Quantum Correlations in Neutrino-dense Environments

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Roadmap

- Introduction to collective neutrino oscillations
- Tools from quantum information science
- Results from quantum many-body calculations

Supernovae: Large ν Sources

- Neutrino luminosity $L_\nu \sim 10^{53}$ ergs/s
- Neutron star temperature $k_B T \sim 10$ MeV

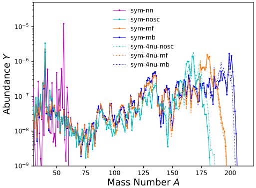
$$\Longrightarrow \sim 10^{58} \text{ neutrinos}$$

- SN envelope: $\ell_{\rm MFP} \gg L_{\rm osc}$ neutrinos evolving coherently;

Large ν Sources & Nucleosynthesis Sites

- Core-collapse SNe, Binary neutron star mergers: sites for nucleosynthesis beyond Fe-56

- Without collective oscillations, expect: $\langle E_{\nu_e} \rangle < \langle E_{\bar{\nu}_e} \rangle < \langle E_{\nu_\mu, \nu_\tau, \bar{\nu}_\mu, \bar{\nu}_\tau} \rangle$



ABB, **MJC**, AVP, RS, XW (2024)

- With collective oscillations: Higher energy $\nu_{\mu,\tau} \to \nu_e \implies$ change n/p $\implies$ affect elemental abundances produced

$$\nu_e + n \longleftrightarrow p + e^-$$

$$\bar{\nu}_e + p \longleftrightarrow n + e^+$$

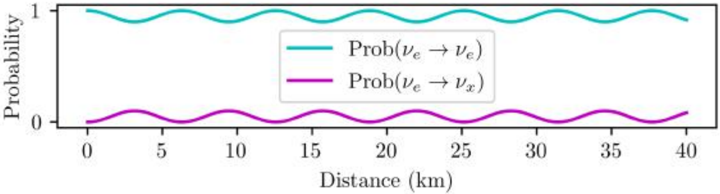
Vacuum Flavor Oscillations

An 1-body Hamiltonian

- Relativistic energy of massive particles:

$$H_\nu = \sum_{\mathbf{p}} (|\mathbf{p}|^2 + m_1^2)^{1/2} a_1^\dagger(\mathbf{p}) a_1(\mathbf{p}) + (|\mathbf{p}|^2 + m_2^2)^{1/2} a_2^\dagger(\mathbf{p}) a_2(\mathbf{p})$$
$$= \sum_{\mathbf{p}} \omega_{\mathbf{p}} \vec{B} \cdot \vec{J}_{\mathbf{p}} + \text{const},$$

where $\omega_{\mathbf{p}} = \dfrac{\Delta m_{21}^2}{2|\mathbf{p}|}$ and $\vec{B} = (0, 0, -1)_{\mathcal{M}} = (\sin 2\theta, 0, -\cos 2\theta)_{\mathcal{F}}$



Geometric Interpretation

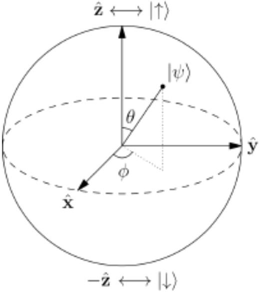
"Polarization" vectors

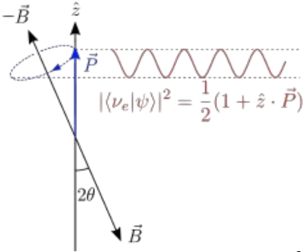
- Define polarization $\vec{P}_{\mathbf{p}} = 2\langle\Psi|\vec{J}_{\mathbf{p}}|\Psi\rangle$
- $\vec{P}_{\mathbf{p}}$: Bloch vector of one neutrino's density $\rho_{\mathbf{p}} = \mathrm{Tr}_{\mathbf{q}(\neq\mathbf{p})}[|\Psi\rangle\langle\Psi|] = \frac{1}{2}(1 + \vec{\sigma}\cdot\vec{P}_{\mathbf{p}})$.
- Non-interacting system: for each ω,

$$\frac{\mathrm{d}}{\mathrm{d}t}\vec{P}_{\mathbf{p}} = \omega_{\mathbf{p}}\vec{B}\times\vec{P}_{\mathbf{p}}$$

Entanglement entropy: $S = -\mathrm{Tr}[\rho\ln\rho]$ inversely related to P

- $P = 1 \iff S = 0$ (Unentangled)
- $P = 0 \iff S = \ln(2)$ (*Maximally*)

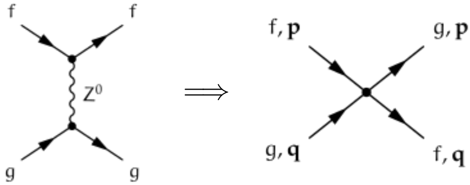




Two-body Hamiltonian

Neutrino-neutrino Interactions

- Low-energy EFT: Z-boson exchange → Fermi 4-point interaction



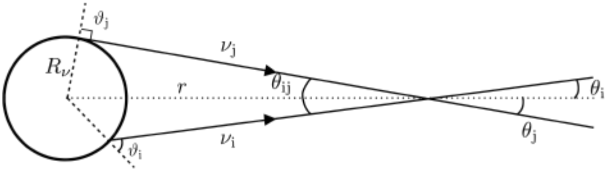
$$H_{\nu\nu} = \frac{\sqrt{2}G_F}{V} \sum_{\mathbf{p},\mathbf{q}} (1 - \widehat{\mathbf{p}} \cdot \widehat{\mathbf{q}}) \sum_{f,g=e,x} a_f^\dagger(\mathbf{p}) a_g(\mathbf{p}) a_g^\dagger(\mathbf{q}) a_f(\mathbf{q})$$

$$= \frac{\sqrt{2}G_F}{V} \sum_{\mathbf{p},\mathbf{q}} (1 - \cos\vartheta_{\mathbf{pq}}) \vec{J}_{\mathbf{p}} \cdot \vec{J}_{\mathbf{q}} + \text{const}$$

Reducing the Two-body Hamiltonian

The "Bulb Model"

- Definite-flavor νs emitted isotropically from spherical surface:



- Make the problem more tractable by averaging over $\theta_{\mathbf{pq}}$;

$$H_{\nu\nu} \approx \frac{\sqrt{2}G_F}{V}\langle 1 - \cos\vartheta_{\mathbf{pq}}\rangle \sum_{\mathbf{p}\neq\mathbf{q}} \vec{J}_{\mathbf{p}} \cdot \vec{J}_{\mathbf{q}}$$

$$= \mu(r) \sum_{\omega,\omega'} \vec{J}_\omega \cdot \vec{J}_{\omega'}, \quad \text{where} \quad \vec{J}_\omega = \sum_{\left\{\mathbf{p}:\omega=\frac{\Delta m^2}{2|\mathbf{p}|}\right\}} \vec{J}_{\mathbf{p}}$$

Hamiltonian, In Summary

Total Hamiltonian

- Hamiltonian for collective neutrino oscillations:

$$H = H_\nu + H_{\nu\nu} \approx \sum_\omega \omega \vec{B} \cdot \vec{J}_\omega + \mu(r) \sum_{\omega,\omega'} \vec{J}_\omega \cdot \vec{J}_{\omega'}$$
$$= \sum_\omega \omega \vec{B} \cdot \vec{J}_\omega + \mu(r) \vec{J} \cdot \vec{J}$$

where

$$\omega = \text{vacuum oscillation frequency}$$
$$\mu(r) = \text{strength of neutrino self-interaction}$$
$$\vec{J} = \sum_\omega \vec{J}_\omega = \text{total isospin op}$$

- What are its eigenvalues and eigenstates?

Proposed solution: Neutrino Flavor Conserved Charges

Richardson-Gaudin Magnets and Bethe ansatz

- N *instantaneously* conserved charges h_ω: $[h_\omega(t), H(t)] = 0$

$$h_\omega = -J^z_\omega + 2\mu \sum_{\omega'(\neq\omega)} \frac{\vec{J}_\omega \cdot \vec{J}_{\omega'}}{\omega - \omega'}$$

- Identities: $\sum_\omega h_\omega = -J^z$, $\sum_\omega \omega h_\omega = H$
- Eigenvalues $(\epsilon_1, \ldots, \epsilon_N)$ uniquely distinguish state
- Related algebraically:

$$h^2_\omega = \mu \sum_{\omega' \neq \omega} \frac{h_{\omega'}}{\omega - \omega'} + c_\omega$$

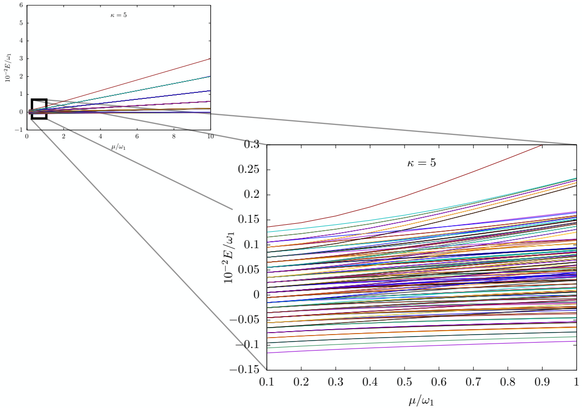
…so their eigenvalues must be, too! (akin to Bethe ansatz)

Bethe Ansatz Results

(Trivial!) Energy Level Crossings

- Many level crossings, even within a total isospin subspace
- *However*, $\epsilon_\omega \leftrightarrow$ eigenvalue of conserved charge $h_\omega(\mu)$; non-degeneracy of $\{\epsilon_\omega\}$ breaks these crossings.

(e.g., $N = 10$) AVP, **MJC**, ABB (2019)



Summary of Many-body Treatment

Adiabatic Evolution

- Consider an initial many-body state, $|\Psi_0\rangle$
  - Example: in the (two-)flavor-basis, $|\nu_e \nu_x \nu_e\rangle$
- Adiabatically evolve with Schrödinger's Eq.

$$\Psi \approx V e^{-\mathrm{i} \int_0^t \Sigma(t') dt'} V_0^\top \Psi_0$$

  - $|E_i(t)\rangle = \sum_{i=1}^{2^N} V_{ji}(t) \, |j\rangle$ map energy states to/from mass product states, parametrized by the solns of Bethe equations
  - $\Sigma \equiv V^\top H \, V = \mathrm{diag}(E_1, \ldots, E_{2^N})$ instantaneous energies; degeneracies are split by time-dependent commuting charges h_ω
  - Obtain both V, Σ efficiently using Bethe ansatz

- Ansatz that relative phases for different ω are random (RPA)
  $\implies$ Mean-field approximation of our Hamiltonian:

$$H_{\nu\nu} = \mu\vec{J}\cdot\vec{J} \underset{MFT}{\approx} \mu\vec{P}\cdot\vec{J} - \frac{1}{4}\mu P^2$$

  where $\vec{P} = 2\left\langle\vec{J}\right\rangle$ is the "mean field" with state $|\psi\rangle$ satisfying

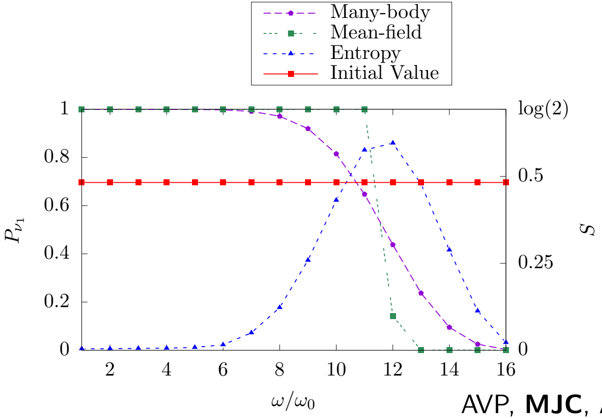
$$\langle\vec{J}_1\cdot\vec{J}_2\rangle = \langle\vec{J}_1\rangle\cdot\langle\vec{J}_2\rangle$$

- "Many-body" wave function simply: $|\Psi\rangle = \bigotimes_\omega |\psi(\omega)\rangle$, $2N$-dim
- ... But can we neglect the other dimensions?!

Final Data of All-Electon Flavor Initial State

$N = 16$ results across the spectrum

- Evolve $|\Psi_0\rangle = |\nu_e\rangle^{\otimes 16}$ to $r \gg R_\nu$ with $\theta = 0.584$
- Compare final P_{ν_1} and S at each ω

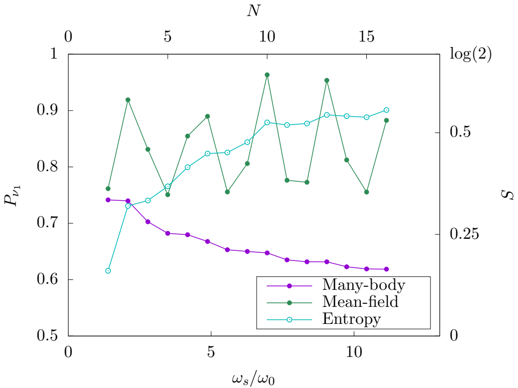


AVP, **MJC**, ABB (2021)

Pinpointing Entanglement

Honing on the spectral split

- Evolve $|\Psi_0\rangle = |\nu_e\rangle^{\otimes N}$ to $r \gg R_\nu$ with $\theta = 0.584$
- Here, spectral split frequency: $\omega_s = \omega_0 N \cos^2(\theta)$
- $P_{\nu_1}(\omega_s)$ & $S(\omega_s)$ vs. N



AVP, **MJC**, ABB (2021)

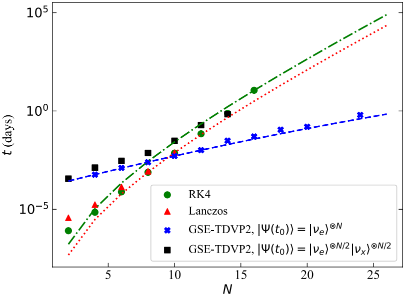
How do we study larger N systems?

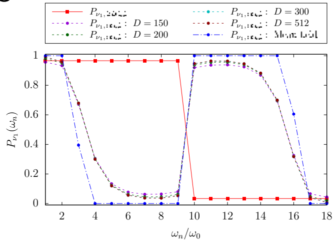
1: Tensor Networks

- (Schmidt) Decompose wave function into product of tensors:

$$\Psi^{\alpha_1 \cdots \alpha_N} = \psi_L^{\alpha_1}(1) \cdots \psi_L^{\alpha_N}(N)$$

- For unentangled system, ψ^α are numbers; o/w matrices
- Dims of matrices $\psi^\alpha \sim$ measure entanglement



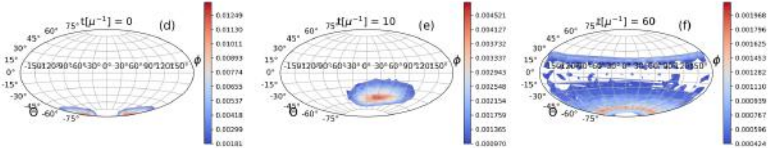


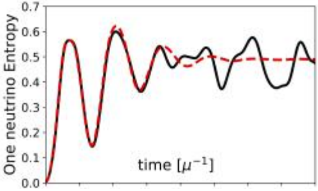
MJC, PS, AVP, ABB, SNC, CWJ (2022)

How do we study larger N systems?

2: Stochastic Mean-field: improving the mean-field approximation

Consider an uniform neutrino beam
SMF: random distribution around initial flavor state
$\rightarrow$ evolve each sample via ordinary mean field (easy!)
$\rightarrow$ average over trajectories (reproduce entanglement!)





DL, ABB, **MJC**, AVP, PS (2022)

How do we study larger N systems?

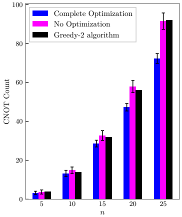
3: Quantum simulation

- Quantum proposal for two flavors: 1 $\nu \longleftrightarrow$ 1 qubit

$$\exp[-it\omega J_z] \longleftrightarrow R_z(2\omega t)$$
$$\exp[-it\mu \vec{J}_1 \cdot \vec{J}_2] \longleftrightarrow \mathrm{SWAP}^{\mu t/2}$$

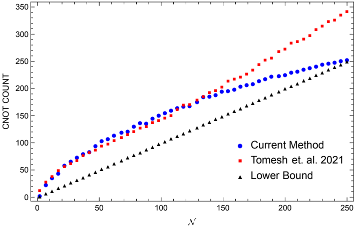
- Three flavors? Qu*tr*it hardware: $|0\rangle, |1\rangle, |2\rangle$? (Ongoing)
- Our qubit circuit compiling is state-of-the-art (on Github):

Diagonalize observables



EMM & **MJC** (2023)

Simulate (diag) Hamiltonian



EMM, **MJC**, HK, AA, PFB (2023)

Summary

- Interacting neutrino problem cast in a many-body perspective
- Additions to a MFT approach measured by quantum entanglement
- Various directions to quantify this entanglement and predict effects in SNe

Some Important Considerations

Future Work

- Tamborra & Shalgar (2023): Dependence upon ν wave packet size
- Cirigliano, Sen, & Yamauchi (2024): Non-forward scattering

- THANK YOU -