Collective Neutrino Oscillations

From spectral swaps/splits to flavor equilibration

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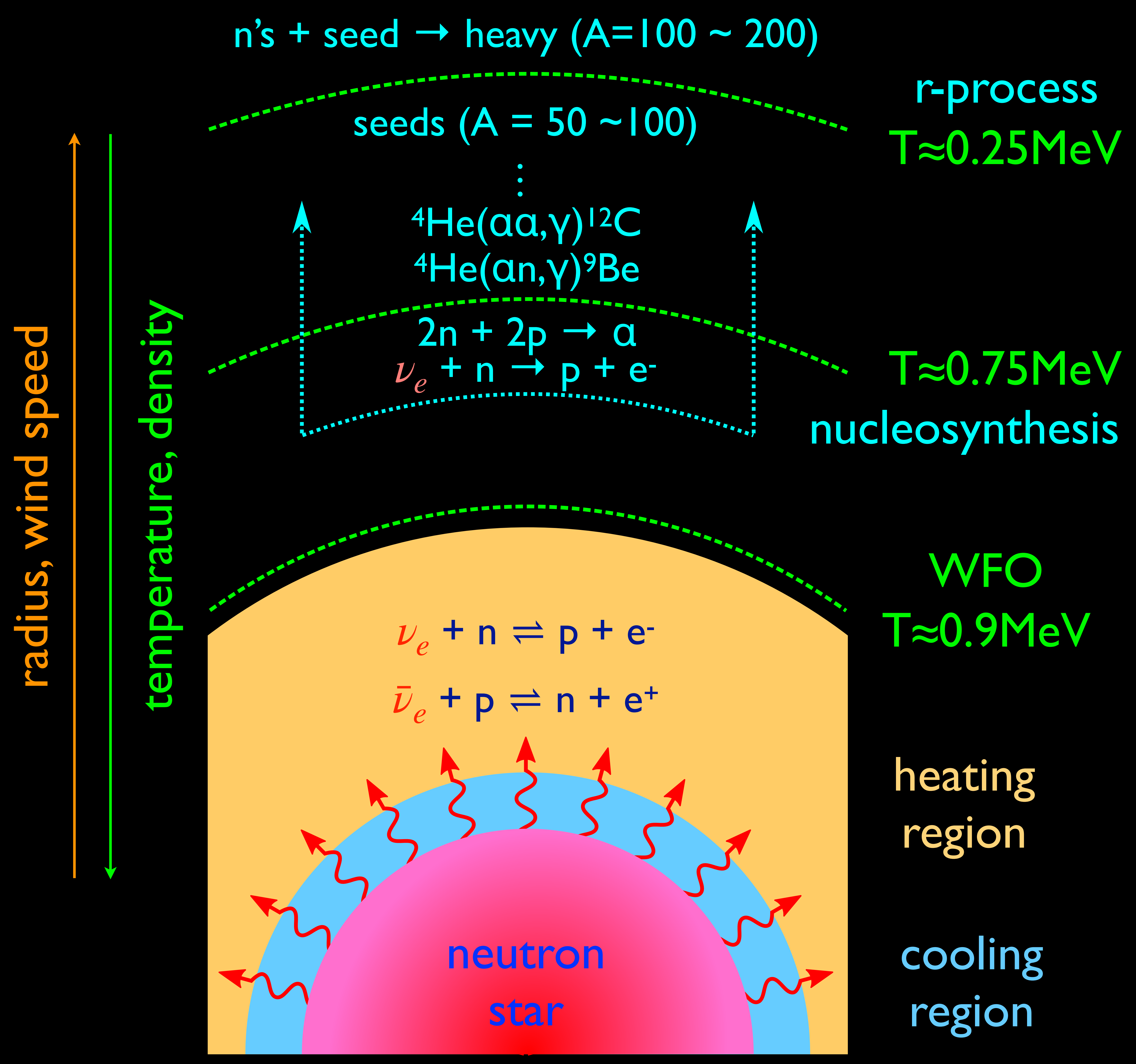
Neutrinos in Physics and Astrophysics

U.S. DEPARTMENT OF ENERGY | Office of Science

Neutrinos in Supernova



n’s + seed → heavy (A=100 ~ 200)

r-process
$T \approx 0.25$MeV

seeds (A = 50 ~100)

⋮

$^{4}He(\alpha\alpha,\gamma)^{12}C$
$^{4}He(\alpha n,\gamma)^{9}Be$

$2n + 2p \rightarrow \alpha$
$\nu_e + n \rightarrow p + e^-$

$T \approx 0.75$MeV
nucleosynthesis

WFO
$T \approx 0.9$MeV

$\nu_e + n \rightleftharpoons p + e^-$
$\bar{\nu}_e + p \rightleftharpoons n + e^+$

heating region

neutron star

cooling region

radius, wind speed

temperature, density

- $\sim 10^{46}$ joules, 10^{58} neutrinos in ~ 10 seconds
- Dominate energetics
- Influence nucleosynthesis
- Probe supernova physics

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RESONANT NEUTRINO OSCILLATIONS AND STELLAR COLLAPSE

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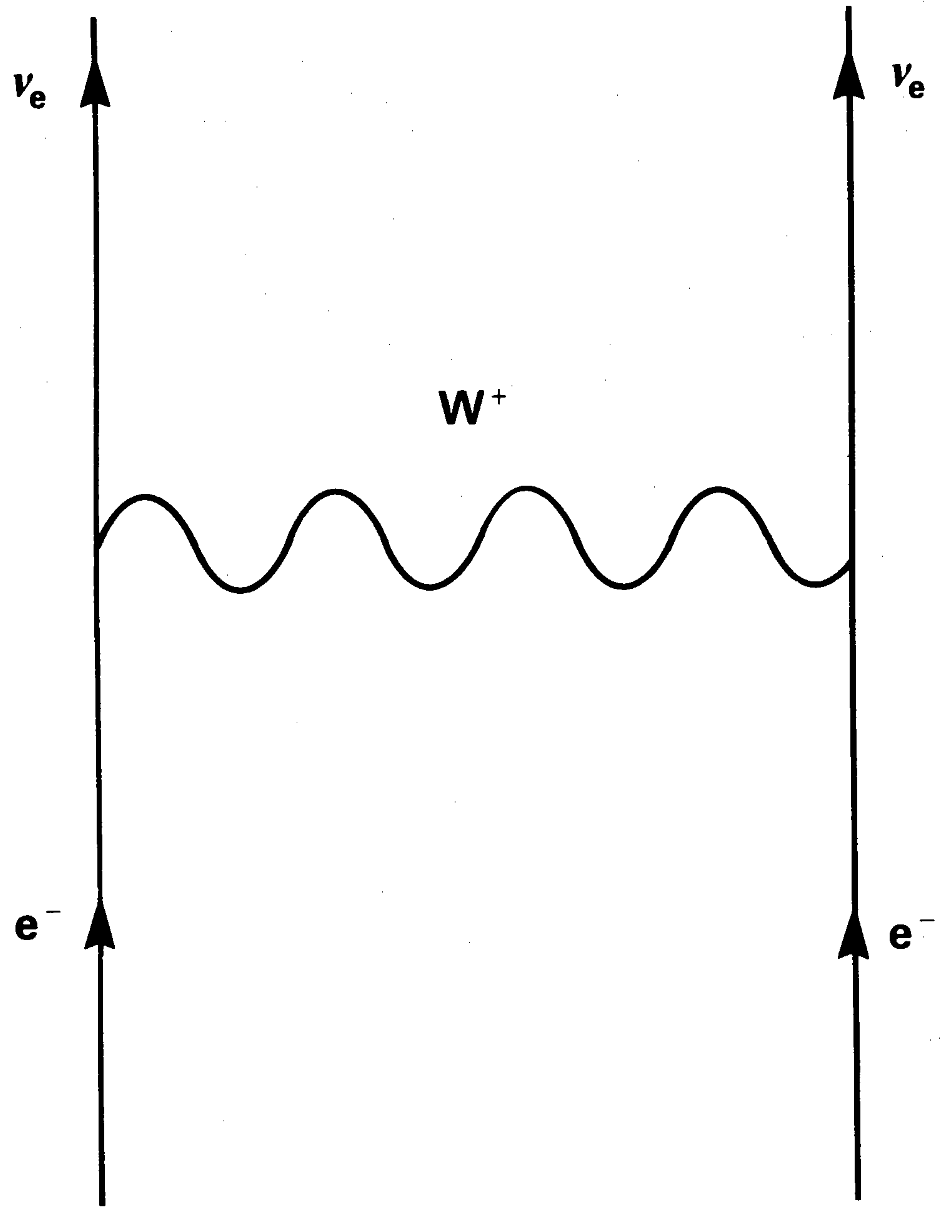
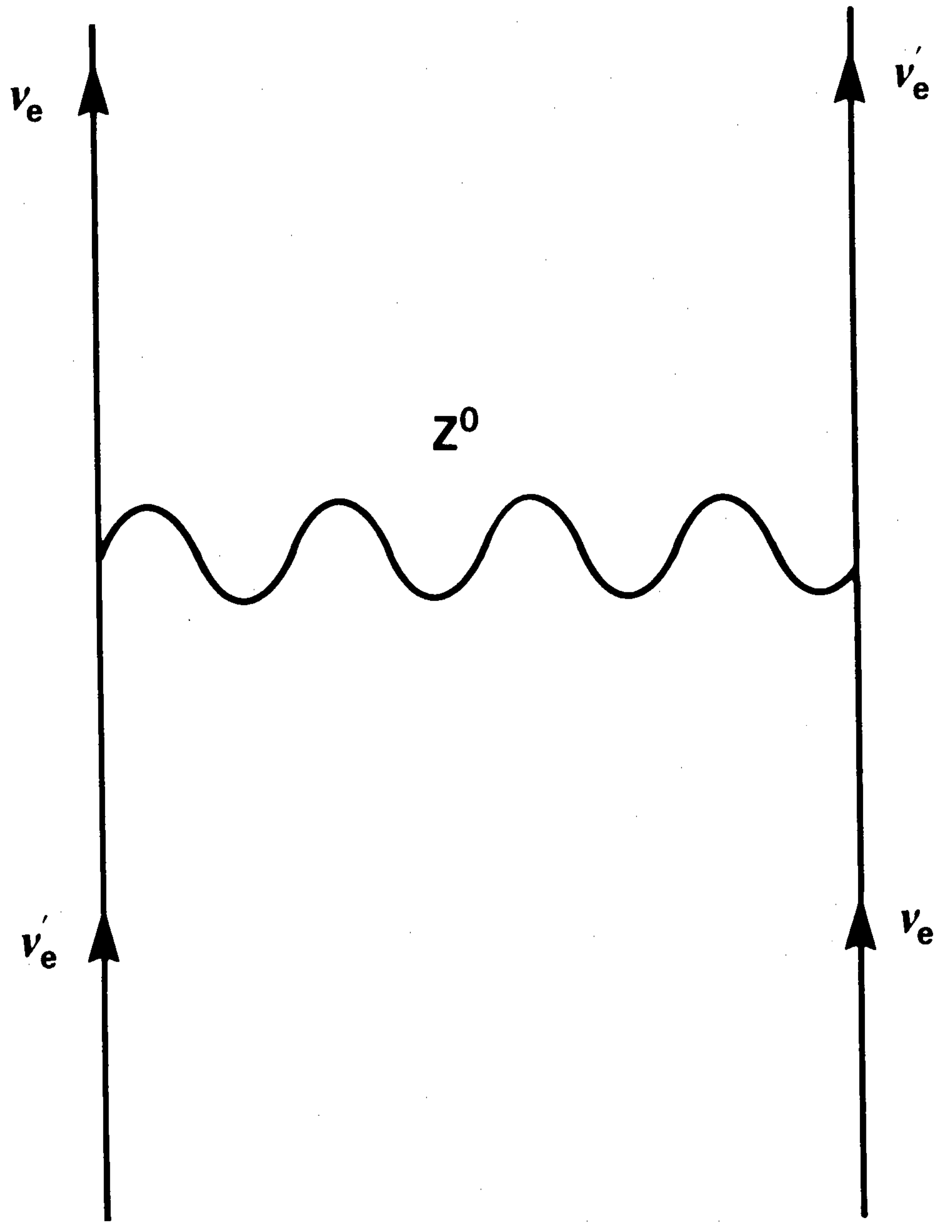
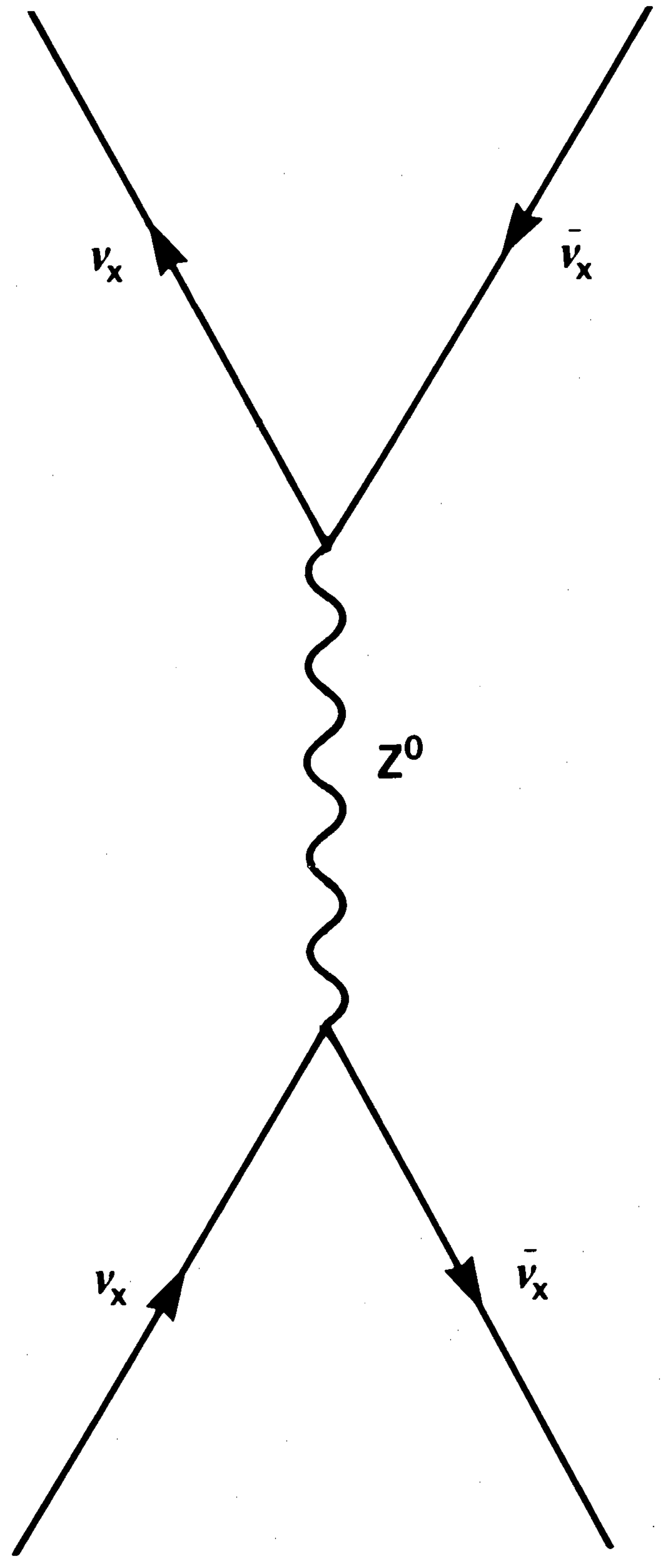


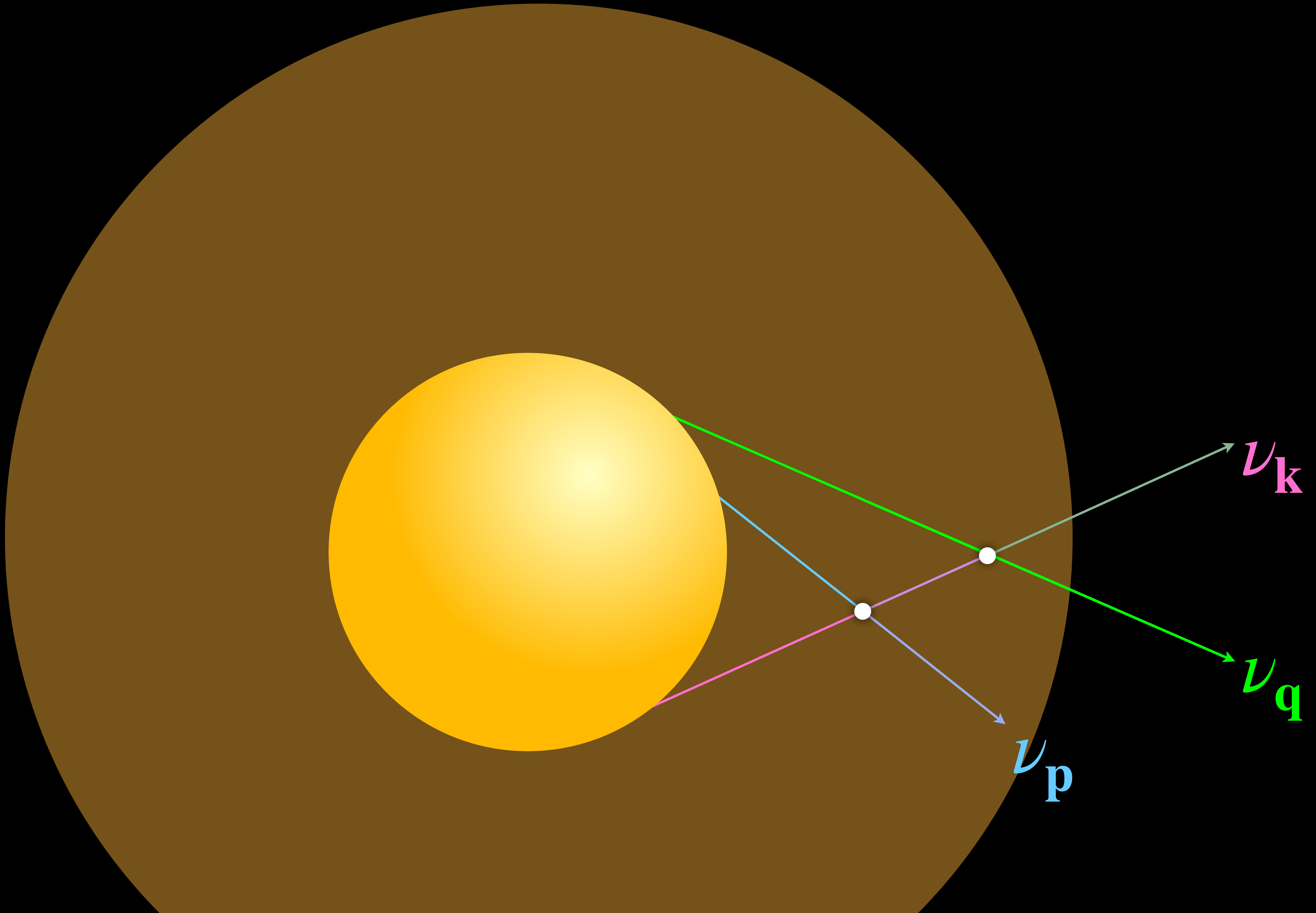




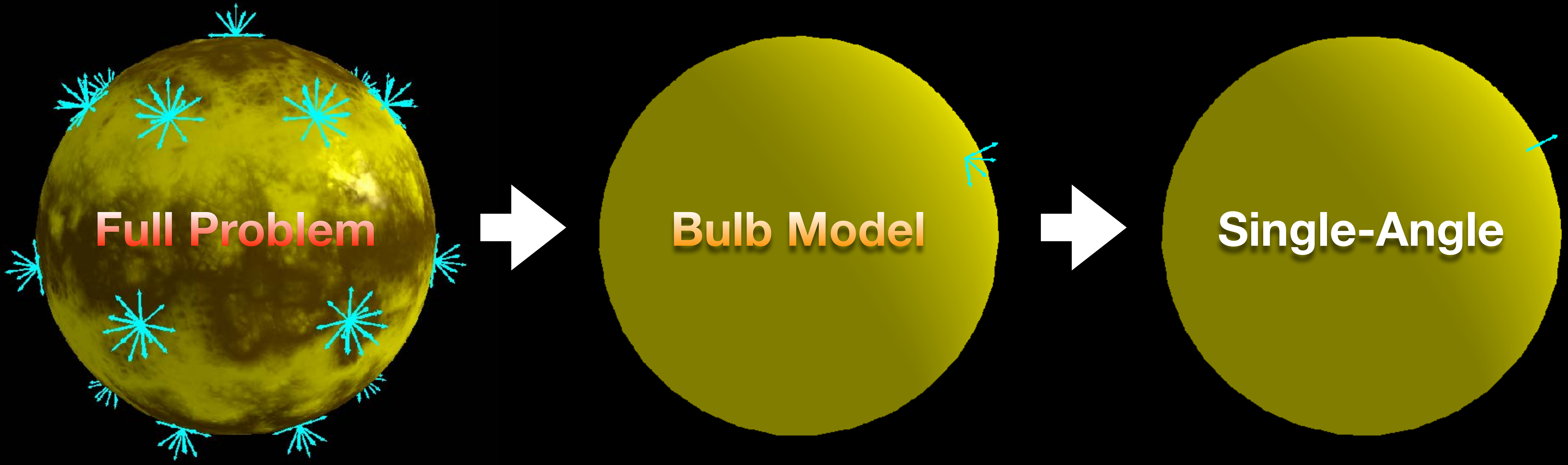
Some Early Works

- MSW-like refraction of the supernova neutrino medium (Fuller+ ’87, Nötzold & Raffelt ’88, Fuller+ ’92, Qian+ ’95, …)
- Flavor coherence term in the neutrino potential (Pantaleone ’92)
- Quantum kinetic equation for neutrino transport (Sigl & Raffelt ’93, Strack & Burrows ’05,…)
- Validity of the single-particle picture (Friedland & Lunardini ’03, Bell +’03, …)
- Bimodal neutrino oscillations with the inverted mass ordering (Kostelecký & Samuel ’93, …)
- Synchronized flavor oscillation and implications (Pastor+ ’02, Dolgov ’02, Abazajian+ ’02, Pastor & Raffelt ’02, Balantekin & Yüksel ’05, …)

Neutrino Oscillations in Supernova



Neutrino Oscillations in Supernova

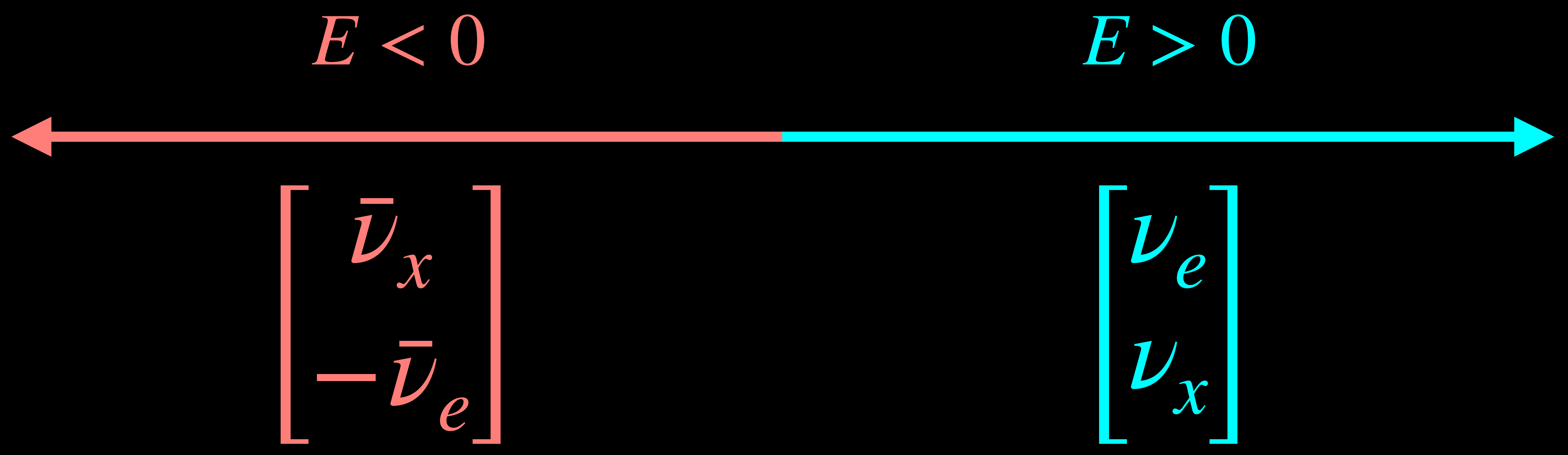


Full Problem → Bulb Model → Single-Angle

Neutrino Flavor (Iso)Spin (NFIS)

Homogeneous and Isotropic Neutrino Gas

$$\mathbf{s} = \psi^{\dagger}\left(\frac{\hat{\boldsymbol{\sigma}}}{2}\right)\psi$$



$E < 0$ $\qquad$ $E > 0$

$$\begin{bmatrix}\bar{\nu}_x \\ -\bar{\nu}_e\end{bmatrix} \qquad \begin{bmatrix}\nu_e \\ \nu_x\end{bmatrix}$$

$$\omega = \frac{\Delta m^2}{2E}$$

Total NFIS

$$\mathbf{S} = \sum_E n(E)\,\mathbf{s}_E$$

$$\dot{\mathbf{s}}_E = (\omega\mathbf{B} + \mu\mathbf{S}) \times \mathbf{s}_E$$

rotating frame $\omega \to \omega'$

$$\Big\downarrow$$

$$\dot{\mathbf{s}}_E = (\omega'\mathbf{B} + \mu\mathbf{S}) \times \mathbf{s}_E$$

Duan, Fuller & Qian (2005)

Matter “Doesn’t Matter”

Homogeneous and Isotropic Neutrino Gas

$$\omega = \frac{\Delta m^2}{2E}$$

$$\lambda = \sqrt{2} G_{\mathrm{F}} n_e$$

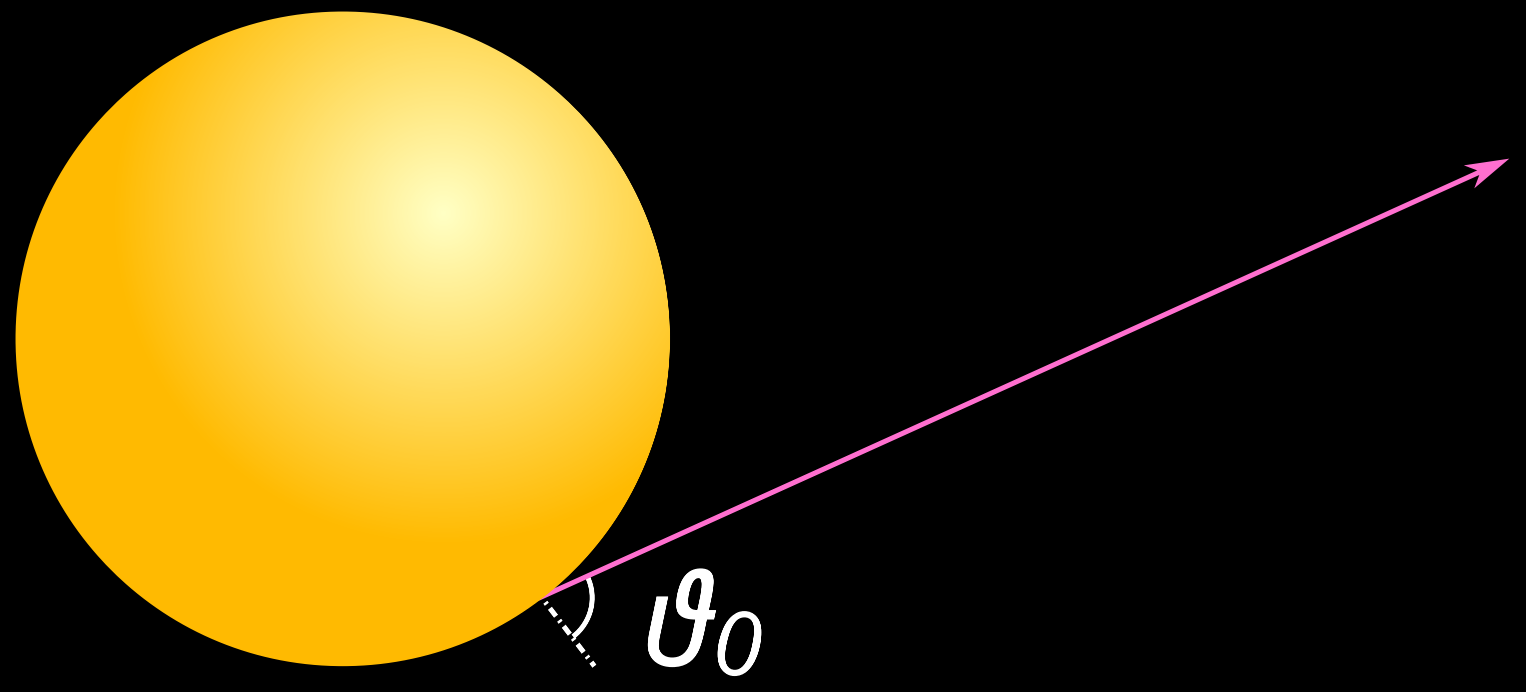
$$\mathbf{S} = \sum_E n(E)\, \mathbf{s}_E$$

$$\dot{\mathbf{s}}_E = (\omega \mathbf{B} + \lambda \mathbf{L} + \mu \mathbf{S}) \times \mathbf{s}_E$$

rotating frame $\omega_{\mathrm{eff}} + \lambda \to \omega_{\mathrm{eff}}$ ⬇ $\lambda \gg |\omega|$

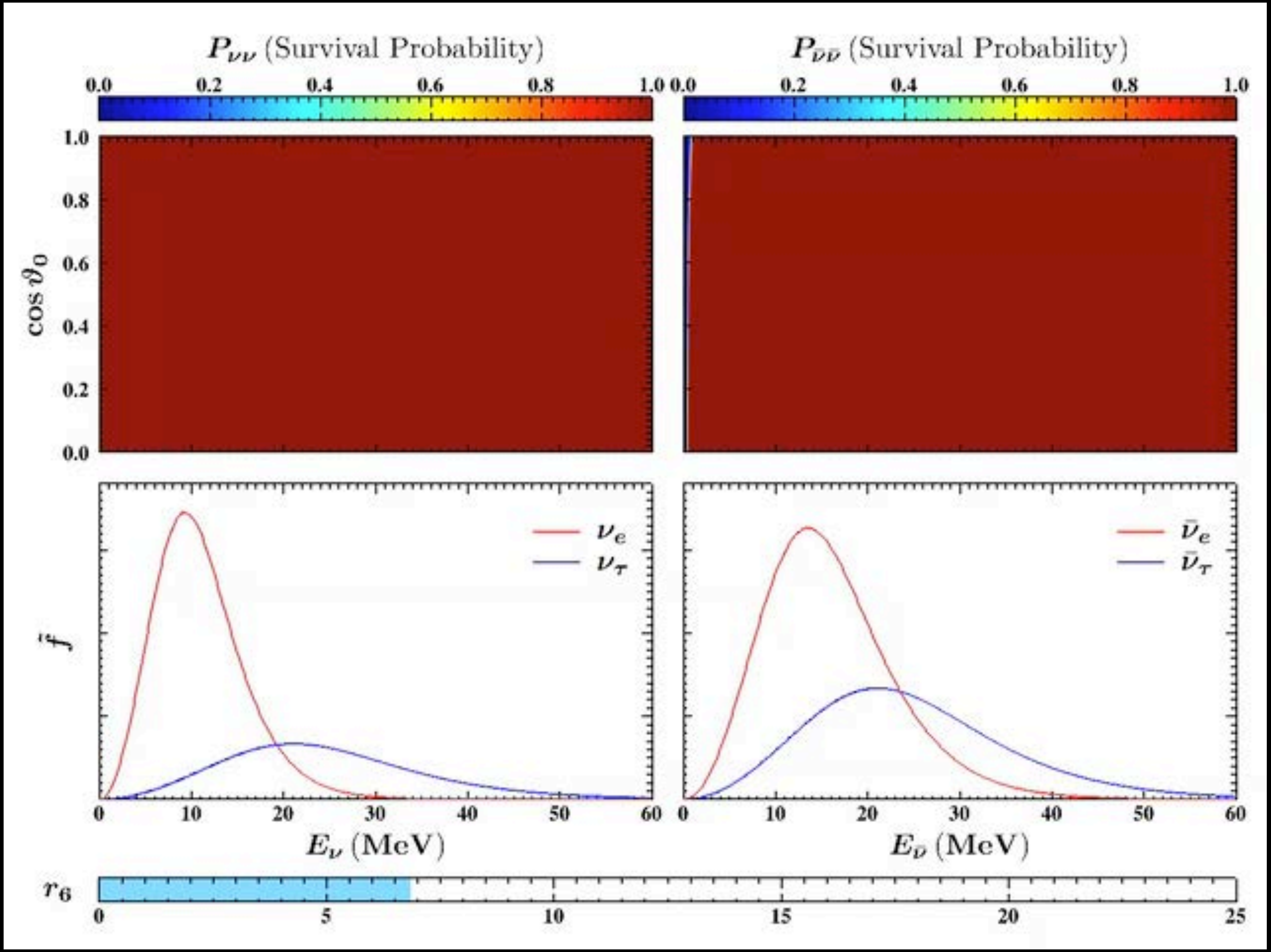
$$\dot{\mathbf{s}}_E \approx (\omega_{\mathrm{eff}} \mathbf{L} + \mu \mathbf{S}) \times \mathbf{s}_E$$

Duan, Fuller & Qian (2005)



The 1st Bulb Model Calculation

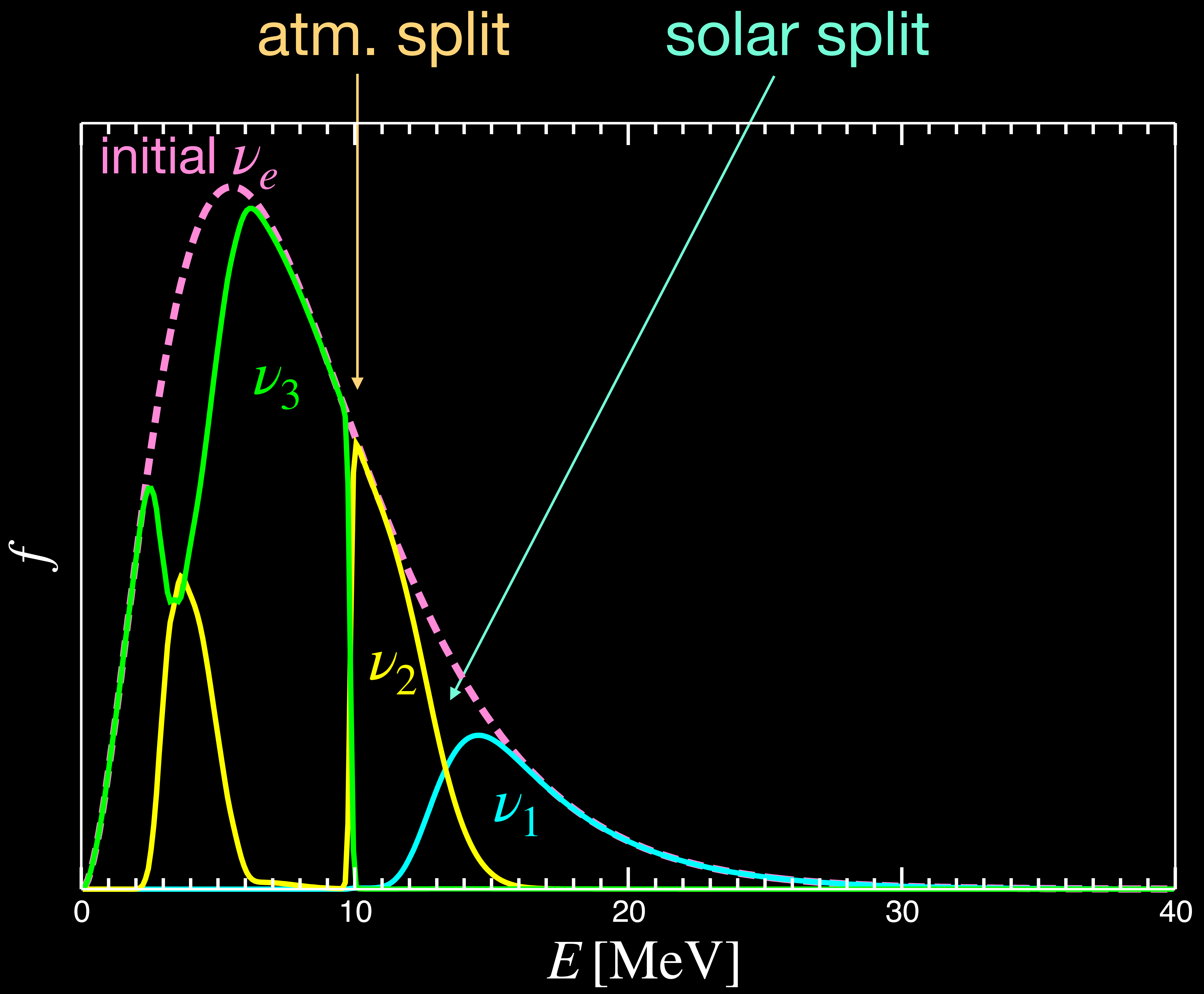
two flavors,
inverted mass
ordering



Duan, Fuller, Carlson & Qian (2006)

Spectral Swaps / Splits

three flavors, single-angle, O-Ne-Mg progenitor, neutralization pulse



Duan, Fuller, Carlson & Qian (2008)

INT PUB 08-01

Stepwise Spectral Swapping with Three Neutrino Flavors

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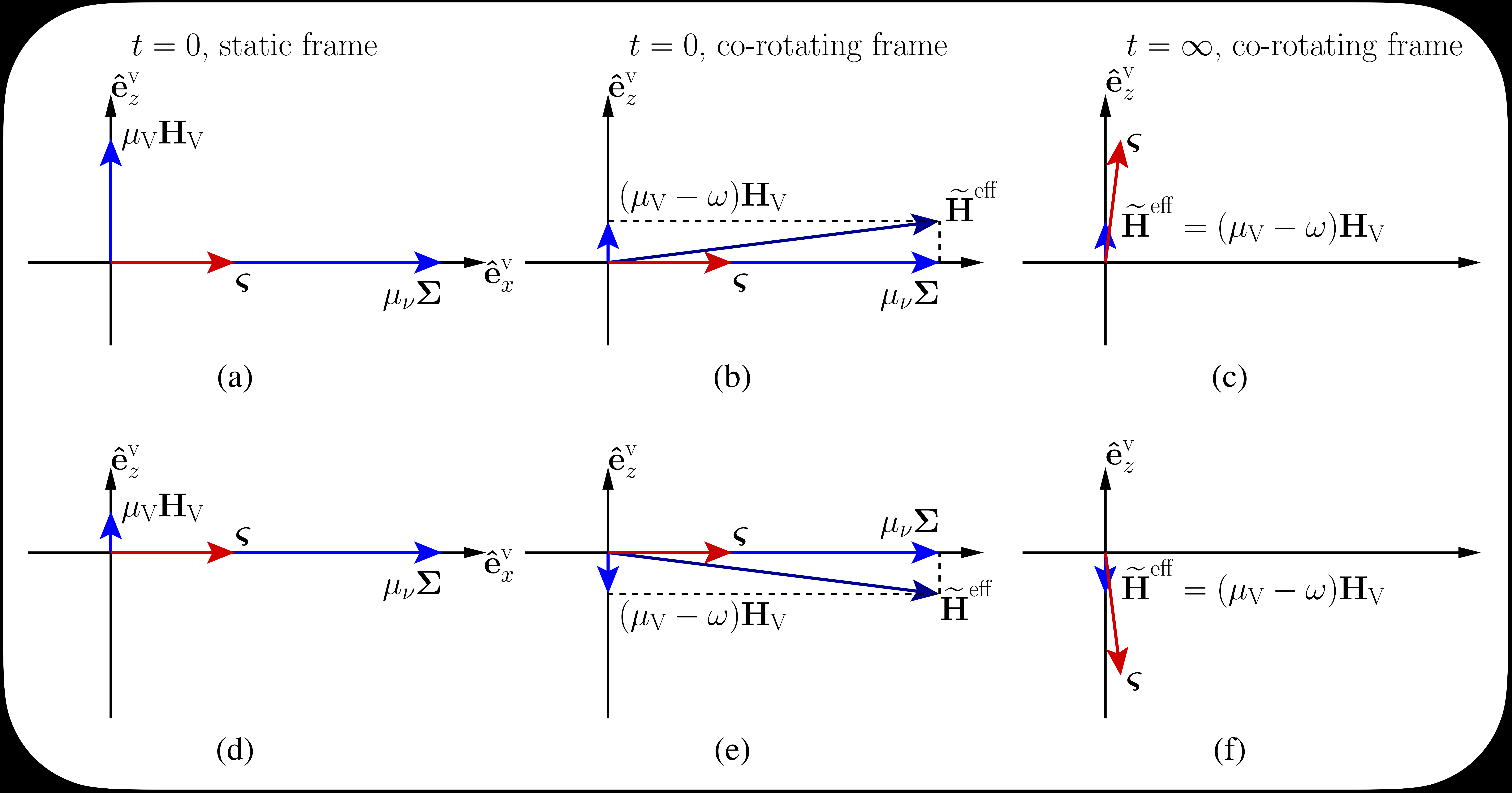
(Dated: September 22, 2008)

We develop a framework for studying collective three-flavor neutrino oscillations based on the density matrix formalism. We show how techniques proven useful for collective two-flavor neutrino oscillations such as corotating frames can be applied readily to three-flavor mixing. Applying two simple assumptions and the conservation of two "lepton numbers" we use this framework to demonstrate how the adiabatic/precession solution emerges. We illustrate with a numerical example how two stepwise spectral swaps appear naturally if the flavor evolution of the neutrino gas can be described by such a solution. For the special case where mu and tau flavor neutrinos are equally mixed and are produced with identical energy spectra and total numbers, we find that one of the spectral swaps in the three-flavor scenario agrees with that in the two-flavor scenario when appropriate mixing parameters are used. Using the corotating frame technique we show how the adiabatic/precession solution can obtain even in the presence of a dominant ordinary matter background. With this solution we can explain why neutrino spectral swapping can be sensitive to deviations from maximal 23-mixing when the "mu-tau" matter term is significant.

Spectral Swaps / Splits



Duan, Fuller, Carlson & Qian (2006); Raffelt & Smirnov (2007)

arXiv:0808.2046v1

INT PUB 08-22

Neutrino Flavor Spin Waves

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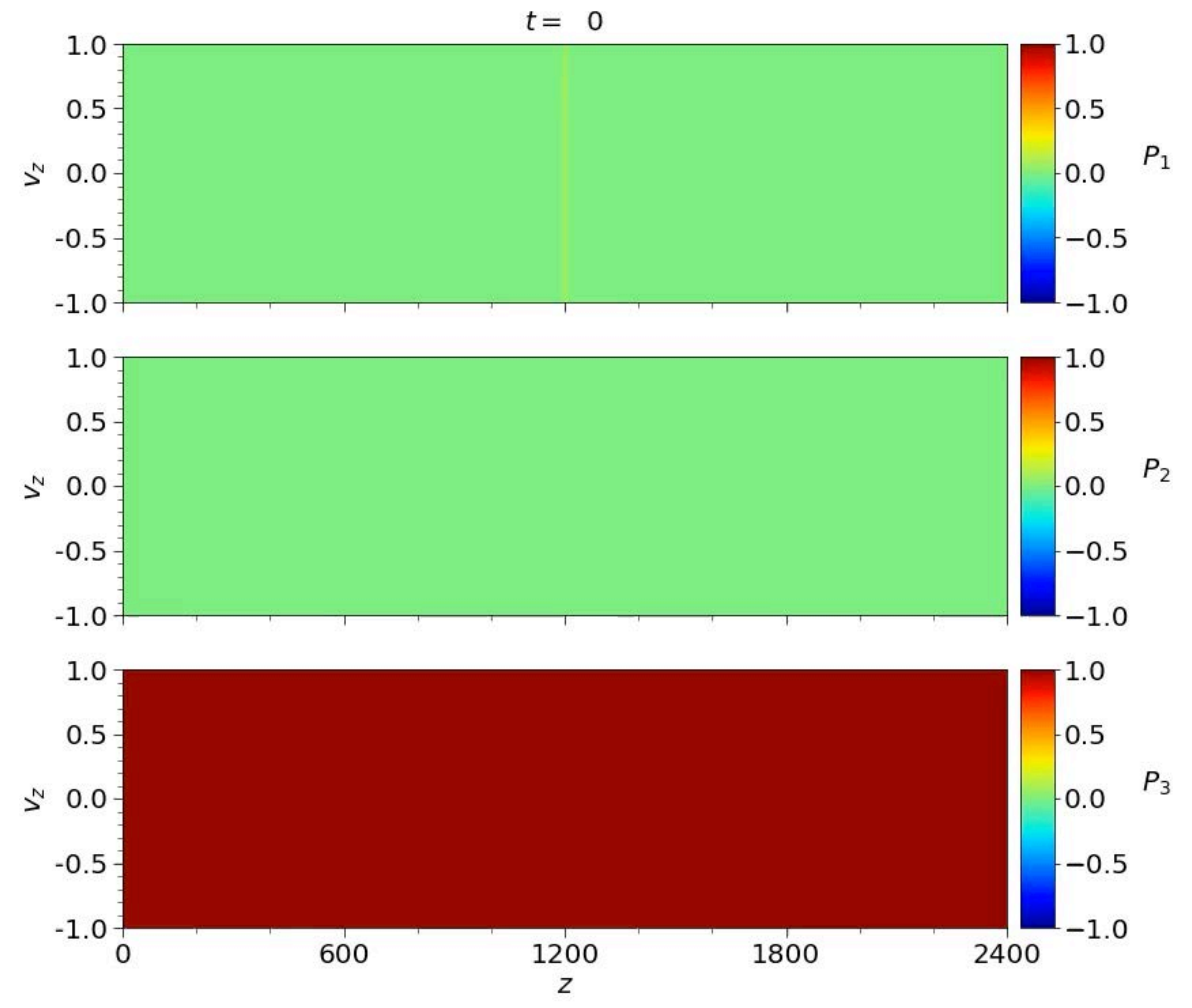
Our calculations reveal a collective neutrino flavor transformation phenomenon in supernovae which is closely akin to spin waves in spin lattices. This "neutrino flavor spin wave", a collective neutrino oscillation mode, can arise in anisotropic neutrino gases because of a symmetry in the equations which govern neutrino flavor evolution. Neutrino flavor transformation with neutrino self-coupling in time-varying, inhomogeneous and anisotropic environments such as supernovae can be described by such flavor spin waves when other non-collective neutrino oscillation modes add up incoherently and average out. We show that the existence of neutrino flavor spin waves in anisotropic environments can explain the stepwise spectral swap (spectral split) phenomenon found in numerical simulations of neutrino flavor transformation in supernovae.

Flavor Oscillation Wave

two flavors,
1D axisymmetric



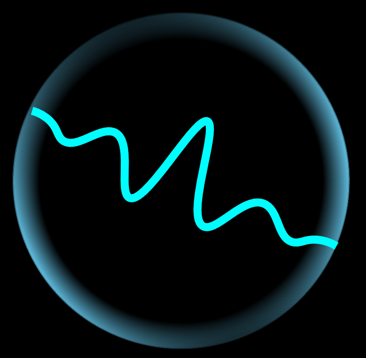
Martin, Yi & Duan (2020)

What about quantum entanglement?

Once-In-a-Lifetime Encounter (OILE) Model

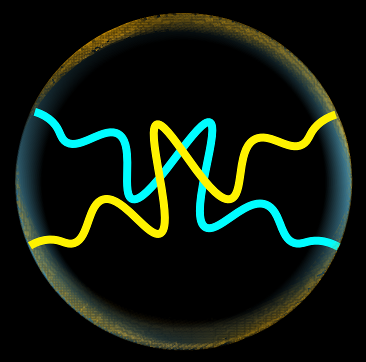
- Only two (or a few) neutrinos interact at each time.
- Each interaction has finite duration Δt.
- None of the neutrinos that participate in the interaction will see each other again.

Once-In-a-Lifetime Encounter (OILE) Model

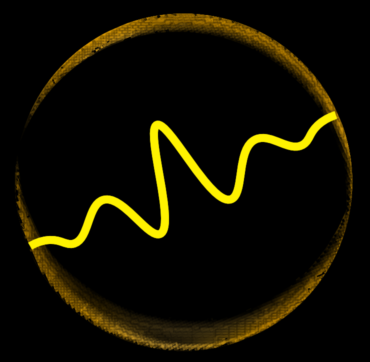


$$\hat{\rho}_2^{\text{after}} = \mathrm{tr}_1(\hat{\rho}_{12}^{\text{after}})$$

$$\hat{\rho}_{12}^{\text{before}} = \hat{\rho}_1^{\text{before}} \otimes \hat{\rho}_2^{\text{before}}$$



$$\hat{\rho}_{12}^{\text{after}} = e^{-\mathrm{i}\hat{H}\Delta t}\,\hat{\rho}_{12}^{\text{before}}\,e^{\mathrm{i}\hat{H}\Delta t}$$



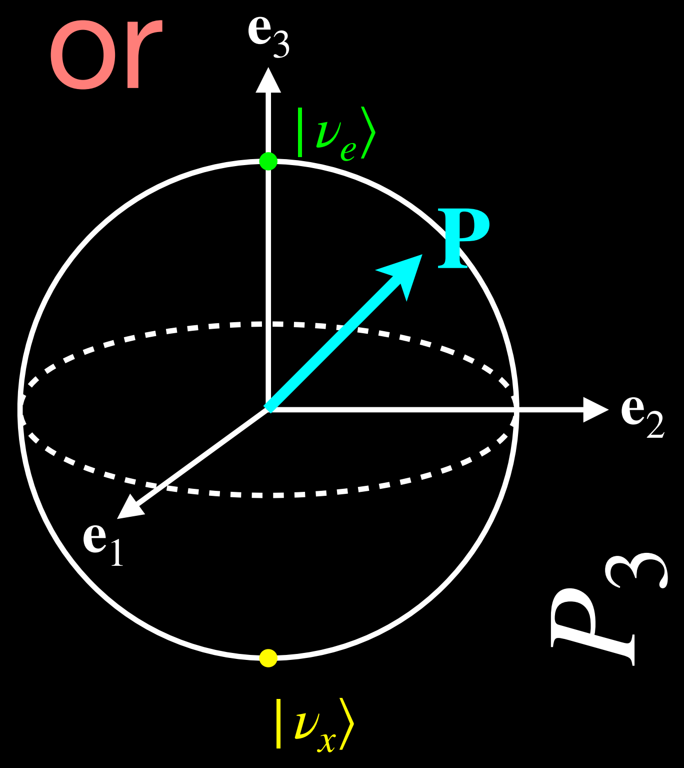
$$\hat{\rho}_1^{\text{after}} = \mathrm{tr}_2(\hat{\rho}_{12}^{\text{after}})$$

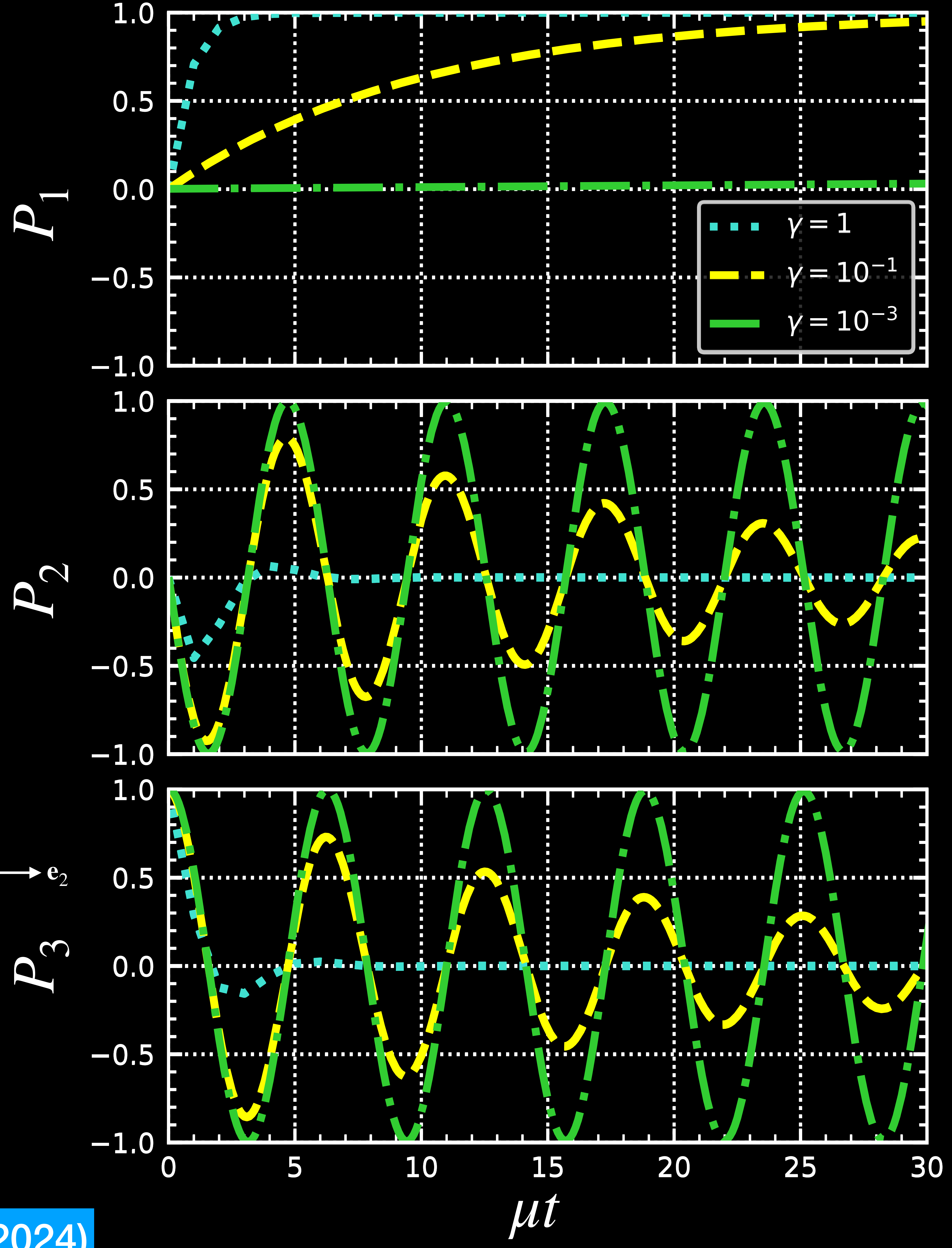
$$\hat{H}_{\nu\nu} = \frac{\sqrt{2}G_{\mathrm{F}}}{V}(1-\vec{v}_1 \cdot \vec{v}_2) \sum_{i=1,2,3} \hat{\sigma}_i \otimes \hat{\sigma}_i$$

OILE Model

Constant Background

- A neutrino passing through a uniform background medium with one encounter at each step of duration Δt.
- $|\psi_{\rm bg}\rangle = (|\nu_e\rangle + |\nu_x\rangle)/\sqrt{2}$ or $\mathbf{P}_{\rm bg} = (1,0,0)$, and $|\psi(0)\rangle = |\nu_e\rangle$ or $\mathbf{P}(0) = (0,0,1)$.
- $\mu = \sqrt{2} G_{\rm F}\, n_{\rm bg}$ and $\gamma = \mu \Delta t$.
- Refraction induced by distinct short $\nu\nu$ scatterings.



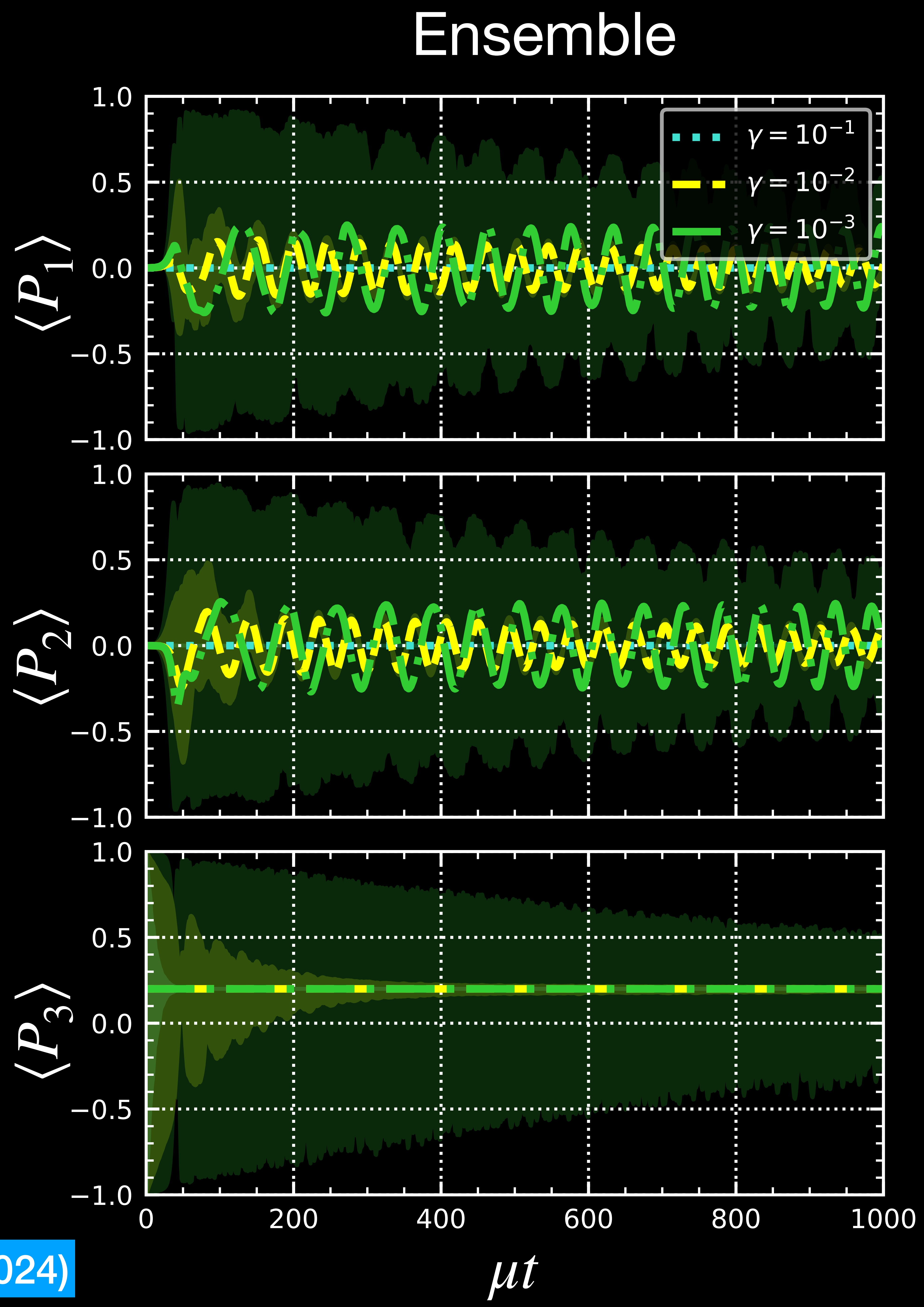


Kost, Johns & Duan (2024)

OILE Model

Random Encounter

- 60 ν_e with $\omega_1 = 0.1\mu$ and 40 ν_x with $\omega_2 = 0.2\mu$ at $t = 0$, all with random $\vec{v}$.
- Flavor equilibration, i.e. $\mathbf{P} \to \langle \mathbf{P} \rangle$, at $t \gg 1/\gamma\mu$.
- Regular precession at intermediate time.



Kost, Johns & Duan (2024)

Summary and Outlook

- Neutrinos can experience collective flavor transformation in super dense environments.
  - A “stellar” collective phenomenon with important consequences.
  - A very rich phenomenon: spectral swaps/splits, flavor oscillation waves, …
- It is possible to incorporate neutrino oscillations into astrophysical simulations if flavor equilibration is reached as a result of
  - the chaotic flavor evolution induced by the non-integrable neutrino-neutrino scattering Hamiltonian at the many-body level, or
  - the fast oscillations at the mean-field level.