Quantum computing for nuclear astrophysics: recent developments

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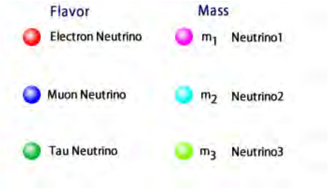
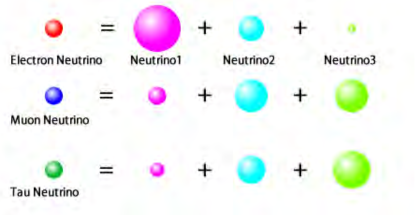
Overview

- Collective neutrino flavor oscillations
- NJL model
- Thermal and ground state preparation

Neutrino Flavor vs Mass

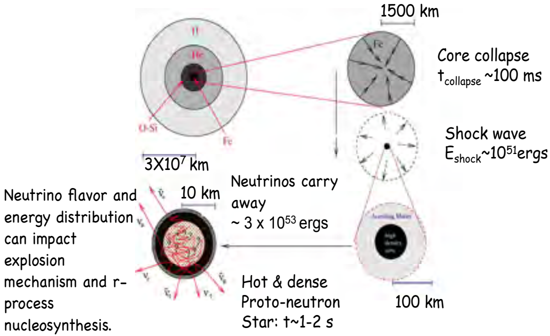
http://www.hyper-k.org/en/neutrino.html





Supernova Explosion

schematic from Sanjay Reddy



- Neutrinos carry $\sim 10^{53}$ erg of energy
- All flavors produced ($\sim 10^{58}$ ν over ~ 10 s)
- $(\nu_e, \overline{\nu}_e)$ can re-invigorate the shock wave and impact nucleo-synthesis
- Flavor oscillations will mix the three flavors

Neutral Currents

"The Universe at the MeV era: neutrino evolution and cosmological observables", Julien Frousty, arxiv 2209.06672

$$\hat{H}^{\nu\nu}_{NC} = \frac{G_F}{\sqrt{2}} m_Z^2 \sum_{\alpha,\beta} \int [\mathrm{d}^3\vec{p}_1][\mathrm{d}^3\vec{p}_2][\mathrm{d}^3\vec{p}_3][\mathrm{d}^3\vec{p}_4]\ (2\pi)^3 \delta^{(3)}(\vec{p}_1 + \vec{p}_2 - \vec{p}_3 - \vec{p}_4)$$
$$\times [\overline{\psi}_{\nu_\alpha}(\vec{p}_1)\gamma_\mu P_L \psi_{\nu_\alpha}(\vec{p}_3)] Z^{\mu\nu}(\Delta) [\overline{\psi}_{\nu_\beta}(\vec{p}_2)\gamma_\nu P_L \psi_{\nu_\beta}(\vec{p}_4)]$$
$$= \frac{1}{4} \sum_{i,j,k,l} \tilde{v}^{ik}_{jl} a_i^\dagger a_k^\dagger a_l a_j$$

Interaction process	$\tilde{v}^{12}_{34} / \left[\sqrt{2} G_F (2\pi)^3 \delta^{(3)}(\vec{p}_1 + \vec{p}_2 - \vec{p}_3 - \vec{p}_4)\right]$
$\nu_\alpha(1)\nu_\beta(2)\nu_\alpha(3)\nu_\beta(4)$	$(1+\delta_{\alpha\beta}) \times [\bar{u}^{h_1}_{\nu_\alpha}(\vec{p}_1)\gamma^\mu P_L u^{h_3}_{\nu_\alpha}(\vec{p}_3)][\bar{u}^{h_2}_{\nu_\beta}(\vec{p}_2)\gamma_\mu P_L u^{h_4}_{\nu_\beta}(\vec{p}_4)]$
$\nu_\alpha(1)\bar{\nu}_\beta(2)\nu_\alpha(3)\bar{\nu}_\beta(4)$	$-(1+\delta_{\alpha\beta}) \times [\bar{u}^{h_1}_{\nu_\alpha}(\vec{p}_1)\gamma^\mu P_L u^{h_3}_{\nu_\alpha}(\vec{p}_3)][\bar{v}^{h_4}_{\nu_\beta}(\vec{p}_4)\gamma_\mu P_L v^{h_2}_{\nu_\beta}(\vec{p}_2)]$
$\nu_\alpha(1)\bar{\nu}_\alpha(2)\nu_\beta(3)\bar{\nu}_\beta(4)$	$(1+\delta_{\alpha\beta}) \times [\bar{u}^{h_1}_{\nu_\alpha}(\vec{p}_1)\gamma^\mu P_L v^{h_2}_{\nu_\alpha}(\vec{p}_2)][\bar{v}^{h_4}_{\nu_\beta}(\vec{p}_4)\gamma_\mu P_L u^{h_3}_{\nu_\beta}(\vec{p}_3)]$

Two flavor operator algebra

J. Phys. G: Nucl. Part. Phys. 45 (2018) 113001

Forward scattering (diagonal elements)

$$J_+(\mathbf{p}) = a_e^\dagger(\mathbf{p}) a_x(\mathbf{p}), \quad J_-(\mathbf{p}) = a_x^\dagger(\mathbf{p}) a_e(\mathbf{p}),$$
$$J_z(\mathbf{p}) = \frac{1}{2}\left(a_e^\dagger(\mathbf{p}) a_e(\mathbf{p}) - a_x^\dagger(\mathbf{p}) a_x(\mathbf{p})\right).$$

$$[J_+(\mathbf{p}), J_-(\mathbf{q})] = 2\delta_{\mathbf{p},\mathbf{q}} J_z(\mathbf{p}), \quad [J_z(\mathbf{p}), J_\pm(\mathbf{q})] = \pm\delta_{\mathbf{p},\mathbf{q}} J_\pm(\mathbf{p}).$$

The Hamiltonian

Mass Term and Forward Scattering with Matter

$$H_\nu = \sum_{\mathbf{p}} \omega_p \mathbf{B} \cdot \mathbf{J_p} - \sqrt{2} G_F \sum_{\mathbf{p}} N_e J^z_{\mathbf{p}},$$

$$\omega_p = \frac{\triangle^2 m}{2p}, \quad \mathbf{B} = \sin(2\theta)\,\hat{x} - \cos(2\theta)\,\hat{z}$$

Forward Scattering Between Neutrinos

$$H_{\nu\nu} = \mu \sum_{\mathbf{pq}} (1 - \cos\vartheta_{\mathbf{pq}}) \mathbf{J_p} \cdot \mathbf{J_q}, \quad \mu = \sqrt{2} \frac{G_F}{V}$$

Collective Oscillations: Approximations

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Mean Field Approximation

$$H_{\nu\nu}^{\mathrm{MF}} = \sum_{\mathbf{pq}} \mu(1 - \cos\vartheta_{\mathbf{pq}})\left(P_{\mathbf{p}} \cdot \mathbf{J}_{\mathbf{q}}\right), \quad \mathbf{P}_{\mathbf{p}} = \langle \Psi(t) | \mathbf{J}_{\mathbf{p}} | \Psi(t) \rangle$$

$$d_t \mathbf{P}_{\mathbf{p}} = \overline{\omega}_p \mathbf{B} \times \mathbf{P}_{\mathbf{p}} + \mu \sum_{\mathbf{q}} (1 - \cos\vartheta_{\mathbf{pq}}) \mathbf{P}_{\mathbf{q}} \times \mathbf{P}_{\mathbf{p}}$$

Collective Oscillations: Approximations

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Mean Field Approximation

$$H_{\nu\nu}^{\mathrm{MF}} = \sum_{\mathbf{pq}} \mu(1-\cos\vartheta_{\mathbf{pq}})\,(P_{\mathbf{p}}\cdot\mathbf{J_q})\,,\quad \mathbf{P_p} = \langle\Psi(t)|\mathbf{J_p}|\Psi(t)\rangle$$

$$d_t\mathbf{P_p} = \overline{\omega}_p\mathbf{B}\times\mathbf{P_p} + \mu\sum_{\mathbf{q}}(1-\cos\vartheta_{\mathbf{pq}})\mathbf{P_q}\times\mathbf{P_p}$$

Mean Field Assumption: $\langle\mathbf{J_iJ_j}\rangle = \langle\mathbf{J_i}\rangle\langle\mathbf{J_j}\rangle$

No Correlations!

Beyond mean field

- Single angle approximation
  - Bethe ansatz PRD 99, 123013 (2019)
  - Tensor networks [Roggero PRD 104, 123023 (2021), Roggero PRD 104 (2021) 103016, Cervia etal PRD 105, 123025 (2022)]
  - Quantum computing [Yeter-Aydeniz etal arxiv 2104.03273, Siwach, Harrison, Balantekin arxiv 2308.09123 (2023)]

- Multi angle treatment
  - Cubic lattice systems, exact, up to 20 ν [Rrapaj, PRC 101, 065805 (2020)]
  - Large N in specific geometries [Roggero, Rrapaj, Xiong PRD 106, 043022 (2022)]
  - Quantum computing [Hall etal Phys. Rev. D 104, 063009 (2021), PRA 106, 052605 (2022)]

Gates and Qubits: Introduction

- Qubit $\equiv$ spin $1/2$, two state system
- n qubits $\equiv$ 2^n states of n spin $1/2$ particle system
- Gate $\equiv$ unitary operator acting on qubits
- Only certain gates are available
  1. One qubit gates $\longrightarrow$ rotations in $SU(2)$
  2. Two qubit gates $\longrightarrow$ controlled rotations
- every other gate can be obtained from them

The state of one qubit

Generic State

$$|\Psi\rangle = \cos(\tfrac{\theta}{2})|0\rangle + e^{-i\phi}\sin(\tfrac{\theta}{2})|1\rangle$$

Z gate (phase flip)

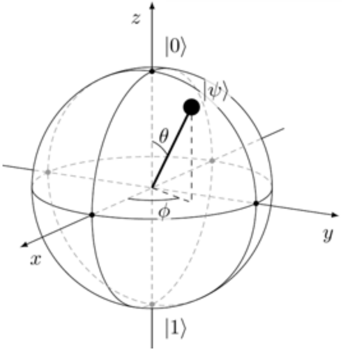
$$\hat{Z}\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

X gate (NOT)

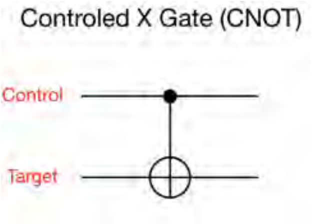
$$\hat{X}|0\rangle = |1\rangle, \ \hat{X}|1\rangle = |0\rangle$$

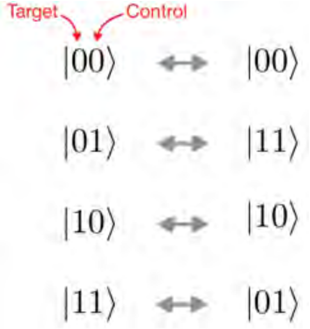
RY gate

$$e^{-i\alpha/2\hat{Y}} = \cos(\tfrac{\alpha}{2})\hat{I} - i\sin(\tfrac{\alpha}{2})\hat{Y}$$



Modifying the state of two qubits





Gate Based Quantum Computing

- Initial state $|\Psi_0\rangle = |0, 0....\rangle$
- Time evolution operator $e^{-i\triangle tH}$ decomposition:
  - Single qubit — rotation ($e^{-i\sigma^{x,y,z}\alpha}$)
  - Two qubit — controlled operation (CNOT)
- Apply gates
- Measure

Three flavors

Hamiltonian, mass basis

$$H = \sum_{\vec{p}} \vec{B}(\vec{p}) \cdot \vec{Q}(\vec{p}) + \mu \sum_{\vec{p},\vec{p}'} \left(1 - \cos\theta_{\vec{p}\vec{p}'}\right) \vec{Q}(\vec{p}) \cdot \vec{Q}(\vec{p}'),$$

$$\vec{B}(\vec{p}) = \{0, 0, -\omega_p, 0, 0, 0, 0, -2\Omega_p/\sqrt{3}\},$$

$$\omega_p = \frac{m_2^2 - m_1^2}{2p}, \Omega_p \approx \frac{m_3^2 - m_1^2}{2p} \approx \frac{m_3^2 - m_2^2}{2p}.$$

Flavor mixing

$$\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix} = U_{\rm PMNS} \begin{pmatrix}\nu_1\\ \nu_2\\ \nu_3\end{pmatrix}, \quad U_{\rm PMNS} = \begin{bmatrix}1 & 0 & 0\\ 0 & c_{23} & s_{23}\\ 0 & -s_{23} & c_{23}\end{bmatrix} \begin{bmatrix}c_{13} & 0 & s_{13}e^{-i\delta_{\rm CP}}\\ 0 & 1 & 0\\ -s_{13}e^{i\delta_{\rm CP}} & 0 & c_{13}\end{bmatrix} \begin{bmatrix}c_{12} & s_{12} & 0\\ -s_{12} & c_{12} & 0\\ 0 & 0 & 1\end{bmatrix}$$

Three flavors & qubits

SU(3) algebra

$$[Q_i, Q_j] = if_{ijk} Q_k, \ \{Q_i, Q_j\} = \frac{1}{3}\delta_{ij} + d_{ijk} Q_k,$$

$$f^{123} = 1, \ f^{147} = -f^{156} = f^{246} = f^{257} = f^{345} = -f^{367} = \frac{1}{2}, \ f^{458} = f^{678} = \frac{\sqrt{3}}{2}.$$

Qubit representation

$$\begin{aligned}
2Q_1 &= \sigma_1 \otimes \sigma_1 + \sigma_2 \otimes \sigma_2, \ 2Q_2 = \sigma_2 \otimes \sigma_1 - \sigma_1 \otimes \sigma_2, \\
2Q_3 &= \sigma_3 \otimes \sigma_0 - \sigma_0 \otimes \sigma_3, \ 2Q_4 = \sigma_1 \otimes \sigma_0 - \sigma_1 \otimes \sigma_3, \\
2Q_5 &= \sigma_2 \otimes \sigma_0 - \sigma_2 \otimes \sigma_3, \ 2Q_6 = \sigma_0 \otimes \sigma_1 - \sigma_3 \otimes \sigma_1, \\
2Q_7 &= \sigma_0 \otimes \sigma_2 - \sigma_3 \otimes \sigma_2, \\
2Q_8 &= \frac{1}{\sqrt{3}}(\sigma_0 \otimes \sigma_3 + \sigma_3 \otimes \sigma_0 - 2\sigma_3 \otimes \sigma_3).
\end{aligned}$$

Three flavors, continued

Physical Subspace

$$|\nu_e\rangle = |01>, \ |\nu_\mu\rangle = |10>, \ |\nu_\tau\rangle = |11>, \ |\tilde{\nu}\rangle = |00>$$

$$p_{\nu_e} = \langle Q_3\rangle + \frac{\langle Q_8\rangle}{\sqrt{3}} + \frac{1}{3}, \ p_{\nu_\mu} = -\langle Q_3\rangle + \frac{\langle Q_8\rangle}{\sqrt{3}} + \frac{1}{3}, \ p_{\nu_\tau} = \frac{1}{3} - \frac{2\langle Q_8\rangle}{\sqrt{3}}$$

Trotter step gate cost

- Naive implementation: > 300 CX
- Symmetries + Subspace: ≤ 34 CX
- Qutrits: ≤ 8 CSUM (CX for 3 levels)

Stay Tuned!

Quantum Chromo Dynamics

- Degrees of freedom: quarks + gluons
- Assymptotic freedom at high Q
- Nonperturbative behavior at $Q \lesssim$ 1 GeV:
  - Confinement of quarks and gluons
  - QCD vacuum has condensations of quarks and gluons
  - Nambu—Goldstone (NG) bosons and dynamical breaking of chiral symmetry (DBCS)
  - $U(1)_A$ anomaly (heavy η' and anomlaous decay $\eta \to 3\pi$)
  - Eplicit $SU(3)_V$ symmetry breaking

Nambu—Jona-Lasinio (NJL) approach

M. Kobayashi and T. Maskawa, Prog. Theor. Phys. 44, 1422 (1970).

Kobayashi and Maskawa (1970) – NJL model needs a term which breaks axial U(1) symmetry to solve the problem of the anomalously large η' mass.

$$\mathcal{L}_6 = g\left[\det\left(\bar{q}_i(1+\gamma_5)q_j\right) + \det\left(\bar{q}_i(1-\gamma_5)q_j\right)\right]$$

S. Weinberg, Phys. Rev. D 11, 3583 (1975).

Weinberg (1975) – Discusses the axial U(1) problem. Unbroken axial U(1) would imply a light isoscalar pseudoscalar boson which has not observed.

G. 't Hooft, Phys. Rev. Lett. 37, 8 (1976); Phys. Rev. D 14, 3432 (1976); Err. 18, 2199 (1978).

't Hooft (1976) – A six-quark interaction of exactly this form arises from instanton physics.

This interaction flips the helicity of the quarks!

"Simple" NJL model

Hatsuda, Kunihiro

Physics Reports Volume 247, Issues 5–6, October 1994, Pages 221-367

$$H = \int dx \ \left\{ \bar{q}\left(-i\gamma \cdot \nabla\right) q + g\left[(\bar{q}q)^2 + (\bar{q}i\gamma^5 q)^2\right] - \mu\left(\bar{q}\gamma^0 q\right) \right\}$$

- $\bar{q}q = \sum_a^{N_C} \bar{q}^{(a)} q^{(a)}$
- N_C = the number of color degree of freedom
- μ = chemical potential
- $g > 0$ (attractive force between quark—anti-quark)
- g has dimension E^{-2} $\longrightarrow$ non-renormalizable, Λ = cut-off

NJL + spin algebra

Interaction term

$$H_g = g \int dx \sum_{\vec{p}_1, s_1} \sum_{\vec{p}_2, s_2} \sum_{\vec{p}_3, s_3} \sum_{\vec{p}_4, s_4} \frac{1}{\sqrt{2E_{p_1}}} \frac{1}{\sqrt{2E_{p_2}}} \frac{1}{\sqrt{2E_{p_3}}} \frac{1}{\sqrt{2E_{p_4}}} \int dx$$

$$\left[\left(a^{\dagger(s_1)}_{\vec{p}_1} \bar{u}^{(s_1)}_{\vec{p}_1} e^{i\vec{p}_1\cdot\vec{x}} + b^{(s_1)}_{\vec{p}_1} \bar{v}^{(s_1)}_{\vec{p}_1} e^{-i\vec{p}_1\cdot\vec{x}} \right) (1-\gamma^5) \left(a^{(s_2)}_{\vec{p}_2} u^{(s_2)}_{\vec{p}_2} e^{-i\vec{p}_2\cdot\vec{x}} + b^{\dagger(s_2)}_{\vec{p}_2} v^{(s_2)}_{\vec{p}_2} e^{i\vec{p}_2\cdot\vec{x}} \right) \right]$$

$$\left[\left(a^{\dagger(s_3}_{\vec{p}_3} \bar{u}^{(s_3)}_{\vec{p}_3} e^{i\vec{p}_3\cdot\vec{x}} + b^{(s_3)}_{\vec{p}_3} \bar{v}^{(s_3)}_{\vec{p}_3} e^{-i\vec{p}_3\cdot\vec{x}} \right) (1+\gamma^5) \left(a^{(s_4)}_{\vec{p}_4} u^{(s_4)}_{\vec{p}_4} e^{-i\vec{p}_4\cdot\vec{x}} + b^{\dagger(s_4)}_{\vec{p}_4} v^{(s_4)}_{\vec{p}_4} e^{i\vec{p}_4\cdot\vec{x}} \right) \right]$$

$=\ldots$Complicated! $\longrightarrow$ Restrict kinematics between particles and antiparticles

Qubit Hamiltonian

$$H = 2N_C \sum_{\vec{p},s} \left[\frac{p}{\Omega} (A^{z,(s)}_{\vec{p}} + B^{z,(s)}_{\vec{p}}) - \frac{\mu}{\Omega} (A^{z,(s)}_{\vec{p}} - B^{z,(s)}_{\vec{p}}) \right]$$

$$+ \frac{2g}{\Omega^2} \sum_{\vec{p}_1, \vec{p}_2, h} \left[\tilde{p}_1^{(-h)} (A^{h(+)}_{\vec{p}_1} + B^{h(+)}_{\vec{p}_1}) + \tilde{p}_1^{(h)} (A^{-h(-)}_{\vec{p}_1} + B^{-h(-)}_{\vec{p}_1}) \right]$$

$$\times \left[\tilde{p}_2^{(-h)} (A^{h(-)}_{\vec{p}_2} + B^{h(-)}_{\vec{p}_2}) + \tilde{p}_2^{(h)} (A^{-h(+)}_{\vec{p}_2} + B^{-h(+)}_{\vec{p}_2}) \right]$$

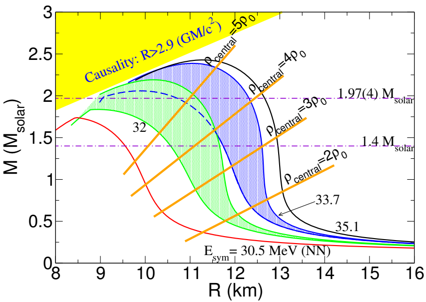
$\rightarrow$ Implement on hardware!

Nuclear potential

S. Gandolfi, J. Carlson, and Sanjay Reddy, PRC 85, 032801(R), (2012)
S.K. Bogner, R.J. Furnstahl, A. Schwenk Prog.Part.Nucl.Phys.65:94-147,2010

$$V(\vec{r}) = \sum_i V_i(\vec{r}) + \sum_{i<j} V_{ij}(\vec{r}) + \sum_{i<j<k} V_{ijk}(\vec{r}) + \ldots$$

Neutron Star Mass-Radius



Three body interactions

QMC calculations for Nuclear Physics

W. M. C. Foulkes, L. Mitas, R. J. Needs, and G. Rajagopal Rev. Mod. Phys. 73, 33 (2001)
J. Carlson, S. Gandolfi, F. Pederiva, Steven C. Pieper, R. Schiavilla, K.E. Schmidt, and R.B. Wiringa Rev. Mod. Phys. 87, 1067 (2015)

Challenges in QMC calculations

- Nuclear physics: many body interactions + complex spin structures
- MC step requires the evolution of $|\Psi_{t+\tau}\rangle \approx e^{-\tau V \hat{\sigma}_a \hat{\sigma}_b \hat{\sigma}_c}|\Psi_t\rangle$

- GFMC $\rightarrow$ fully entangled wavefunction
- AFDMC exploits decoupling $|\Psi_{t+\tau}\rangle \approx \sum_{k\epsilon\{a,b,c\}} p_k e^{-\tau V \hat{\sigma}_k}|\Psi_t\rangle$

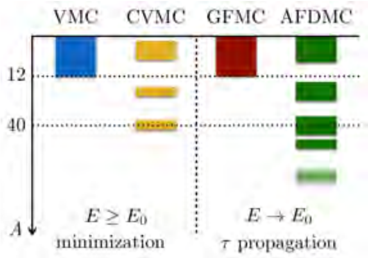


figure from Diego Lonardoni

Auxiliary fields

R. L. Stratonovich, Soviet Physics Doklady 2, 416 (1957)
J. Hubbard, Phys. Rev. Lett. 3, 77 (1959)
J. E. Hirsch, Phys. Rev. B 28, 4059 (1983)
E. Rrapaj, A.Roggero PRE 103, 013302 (2021)

Classical Computing

- Hubbard-Stratonovich: $\exp\left(-\frac{\tau}{2}\hat{O}^2\right) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} dh e^{-\left(h^2/2+\sqrt{-\tau}h\hat{O}\right)}, \ \tau < 0$
- Hirsch: $\exp\left(-\tau\hat{\rho}_\mu\hat{\rho}_\nu\right) = \sum_{h=\pm1} P(h) e^{h(A_\mu\hat{\rho}_\mu + A_\nu\hat{\rho}_\nu)}, \ \tau \epsilon R$
- Higher order interactions $\rightarrow$ Restricted Boltzmann Machine

Quantum Computing

Apply Boltzmann machines to quantum hardware:

- Finite temperature
- Ground state preparation

Thank you

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