Modeling Neutron Star Structure with a TOV Solver Using Piecewise Polytropic Equations of State

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Neutron stars

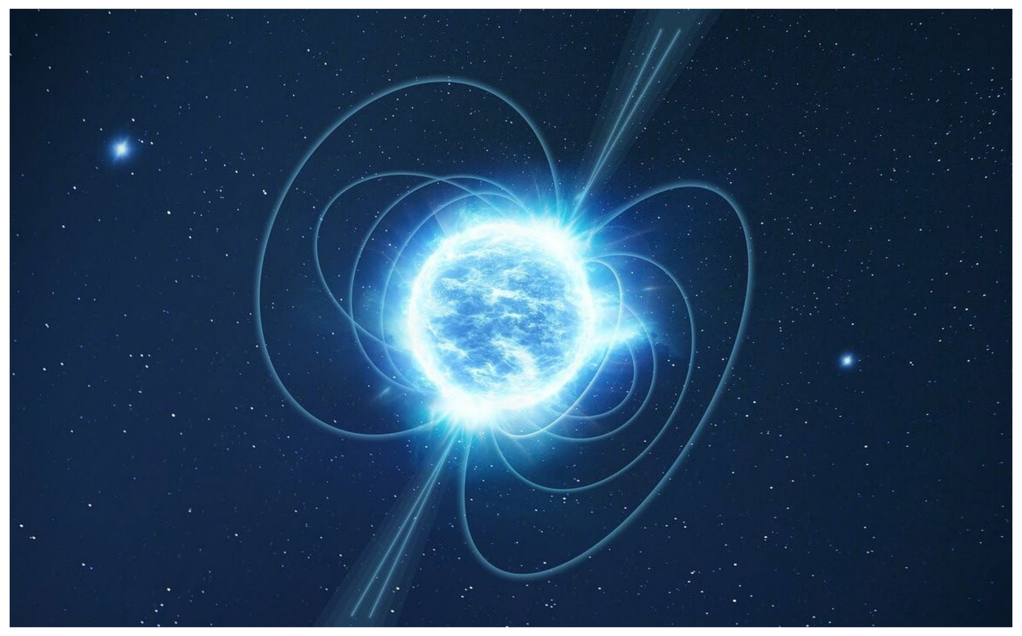


Figure 1: Neutron star [1]

Neutron stars are among the most compact material objects in the universe. They have average radii around 10km, but mass can reach $2M_{\odot}$. Thus, their density exceeds nuclear saturation at $\rho \gtrsim 10^{14} g/cm^3$[2]. These extreme conditions make neutron starts natural laboratories for studying dense matter and testing theories of gravity, and quantum chromodynamics (QCD).

Equation of states (EOS) of Neutron stars

The EOS of neutron stars encodes the thermodynamic behavior of matter at ultra-high densities where QCD becomes non-perturbative. Since direct calculations from QCD are limited at finite baryon density, neutron stars allow indirect probing of the QCD phase diagram, including possible phase transitions, exotic matter, and the properties of strongly interacting matter [3].

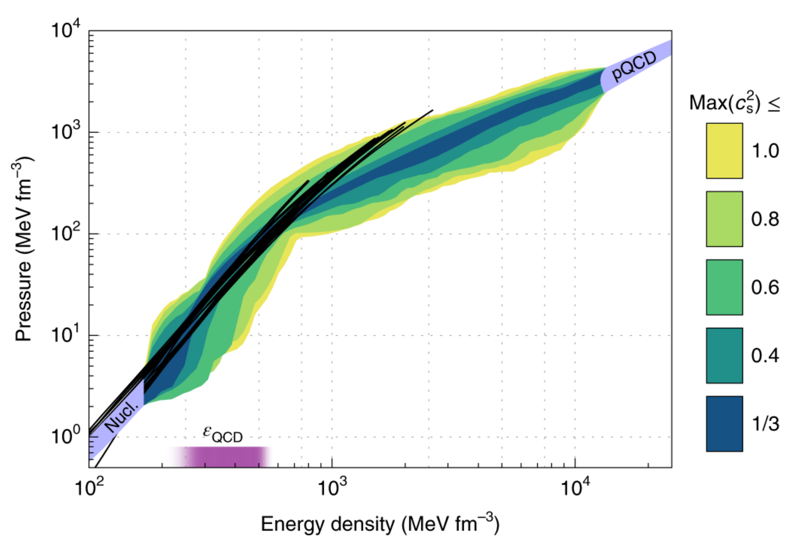


Figure 2: Range of allowed NS-matter EOSs [4]

Piecewise Polytropic EOS

$$\mathrm{P}(\rho) = K_i \rho^{\Gamma_i} \qquad \epsilon = \rho + \frac{K_i \rho^{\Gamma_i}}{\Gamma_i - 1}$$

Γ_i: Adiabatic index in the i-th segment
K_i: Polytropic constant

A piecewise polytropic EOS is a flexible model used to approximate the behavior of dense nuclear matter in neutron stars. It divides the EOS into density intervals.

- Choose density breakpoints: $\rho_1, \rho_2, \rho_3, \ldots$
- Assign Γ_i to each segment
- Set K_1 then calculate other K_i by matching P at each boundary [5]

Enthalpy formulated Tolman-Oppenheimer-Volkoff (TOV) solver

The standard TOV equation, formulated in terms of pressure, is widely used to model the equilibrium structure of neutron stars.
An enthalpy-based formulation offers improved numerical stability especially near the neutron star surface. [6]

$$h(p) = \int_0^P \frac{dp'}{\epsilon(p') + p'}$$

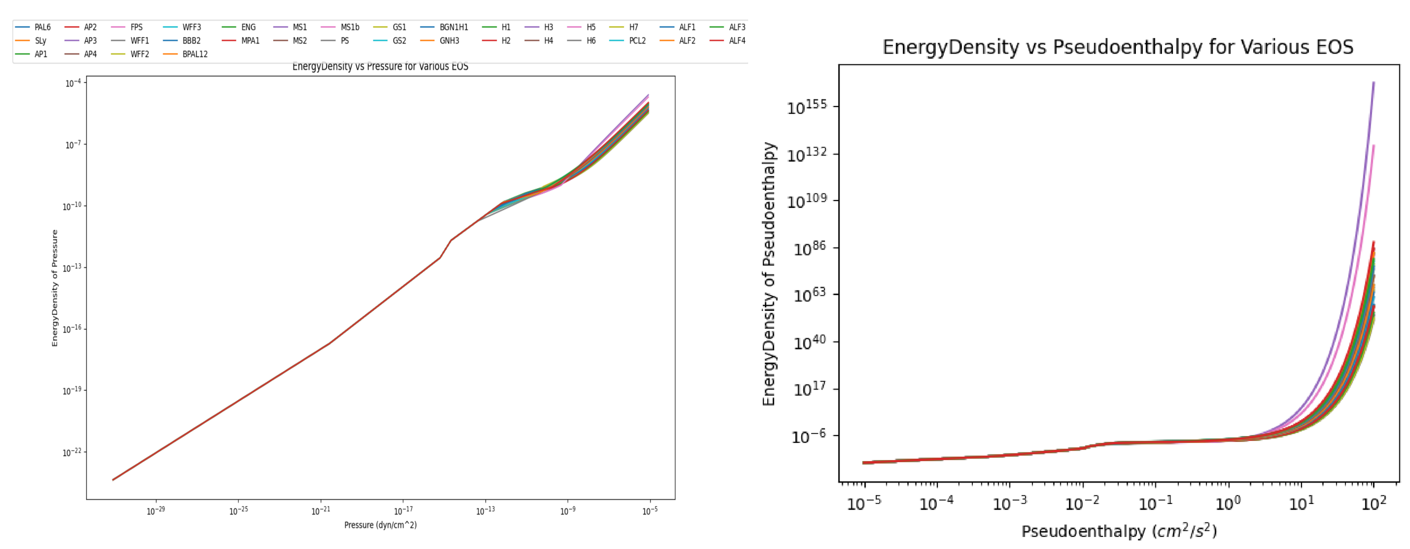
$$\frac{dm}{dr} = 4\pi r^2 \rho \qquad \frac{dp}{dr} = -(\rho + p)\frac{m + 4\pi r^3 p}{r(r - 2m)}$$

$$\longrightarrow$$

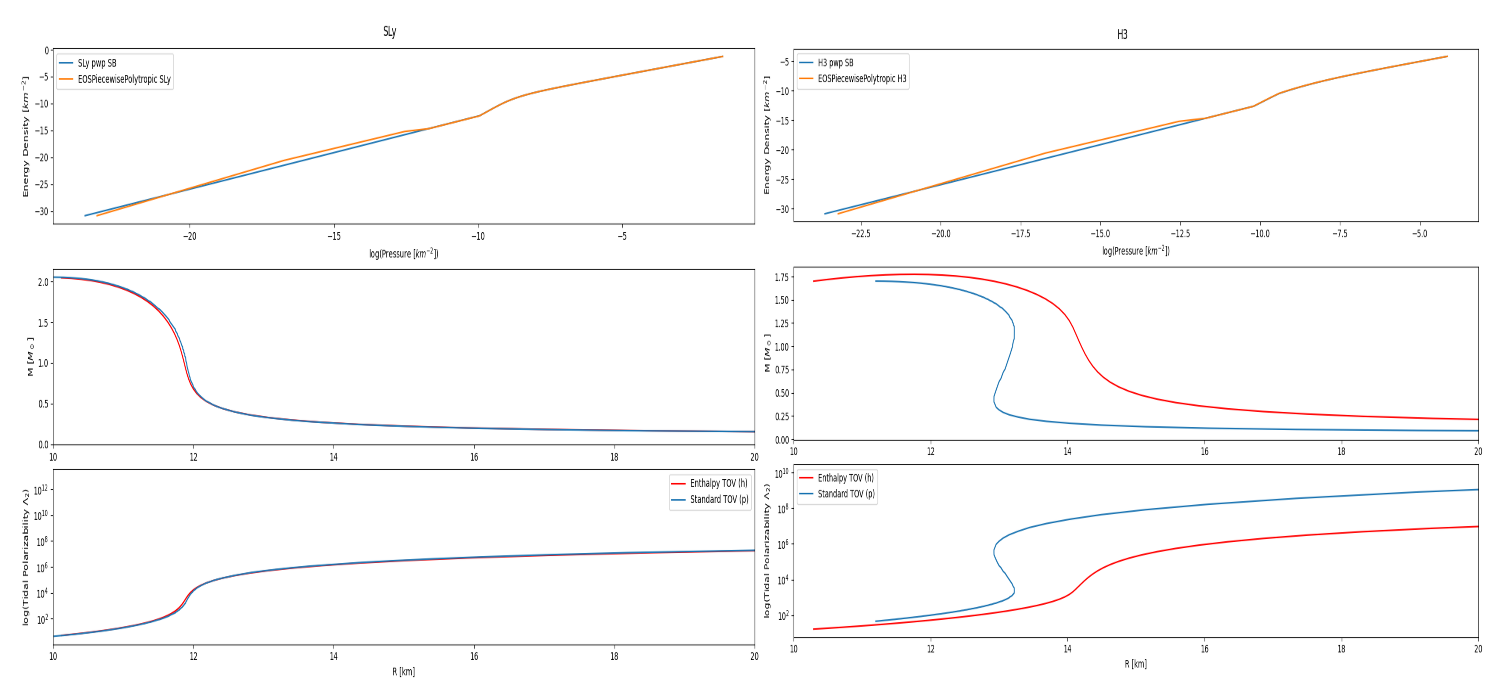
$$\frac{dm}{dh} = -\frac{4\pi\epsilon(h) r^3 (r - 2m)}{m + 4\pi r^3 P(h)} \qquad \frac{dr}{dh} = -\frac{r(r - 2m)}{m + 4\pi r^3 P(h)}$$

- Compute $h(\rho)$ with $P(\rho)$, $\epsilon(\rho)$, and ρ
- Invert the relationship using interpolation to express EOS in terms of enthalpy, finding $P(h)$, $\epsilon(h)$, and $\rho(h)$
- Integrate $\frac{dr}{dh}$ and $\frac{dm}{dh}$ from $h = h_c$ (center) to $h = 0$ (surface)
- Vary the central enthalpy to generate the full mass-radius relation and tidal polarizability. [7]

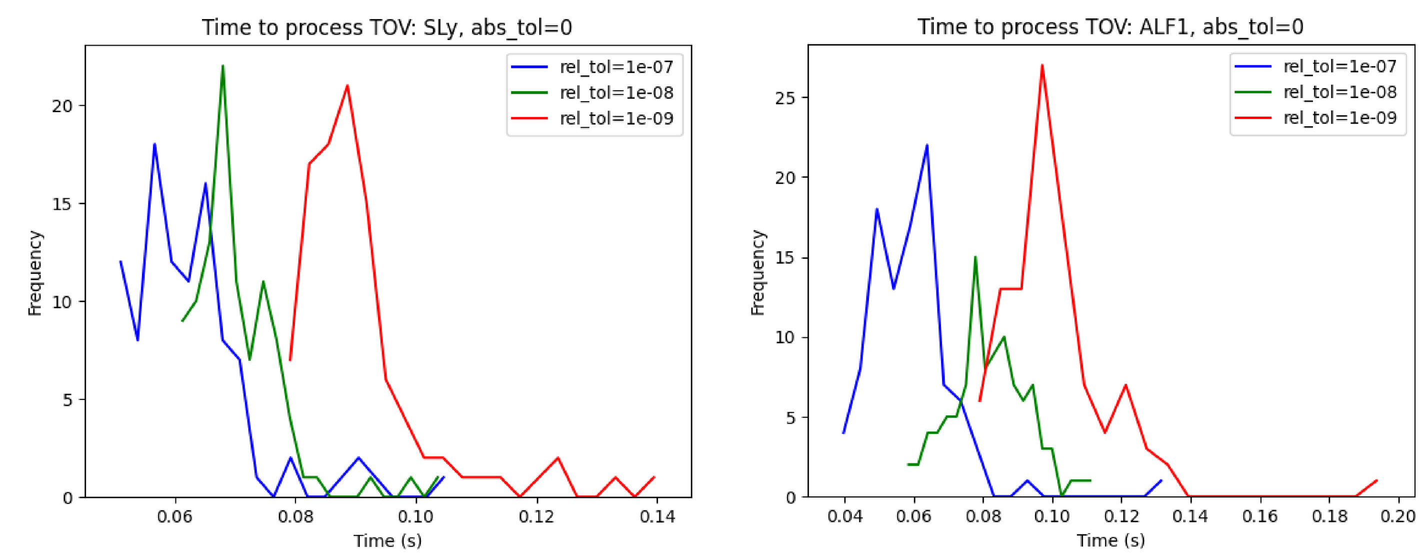
Result



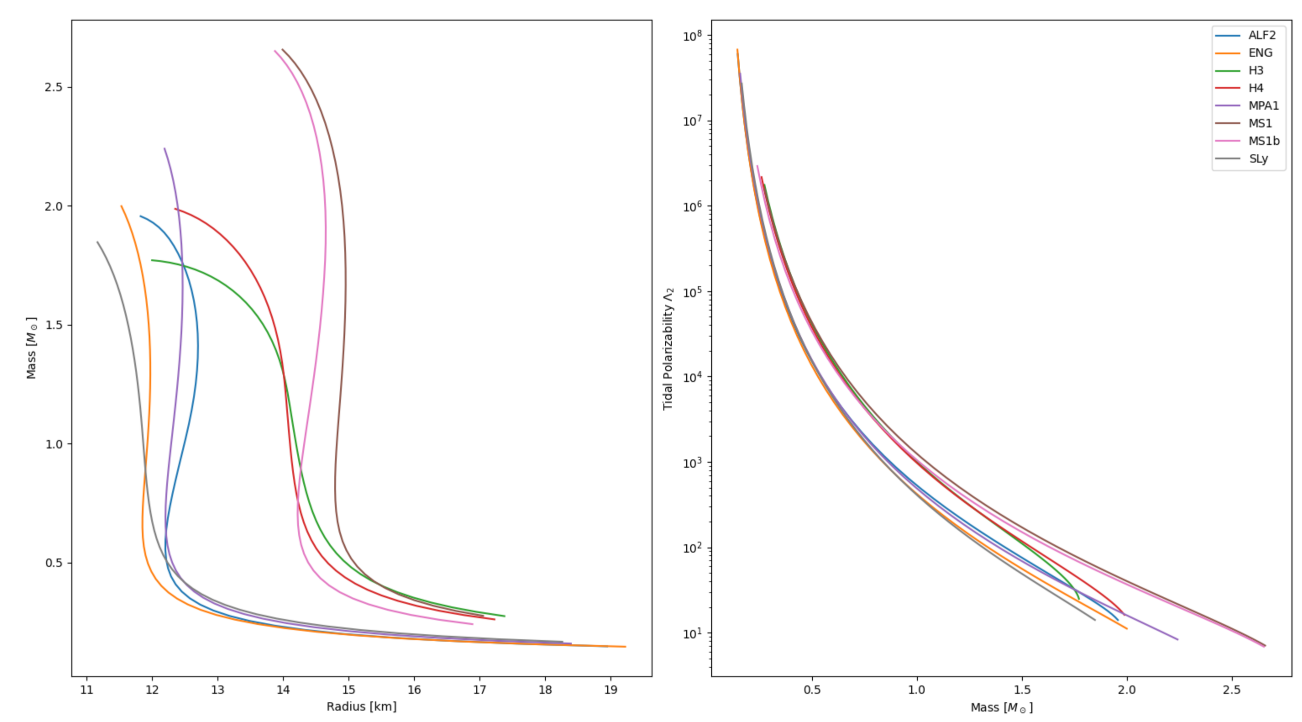
The way how various nuclear and hybrid EOS behave when re-expressed in terms of pseudoenthalpy. The EOS functions $P(h)$, $\epsilon(h)$, and $\rho(h)$ derived here are directly used in the enthalpy-based TOV solver, providing smooth and stable integration from core to surface. The low-density crust region is fixed to the SLy EOS across all models.



The M-R sequence of Sly with the standard TOV solver achieves a nearly exact form compared to the original [8], while H3 shows visible differences in the mass-radius and tidal deformability curves. Despite these differences, the curves do not exhibit unphysical trends and are qualitatively similar to the standard TOV; thus, the result remains physically plausible.



Across different relative error settings $(10^{-7}, 10^{-8}, 10^{-9})$, the resulting mass-radius outputs are generally equivalent up to the 8$^{\text{th}}$ significant digit. This justifies the use of a higher relative tolerance for practical simulations, offering a significant speedup without sacrificing meaningful accuracy.



The trends in both panels are consistent with theoretical expectations: stiffer EOSs produce neutron stars with larger radii and higher tidal polarizability, while softer EOSs yield more compact configurations [9]. This confirms that our enthalpy-based TOV framework, when supplied with pseudoenthalpy-parameterized EOS input, reliably reproduces neutron star structure and observables across a wide range of physical models. In the future, the model will be applied to constrain the EOS using NICER data.

References

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