Optimizing Ground-Based Gravitational Wave Detectors—Resonant Spheres

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Introduction

Compact Binary Inspiral Gravitational Waves, produced from inspiraling and coalescing binary systems, have been the primary type of GW detection by the Laser Interferometer Gravitational-Wave Observatory (LIGO).

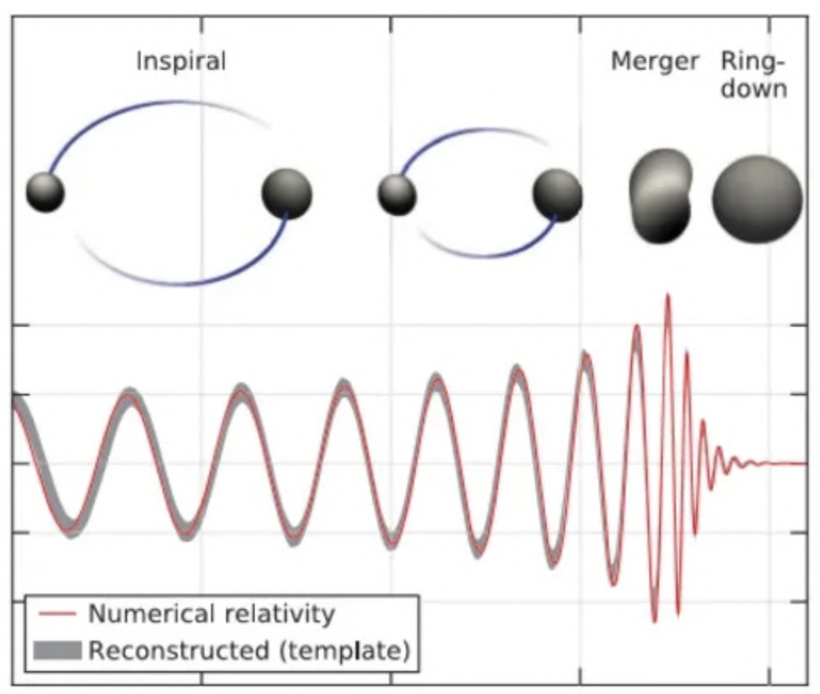

Figure 1. Stages of a Compact Binary Coalescence

LIGO, being optimized to detect such GWs, is most sensitive near ~ 200 Hz. it perfectly captures the signal from the inspiral range right up to the beginning of the binary system's coalescence phase. However, above a certain frequency, the LIGO sensitivity faces significant limitations imposed primarily by photon shot noise, losing the ability to precisely capture signals sourced from a higher frequency regime. Thus, we turn to other ground-based detection methods designed and optimized for higher-frequency detection: resonant spheres.

Spherical Resonant Mass Detectors

An incoming GW will displace the resonant mass detector and with the help of a transducer, convert the displacement into an electrical signal that can be read out.

- Compared to resonant bars, they have relatively large cross sections for GW absorption and therefore increased sensitivity.
- The geometric configuration enables isotropic sensitivities, providing extensive coverage and virtually eliminating blind spots.
- The spherical geometry makes them multi-mode detectors. Using the sphere's different quadrupolar modes, one can reconstruct the arrival direction and polarization of a GW.

Figure 2. MiniGRAIL Resonant Mass Antenna

Resonant mass detectors are limited to a smaller bandwidth than laser interferometers determined by the resonant frequency of the sphere itself. Thus, designing instrumentation within specific parameters is imperative.

Resonant Frequencies of a Hollow Sphere

Initiating the quest for solutions to the sphere's resonant frequencies, w, our approach diverged from employing a solid sphere model to embracing the broader framework of a hollow sphere.

An infinitesimal volume element will be displaced by the action of an external force, such as that from a GW, to a new position $\mathbf{x} + \mathbf{u}(\mathbf{x}, t)$. Under elasticity theory, the dynamics of the displacement, $\mathbf{u}(\mathbf{x}, t)$, are governed by

$$\rho \partial_t^2 \mathbf{U} = (\lambda + \mu)\nabla(\nabla \cdot \mathbf{U}) + \mu \nabla^2 \mathbf{U} + \boldsymbol{f}_{ext} \quad (1)$$

where $\boldsymbol{f}$ is the force per unit volume acting on the elastic body, ρ is the material's density, and λ and μ are the material's characteristic Lamé coefficients.

The ansatz $\mathbf{U}(\mathbf{x}, t) = \mathbf{U}(\mathbf{x})e^{i\omega t}$ is employed where the wavenumber $k = \sqrt{\omega^2 \rho / \mu}$, allowing for the additional value, q, to be defined as $k^2\mu/(\lambda + 2\mu)$ for simplification purposes. After satisfying the boundary conditions at the inner (a) and outer (R) radii of the spherical shell, taking into account that GWs only couple to spherical modes with $l = 2$, we derive a system of equations, represented by the 4×4 matrix $\mathbf{M}$ with the following elements:

$$\begin{aligned}
M_{11} &= q^2(2\mu j_l''(aq) - \lambda j_l(aq)) & M_{31} &= \frac{2\mu(aqj_l'(aq) - j_l(aq))}{a^2} \\
M_{12} &= \frac{2l(l+1)\mu(j_l(ak) - akj_l'(ak))}{a^2} & M_{32} &= -k^2\mu j_l''(ak) - \frac{(l(l+1) - 2\mu j_l(ak))}{a^2} \\
M_{13} &= M_{11}|_{j_l \to y_l} & M_{33} &= M_{31}|_{j_l \to y_l} \\
M_{14} &= M_{12}|_{j_l \to y_l} & M_{34} &= M_{32}|_{j_l \to y_l} \\
M_{2i} &= M_{1i}|_{a \to R} & M_{4i} &= M_{3i}|_{a \to R}
\end{aligned} \quad (2)$$

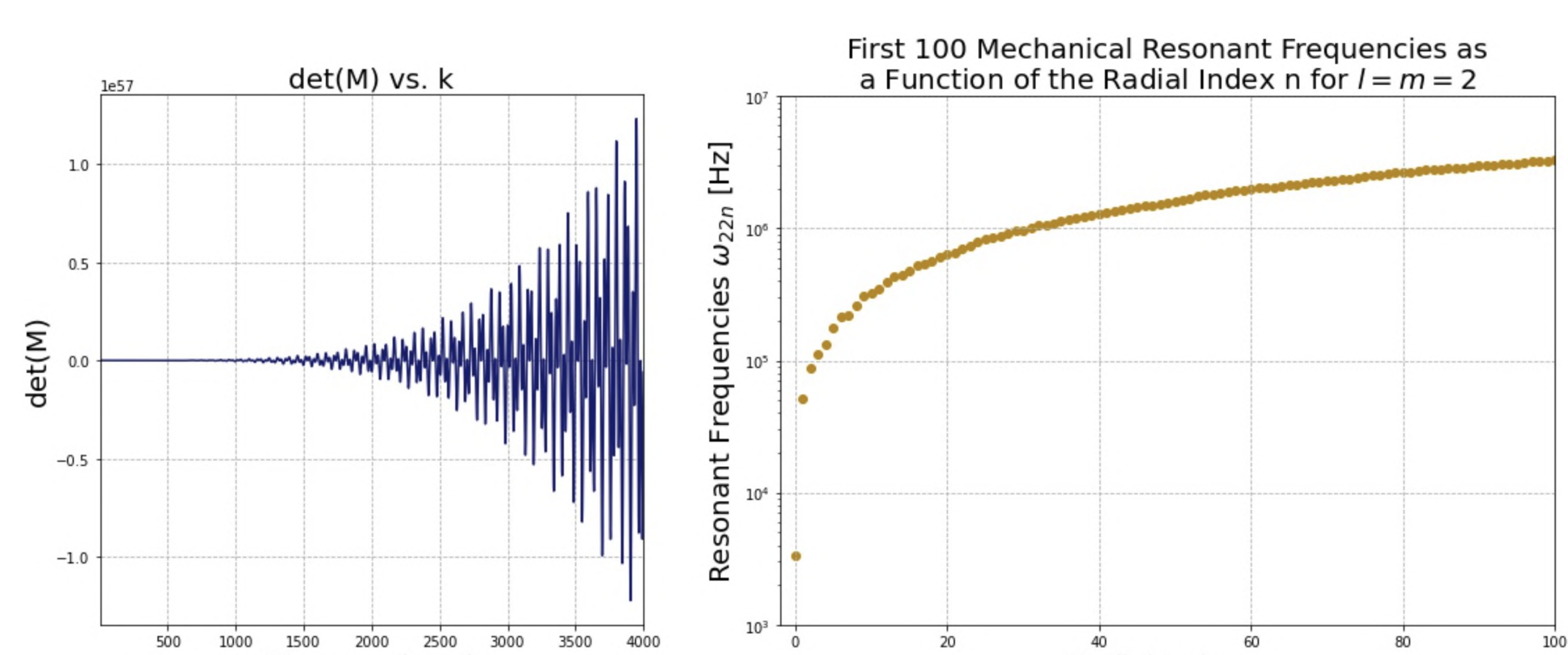

Figure 3. The determinant of $\mathbf{M}$ as a function of its wavenumber k (left) and its first 100 mechanical resonant frequencies (right) for parameters $\mu = 3.75 \times 10^{10}$ Pa, $\lambda = 1.475 \times 10^{11}$ Pa, $\rho = 8570\,\mathrm{kg/m^3}$, $a = 0.40$ m; $R = 0.55$ m, and $l = m = 2$.

By taking the determinant of $\mathbf{M}$ and setting it to zero, we numerically calculated various $k-$values to determine the possible resonant frequencies for a given set of parameters.

Exploring Correlations—Inner and Outer Radii

The strain sensitivity of a detector, a dimensionless quantity describing the minimum detectable change a detector can observe, depends on the resonant frequency. Having the methodology to calculate the resonant frequencies of the detector, we then set out to explore how different parameters correlate to different w-values since they, given the 4×4 matrix, are not exactly obvious.

Beginning with varying the inner and outer radii while holding the other parameters fixed, we solved the determinant of matrix $\mathbf{M}$ and numerically found the first zeros, offering us the lowest resonant frequency for every inner and outer radii pair calculated.

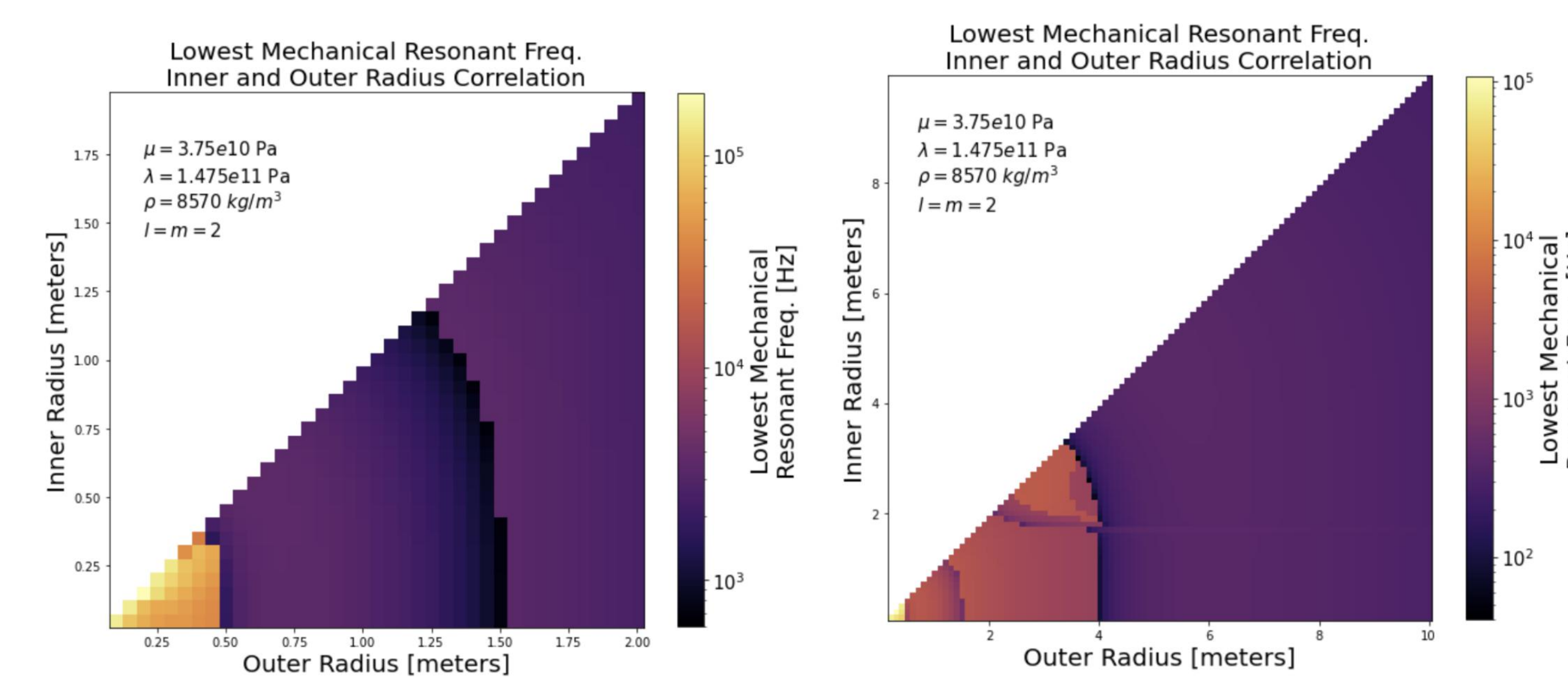

Figure 4. Lowest mechanical resonant frequencies for various a and R combinations.

This proved to be more computationally intensive than expected, and a mapping of the various w for the numerous radii-combination produced intriguing results, for which further inspection and cross-examination are required.

Going Forth

Going forth, our aim is to uncover additional correlations and leverage them to derive precise strain sensitivity spectra for the resonant mass detector, aiming for the utmost sensitivity theoretically attainable. However, it's essential to emphasize that the complexity of the numerical analysis involved necessitates a thorough reassessment of the existing theoretical framework before proceeding further.

Additionally, we are actively exploring various approaches to enhance the performance of the detector's readout component, particularly the transducer element. However, before delving into optimization strategies, it's imperative to deepen our understanding of the underlying apparatus and hardware.

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