SIMP Miracles and WIMP Dead Ends: Navigating the Freeze-Out of MeV Dark Matter

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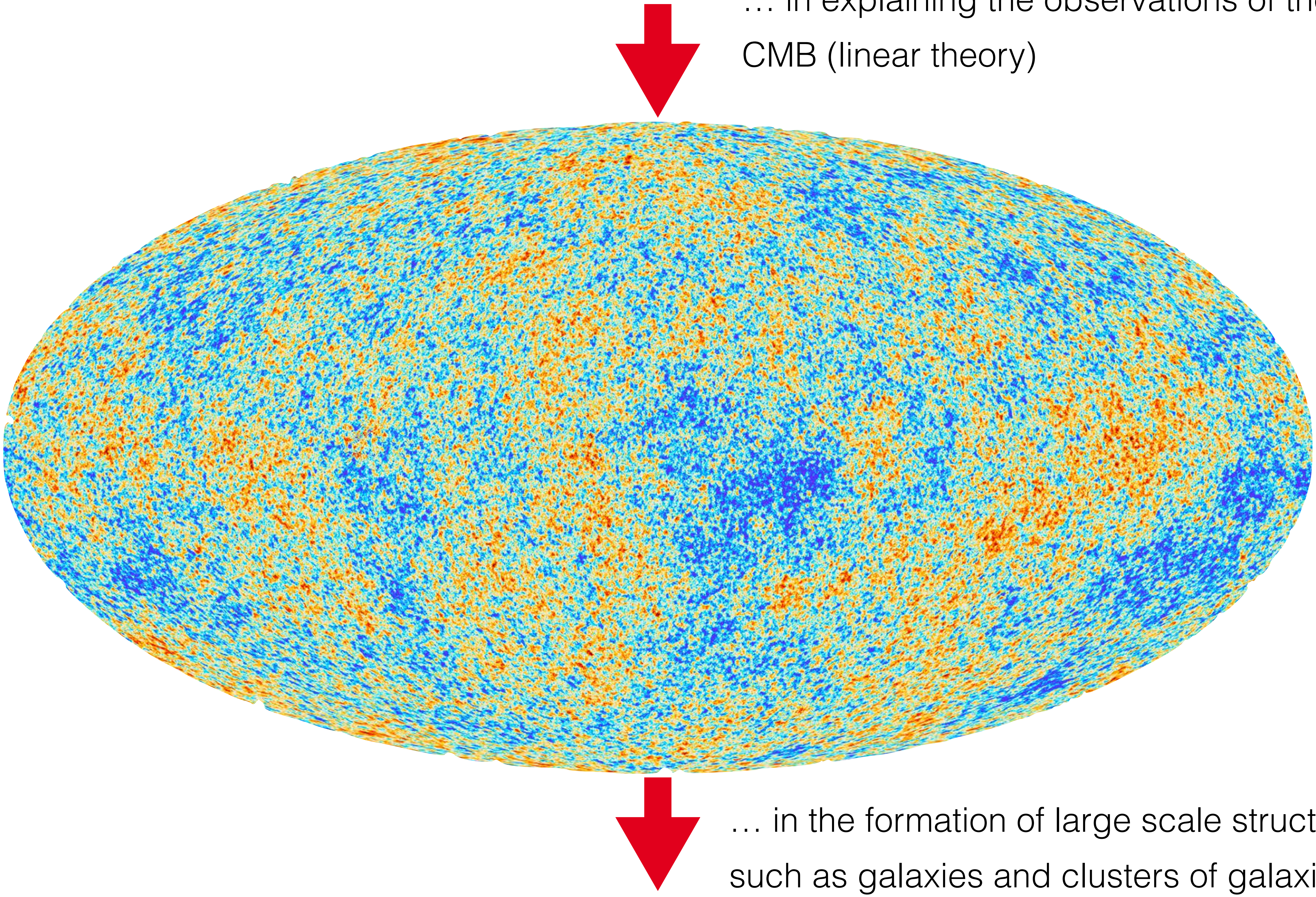
Dark Matter is key…

… in explaining the observations of the CMB (linear theory)



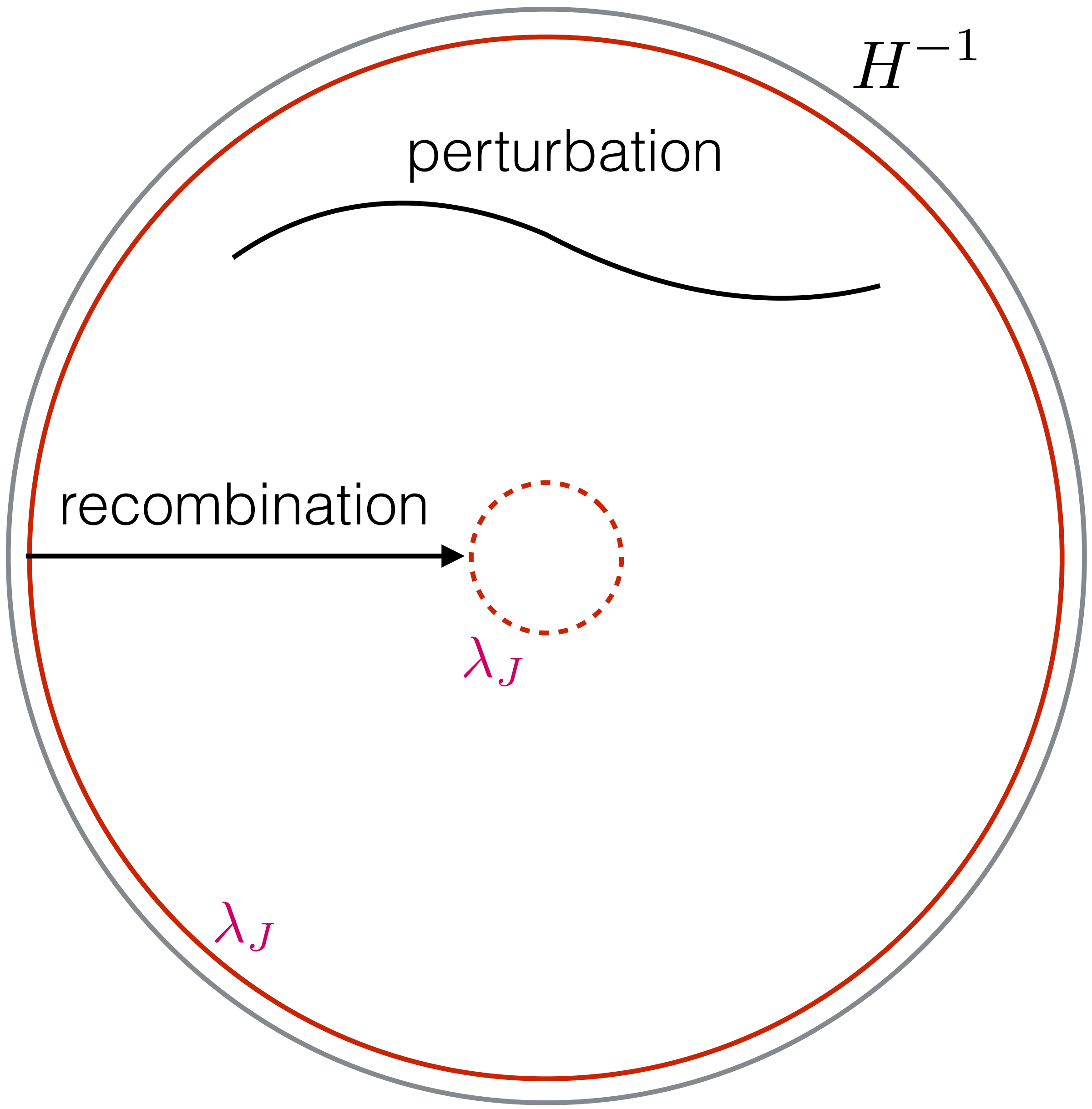
… in the formation of large scale structure such as galaxies and clusters of galaxies

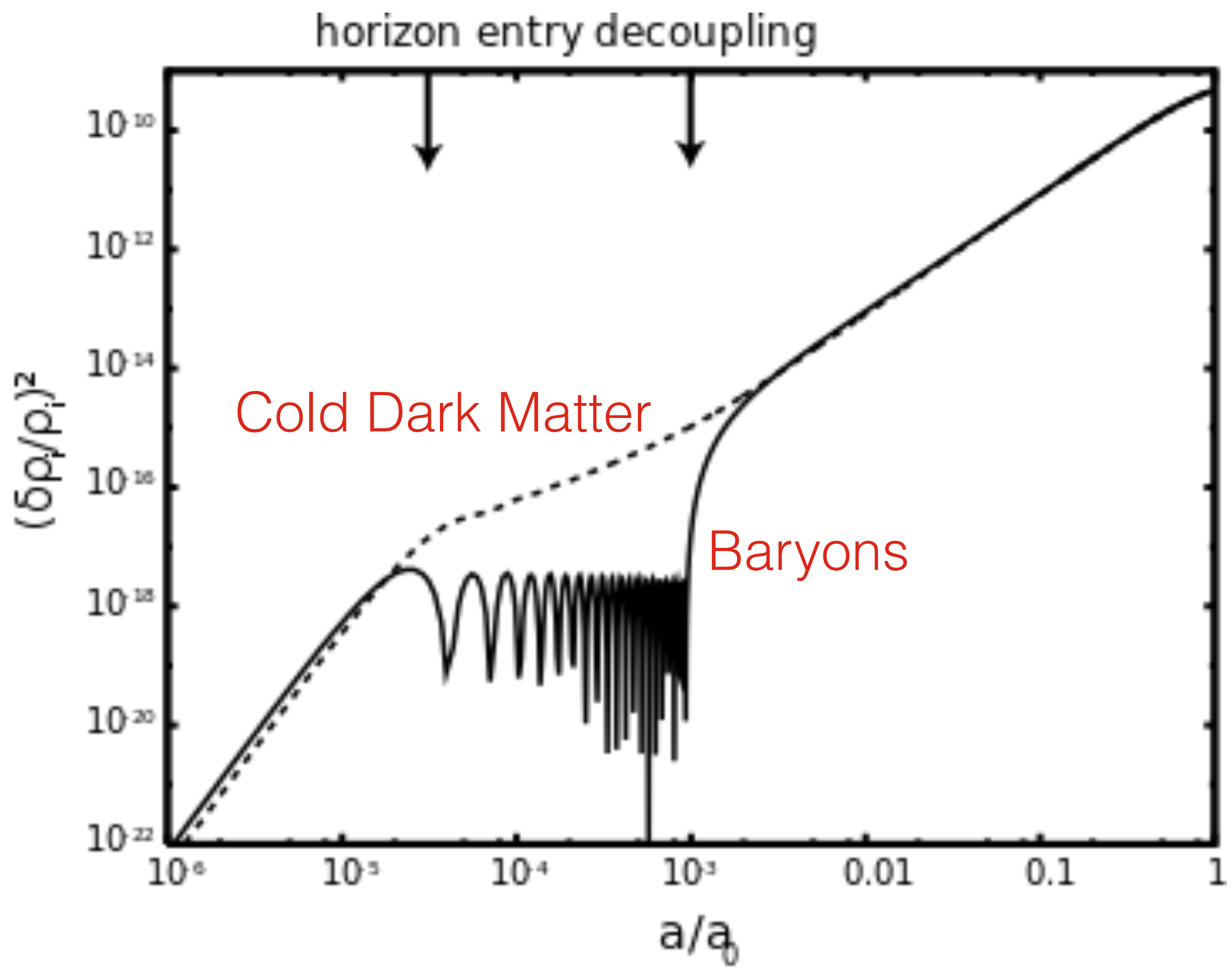
Dark Matter is key…

explaining the origin of structure

$$\ddot{\delta} + [\text{Pressure} - \text{Gravity}]\,\delta = 0$$

baryons fall into the potential wells created by dark matter.

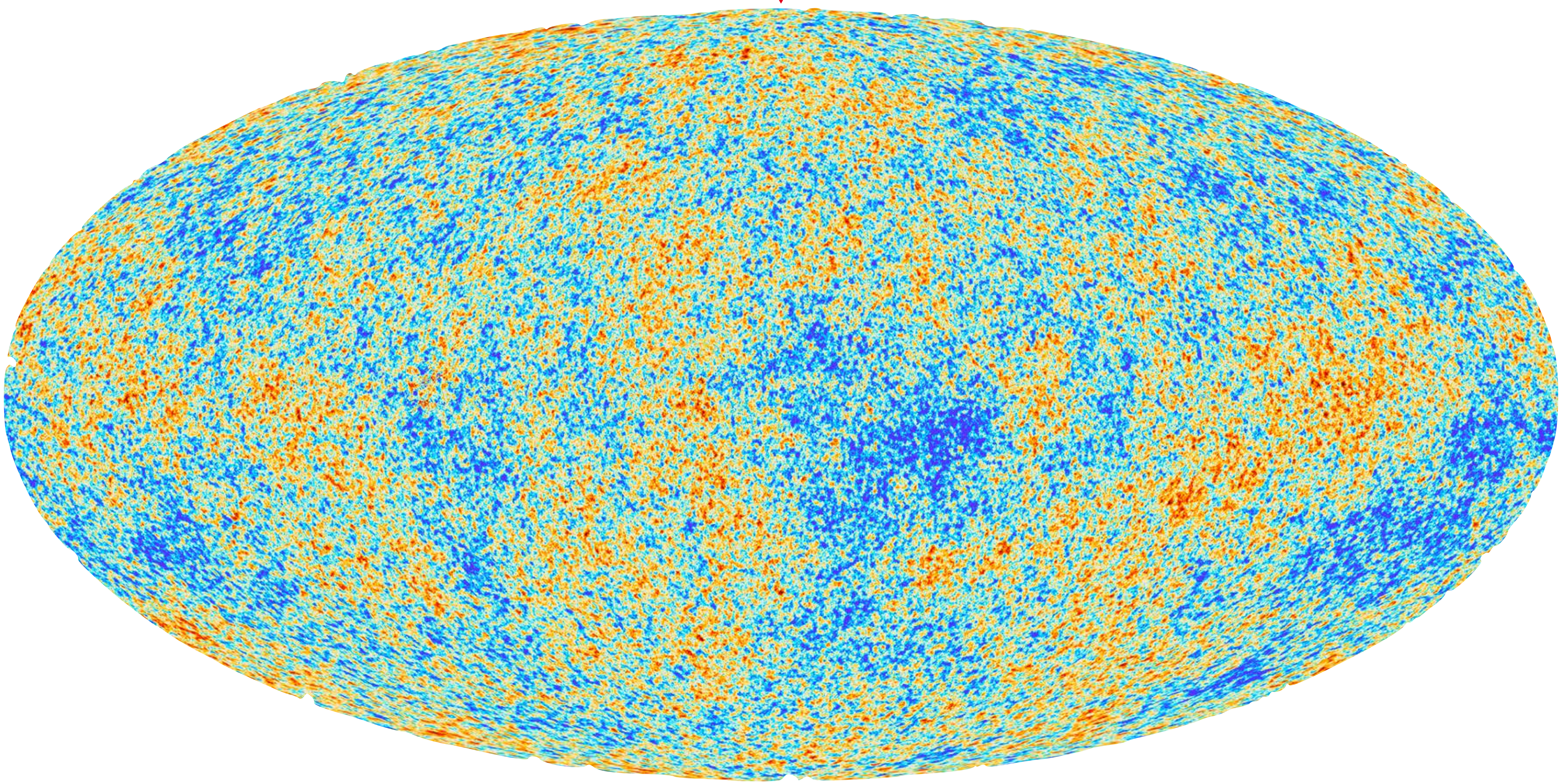




Dark Matter is key…

Important observable: Neff

… in explaining the observations of the CMB (linear theory)



Important observable: small scale structure

… in the formation of large scale structure such as galaxies and clusters of galaxies

Outline

1 SIMPs

On the freeze-out of strongly interacting dark matter candidates (SIMPs)

=> SIMP miracles revived

arXiv:2401.12283 (PRL)

w/ Xiaoyong Chu, Marco Nikolic

2 WIMPs

On the freeze-out of weakly interacting dark matter

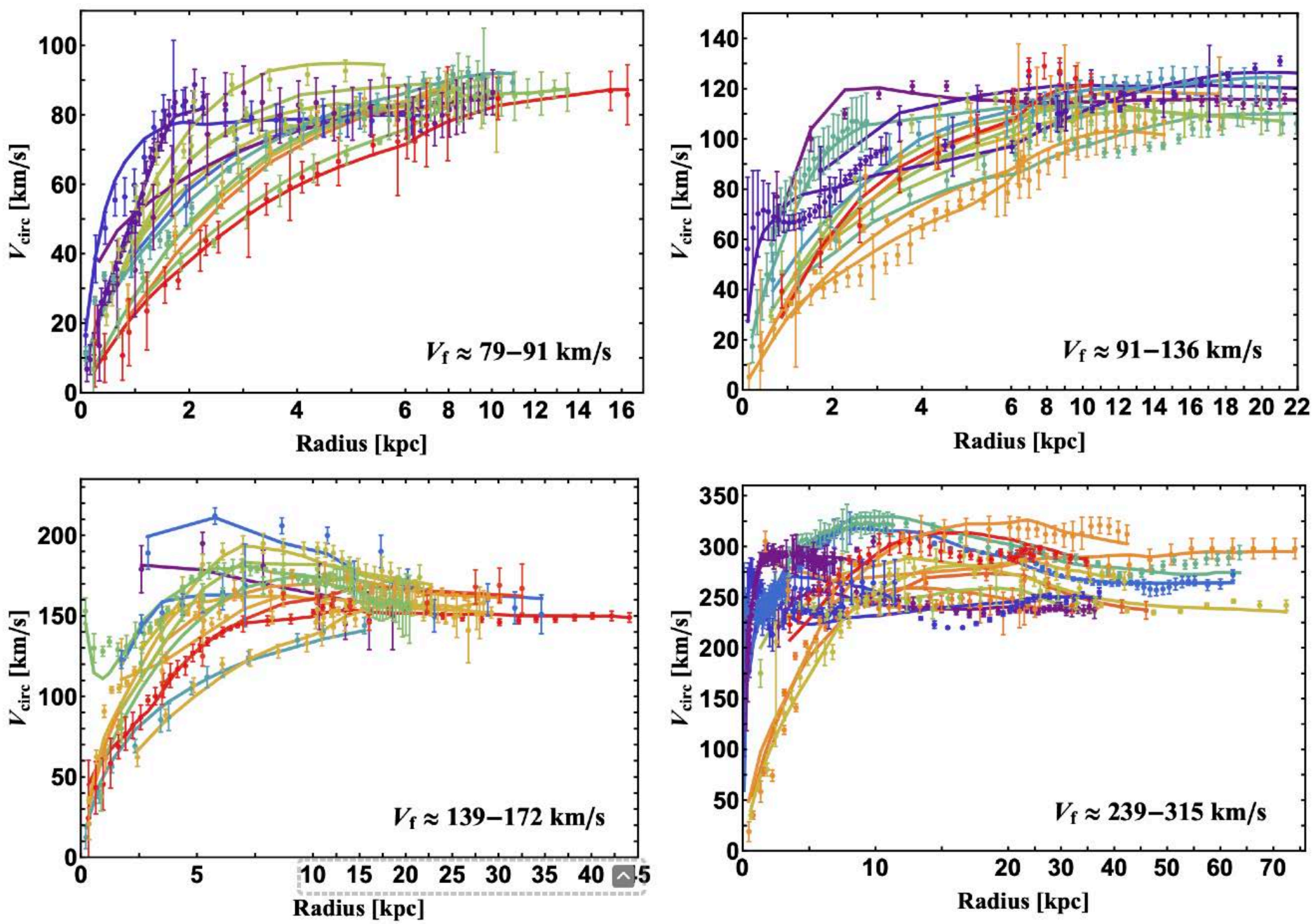
=> what is the minimal thermal DM mass? The WIMP dead end.

arXiv:2205.05714 (PRD)
arXiv:2310.06611 (PRD)

w/ Xiaoyong Chu, Jui-Lin Kuo

Motivation for SIMPs

Small scale structure problems in LCDM (core-cusp, diversity)



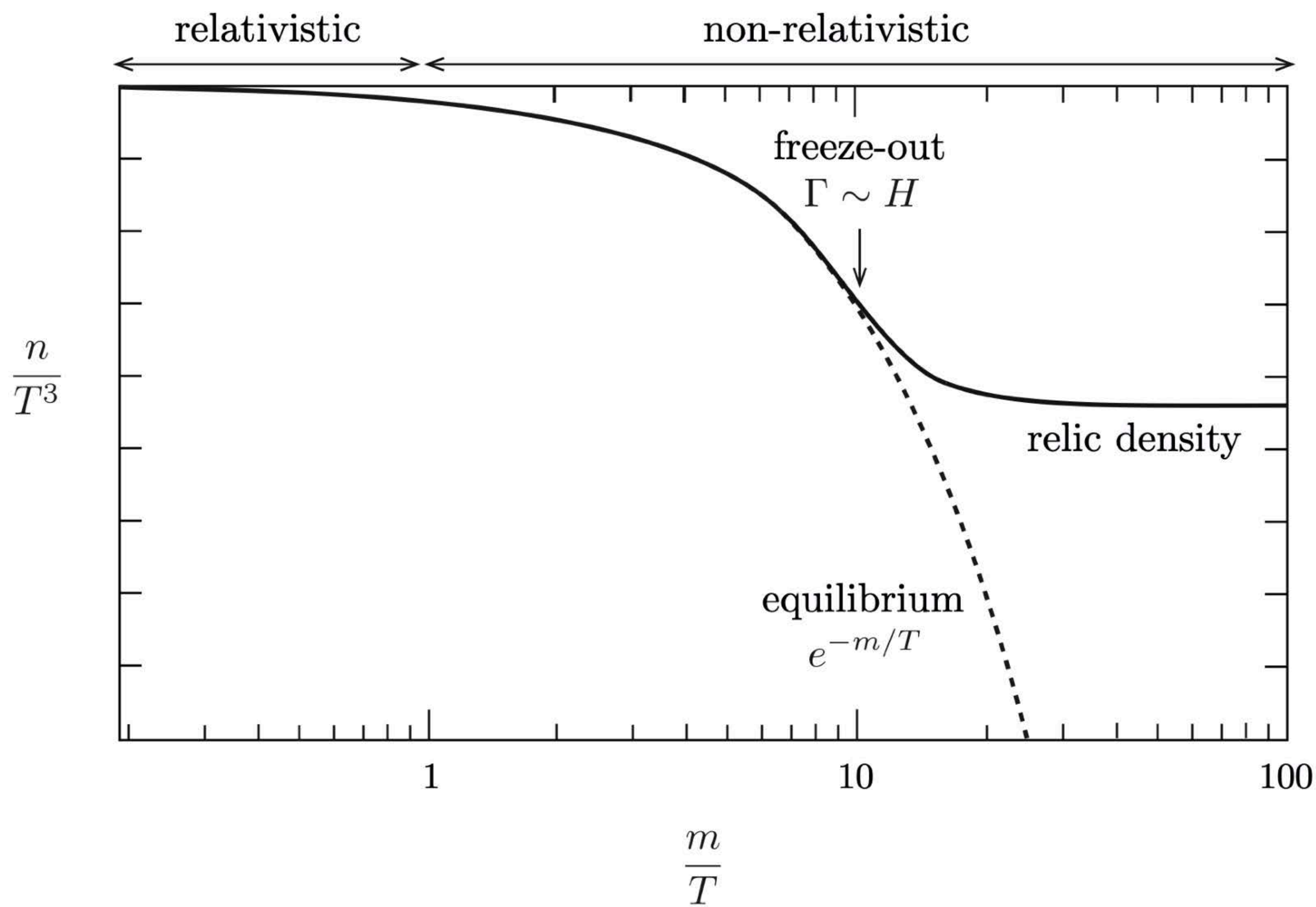
self-interactions lead to heat transfer in the halo, diversifying the halo density in the central regions of galaxies

e.g. Ren, Kwa, Kaplinghat, Yu [2019]

$$\sigma/\mathrm{m} = 3\ \mathrm{cm}^2/\mathrm{g}$$

Weakly interacting massive particles (WIMPs)

Freeze out when 2 -> 2 annihilation rate ~ Hubble rate



WIMPs

"Weakly Interacting Massive Particles"

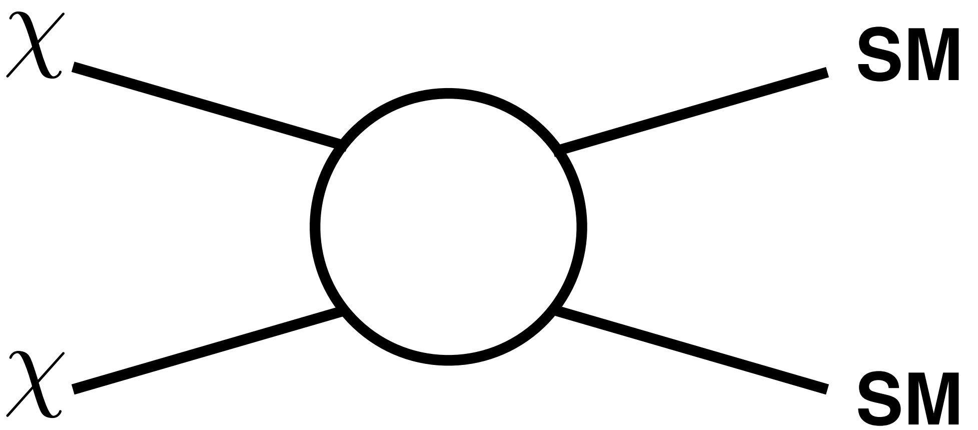
Freeze out when 2 -> 2 annihilation rate ~ Hubble rate

$$\Gamma_{2\to 2}(T_f) = \langle \sigma v \rangle n_\chi(T_f) \sim H(T_f)$$

$$\langle \sigma v \rangle \sim \frac{\alpha^2}{m_\chi^2}$$

$$n_\chi(T_f) = \frac{\rho_\chi(T_f)}{m_\chi} = \frac{T_{eq} m_\chi^2}{x_f^3}$$

$$H(T_f) \sim \frac{T_f^2}{M_P} = \frac{m_\chi^2}{x_f^2 M_P}$$



$$x_f = m_\chi / T_f \sim 20$$

$$m_\chi \sim \frac{\alpha}{\sqrt{x_f}} \sqrt{T_{eq} M_P} \sim \alpha (30 \text{ TeV})$$

=> points to electroweak scale

SIMPs

"Strongly Interacting Massive Particles"

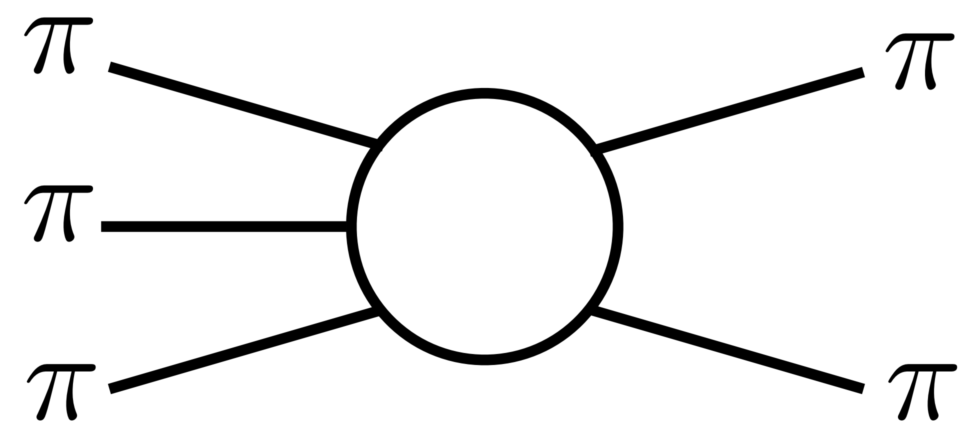
Freeze out when **3 -> 2** annihilation rate ~ Hubble rate

$$\Gamma_{3\to 2}(T_f) = \langle \sigma v^2 \rangle n_\pi^2(T_f) \sim H(T_f)$$

$$\langle \sigma v^2 \rangle \sim \frac{\alpha^3}{m_\chi^5}$$

collision term or

"cross section" of mass dimension -5



$$m_\pi \sim \alpha (T_{eq}^2 M_P)^{1/3} \sim \alpha (100 \text{ MeV})$$

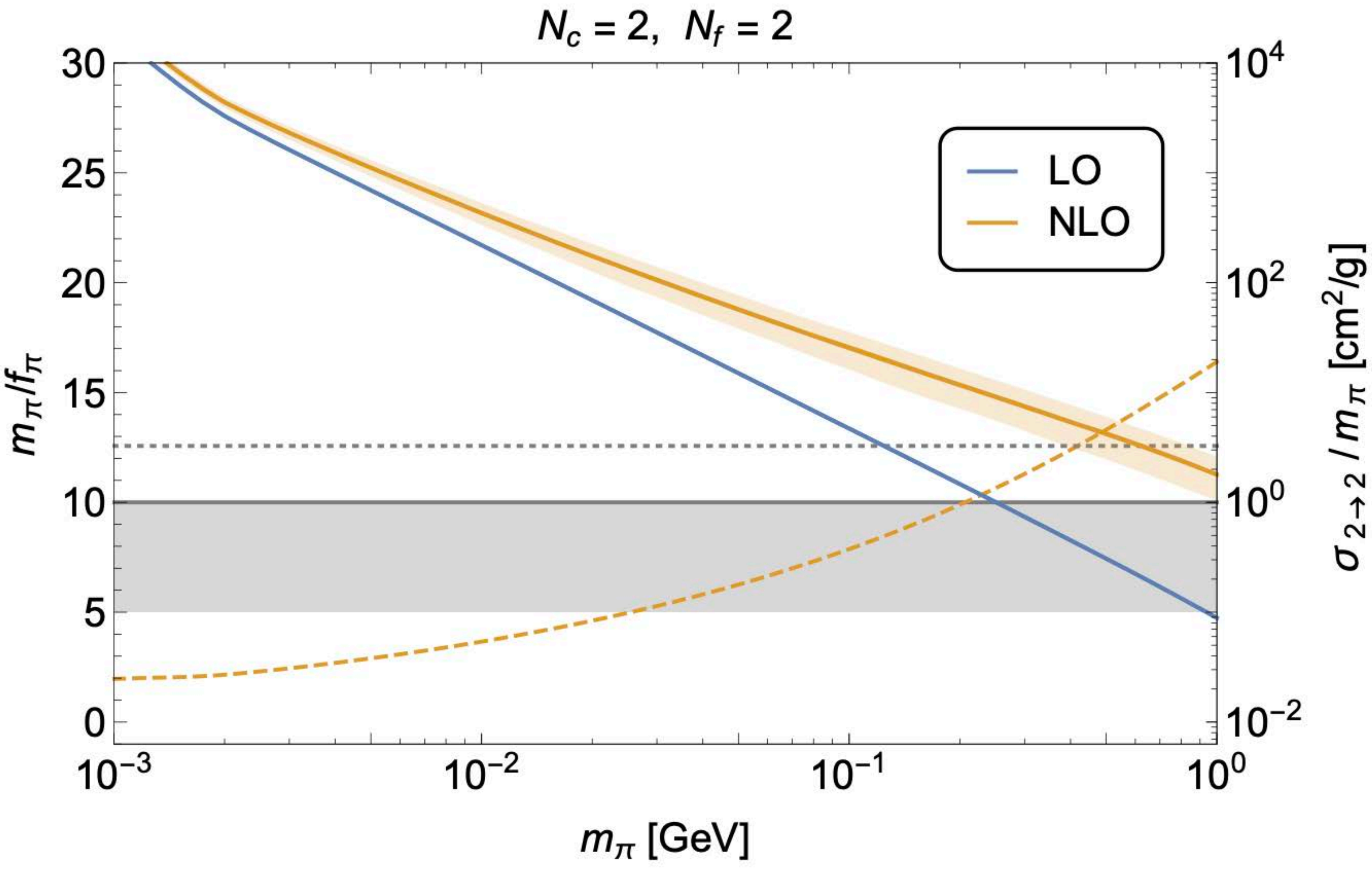
=> points to strong interactions

=> MeV scale DM

[Hochberg et al 2015, …]

A SIMP miracle?

Right relic density AND interesting self-scattering cross section?

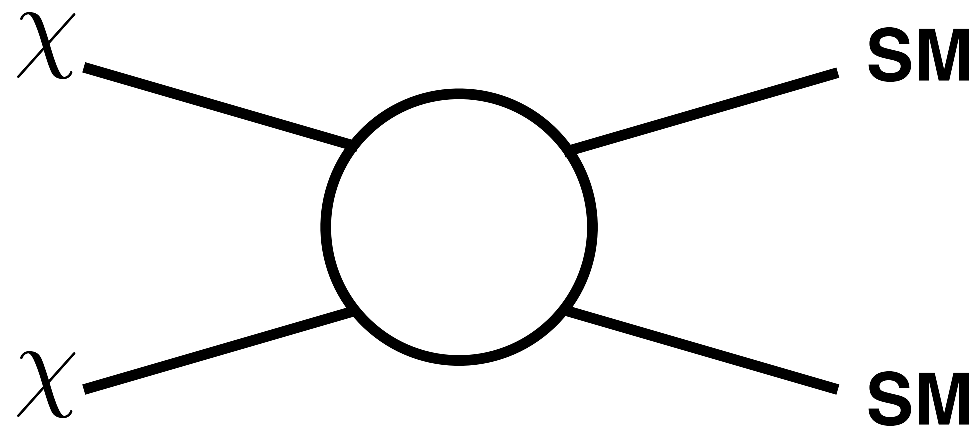


[Hansen, Langaeble, Sannino 2016]

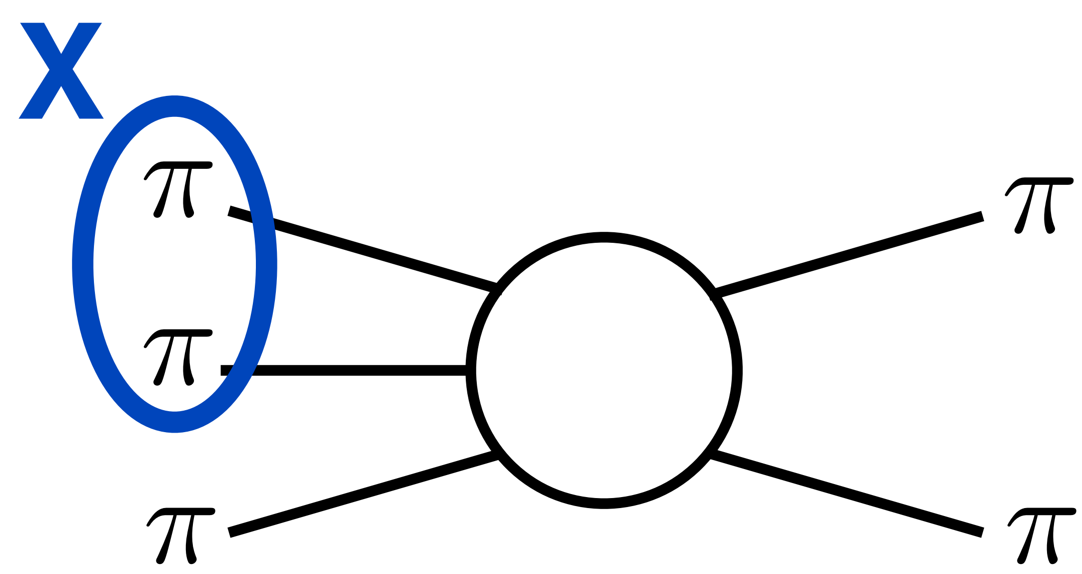
tension in the joint "miracle" solution

WIMPs vs. SIMPs

$$m_\chi \sim \frac{\alpha}{\sqrt{x_f}} \sqrt{T_{eq} M_P} \sim \alpha (30 \text{ TeV})$$



what if we make a stable bound state?



$$m_\pi \sim \alpha (T_{eq}^2 M_P)^{1/3} \sim \alpha (100 \text{ MeV})$$

SIMP prototype model

Dark Matter as Goldstone bosons of a confining dark sector

For example, two flavor $N_f = 2,\ Sp(4)_c$ gauge group

Kulkarni, Maas, Mee, Nikolic, JP, Zierler SciPost Phys. 14 (2023) 3, 044,

$$\mathcal{L}^{\mathrm{UV}} = -\frac{1}{2}\,\mathrm{Tr}\left[G_{\mu\nu}G^{\mu\nu}\right] + \bar{u}\left(\gamma_\mu D_\mu + m_u\right)u + \bar{d}\left(\gamma_\mu D_\mu + m_d\right)d$$

Quarks are in pseudoreal representation of color group $(\tau^a)^T = S\tau^a S$

Flavor:

$$\Psi \equiv \begin{pmatrix} u_L \\ d_L \\ \sigma_2 S u_R^* \\ \sigma_2 S d_R^* \end{pmatrix}$$ => $$\mathcal{L}_{\mathrm{kin}}^{\mathrm{UV,f}} = i\Psi^\dagger \bar{\sigma}_\mu D^\mu \Psi.$$ => SU(4)

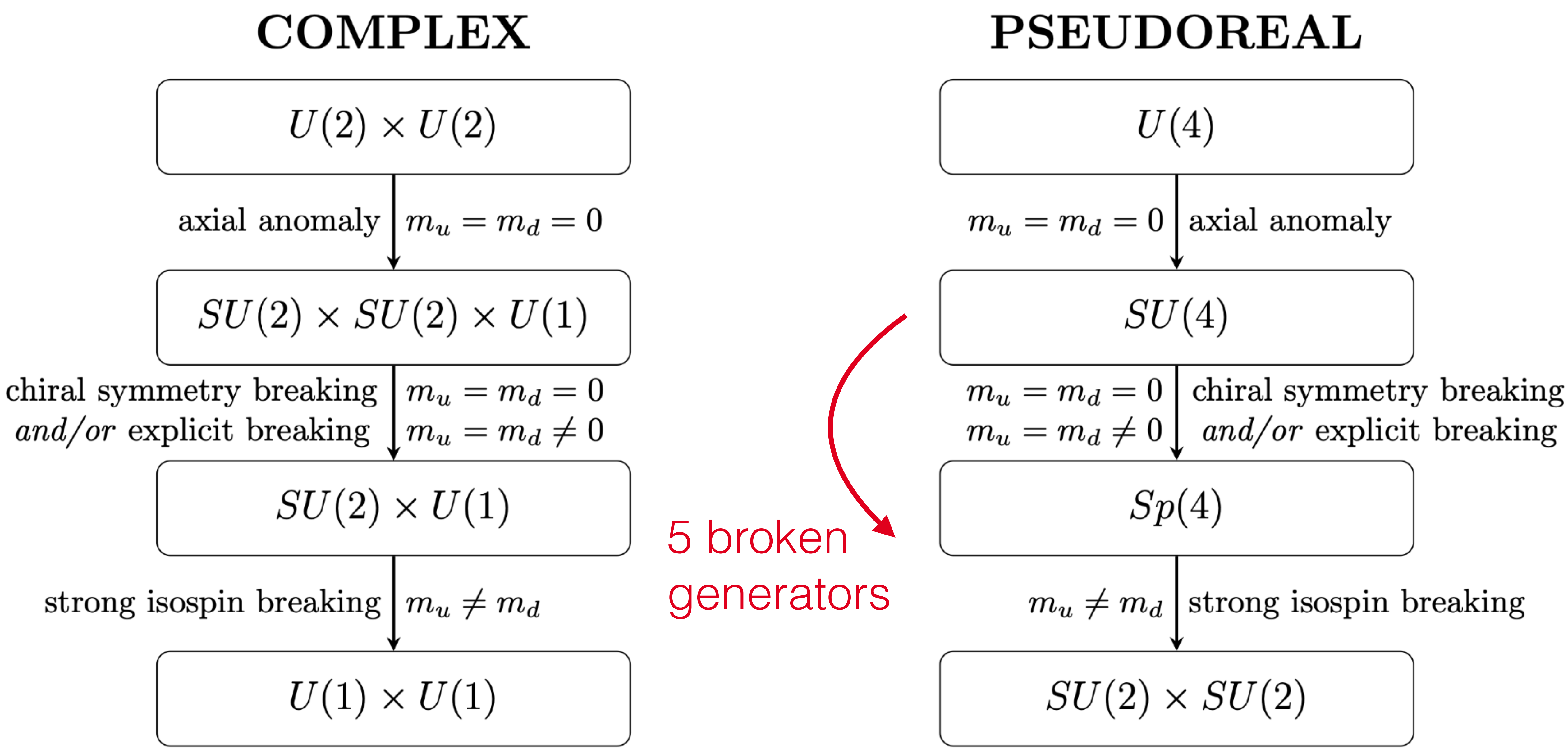
$$\bar{u}u + \bar{d}d = -\frac{1}{2}\Psi^T \sigma_2 S E \Psi + \text{ h.c.} \qquad (m_u = m_d)$$

$$E = \begin{pmatrix} 0 & \mathbb{1}_{N_f} \\ -\mathbb{1}_{N_f} & 0 \end{pmatrix} \qquad U^T E U = E$$ => Sp(4)

Flavor breaking pattern

QCD-like

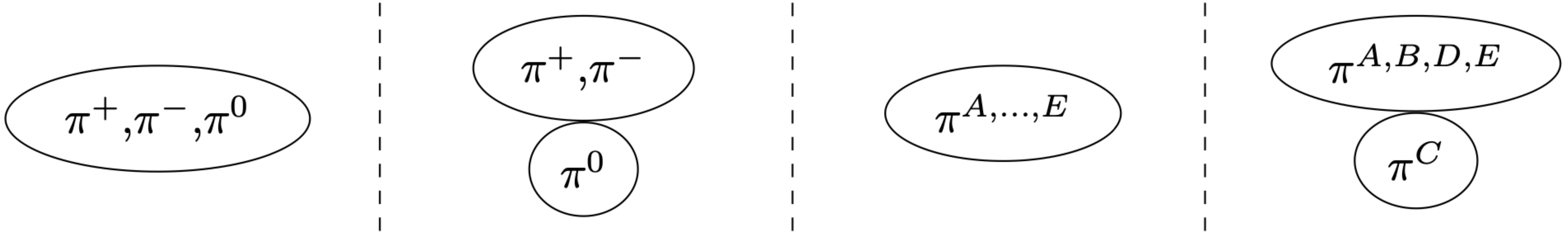
this example



=> 5 Goldstone bosons

Meson multiplet structure

$SU(3)_c$	$SU(3)_c$	$Sp(4)_c$	$Sp(4)_c$
$m_u = m_d$	$m_u \neq m_d$	$m_u = m_d$	$m_u \neq m_d$



$$\pi^D = \bar{d}\gamma_5 SC\bar{u}^T$$

$$\pi^E = d^T SC\gamma_5 u$$

$$\pi = \sum_{i=1,\ldots,5} \pi_a T^a = \sum_{N=A,\ldots,E} \pi_N T^N = \frac{1}{2}\begin{pmatrix} \pi^C & \pi^B & 0 & \pi^E \\ \pi^A & -\pi^C & -\pi^E & 0 \\ 0 & -\pi^D & \pi^C & \pi^A \\ \pi^D & 0 & \pi^B & -\pi^C \end{pmatrix}$$

=> 5 Goldstone bosons

Prototype SIMP theory

Low energy description

chiral field $\Sigma = e^{i\pi/f_\pi}\Sigma_0 e^{i\pi^T/f_\pi}$ $\pi = \sum_{n=1} \pi_n T^n$ ← broken generators

$$\mathcal{L} = \frac{f_\pi^2}{4}\mathrm{Tr}\left[\partial_\mu\Sigma\partial^\mu\Sigma^\dagger\right] - \frac{\mu^3}{2}\left(\mathrm{Tr}\left[M\Sigma\right] + \mathrm{Tr}\left[\Sigma^\dagger M^\dagger\right]\right) + \ldots.$$

↓ expansion yields 4-point, 6-point, etc interactions

$$\mathcal{L}_{\mathrm{int}}^{\mathrm{even}} \supset -\frac{1}{3f_\pi^2}\mathrm{Tr}\left([\pi,\partial_\mu\pi][\pi,\partial^\mu\pi]\right) + \frac{m_\pi^2}{3f_\pi^2}\mathrm{Tr}\left[\pi^4\right]$$

even-numbered only

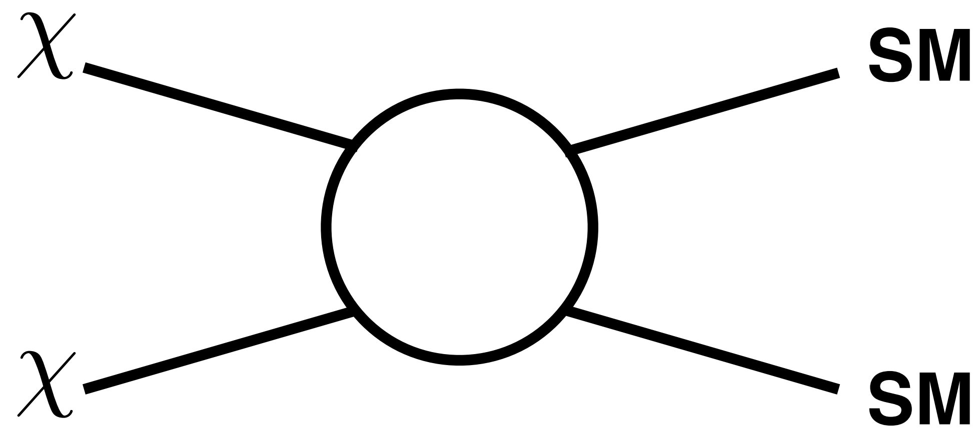
Wess-Zumino-Witten term when coset space has non-trivial fifth homotopy group

$$\mathcal{L}_{\mathrm{int}}^{\mathrm{odd}} = \frac{2N_c}{15\pi^2 f_\pi^5}\epsilon^{\mu\nu\rho\sigma}\mathrm{Tr}\left[\pi\partial_\mu\pi\partial_\nu\pi\partial_\rho\pi\partial_\sigma\pi\right].$$

odd-numbered

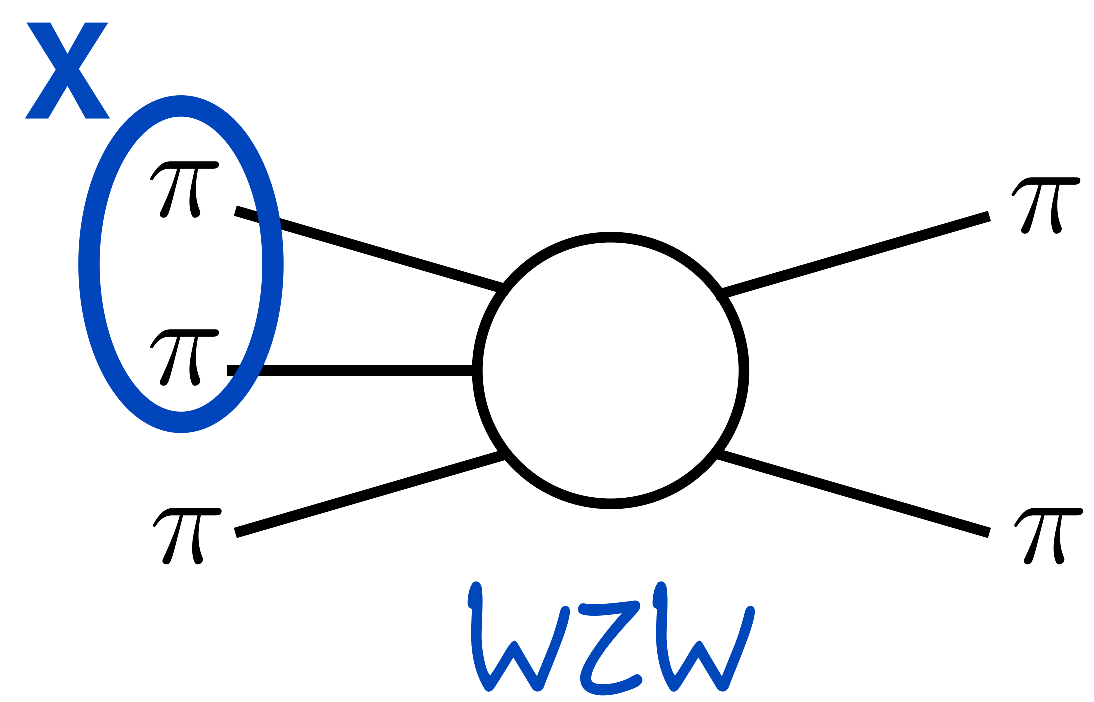
WIMPs vs. SIMPs

$$m_\chi \sim \frac{\alpha}{\sqrt{x_f}}\sqrt{T_{eq}M_P} \sim \alpha(30\ \text{TeV})$$



what if we make a stable bound state?

$$m_\pi \sim \alpha(T_{eq}^2 M_P)^{1/3} \sim \alpha(100\ \text{MeV})$$



SIMP bound states

X = [π π] must exist

- considering SIMPs as pseudo-Nambu-Goldstone bosons of a strongly interacting theory we require a molecular state with negative binding energy such that $m_X \leq 2m_\pi$

  QCD with $m_q \ll \Lambda_{\text{strong}}$ has a mass gap, hence not prospective

  => better consider a dark confining theory with $m_q \sim \Lambda_{\text{strong}}$ and

  => make SIMP-onium

  => or take $m_q \gg \Lambda_{\text{strong}}$: Glueball dark matter $J^{PC} = 0^{++} \text{or } 0^{-+}$ e.g. [Soni, Zhang, 2016]

$$V(G) = \frac{1}{4}\frac{m_G^2}{\Lambda_G^2}\left(G^4 \ln\left|\frac{G}{\Lambda_G}\right| - \frac{G^4}{4}\right)$$

  => yields odd G^3 interactions

  => 3-to-2 SIMP mechanism

  => make Glueball-onium for G-bound states see [Giacosa, Pilloni, Trotti 2021]

- one may also use a Yukawa force with sizable coupling; options exist

  e.g. [G. Kribs and E. Neil 2016, Y. Tsai, R. McGehee, H.Murayama 2020, R. Mahbubani, M. Redi and A. Tesi 2020, ….].

Bound-state assisted freeze-out

Catalysis

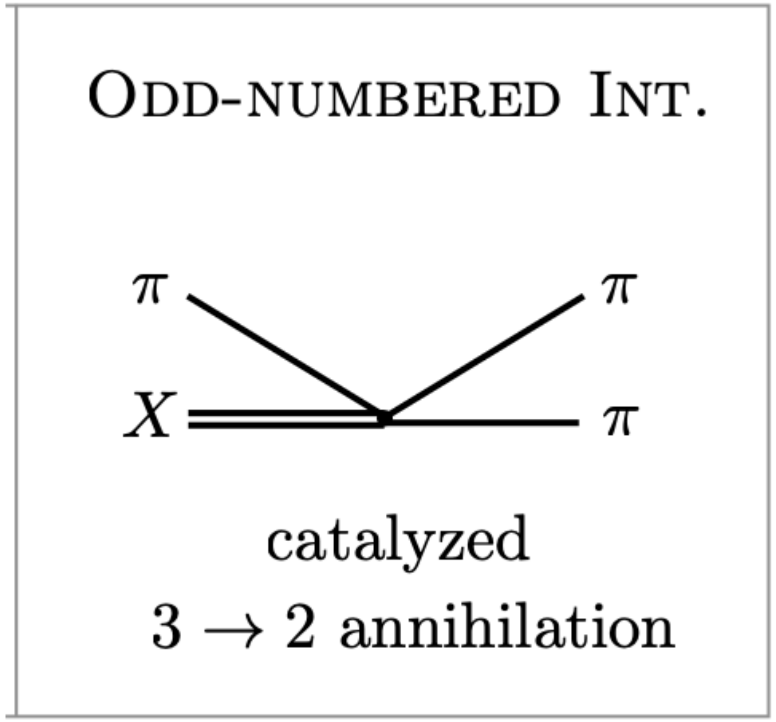
Probability of two particles finding each other

in a bound state vs. as free particles

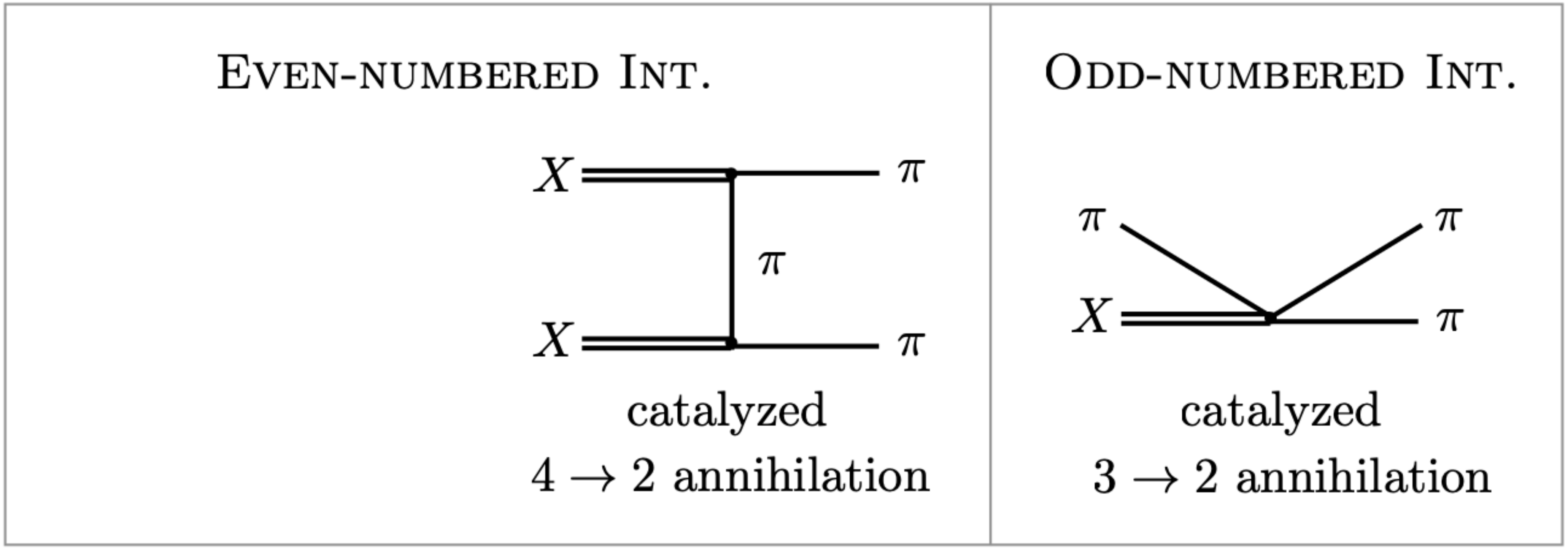
$$\frac{n_X |\psi(0)|^2}{n_\pi^2} \approx \underbrace{2\sqrt{2}\pi^{3/2} x_{\rm f}^{3/2} e^{\kappa x_{\rm f}}}_{\approx 10^3} \underbrace{\frac{|\psi(0)|^2}{m_\pi^3}}_{O(1)}$$

$$x_{\rm f} = 20$$

$$\kappa \equiv E_B/m_\pi \sim 0.1$$



Bound-state assisted freeze-out

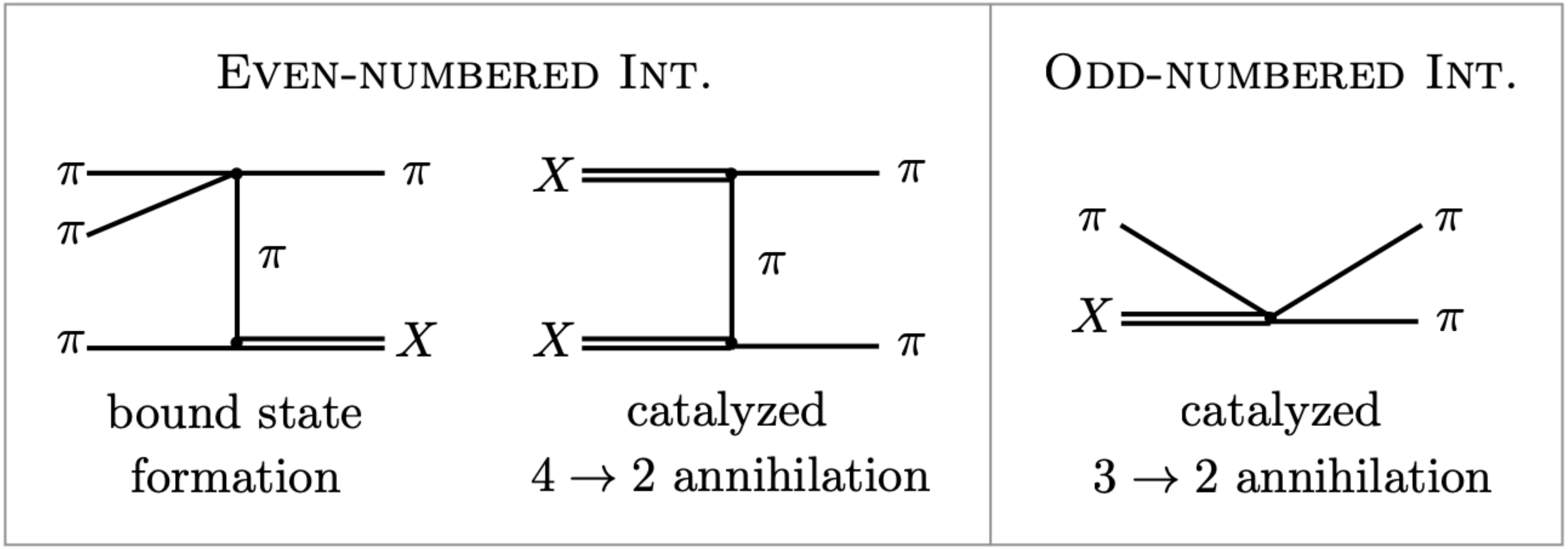


WZW-free SIMP mechanism

self-depletion of mass density in the early Universe possible
with even-numbered interactions only!

=> relaxes the requirement on the topological structure of the theory

Bound-state assisted freeze-out



guaranteed X formation

Comparing the rates of X-formation to free

$$\frac{\Gamma_{3\pi\to X\pi}}{\Gamma_{3\pi\to 2\pi}} = \frac{\langle \sigma_{3\pi\to X\pi} v^2\rangle}{\langle \sigma_{3\pi\to 2\pi} v^2\rangle} \approx \frac{|\psi(0)|^2 f_\pi^2}{m_\pi^5} x_{\rm f}^2 .$$

easily exceeds unity

Bound-state assisted freeze-out

Expectations/guesses for $|\psi(0)|^2$

In analogy to QED, one may posit a scale a_B "Bohr radius"

For perturbative couplings α $\quad a_B \sim 1/(\alpha\mu) = 2/(\alpha m_\pi) \geq 2/m_\pi$

Radial profiles (for n=1)

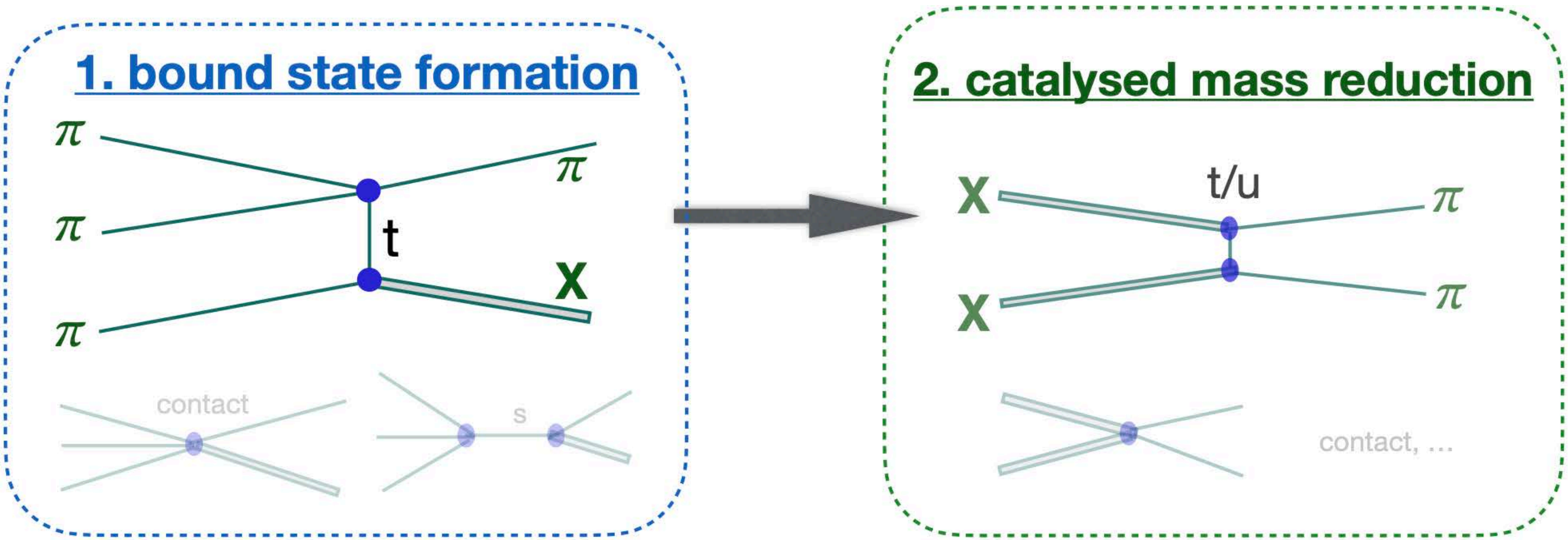
$$R_s(r) \simeq R_s(0)\, e^{-(r/2a_B)}, \qquad R_p(r) \simeq R'_p(0)\, r\, e^{-(r/2a_B)},$$

s-wave (l=0) p-wave (l=1)

$$R_s(0) = \frac{1}{\sqrt{2a_B^3}} \sim 0.25(\alpha m_\pi)^{3/2}, \quad R'_p(0) = \frac{1}{\sqrt{24a_B^5}} \sim 0.035(\alpha m_\pi)^{5/2}$$

=> $|\psi(0)|/m_\pi^{3/2} \sim 0.9\alpha^{3/2}$

CASE 1: even-numbered interactions only



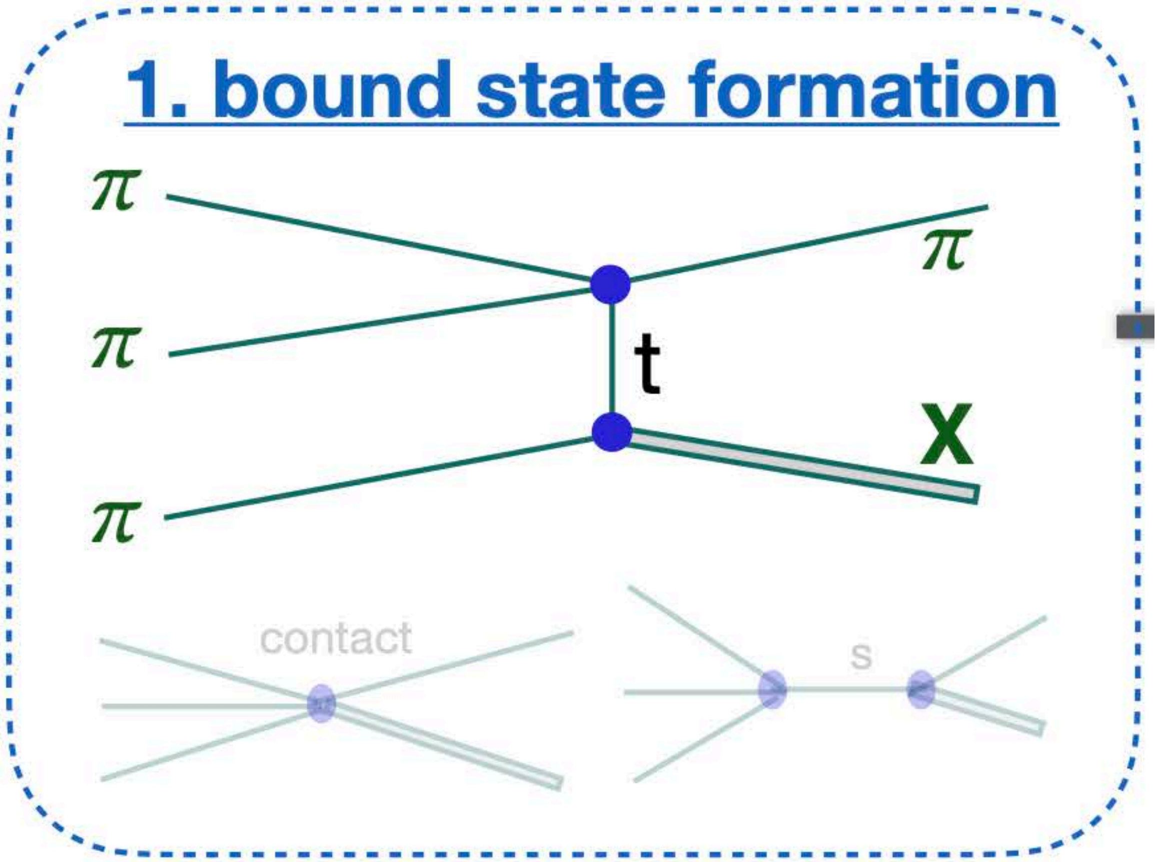
Working hypothesis:

X is a weakly bound (non-relativistic) state, such as a hadronic molecule

Bethe-Salpeter wave functions => non-relativistic Schroedinger equation

[e.g. K.Petraki, M.Postma, J.de Vries 2016, …]

CASE 1: even-numbered interactions only



In the non-relativistic limit, one obtains a t-channel **resonance**:

$$\frac{s}{t-m_\pi^2} \propto \frac{m_\pi^2}{m_X^2-4m_\pi^2} \propto \frac{m_\pi}{E_B} \gg 1$$

radial wave function of X (s-wave)

$$i\mathcal{M}(p_1,p_2,p_3 \to k,Q)_{3\pi\to\pi X} \simeq \langle \sigma_{3\pi\to\pi X} v^2 \rangle \simeq \frac{57\,041}{1\,310\,720\sqrt{3}\pi^2} \frac{R_S^2(0)}{f_\pi^8} \left(\frac{m_\pi}{E_B}\right)^{3/2} \quad \to k, Q/2+q, Q/2-q)$$

additional t-channel enhancement

CASE 1: even-numbered interactions only

Total mass density (free and bound) reduces

mass-reduction rate

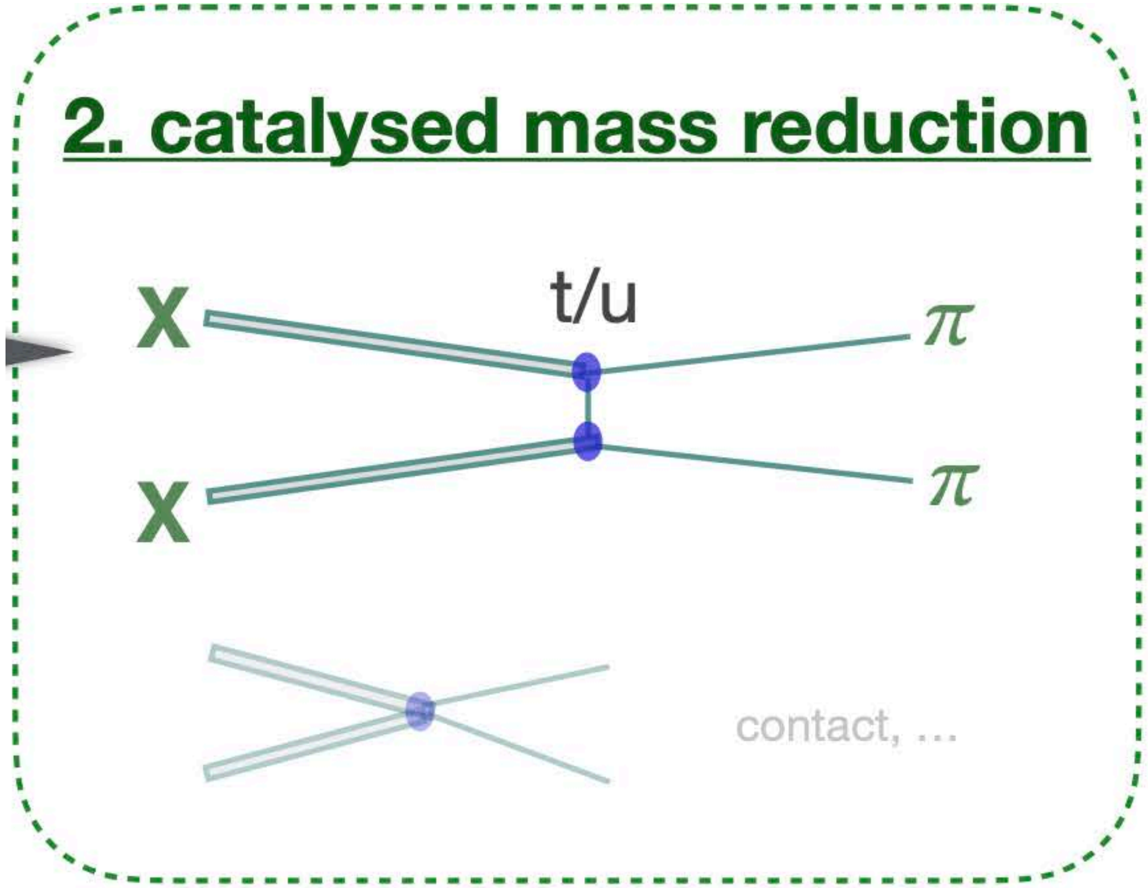
$$\Gamma_{XX\to\pi\pi} = \frac{n_X^2 \langle \sigma_{XX\to\pi\pi} v \rangle}{n_\pi}$$

In practice, 2X→2**π** changes **π**-abundance

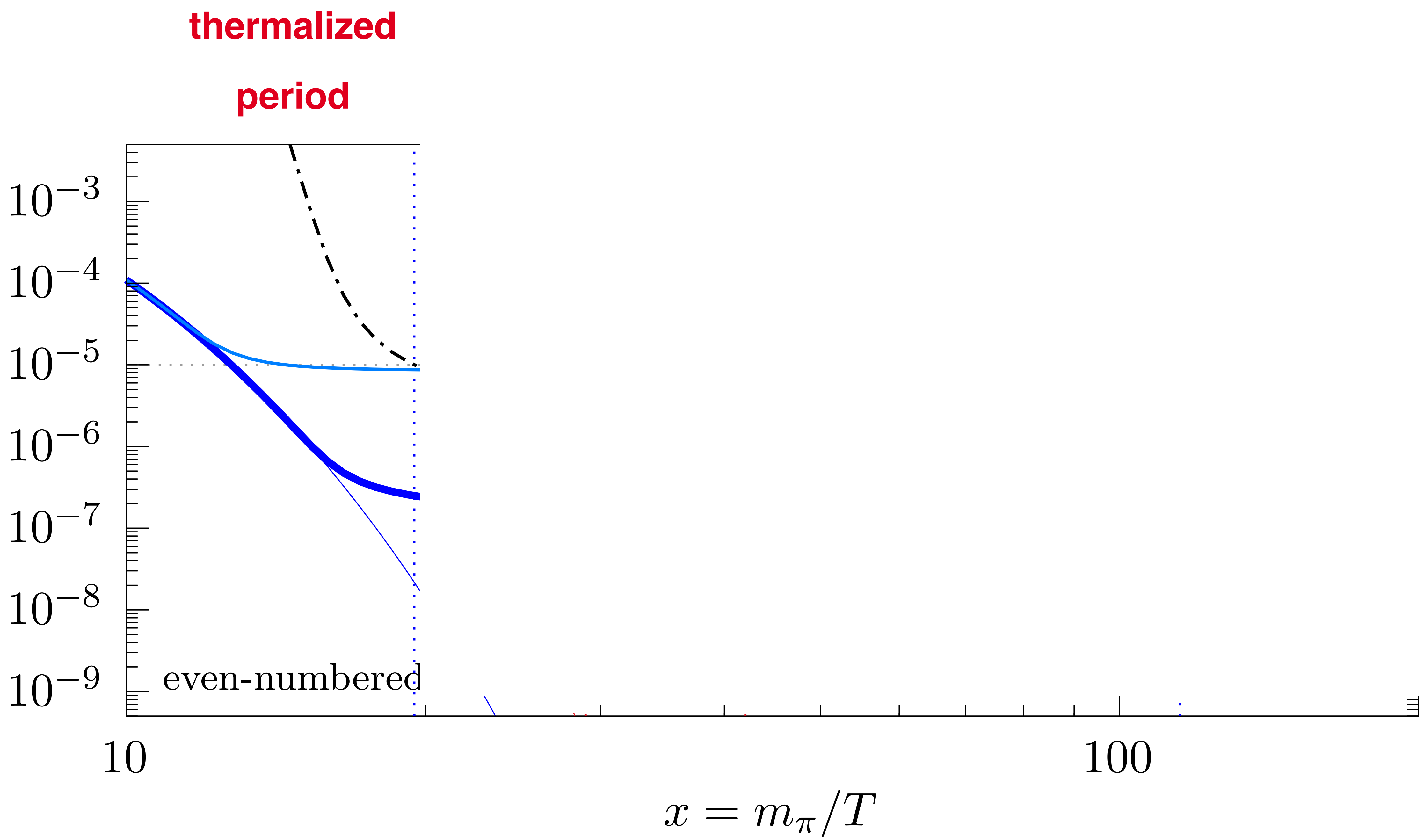
fast enough when $\Gamma_{XX\to\pi\pi} > H$

Cross section is s-wave

$$\langle \sigma_{XX\to\pi\pi} v \rangle \simeq \frac{2\,529\,757}{424\,673\,280\sqrt{3}\,\pi^3} \frac{R_S^4(0)}{f_\pi^8}$$

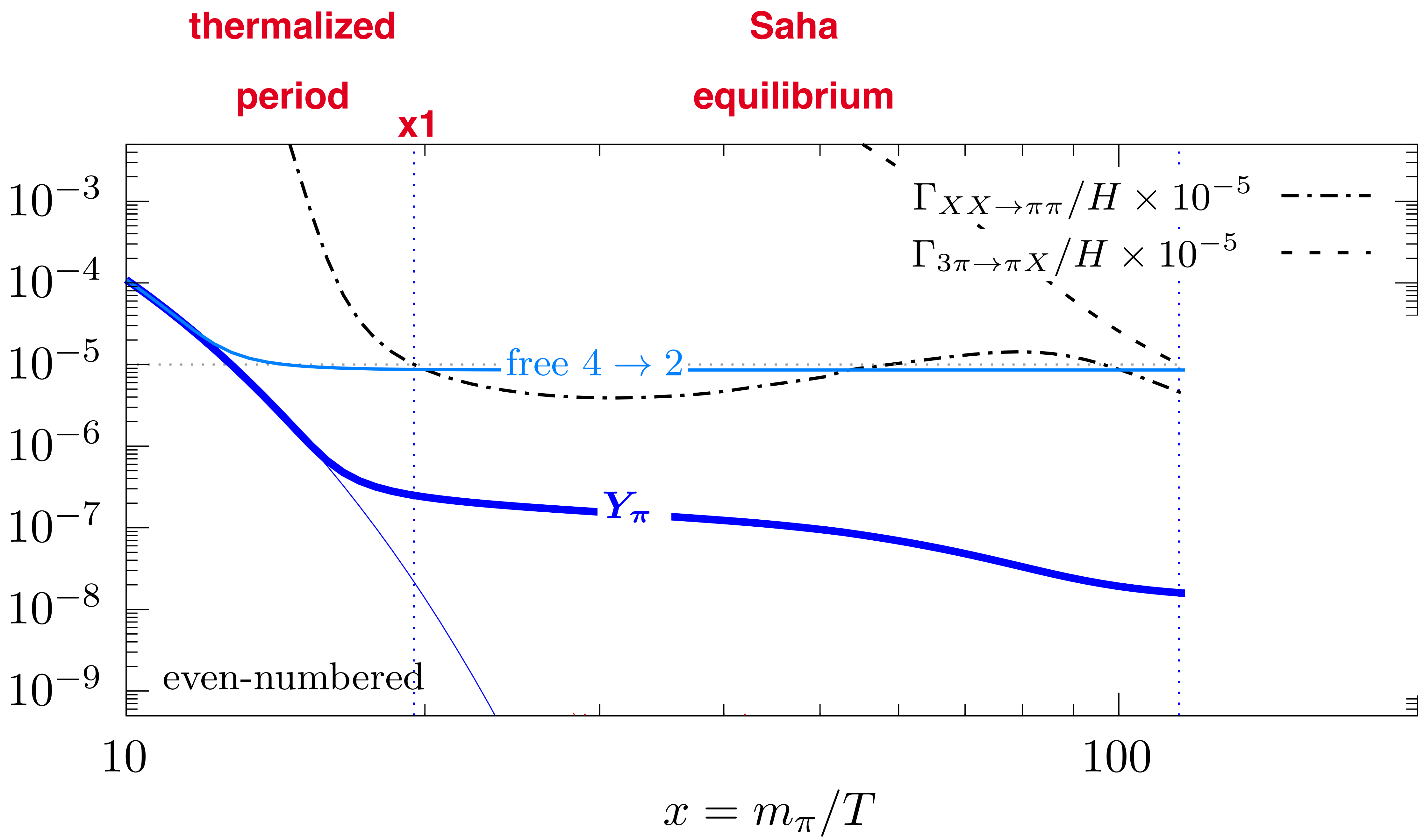


CASE 1: even-numbered interactions only



$$Y_{\pi,X} = Y_{\pi,X}^{\text{eq}}$$

CASE 1: even-numbered interactions only

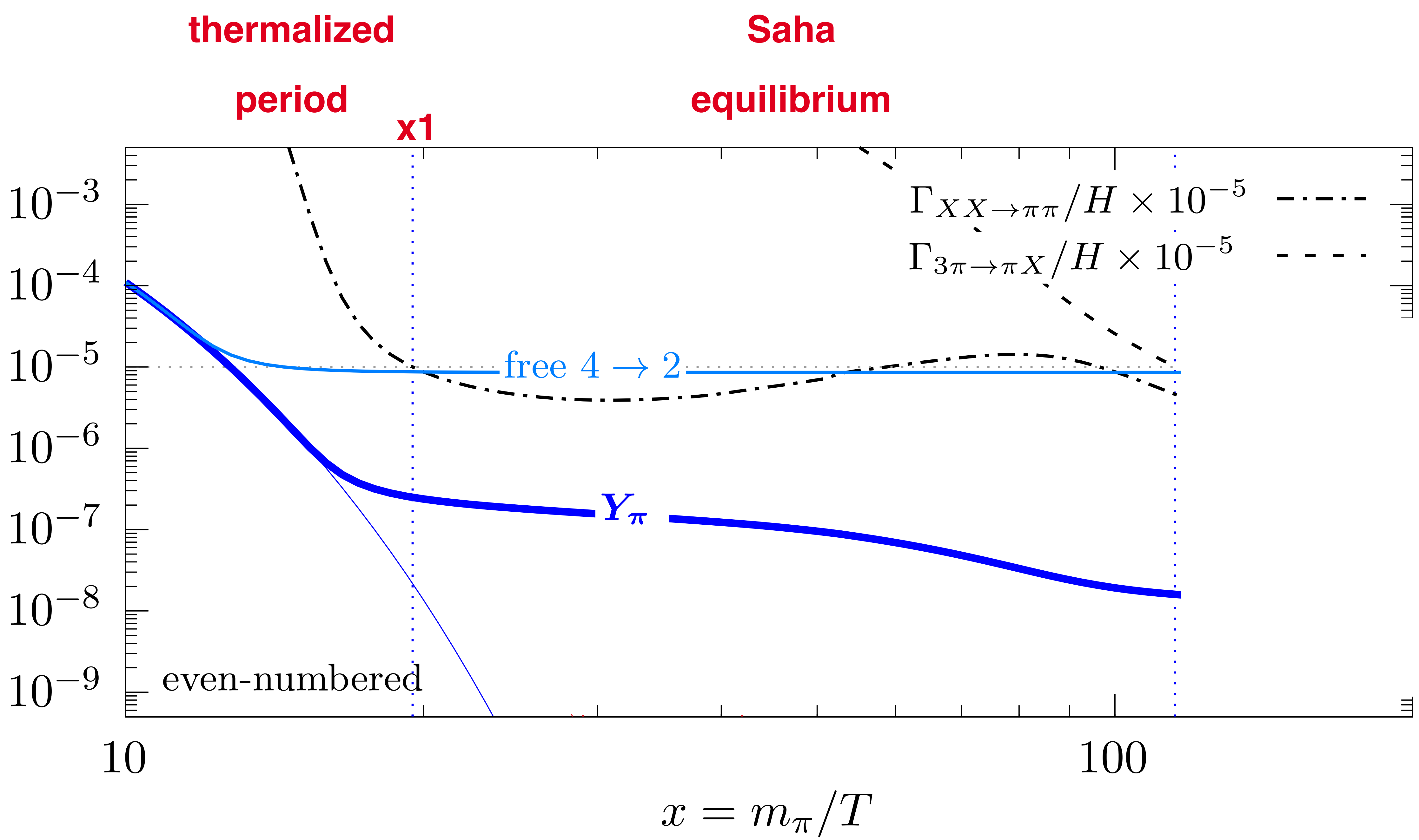


Bound-state formation maintains

Saha equilibrium between X and pi

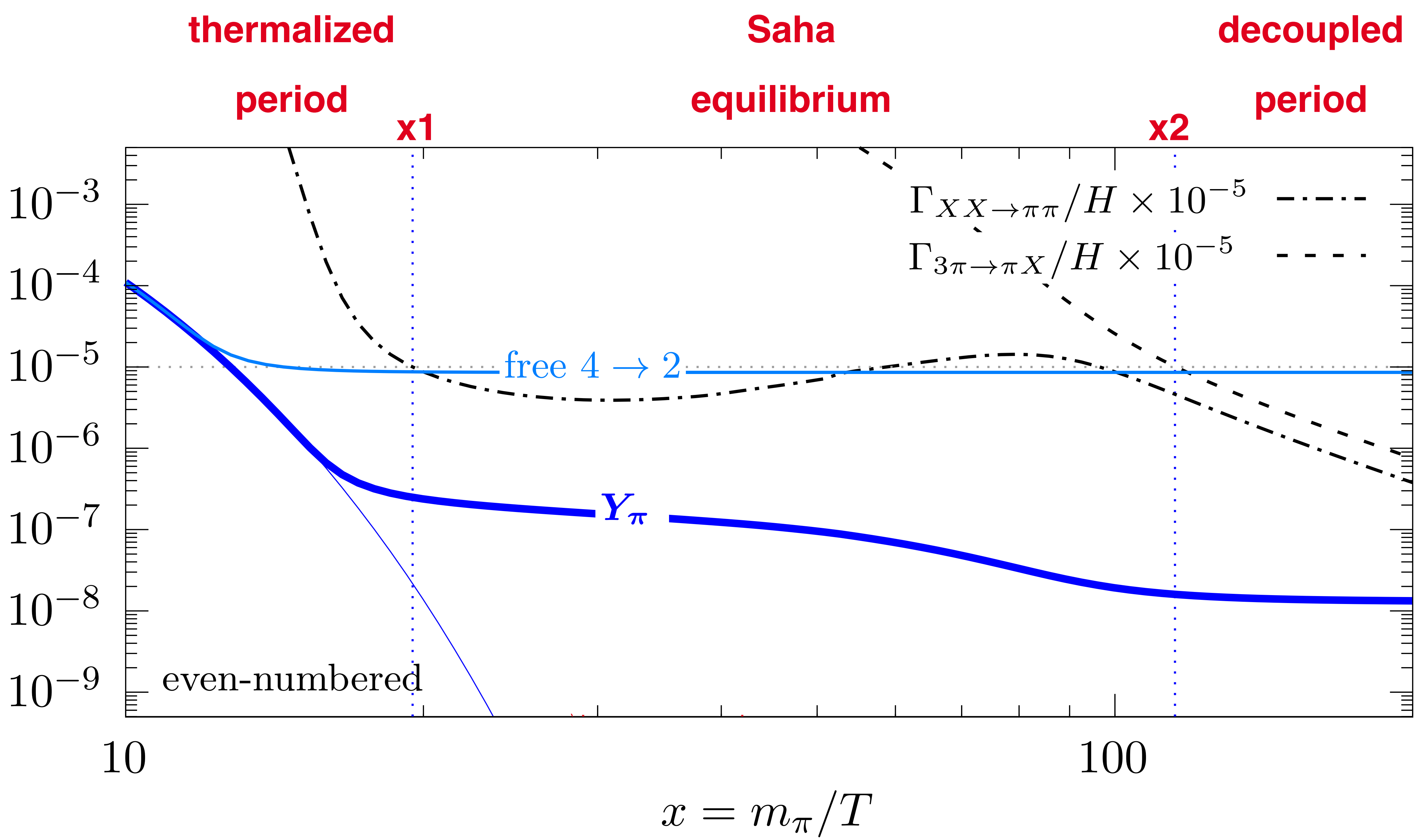
$$Y_X = \frac{Y_\pi^2 Y_X^{\rm eq}}{(Y_\pi^{\rm eq})^2}$$

CASE 1: even-numbered interactions only



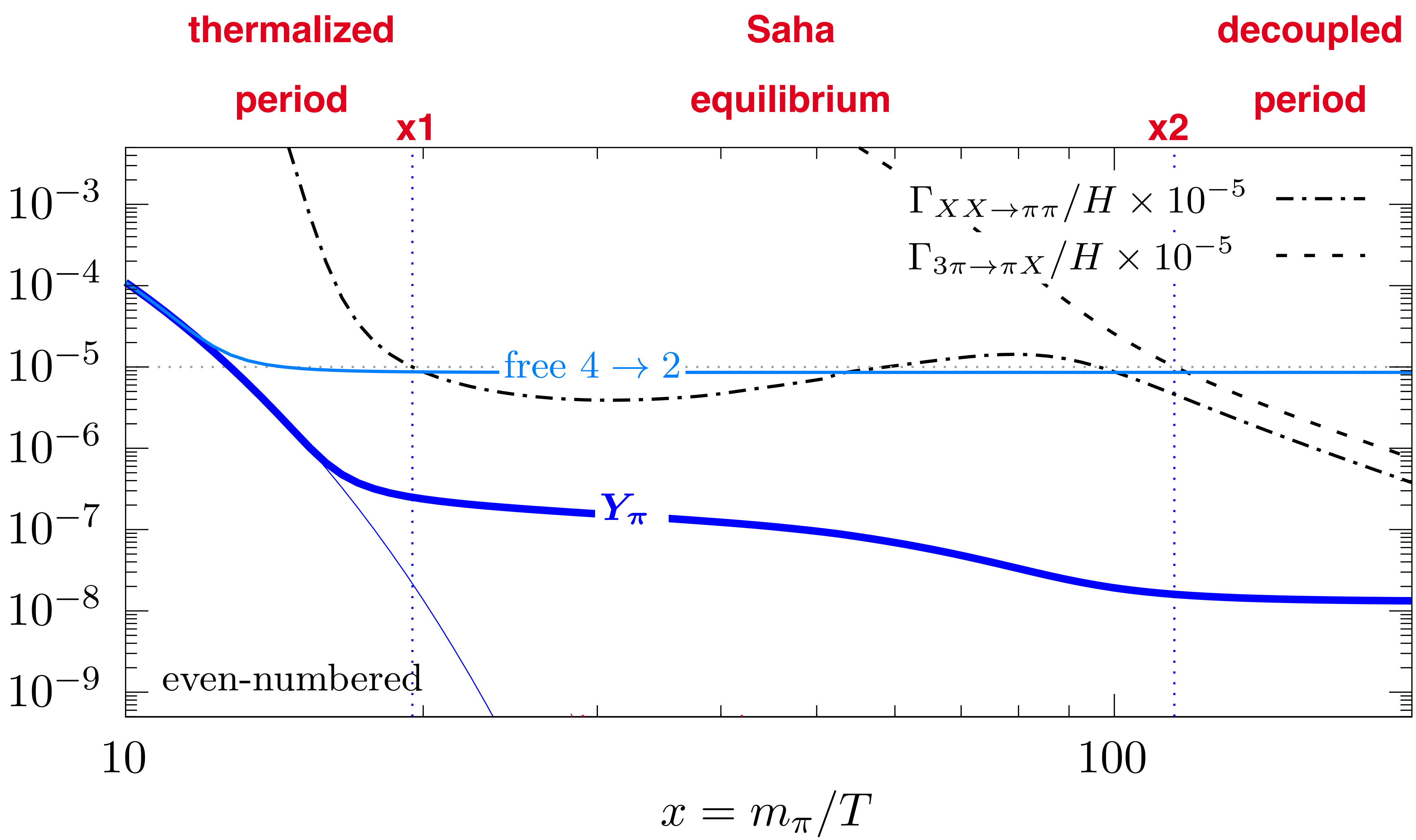
mass depletion $$\frac{d(Y_\pi + 2Y_X)}{dx} = -\frac{2\langle\sigma_{XX\to\pi\pi}v\rangle Y_X^2 s(x)}{xH(x)} \quad (x > x_1)$$

CASE 1: even-numbered interactions only



$$Y_\pi^{-3}(x_2) \simeq \frac{256\sqrt{2}\pi^8 g_*^{5/2} m_\pi M_P \langle\sigma_{XX\to\pi\pi}v\rangle}{6075\sqrt{5}x_2^4}\frac{N_X^2}{N_\pi^4} \times \left[8(\kappa x_2)^4\,\mathrm{Ei}(2\kappa x_2) - e^{2\kappa x_2}\left(3 + 2\kappa x_2 + 2\kappa^2 x_2^2 + 4\kappa^3 x_2^3\right)\right]$$

CASE 1: even-numbered interactions only

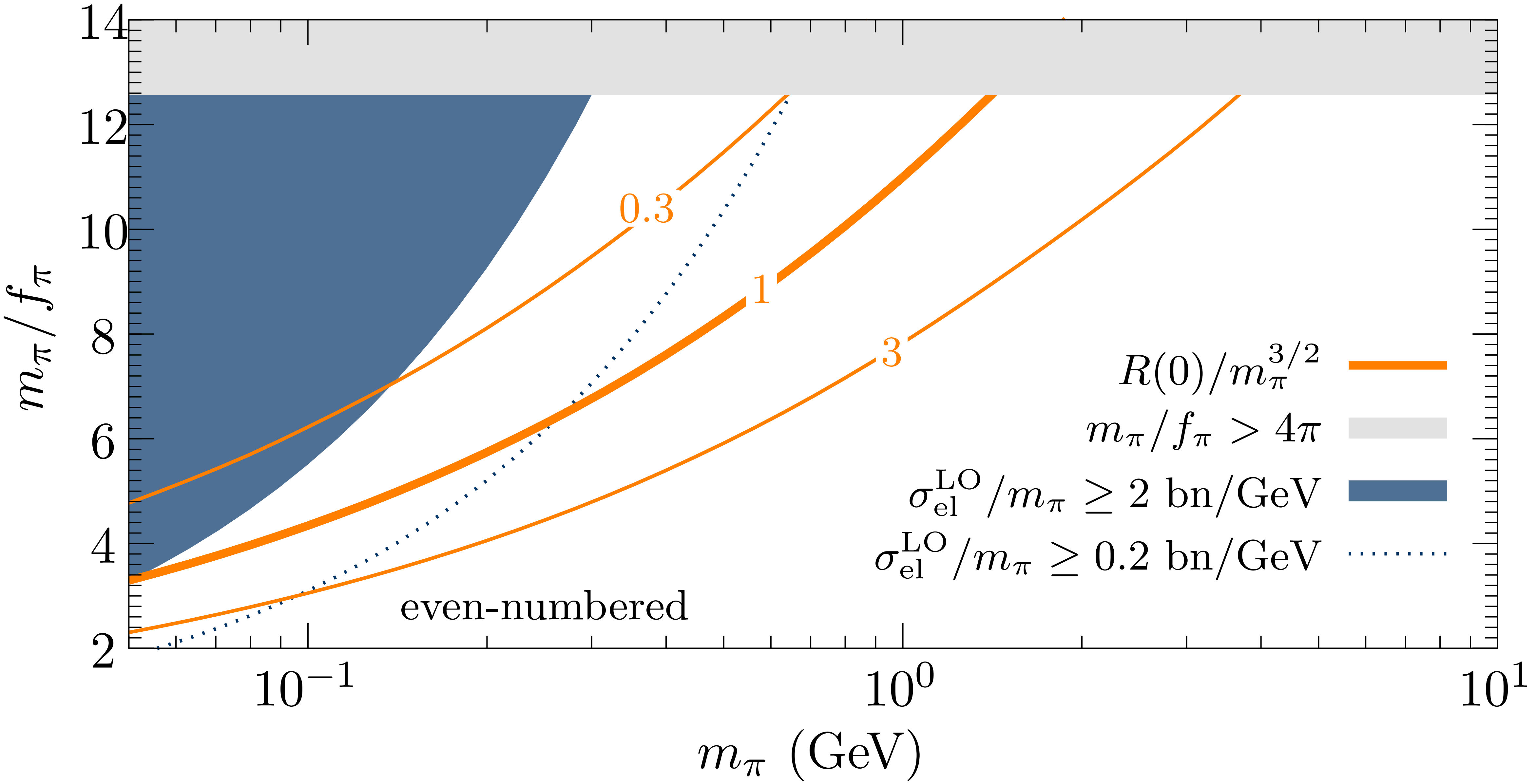


$$\Omega_\pi^{\rm even} \sim 0.2 \left(\frac{10^3}{\kappa^4 e^{\kappa x_2}} \frac{\rm bn/GeV}{\langle \sigma_{XX\to\pi\pi} v\rangle/m_\pi} \frac{m_\pi}{\rm GeV} \right)^{1/3}$$

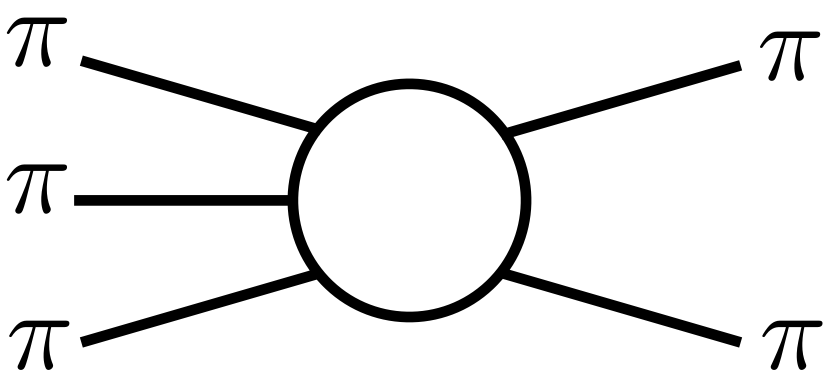
x2 - dependent!

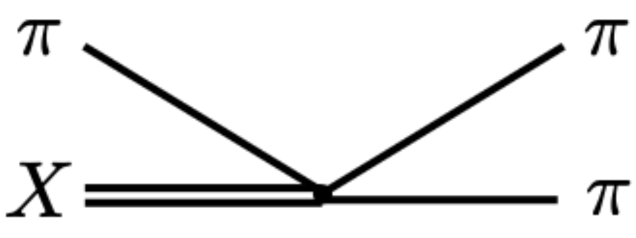
Even SIMP miracles are possible!

coincidence of correct relic density + interesting self scattering ballpark



CASE 2: odd-numbered interactions





catalyzed
$3 \to 2$ annihilation

standard WZW annihilation **(d-wave)**

$$\langle \sigma_{3\pi\to 2\pi} v^2 \rangle = \frac{\sqrt{5} N_c^2 m_\pi^3 T^2}{12800 \pi^5 f_\pi^{10}}$$

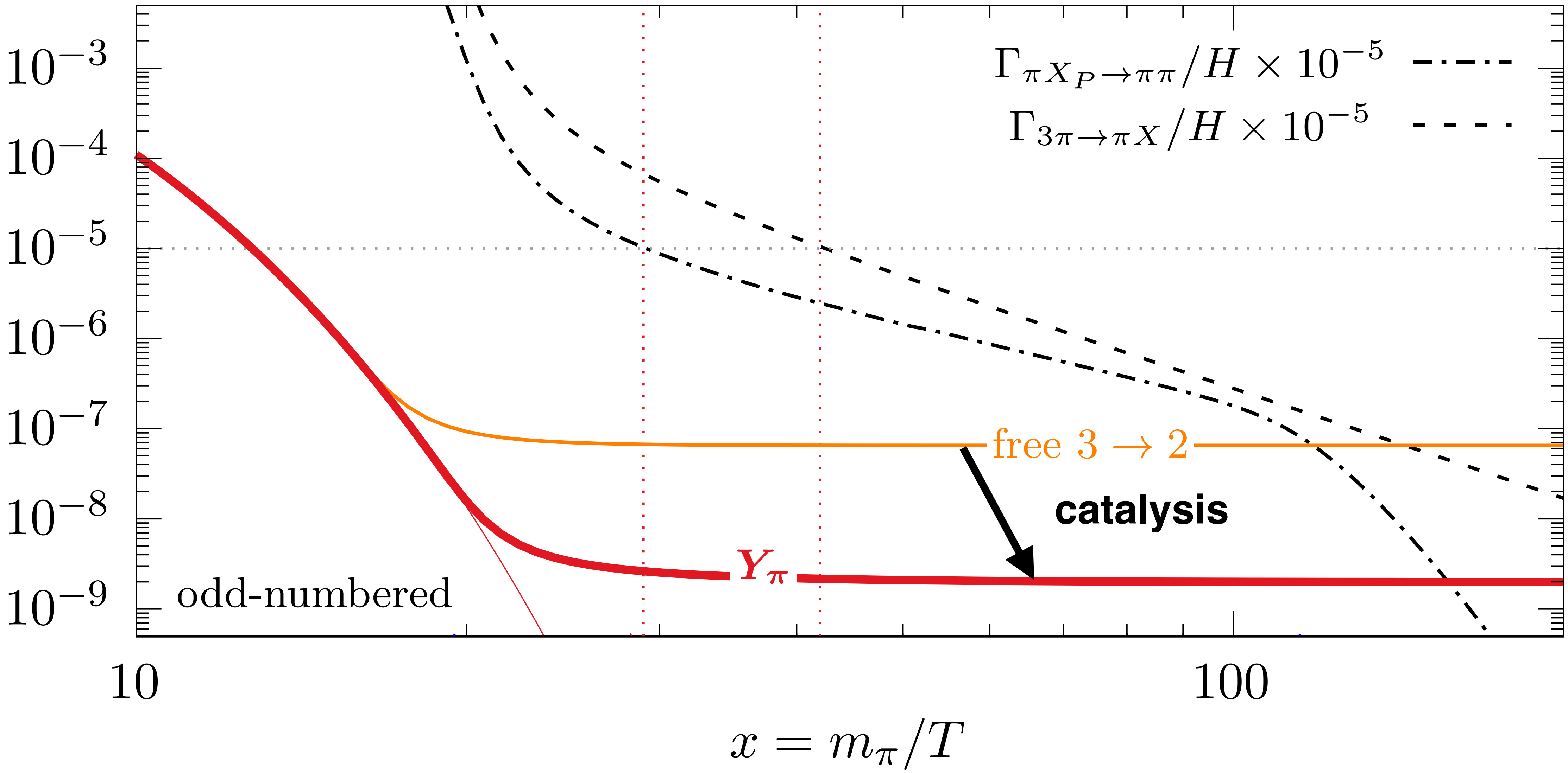
derivative of radial wave function of X **(p-wave)**

$$\langle \sigma_{\pi X \to 2\pi} v \rangle = \frac{\sqrt{5} N_c^2 R'(0)^2 m_\pi^2}{512 \pi^6 f_\pi^{10}} T$$

p-wave X are available through collisional excitation

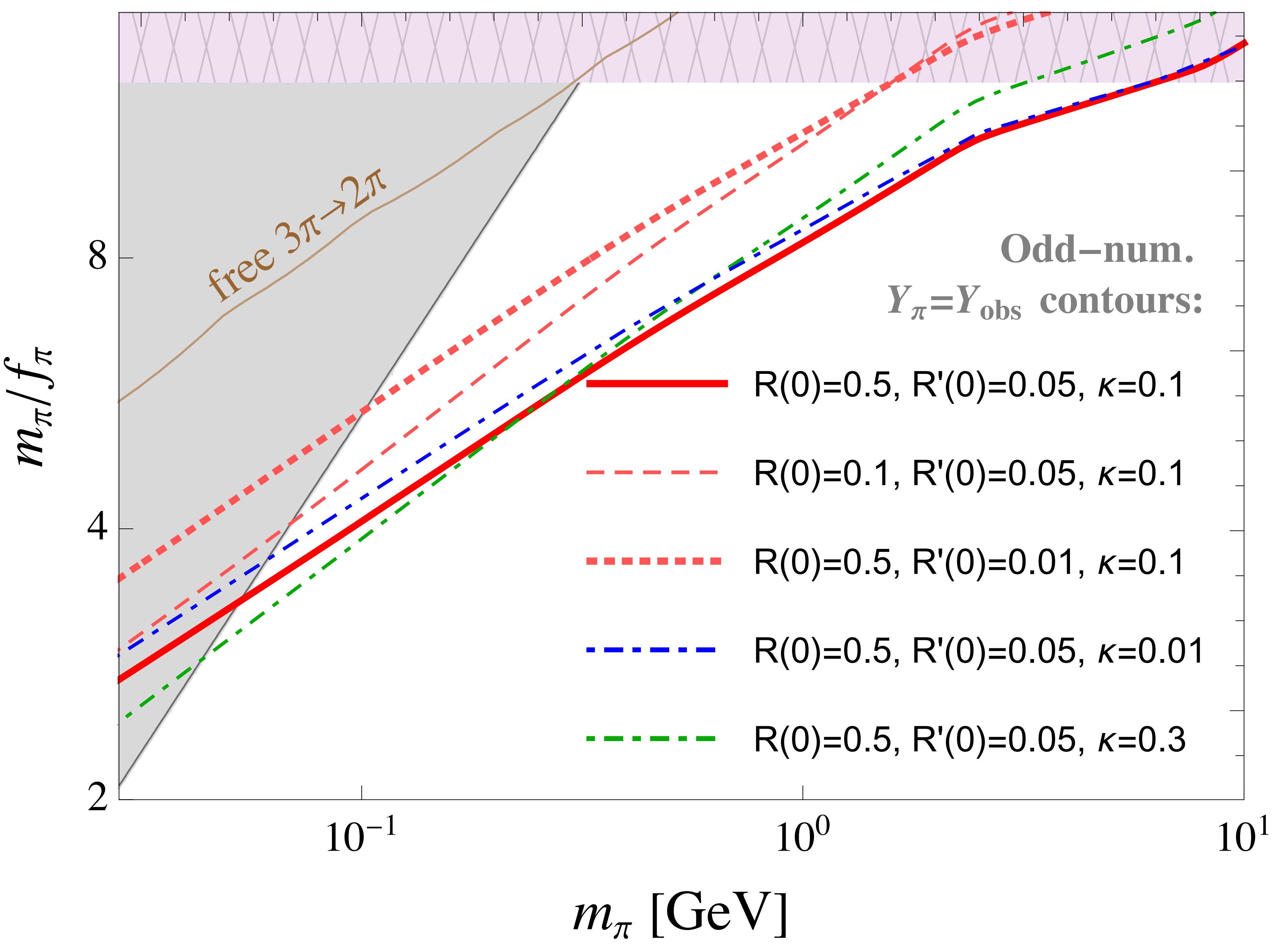
$$\frac{n_{X_P}}{n_{X_S}} = 3 e^{-|E_S - E_P| T / m_\pi}$$

CASE 2: odd-numbered interactions

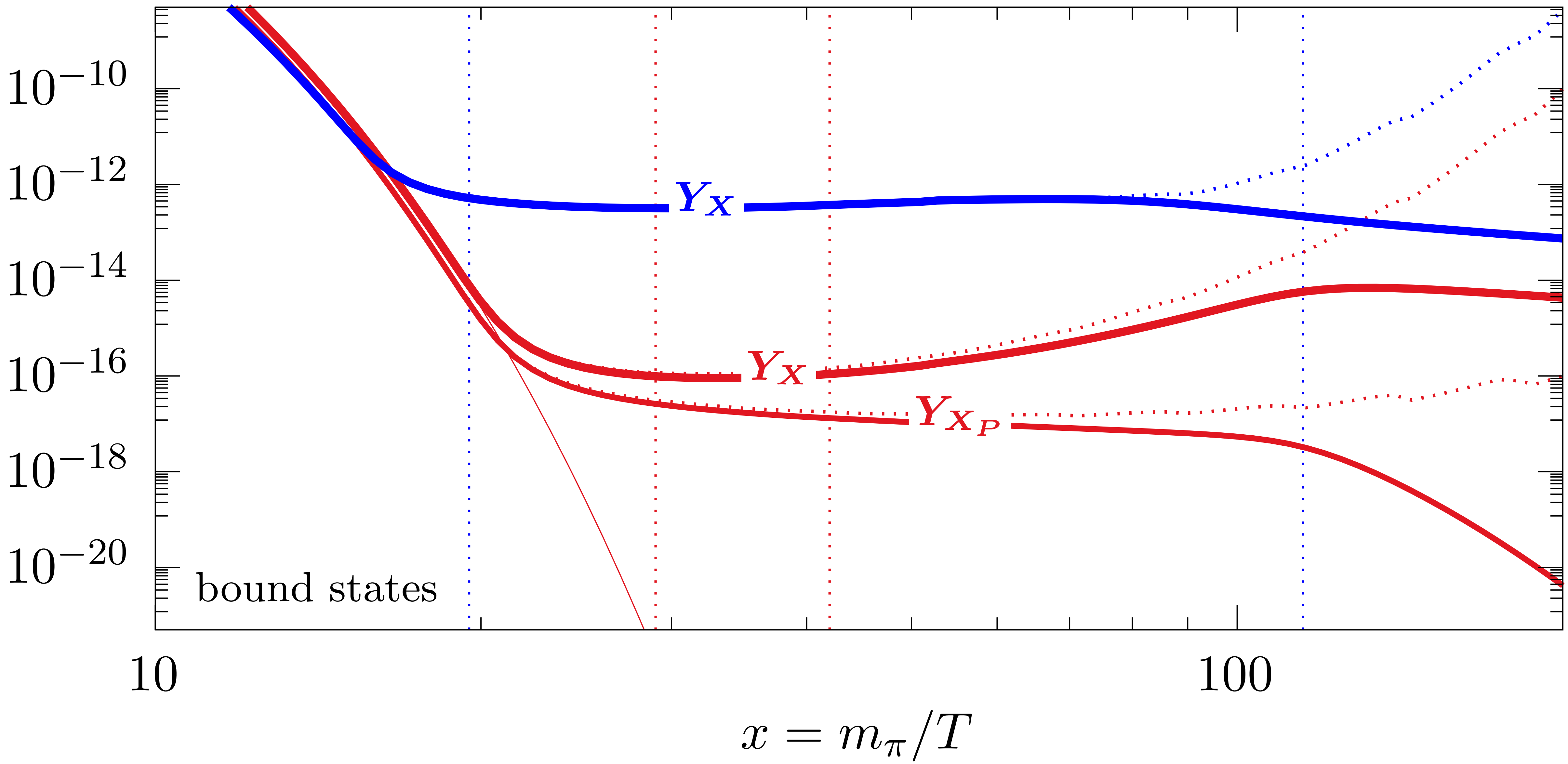


$$\Omega_\pi^{\mathrm{odd}} \simeq 0.2 \left(\frac{x_1}{20}\right)^{5/4} \left(\frac{e^{-\kappa_P x_1}\, 10^{-3}\, \mathrm{bn/GeV}}{\langle \sigma_{\pi X_P \to \pi\pi} v\rangle / m_\pi}\right)^{1/2}$$

CASE 2: odd-numbered interactions



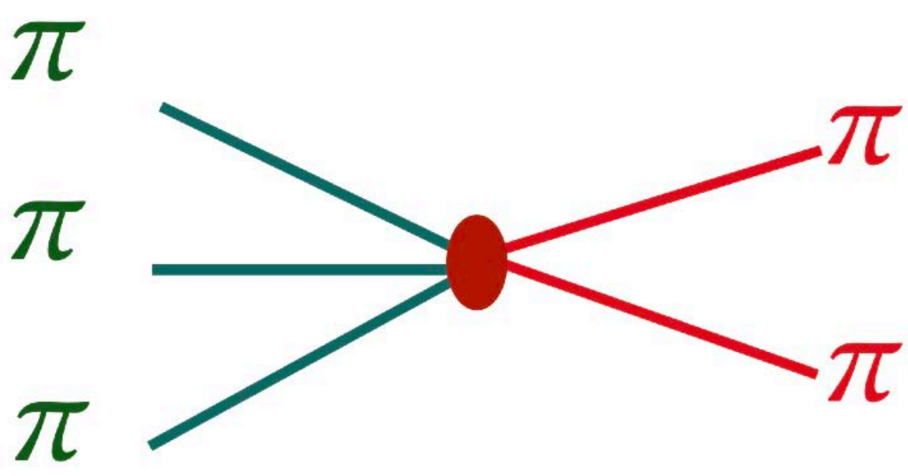
What bound states do



two-body process remains efficient even after pions are frozen out

$$n_X \langle \sigma_{XX\to\pi\pi} v \rangle > H(x_2)$$

Coupling to Standard Model



SIMPs in isolation lead to HOT dark matter (excluded)

SIMPs must come into kinetic equilibrium with the SM plasma (=share the same temperature)

$\pi\,\mathrm{SM}_i \to \pi\,\mathrm{SM}_i$ with $\Gamma_{\pi\,\mathrm{SM}} = \langle \sigma_{\pi\,\mathrm{SM}} c \rangle n_i > H$

=> typically enables $\pi\pi \to \mathrm{SM}_i \overline{\mathrm{SM}}_i$ but OK, because $n_i/n_\pi \gg 1$

HERE: destabilizes the bound state

$$X = [\pi\pi] \to \mathrm{SM}_i \overline{\mathrm{SM}}_i$$

Meta-stability of X

$$X = [\pi\pi] \to \mathrm{SM}_i\overline{\mathrm{SM}}_i$$

Noting that $|\psi(0)|^2 v$ has units of particle flux => $\Gamma_X \sim |\psi(0)|^2(\sigma_{\rm ann} v)$

$$\Gamma_X/H < 1 \quad \Rightarrow \quad \sigma_{\rm ann} v \lesssim 10^{-3}\mathrm{pb}\; x^{-2}\left(\frac{m_\pi}{100\ \mathrm{MeV}}\right)^2 \frac{\mathrm{MeV}^3}{|\psi(0)|^2}$$

Taking $\sigma_{\pi\,\rm SM} c \sim \sigma_{\rm ann} v$, the stability requirement (X lives beyond freeze out) imposes upper limit on the elastic scattering rate that is needed to make Dark Matter "cold".

$$1 \lesssim \frac{\Gamma_{\pi\,\rm SM}}{H} \lesssim \frac{10^6}{x^3}\left(\frac{m_\pi}{100\ \mathrm{MeV}}\right)^3 \frac{\mathrm{MeV}^3}{|\psi(0)|^2}$$

=> can easily be satisfied: retain kinetic equilibrium while maintaining sufficient longevity of X, paired with sub-Hubble two-body annihilation

=> no escalated model building requirements in comparison to original works on the SIMPs

=> previously explored phenomenology remains in place

X-catalyzed SIMP mechanism

When coupled to SM

additional X formation and breakup reactions may open

=> the detailed balancing condition

$$Y_X = \frac{Y_\pi^2 Y_X^{\rm eq}}{\left(Y_\pi^{\rm eq}\right)^2}$$ **remains unaltered**

=> If the new processes dominate over $3\pi \leftrightarrow \pi X$, detailed balancing retains its validity longer

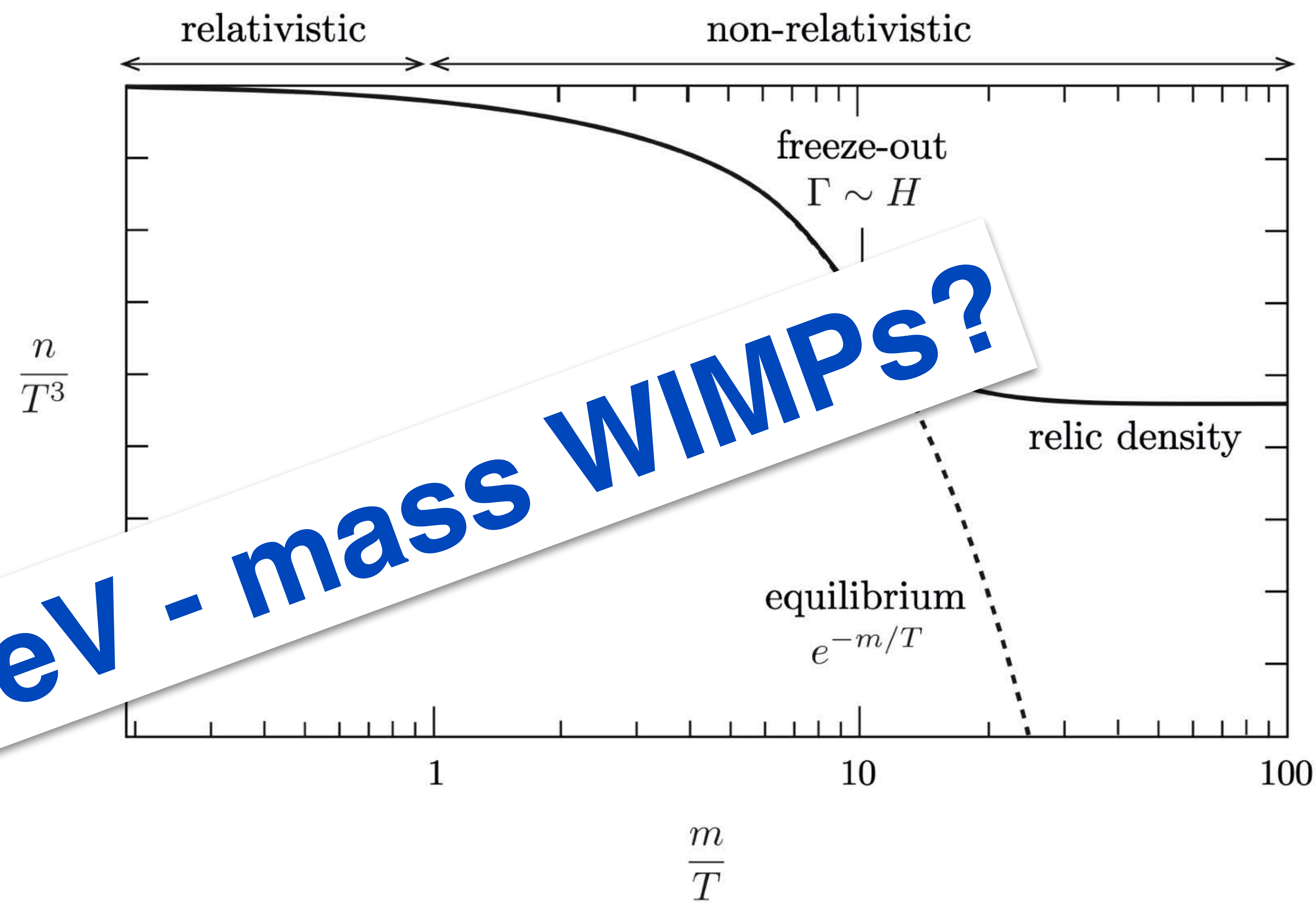
=> x2 will be larger, and **relic density smaller**

Introduction of **SM-interactions** harbor the potential to make **X-assisted freeze-out even more efficient**, without jeopardizing the overall picture!

2. WIMP dead end

Chu, Kuo, JP, PRD 2022
Chu, JP PRD 2024

OR: what is the lightest thermal DM mass?



MeV - mass WIMPs?

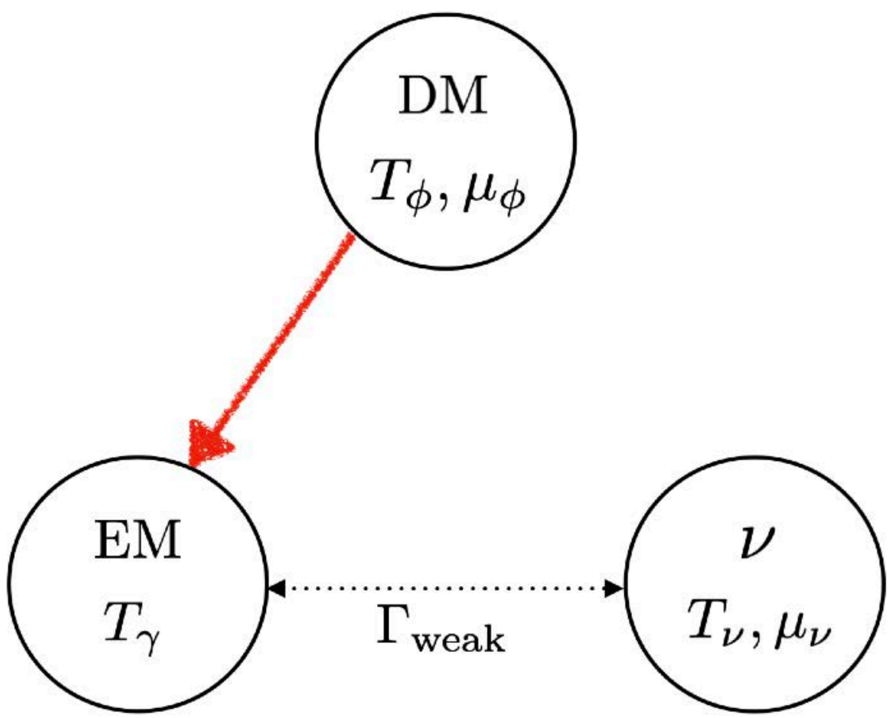
2. Thermal MeV DM

Chu, Kuo, JP, PRD 2022
Chu, JP PRD 2024

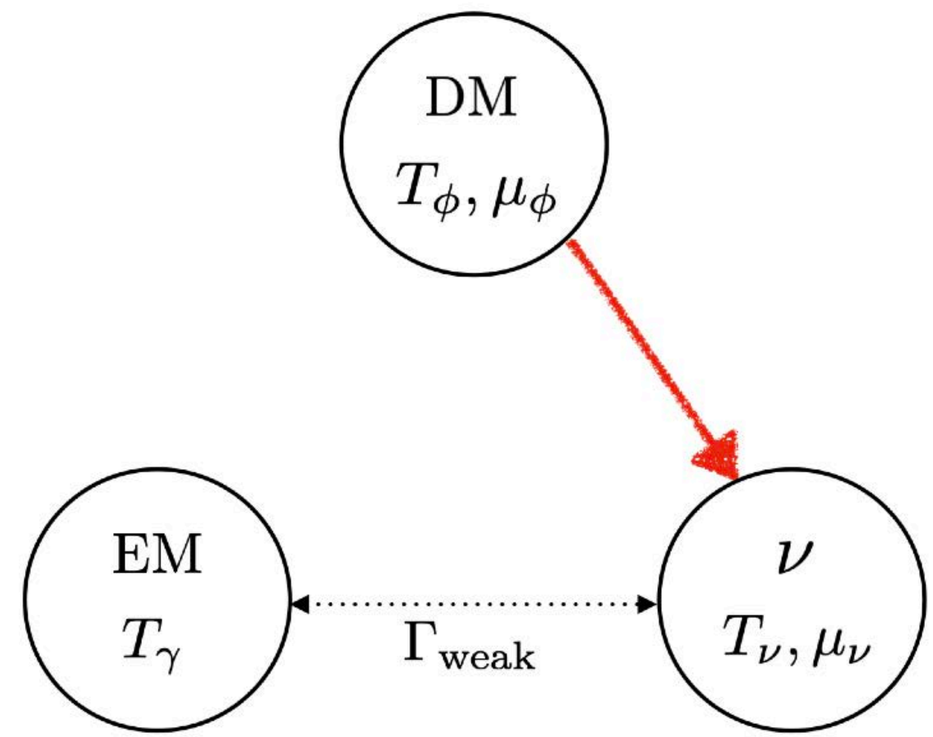
OR: what is the lightest thermal DM mass?

Well known that MeV-DM subject to Neff bound from heating by annihilation

Previous treatments had to assume a branching either into EM-sector OR neutrinos:



OR



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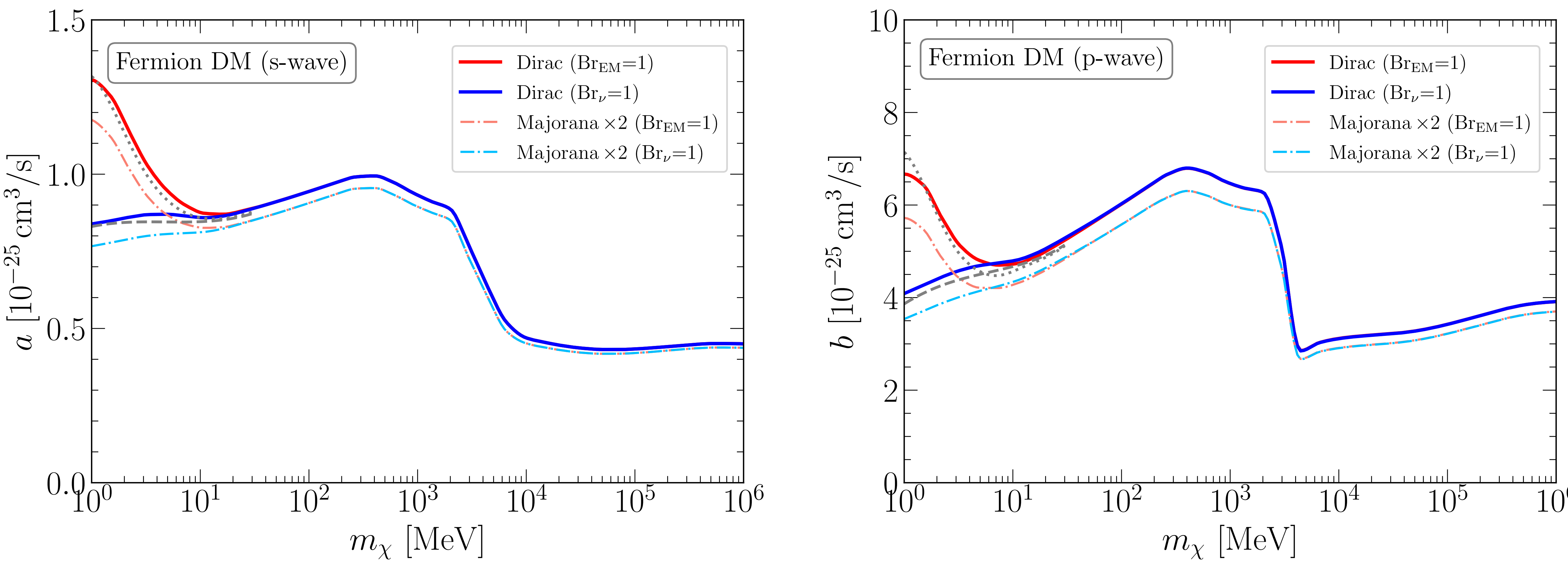

$$\langle \sigma_{\rm ann} v \rangle = a + b\,(6T/m_{\phi,\chi})$$

2. Thermal MeV DM

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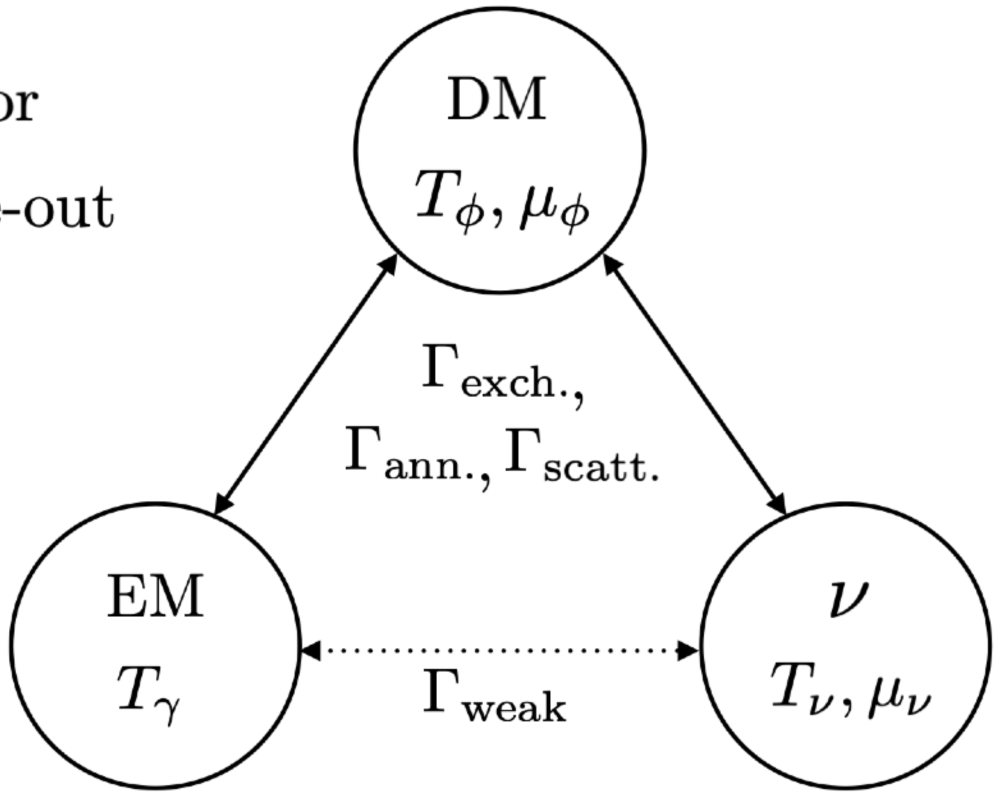
In the full picture, joint treatment of the three coupled sectors is necessary

three-sector
DM freeze-out



$$\Gamma_{\text{weak}} \equiv n_e G_F^2 T_\gamma^2 \,,$$
$$\Gamma_{\text{ann.}} \equiv n_\phi \langle \sigma_{\text{ann.}} v \rangle \,,$$
$$\Gamma_{\text{exch.},i} \equiv n_\phi^2 \langle \sigma_{\text{ann.},i} v \delta E \rangle / \rho_i \,,$$
$$\Gamma_{\text{scatt.},i} \equiv n_i \langle \sigma_{\text{scatt.}}^{\phi i} v \rangle \,.$$

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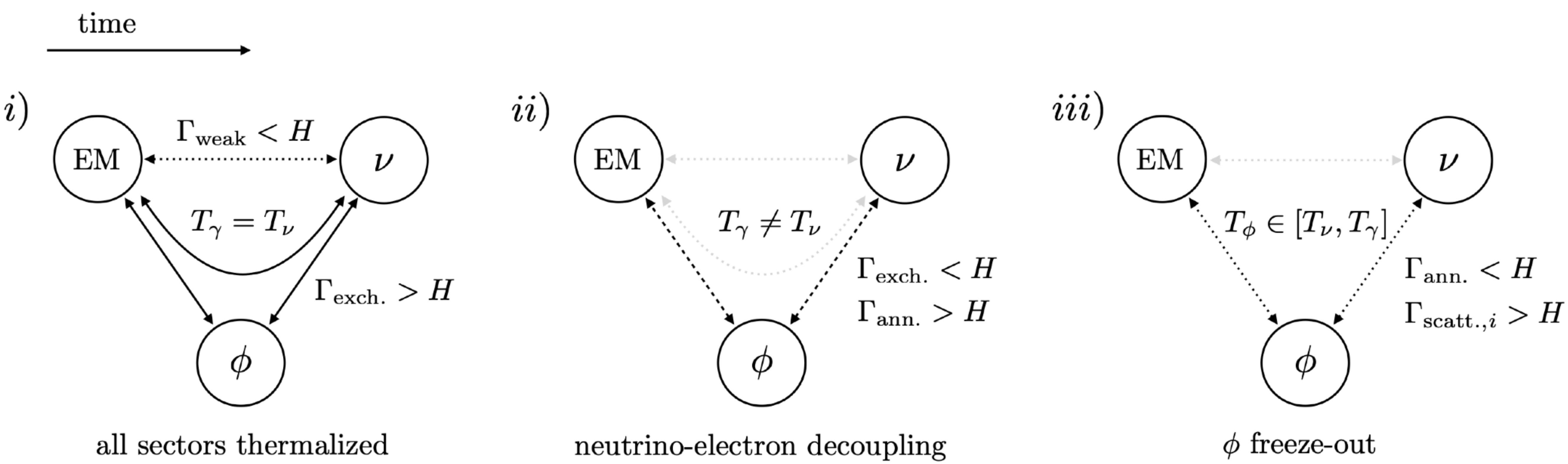
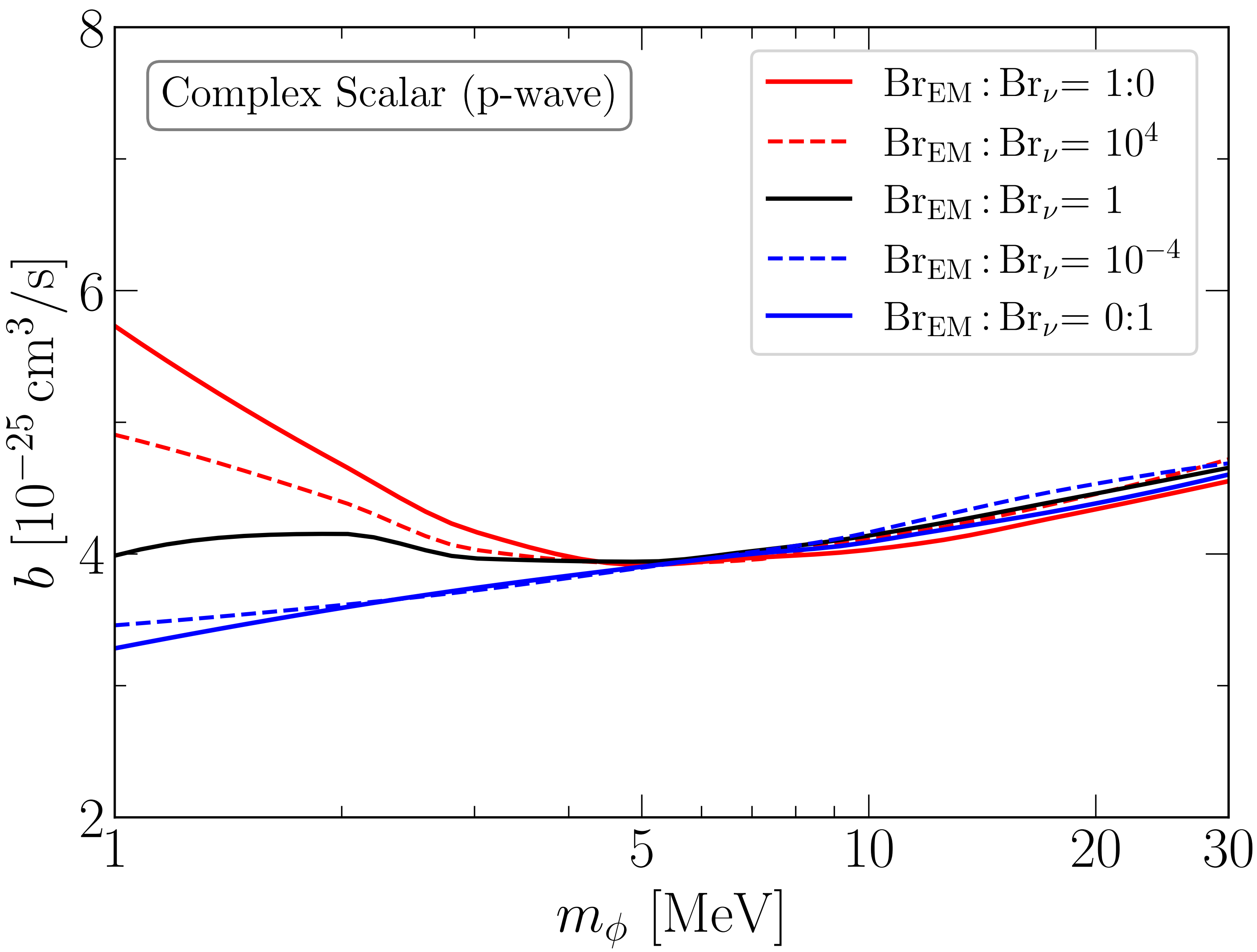


Light DM freeze out

Thermal cross section

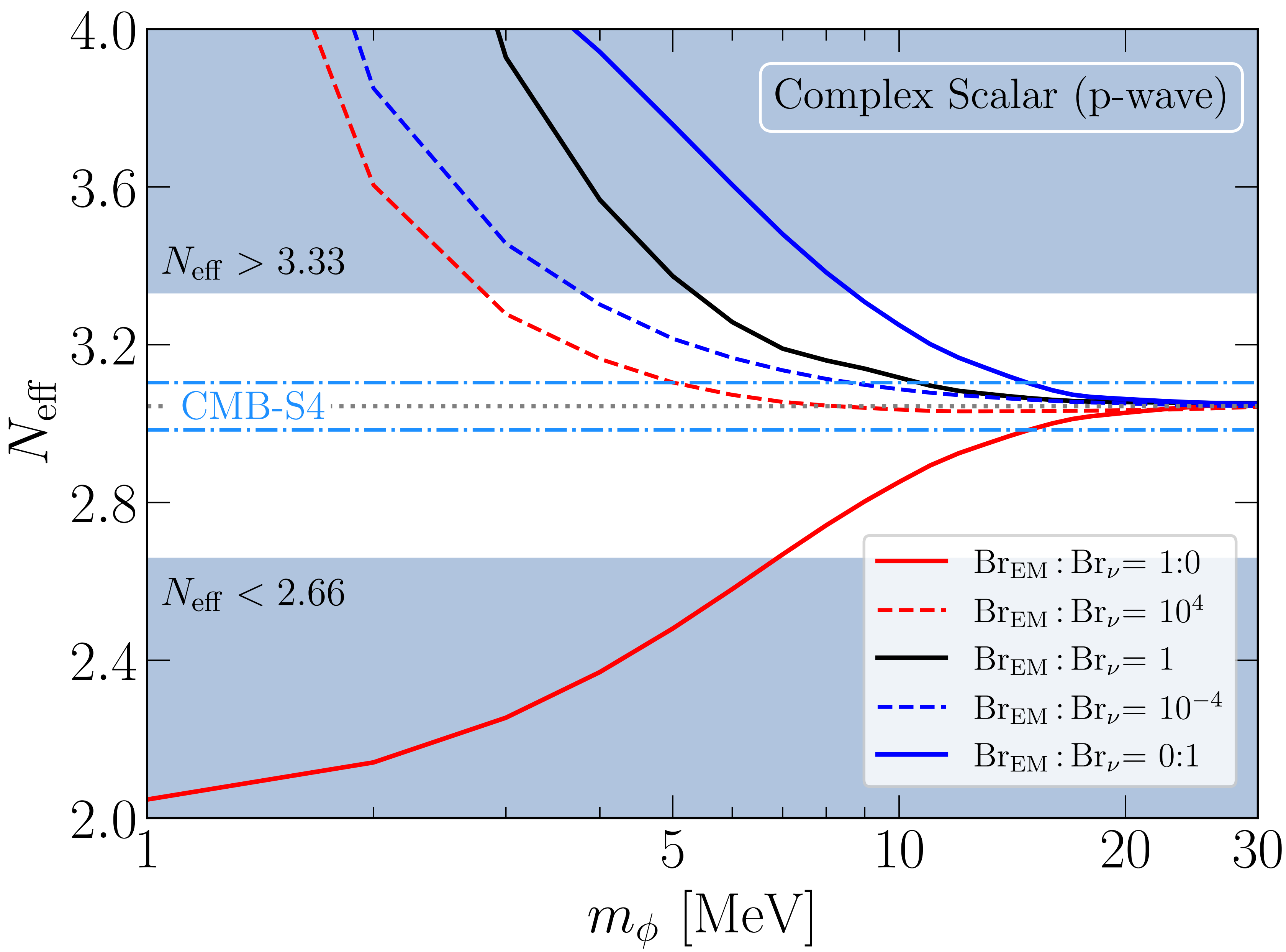


Example: p-wave annihilation

$$\mathcal{L}_{Z'}^{\rm int} = g_\phi^2 Z'^\mu Z'_\mu \phi^* \phi - i g_\phi Z'^\mu (\phi^* \overleftrightarrow{\partial}_\mu \phi) - g_l Z'^\mu \bar{l} \gamma_\mu l \,.$$

Light DM freeze out

What is the lightest thermal DM mass?

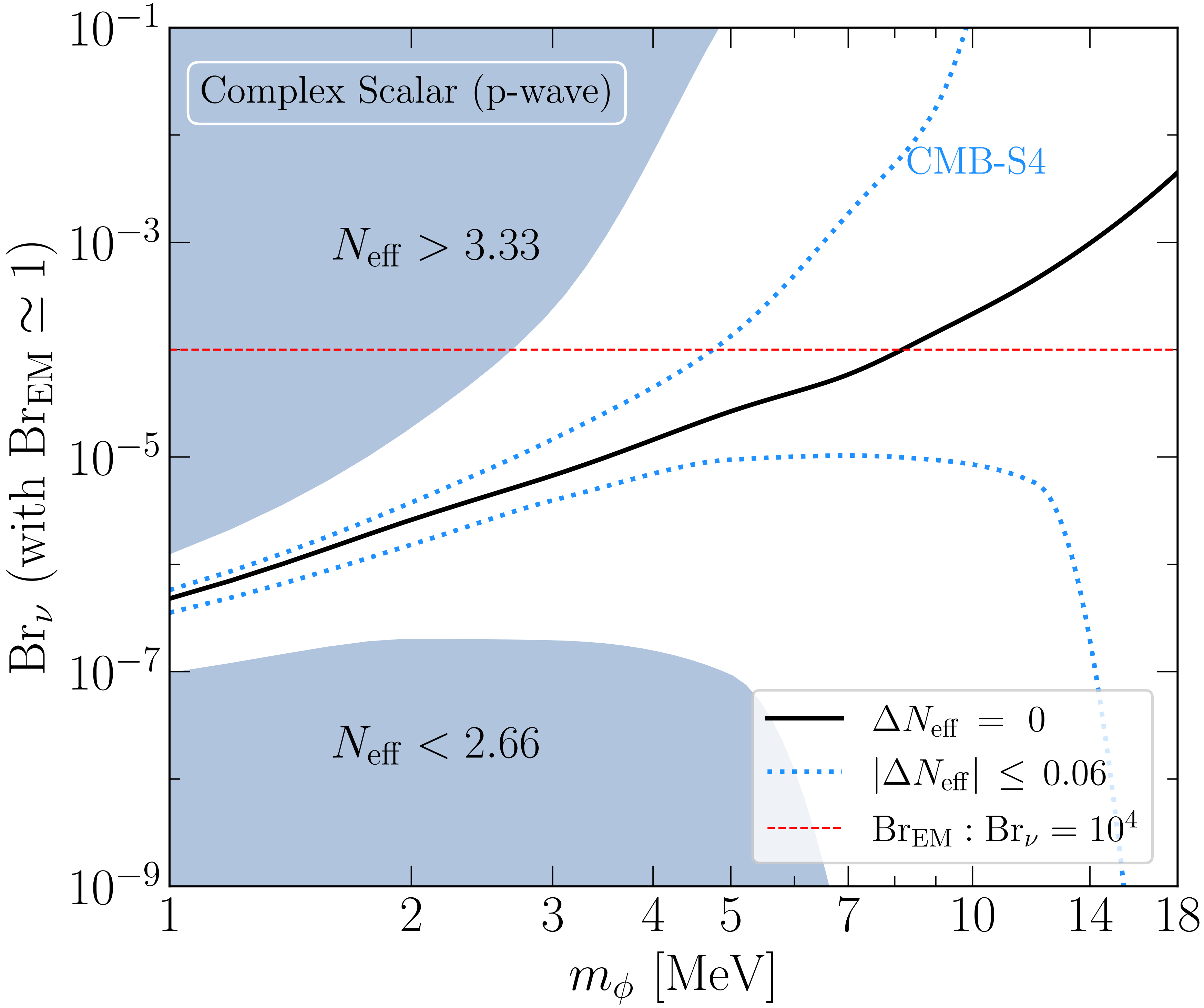


Example: p-wave annihilation

$$\rho_{\rm rad} = \left[1 + \frac{7}{8}\left(\frac{4}{11}\right)^{4/3} N_{\rm eff}\right]\rho_\gamma$$

Evading Neff bound

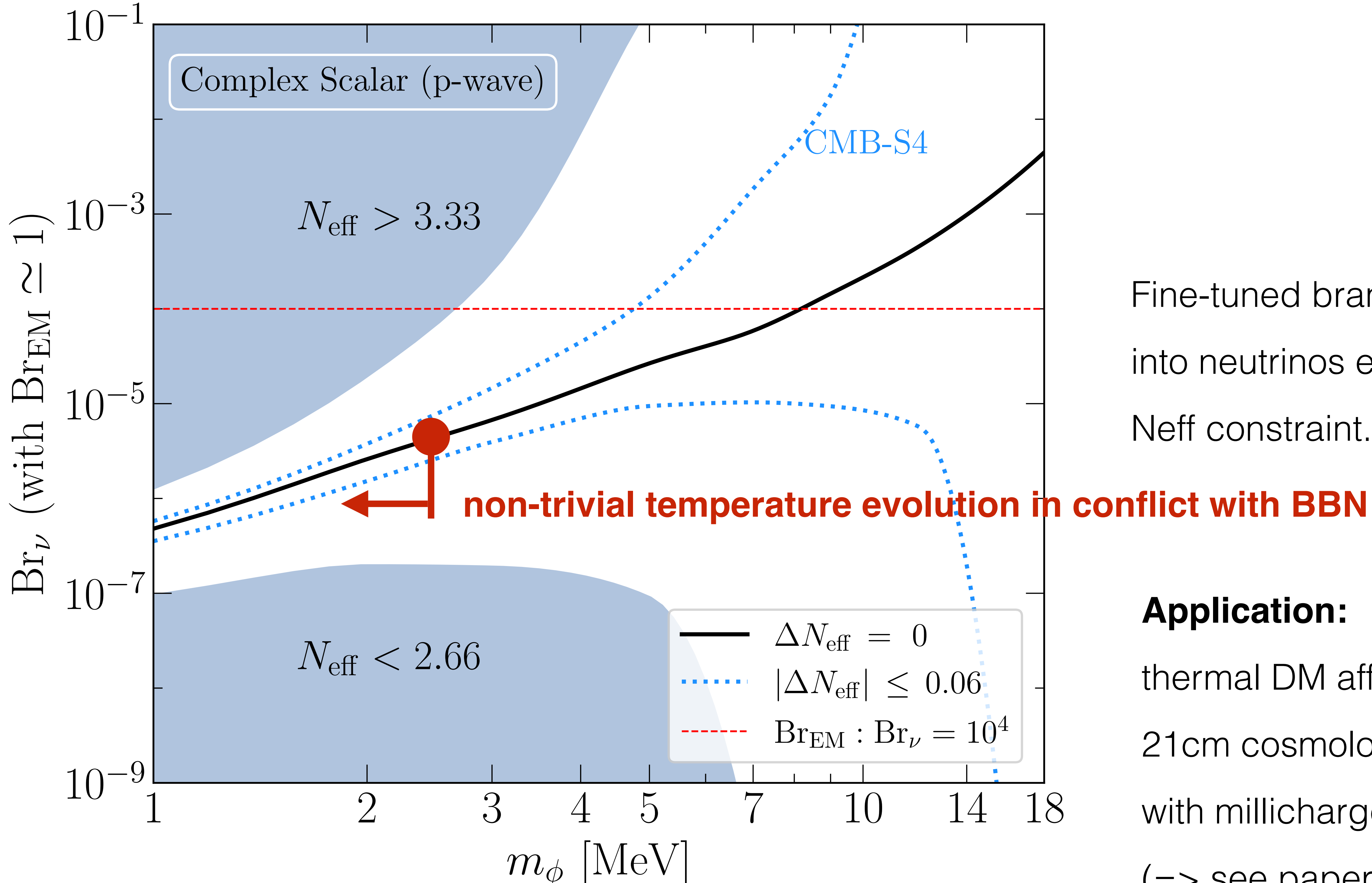
OR: How low can you go?



Fine-tuned branching into neutrinos evades Neff constraint.

Evading Neff bound

OR: How low can you go?



Fine-tuned branching into neutrinos evades Neff constraint.

Application:

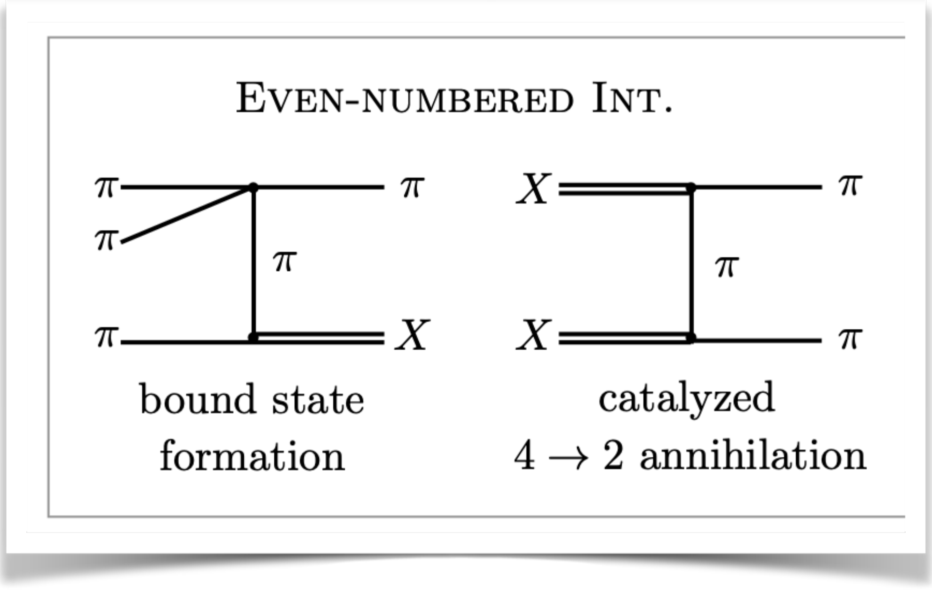
thermal DM affecting 21cm cosmology with millicharged DM

(=> see paper)

Summary

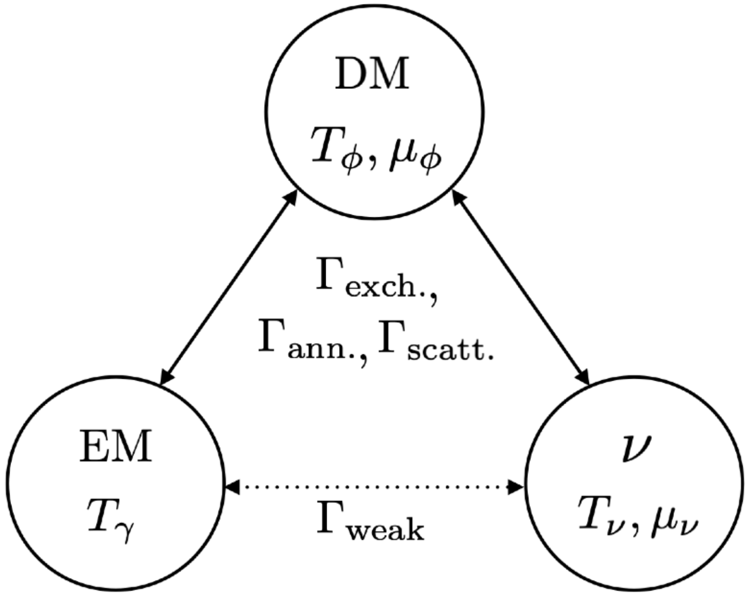
Freeze-out of MeV-mass DM candidates

- Small-scale structure problems pertinent to LCDM may be a hint for DM self-interactions, naturally realized in theories with strongly interacting particles (SIMPs)
- When SIMPs regulate their relic abundance in N->2 processes, bound states — should they exist — significantly alter the standard picture.
- Even-numbered SIMP-mechanism is possible



……………………

- A comprehensive assessment of thermal MeV-scale DM necessitates a three-sector treatment of vastly changing rates => systematic formulation
- nice application for DM affecting 21cm cosmology



Thank you