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Anomalous Diffusion in MRI: *Fractional Derivatives versus Relaxation Spectra*

N3AS Network for Neutrinos, Nuclear Astrophysics, and Symmetries
PHYSICS FRONTIER CENTER

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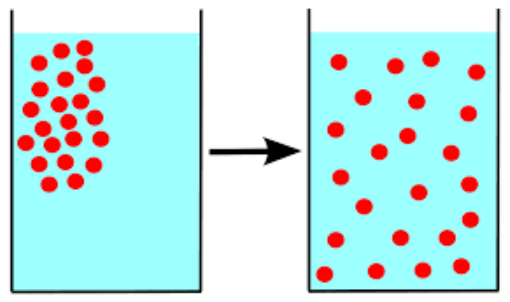
Diffusion

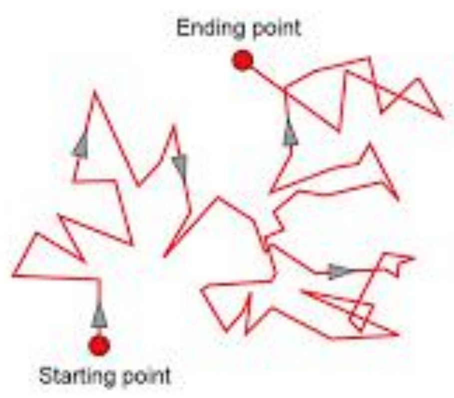
- Diffusion: Describes the way in which particles move

-To the right is the most common example: particles spreading out to equilibrium



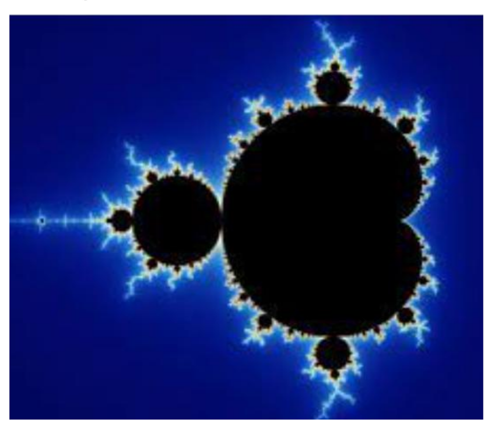
- Diffusion is an appropriate tool used where we see unresolved motion at small scales

- Brownian Motion gives evidence of this, as it is modeled by random walks



-Diffusion equations describe motion of many Brownian particles

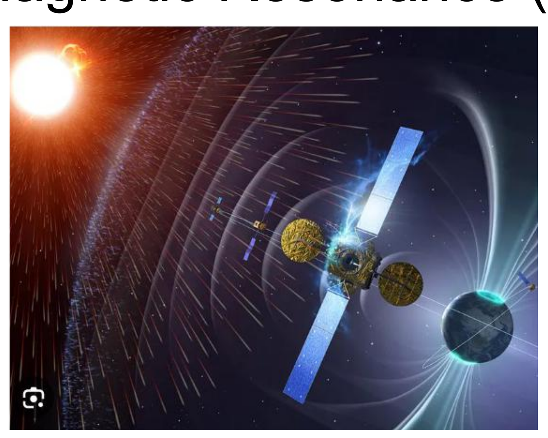
- When Random walks include infinitesimal jumps, their trajectories look like fractals



- When this occurs, we have to adapt our equation to include anomalous diffusion

- Anomalous diffusion is a powerful tool for describing a wide range of complex systems

- Common examples of using complex Diffusion include: Cosmic Rays Nuclear Magnetic Resonance (NMR)



Relaxation Spectrum: Stable Distributions

-Bloch-Torrey: Equations of motion of nuclear magnetization as a function of time controlled by relaxation rates T1 and T2

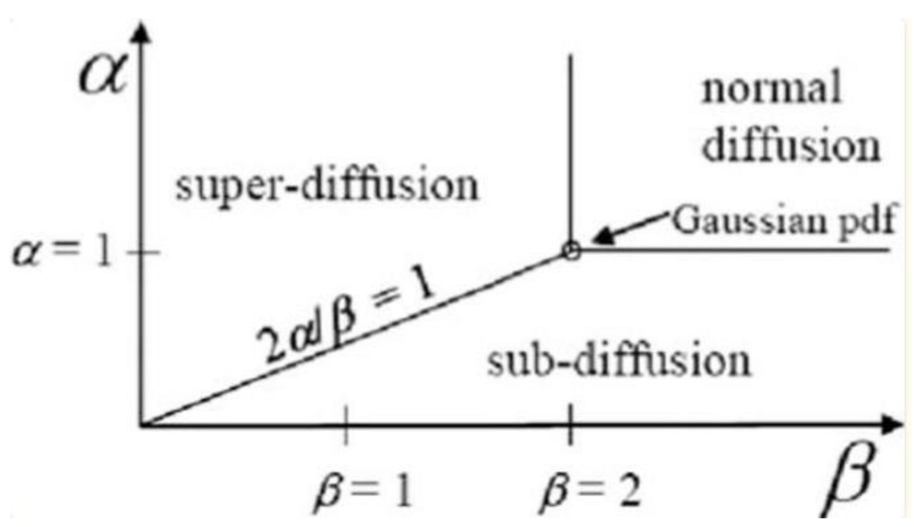
$$\frac{d\mathbf{M}}{dt} = \gamma \mathbf{M} \times \mathbf{B} - \frac{M_x \mathbf{i} + M_y \mathbf{j}}{T_2} - \frac{M_z - M_0}{T_1}\mathbf{k}$$

- T1 (spin-lattice relaxation): rate of transfer of energy from nuclear spin system to molecules.

-T2 (spin-spin relaxation): the loss of coherence

Tissue	T_1 (msec)	T_2 (msec)
Gray matter (GM)	950	100
White matter (WM)	600	80
Muscle	900	50
Cerebrospinal fluid (CSF)	4500	2200
Fat	250	60
Blood	1200	100-200

- when incorporating anomalous diffusion into the Bloch-Torrey equation, Magin observed non-exponentials that suggest a diffusion phase transition



-α – fractional derivative in space

-β – fractional derivative in time

- Magin's observation raises the question of what α and β, prompting an investigation into the motivation of choice

- Taking a statistical approach, we assume that we can replace fractionals with a fine-grained inhomogeneity equation to try to observe non-exponential behavior

- When in the "super-diffusion" phase, we would expect a Cauchy distribution, "normal-diffusion" is Gaussian, and "sub-diffusion" is levy distribution

- Relaxation Spectrum: A set of eigenvalues of an equation describing the relaxation or equilibration

-Relaxation can be decomposed into exponentials, needed anomalous diffusion and phase transitions

- If spectrum is non-gaussian, we will have a distribution with diverging moments that may allow us to tune parameters to observe qualitative change in distribution

-We sampled alpha stable distributions since they generalize Gaussians without finite mean invariance

-this allowed us to look for partition zeros on a complex time plane, but at finite time, this did not exist

- different approach was needed

Relaxation Spectrum: Bloch Torrey

- Our current path to finding the relaxation spectrum is to derive the eigenvalues directly from our Bloch-Torrey equation

- Consider nonlinear Bloch-Torrey by assuming diffusion coefficient, and/or T1 and T2 are functions of local magnetization

- linearize it and solve for a time-dependent relaxation spectrum

-Since magnetization is inhomogeneous, nonlinearity automatically introduces inhomogeneity to coefficients of the equation

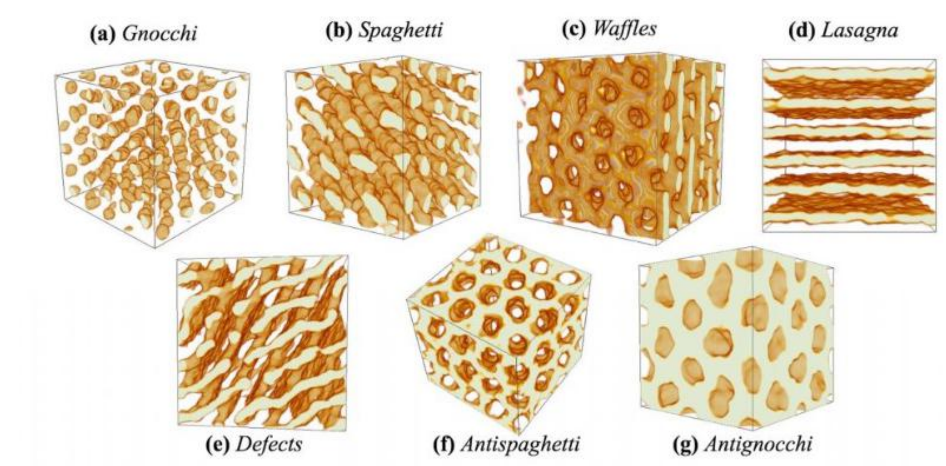
Relevance/Future Direction

Going forward:
- We will examine how the magnetization dynamics change as diagnosed by the linearized relaxation spectrum as we add inhomogeneity and nonlinearity

Relevance within N3AS:
- Magnetization transport parallels neutrino flavor transport

- Exploring anomalous transport in a general sense can lead to new applications within particle astrophysics

- For example ,we speculate this may be relevant in neutrino transport in nuclear pasta within neutron stars



References

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