Microscopic calculations of neutrino scattering and absorption in warm dense matter

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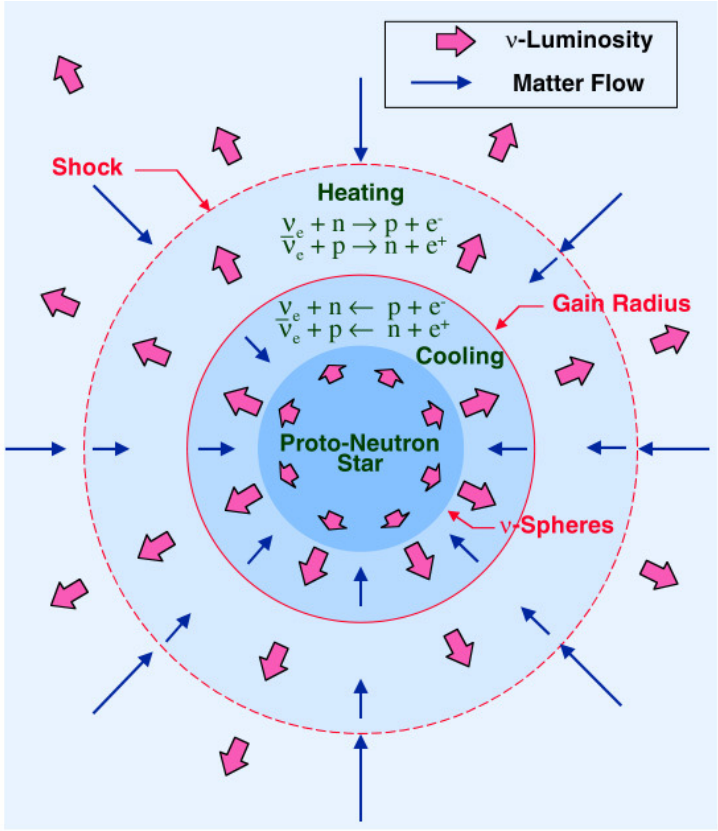
Texas A&M University, College Station

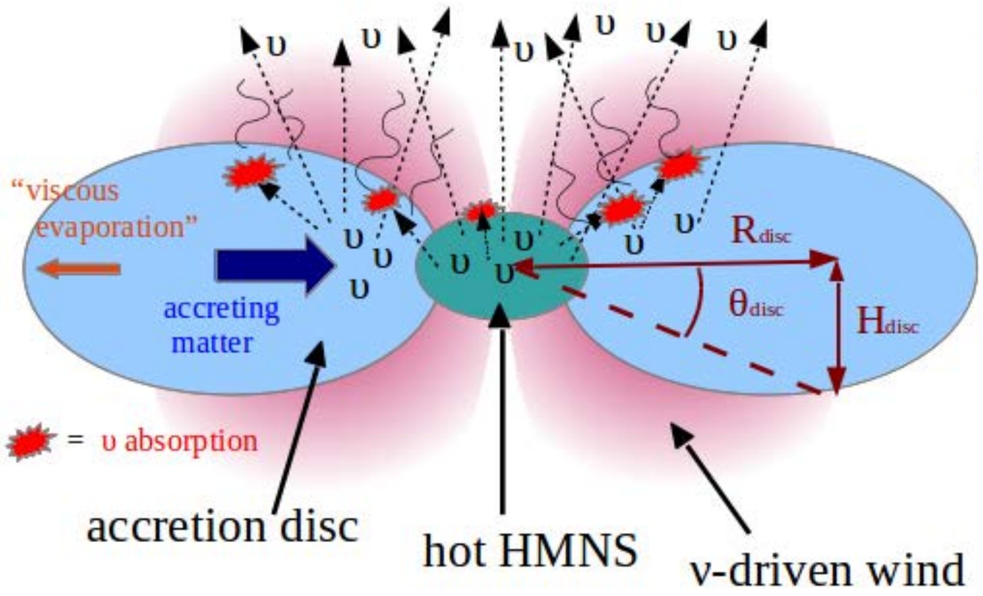
*E. Shin, E. Rrapaj, J. W. Holt and S. Reddy, PRC 109 (2024) 015804

N3AS Seminar: 4/23/24

Neutrinos in core-collapse supernovae and neutron star mergers



Mezzacappa (2005)



Perego (2014)

- Hydrodynamic evolution (energy, momentum, lepton number transport)
- Composition (proton fraction) through charged-current reactions
- Nucleosynthesis (weak & strong r-process, νp process,…)
- Deep probe of supernova dynamics for Earth-based detectors

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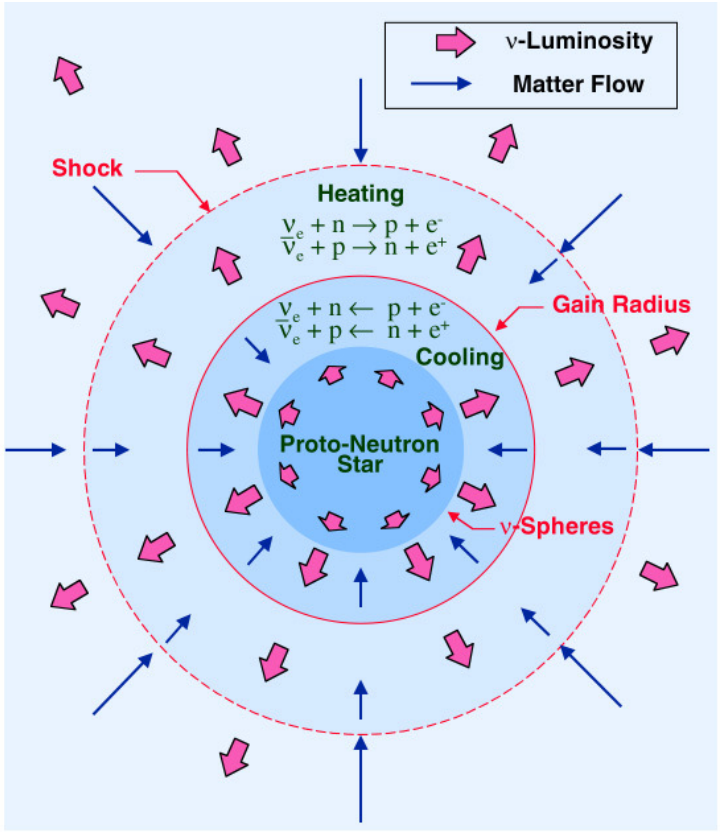

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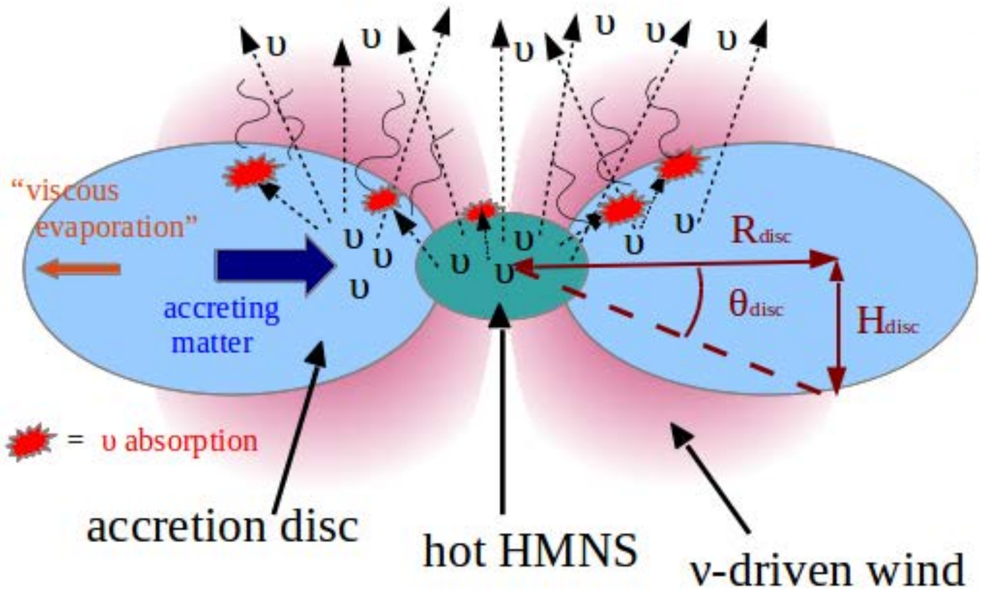

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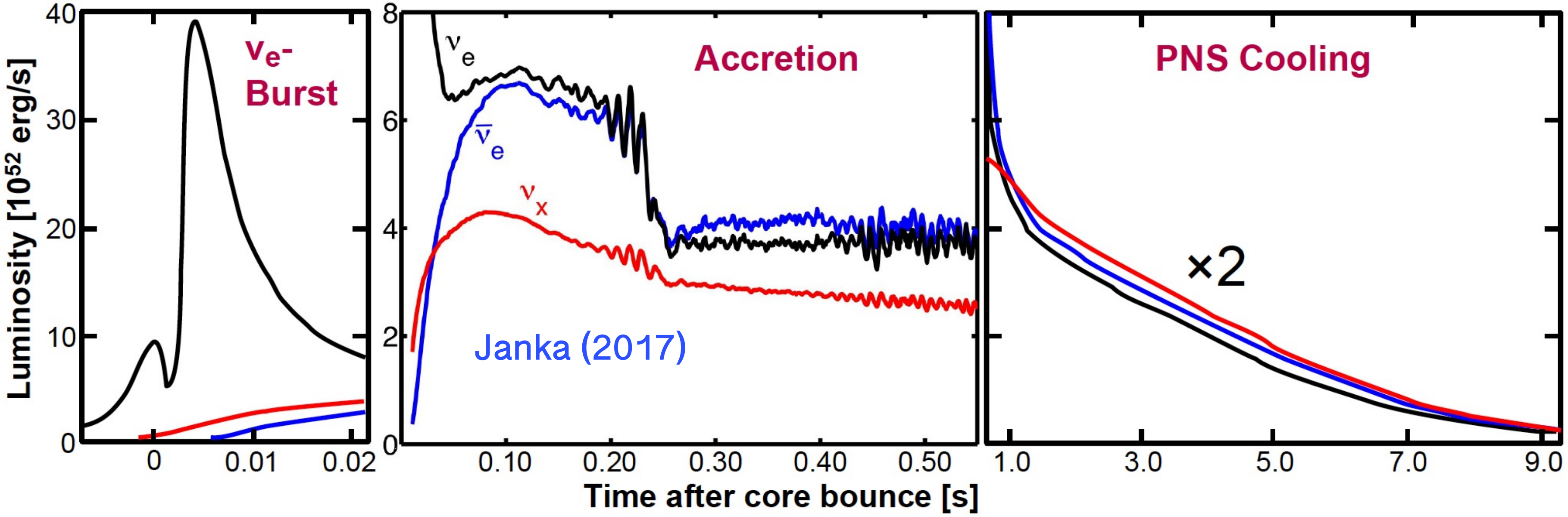
- Deep probe of supernova dynamics for Earth-based detectors

Sensitive to decoupling energies, luminosities,…

Neutrinos from core-collapse supernovae



Beta processes:

- $e^- + p \rightleftharpoons n + \nu_e$
- $e^+ + n \rightleftharpoons p + \bar{\nu}_e$
- $e^- + A \rightleftharpoons \nu_e + A^*$

Neutrino scattering:

- $\nu + n, p \rightleftharpoons \nu + n, p$
- $\nu + A \rightleftharpoons \nu + A$
- $\nu + e^\pm \rightleftharpoons \nu + e^\pm$

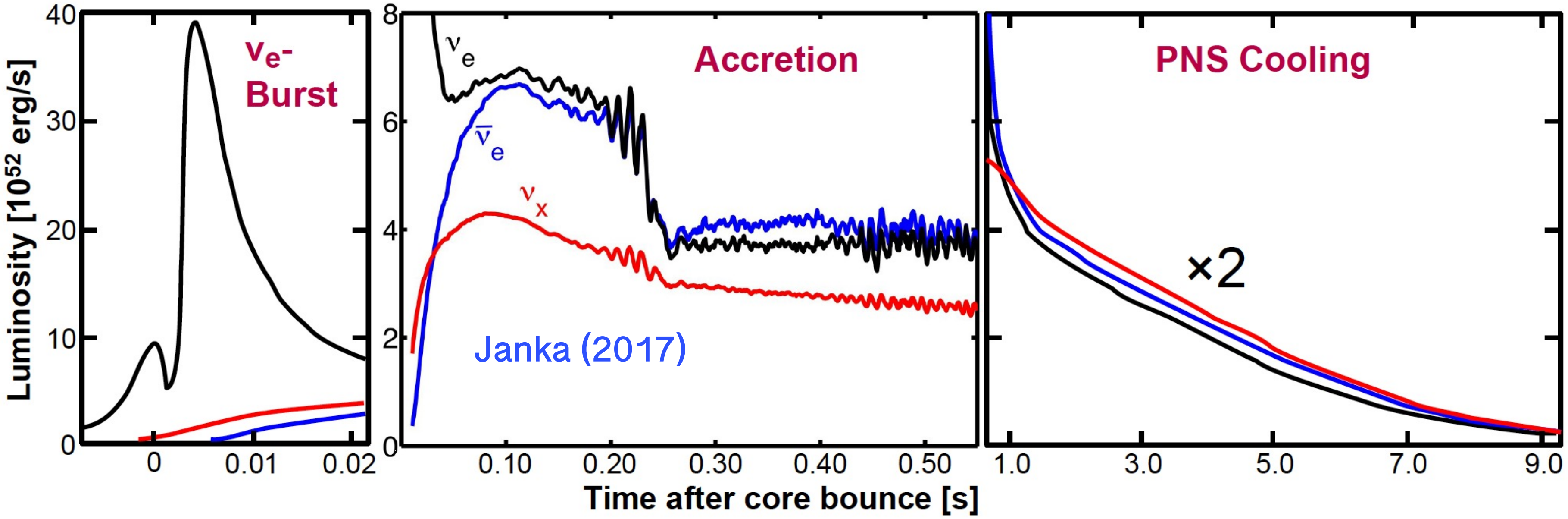
Thermal pair processes:

- $N + N \rightleftharpoons N + N + \nu + \bar{\nu}$
- $e^+ + e^- \rightleftharpoons \nu + \bar{\nu}$

Neutrino-neutrino reactions:

- $\nu_x + \nu_e, \bar{\nu}_e \rightleftharpoons \nu_x + \nu_e, \bar{\nu}_e$
  $(\nu_x = \nu_\mu,\ \bar{\nu}_\mu, \nu_\tau,$ or $\bar{\nu}_\tau)$
- $\nu_e + \bar{\nu}_e \rightleftharpoons \nu_{\mu,\tau} + \bar{\nu}_{\mu,\tau}$

Neutrinos from core-collapse supernovae



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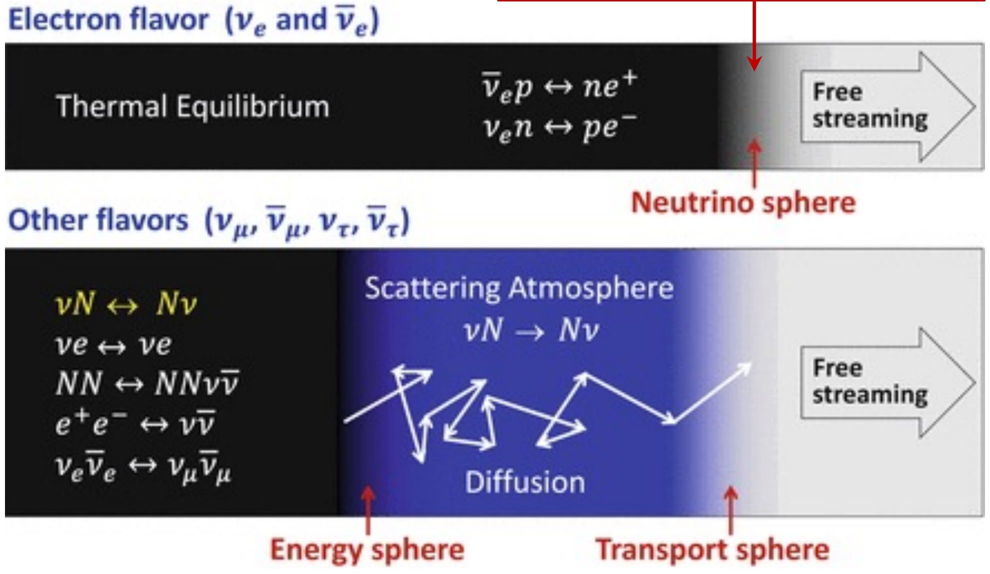
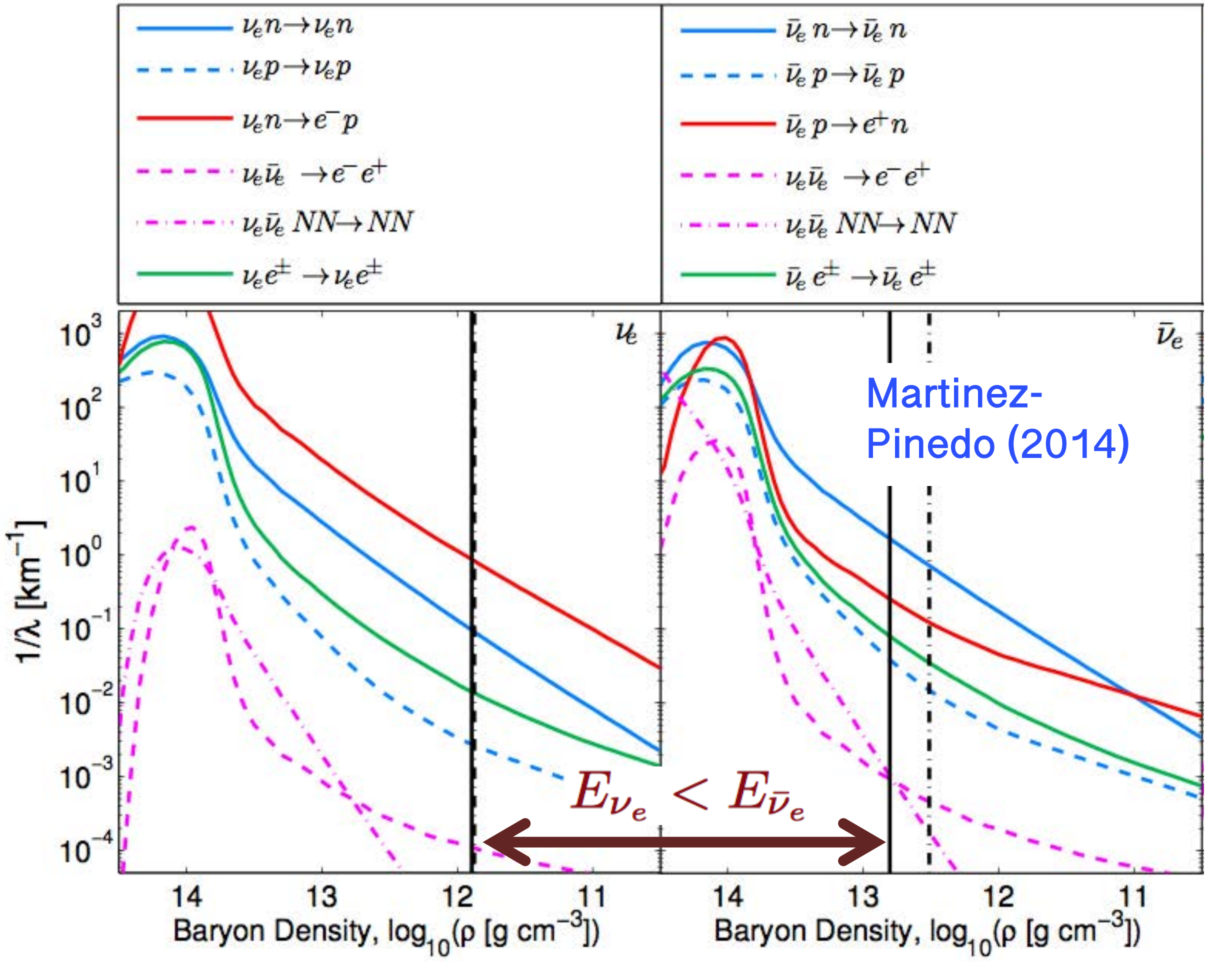
Supernova neutrino opacities

Neutrinosphere
T = 5 – 10 MeV,
n = $10^{11} - 10^{13}$ g/cm^3,
$Y_p \sim 0.05 - 0.10$



Janka (2017)

Neutrino opacity | Anti-neutrino opacity

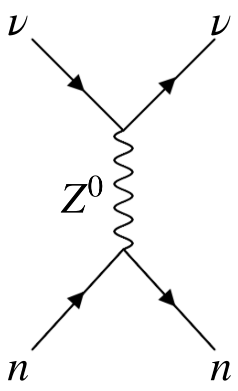


Martinez-Pinedo (2014)

Governs energies of free-streaming neutrinos

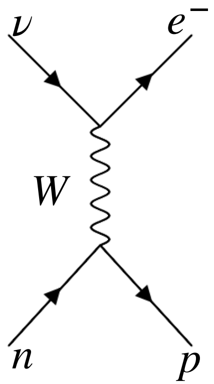
Differential scattering cross sections

- Neutrino-nucleon scattering (weak neutral-current reaction)



$$\frac{1}{V}\frac{d^2\sigma}{d\cos\theta\, d\omega} = \frac{G_F^2}{4\pi^2}E_3^2\left[c_V^2(1+\cos\theta)S_V(\omega,q) + c_A^2(3-\cos\theta)S_A(\omega,q)\right]$$

- Neutrino absorption (weak charged-current reaction)



$$\frac{1}{V}\frac{d^2\sigma}{d\cos\theta\, d\omega} = \frac{G_W^2}{4\pi^2}p_e E_e\left(1 - f_e(E_e)\right)\left[g_V^2(1+\cos\theta)S_V^\tau(\omega,q) + g_A^2(3-\cos\theta)S_A^\tau(\omega,q)\right]$$

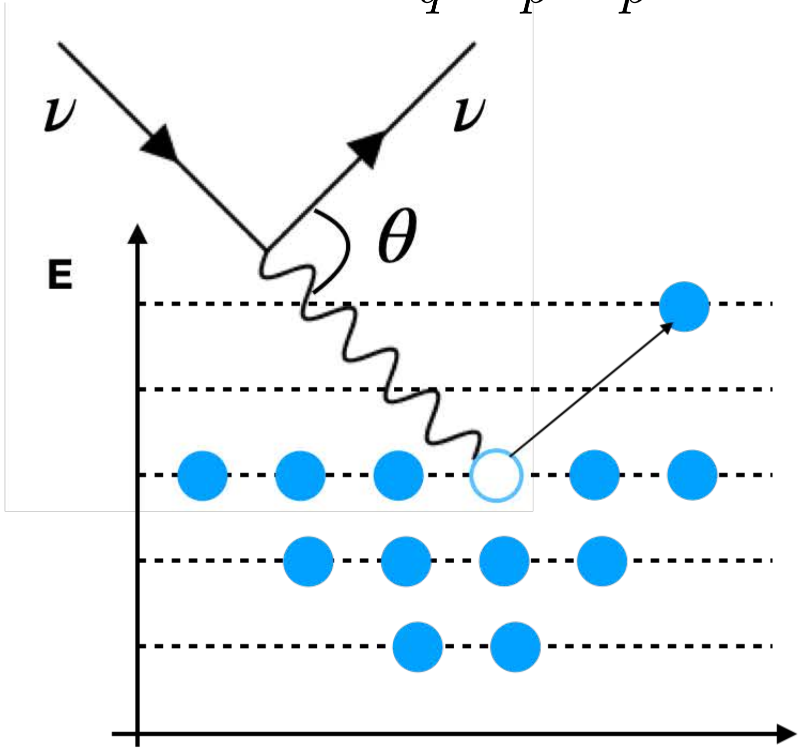
- Relation to response function χ (fluctuation-dissipation theorem):

$$S(\omega,q) = \frac{-2}{n}\frac{1}{1-e^{-\beta\omega}}\mathrm{Im}\,\chi(q,\omega)$$

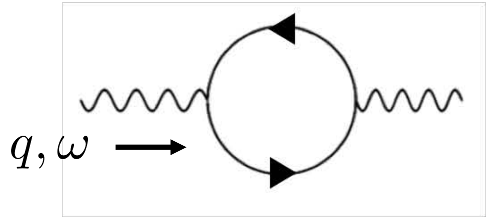
Response functions in many-body perturbation theory

$$\omega = E_\nu - E'_\nu$$

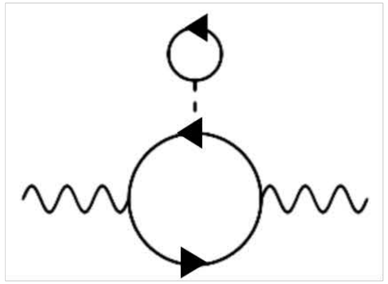
$$\vec{q} = \vec{p} - \vec{p}\,'$$

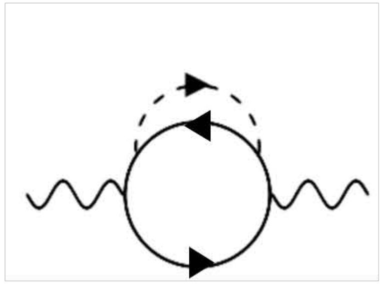


0th order

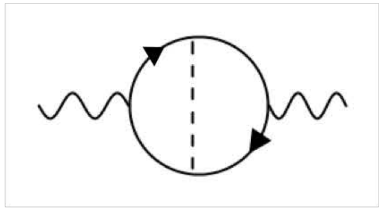


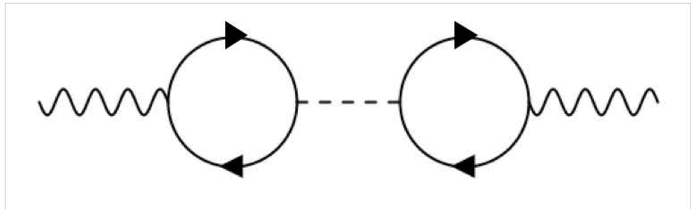
1st order mean field correction





1st order vertex correction





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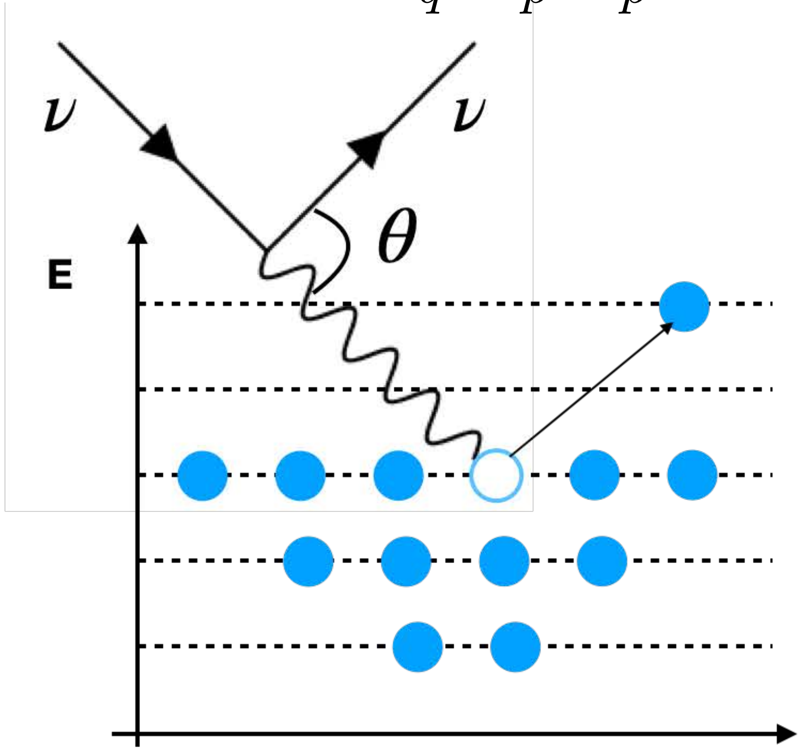
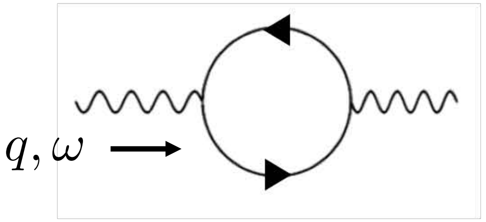


- Mean field models (Skyrme, RMF)
- Fermi liquid theory
- Virial expansion
- Microscopic nuclear forces (e.g., chiral EFT)?

0th order



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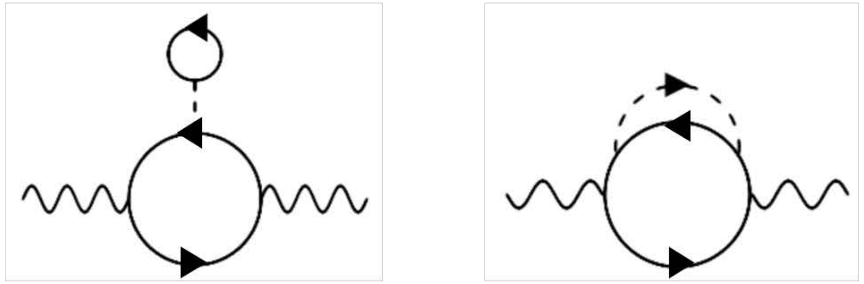

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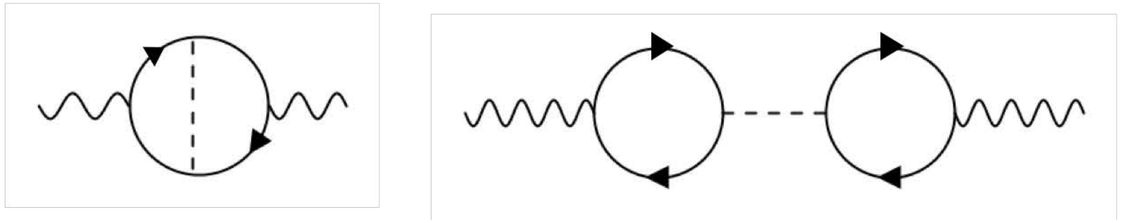
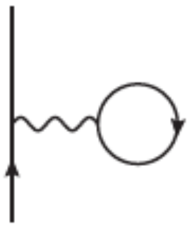


1. Role of nuclear mean fields



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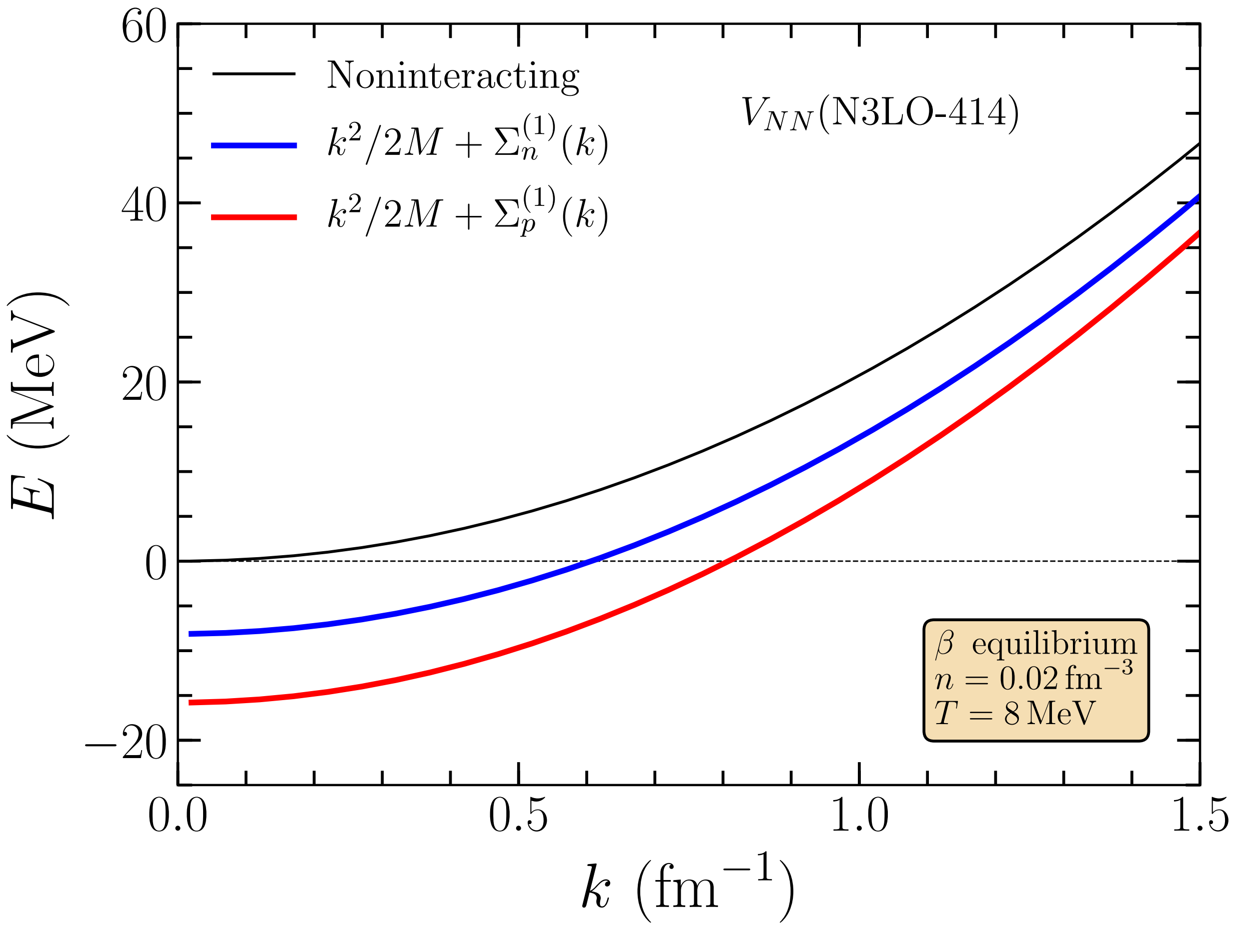

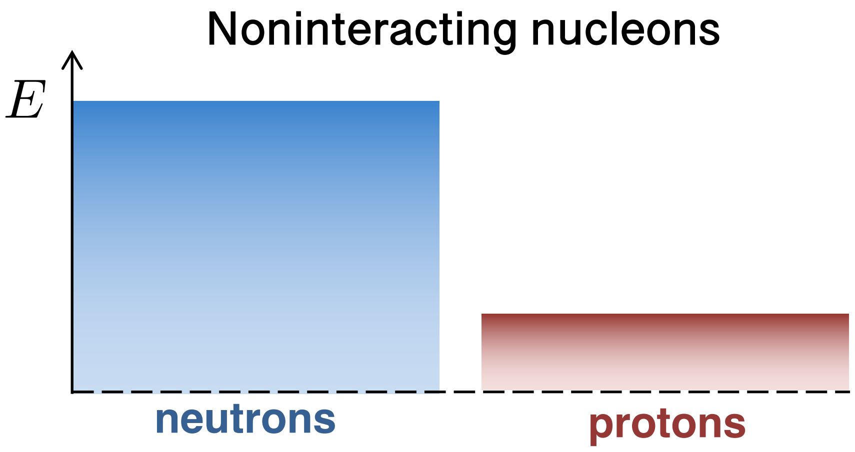

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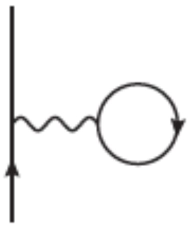

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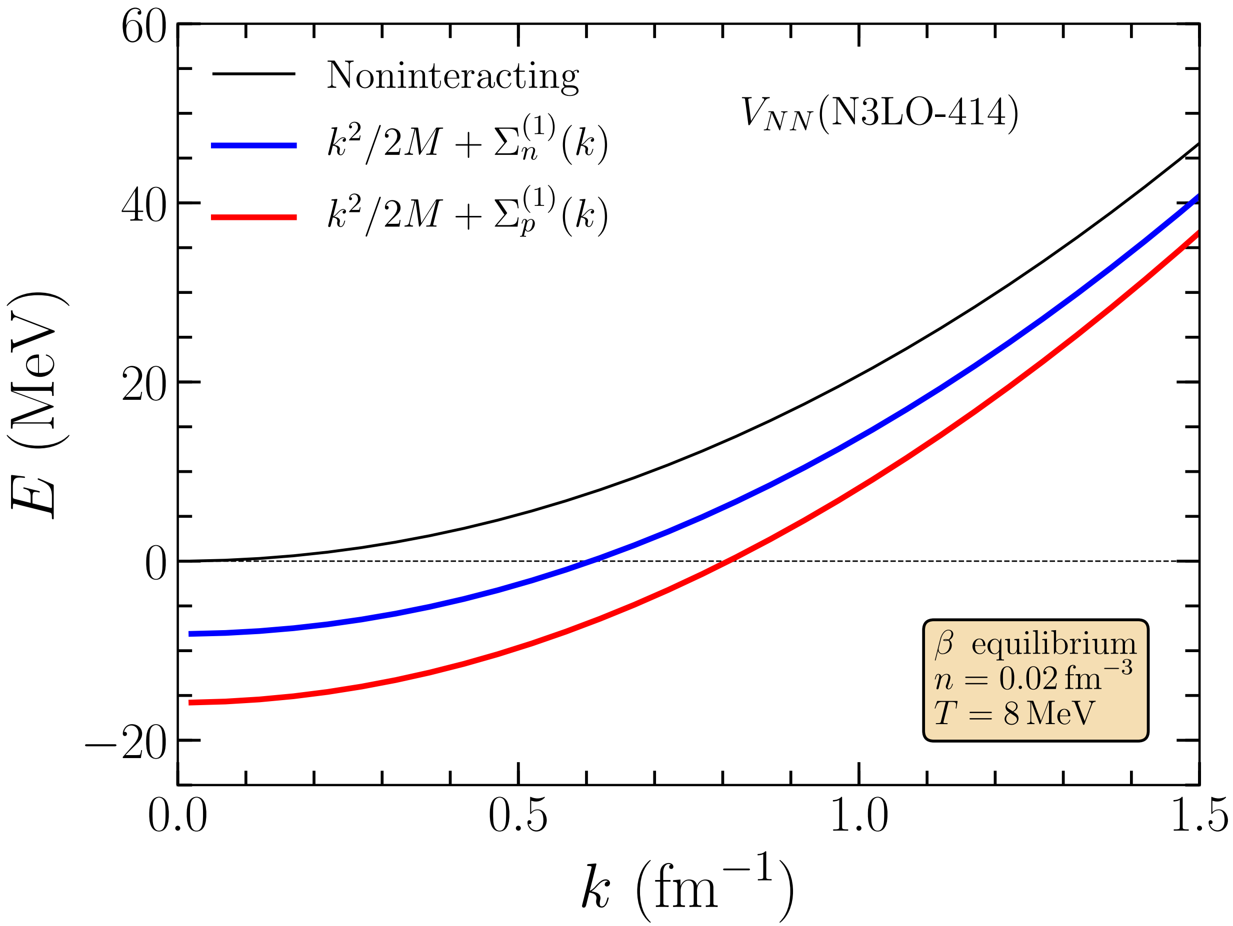

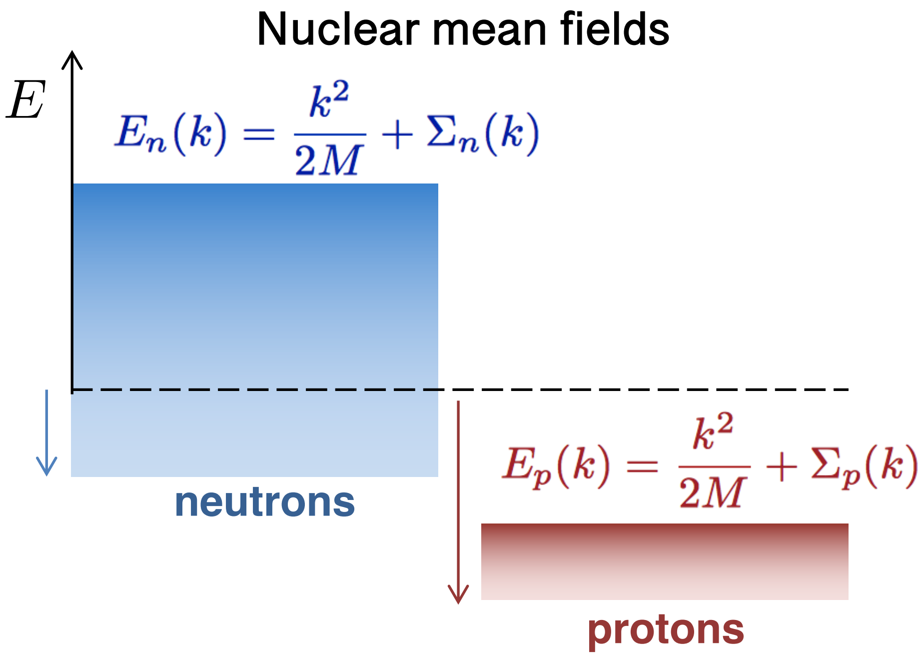

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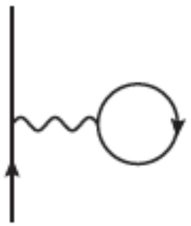

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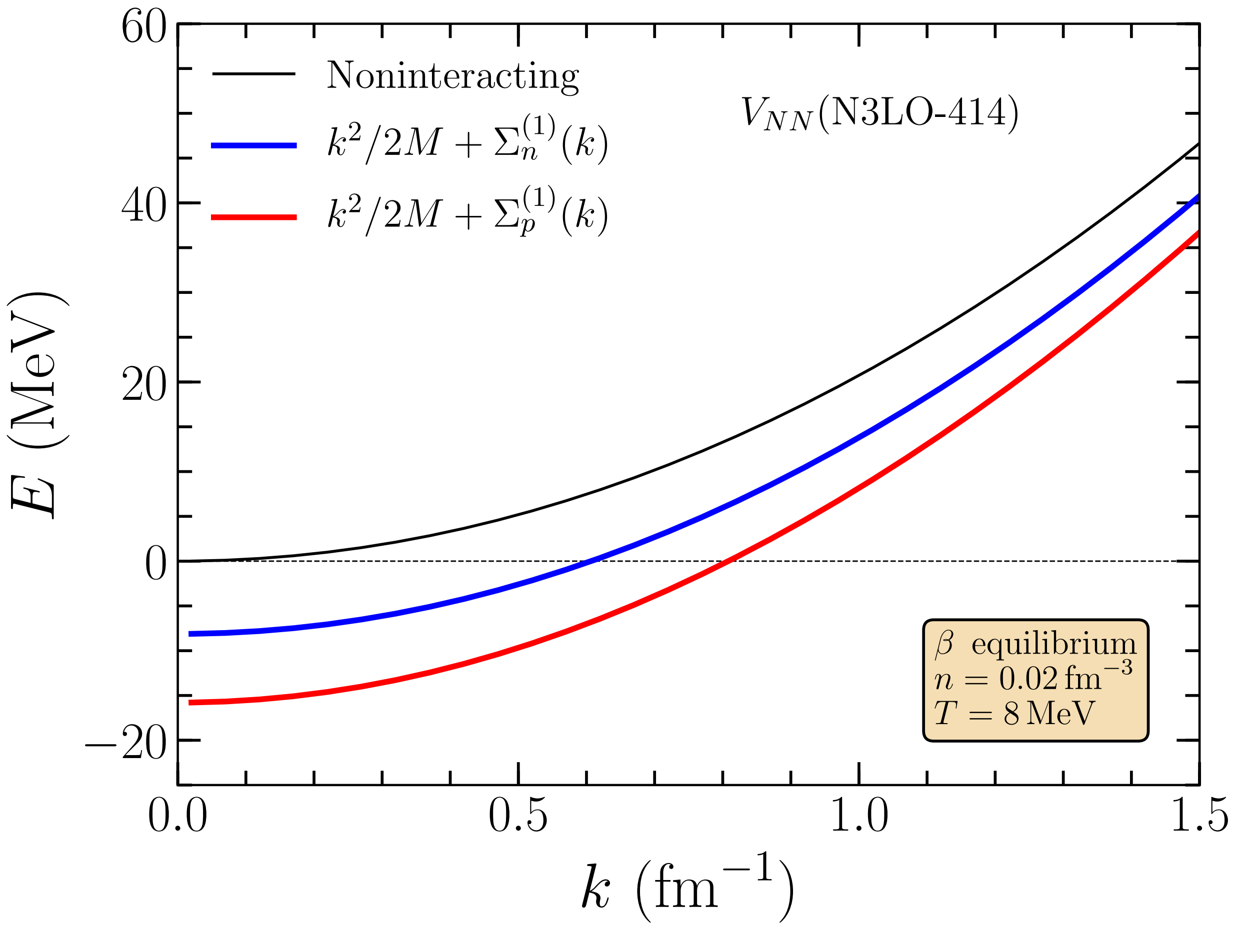
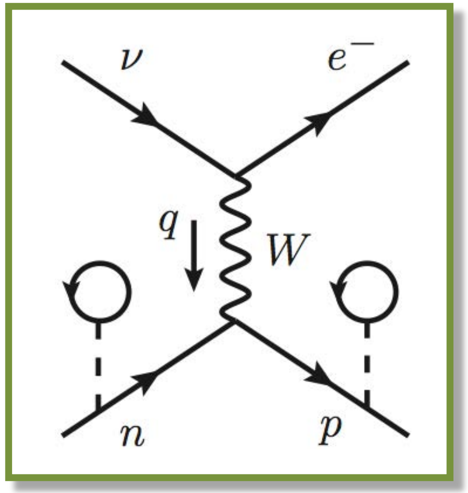


$$\chi^{(0)}_{\tau\rho}(\vec{q}, \omega) = \sum_{s_1 s_2} \int \frac{d\vec{k}}{(2\pi)^3} \frac{f_{\vec{k},n} - f_{\vec{k}+\vec{q},p}}{\omega + e_{\vec{k},n} - e_{\vec{k}+\vec{q},p} + i\eta} \delta_{s_1,s_2},$$

Mean-field effects



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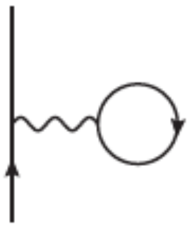

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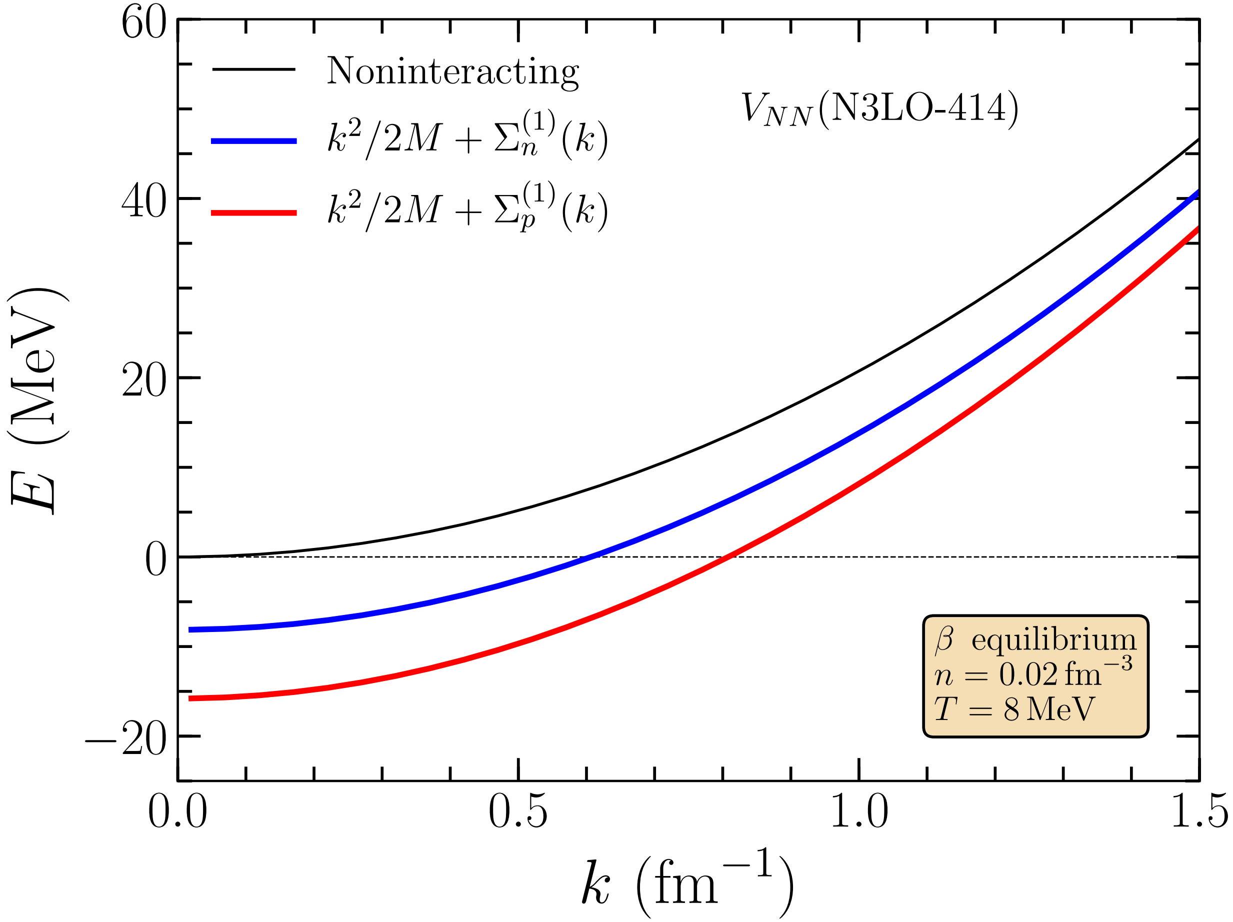

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$$f_{\vec{k},n} = \left[1 + e^{(k^2/2M^* + U_n - \mu_n)/T}\right]^{-1}$$

$$\mu^{MF} \simeq \mu^{(0)} - U_n$$

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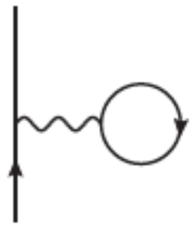

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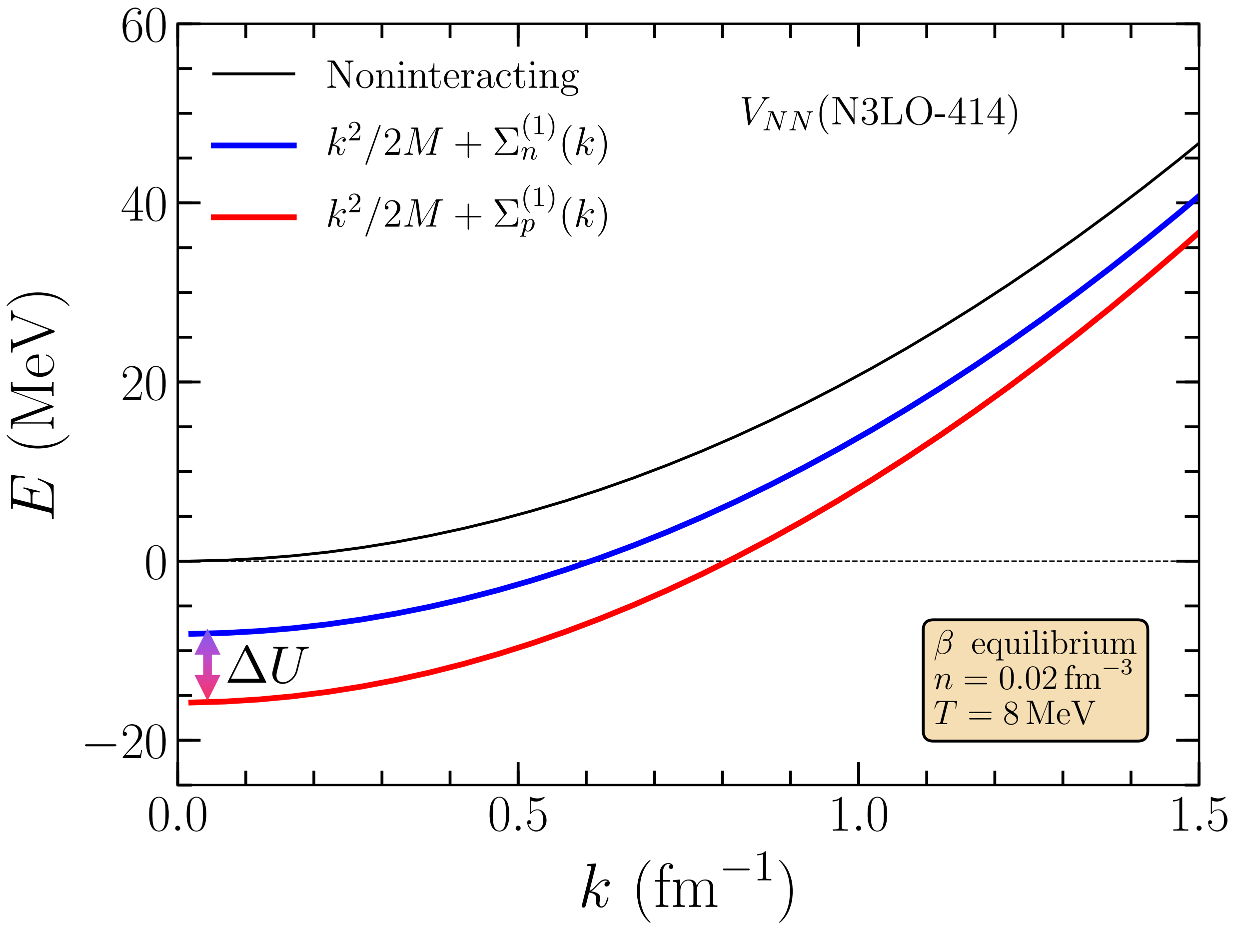

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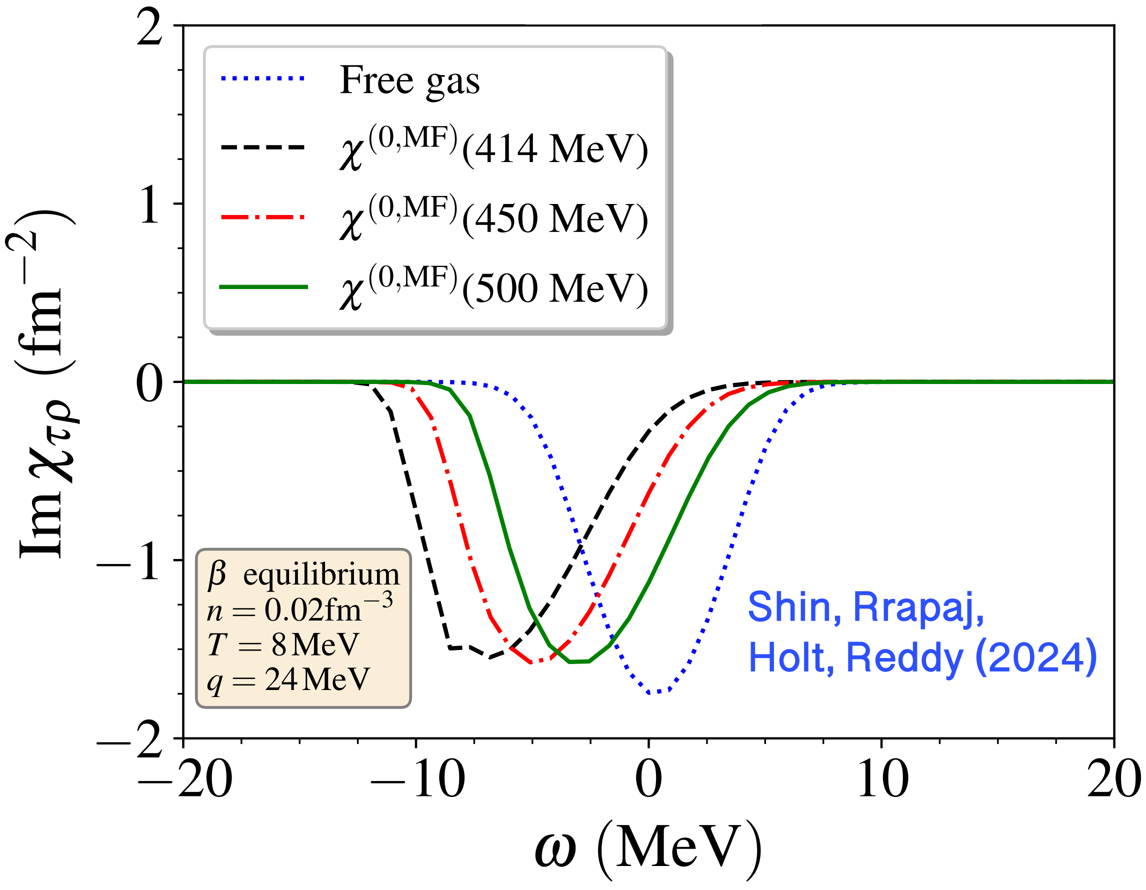
Energy conservation

$$(\omega + U_n - U_p) + \frac{k^2}{2M_n^*} - \frac{|\vec{k} + \vec{q}|^2}{2M_p^*} = 0$$

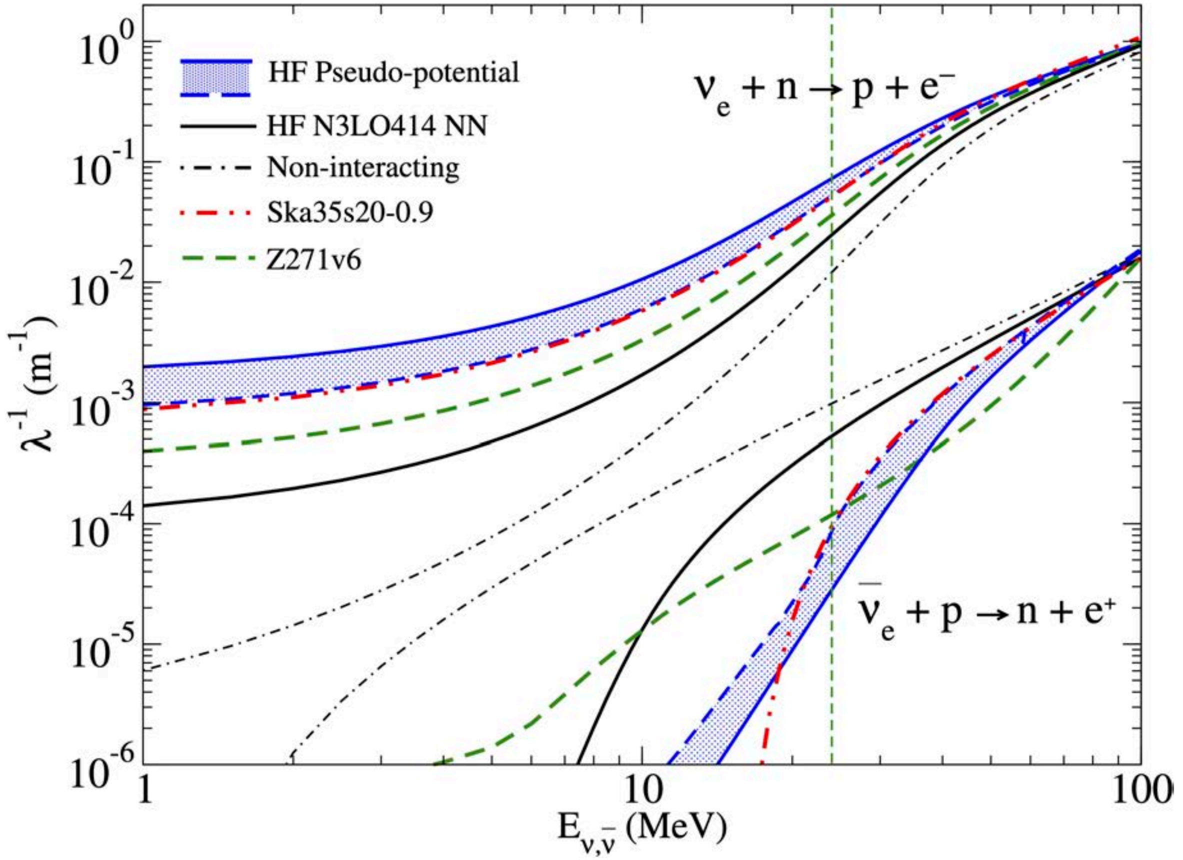
$$\omega^{MF} \simeq \omega^{(0)} - \Delta U$$

1. Role of nuclear mean fields



Shin, Rrapaj, Holt, Reddy (2024)

Roggero, Rrapaj, Holt (2016)



Outgoing electron energy increases: $E_e = E_\nu - \omega$

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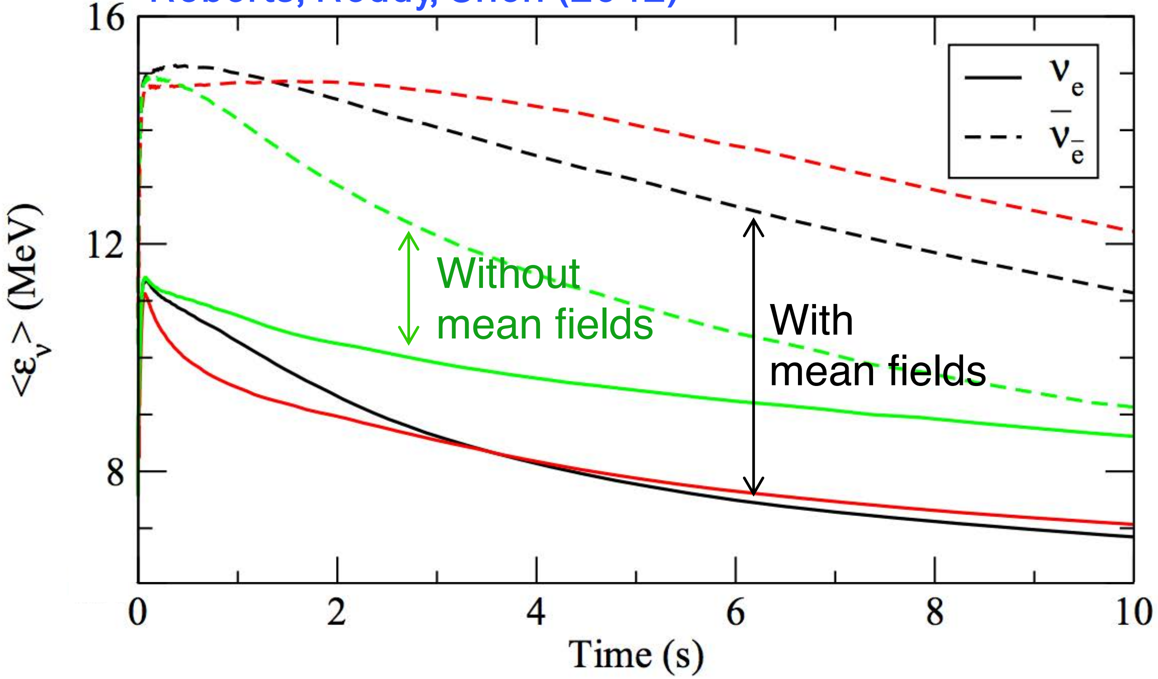
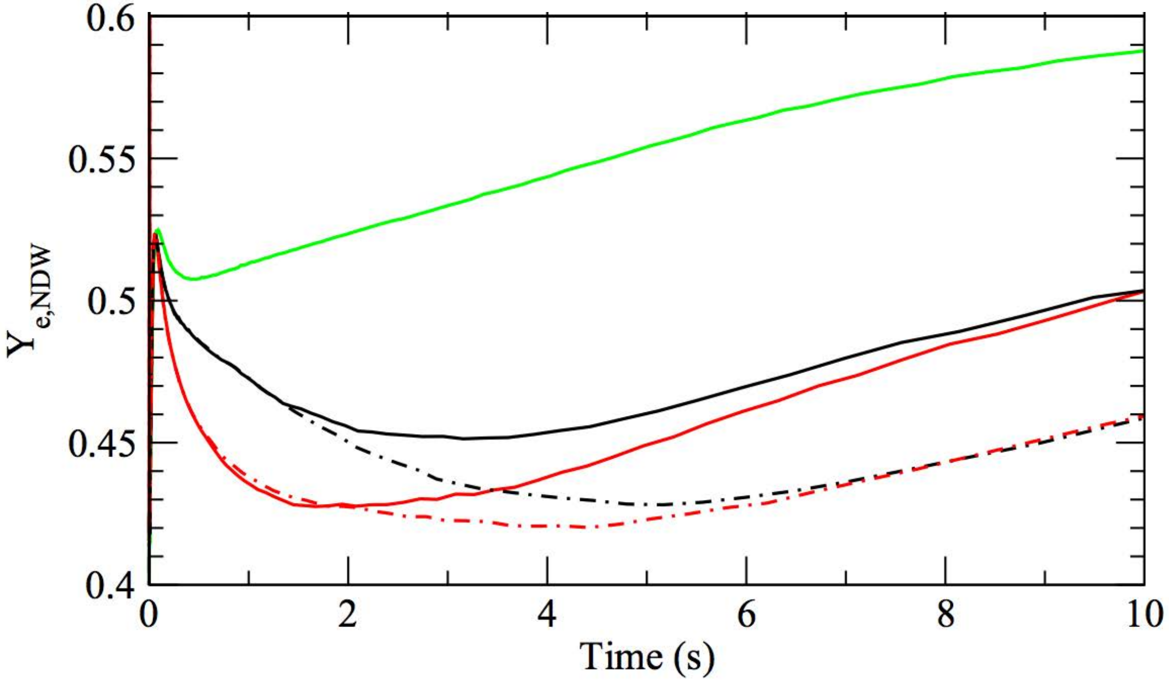
Effect on supernova neutrino energies and outflow composition

Roberts, Reddy, Shen (2012)





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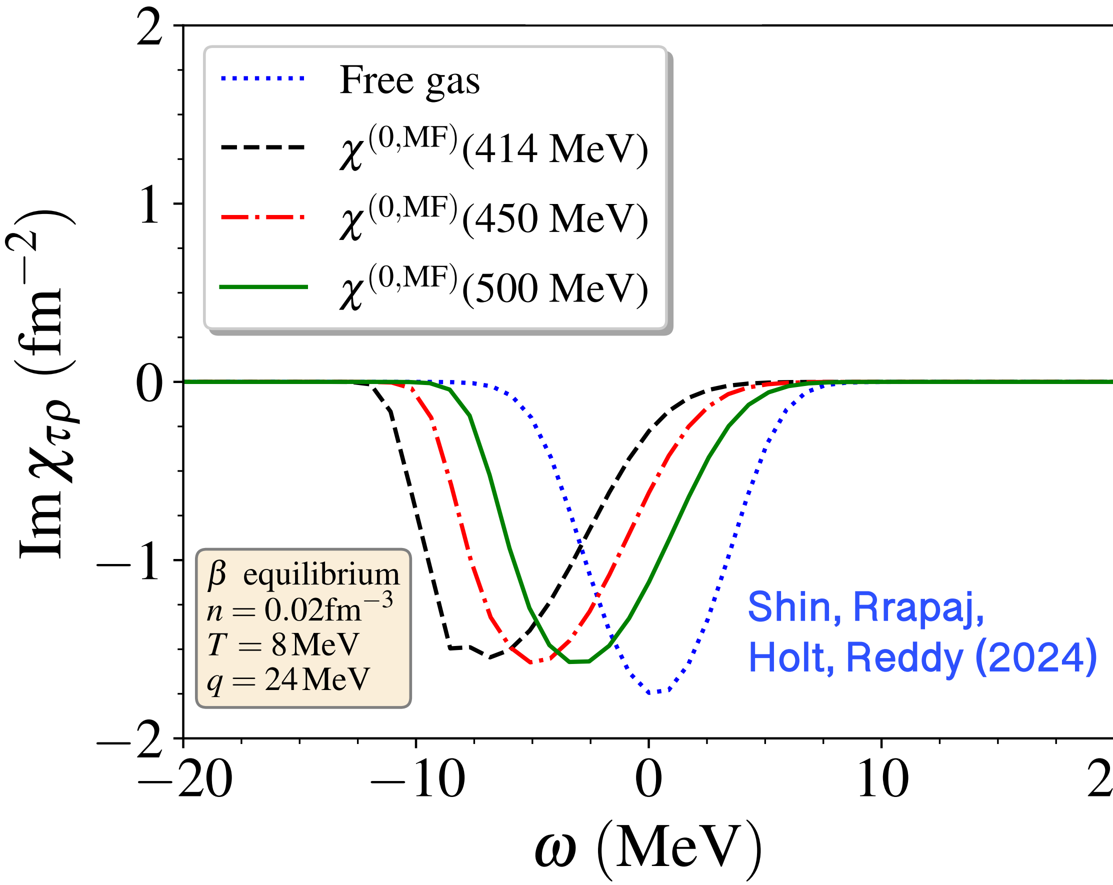

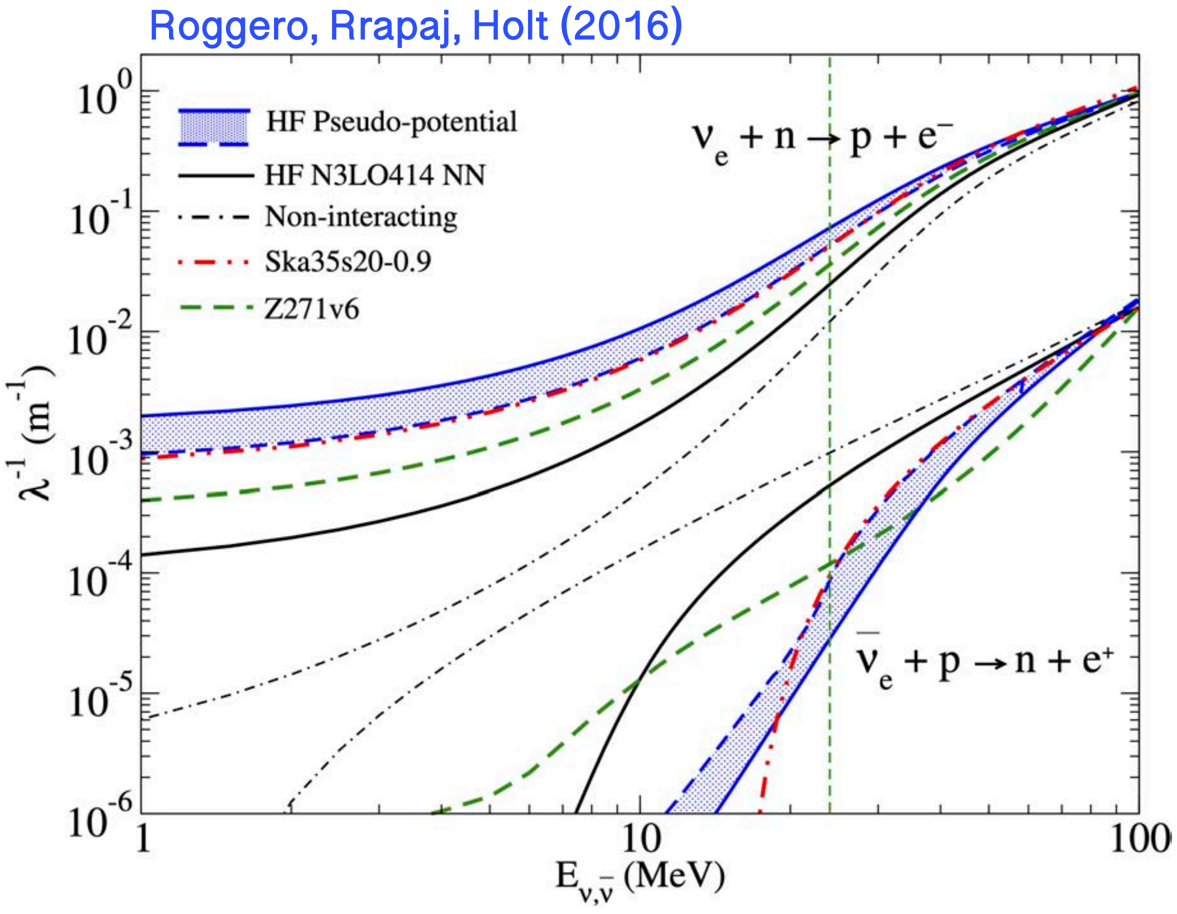

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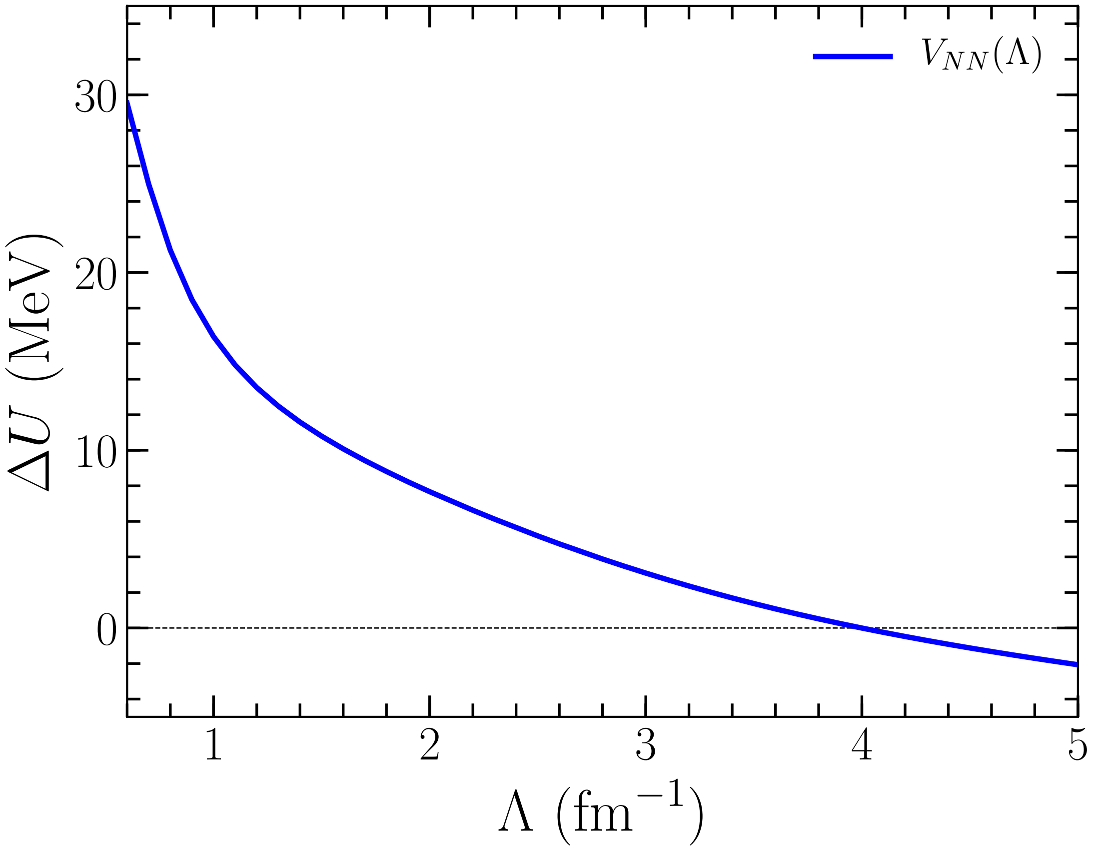
HF Pseudo-potential	4.9%	0.65	0.42	22	55	33
HF Pseudo-potential (mod)	3.8%	0.78	0.57	18	42	23
HF N3LO414	2.2%	0.95	0.89	8	16	8
RMF: Z271v6	2.8%	0.96	0.96	9	22	13
Skyrme: Ska35s20-0.9	3.3%	0.98	1.0	9	26	17

- Microscopic (HF) mean fields smaller than phenomenological interactions

Low-resolution nucleon-nucleon interactions

- **Phase-shift equivalent potentials:**

$$T(k',k;k^2) = V_{\rm low\ k}(k',k) + \frac{2}{\pi}\,\mathcal{P}\int_0^{\Lambda} \frac{V_{\rm low\ k}(k',p)\;T(p,k;k^2)}{k^2-p^2}\;p^2dp$$



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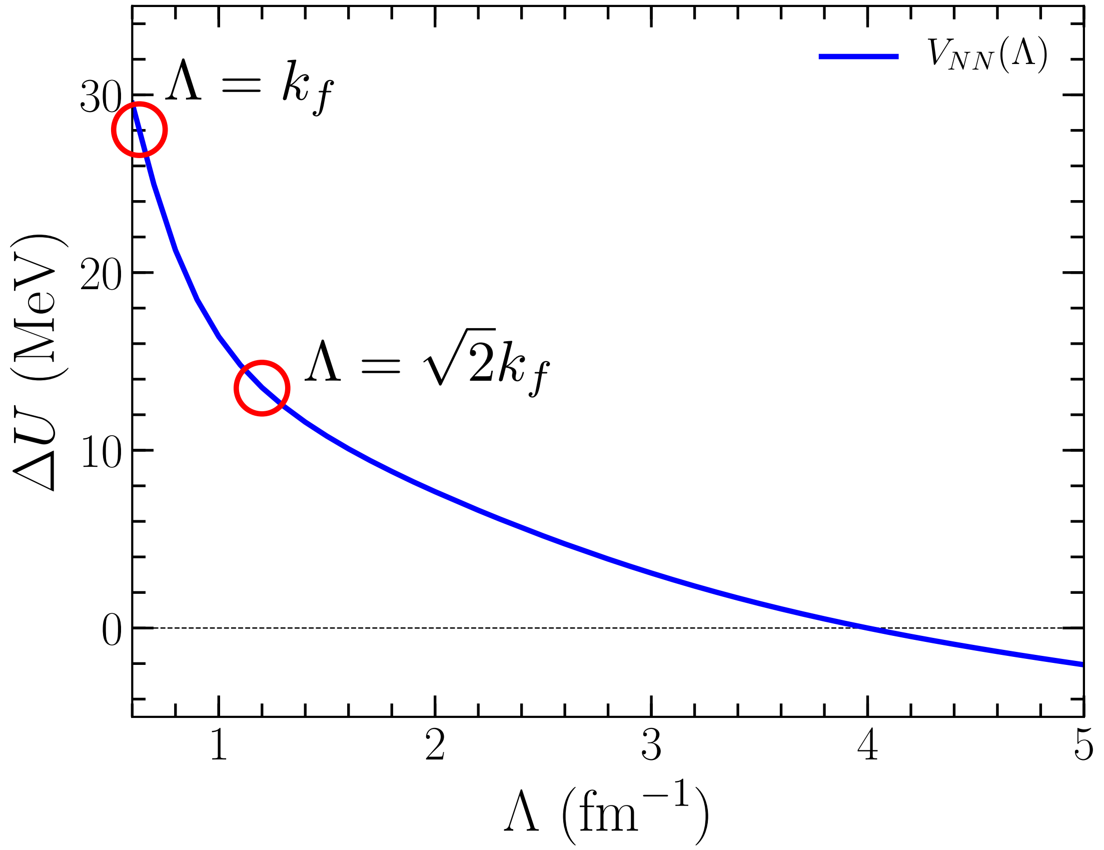

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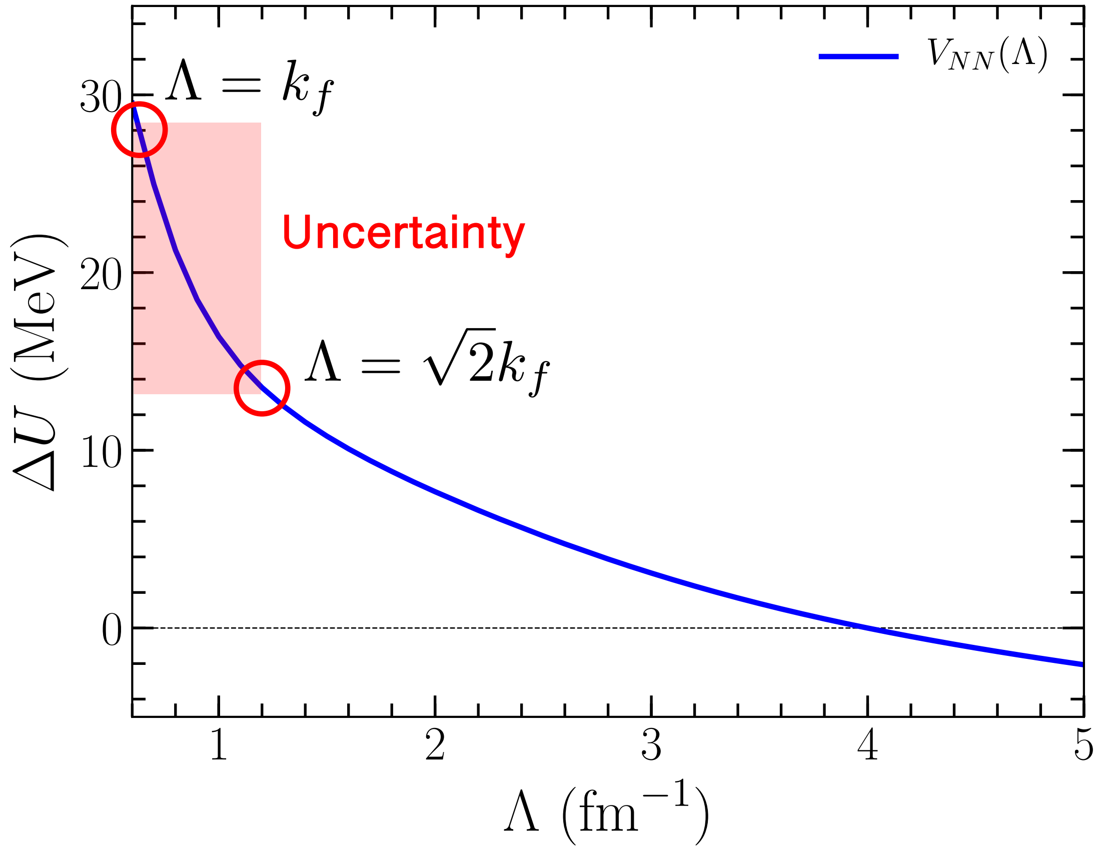

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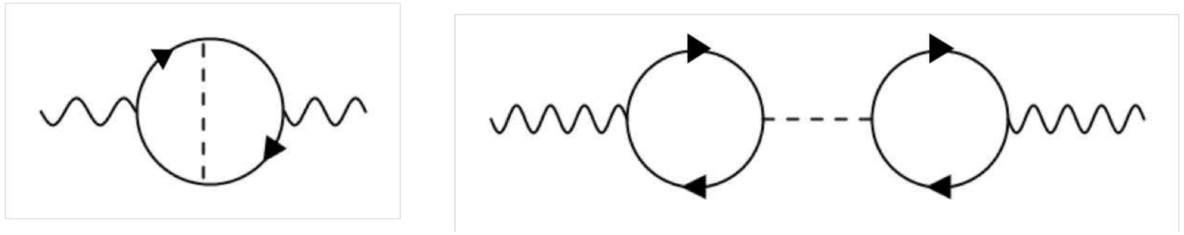
2. Vertex corrections to response functions

1st-order vertex correction



Vertex corrections require off-shell matrix elements:

$$\chi^{(1)}_{\rho\rho}(\omega,\vec{q}) = \frac{M^2}{8\pi^4 q^2}\int dk_1 k_1 \int d\cos\theta_1 \left[\frac{n_{k_1}-n_{k_1+q}}{\cos\theta_1 - \frac{M\omega}{k_1 q}+\frac{q}{2k_1}-i\eta}\right]\int dk_2 k_2 \int d\cos\theta_2 \left[\frac{n_{k_2}-n_{k_2+q}}{\cos\theta_2 - \frac{M\omega}{k_2 q}+\frac{q}{2k_2}-i\eta}\right]$$
$$\int d\phi_2 \sum_{LSJ}(2J+1)P_L(\hat{q}_1\cdot\hat{q}_2)(1-(-1)^{L+S+1})\underline{\langle q_1 LSJ|V|q_2 LSJ\rangle}.$$

- Especially large off-shell matrix elements associated with tensor force from $\mathbf{1\pi}$ –exchange

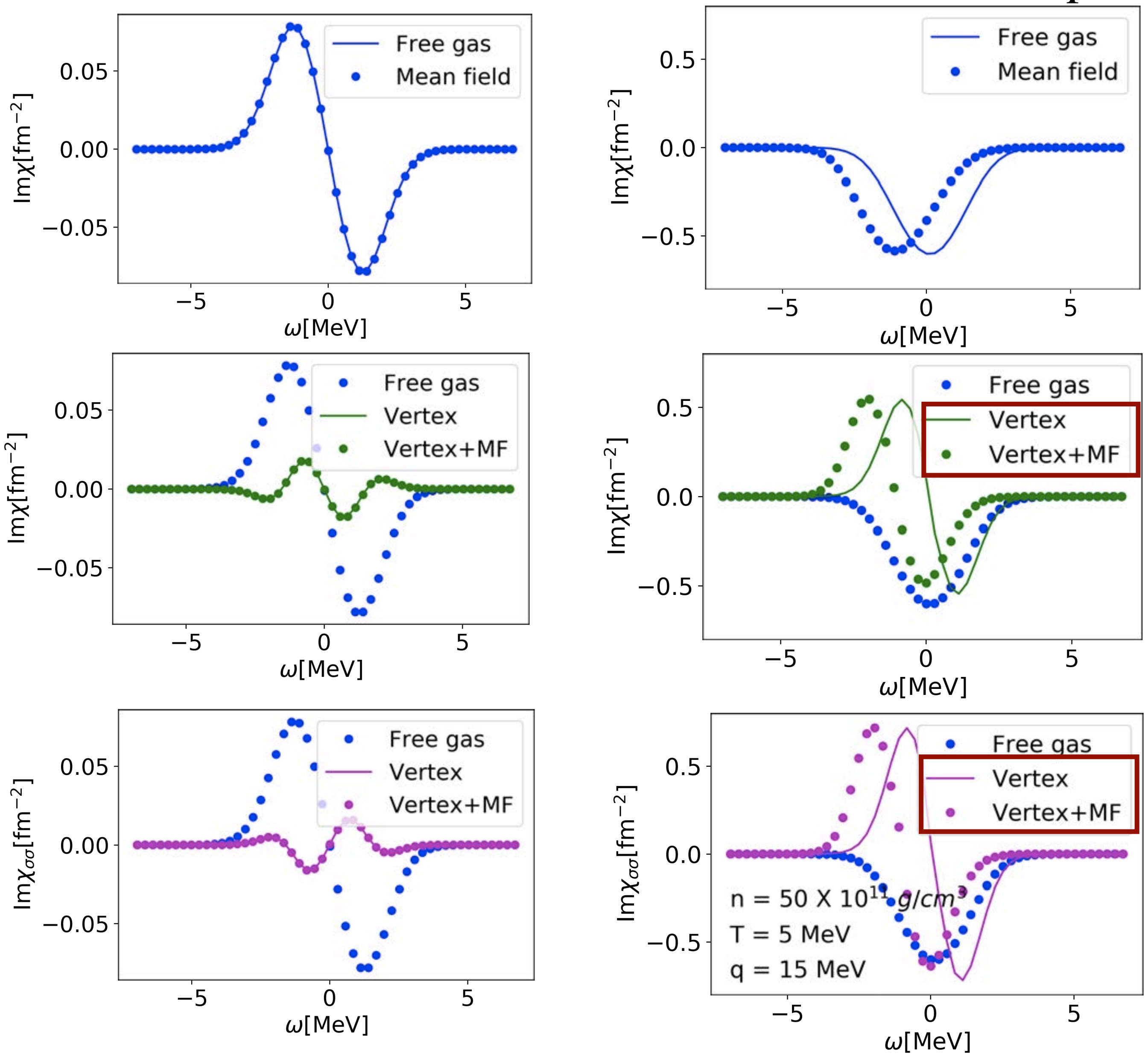
1st-order vertex corrections

$\nu + n \to \nu + n$ | $\nu + n \to e^- + p$

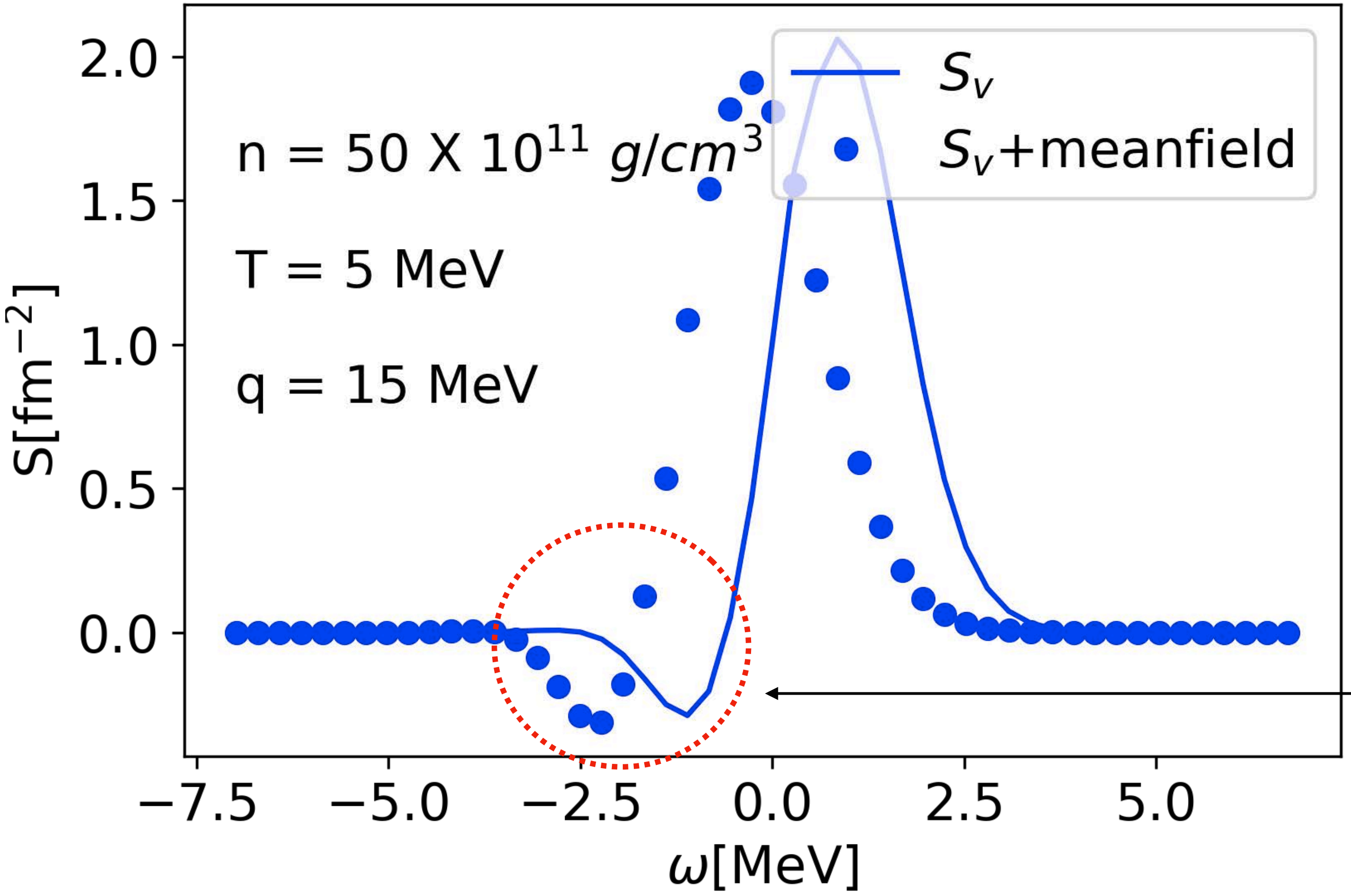
$Im\chi_{nn}^{MF}$

$Im\chi_{nn}^{Vertex}$

$Im\chi_{\sigma\sigma}$



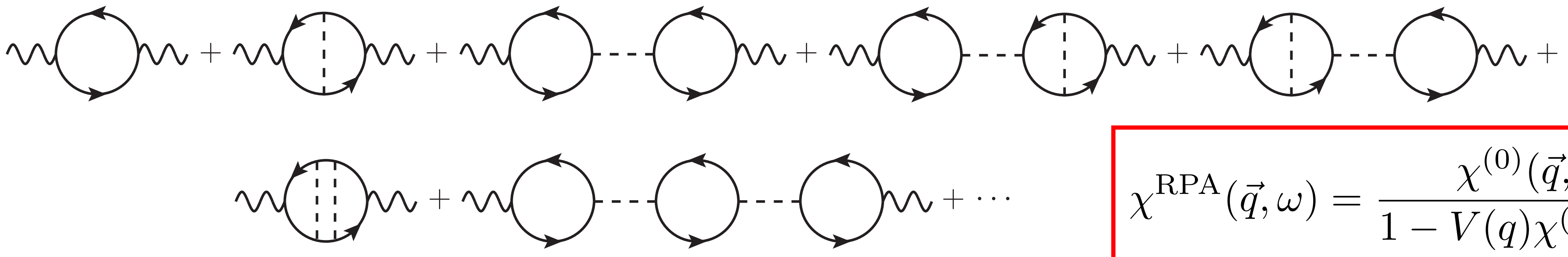
Caveat: nonphysical behavior in dynamic structure functions



S > 0 expected
But S < 0 for -4 < w < 0

Consider higher order contribution
e.g. RPA

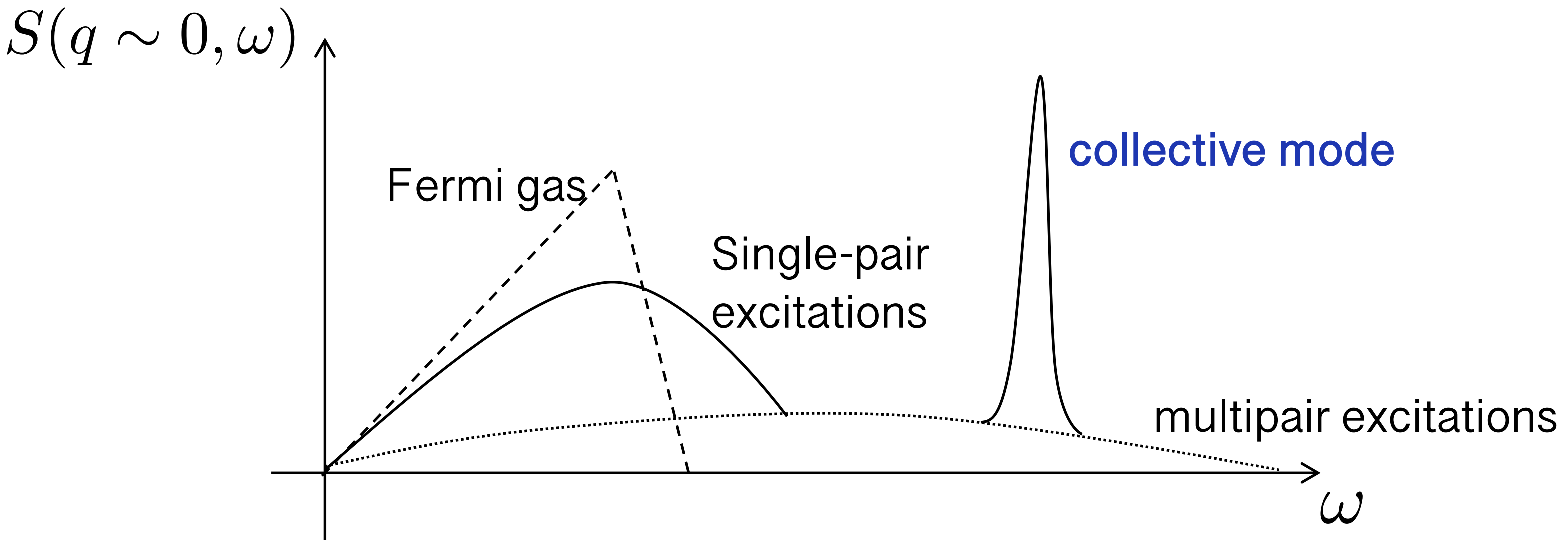
RPA vertex corrections to response functions



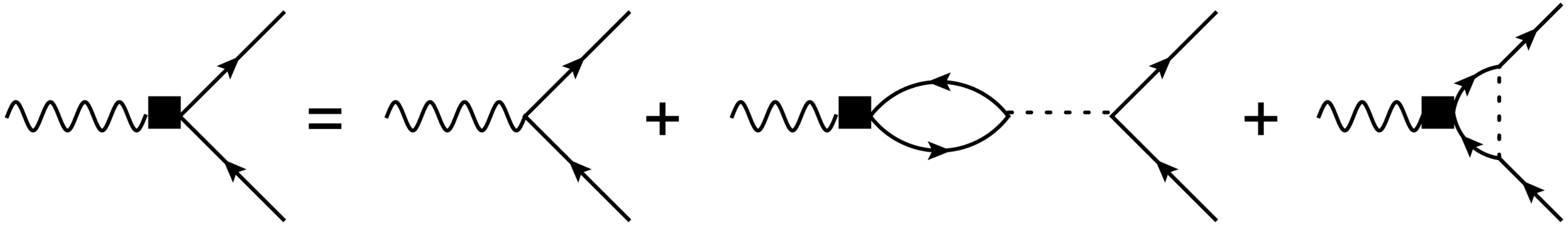
$$\chi^{\mathrm{RPA}}(\vec{q},\omega) = \frac{\chi^{(0)}(\vec{q},\omega)}{1 - V(q)\chi^{(0)}(\vec{q},\omega)}$$

Local interactions resummed in the direct channel

- RPA + HF mean field theory is a "conserving approximation" guaranteed to yield physical structure functions
- RPA can account for collective modes (denominator above approaches 0)



RPA vertex function



$$L(\vec{k}s;\vec{q}\omega) = L_0(\vec{k}s;\vec{q}\omega) + L_0(\vec{k}s;\vec{q}\omega)\sum_{s'}\int\frac{d\vec{k}'}{(2\pi)^3}\langle \vec{k}\vec{k}'+\vec{q}, ss', np|\bar{V}|\vec{k}+\vec{q}\vec{k}', ss', pn\rangle L(\vec{k}'s';\vec{q}\omega)$$

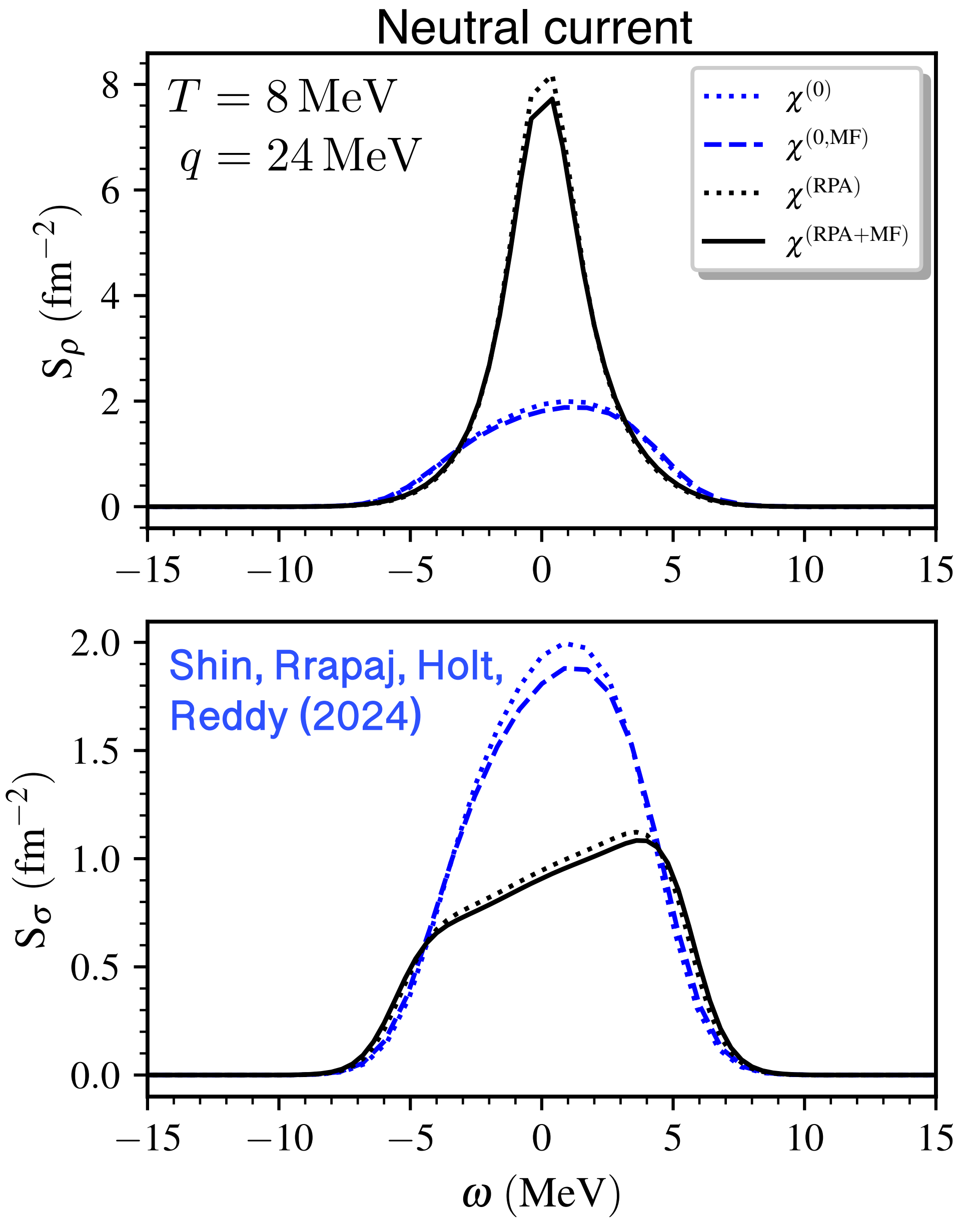
- Discretize integral and recast as a matrix equation with eigenvectors $|\ell\rangle$ and eigenvalues ω_ℓ

$$\mathrm{Im}\,\chi^{\mathrm{RPA}}_{\tau\rho}(\vec{q},\omega) = -i\pi\sum_{\ell}\frac{\langle B|\ell\rangle^2}{\langle\ell|N^{-1}|\ell\rangle}\delta(\omega-\omega_\ell)$$

- Need to introduce smeared **δ**-functions to account for discretization

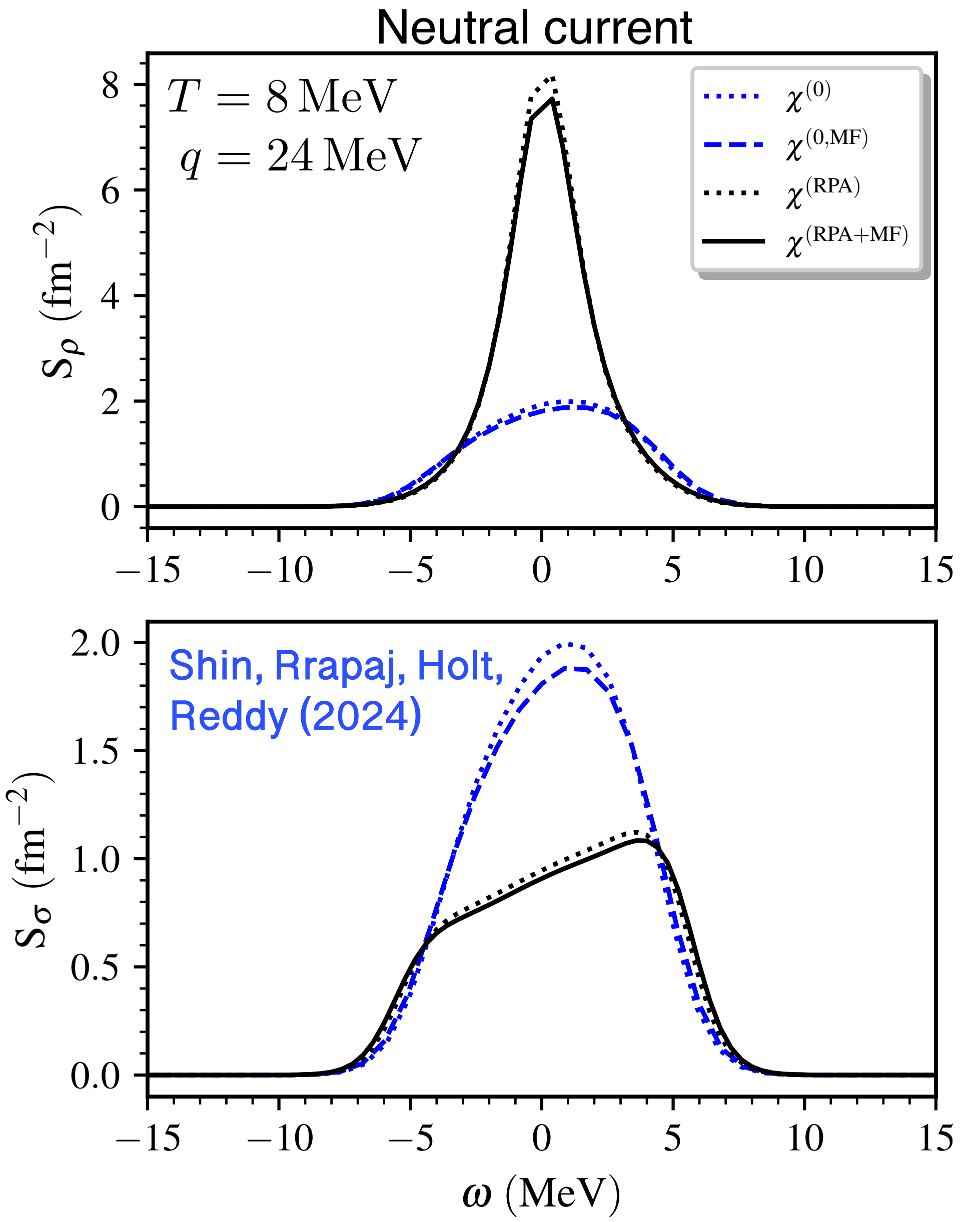
$$\delta_\epsilon(\omega) = \frac{1}{\sqrt{2\pi\epsilon}}e^{-\omega^2/2\epsilon}$$

RPA vertex corrections to response functions



- Neutral-current density structure factor enhanced by RPA correlations
- Neutral-current spin structure function suppressed by RPA correlations

RPA vertex corrections to response functions



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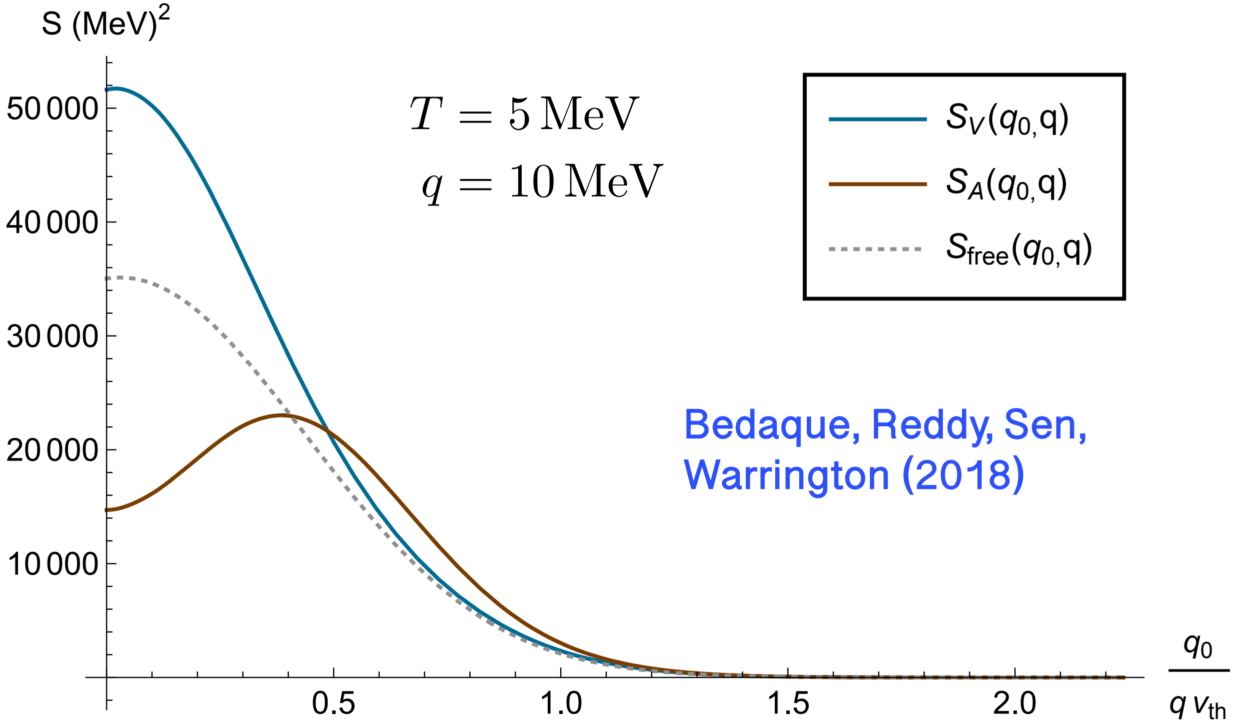
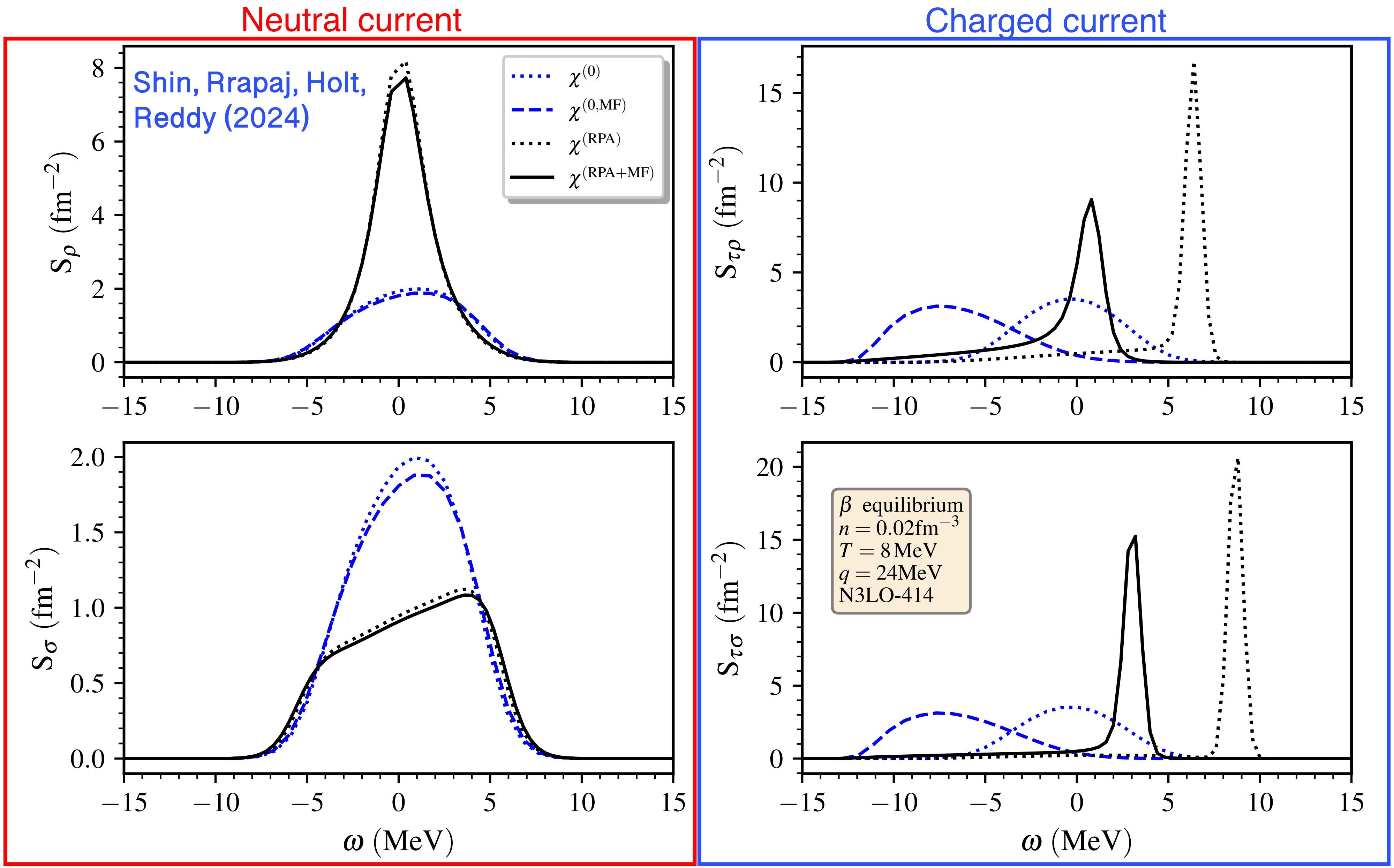
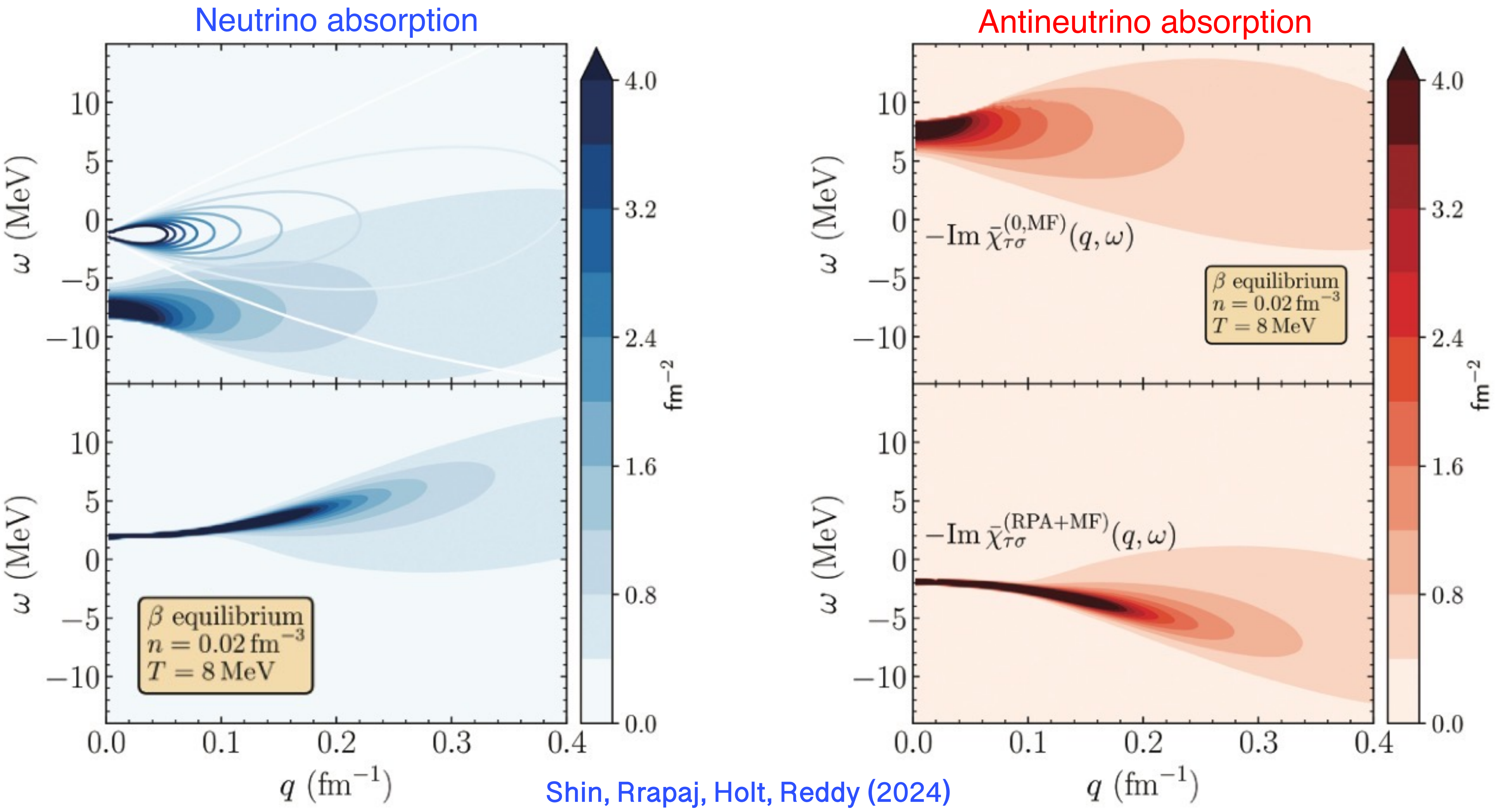


RPA vertex corrections to response functions



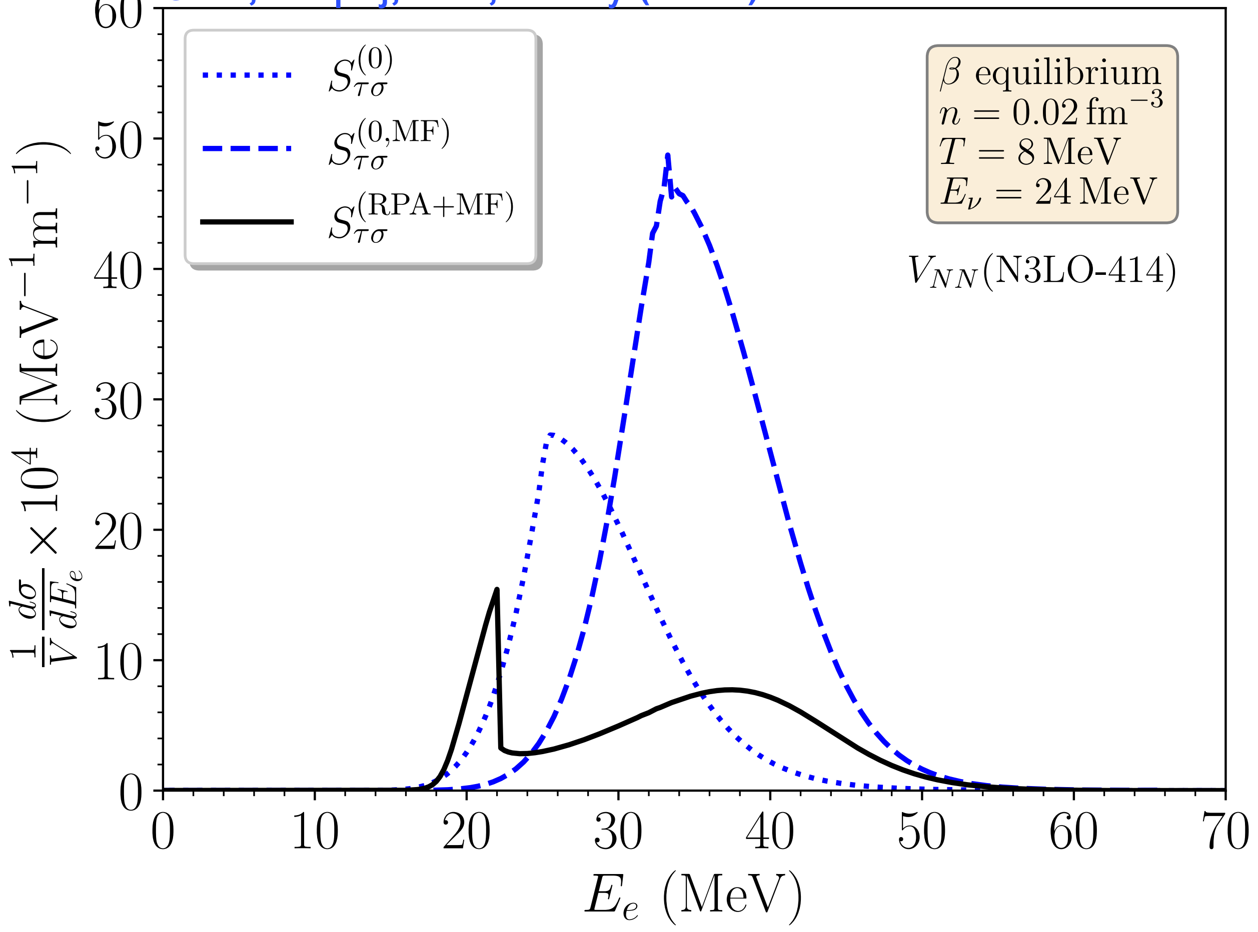
Imaginary response for electron neutrino/antineutrino absorption

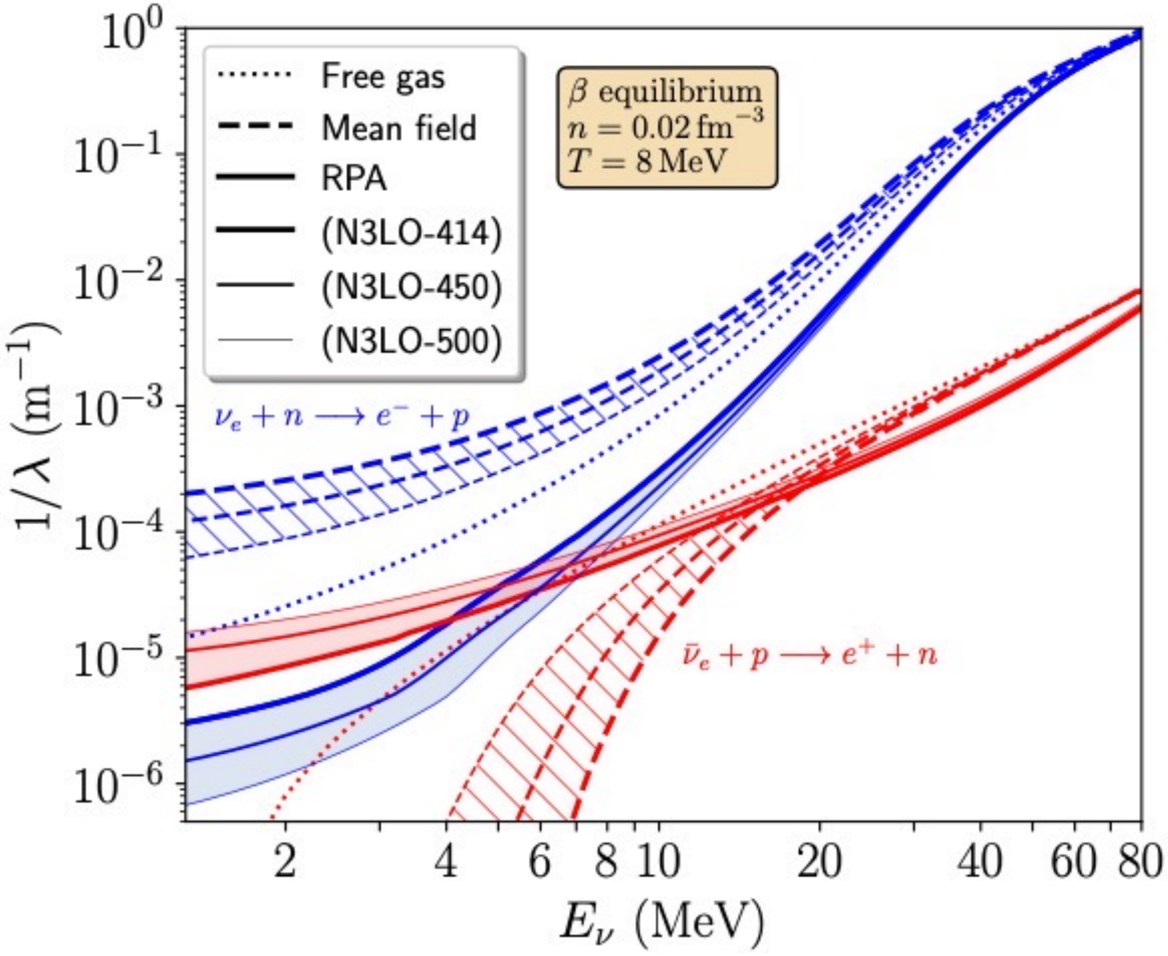


Shin, Rrapaj, Holt, Reddy (2024)

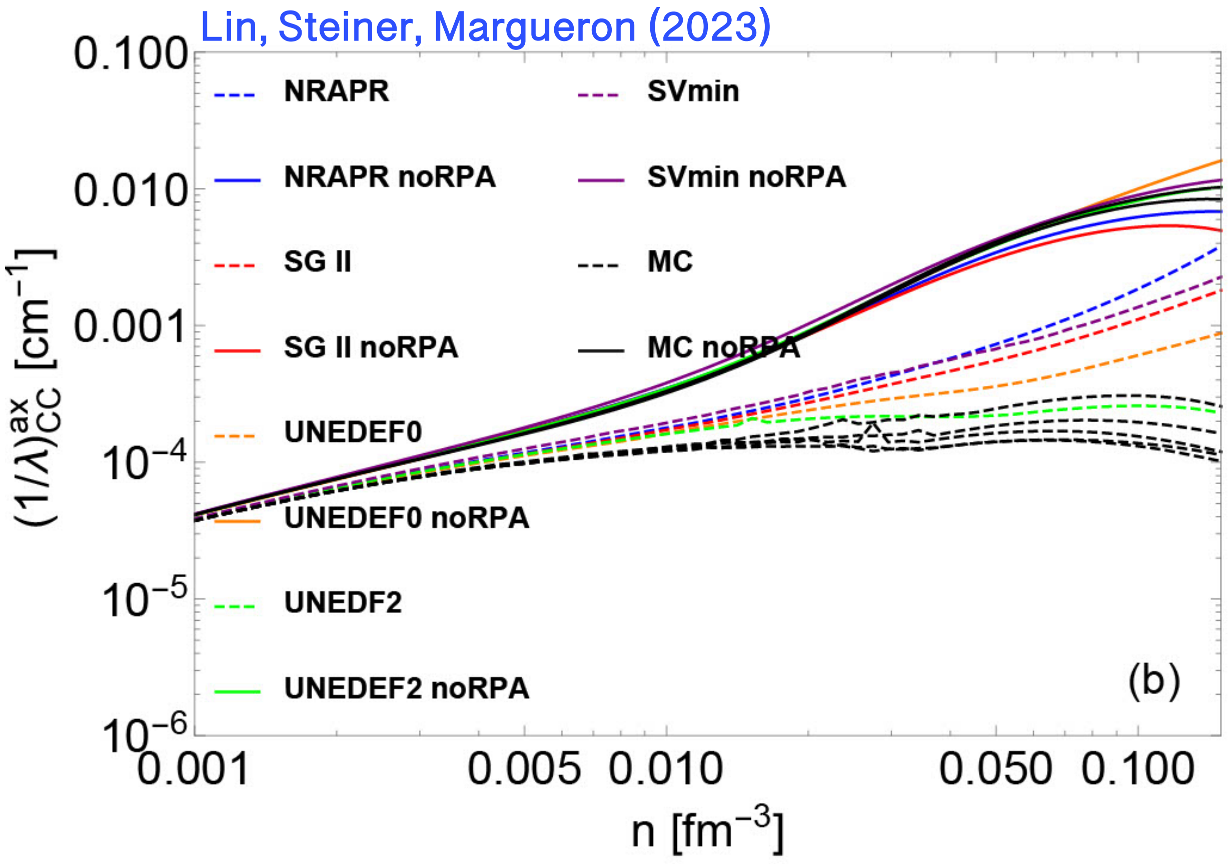
Differential cross section and scattering length

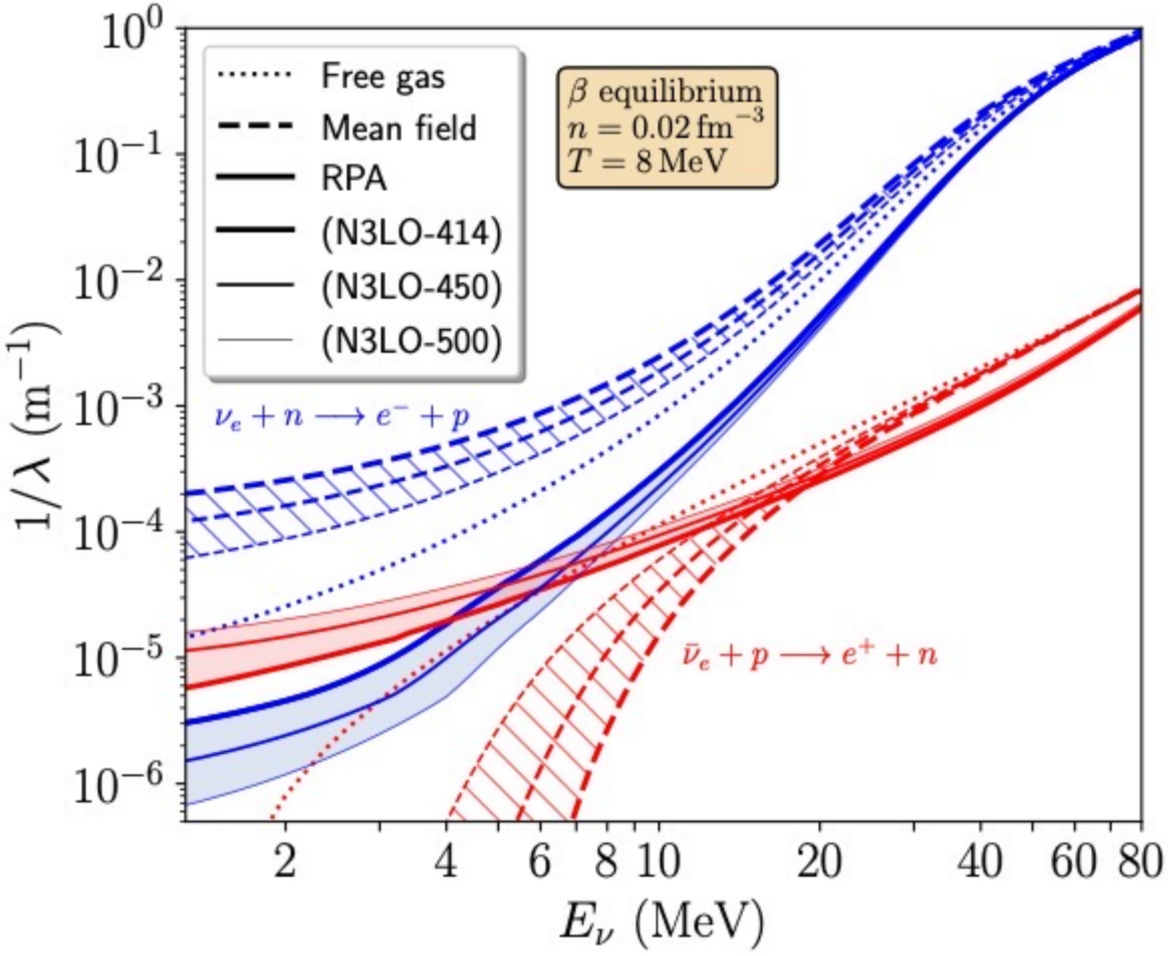
Shin, Rrapaj, Holt, Reddy (2024)





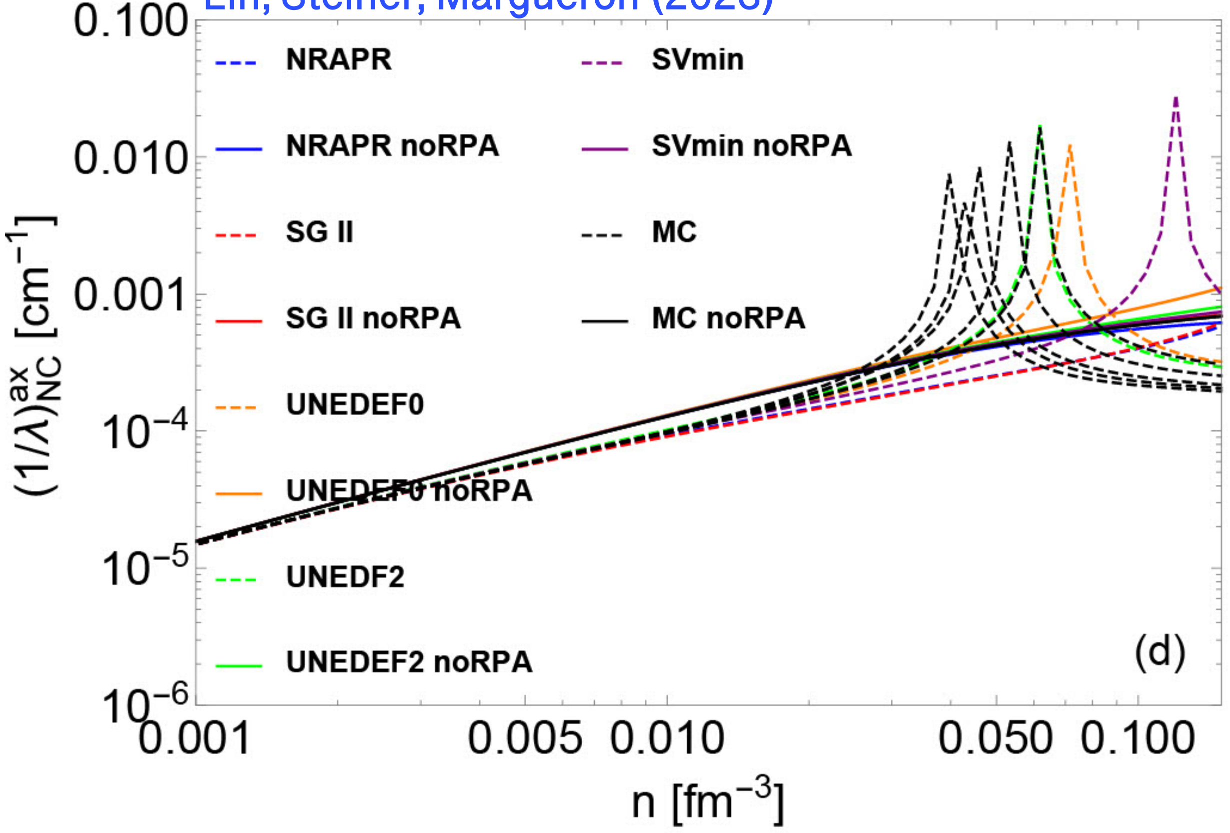
Differential cross section and scattering length

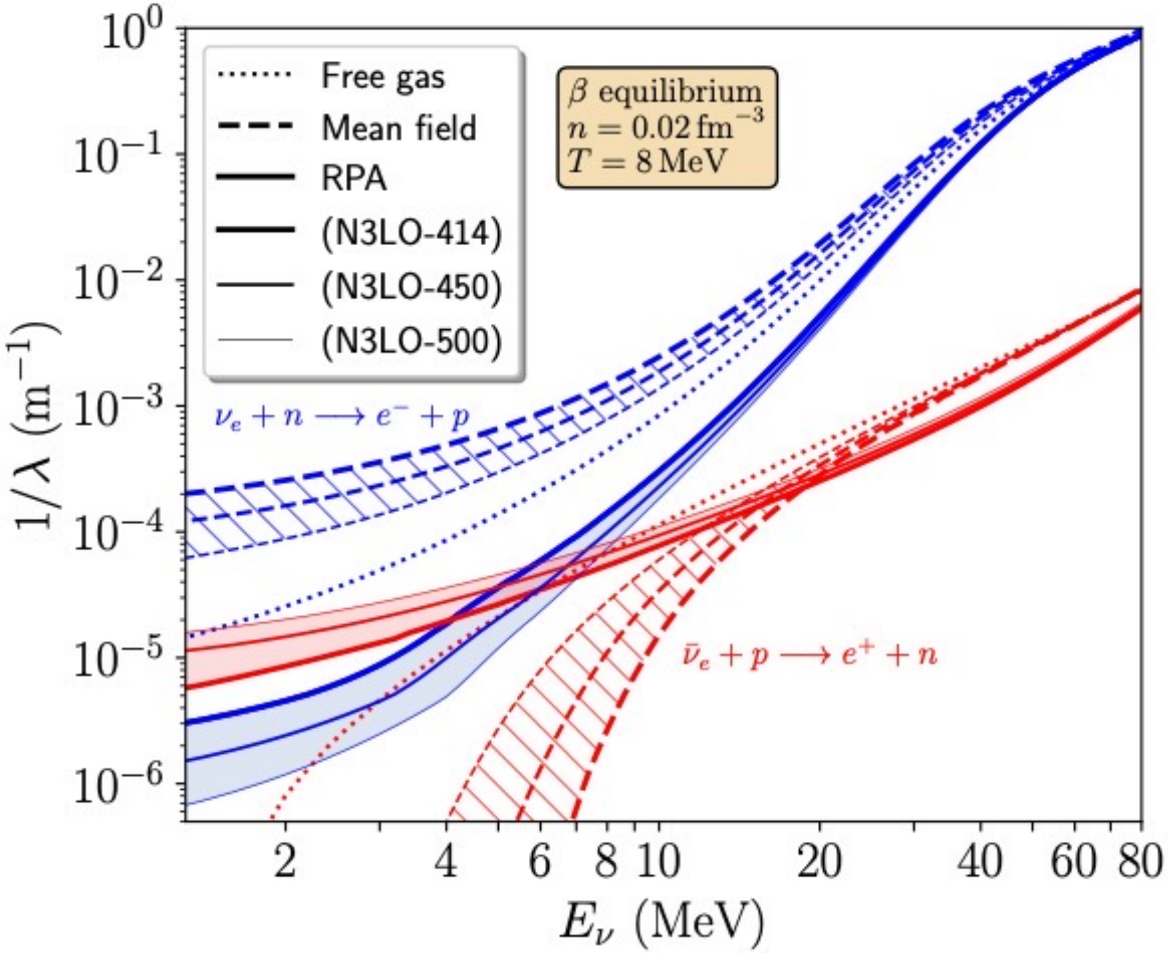




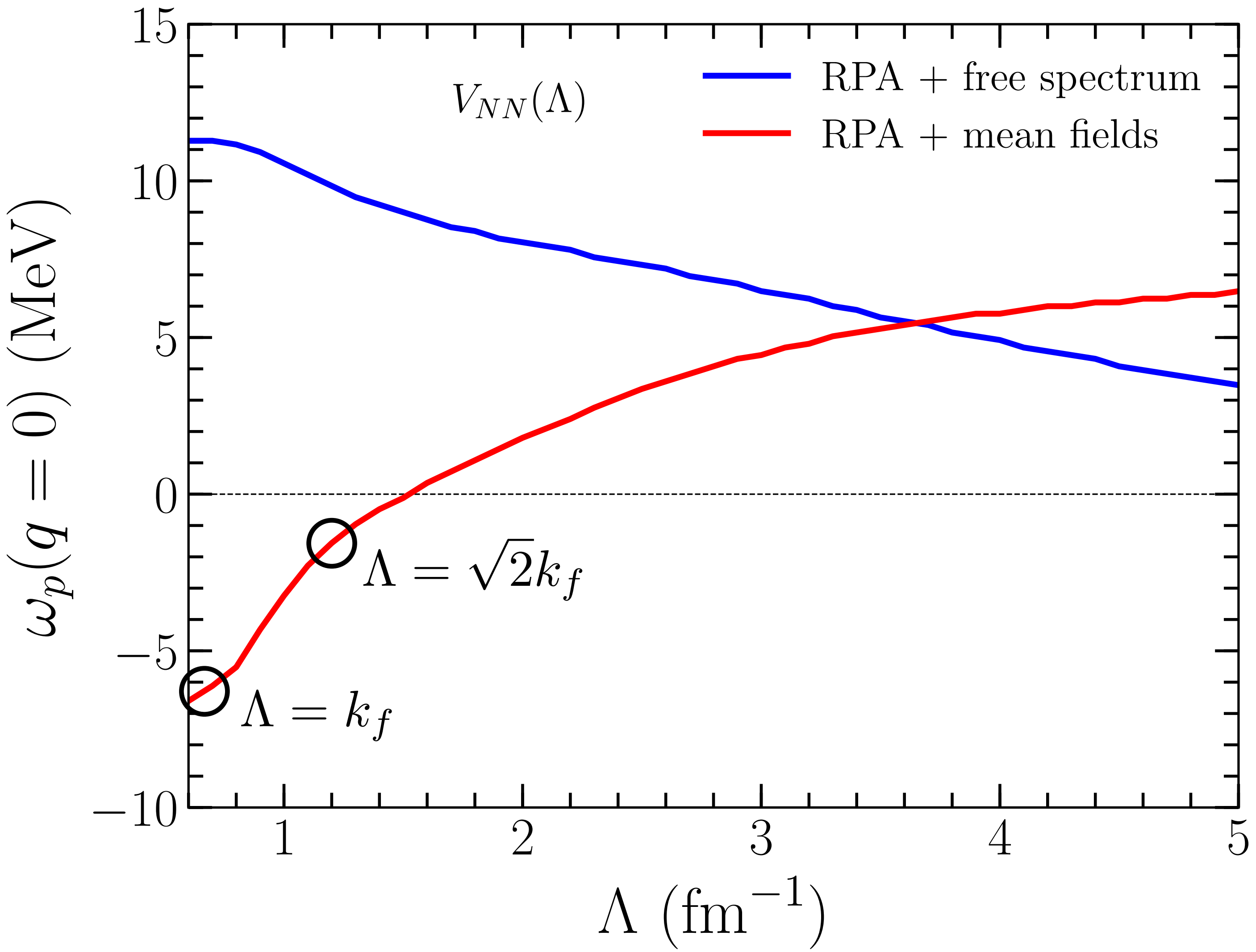
Differential cross section and scattering length

Lin, Steiner, Margueron (2023)





Energy of collective mode



- Mean field and vertex corrections both increase with decreasing **Λ**
- Mean field effects are generically stronger

Summary/Conclusions

- RPA vertex corrections crucial for modeling the effects of **nuclear collective excitations**
- The coupling to collective modes suppresses electron-neutrino absorption across all energies
- The coupling to collective modes enhances electron-antineutrino absorption at low energies and suppresses absorption at high energies
- At low energies, mean fields and correlations shift strength into opposite energy regions, and therefore both effects should be included on a consistent footing
- Need to develop improved microscopic interactions that can better capture resonant features at low momenta (large S-wave scattering lengths, bound states in np channel)
- **Future**: implications for neutrino luminosities, explosion dynamics, nucleosynthesis, neutrino flavor oscillation,…