Congratulations and thank you, Baha & George for your scientific contributions and your community leadership.

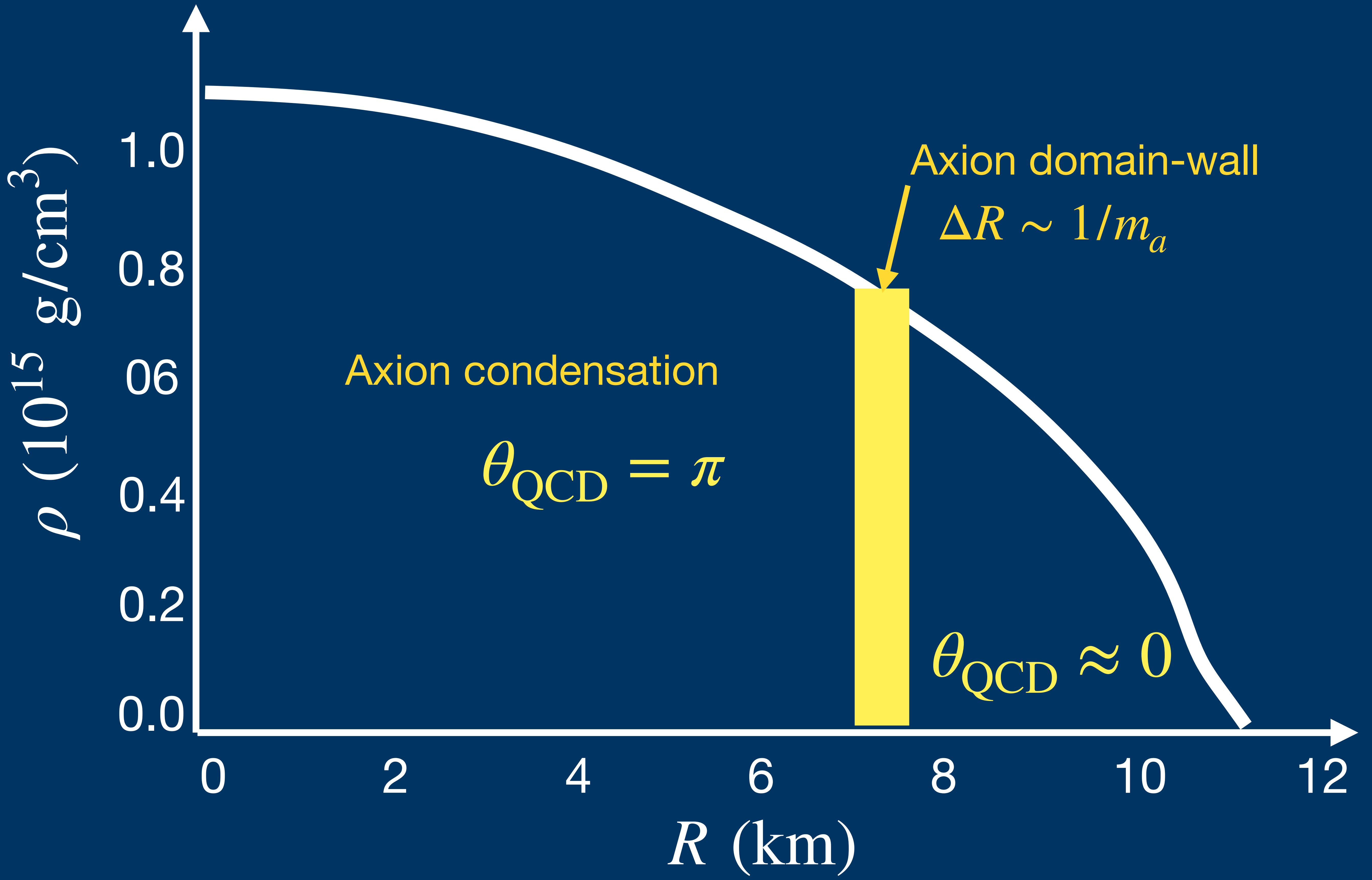
The youngest wise uncle

The oldest brilliant school boy

π in the Sky: Axion Condensation in Neutron Stars

Sanjay Reddy, Institute for Nuclear Theory, University of Washington, Seattle.

Collaborators: M. Kumamoto, J. Huang, C. Drischler, M. Baryakhtar



Neutrinos in Physics and Astrophysics, Celebrating the Contributions of Baha Balantekin and George Fuller, UC Berkeley, 1/16/25

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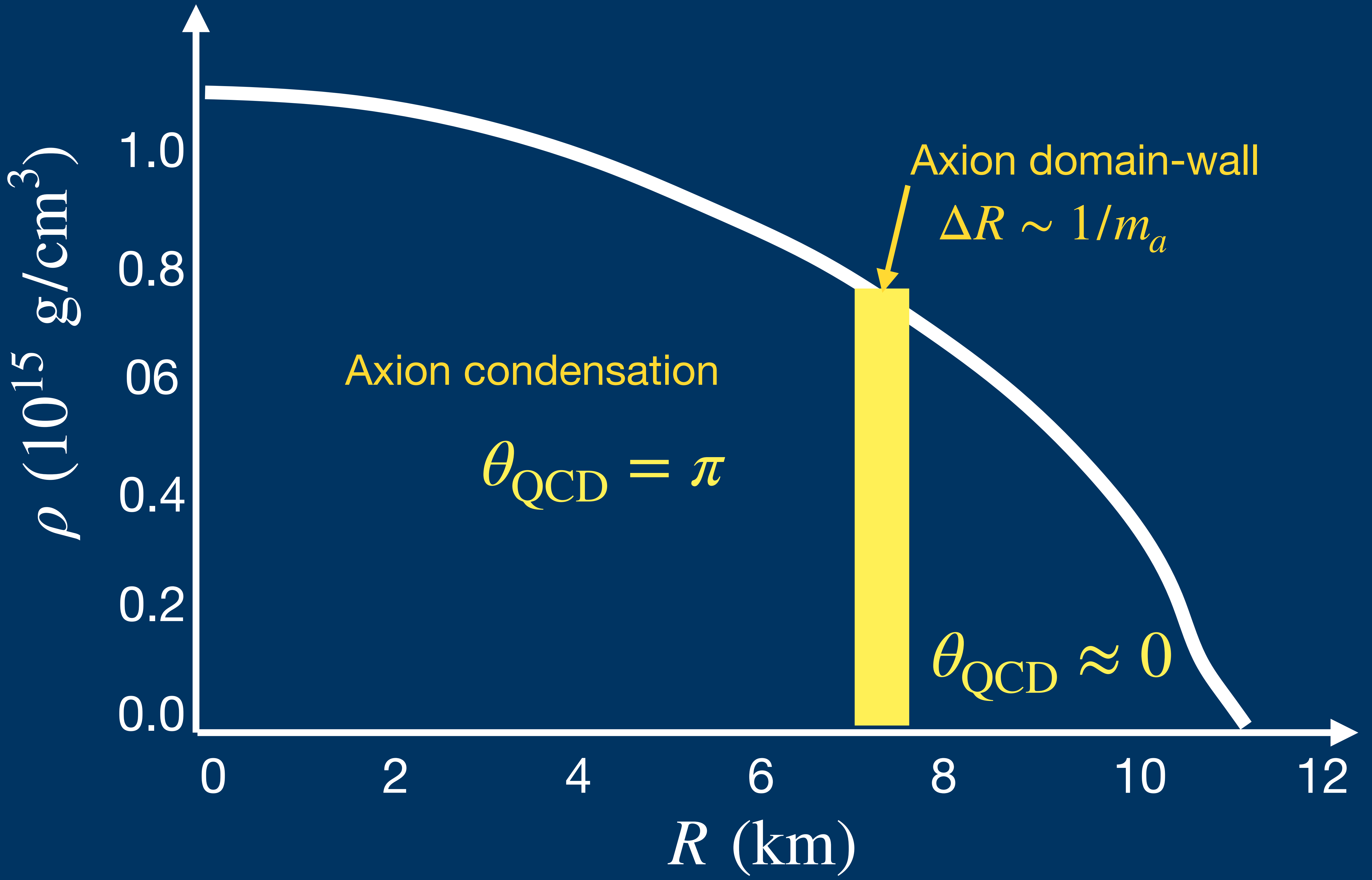

- Introduction & motivation.
- Axion condensation in neutron stars.
- Quark mass in EFT and pion coupling to two nucleons.
- Conclusions.

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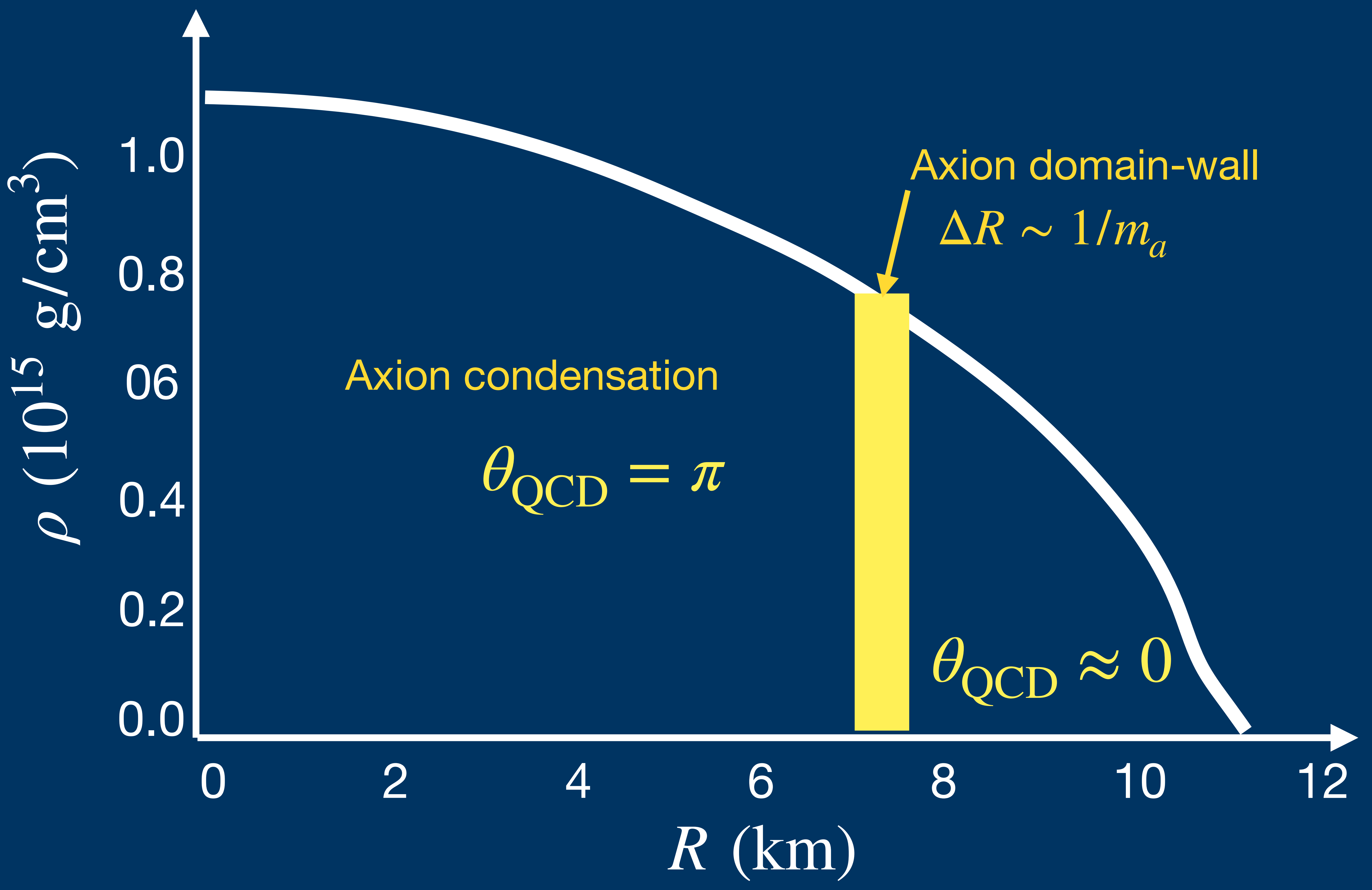

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QCD

A Simple Lagrangian with Marvelous Emergent Complexity at Low-Energy:

$$\mathscr{L} = \sum_f \bar{\psi}_{\alpha f} \left(i\gamma^\mu (\delta_{\alpha\beta} \partial_\mu - g \ (T_a G_\mu^a)_{\alpha\beta}) + m_f \right) \psi_{\beta f} - \frac{1}{4} G_{\mu\nu}^a G_a^{\mu\nu}$$

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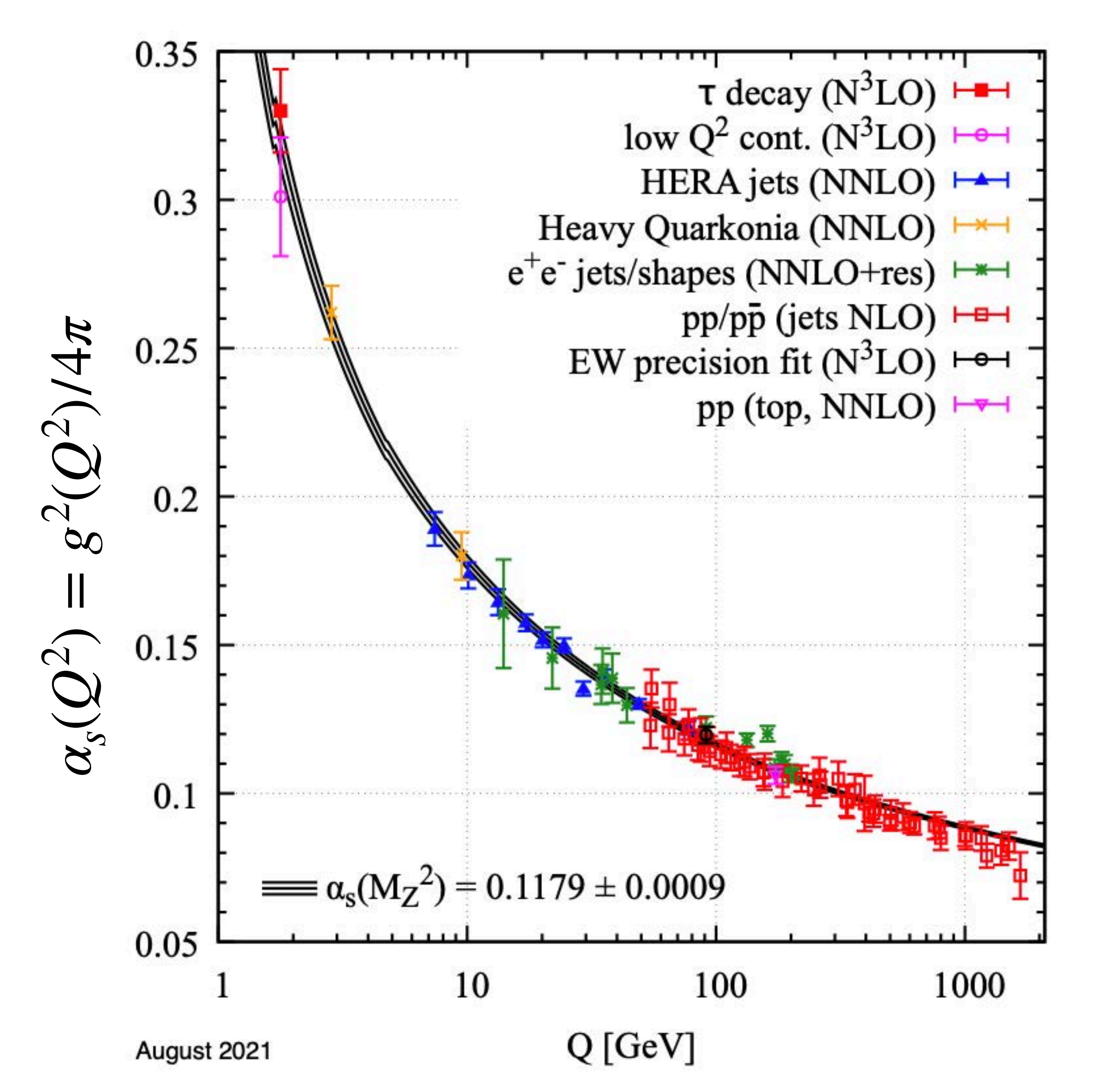
Running Coupling



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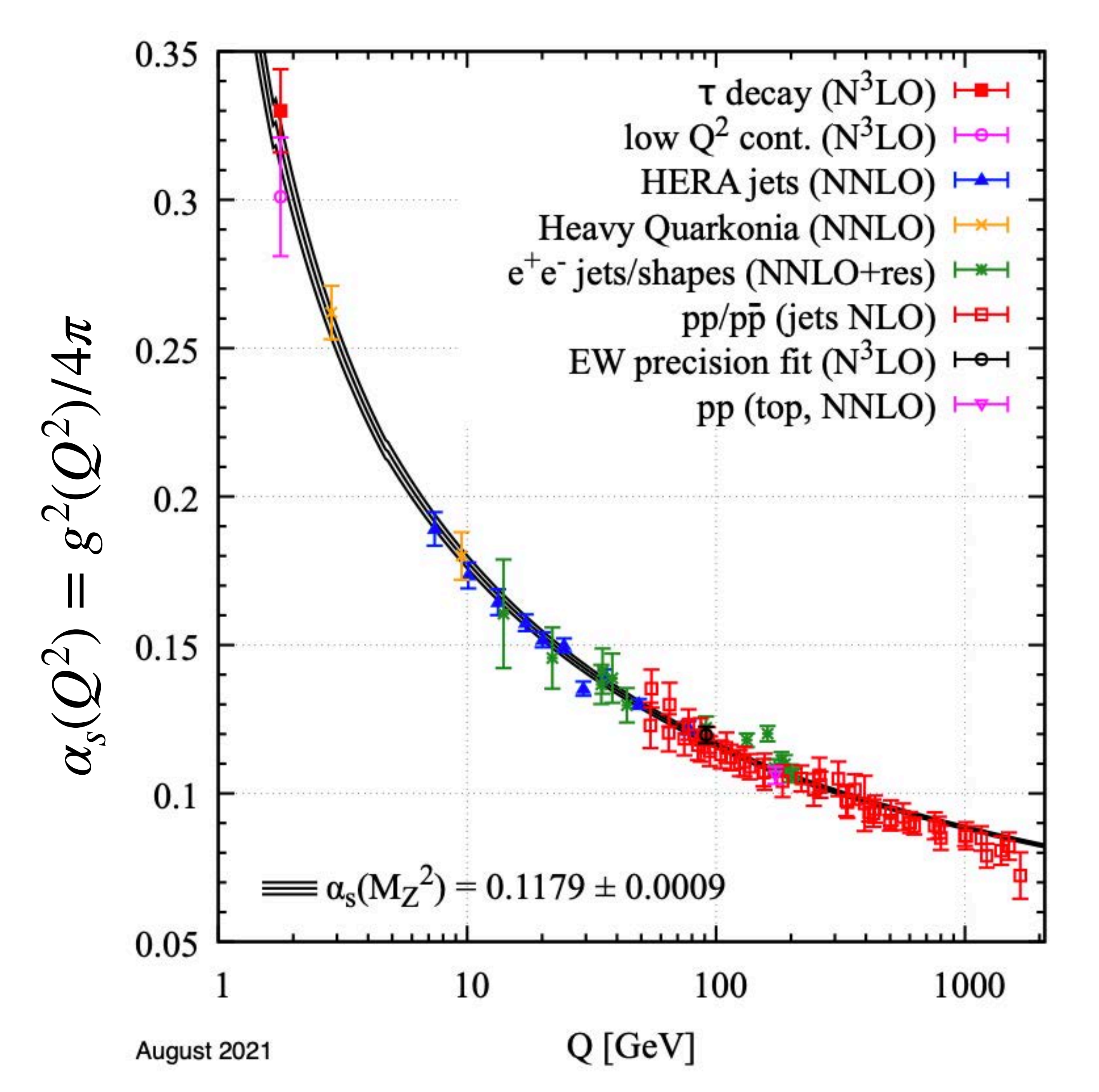

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Quark Mass Matrix

$$\begin{pmatrix} m_u \approx 2.5 \text{ MeV} & 0 & 0 \\ 0 & m_d \approx 5 \text{ MeV} & 0 \\ 0 & 0 & m_s \approx 100 \text{ MeV} \end{pmatrix}$$

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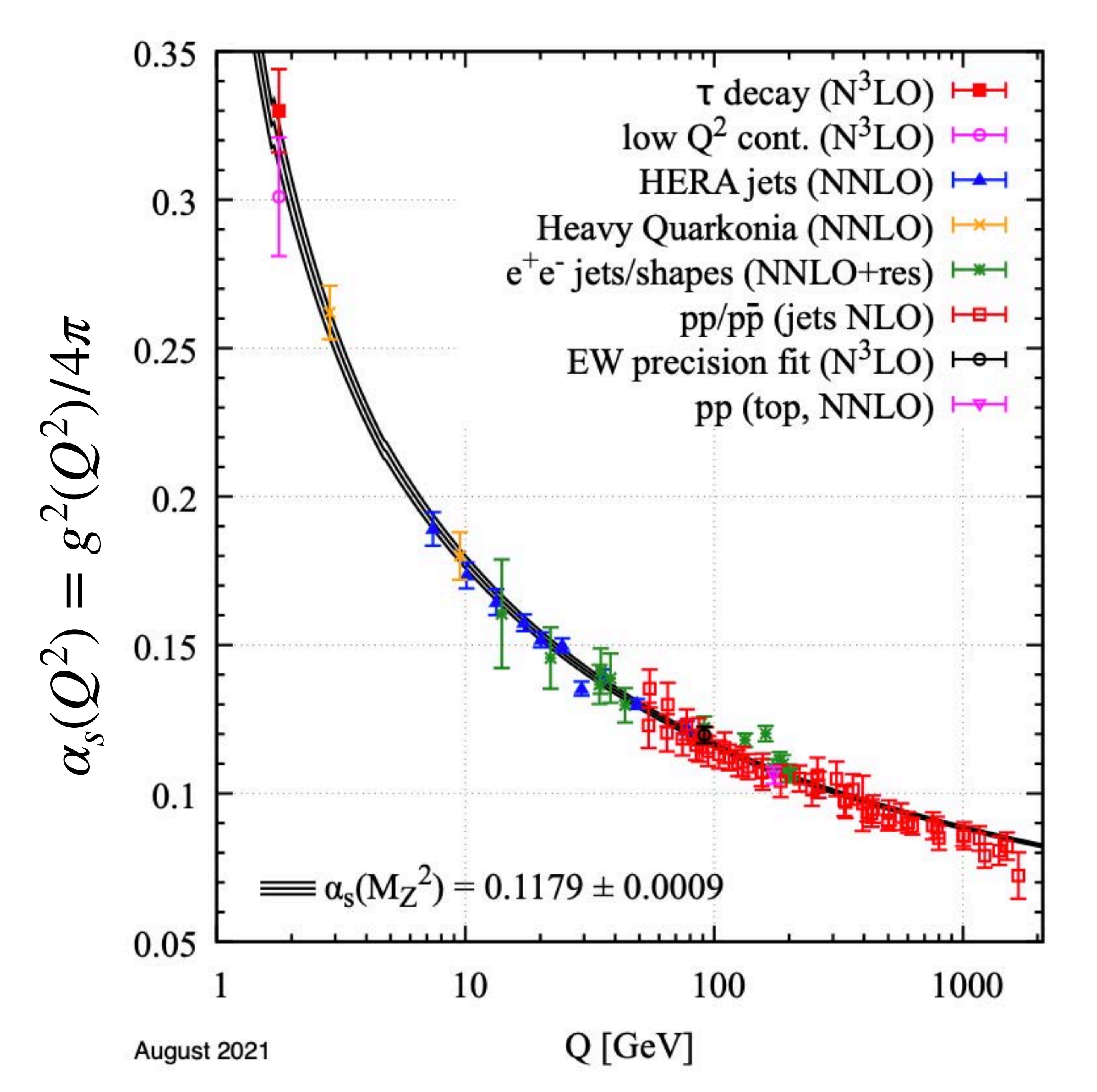

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$m_c \approx 1.3$ GeV $\quad m_b \approx 4$ GeV $\quad m_t \approx 170$ GeV

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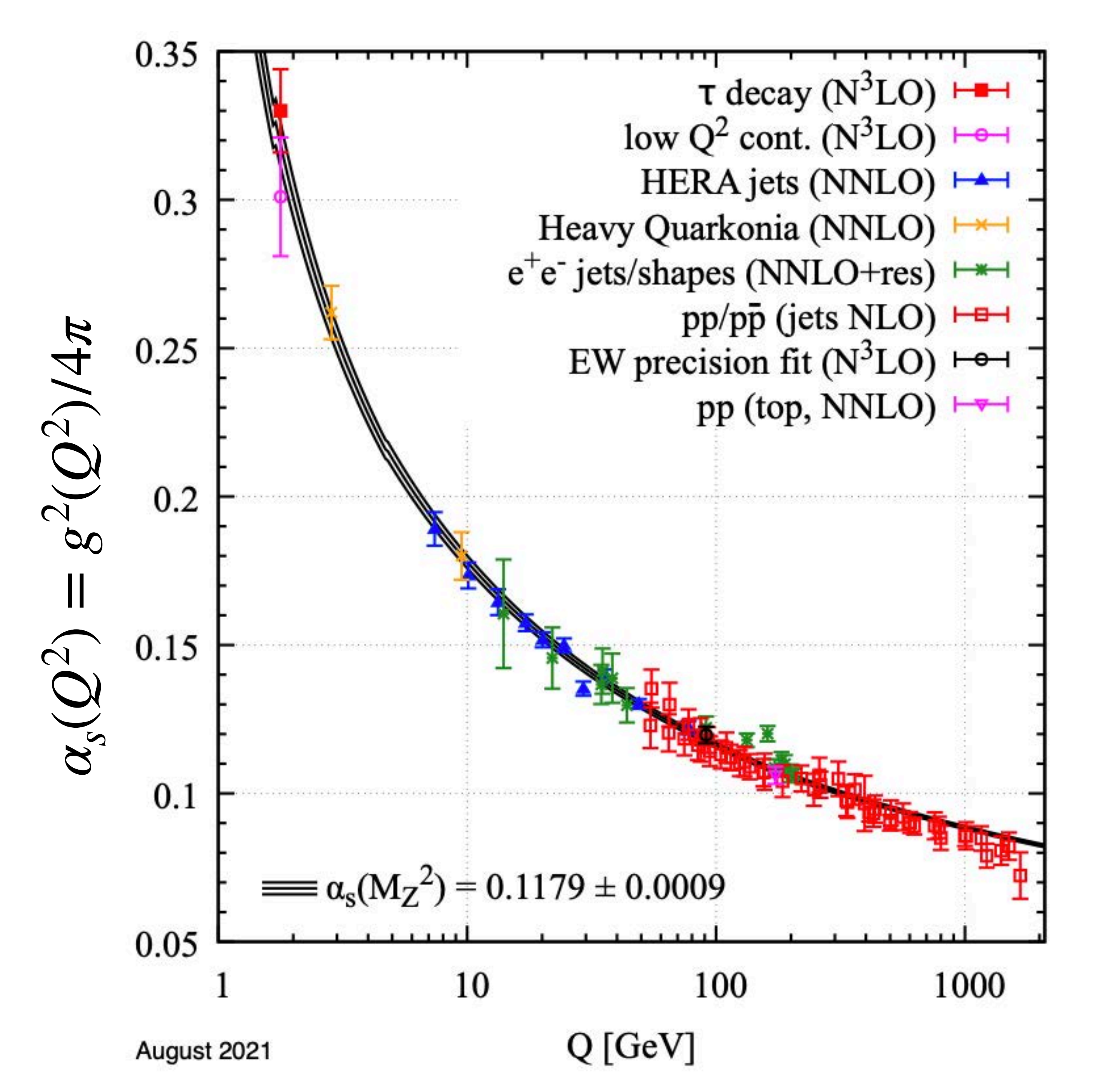

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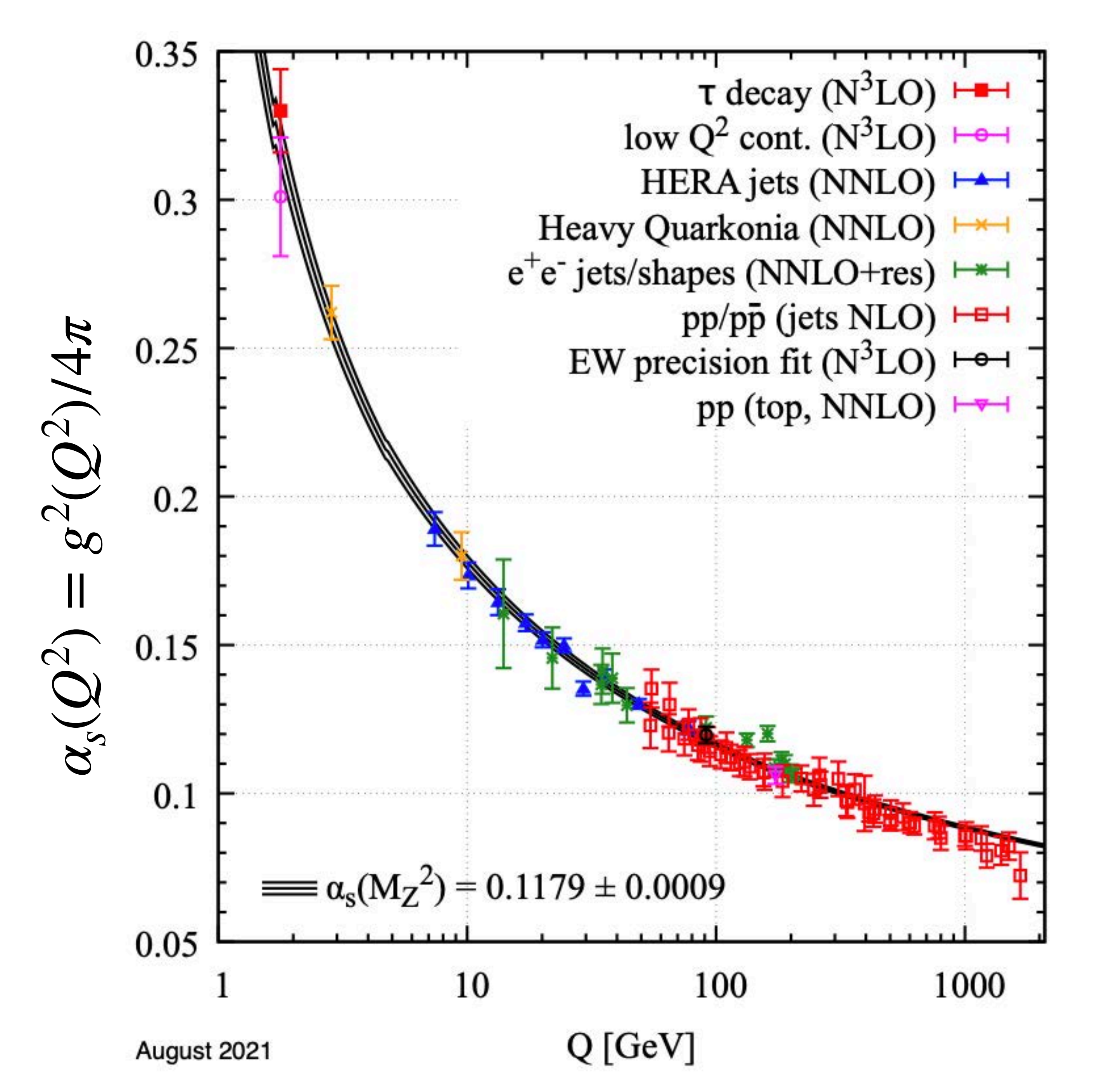

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θ-QCD

- Source of CP violation
- Induces neutron EDM: $d_n \approx 3 \times 10^{-16}\ \theta$ e cm
- Experimental bound: $d_n \lesssim 10^{-26}$ e cm or $\theta \lesssim 10^{-10}$

θ and Axions

$$\mathscr{L}_\theta = \theta \, \frac{g^2}{32\pi^2} G^a_{\mu\nu} \tilde{G}^{\mu\nu}_a$$

To explain $\theta < 10^{-10}$, θ is promoted to a dynamical quantity through the Pecci and Quinn mechanism. A new field that relaxes to zero to minimize the vacuum free energy:

$$\theta = \frac{a}{f_a}$$

Axion field

A new high energy scale

R. Peccei and H. R. Quinn (1977), S. Weinberg (1978), F. Wilczek (1978)

Axion Mass and Energy

The axion coupling to gluons can be elimanted by a transformation of the quark mas matrix M_q

$$M_q \to M_q \exp\left(2i\frac{a}{f_a} Q_a\right) \quad \text{where} \quad Q_a = \frac{M_q^{-1}}{\mathrm{Tr}\, M_q^{-1}} = \frac{1}{m_u + m_d}\begin{pmatrix} m_d & & \\ & m_u & \\ & & 0 \end{pmatrix} + \mathcal{O}[m_u/m_s, m_d/m_s]$$

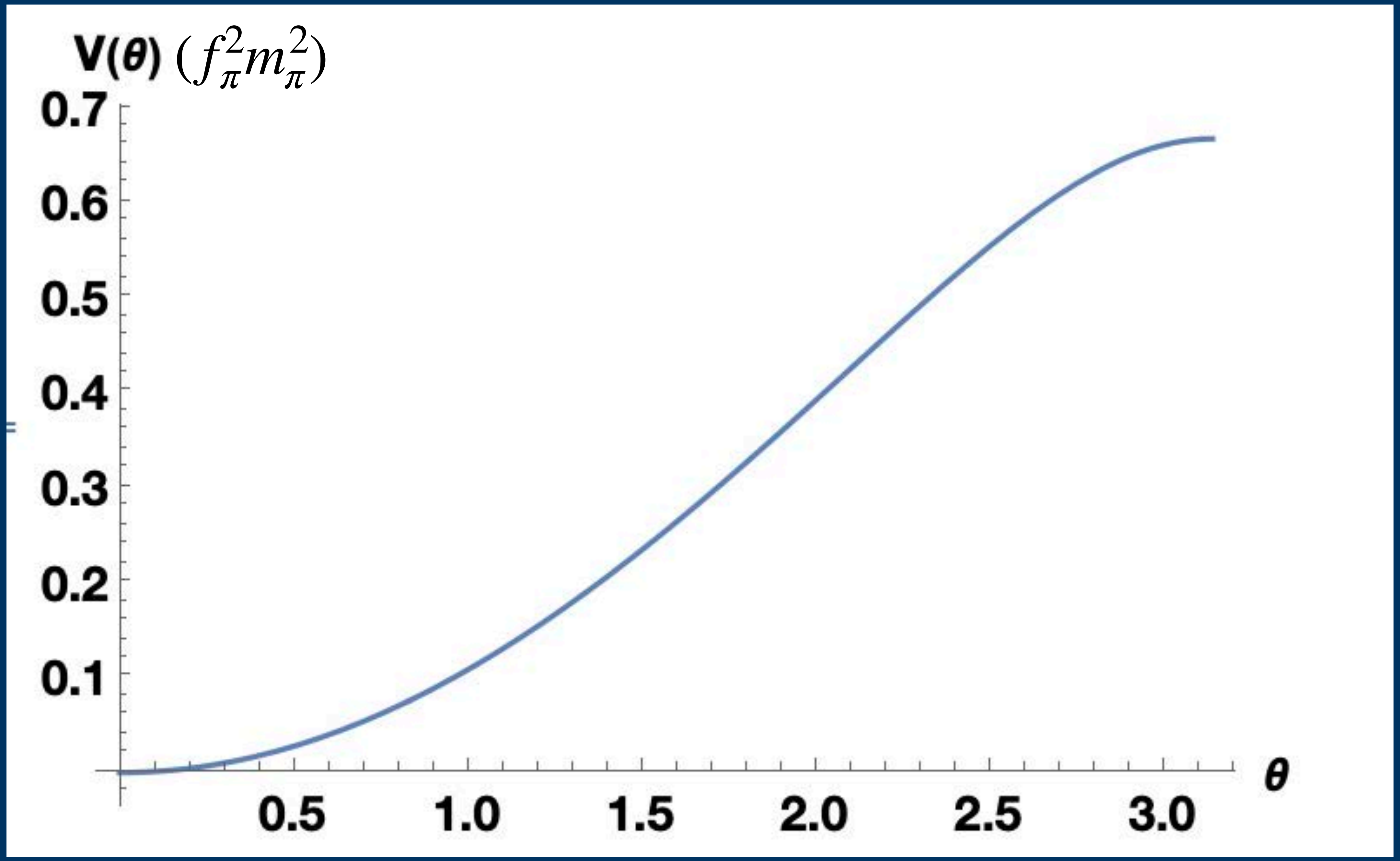
This leads to an axion mass which can be calculated from Chiral Perturbation Theory

$$m_a^2 = \frac{f_\pi^2}{f_a^2}\left(\frac{m_u m_d}{(m_u + m_d)^2}\right) m_\pi^2$$

The corresponding contribution to the energy density or an axion potential

$$V\left(\theta = \frac{a}{f_a}\right) = f_\pi^2 m_\pi^2 \left[1 - \sqrt{1 - \frac{4 m_u m_d}{(m_u + m_d)^2}\sin^2\left[\frac{\theta}{2}\right]}\right]$$

$$= \frac{1}{2} f_a^2 m_a^2 \theta^2 + \cdots$$



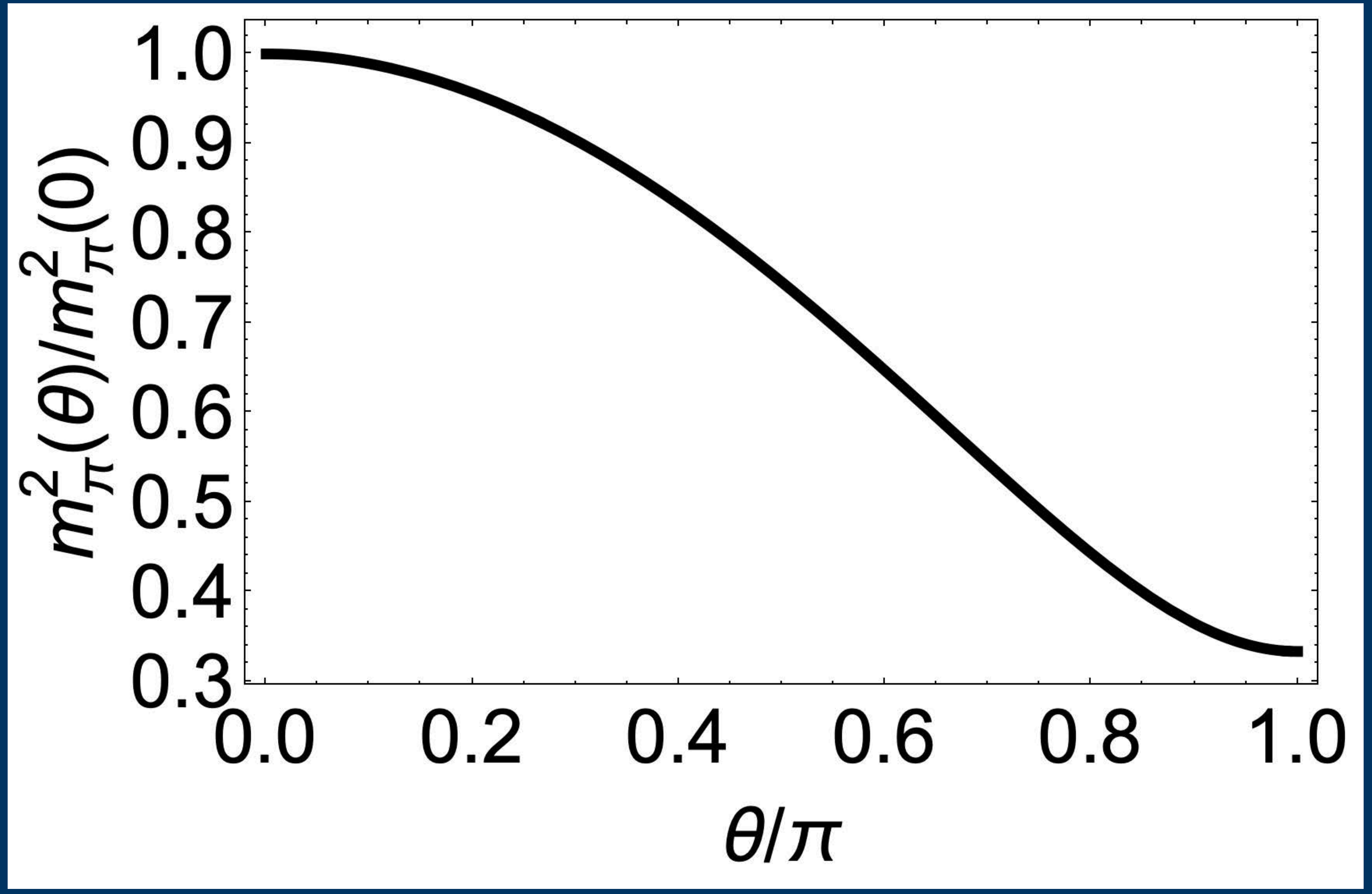
Hadrons at $\theta \neq 0$

Using $M_q \to M_q \ \exp\left(2i\theta\, Q_a\right)$

The pion mass at finite θ can be calculated in ChiPT

$$m_\pi^2(\theta) = m_\pi^2(\theta=0)\sqrt{1 - \frac{4m_u m_d}{(m_u+m_d)^2}\sin^2\left[\frac{\theta}{2}\right]}$$

$$\Rightarrow \quad \frac{m_\pi^2(\theta=\pi)}{m_\pi^2(\theta=0)} = \frac{m_d - m_u}{m_d + m_u} \approx \frac{1}{3}$$

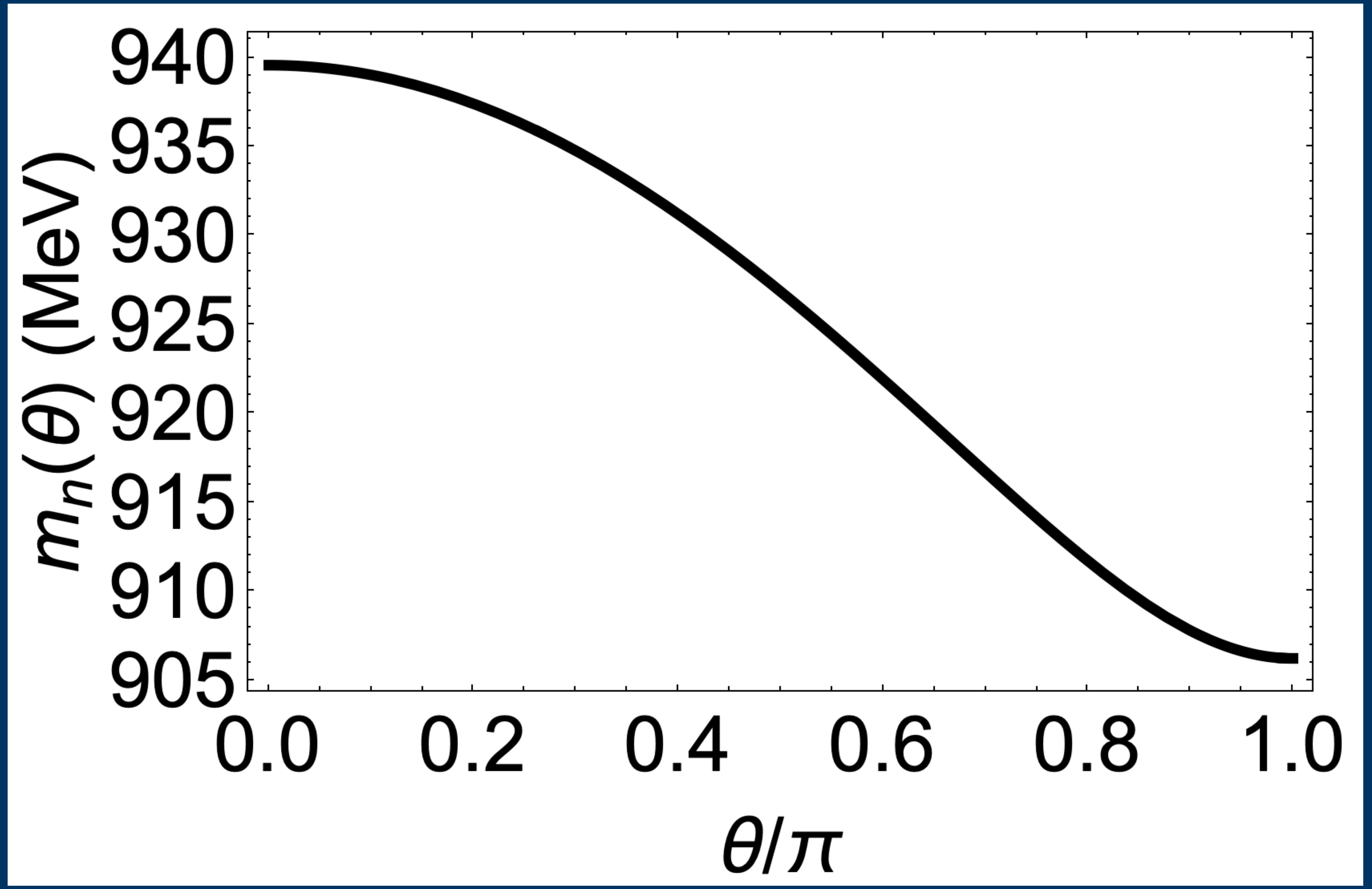


Note at $\theta=\pi$: CP is conserved (M_q is real).

The only change is $m_u \to -m_u$

The resulting decrease in the nucleon mass

$$m_n(\theta) = m_0 + \sigma_{\pi n}\frac{m_\pi^2(\theta)}{m_\pi^2(\theta=0)} + \cdots$$



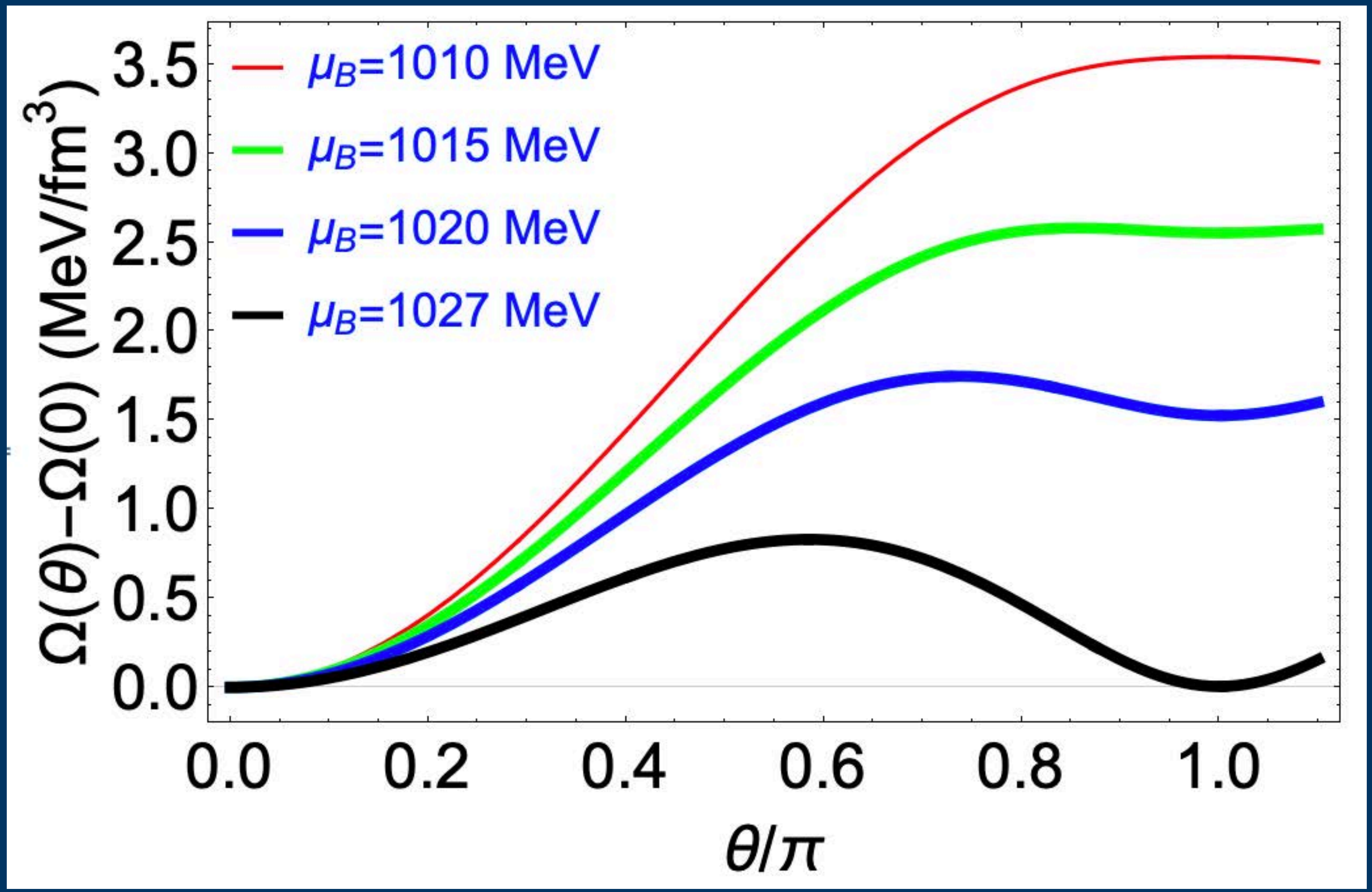
Axion Condensation

The decrease in the nucleon mass $m_n(\theta) = m_0 + \sigma_{\pi n}\dfrac{m_\pi^2(\theta)}{m_\pi^2(\theta=0)} + \cdots$

favors a first-order transition to a ground state with $\theta = \pi$

Neglecting nuclear interactions the energy gain per nucleon is

$$\Delta E \simeq \sigma_{\pi n}\left(1 - \frac{m_\pi^2(\theta=\pi)}{m_\pi^2(\theta=0)}\right) \simeq \frac{2}{3}\sigma_{\pi n}$$



The energy cost (due to axion potential) per nucleon is

$$\Delta E = \frac{V(\theta=\pi)}{n_B} \simeq \frac{2}{3}\frac{f_\pi^2 m_\pi^2}{n_B}$$

The Condensation occurs when $\sigma_{\pi N} n_B > f_\pi^2 m_\pi^2$

For $\sigma_{\pi n} = 50$ MeV → $n_B^c \simeq 2.6\ n_{\rm sat}$

A. Hook and J. Huang (2018) R. Balkin, J. Serra, K. Springmann, and A. Weiler, (2020) R. Kumamoto, J. Huang, C. Drischler, M. Baryakthar, and S. Redd (2024)

Role of nuclear interactions

Modification of nuclear interactions at $\theta = \pi$ can alter the critical density because it can be comparable to the energy gain

$$\Delta E \simeq \sigma_{\pi n} \left(1 - \frac{m_\pi^2(\theta = \pi)}{m_\pi^2(\theta = 0)} \right) \simeq \frac{2}{3} \sigma_{\pi n} \approx 30 - 40 \text{ MeV}$$

At $\theta = \pi$, the dominant effect is the reduction in the pion mass.

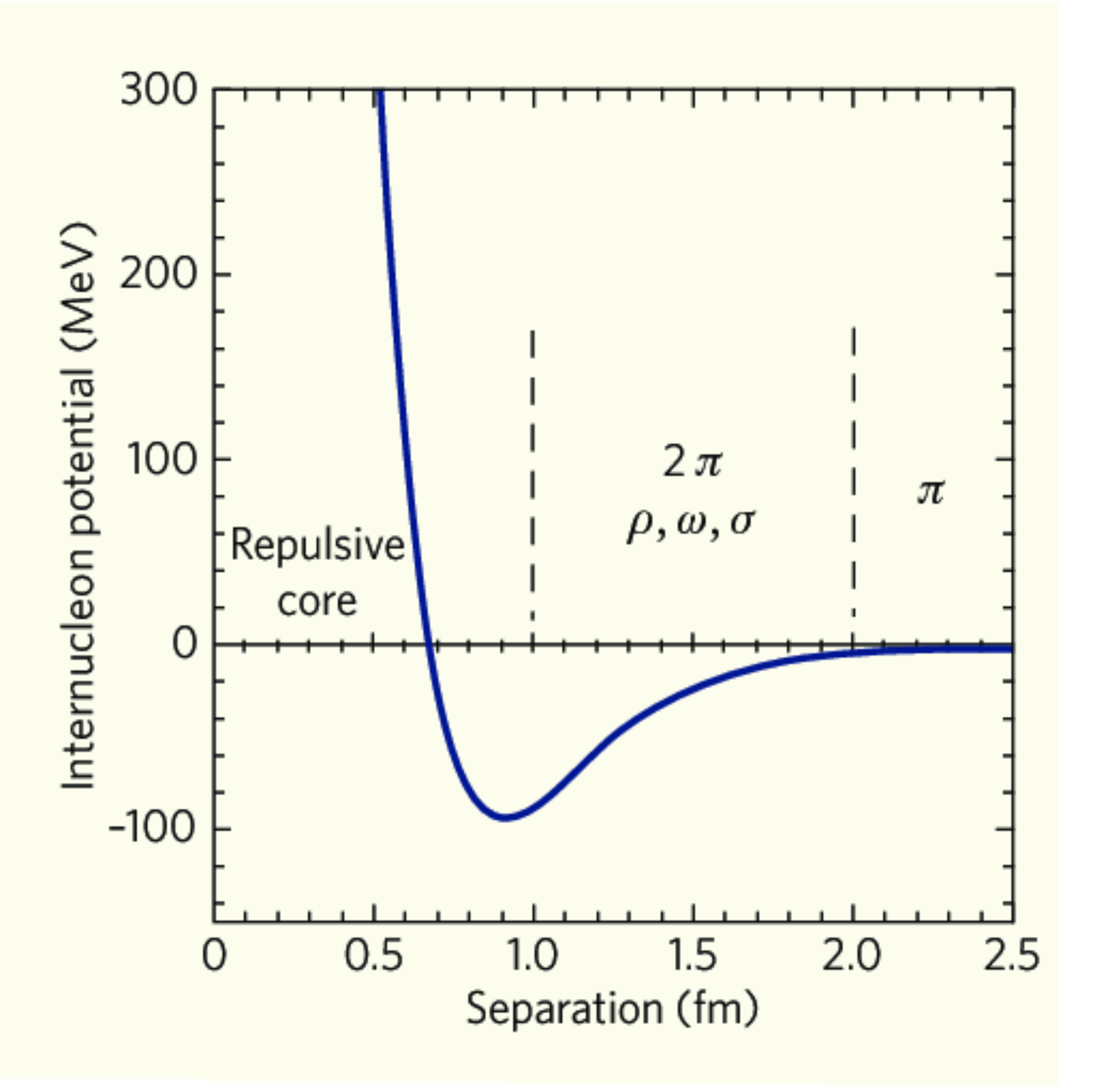
What is nuclear physics look like at $m_\pi \simeq 80 \text{ MeV}$?

If nuclear forces are more attractive at $m_\pi \simeq 80 \text{ MeV}$ axion condensation will occur at $n_B < 2.6\ n_{\text{sat}}$.

How do changes to M_q affect nuclear interactions?

Short answer: We do not really know.

- The effect on pion-exchange is easy to implement, but effects at short distances are not.



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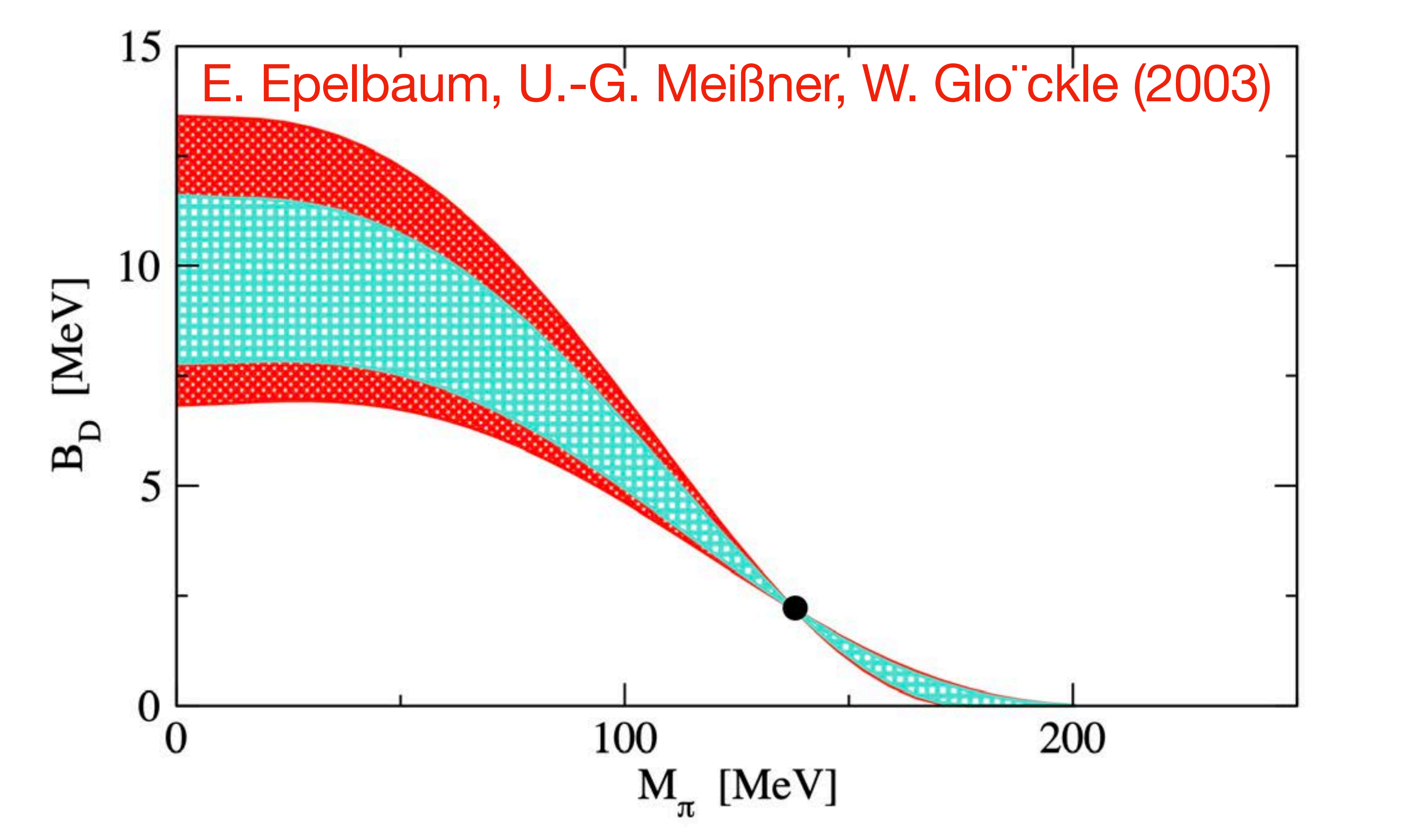

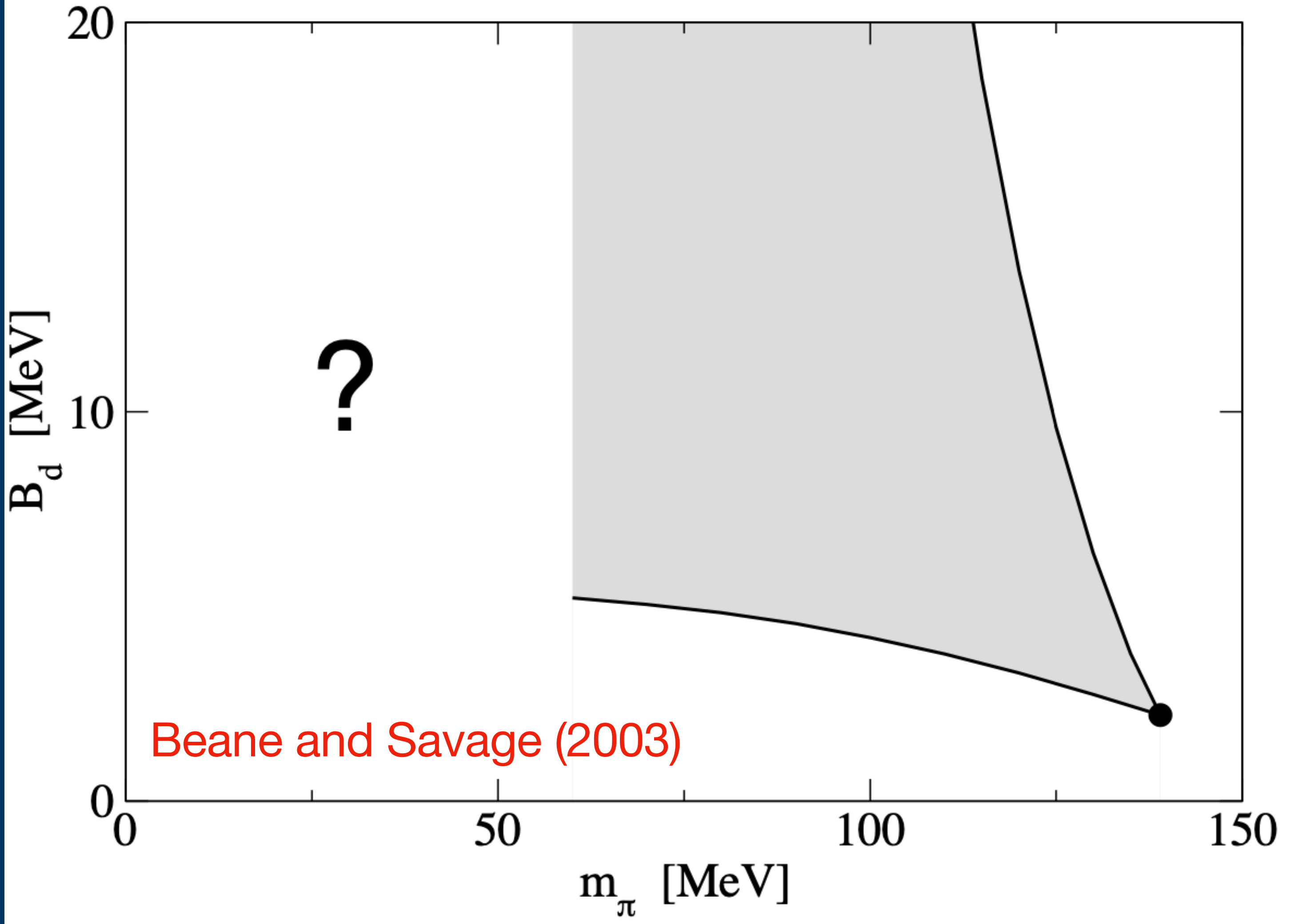

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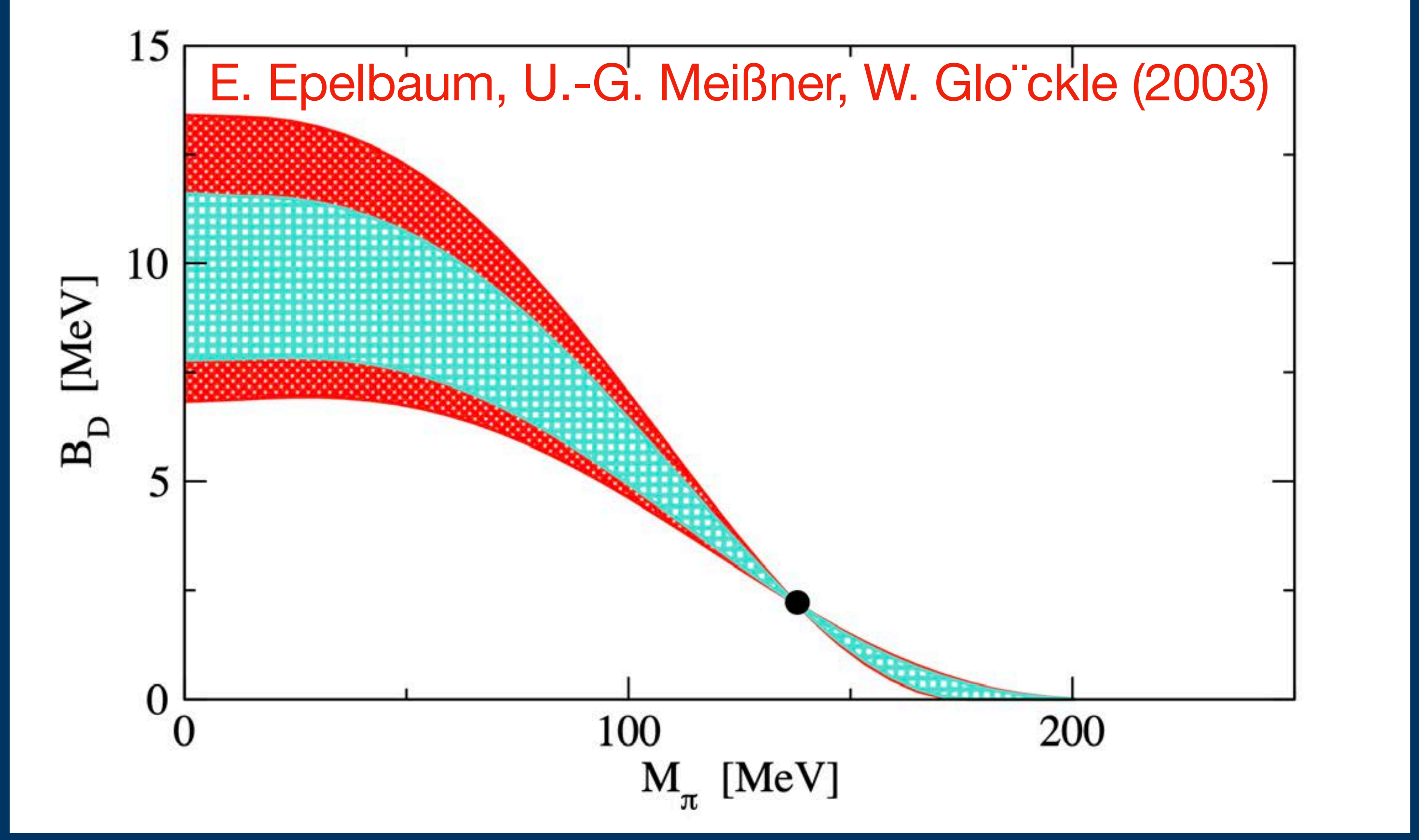

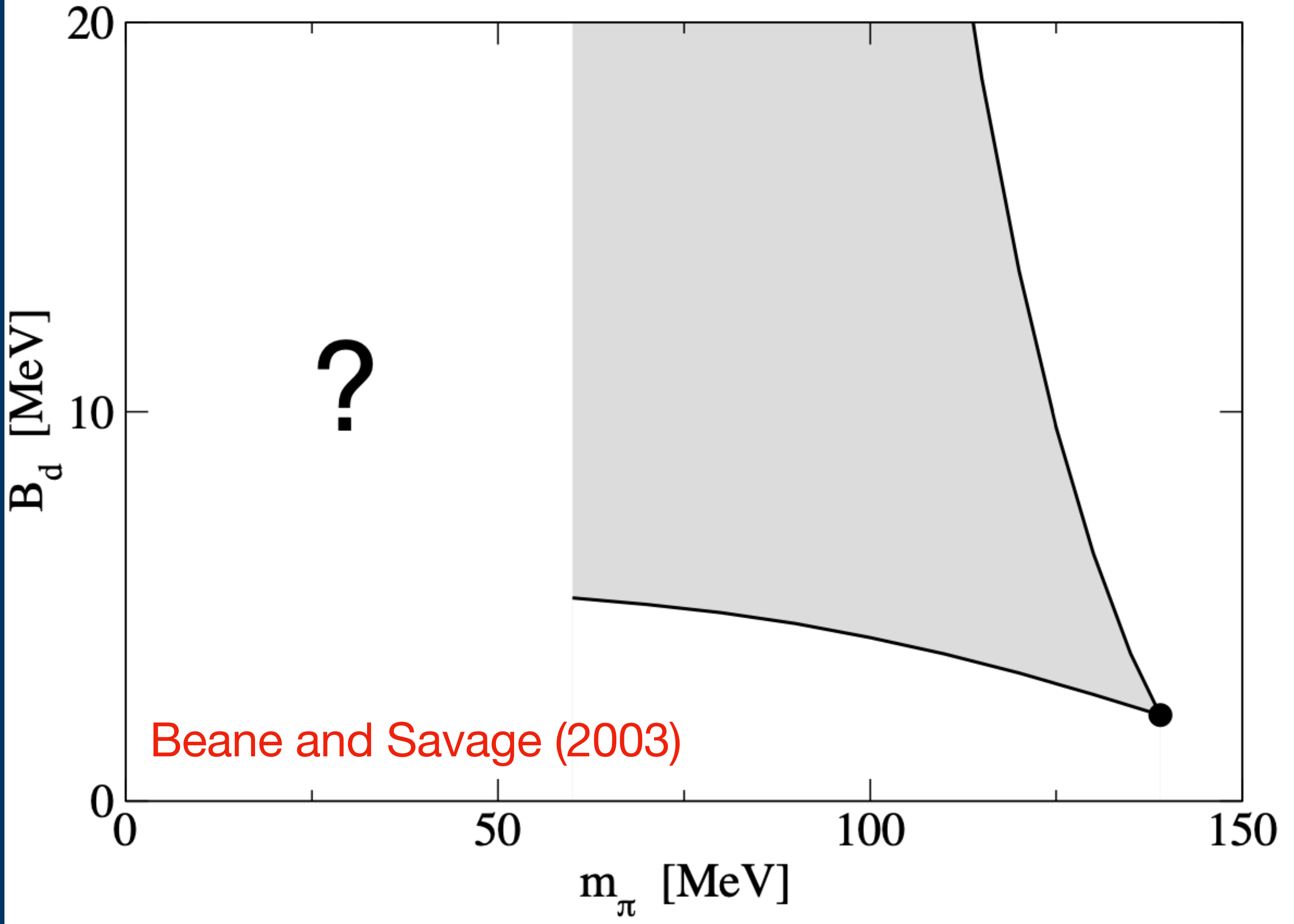

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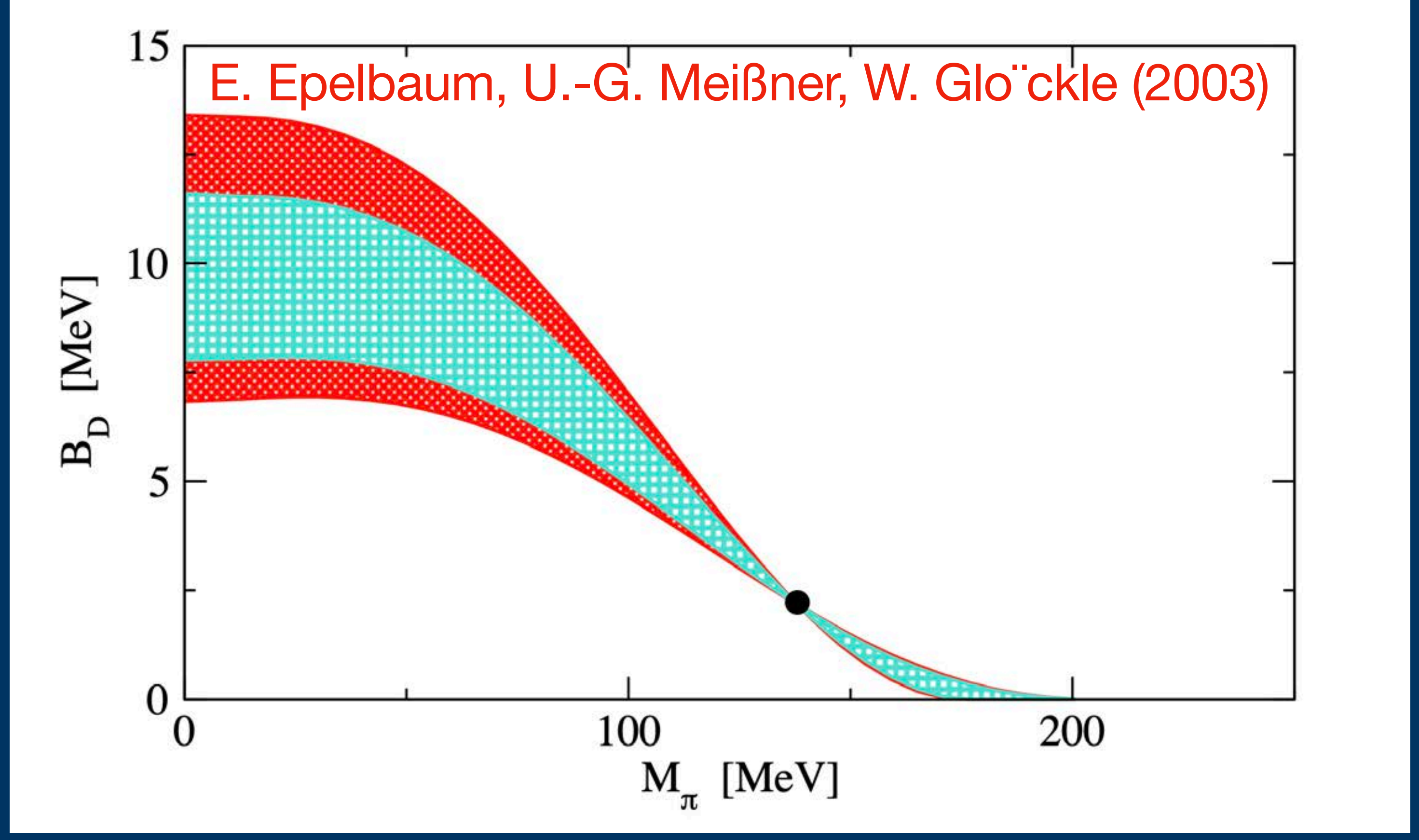
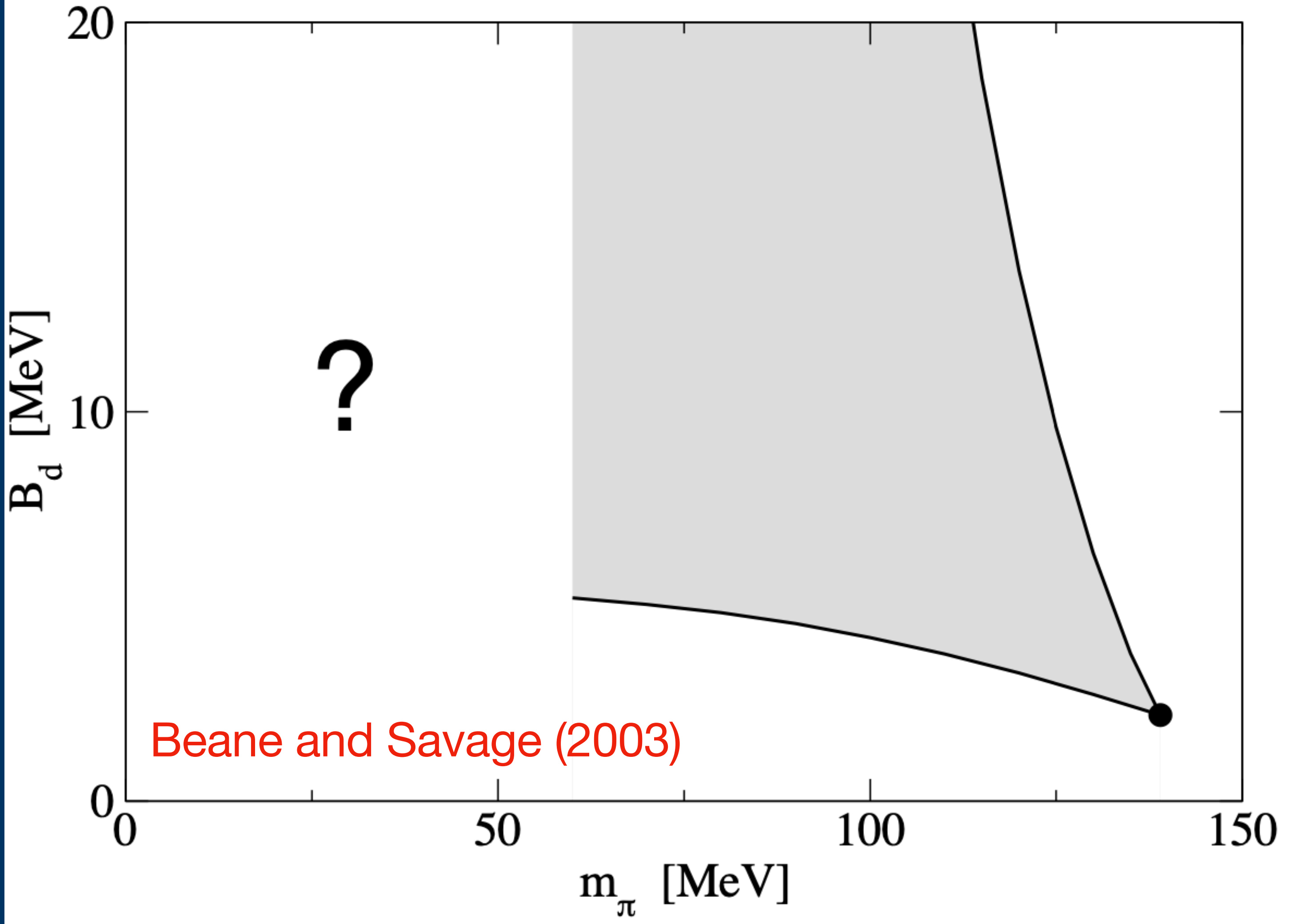
Effect of quark mass (pion mass) on the scattering length:

$$K_{a_s} = \frac{m_\pi^2}{a_s}\frac{\delta a_s}{\delta m_\pi^2} \simeq 2.4 \pm 3$$ J. C. Berengut, E. Epelbaum, et al. (2013)

$$\simeq 5 \pm 5$$ Beane and Savage (2003)

$$\simeq 2.3 \pm 1.9$$ E. Epelbaum, U.-G. Meißner, W. Glo¨ckle (2003)





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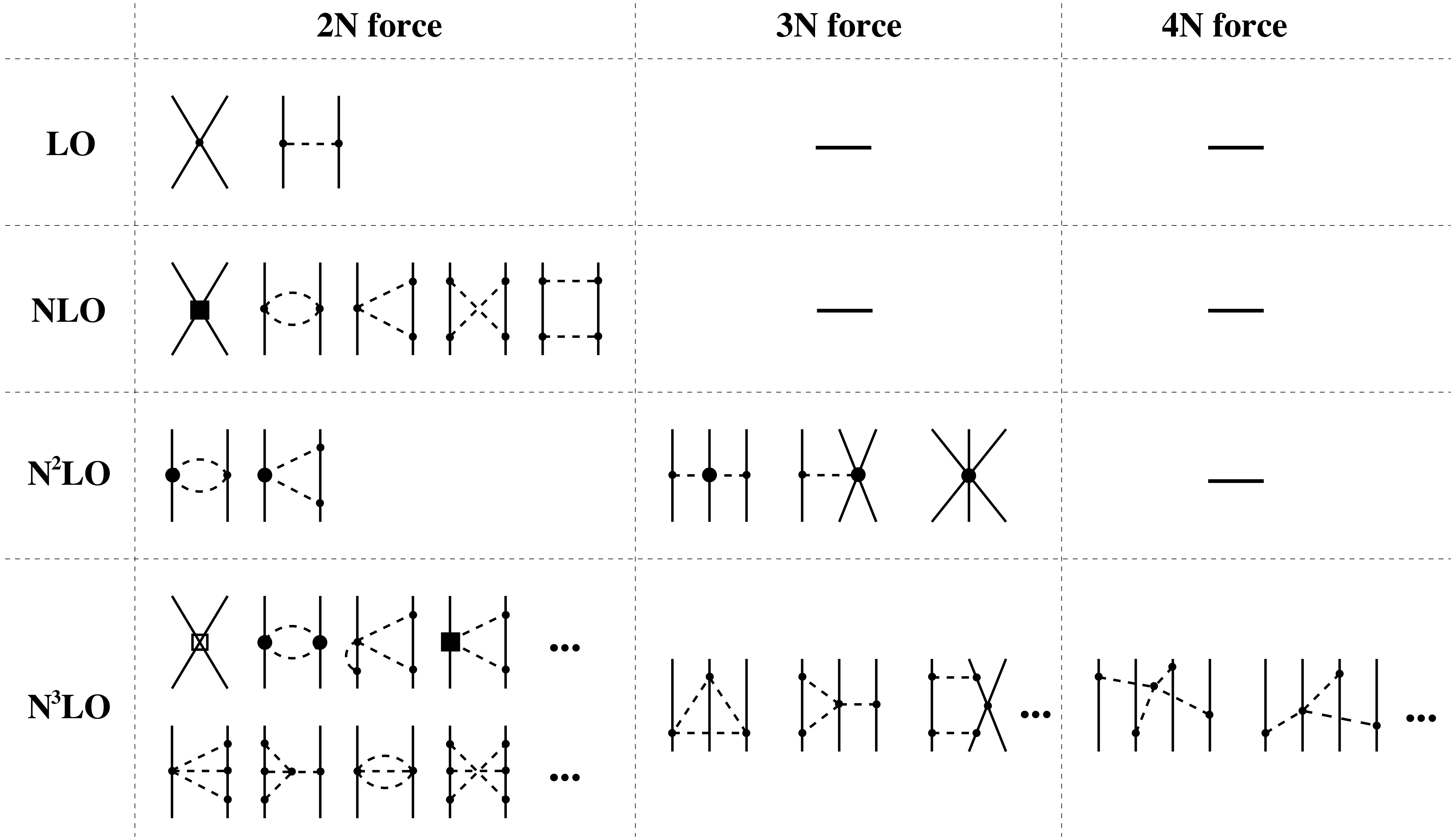
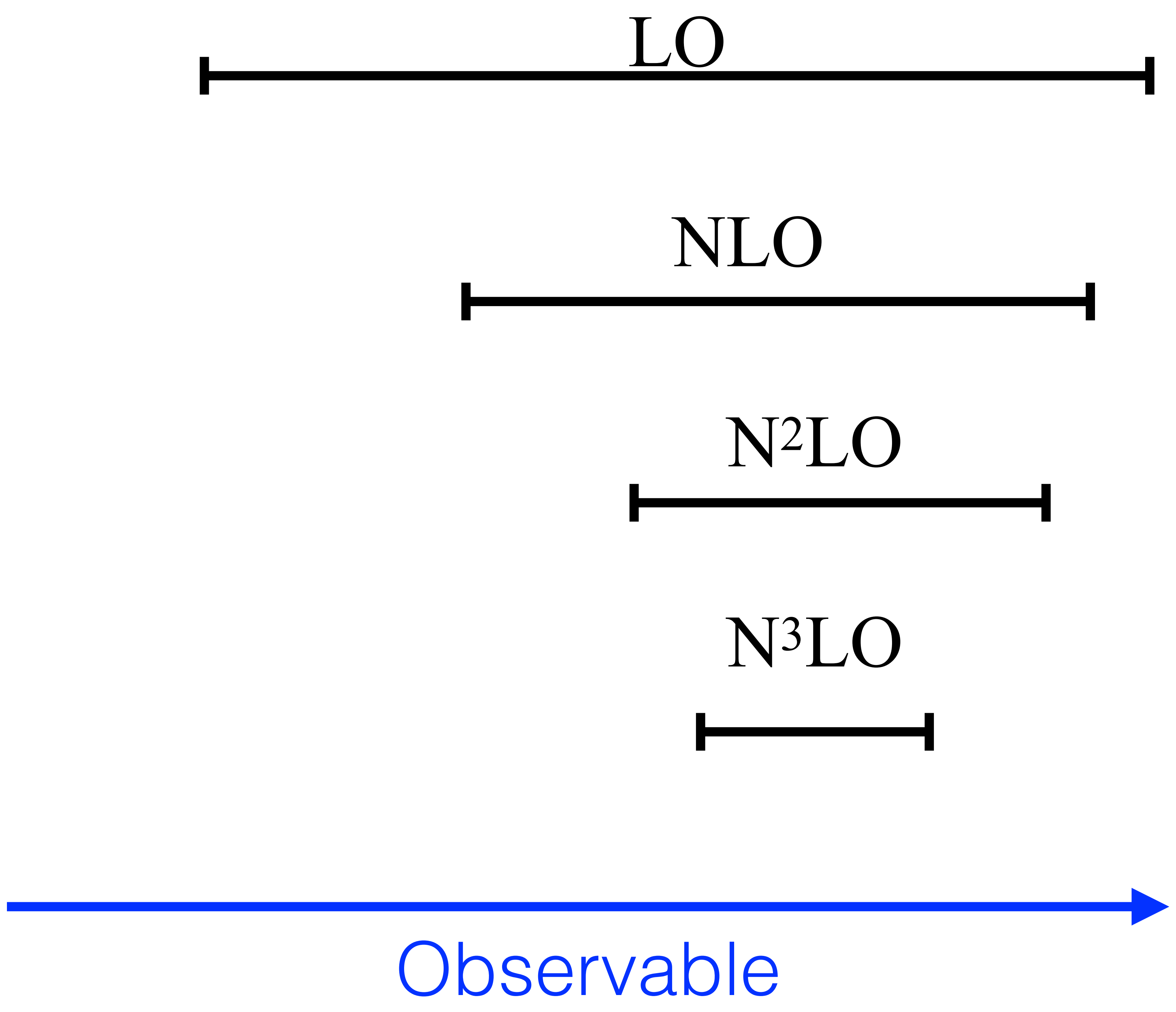
Nuclear Forces from Effective Field Theory (EFT)

EFT Hamiltonians organizes operators in powers of the momentum: $\frac{Q}{\Lambda_B}$





Allows for error estimation*. Provides guidance for the structure of three and many-body forces.

Beane, Bedaque, Epelbaum, Kaplan, Machliedt, Meisner, Phillips, Savage, **van Klock**, **Weinberg,** Wise ..

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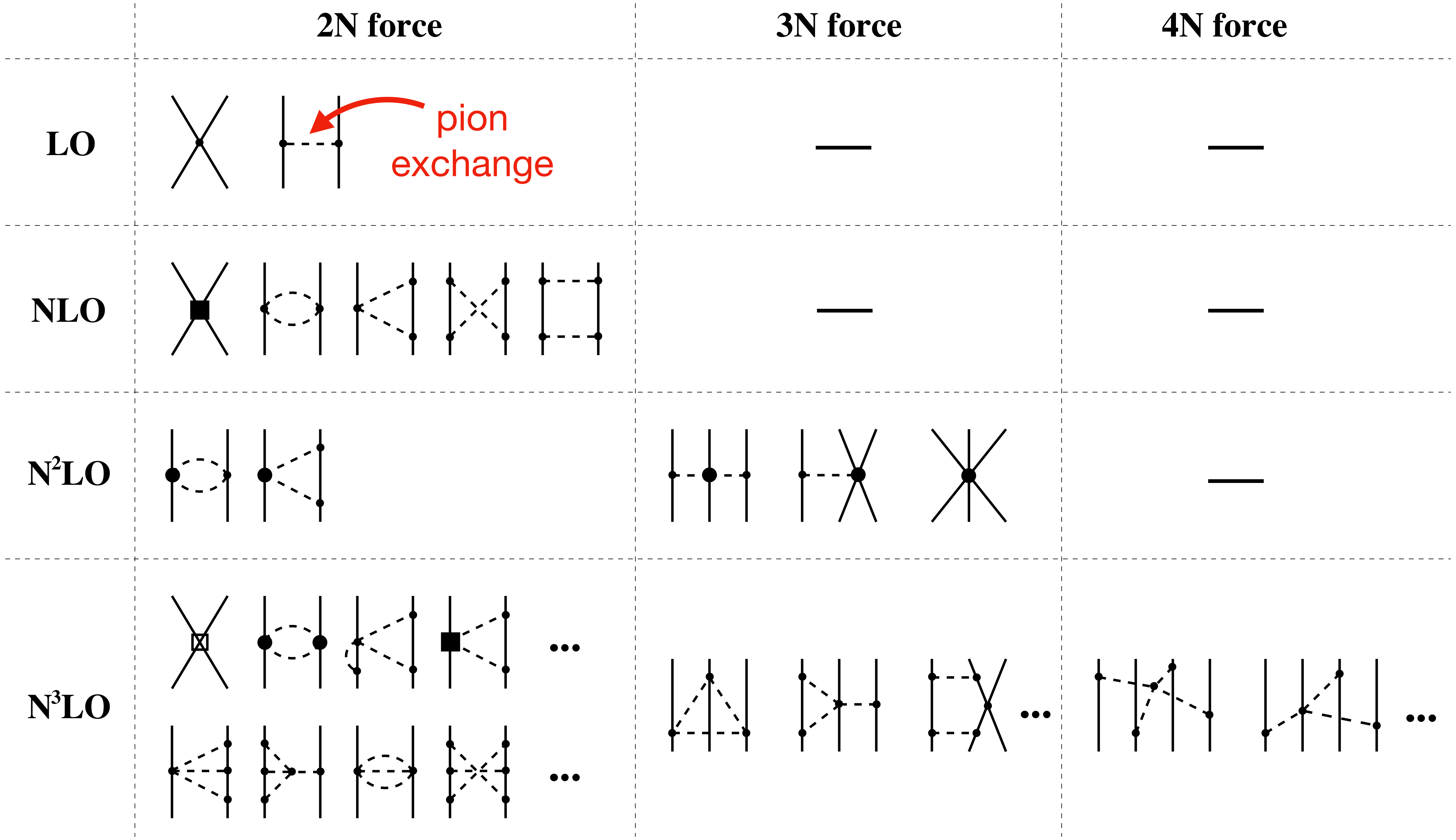

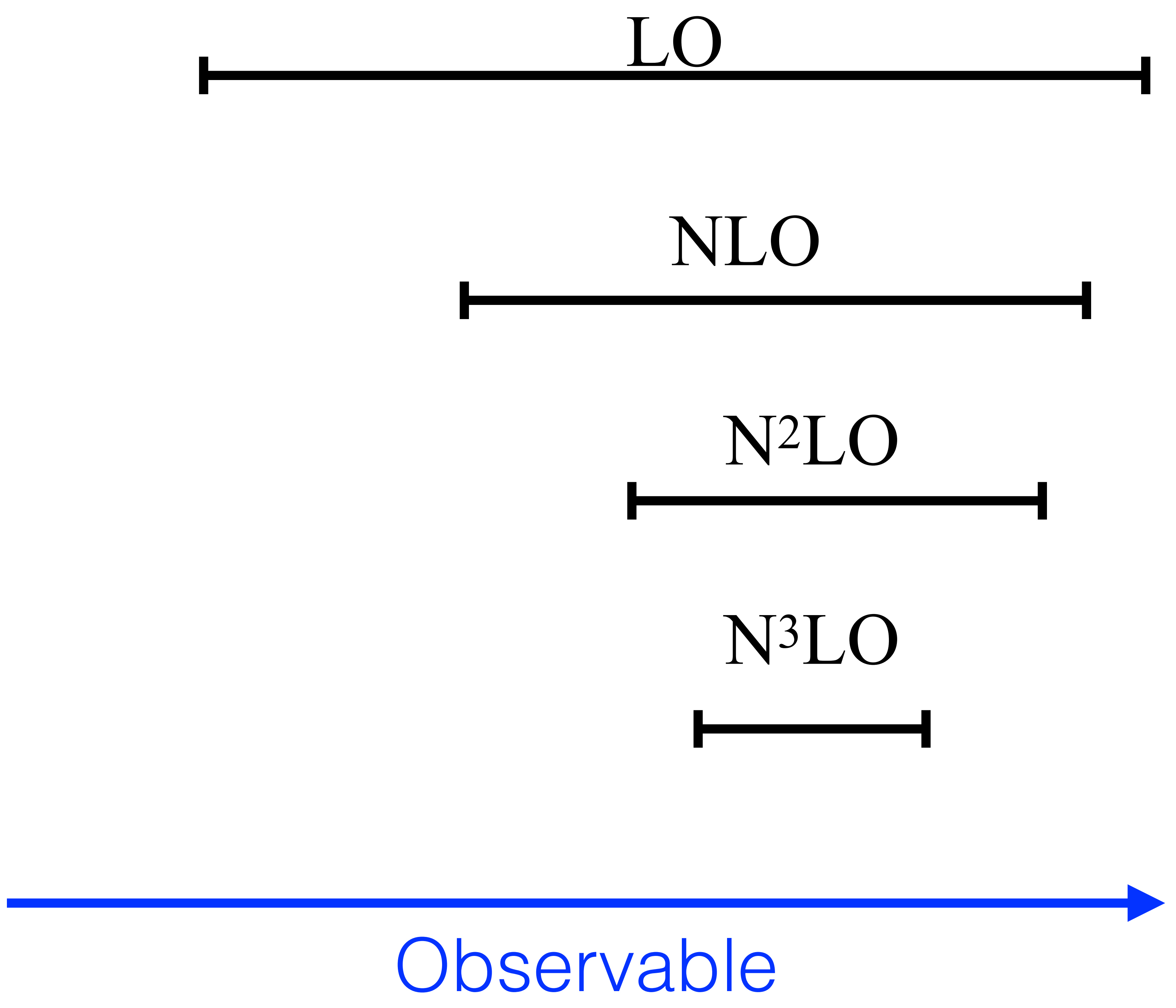

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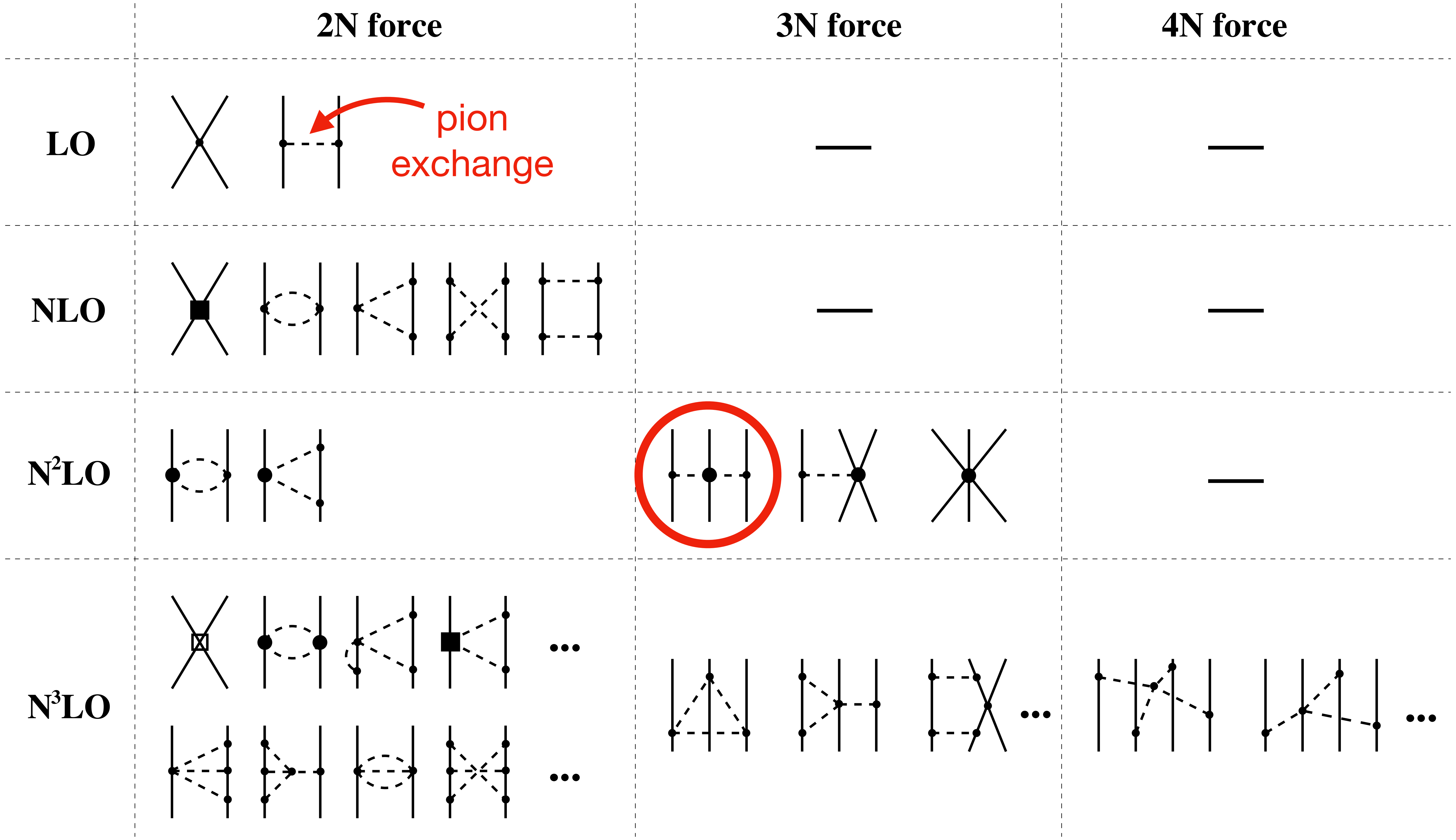

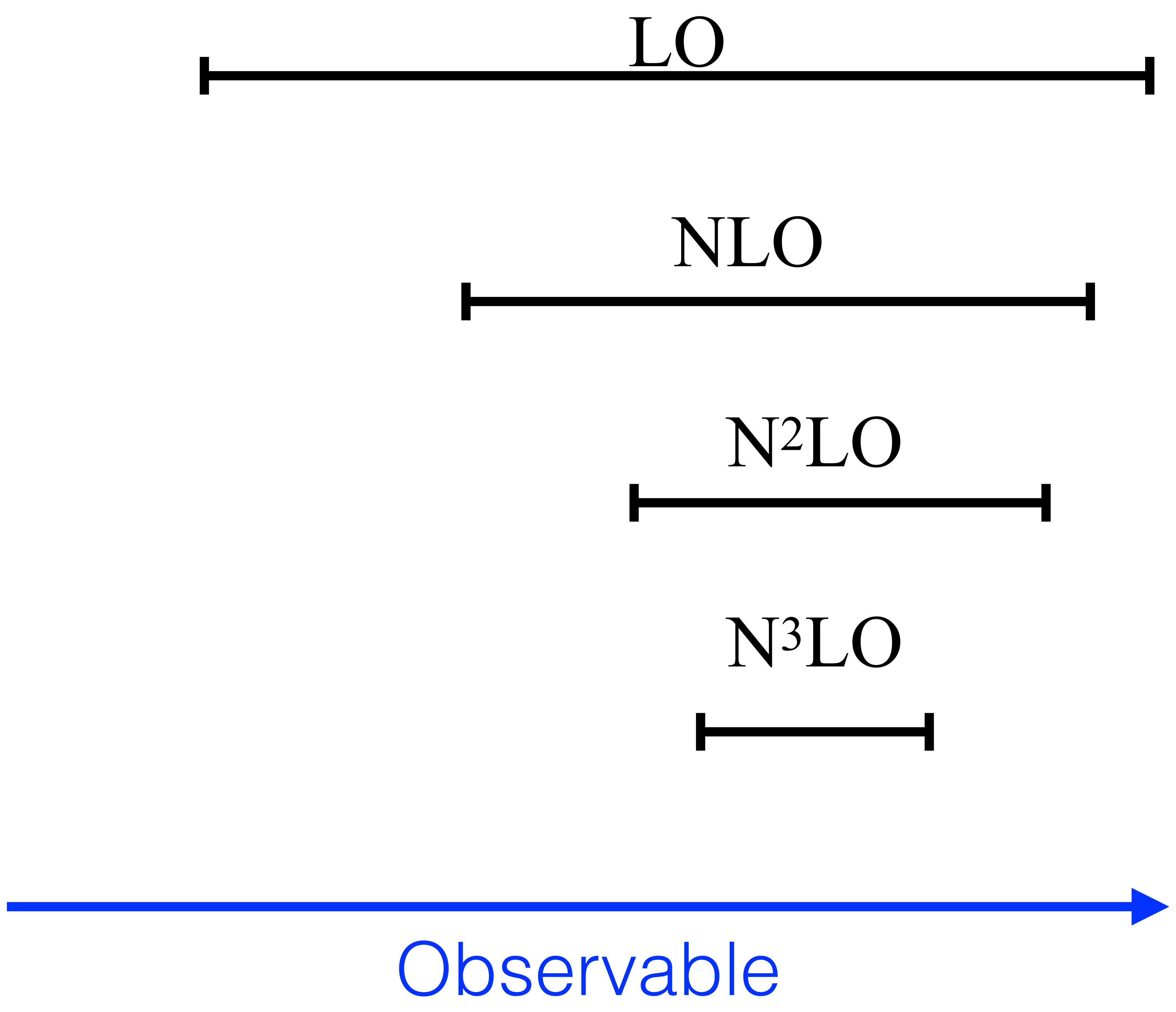

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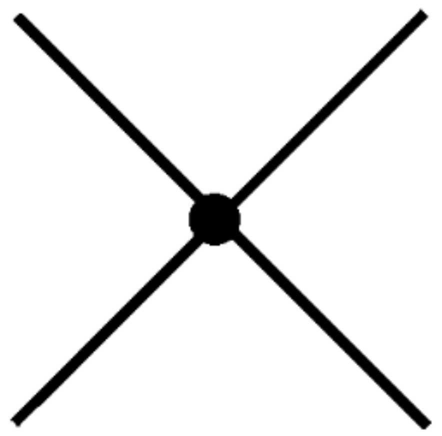
Quark (pion) mass-dependence of NN interaction in EFT



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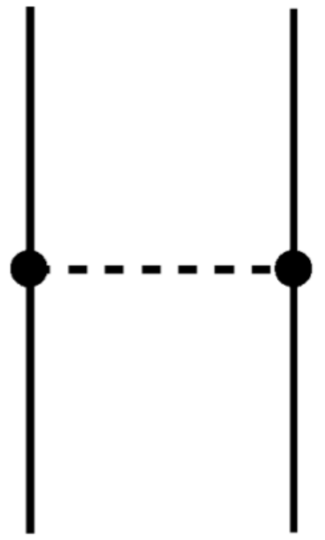

$$V_{\mathrm{LO}}(q) = C_0 + D_2\, m_\pi^2 + \frac{g_A^2}{4f_\pi^2}\frac{\sigma_1\cdot\mathbf{q}\;\sigma_2\cdot\mathbf{q}}{\mathbf{q}^2+m_\pi^2}\,\tau_1\cdot\tau_2$$

Analysis of 2-nucleon scattering in Lattice QCD for different values m_π could, in principle, determine D_2 but systematics are too large at this time.

Beane, Bedaque, Detmold, Savage (NPLQCD), Walker-Loud (Cal-Lat), Aoki, Hatsuda, Ishii (HAL QCD Collaboration), ….

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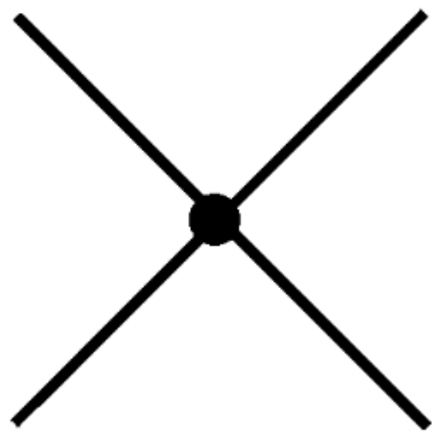

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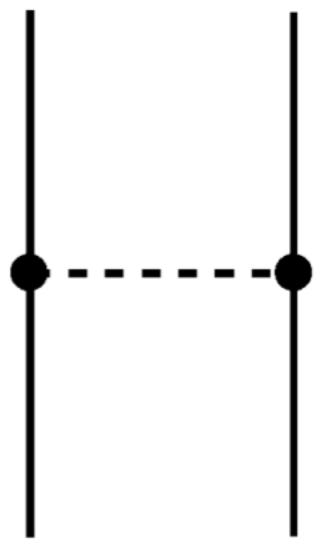

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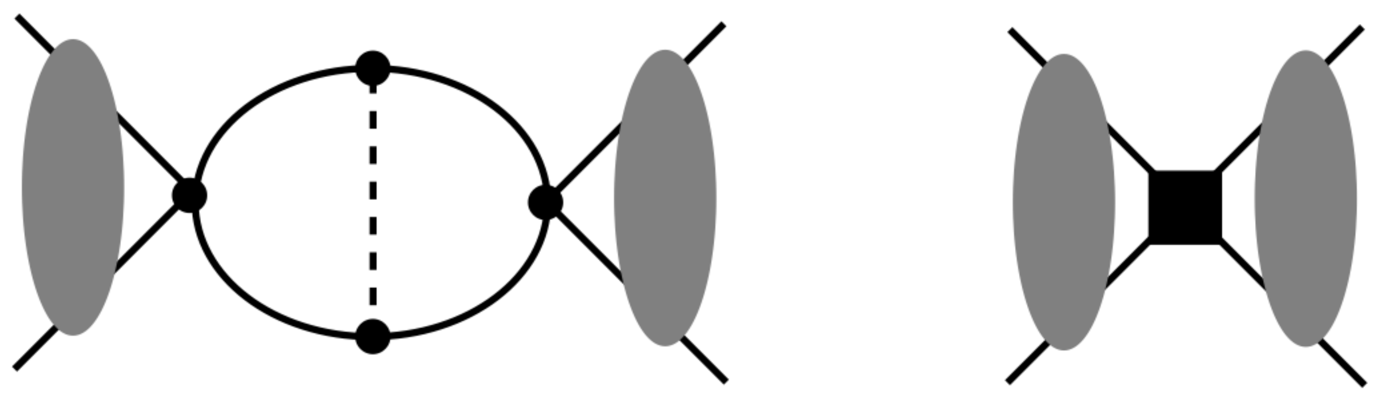
Renormalization requires D_2 :

Kaplan, Savage, Wise (1998)

To obtain a scattering amplitude that is independent of regularization or cut-off Λ requires:



$$\Lambda\frac{d}{d\Lambda}\left(\frac{D_2}{C_0^2}\right)_{\rm KSW} = \frac{g_A^2 m_N^2}{64\pi^2 f_\pi^2} \quad \Rightarrow \quad \frac{|D_2|}{C_0^2} \approx \frac{g_A^2 m_N^2}{64\pi^2 f_\pi^2} \simeq \frac{1}{4}$$

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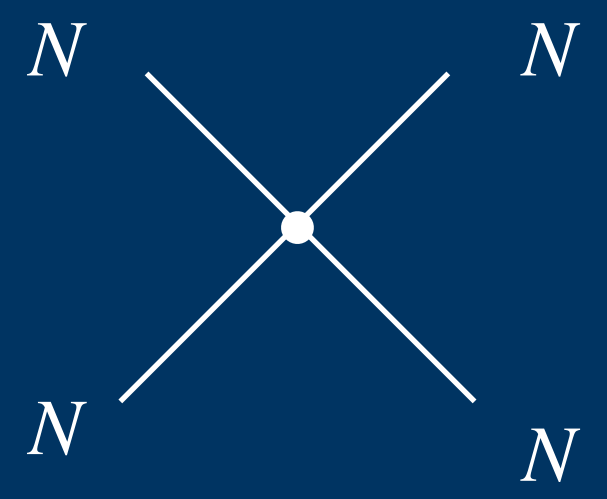
D_2 and Coupling to Pions

Chiral symmetry requires that pion mass terms only appear in a specified form:

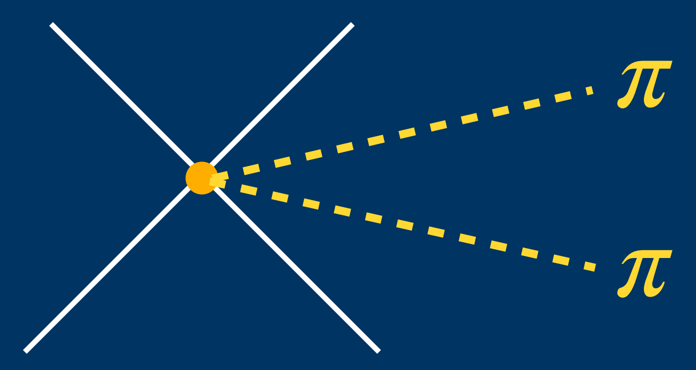
$$m_\pi^2 \Rightarrow m_\pi^2 \left(1 + \frac{\pi_a \pi_b}{2 f_\pi^2} \, \delta_{ab} + \cdots \right)$$

This induces a coupling of pions to two-nucleons:



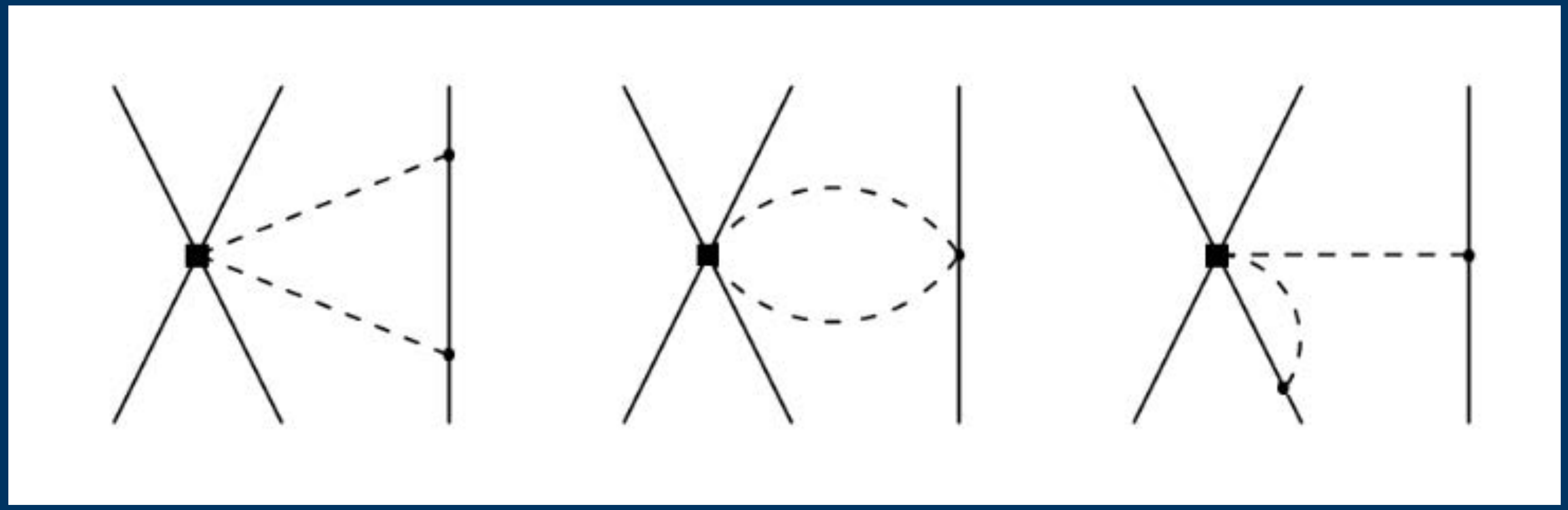
$D_2 \, m_\pi^2$

+



$D_2 \, \frac{m_\pi^2}{2 f_\pi^2}$

A New Class of Three Nucleon Forces



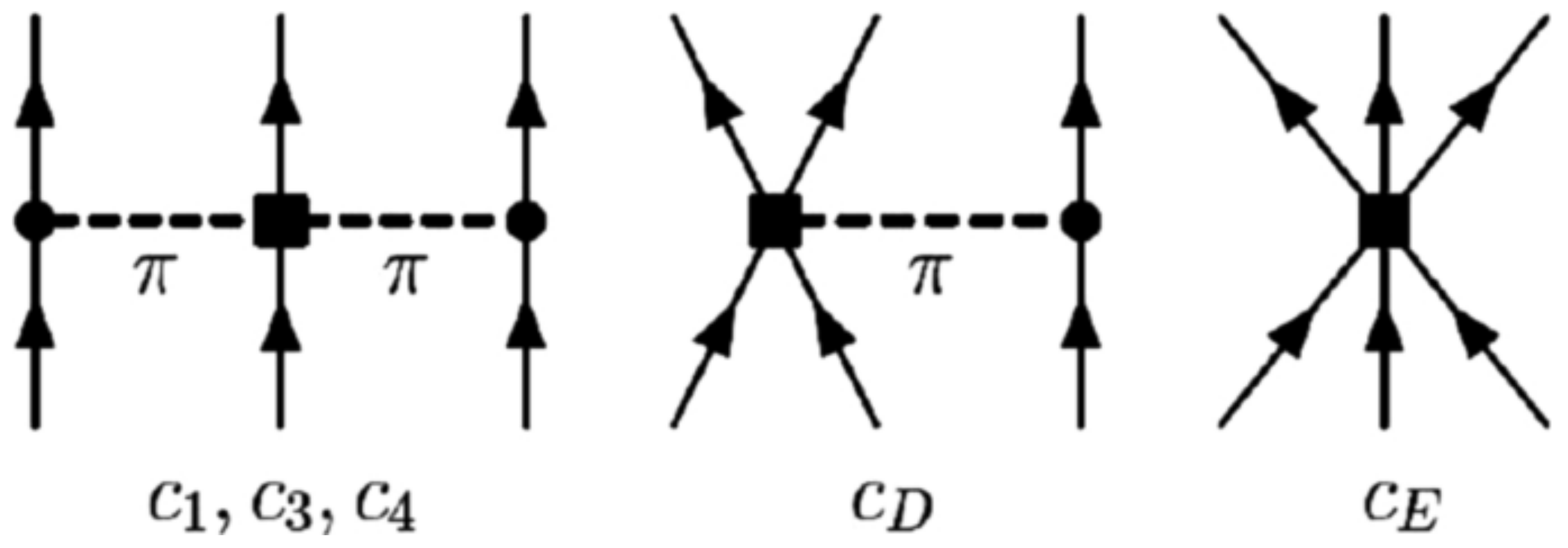
3NF due to pion coupling to two nucleons

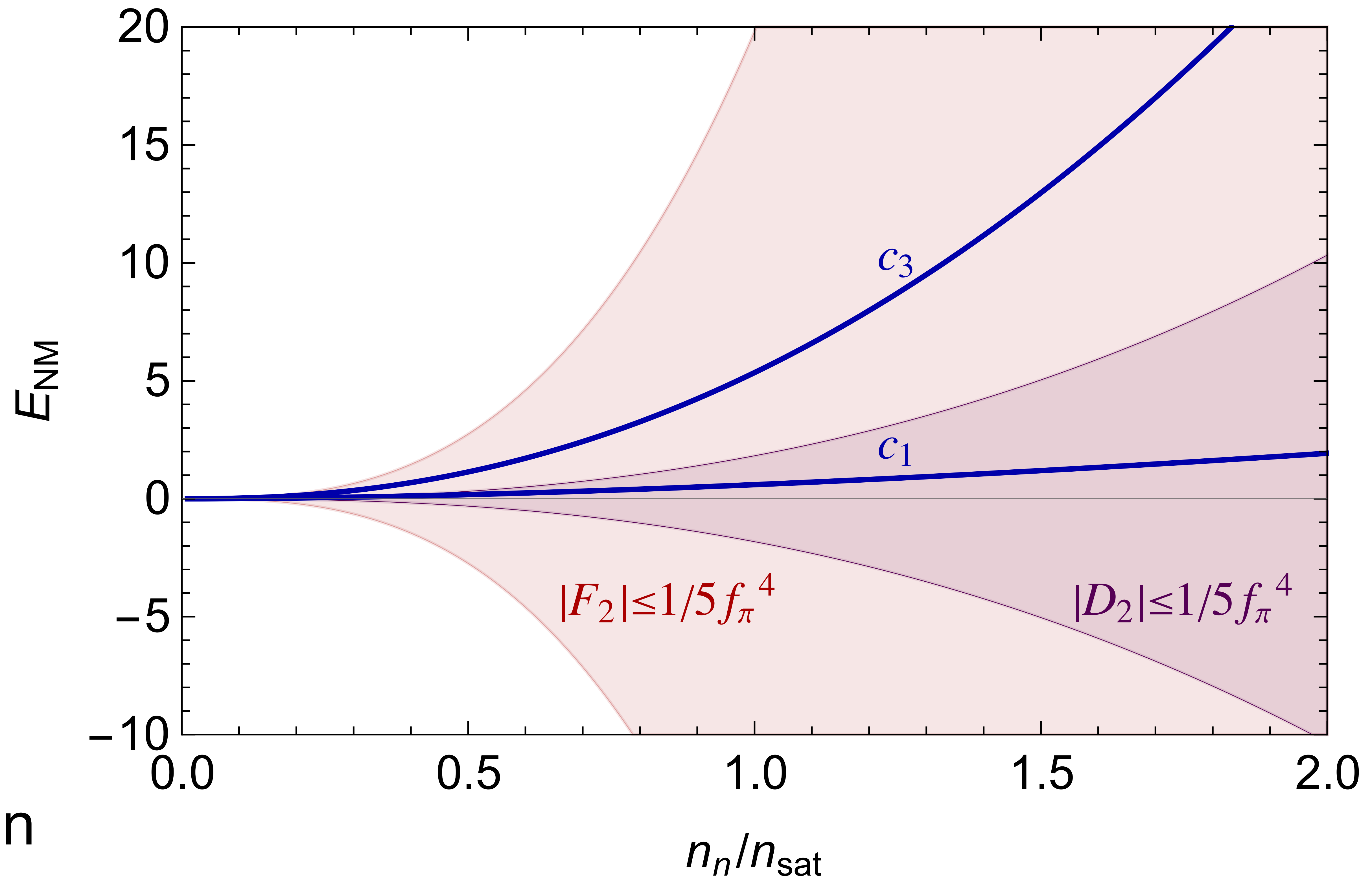
$$V_{ijk}^{i'j'k'}(\vec{q}_1, \vec{q}_2, \vec{q}_3) = -\frac{9g_A^2 D_2 m_\pi^3}{128\pi f_\pi^4}\kappa_{ij}^{i'j'}\delta_{kk'}\,\mathcal{I}\left(\frac{\vec{q}_3^{\,2}}{4m_\pi^2}\right) \quad \text{where} \quad \mathcal{I}(b) = \frac{2}{3}\left(1 + \left(\frac{1}{2\sqrt{b}} + \sqrt{b}\right)\cot^{-1}(1/\sqrt{b})\right)$$

V. Cirigliano, M. Dawid, W. Dekens, S. Reddy (2024)

D_2 and F_2 Contributions to the Energy are Large

In neutron and nuclear matter, the leading 3NF plays a critical role.





The new 3NF involving the can be large enough to compete with the NNLO forces currently employed in Chiral EFT.

The uncertainty is large because D_2 & F_2 are not yet known.

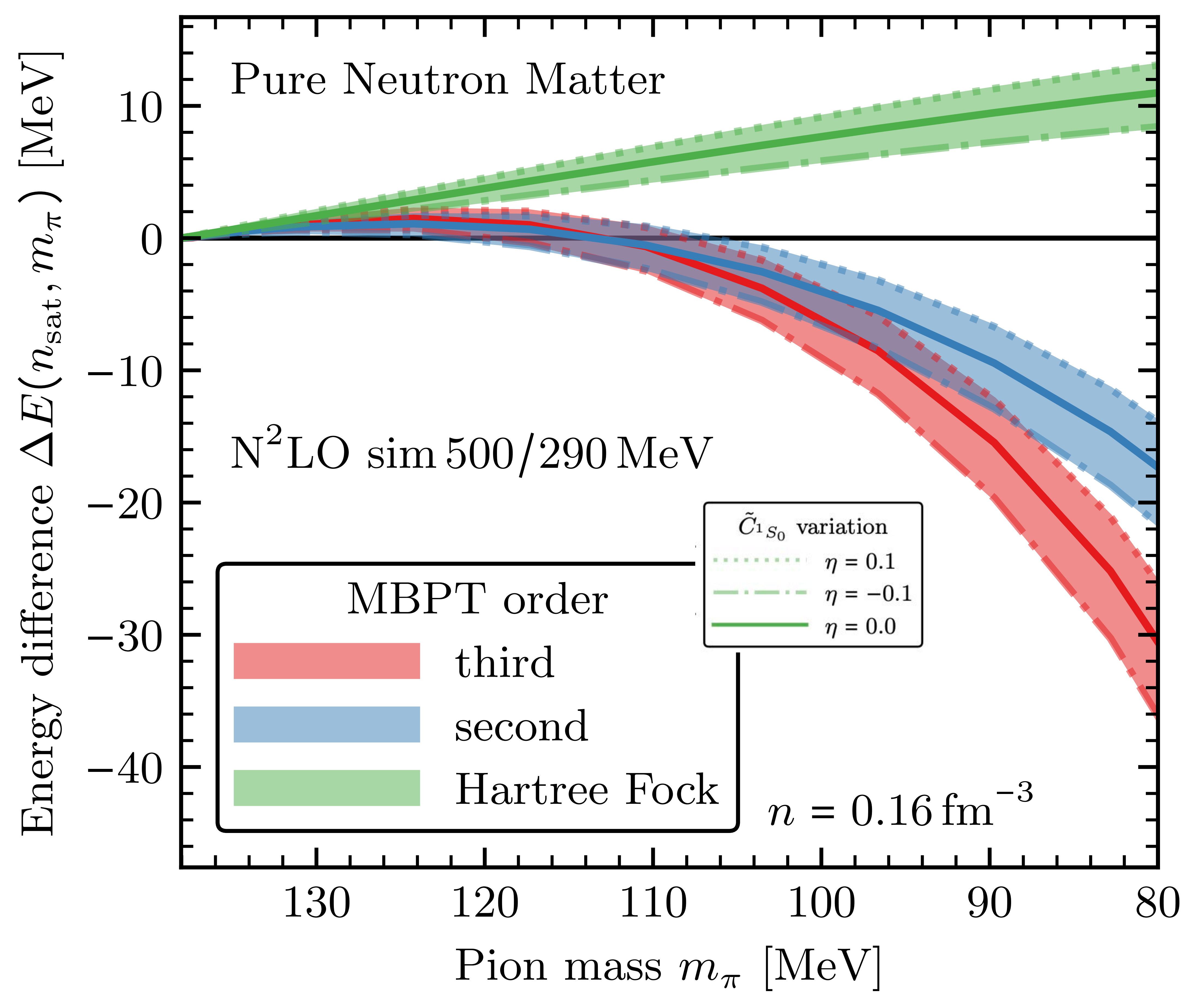
V. Cirigliano, M. Dawid, W. Dekens, S. Reddy (2024)

Can interactions favor axion condensation?

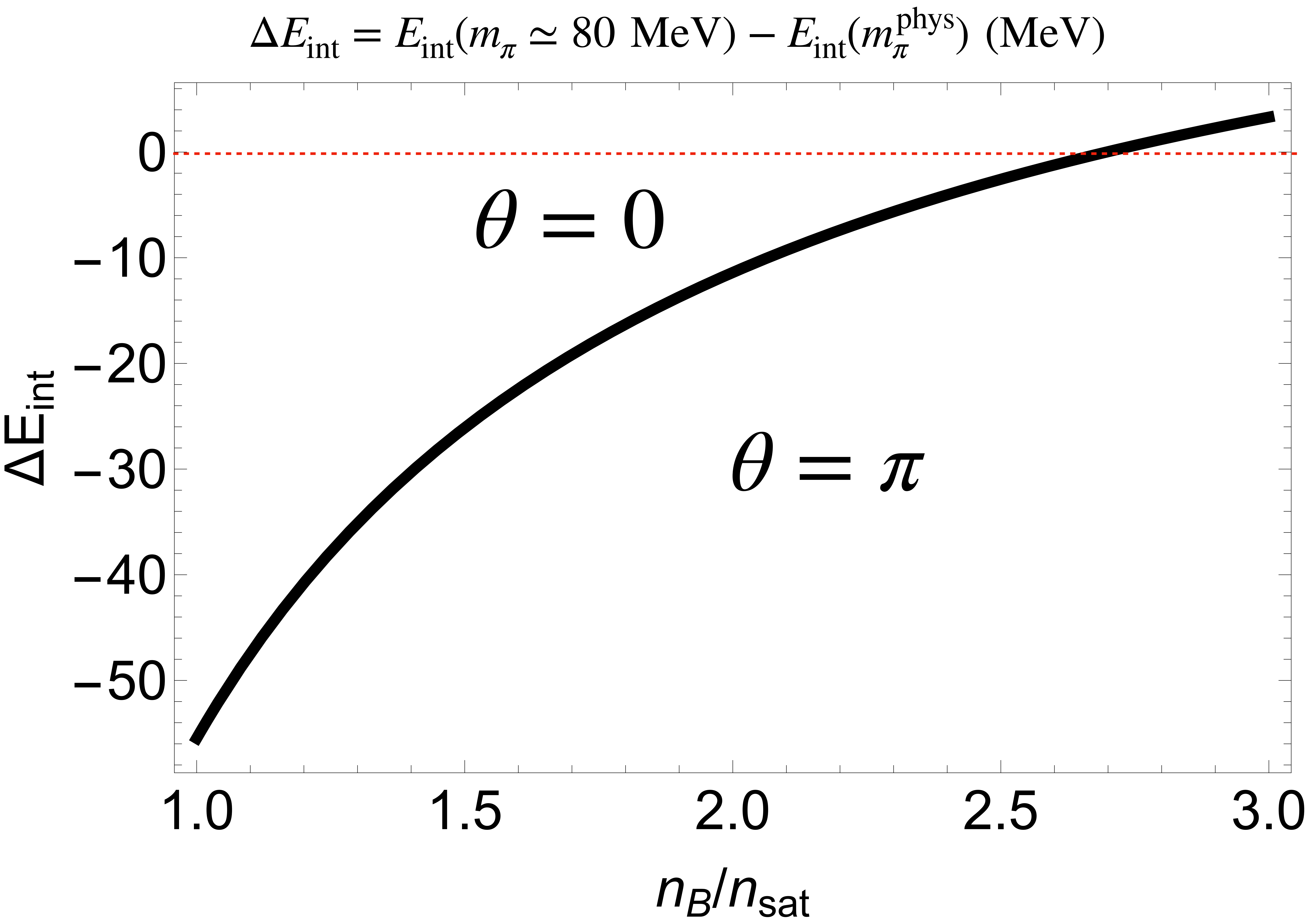
How does the interaction energy at nuclear density change with m_π?

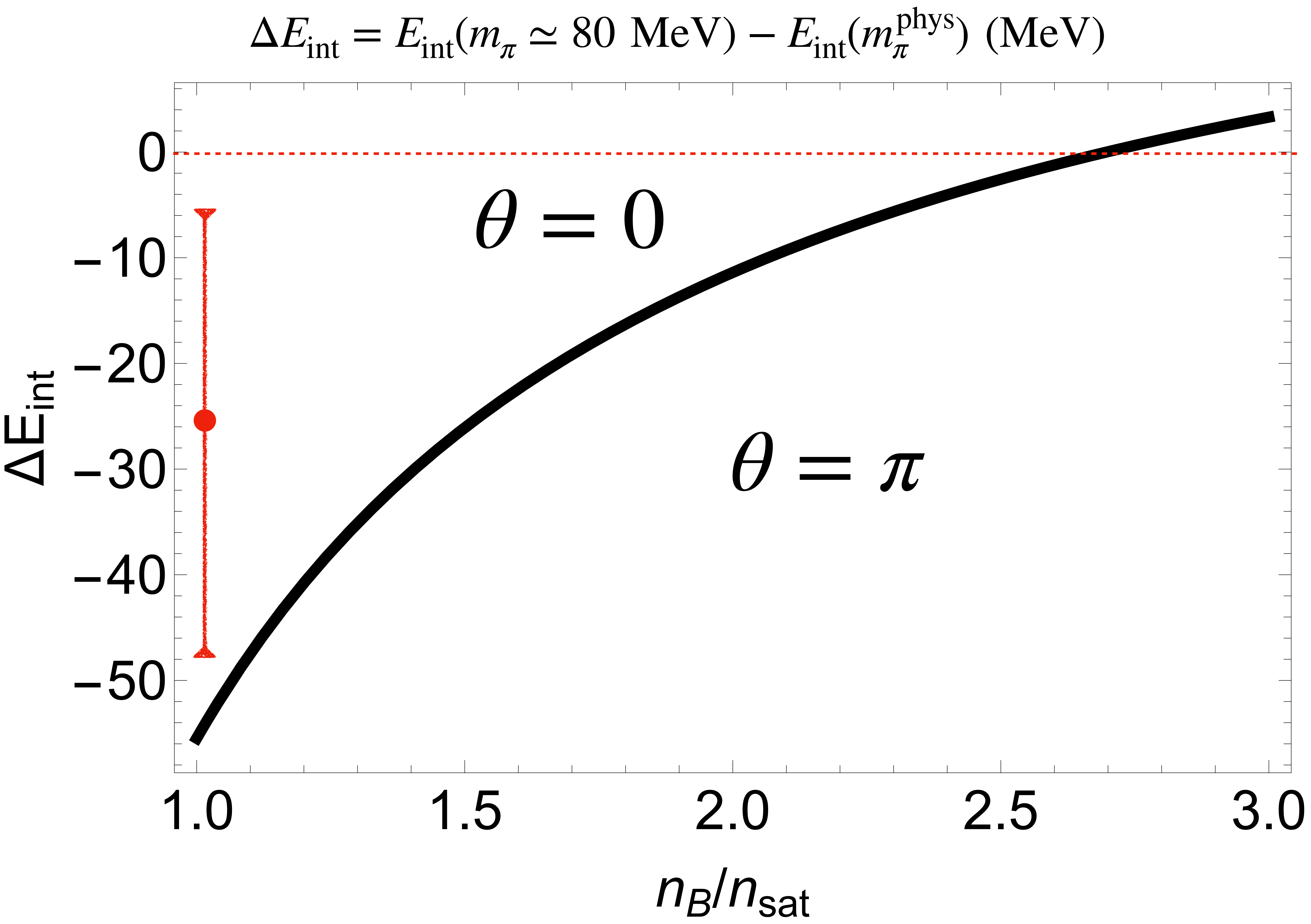
$$\Delta E_{\text{int}} = E_{\text{int}}(m_\pi) - E_{\text{int}}(m_\pi^{\text{phys}})$$

- In ChiEFT, interaction increases at first, and then decreases with decreasing m_π
- A large cutoff dependence suggets that a systematic study of the pion-mass dependence of short-range components (LECs) is warranted.
- The contribution from three body forces can be especially important and not all of them have been accounted for.



M. Kumamoto, J. Huang, C. Drischler, M. Baryakhtar, S. Reddy (2024)


$\Delta E_{\rm int} = E_{\rm int}(m_\pi \simeq 80 \text{ MeV}) - E_{\rm int}(m_\pi^{\rm phys})$ (MeV)
$\Delta E_{\rm int}$
0
−10
−20
−30
−40
−50
$\theta = 0$
$\theta = \pi$
1.0
1.5
2.0
2.5
3.0
$n_B/n_{\rm sat}$

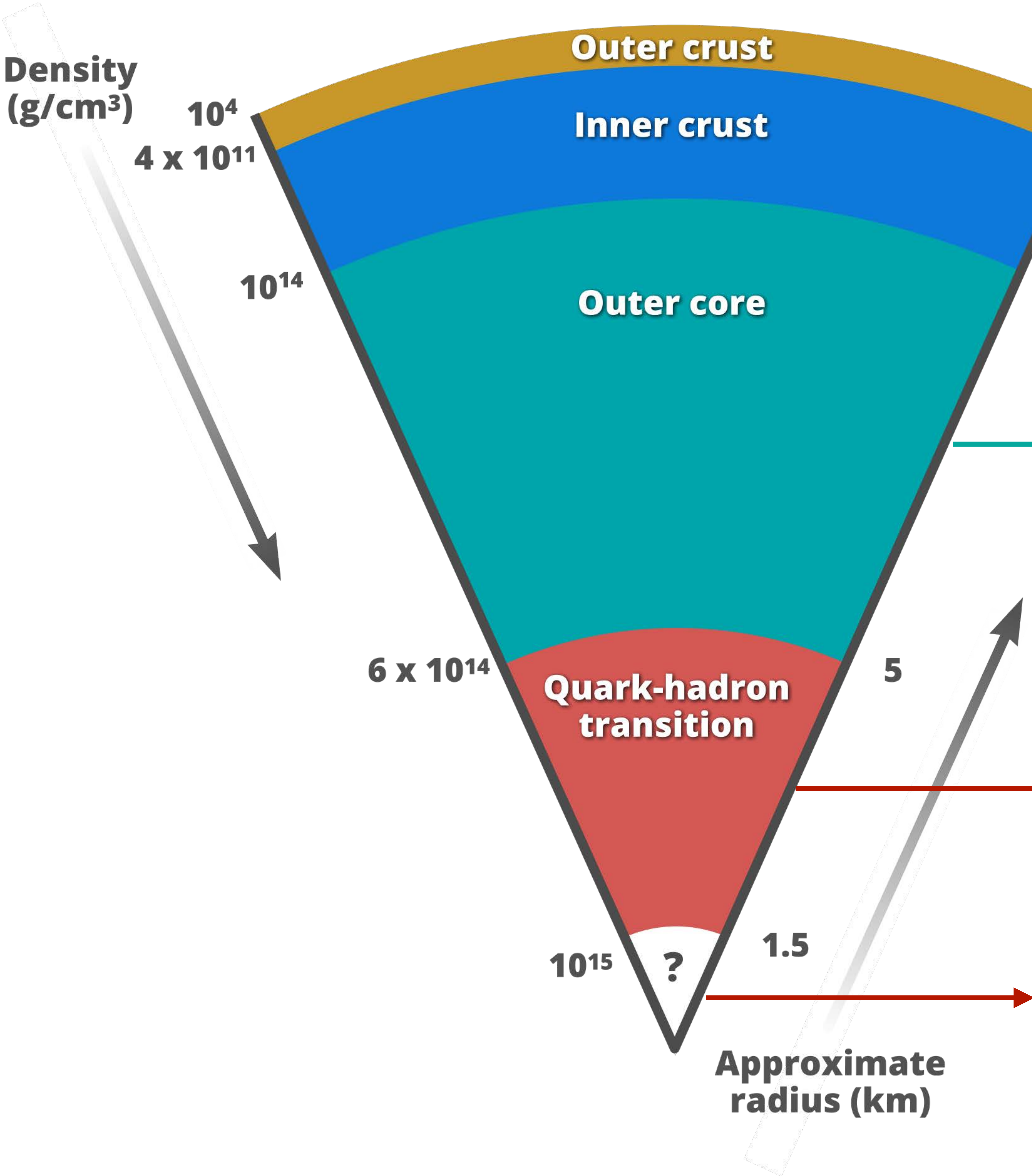


$\Delta E_{int} = E_{int}(m_\pi \simeq 80 \text{ MeV}) - E_{int}(m_\pi^{phys})$ (MeV)
0
−10
−20
−30
−40
−50
ΔE_{int}
$\theta = 0$
$\theta = \pi$
1.0
1.5
2.0
2.5
3.0
n_B/n_{sat}

Conclusions

- Understanding the quark or pion mass dependence of nuclear forces is important to address the possibility of axion condensation in neutron stars.
- Preliminary calculations suggest that the interaction energy decreases with pion mass and favors axion condensation.
- Three nucleon forces including those that originate from pion coupling to two nucleons will likely play a key role and warrant further investigation.

Inside Neutron Stars



Nuclei and relativistic electrons.

Neutron-rich nuclei, relativistic electrons, superfluid neutrons

Neutrons (~ 90%), protons, relativistic electrons, muons. Description in terms of baryons remains useful. Superfluid neutrons & superconducting protons.

Complex strongly interacting relativistic matter. Description not simple in terms of either baryons or quarks. Quarkyonic Matter?

Ordered quark matter? color superconductor?

An Aside: Exceptionally light QCD axions

There has been recent interest in more exotic scenarios involving a large number of BSM gauge fields that also couple to the QCD axion.

In these scenarios the axion potenatial takes the form

$$V\left(\theta = \frac{a}{f_a}\right) = \epsilon\, f_\pi^2 m_\pi^2 \left[1 - \sqrt{1 - \frac{4 m_u m_d}{(m_u + m_d)^2} \sin^2\left[\frac{\theta}{2}\right]}\right]$$

where the new parameter $\epsilon \ll 1$ leads to a lighter axion.

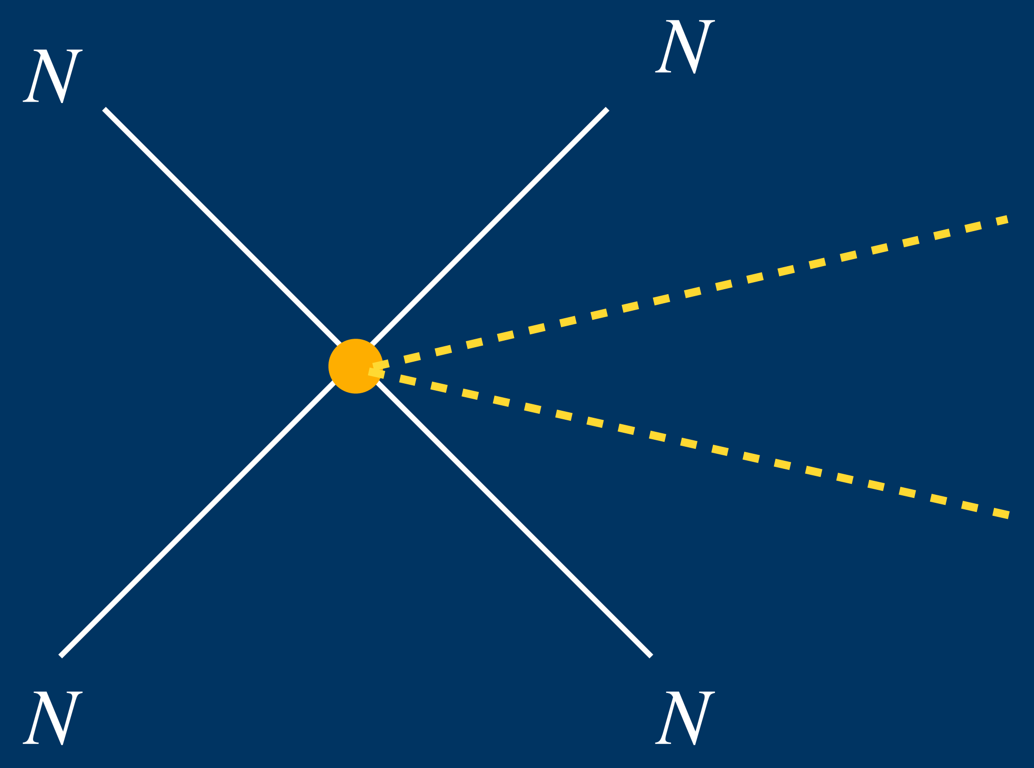
Such axions would condense when $\sigma_{\pi N} n_B > \epsilon f_\pi^2 m_\pi^2$

Or when $n_B^c = \epsilon\, \frac{f_\pi^2 m_\pi^2}{\sigma_{\pi n}}$ $\longrightarrow$ $n_B^c \simeq 2.6\ \epsilon\ n_{\text{sat}}$

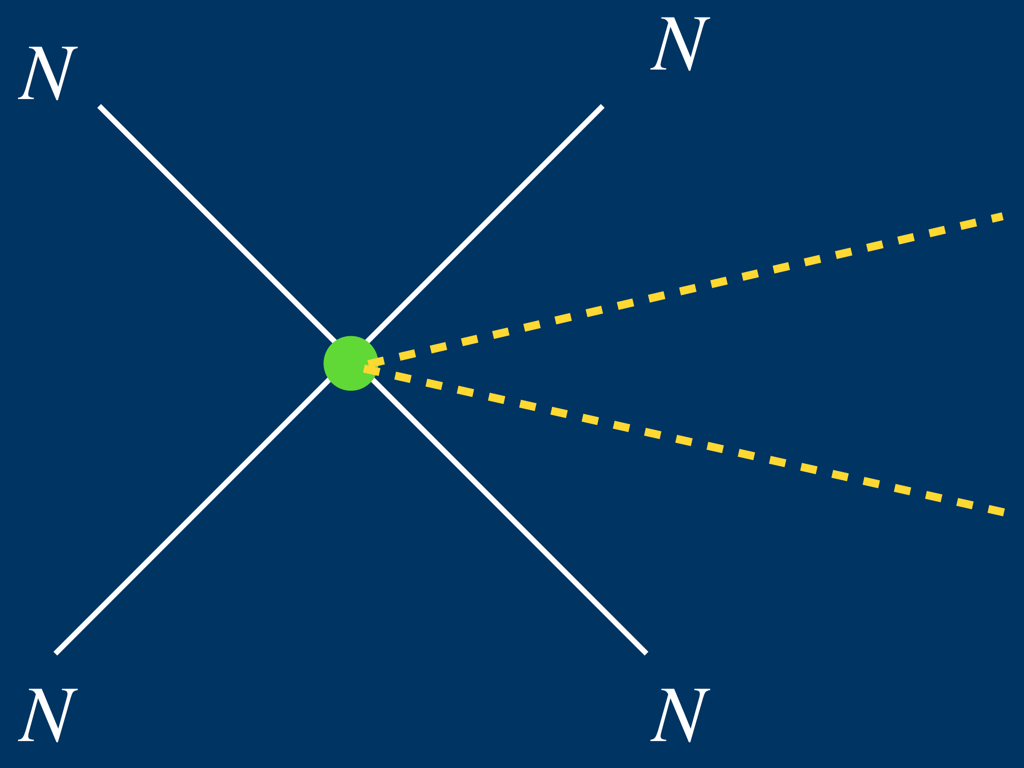
A. Hook (2018)

Two more enhanced pion-two-nucleon couplings: E_2 and F_2

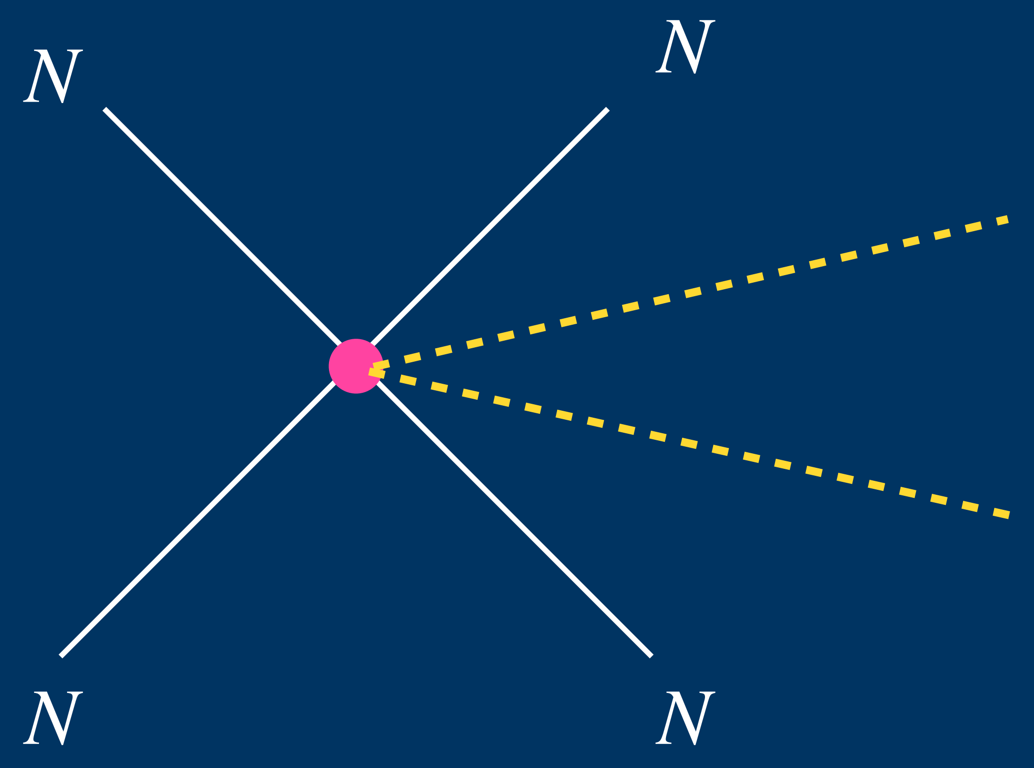
B. Borasoy and H. W. Griesshammer (2001), (2003)



$D_2\ m_\pi^2$



$E_2\ \omega^2$



$F_2\ q^2$

$D_2, E_2,$ & F_2 are enhanced for the same reason and are apriori expected to be of similar size.

Typical size of these LECs: $D_2 \approx E_2 \approx F_2 \approx \dfrac{1}{5 f_\pi^4}$