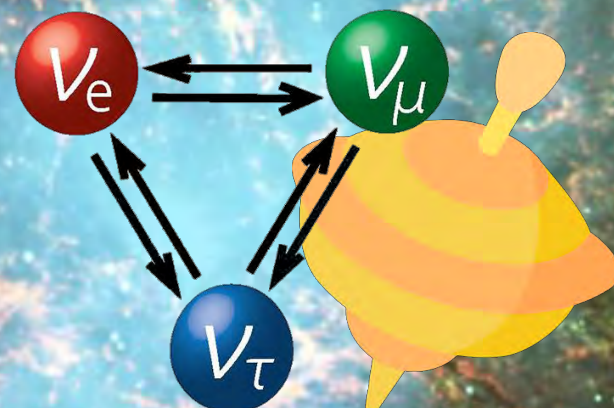
From Neutrino Dispersion to Flavor Waves

The first thirty years

Balantekin & Fuller Fest 16–18 Jan 2025



Georg Raffelt

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MAX-PLANCK-INSTITUT FÜR PHYSIK

SFB 1258 | Neutrinos Dark Matter Messengers

ORIGINS Excellence Cluster

COSMIC WISPERS CA21106

Collective Neutrino Flavor Conversion

Mass matrix → $\frac{\mathbb{M}^2}{2E}$; Charged lepton density → $\sqrt{2}G_F\mathbb{N}$; Neutrino neutrino refraction → $\mathbb{H}_{\nu\nu}$

$$i\partial_t \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \left(\frac{\mathbb{M}^2}{2E} + \sqrt{2}G_\mathrm{F}\mathbb{N} + \mathbb{H}_{\nu\nu} \right) \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}$$

George Fuller

31 Research papers
1 Influential review
(solicited by me)

- Refractive back reaction of neutrinos on each other in neutrino-dense environments
- Collective flavor conversion effects

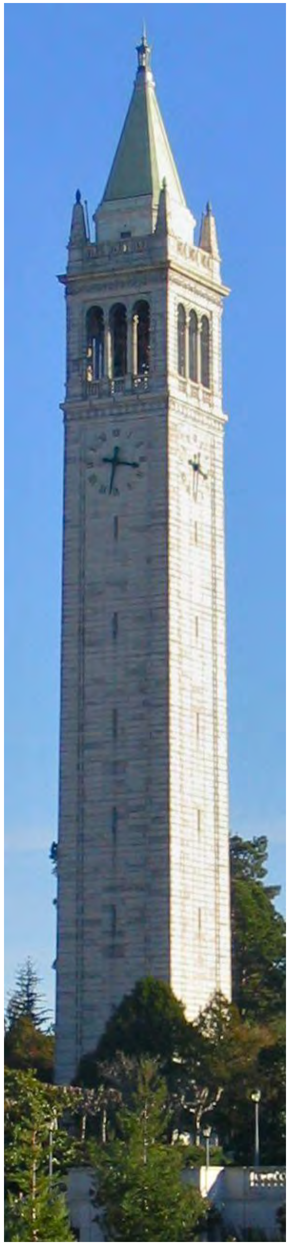
Baha Balantekin

17 Research papers
Focus on quantum many-body approach

Small but active research field

- ca 40 papers/year recently
- Dedicated workshops every year (eg organized by B&F and others)
- Fantastic young researchers especially of B&F groups

Returning to the Scene of the Crime



- Georg Raffelt – Postdoc 1986–90 at IGPP (LLNL) & Astr. Dept. (Joe Silk)
  Working with David Dearborn on axion emission from normal stars
- George Fuller – IGPP, 1986–88 (then moving to UCSD)
  (cosmo quark-hadron phase trans., majorons & flavor osci in SNe …)
- Grant Mathews – LLNL 1981–94

But GR did not directly scientifically overlap or collaborate w/ GF & GM

For GR, 1987 in Berkeley, *annus mini mirabilis*
(3 of 5 most often-cited research papers)

- **Mixing of the Photon with Low Mass Particles**
  GR & L. Stodolsky, PRD 37 (1988) 1237
  (Explaining axion-photon conversion as flavor oscillation)
- **Bounds on Exotic Particle Interactions from SN 1987A**
  GR & D. Seckel, PRL 60 (1988) 1793
  (Constraining anomalous PNS energy loss from neutrino signal duration)
- **Neutrino Dispersion at Finite Temperature and Density**
  D. Nötzold & GR, Nucl. Phys. B 307 (1988) 924
  (Neutrino dispersion beyond low-E limit, especially CP symmetric medium)

Flavor Evolution in Astrophysics

- **Multi-messenger astronomy**

  Learn about neutrinos & sources
  Beginning w/ solar & atm neutrinos, detection of flavor oscillations, 2 Nobel Prizes

- **Core-collapse SNe, neutron-star mergers**

  Flavor & lepton number transport
  3-flavor (collective) oscillation problem w/ collisions, thus far unresolved
  Explosion physics, nucleosynthesis, detectable neutrino signal, …

- **Neutrino decoupling around BBN**

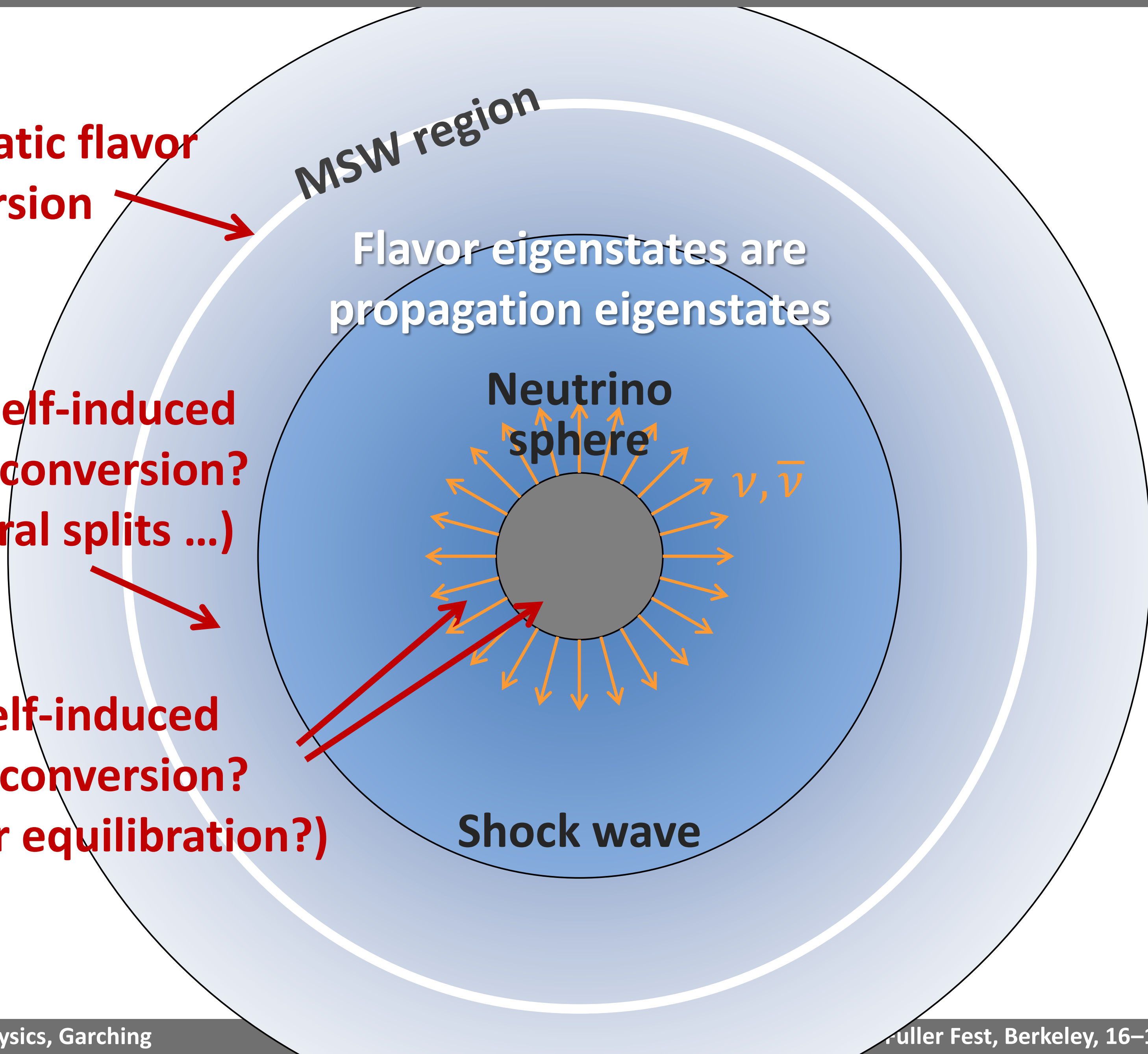
  Prediction of Neff, constraints on neutrino degeneracy
  First study of collective flavor evolution (Kostelecký, Pantaleone & Samuel 1993)
  For latest see Froustey & Pitrou, arXiv:2405.06509, and references

- **Flavored Leptogenesis**

  Lepton asymmetry from heavy Majorana neutrino decay, flavor effects
  Different kinetic equations from SN physics, nearly disconnected literature
  eg Davidson, Nardi & Nir, arXiv:0802.2962, Dev+, arXiv:1711.02861

Flavor Conversion in Core-Collapse Supernovae

- **Adiabatic flavor conversion**
- **Slow self-induced flavor conversion? (Spectral splits …)**
- **Fast self-induced flavor conversion? (Flavor equilibration?)**



Kinetic Equation for Neutrino Transport

Flavor-dependent phase-space densities (occupation number matrices)

$$\varrho = \begin{pmatrix} f_{\nu_e} & f_{\langle\nu_e|\nu_\mu\rangle} & f_{\langle\nu_e|\nu_\tau\rangle} \\ f_{\langle\nu_\mu|\nu_e\rangle} & f_{\nu_\mu} & f_{\langle\nu_\mu|\nu_\tau\rangle} \\ f_{\langle\nu_\tau|\nu_e\rangle} & f_{\langle\nu_\tau|\nu_\mu\rangle} & f_{\nu_\tau} \end{pmatrix}$$

Diagonal: Usual occupation numbers
Off-diag: Flavor coherence information

and similar for $\overline{\nu}$

Transport equation

$$\left(\partial_t + \vec{v}\cdot\vec{\nabla}_x - \vec{F}\cdot\vec{\nabla}_p\right)\varrho(t,\vec{x},\vec{p}) = -\mathrm{i}\left[\mathbb{H}(t,\vec{x},\vec{p}),\varrho(t,\vec{x},\vec{p})\right] + \mathbb{C}\left[\varrho(t,\vec{x},\vec{p}),\overline{\varrho}(t,\vec{x},\vec{p})\right]$$

Streaming

Gravitational forces (redshift, deflection)

Flavor oscillations (vacuum, matter, $\nu\nu$)

Collisions

β
- $e^- + p \rightleftharpoons n + \nu_e$
- $e^+ + n \rightleftharpoons p + \bar{\nu}_e$
- $e^- + A \rightleftharpoons \nu_e + A^*$

- $\nu + n, p \rightleftharpoons \nu + n, p$
- $\nu + A \rightleftharpoons \nu + A$
- $\nu + e^\pm \rightleftharpoons \nu + e^\pm$
- $N + N \rightleftharpoons N + N + \nu + \bar{\nu}$
- $e^+ + e^- \rightleftharpoons \nu + \bar{\nu}$
- $\nu_x + \nu_e, \bar{\nu}_e \rightleftharpoons \nu_x + \nu_e, \bar{\nu}_e$
  ($\nu_x = \nu_\mu$, $\bar{\nu}_\mu, \nu_\tau$, or $\bar{\nu}_\tau$)
- $\nu_e + \bar{\nu}_e \rightleftharpoons \nu_{\mu,\tau} + \bar{\nu}_{\mu,\tau}$

Typical approximations in numerical simulations:

- **Reducing 6+1 dimensions (Angular moments, ray-by-ray, …)**
- **No gravitational deflection**
- **No flavor conversion (large matter effect!)**
- **Until recently, no muons**
- **Often 3-species transport: $\nu_e, \overline{\nu}_e, \nu_x$**

Flavor Conversion as a Spin Precession

- **Vacuum evolution of flavor states**

$$i\partial_t \begin{pmatrix}\nu_e\\ \nu_x\end{pmatrix} = \frac{\mathbb{M}^2}{2E}\begin{pmatrix}\nu_e\\ \nu_x\end{pmatrix}$$

- **Flavor density matrices instead**

$$i\partial_t \varrho = \left[\frac{\mathbb{M}^2}{2E}, \varrho\right]$$

$$\varrho = \begin{pmatrix} f_{\nu_e} & \psi^*_{\nu_e\nu_x} \\ \psi_{\nu_e\nu_x} & f_{\nu_x}\end{pmatrix}$$

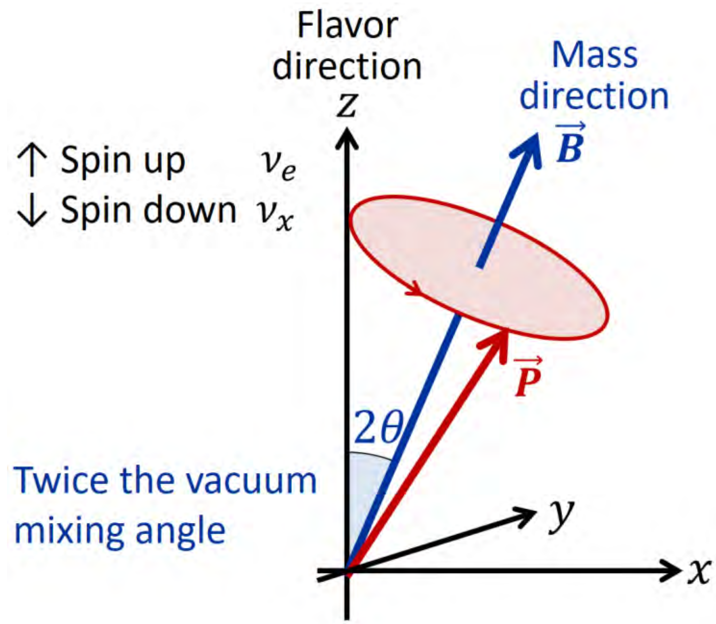
Occupation number

- Field of flavor coherence
- Depends on $(\vec{p}, \vec{r}, t)$
- Can develop unstable flavor waves in the presence of $\nu\nu$ refraction

- **Flavor isospin precession**

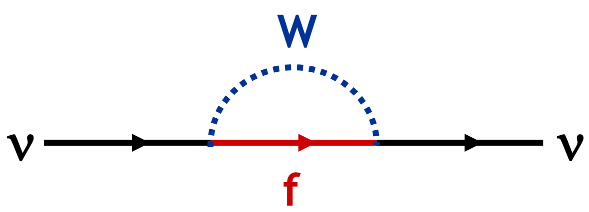
$$\frac{\mathbb{M}^2}{2E} = \frac{\Delta m^2}{2E}\frac{\vec{B}\cdot\vec{\sigma}}{2} \qquad \varrho = \frac{\vec{P}\cdot\vec{\sigma}}{2}$$

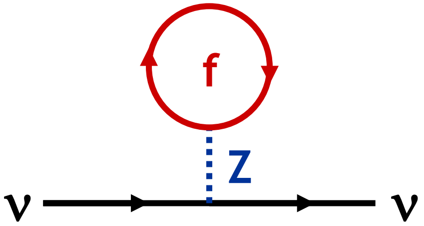
$$\partial_t \vec{P} = \frac{\Delta m^2}{2E}\vec{B}\times\vec{P}$$

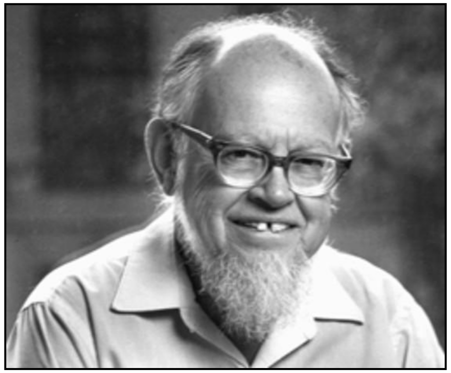


Matter Suppression of Flavor Conversion

Neutrinos in a medium suffer flavor-dependent refraction



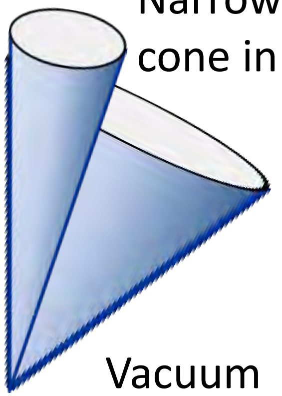




Lincoln Wolfenstein

$$V_{\rm weak} = \sqrt{2}G_{\rm F} \times \begin{cases} N_{\rm e} - N_{\rm n}/2 & \text{for } \nu_{\rm e} \\ -N_{\rm n}/2 & \text{for } \nu_\mu \end{cases}$$

Crucial for flavor oscillations (Wolfenstein 1978, 6000 cites)



Earth: $\rho \approx 5\ {\rm g/cm^3}$ $\quad \Delta V_{\rm weak} \approx 2 \times 10^{-13}\ {\rm eV}$

SN core: $\rho \approx 3 \times 10^{14}\ {\rm g/cm^3}$ $\quad \Delta V_{\rm weak} \approx 10\ {\rm eV}$

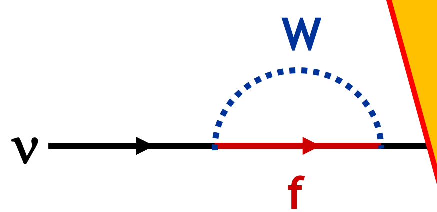
Mass splitting (solar) $\quad \frac{\Delta m^2}{2E} \approx 2 \times 10^{-12}\ {\rm eV}$

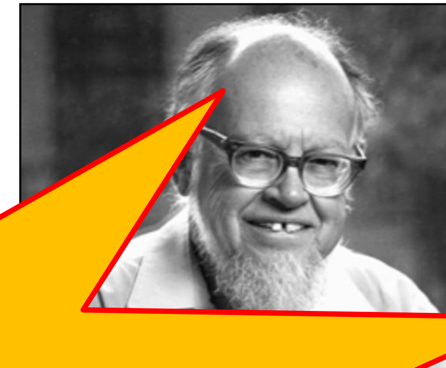
Wolfenstein, *Neutrino oscillations and stellar collapse*, PRD 20 (1979) 2634

"It is shown that even if vacuum neutrino oscillations exist, they are effectively inhibited from occurring in collapsing stars because of the high matter density."

Matter Suppression of Flavor Conversion

Neutrinos in a medium suffer flavor-d… …pendent re… …n





…nstein

ν_W…

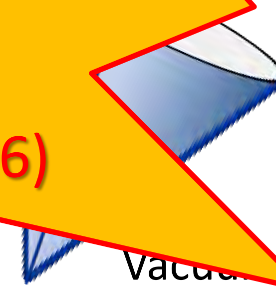
Crucial for fl…

Earth…

SN co…

Mass splittin…

Narrow precession
…ne in medium



That's wrong!

Strong flavor conversion by
nu-nu refraction still possible!
(Duan, Carlson, Fuller, Qian 2005–2006)

Wolfenstein, *Neut… …stell… …ollapse*, PRD 20 (1979) 2634

"It is shown that ev… …acuum …eutrino oscillations exist, they are effectively inhibited from occ… …ng in collapsing stars because of the high matter density."

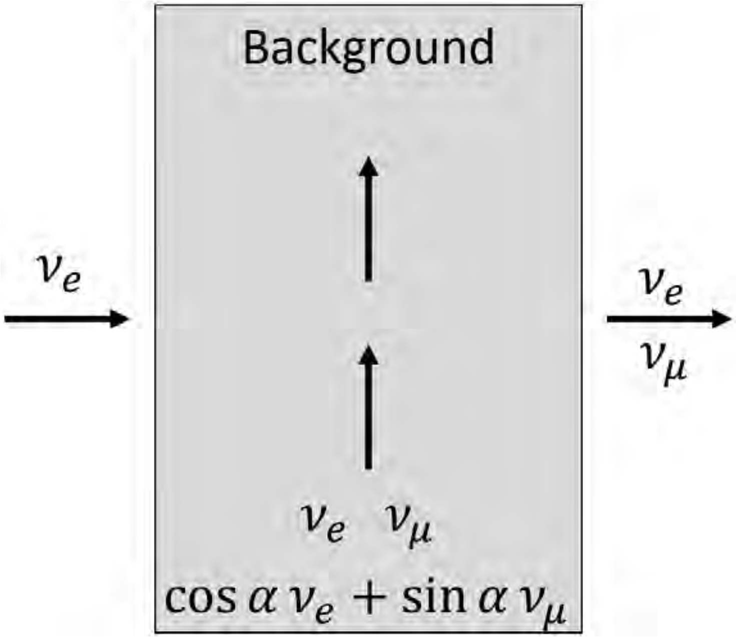
Pair-wise flavor conversion

- **True flavor conversion by neutrino masses**
  - Large mixing angle $\rightarrow$ large flavor conversion
  - Suppressed by matter refraction
  - Trapped electron lepton number remains conserved

- **Collective flavor conversion**
  - Redistributes flavor in neutrino phase space
  - Most importantly: $\boldsymbol{\nu_e \overline{\nu}_e \leftrightarrow \nu_x \overline{\nu}_x}$
  - Happens anyway by "hard" collisions (order G_{F}^2)
  - **Speed up by refraction:**
    Much faster (order G_{F}) by forward scattering

Flavor-off-diagonal neutrino-neutrino refraction

Pantaleone, PLB 287 (1992) 128
Framed as in Friedland & Lunardini, PRD 68 (2003) 013007



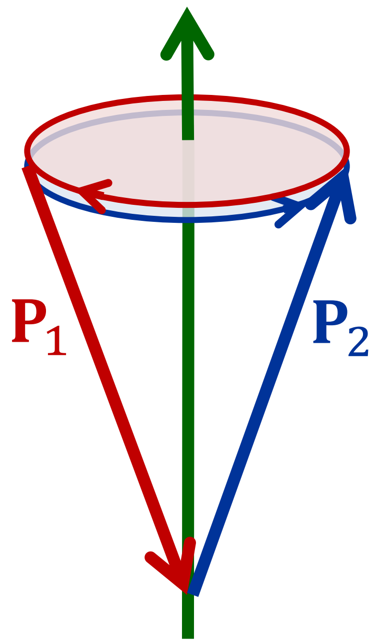
Original ν_e can emerge as ν_μ

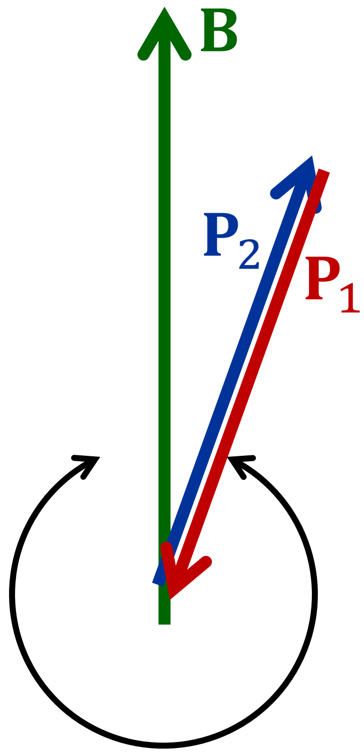
- **By a "hard" collision:**
  - Exchange flavor w/ one ν_μ from background
  - **Maximum effect for ν_μ background**
- **Refractively (forward scattering):**
  - Exchange flavor coherently w/ background
    Entire background "recoils" in flavor space
  - Not possible for statistical background mixture
  - **Maximum effect for coherent background of $(\nu_e + \nu_\mu)/\sqrt{2}$**

- **Background of coherent flavor mixture: flavor off-diagonal weak potential, $\rightarrow$ flavor conversion of other neutrinos, even without vacuum mixing**
- **Neutrino-neutrino refraction $\rightarrow$ self-induced flavor conversion**

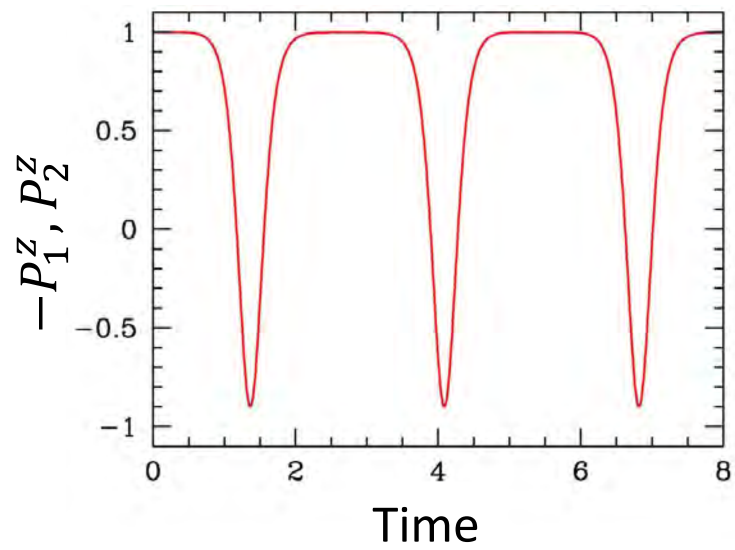
Self-induced flavor instability

Two spins initially with opposite orientation





Even for strong matter effect, large-amplitude flavor oscillations



No interaction:
Free precession
in opposite direction

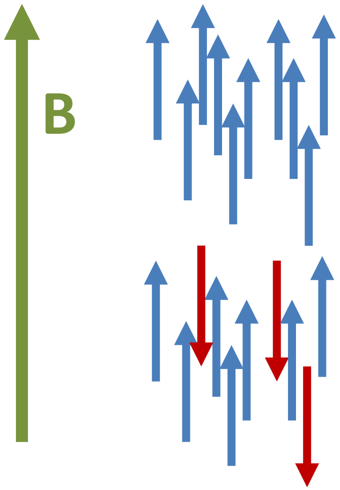
Strong interaction
Pendular motion
Not suppressed by matter effect

Duan, Fuller, Qian, Carlson (2005–2006)

B + two interacting spins
→ Spinning top
(Also 3 spins with different interaction strength: fast flavor pendulum)

Gas of interacting flavor spins

Angular momentum conservation along B (“mass direction”)

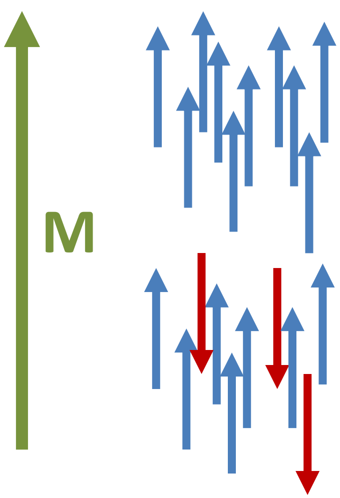


- Nearly aligned
  $\to$ spin waves, no instability

Different initial orientations (“crossed distribution”)
Nus and antinus of same flavor: Opposite (iso)spins
$\to$ Unstable spin waves possible
- Bimodal instability (Samuel 1995, Duan+ 2005)
- Collisional instability (Johns 2021)

$\to$ Leads to equilibration? (ergodic hypothesis – Luke Johns)

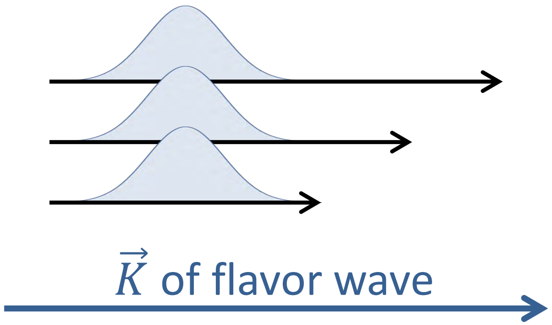
No masses, no mixing, only nu-nu interaction



- “Magnetization M” from all flavor spins $\leftrightarrow$ Role of B
- Nearly aligned $\to$ spin waves, no instability

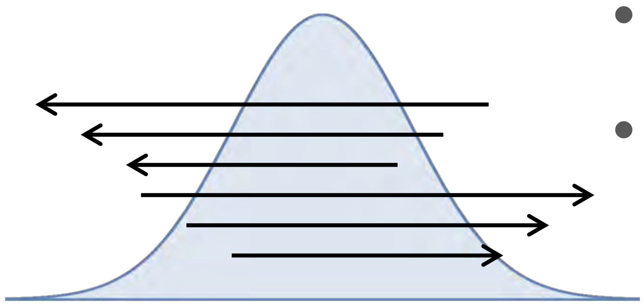
Crossed angle distribution: Instabilities must occur
“Fast flavor instability” (without masses or mixing)
(Sawyer 2005–15, Chakraborty+ 2016, Morinaga 2022)

Non-collective modes:



- Infinitely many neutrino velocity projections on $\vec{K}$
- Each carries along its initial flavor coherence
- Kinematical decoherence of initial wave packet (Does not happen in two-beam model)
- Fast dissipation of any initial wave packet

Collective modes



Wave packet of flavor coherence

- Infinitely many neutrino velocity projections on $\vec{K}$
- Move through wave packet (here taken with vanishing central wave number)
- Wave packet moves in neutrino gas, independently of velocities of neutrino “beams”

Fast flavor waves in analogy to plasma waves

Damiano Fiorillo

Linear quantum kinetic equation for flavor coherence $\psi_{\vec{v}}(t,\vec{r})$

$$\left(\partial_t + \vec{v}\cdot\vec{\nabla}_{\vec{r}}\right)\psi_{\vec{v}} = -i\sqrt{2}G_{\rm F}\int d\vec{v}'(1-\vec{v}\cdot\vec{v}')\left(G_{\vec{v}}\psi_{\vec{v}'} - G_{\vec{v}'}\psi_{\vec{v}}\right)$$

Vlasov operator — Integral over velocity distribution

In Fourier space

$$\left(\omega - \vec{v}\cdot\vec{k}\right)\psi_{\vec{v}} = \cdots$$

Vanishes for Cherenkov condition:

Flavor wave $\left(\omega,\vec{k}\right)$ on resonance with some neutrinos: $\left(\omega - \vec{v}\cdot\vec{k}\right) = 0$

- Landau damping or exponential growth, depending on velocity distribution $G_{\vec{v}}$
- Interaction of collective waves with individual particles
- Interpret dispersion relation in terms of “flavor susceptibility” (system response to applied flavor field)

Fiorillo & Raffelt (2023–2024), long list of papers, eg arXiv:2406.06708

Vlasov vs. Landau picture of plasma waves

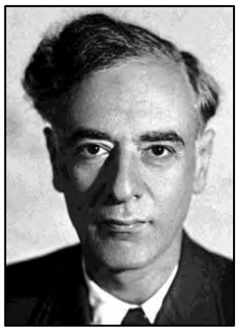
A.A.Vlasov

Kinetic eq. for electron distribution, self-affected by E field of disturbance

$$\left(\partial_t + \vec{v}\cdot\vec{\partial}_{\vec{r}}\right) f_{\vec{v}} = -\frac{e n_e}{m_e}\vec{E}[f]\cdot\vec{\partial}_{\vec{v}} f^0_{\vec{v}}$$

$$1 = -\frac{\omega_P^2}{\vec{k}^2}\int d^3\vec{v}\,\frac{\vec{k}\cdot\vec{\partial}_{\vec{v}} f^0_{\vec{v}}}{\omega - \vec{v}\cdot\vec{k}}$$

Dispersion relation for plasma waves
- Normal modes
- Assume principal value (Vlasov 1945)



L.D.Landau

"This is mostly wrong" (Landau 1946)
Consider causal evolution of initial perturbation
Deal systematically with "Cherenkov singularity"

$$1 = -\frac{\omega_P^2}{\vec{k}^2}\int d^3\vec{v}\,\frac{\vec{k}\cdot\vec{\partial}_{\vec{v}} f^0_{\vec{v}}}{\omega - \vec{v}\cdot\vec{k} + i\epsilon}$$

Physical waves are "Landau damped"
Interaction with individual electrons

N.v.Kampen

Any initial disturbance can be expanded in normal modes
- Collective modes (plasma waves)
- Non-collective modes with singular wave functions "Case–van Kampen modes" (~ individual particles)

(N. van Kampen 1955, K.M. Case 1959)
Landau damping ~ de-phasing of CvK modes

Different Bubbles for Collective Flavor Evolution

Boltzmann-like quantum kinetic transport equation

Connections to Plasma Physics

Exact Many Body (Entanglements, but few particles in small boxes, no transport)

Thermodynamics of mixed nus

Physics-informed parametric implementation in numerical codes

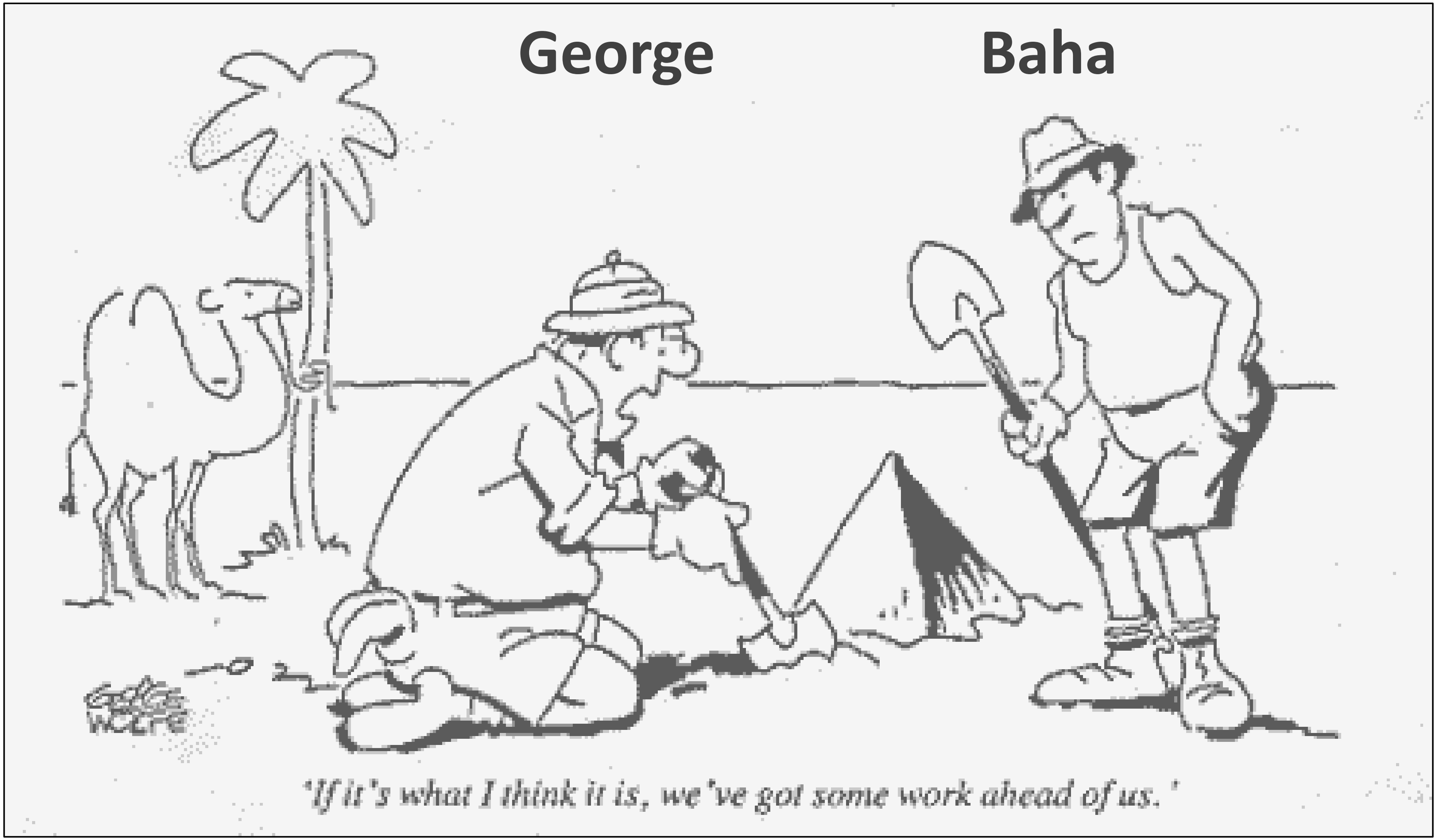
A problem worthy of attack proves its worth by fighting back

Old Adage
(Paul Erdos?, Piet Hein?)

Many Open Questions ... It is Only the Beginning

'If it's what I think it is, we've got some work ahead of us.'

Many Open Questions ... It is Only the Beginning



Short History of Collective Neutrino Oscillations

1992–1993 Off-diagonal refraction (Pantaleone 1992),
QKEs (Dolgov 1981, Rudzsky 1990, Sigl & Raffelt 1993)

1993–2005 Self-induced coherence, early universe (Samuel 1993)
Homogeneous systems evolving in time
But spontaneous breaking of homogeneity

2005–2015 Supernova neutrinos, bulb model: **static solutions evolving in space**
(Duan, Fuller, Qian 2005, Duan, Fuller, Carlson, Qian 2006)
Looking for signatures (spectral splits, ...)
But many symmetries ... get spontaneousy broken

2015–today **Full space-time problem** (everything done before is wrong?)
Time-dependent solutions (Abbar, Duan 2015, Dasgupta, Mirizzi 2015)
Fast flavor conversion (Sawyer 2015)
Dispersion relation in linear regime (Izaguirre, Raffelt, Tamborra 2017)
Similar to plasma physics (Capozzi+ 2017, Fiorillo & Raffelt 2024)

FFC local effect? Connection to slow oscillations?
Leads to equilibrium, even thermalization?
Effective implementation in SN codes?