrt{2}G_F E\,\rho_e(x(t))}{\delta m_{12}^2} - \cos 2\theta_{12}$$

$$\frac{d}{dt}\begin{pmatrix} a_e(t) \\ a_\mu(t) \end{pmatrix} = \frac{i}{\hbar}\frac{\delta m_{12}^2}{4E}\begin{pmatrix} X(t) & \sin 2\theta_{12} \\ sin2\theta_{12} & -X(t) \end{pmatrix}\begin{pmatrix} a_e(t) \\ a_\mu(t) \end{pmatrix}$$

Instantaneous Eigenbasis

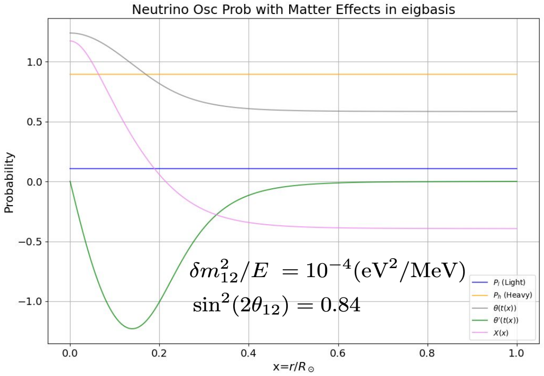
We use a basis that changes in time so that it is always the eigenstate basis.

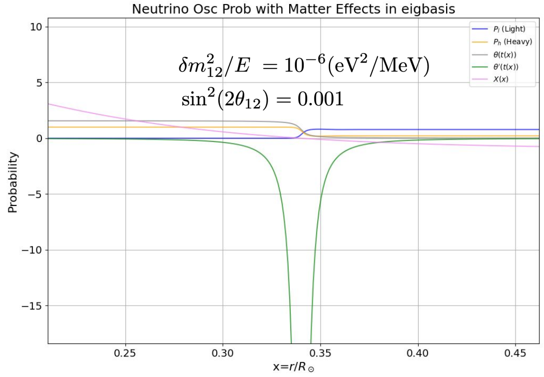
θ(t) is the angle relating the flavor basis to the instantaneous eigenstate basis.   It is a function of density through t or x=ct.

$$\begin{pmatrix} a_L(t) \\ a_H(t) \end{pmatrix} = \begin{pmatrix} \cos\theta(t) & -\sin\theta(t) \\ \sin\theta(t) & \cos\theta(t) \end{pmatrix} \begin{pmatrix} a_e(t) \\ a_\mu(t) \end{pmatrix} = U_{LH}(t) \begin{pmatrix} a_e(t) \\ a_\mu(t) \end{pmatrix}$$

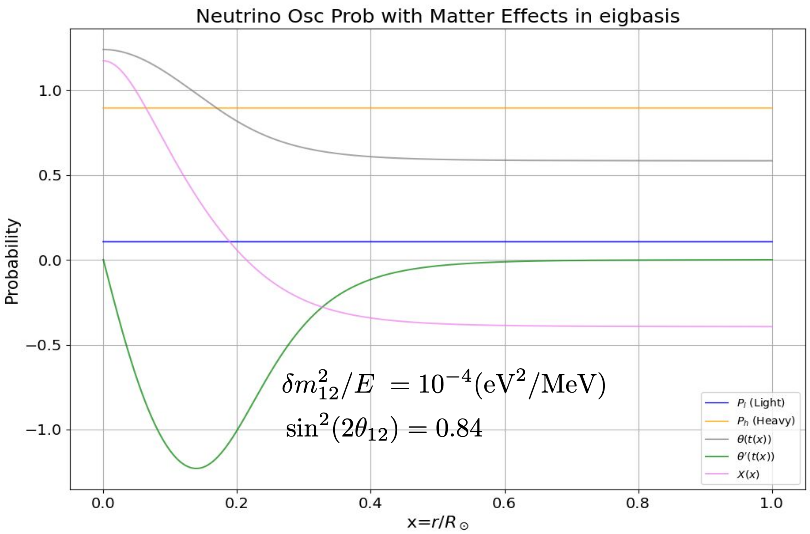
$$\lambda(t) = X(t)\cos(2\theta(t)) - \sin(2\theta_{12})\sin(2\theta(t))$$

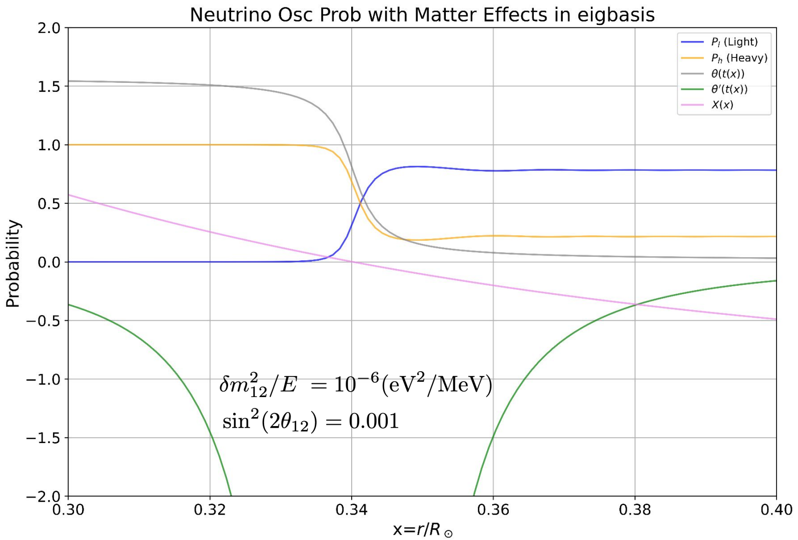
$$\frac{d}{dt}\begin{pmatrix} a_L(t) \\ a_H(t) \end{pmatrix} = -\frac{i}{\hbar}\left[\frac{m_{21}^2}{4E}\lambda(t)\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} + i\hbar\theta'(t)\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\right]\begin{pmatrix} a_L(t) \\ a_H(t) \end{pmatrix}$$





Instantaneous Eigenbasis Figures





Future Plans (Summer 2025)

- Will extend simulations from 2-flavor to 3-flavor neutrino evolution
  - Incorporate muon density effects in proto-neutron stars
- Will optimize basis rotation encoding and factor out rapidly evolving phases in the adiabatic regime
- Will transition from solar to supernova conditions and implement time-resolved simulations for dynamic environments

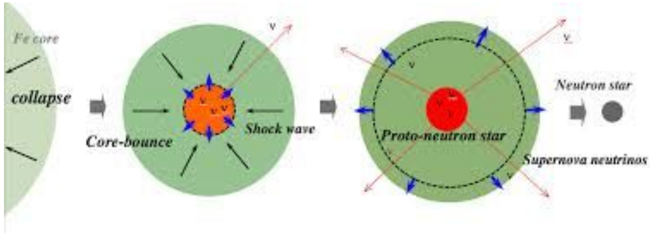
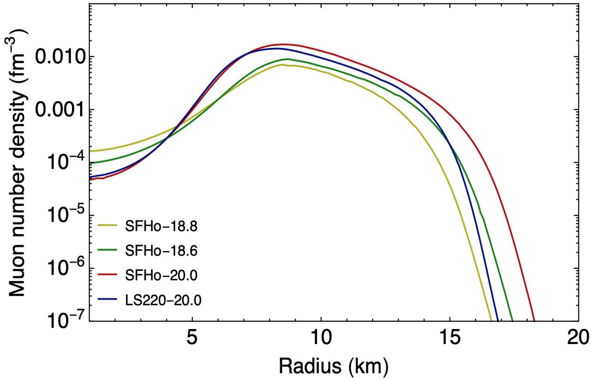
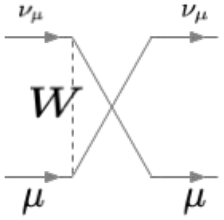


Fig 1. From K. Sumiyoshi (2019)



1 sec post bounce, 10^{36} muons/cm^3

Phys. Rev. Lett. 125, 051104 (2020)



charged current interaction, muon neutrino with muons

Conclusion/Learning Recap

Main Conclusions

- Neutrino propagation is predominantly adiabatic under realistic conditions
- The instantaneous eigenstate basis provides the most effective framework for analysis
  - In this basis, the Hamiltonian becomes nearly diagonal, simplifying calculations
    - I expect that this approach significantly improves computational efficiency

Learning Recap

- Gained proficiency in Python and GitHub for scientific computing
  - Developed and applied differential equation solvers for physics problems
- Modeled neutrino flavor evolution in flavor and eigenstate bases
- Explored vacuum vs. matter effects in neutrino flavor evolution

Reference

1. M. Brüggen, W. C. Haxton, and Y.-Z. Qian, *Landau-Zener treatments of solar neutrino oscillations*, Phys. Rev. D 51, 4028 (1995).
2. W. C. Haxton, *Analytic treatments of matter-enhanced solar-neutrino oscillations*, Phys. Rev. D 35, 2352 (1987).
3. J. N. Bahcall, Solar Models
4. K. Sumiyoshi, *Neutrino emission and equation of state in core-collapse supernovae*, presented at MICRA 2019, Jena, Germany, August 14, 2019.