Probing Neutron Star Interior with Non-Radial Oscillations

Tianqi Zhao

University of California, Berkeley & Ohio University

N3AS Network for Neutrinos, Nuclear Astrophysics, and Symmetries PHYSICS FRONTIER CENTER

Detection of Neutron Star Oscillation

The gravitational waves produced by neutron star mergers provide us with crucial information about the properties of these celestial objects. The chirp mass and tidal deformability have been analyzed using data from LIGO and Virgo observations of GW170817. The third generation of gravitational wave detectors may be able to detect direct signals of oscillating neutron stars in[1]:

- BNS merger remnant
- Core-collapse SNe
- Star quake (glitches)
- NS close encounter

$$h(t) = h_0 e^{-t/\tau}\cos\omega t, \quad F(t) = \frac{c^3\omega^2 h_0^2}{16\pi G}e^{-2t/\tau}$$

$$E = \frac{c^3\omega^2 h_0^2 \tau D^2}{8G} = 4.27\times 10^{49}\left(\frac{\nu}{\mathrm{kHz}}\right)^2\left(\frac{h_0}{10^{-23}}\right)^2\left(\frac{\tau}{0.1\,\mathrm{s}}\right)$$

, or BNS mergers via indirect signal of mode excitation during the inspiring phase. Orbital energy decay due to resonant tidal energy transfer is,

$$\frac{\Delta E_{orbit}}{E_{orbit}} = \frac{\pi^2}{512}\frac{GM^2}{R}\omega_{nl}^{1/3}Q_{nl}^2\beta^{-5/2}q\left(\frac{2}{1+q}\right)^{5/3}$$

Classification of Neutron Star Oscillations

Radial oscillation (l=0): don't couple to gravitational waves:

$$\varepsilon^r = R_n^r(r)e^{\mathrm{i}\omega t}, \quad \varepsilon^{\theta,\,\phi} = 0$$

Further classify to f-mode (fundamental n=0), p-modes (pressure n=1,2,...)[2, 3].

Non-radial oscillation (l>=2):

$$\varepsilon^{r,\,\theta,\,\phi} = \partial_{r,\,\theta,\,\phi}\left(R_n^{r,\,\theta,\,\phi}(r)Y_m^l(\theta,\phi)e^{\mathrm{i}\omega t}\right)$$

Further classify to f-mode, p-modes, g-modes (gravity n=1,2,...).

	ν(kHz)	τ(s)
f-mode	1.3-2.8	0.1-1
p-mode	>2.7	1-1000
g-mode	<0.8	>100

Table 1. Frequency and damping time

Frequency, damping time and oscillation profile of these oscillations are sensitive to the equation of state (EOS) of a neutron star which is crucial in relating the properties of heavy nuclei, dense uniform nuclear matter and neutron stars.

Method: Linearized Full General Relativity

Even parity perturbation of the Regge-Wheeler metric,

$$ds^2 = -e^{\nu(r)}(1+r^\ell H_0(r)e^{i\omega t}Y_{\ell m}(\phi,\theta))c^2dt^2 + e^{\lambda(r)}(1-r^\ell H_0(r)e^{i\omega t}Y_{\ell m}(\phi,\theta))dr^2$$
$$+(1-r^\ell K(r)e^{i\omega t}Y_{\ell m}(\phi,\theta))r^2d\Omega^2 - 2i\omega r^{\ell+1}H_1(r)e^{i\omega t}Y_{\ell m}(\phi,\theta)dtdr$$

where H_0, H_1, and K are metric perturbation functions. ω is the complex oscillation frequency.

Fluid perturbations with even parity are described by the Lagrangian displacement vectors

$$\xi^r = r^{\ell-1}e^{-\lambda/2}WY_m^\ell e^{i\omega t}$$
$$\xi^\theta = -r^{\ell-2}V\partial_\theta Y_m^\ell e^{i\omega t}$$
$$\xi^\phi = -\frac{r^{\ell-2}}{\sin^2\theta}V\partial_\phi Y_m^\ell e^{i\omega t},$$

which define the fluid perturbation amplitudes W and V. Einsteins' equation reduce to linearized equation of motion (EOM), defining the eigenvalue problem of standing waves.

g-mode

1. g-mode due to density discontinuity

Discontinuity g-mode is similar to surface wave in the ocean. **Its frequency is sensitive to phase transition properties in the core of NS such as density discontinuity and the location of the discontinuity[1],**

$$\Omega_g^2 \approx \frac{\beta^3(M_t/M)(R/R_t)^3(\Delta\varepsilon/\varepsilon_t)D\tanh[D]}{1+\Delta\varepsilon/\varepsilon_t+\tanh[D]/\tanh[D(R/R_t-1)]}, \tag{1}$$

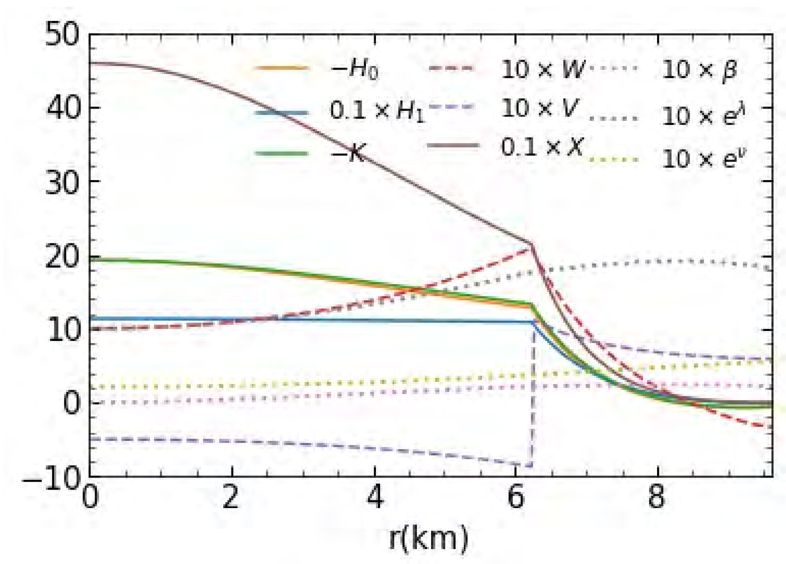

Figure 1. Perturbation profiles of a typical discontinuity g-mode.

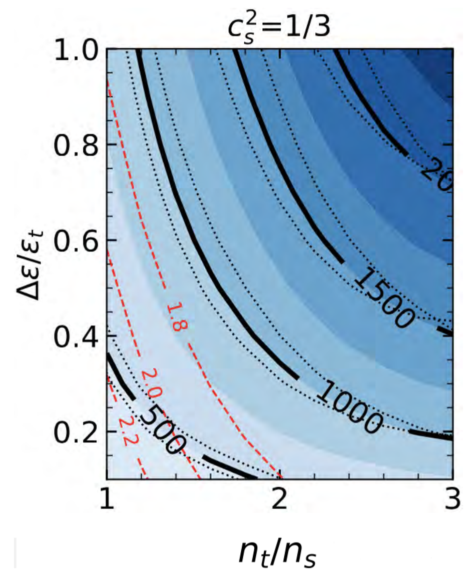

Figure 2. g-mode frequency varying phase transition properties[1].

2. g-mode due to composition gradient

Equilibrium equation of state is $p(n_B(r), Y_i(r))$, where both pressure p, baryon number density n_B and chemical composition Y_i is a function of position inside neutron star. Therefore, adiabatic index of compression can be defined,

$$\Gamma_{ad} = \frac{\partial\ln p}{\partial\ln n_B} \tag{2}$$

$$\Gamma_{eq} = \frac{d\ln p}{d\ln n_B} = \Gamma_{ad} + \frac{\partial\ln p}{\partial\ln Y_i}\left(\frac{d\ln Y_i}{dr}\right)\left(\frac{d\ln n_B}{dr}\right)^{-1} \tag{3}$$

The difference between two adiabatic index $\Delta(c^{-2}) = (1/\Gamma_{eq} - 1/\Gamma_{ad})\frac{\varepsilon+p}{p}$, leads to the local gravity oscillation with frequency known as the Brunt-Väisälä frequency,

$$\nu_g^2 = g^2 e^{\nu-\lambda}\Delta(c^{-2}), \tag{4}$$

where ν and λ are the temporal and radial metric functions, and $g = (dp/dr)(\varepsilon+p)^{-1}$ is the local gravity.

The global g-mode frequency is sensitive to the chemical composition, e.g. lepton fraction Y_{lep},

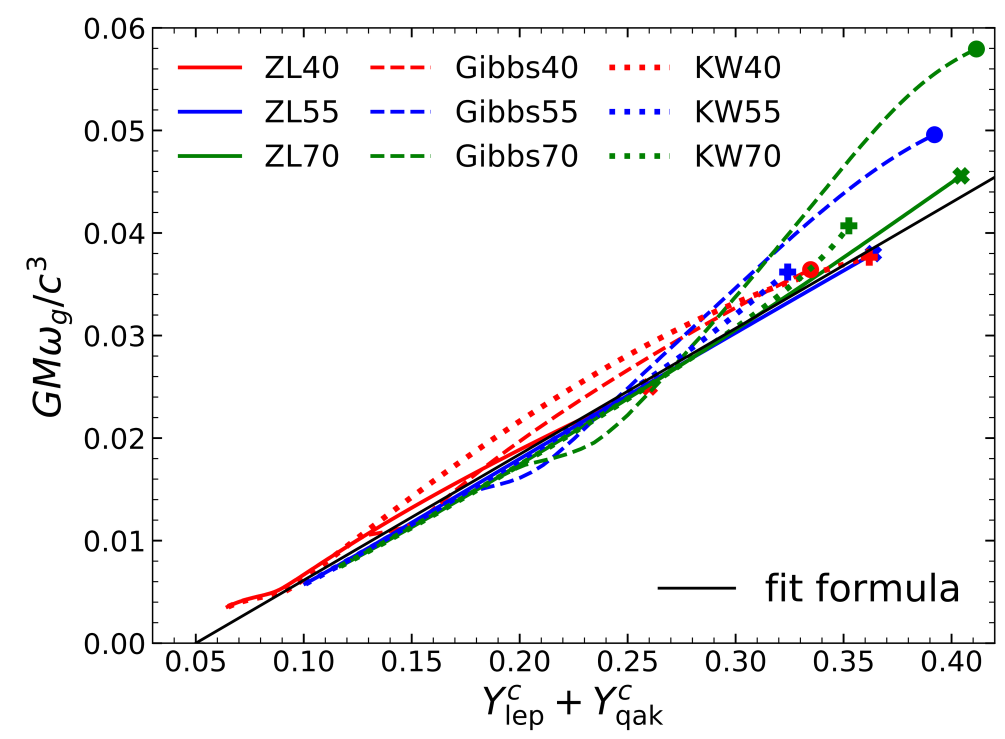

Figure 3. Frequency of compositional g-mode vs sum of lepton and quark fractions[4].

f-mode and p-modes

f-mode and p-modes are pressure supported oscillation modes. The fluid perturbations, W and V increases with radius. The mode frequencies increase with stellar compactness.

- f-mode is the dominant mode from Core-collapse SNe and NS merger simulations, since it have lowest number of radial nodes (usually zero).
- While p-modes have more radial nodes, $n = 1, 2, \ldots$ and higher frequency. Therefore, p-modes are harder to excite and observe.

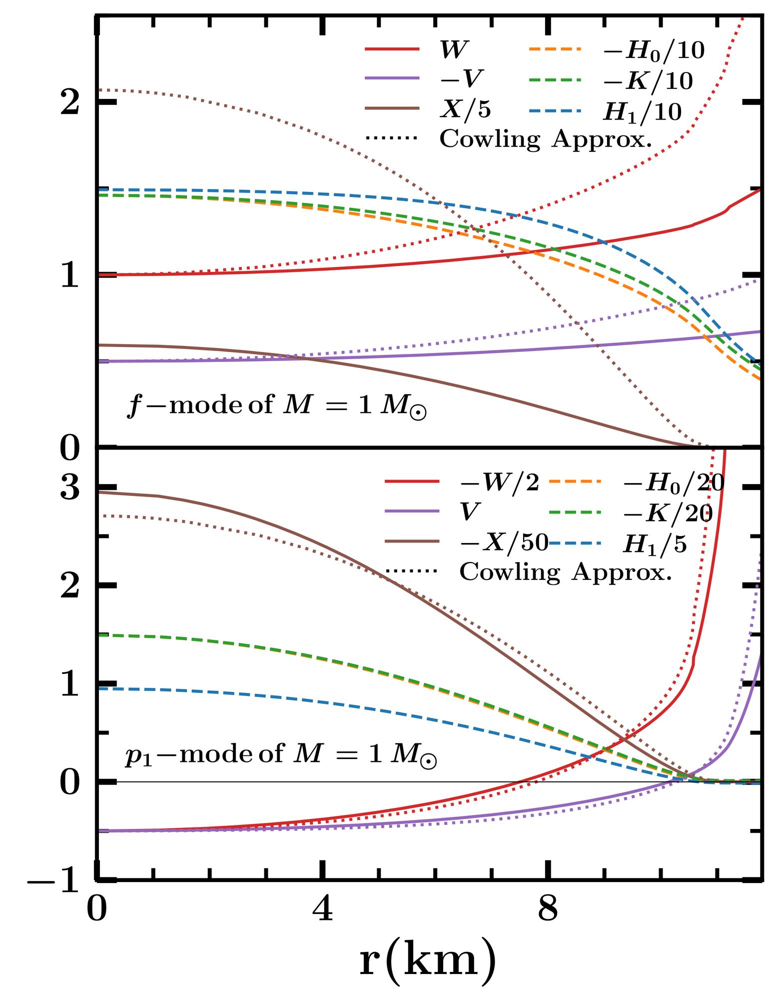

Figure 4. Perturbation profiles of typical f-mode and p_1-mode[5].

f- and p-modes are sensitive to pressure at saturation density and twice saturation density respectively.

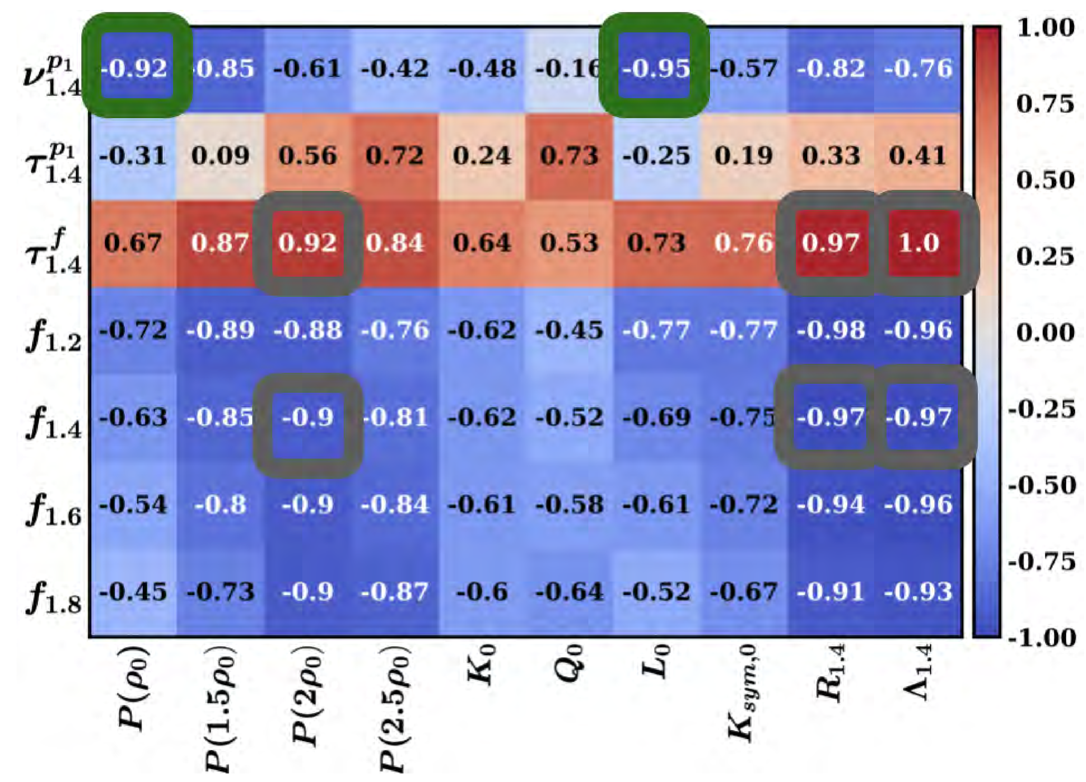

Figure 5. Correlation coefficient between frequency(damping time) nuclear properties[5].

References

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