Nuclear Effective Theory of $\mu \to e$ Conversion

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Introduction

- Neutrino flavor oscillations imply **lepton flavor** is **not conserved**
- Beyond Standard Model (BSM) physics can produce **observable charged lepton flavor violation** (CLFV)
- **Limits** on the CLFV $\mu \to e$ **conversion branching ratio**

$$B(\mu^- + (A,Z) \to e^- + (A,Z)) \equiv \frac{\Gamma(\mu^- + (A,Z) \to e^- + (A,Z))}{\Gamma\left(\mu^- + (A,Z) \to \nu_\mu + (A,Z-1)\right)}$$

could improve by **4 orders of magnitude** at next-generation experiments **Mu2e** at Fermilab and **COMET** at J-PARC

τ^-	ν_τ
μ^-	ν_μ
e^-	ν_e

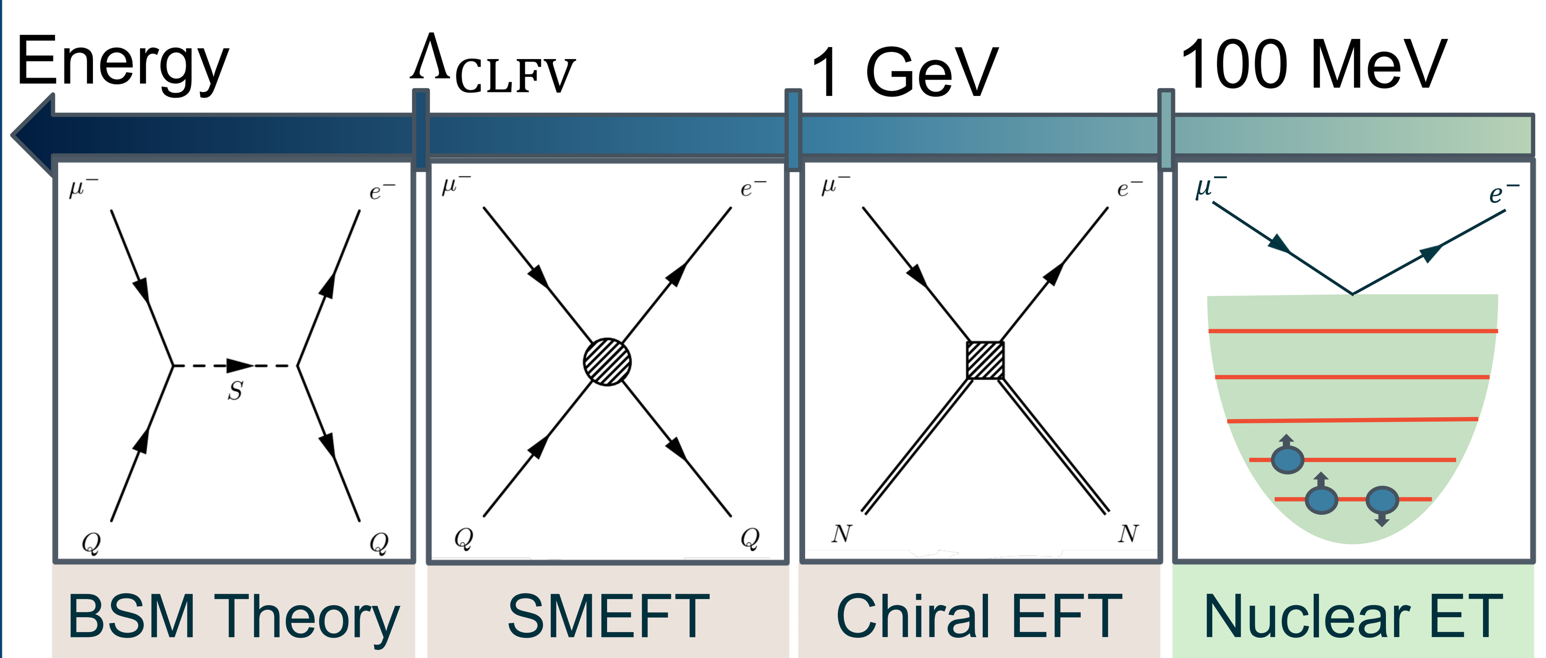
- Elastic $\mu \to e$ conversion:
  - **Nucleus** remains in **ground state**
  - **Maximizes energy** of outgoing **electron**
  - **Parity P** and **time-reversal T symmetries of nuclear ground state restrict operators** that can contribute
  - **Nuclear** physics makes **interpretation** of results **difficult**

How can we extract the **most information** about underlying **CLFV operators** from measurements of **elastic $\mu \to e$ conversion in nuclei**?

Why a Nucleon-level Effective Theory?

The process of $\mu \to e$ **conversion** spans a **wide range** of **energies**, from the **nuclear scale** where **experiments** are performed up to the **high-energy** realm ($\Lambda_{\rm CLFV} \gtrsim 1$ TeV) of candidate **UV CLFV theories**. One may formulate an effective theory description of the process in each of these energy ranges. The **nuclear-scale effective theory** is the **most natural**:

- **Interfaces** directly with **experiments**
- **Factorizes CLFV** physics from **nuclear** physics
- **Many-body calculations** require **nucleon** degrees of freedom



Above: Sketch of the energy scales that are relevant in $\mu \to e$ conversion and the corresponding theoretical descriptions that are applicable in each energy range.

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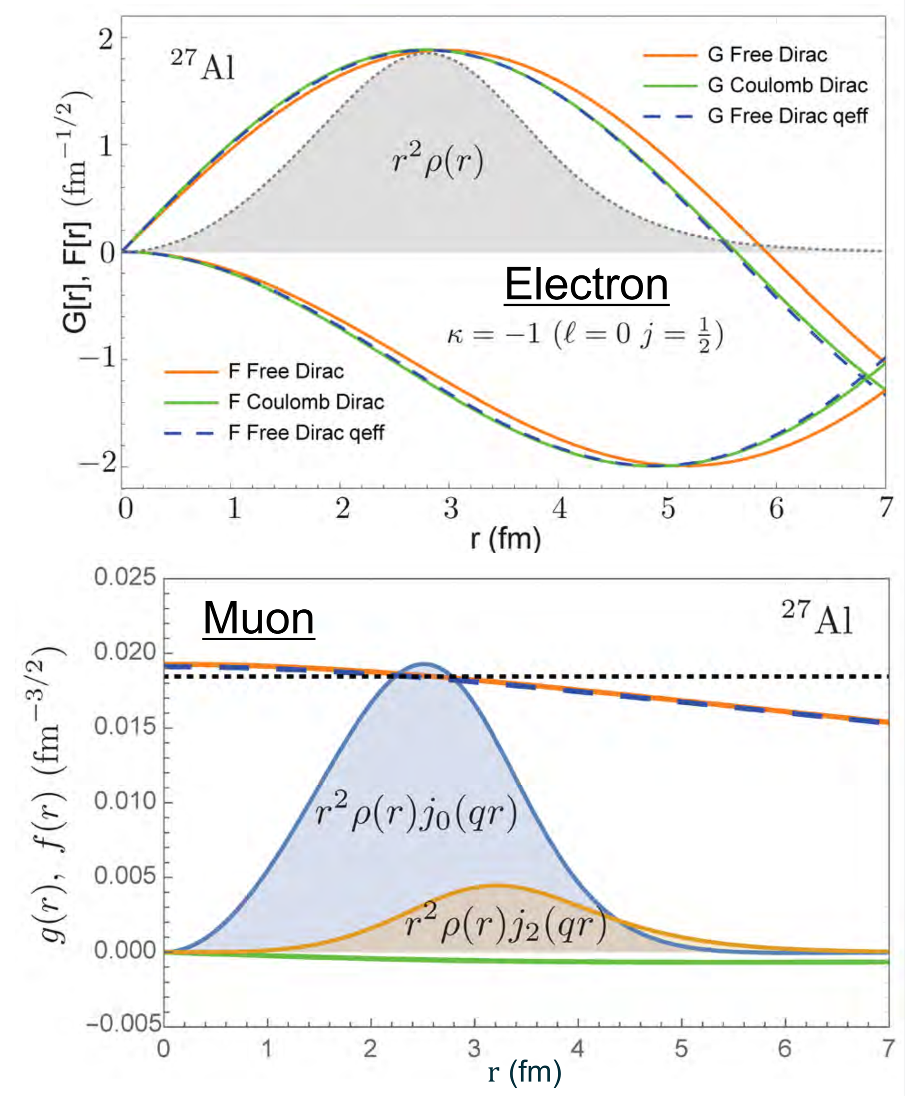
Lepton Wave Functions

- High precision numerical solutions of the **Dirac equation** can be computed for **bound muon** and **outgoing electron**
- **Effective momentum approximation** treats **electron as Dirac plane wave** with $q_{\rm eff} = q - \vec{V}_C$
- **Muon** is **approximately constant** over nuclear size
- **Relativistic effects** from muon lower component yield $\lesssim 5\%$ **corrections**



Top right: The exact $\kappa = -1$ Dirac electron wave function (green) for the target ^{27}Al is compared to the free Dirac plane wave solution (orange) and the effective momentum approximation solution (blue dashed). Nuclear charge density (with arbitrary normalization) is shown in gray.

Bottom right: The Schrodinger (blue dashed) and Dirac (orange) muon wavefunctions for the target ^{27}Al are compared. The dashed black line shows the constant approximation. The lower component is shown in green. Shaded regions correspond to nuclear monopole and quadrupole densities.

Effective Theory and Nuclear Embedding

- **16** independent **single-nucleon** CLFV operators $\mathcal{O}_i$ ($\times$ 2 for isospin)
- **Coefficients c_i** of single-nucleon operators $\mathcal{O}_i$ are approximately **target-independent**

Single Nucleon Level: 16 Operators

$\mathcal{O}_1 = 1_L 1_N$
$\mathcal{O}_{11} = i\hat{q} \cdot \vec{\sigma}_L\, 1_N$

Building blocks:
$1_L,\ 1_N,\ i\hat{q},\ \vec{v}_N,\ \vec{\sigma}_L,\ \vec{\sigma}_N$

$\mathcal{O}_7 = 1_L\ \vec{v}_N \cdot \vec{\sigma}_N$
$\mathcal{O}_{14} = i\hat{q} \cdot \vec{\sigma}_L\ \vec{v}_N \cdot \vec{\sigma}_N$

$\mathcal{O}_4 = \vec{\sigma}_L \cdot \vec{\sigma}_N$
$\mathcal{O}_6 = i\hat{q} \cdot \vec{\sigma}_L\ i\hat{q} \cdot \vec{\sigma}_N$
$\mathcal{O}_9 = \vec{\sigma}_L \cdot (i\hat{q} \times \vec{\sigma}_N)$
$\mathcal{O}_{10} = 1_L\ i\hat{q} \cdot \vec{\sigma}_N$

$\mathcal{O}_2' = 1_L\ i\hat{q} \cdot \vec{v}_N$
$\mathcal{O}_5 = \vec{\sigma}_L \cdot (i\hat{q} \times \vec{v}_N)$
$\mathcal{O}_8 = \vec{\sigma}_L \cdot \vec{v}_N$
$\mathcal{O}_{16}' = i\hat{q} \cdot \vec{\sigma}_L\ i\hat{q} \cdot \vec{v}_N$

$\mathcal{O}_3 = 1_L\ i\hat{q} \cdot (\vec{v}_N \times \vec{\sigma}_N)$
$\mathcal{O}_{12} = \vec{\sigma}_L \cdot (\vec{v}_N \times \vec{\sigma}_N)$
$\mathcal{O}_{13}' = \vec{\sigma}_L \cdot [i\hat{q} \times (\vec{v}_N \times \vec{\sigma}_N)]$
$\mathcal{O}_{15} = i\hat{q} \cdot \vec{\sigma}_L\ i\hat{q} \cdot (\vec{v}_N \times \vec{\sigma}_N)$

Above: The nucleon-level effective theory is constructed from four Hermitian operators: the direction of the outgoing electron $i\hat{q}$, the nucleon velocity operator $\vec{v}_N$, and the lepton $\vec{\sigma}_L$ and nucleon $\vec{\sigma}_N$ spin operators.

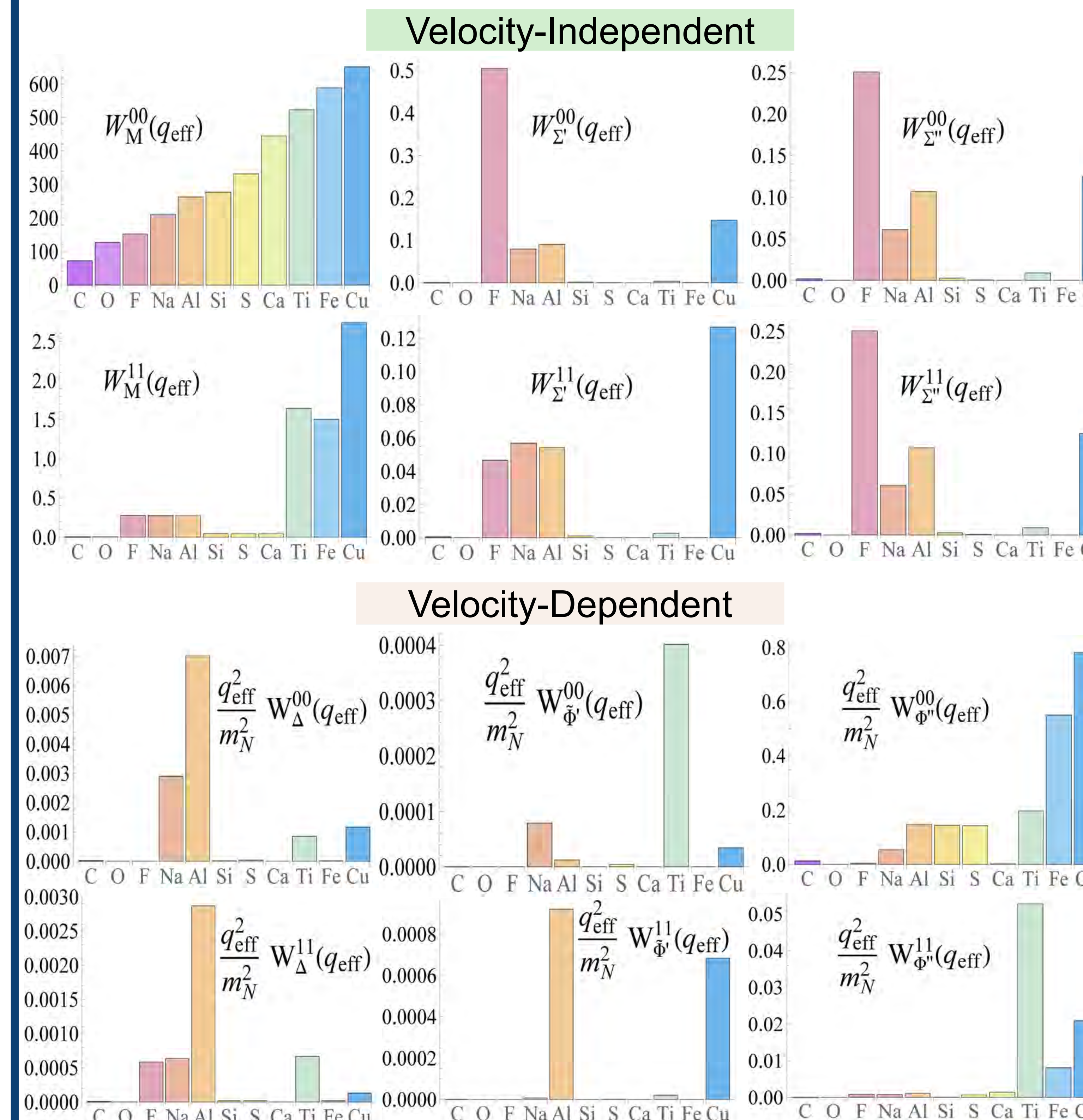
Below: The nuclear embedding imposes **P** and **T** symmetries of the nuclear ground state, restricting the operators (and multipolarity **J**) that can contribute. Some operators are enhanced by coherence; others are reduced by selection rules.

Nucleus Level: 6 Response Functions

Response	Operators	J
M_J (COHERENT)	$\mathcal{O}_1, \mathcal{O}_{11}$	Even J
Σ_J'	$\mathcal{O}_4, \mathcal{O}_9$	Odd J
Σ_J''	$\mathcal{O}_4, \mathcal{O}_6, \mathcal{O}_{10}$	Odd J
Δ_J	$\mathcal{O}_5, \mathcal{O}_8$	Odd J
$\tilde{\Phi}_J'$	$\mathcal{O}_{12}, \mathcal{O}_{13}'$	Even J
Φ_J'' (COHERENT)	$\mathcal{O}_3, \mathcal{O}_{12}, \mathcal{O}_{15}$	Even J

- **16 Single-nucleon operators** $\to$ (**6 Nuclear response functions**, **2 Interference terms**)
- **Conversion rate factorizes** $\omega_{\mu\to e} \propto$ **CLFV $R(c)$** $\times$ **Nuclear $W(q_{\rm eff})$**

Nuclear Responses



Above: Nuclear response functions evaluated using shell-model wave functions in various $\mu \to e$ conversion targets. Superscript "00" ("11") denotes the pure isoscalar (isovector) response.

Conclusions

A program of **$\mu \to e$ conversion measurements** can place up to **16 constraints on CLFV operators** by varying the **nuclear response functions W** through **target selection**.

Using state-of-the-art **shell-model** calculations, we explored the **sensitivity** to the underlying **CLFV operators** of **11 potential targets** including **^{27}Al**, the **chosen target** of the next-generation experiments Mu2e and COMET. The **effective theory** that we have developed provides a clear **factorization** between the **nuclear** physics and the **CLFV** physics, sequestering the latter quantity into **unknown coefficients** (the c_i's) that are **directly probed** by experiment. Finally, we have distilled the **nuclear effective theory** into publicly-available **Mathematica** and **Python** scripts.

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