Calculation of Neutrino Propagation Through the Sun

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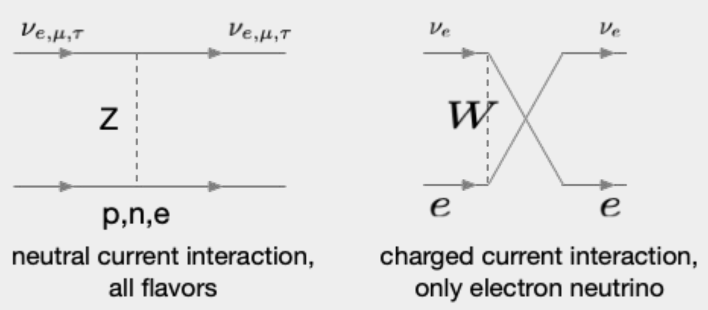
Background

Neutrino propagation in a vacuum can be characterized by two parameters: a mixing angle θ_{12} which relates the flavor basis (e/μ) to the mass eigenstate basis, and a difference of the squared masses δm_{12}^2.

When neutrinos propagate through matter, all flavors interact with the background electrons through a neutral weak current carried by the Z boson. This affects the masses of all the flavors (e/μ/τ) in the same way and can be ignored. Electron neutrinos however, interact in a special way because this interaction is via the W boson which carries an electric charge and shifts only the effective electron neutrino mass .

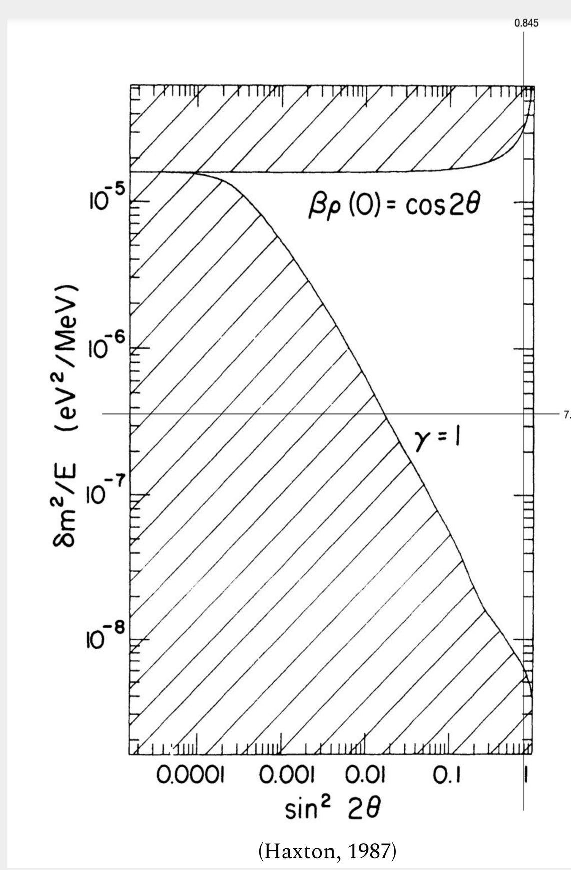


The two-flavor (e/μ) propagation of neutrinos through matter can be described by the equation below. E is the energy of the neutrino being modeled. The term $2E\sqrt{2}G_F\rho_e(x)$ represents the electron neutrino gaining an effective higher mass as a function of the density $\rho_e(x)$. The common neutral coupling via the Z has the same contribution for all flavors and can be ignored.

$$\frac{d}{dt}\begin{pmatrix}a_e(t)\\a_\mu(t)\end{pmatrix}=-\frac{i}{\hbar}\frac{1}{4E}\begin{pmatrix}-\delta m_{21}^2\cos 2\theta_{12}+2E\sqrt{2}G_F\rho_e(x) & \delta m_{21}^2\sin 2\theta_{12}\\ \delta m_{21}^2\sin 2\theta_{12} & \delta m_{21}^2\cos 2\theta_{12}-2E\sqrt{2}G_F\rho_e(x)\end{pmatrix}\begin{pmatrix}a_e(t)\\a_\mu(t)\end{pmatrix}$$

The adiabatic theorem describes the evolution of a system for a perturbation that slowly acts on the system. As the eigenstates evolve, their occupation remains fixed. Relating this to the sun, the Hamiltonian changes slowly from the core to a critical density which is where the adiabatic theorem is no longer valid because the electrons become degenerate. Figure 1 shows where the adiabatic theorem is valid (the shaded region) whereas unshaded represent where the adiabatic theorem is valid (Haxton) and the marked region are the current values for $\sin^2(2\theta_{12})$ and $\delta m_{12}^2/E$ with neutrino energy E=10 MeV.

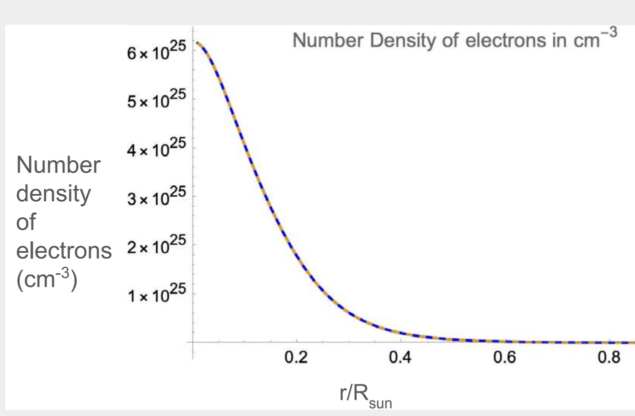
Fig. 1



(Haxton, 1987)

Neutrinos propagating in the Sun and supernovae see a higher density of electrons. The graph on the right is of the number density in cm^{-3} in the sun as a function of x=r/R$_{sun}$.

Fig. 2



Abstract

Neutrinos propagate in a vacuum in mass states (a mix of their flavor states). When electron neutrinos propagate through dense matter with electrons (such as the sun) their effective mass changes. This is important because it significantly reduces the amount of electron neutrino flux that we receive from the sun on earth. Neutrinos interact with electrons via the weak current from the W boson which increases the effective electron neutrino mass. Most of the propagation where the mass eigenstates are distinct can be understood using the Adiabatic Theorem. The electron density point where the mass eigenstates are nearly degenerate has to be handled separately. The goal of this project is to investigate mathematical methods to improve the accuracy and efficiency of computing the propagation of neutrinos in varying density matter like the sun or supernovae.

Methods

In order to determine the number of neutrinos that propagate from the core of our sun to the edge, we need to evolve the Hamiltonian over time

1. Calculated the electron density with respect to the radius from the core of the sun using Bachall's table (Bachall).
2. Solved for the critical radius where the electrons become degenerate
   - Found where the mass eigenstates of the neutrino were closest to each other using Eq. 3, the Hamiltonian
   - This happens when X(t)=0 (Eq. 2), so then we were able to solve for density
   - Can use electron density function to find critical radius that corresponds to critical density

Eq. 1 $$\begin{pmatrix}a_e(t)\\a_\mu(t)\end{pmatrix}=\exp\left(-\frac{i}{\hbar}\begin{pmatrix}-\delta m_{21}^2\cos 2\theta_{12}+4E\sqrt{2}G_F\rho_e(x) & \delta m_{21}^2\sin 2\theta_{12}\\ \delta m_{21}^2\sin 2\theta_{12} & \delta m_{21}^2\cos 2\theta_{12}\end{pmatrix}t\right)\begin{pmatrix}a_e(0)\\a_\mu(0)\end{pmatrix}$$

Eq. 2 $$X(t)=\frac{2\sqrt{2}G_F E\rho(x)}{\delta m_{21}^2}-\cos 2\,\theta_{12}$$

Eq. 3 $$\frac{d}{dt}\begin{pmatrix}a_e(t)\\a_\mu(t)\end{pmatrix}=-\frac{i}{\hbar}\frac{m_{21}^2}{4E}\begin{pmatrix}X(t) & \sin 2\theta_{12}\\ \sin 2\theta_{12} & -X(t)\end{pmatrix}\begin{pmatrix}a_e(t)\\a_\mu(t)\end{pmatrix}$$

Rather than calculating the Hamiltonian for the electron-muon basis, we want to calculate it in the instantaneous basis

1. Using Eq. 4 (the Hamiltonian) we can diagonalize it to give the energy states which we would use as our basis for the instantaneous Hamiltonian
2. To work in the instantaneous eigenstate basis, we need to rotate the electron-muon basis which would give us Eq. 6.
   - Rotate the electron-muon basis to the light/heavy basis by multiplying it by the first matrix in Eq. 4
   - The second term in brackets of Eq. 6 tells us when adiabatic propagation is valid
     - ➢ If this term is smaller, then adiabatic propagation occurs
     - ➢ If the term is larger, then Landau-Zener hopping from the heavy state to the light state occurs
   - θ(t) describes the rotation to the light-heavy basis, and θ'(t) tells us how fast it changes.

Eq. 4 $$\begin{pmatrix}a_L(t)\\a_H(t)\end{pmatrix}=\begin{pmatrix}\cos\theta(t) & -\sin\theta(t)\\ \sin\theta(t) & \cos\theta(t)\end{pmatrix}\begin{pmatrix}a_e(t)\\a_\mu(t)\end{pmatrix}$$

Eq. 5 $$\lambda(t)=X(t)\cos(2\theta(t))-\sin(2\theta_{12})\sin(2\theta(t))$$

Eq. 6. $$\frac{d}{dt}\begin{pmatrix}a_L(t)\\a_H(t)\end{pmatrix}=-\frac{i}{\hbar}\left[\frac{m_{21}^2}{4E}\begin{pmatrix}\lambda(t) & 0\\ 0 & -\lambda(t)\end{pmatrix}+i\hbar\theta'(t)\begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix}\right]\begin{pmatrix}a_L(t)\\a_H(t)\end{pmatrix}$$

Eq. 7 $$\theta(t)=\arctan\left(\frac{X(t)+\sqrt{X(t)^2+\sin^2 2\theta_{12}}}{\sin 2\theta_{12}}\right)$$

Results

Sections below show the differences of working in different eigenstate bases.

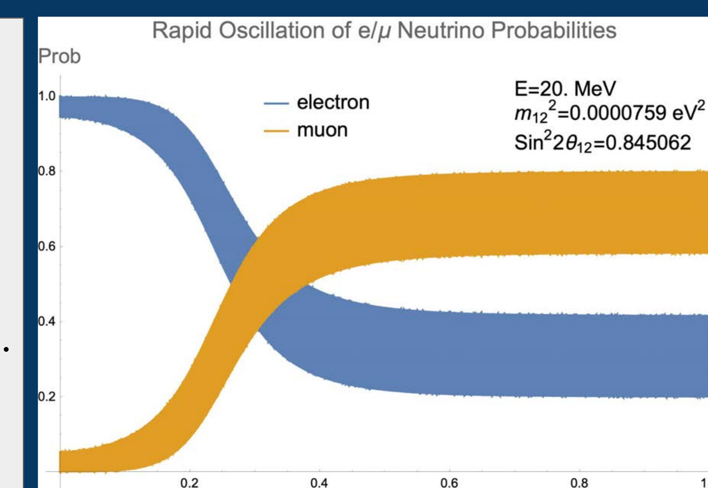
- Figure 1 is working in the electron-muon basis while Figure 2 is working in the instantaneous eigenstate basis
- Below that section shows what happens when Landau-Zener hopping occurs, a phenomena which was discussed previously in the methods section using Eq. 6

Electron-Muon Neutrino Basis & Instantaneous Eigenstate Basis

Rapid oscillation indicates the use of the wrong basis.

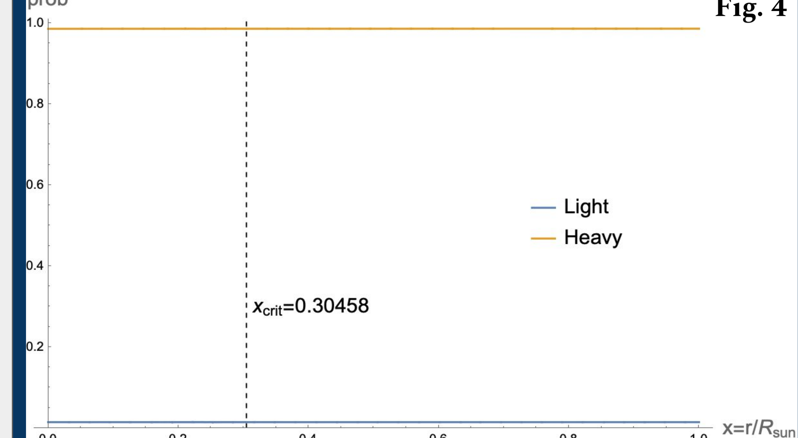
The average electron probability (blue band) is about ⅓ on exit from the sun.

Fig. 3



Components in a local eigenbasis do not oscillate. In the adiabatic case the occupation of the light and heavy states are fixed, but their definition evolves with density – Very Efficient! A basis change back to (e/μ) yields the same ⅓ e-neutrino average.

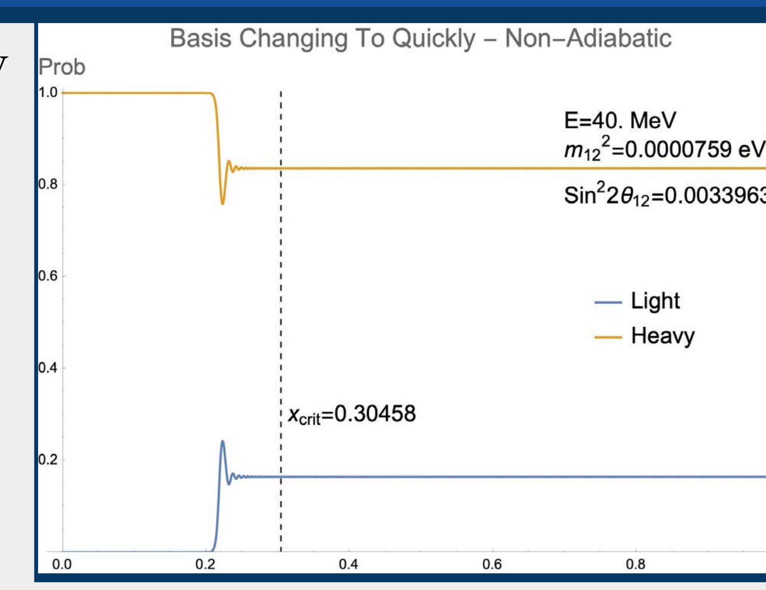
Fig. 4



Landau-Zener Hopping

As the critical density is approached, hopping from the heavy to the light state occurs. This hopping reduces the conversion from electron neutrinos to muon neutrinos.

Fig. 5



Landau Zener hopping occurs when the eigenstates of H become nearly degenerate and θ(t) is changing rapidly so the second term of the L/H differential equation becomes relatively large. The second term is off-diagonal, connecting the two states. While this process does not occur in the sun, it does happen in supernovae with higher energy neutrinos suppressing the first term.

Discussion

In situations where adiabatic propagation is valid, the initially generated electron neutrino can be transformed into the local L/H basis and the occupation of these states will be constant all the way to vacuum.

- We can then transform back to the electron/muon neutrino basis.
- The resulting probability of detecting an electron neutrino is then averaged over one oscillation cycle, yielding approximately ⅓ in the Sun.
- Most electron neutrinos are converted to muon neutrinos.

Fig. 3 shows the complexity caused by working in the wrong basis.

- A numerical differential equation solver has to reproduce rapid oscillations between electron and neutrino flavors
- Fig 4. Shows a boring constant graph in the L/H basis.
  - This is much easier for a differential equation solver.

In other environments the adiabatic constraints are violated.

- In a region around the critical density where the eigenstates are nearly degenerate, the second term in the L/H basis Hamiltonian couples the two states. A tricky aspect is knowing how wide the region is.
- Fig. 5 shows the hopping occurring before the critical density is reached.
- With a reliable answer for the width, adiabatic propagation can be efficiently used outside the region and a differential equation solver inside the non-adiabatic region.
- The shape of the density curve can change the results, motivating the use of the actual density profile instead of an approximation.

Modern neutrino evolution codes for supernovae work in time steps with each time step using a fixed snapshot of the local basis.

- This local basis is only updated when it becomes too different from the instantaneous basis.
- We would suggest that working in a continuously changing basis is much more efficient.

Future Work

I would like to extend the local eigenstate formulation to 3 neutrinos and try to understand how to use it with the additional neutrino-neutrino interactions that become important in supernovae.

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Sources

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