Nucleosynthesis: Connecting Nuclear Properties and Observations - I
Outline for lecture I

• How can we study nuclear physics in astrophysics? Some observables [3-7]
• Some basic nuclear physics: masses, decays, reactions, reverse reaction rates, and equilibria [9-19]
• Reaction networks (BBN example and heavy element nucleosynthesis example) and using hydro simulations [21-30]
• Solar fusion [32-36]
• Stellar burning and stellar evolution [38-45]
The solar composition can be decomposed into many processes at multiple nucleosynthesis sites enriched the solar system.
Stellar spectroscopy

High-resolution full visible spectrum of our Sun
(50 slices with wavelength increasing from left to right and bottom to top, starting at 4000-7000 Angstroms)

N.A.Sharp, NOAO/NSO/Kitt Peak FTS/AURA/NSF
Meteorites

- CI Chondrites: fragile and rare types of meteorites, most important for studying solar system composition (only 5 known of ~1000 recorded meteorites observed to fall)
- When comparing to spectroscopic abundances from the solar photosphere, CI chondrites show the best agreement
- Allende meteorite most studied and dated (Mexico 1969): 4.6 billion years old

Deep sea ocean crusts

- Plutonium-244 (half-life 81 Myr) detection in Earth’s deep sea ocean floor implies an extraterrestrial source of Pu arriving on Earth during the last ~25 Myr
Multi-messenger events

SN1987A: A famous core-collapse supernova

Neutrinos

SN1987A: Fig. 2.—Bolometric evolution of SN 1987A through day 1444. The solid line from day 1 to 500 represents the estimates of the bolometric flux based on the U to M integration (see Suntzeff & Bouchet 1990), and the filled circles represent the U to 20 μm integration. The dotted lines represent the estimates for the energy deposition (WPH89) for the four radioactive nuclides, except that the 57Co has been raised to 5 times solar. The initial masses of these radioactive nuclides are: 56Ni (and subsequently 56Co), 0.075 M⊙; 44Ti, 1 x 10^-4 M⊙; 22Na, 2 x 10^-6 M⊙; and 57Co, 0.009 M⊙ (5 times solar). By day 700, the observed bolometric flux is greater than the amount of energy deposited by 56Co and another energy source is present.

From the early-time evolution of SN 1987A, we know the amount of 56Ni (and its daughter 56Co) synthesized in the explosion was 0.069 ± 0.0012 M⊙ (Bouchet et al. 1991b). In order to characterize the energy sources that dominate the late-time evolution, we have subtracted the energy deposition that corresponds to 0.069 M⊙ of 56Co from the observed bolometric luminosity. In addition, in order to understand how the uncertainties in the assumed amount of 57Co affect the results, we have also done the same calculation for the 3σ limits of 56Co (0.073 and 0.065 M⊙). All three calculations show that the non-56Co energy source declined by at least a factor of 20 from day 600 to 1500. The calculation for the 0.065 M⊙ of 56Co, however, gave a very large excess for days 600 to 800 that could not be fit by the models described below, so we have ignored this fit in the following discussion.

In Figure 3, we plot this energy “excess,” that is, the difference between the observed bolometric luminosity and the predicted energy deposition from 0.069 or 0.073 M⊙ of 56Co. One of the major conclusions to be drawn from Figure 3 is that the energy excess must be predominantly from an energy source that is exponentially declining in time. This is inconsistent with a constant energy source, such as that envisioned from a X-ray pulsar such as Her X-l or SMC X-l, or an embedded radio pulsar similar to the Crab pulsar (WPH89; Kumagai et al. 1989). The energy deposition from an X-ray pulsar with the spectral energy distribution of Her X-l or SMC X-l is expected to decline by less than 40% from day 600 to 1500 due to the changing optical depth in the homologously expanding envelope. The Crab pulsar, which has a softer spectral energy distribution in the X-ray region, is expected to decline by a factor of 2.6 during this time interval as compared with an observed decrease of roughly a factor of 20.

The exponential decline shown in Figure 3 can be explained naturally by the energy deposition from a radioactive nuclide. The solid lines refer to the best fit to the data for the energy deposition from 57Co (enhanced by factors of 5 and 6 over the predicted or “solar” value), and “solar” values for 44Ti (1 x 10^-4 M⊙) and 22Na (2 x 10^-6 M⊙). Over this time range, the latter two radioactive nuclides deposit negligible energy into the bolometric luminosity for such large enhancements.

Light curve

Kamiokande

IMB

Baksan

Woosley&Janka 06

Suntzeff+92

56Ni → 56Co + γ (T1/2 ~6 days)

56Co → 56Fe + γ (T1/2 ~77 days)
A new kind of messenger: gravitational waves

GW170817 & AT2017gfo: Binary neutron star merger

Over ~70 observing teams (~1/3 of the worldwide astronomical community) followed up on the merger event!

Ultraviolet (left, NASA Swift satellite)
Infrared (middle, Gemini South telescope)
Radio (right, Very Large Array)
γ-ray, X-ray, and optical also observed
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Nuclear masses and binding energy

The total nuclear masses is *less than* the sum of the masses of its constituent neutrons and protons:

$$M_{\text{nuc}} = Z M_p + N M_n - \Delta m$$

“mass defect” defines binding energy through $\Delta E = \Delta m \, c^2$; that is

$$BE(Z,N) = (Z M_p + N M_n - M_{\text{nuc}})c^2$$

*Nuclear processes liberate energy as long as the binding energy per nucleon of the final products exceeds the binding energy per nucleon of the initial constituents*
Nuclear masses and binding energy

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*Nuclear processes liberate energy as long as the binding energy per nucleon of the final products exceeds the binding energy per nucleon of the initial constituents.

\[ \frac{BE}{A} (^{12}C) = 7.68 \text{ MeV} \]
\[ \frac{BE}{A} (^{4}He) = 7.07 \text{ MeV} \]
\[ \frac{BE}{A} (^{16}O) = 7.98 \text{ MeV} \]

The energy release is:

\( (127.68 - 92.16 - 28.28) \text{ MeV} = 7.24 \text{ MeV} \)
Nuclear masses and binding energy

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*Nuclear processes liberate energy as long as the binding energy per nucleon of the final products exceeds the binding energy per nucleon of the initial constituents

Example 2

\[ \frac{BE}{A} (^{28}\text{Si}) = 8.45 \text{ MeV} \]

\[ \frac{BE}{A} (^{56}\text{Fe}) = 8.79 \text{ MeV} \]
Nuclear Decays

Decay constant (decay rate) $\lambda$ (and half-life $T_{1/2} = \frac{\ln(2)}{\lambda}$) defined by:

$$N_0 e^{-\lambda t}$$

Fermi’s Golden Rule:

$$\lambda = \frac{2\pi}{\hbar} |\langle f | H_{\text{int}} | i \rangle|^2 \rho(E)$$

$f, i$ are final and initial state wavefunctions and $M_{fi} = \langle f | H_{\text{int}} | i \rangle$ is the “matrix element”

$H_{\text{int}}$ is the weak interaction Hamiltonian

$\rho(E)$ is the density of states of the final particles (a process is more likely to happen if there is a larger choice of final states)
Cross sections and stellar reaction rates: \( B + x \rightarrow C + D \)

The cross section depends on the matrix element \( M_{fi} = \langle f | H_{\text{int}} | i \rangle \)
and can be understood schematically as:

\[
\sigma = \frac{\text{# of interactions per time}}{\text{(# of incident particles per area per time) (# of target nuclei in beam)}}
\]

Since cross sections are dependent on the incident energy (velocity),
\textit{in astrophysical plasmas must average over a velocity distribution}
to get the thermally averaged cross section:

\[
\langle \sigma v \rangle = \int \sigma \ v \ f(v) \ dv
\]

where if nuclei are non-relativistic and non-degenerate velocities
described by Maxwell-Boltzmann distribution \( \sim e^{-E/kT} \) giving

\[
f(v) = \left( \frac{m}{2\pi kT} \right)^{3/2} 4\pi v^2 e^{-\frac{mv^2}{2kT}} \quad \text{with} \quad m = \frac{m_B m_x}{m_B + m_x} \quad \text{(the reduced mass)}
\]
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\]

Interaction rate or reaction rate [cm\(^{-3}\) s\(^{-1}\)]:

\[
\tau_{Bx} = \frac{n_B n_x}{1 + \delta_{Bx}} \langle \sigma v \rangle
\]

“Stellar reaction rate” (per target nucleus) [s\(^{-1}\)]:

\[
\lambda_{Bx} = \frac{1}{1 + \delta_{Bx}} n_B \langle \sigma v \rangle
\]
Putting it all together: consider $B + x \rightarrow C + D$

$Q = (M_B + M_x - M_C - M_D) c^2$

$Q =$ energy released ($+$) or absorbed ($-$), aka Q-value [MeV]

$S_n(Z, A + 1) = M_{Z,A} + M_n - M_{Z,A+1}$

$S_n =$ one neutron separation energy [MeV]

$n_B = \rho N_A \frac{X_B}{A_B} = \rho N_A Y_B$

$n_B =$ number density [cm$^{-3}$], $\rho =$ density [g · cm$^{-3}$], $N_A =$ Avogadro's number ($6.022 \times 10^{23}$) [g$^{-1}$]

$X_B \over A_B = \frac{\text{mass fraction (} \sum_i x_i=1 \})}{\text{mass number (# protons + # neutrons)}}, Y_B =$ abundance

$Y_e = \sum_i Z_i Y_i = \frac{n_p}{n_p + n_n}$

$Y_e =$ electron fraction (formula assumes charge neutrality); lower $Y_e$ is more neutron rich

$\langle \sigma v \rangle =$ thermally averaged cross section $= \int \sigma \cdot v \cdot f(v) \, dv$ where $f(v) = \left( \frac{m}{2\pi kT} \right)^{3/2} 4\pi v^2 e^{-mv^2/2kT}$ is the Maxwell-Boltzmann distribution ($\sim e^{-E/kt}$) and $m = \frac{m_B m_x}{m_B + m_x}$ (the reduced mass)

$r_{Bx} = \frac{n_B n_x}{1 + \delta_{Bx}} \langle \sigma v \rangle$

$r =$ interaction rate or reaction rate [cm$^{-3}$ s$^{-1}$], $\lambda =$ “stellar reaction rate” (per target nucleus) [s$^{-1}$] (Note units of $N_A \langle \sigma v \rangle =$cm$^3$/s/g)

$\lambda_{Bx} = \frac{1}{1 + \delta_{Bx}} Y_B \rho N_A \langle \sigma v \rangle$
Recall definitions for $B + x \rightarrow C + D$

$$Q = (M_B + M_x - M_C - M_D)c^2$$

$$r_{Bx} = \frac{n_B n_x}{1 + \delta_{Bx}} \langle \sigma v \rangle$$

$$n_B = \rho N_A \frac{X_B}{A_B} = \rho N_A Y_B$$

$$\lambda_{Bx} = \frac{1}{1 + \delta_{Bx}} Y_B \rho N_A \langle \sigma v \rangle$$

**Reverse rate for $C + D \rightarrow B + x$** from detailed balance (equilibrium): Saha Equation

If $B \neq x$ and $C \neq D$ with all being nuclei:

$$r_{Bx} = r_{CD} \Rightarrow \frac{n_{CD}}{n_{Bx}} = \frac{\langle \sigma v \rangle_{Bx}}{\langle \sigma v \rangle_{CD}}$$

along with

$$\frac{\sigma_{Bx}}{\sigma_{CD}} = \frac{g_C g_D A_C A_D E_{CD}}{g_B g_X A_B A_X E_{Bx}}$$

where $g=2J+1$; can then obtain:

$$\frac{n_{CD}}{n_{Bx}} \frac{\langle \sigma v \rangle_{Bx}}{\langle \sigma v \rangle_{CD}} = \frac{g_C g_D}{g_B g_X} \left( \frac{A_C A_D}{A_B A_X} \right)^{3/2} e^{+Q/kT}$$

*See Fowler, Caughlan, and Zimmerman (1967) for more details*
Recall definitions for \( B + x \rightarrow C + D \)

\[
Q = (M_B + M_x - M_C - M_D)c^2 \quad r_{Bx} = \frac{n_{Bn_x}}{1+\delta_{Bx}} \langle \sigma \nu \rangle
\]

\[
n_B = \rho N_A \frac{X_B}{A_B} = \rho N_A Y_B \quad \lambda_{Bx} = \frac{1}{1 + \delta_{Bx}} Y_B \rho N_A \langle \sigma \nu \rangle
\]

**Reverse rate for \( C + D \rightarrow B + x \)** from detailed balance (equilibrium): Saha Equation

If \( B \neq x \) and \( C \neq D \) with all being nuclei:

\[
r_{Bx} = r_{CD} \Rightarrow \frac{n_{CnD}}{n_{Bn_x}} = \frac{\langle \sigma \nu \rangle_{Bx}}{\langle \sigma \nu \rangle_{CD}} \quad \text{along with} \quad \frac{\sigma_{Bx}}{\sigma_{CD}} = \frac{g_C g_D A_C A_D E_{CD}}{g_B g_X A_B A_X E_{BX}}
\]

where \( g = 2J + 1 \); can then obtain:

\[
\frac{n_{CnD}}{n_{Bn_x}} = \frac{\langle \sigma \nu \rangle_{Bx}}{\langle \sigma \nu \rangle_{CD}} = \frac{g_C g_D A_C A_D}{g_B g_X A_B A_X} \left( \frac{A_D}{A_B A_X} \right)^{3/2} e^{+Q/kT}
\]

If instead \( C \) is a photon:

\[
r_{Bx} = r_{DY} \Rightarrow \frac{n_D}{n_{Bn_x}} = \frac{\langle \sigma \nu \rangle_{Bx}}{\lambda_y}
\]

Gives:

\[
\frac{n_D}{n_{Bn_x}} = \frac{\langle \sigma \nu \rangle_{Bx}}{\lambda_y} = \frac{g_D}{g_B g_X} \left( \frac{A_D}{A_B A_X} \right)^{3/2} \left( \frac{2\pi \hbar^2}{mkT} \right)^{3/2} e^{+Q/kT}
\]

*See Fowler, Caughlan, and Zimmerman (1967) for more details*
Example: \((n, \gamma) \Leftrightarrow (\gamma, n)\) equilibrium + steady \(\beta\) flow

Assume \((n, \gamma) \Leftrightarrow (\gamma, n)\) equilibrium to obtain relative abundances of neighboring isotopes:

\[
\frac{Y_{A+1}}{Y_{A}} = \frac{n_{A+1}}{n_{A}} \approx n_{n} \frac{g_{A+1}}{g_{A}} \left(\frac{A+1}{A}\right)^{3/2} \left(\frac{2\pi \hbar^{2}}{Am_{n}m_{n}kT} (A + 1)m_{n}\right)^{3/2} e^{+S_{n}/kT}
\]

The evolution of abundances is determined from flow of \(\beta\)-decay:

\[
\frac{dn(Z)}{dt} = \lambda_{Z-1} n(Z - 1) - \lambda_{Z} n(Z) \quad \text{where} \quad \lambda_{Z} = \sum_{A} n(Z, A)\lambda_{\beta}(Z, A)
\]

*Sets the relative abundances along an isotopic chain

*Allows for the chain to move to elements with higher proton numbers or in the case of steady flow sets relative \(Z\) abundances
Nuclear Statistical Equilibrium (NSE)

If the environment is hot enough to overcome Coulomb barriers and has high energy photons, neutron and proton captures on \((Z,N)\) are in chemical equilibrium with reverse photodissociations:

\[
N \text{n neutrons} + Z \text{protons} \rightleftharpoons (Z,N)
\]

\[
N\mu_n + Z\mu_Z = \mu_{Z,N}
\]

where \(\mu\) is the chemical potential; nucleons and nuclei are described by Maxwell-Boltzmann distributions (note \(G_i\) is the partition function):

\[
\mu_i = m_i c^2 + kT \ln \left[ \rho N_A \frac{Y_i}{G_i} \left( \frac{2\pi\hbar^2}{m_i kT} \right)^{3/2} \right]
\]

*The above equations are used along with \(\sum_i A_i Y_i = 1\) and \(\sum Z_i Y_i = Y_e\) to solve for abundances at a given \(\rho, T, Y_e\)

For high temperatures, favors a composition of n, p, and \(\alpha\) due to photodissociation, for lower temperatures nuclei with the highest binding energy are favored (\(^{56}\text{Fe}\) for \(Y_e < 0.5\) and \(^{56}\text{Ni}\) for \(Y_e = 0.5\))
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A short intro to reaction networks

For the two-body reaction

\[ B + x \rightarrow C \]

\[ \frac{dn_B}{dt} = -n_B \lambda_{Bx} = -n_B Y_x \rho N_A \langle \sigma v \rangle \]

\[ \frac{dn_C}{dt} = +n_B \lambda_{Bx} \]
A short intro to reaction networks

For the two-body reaction

\[ B + x \rightarrow C \]

\[ \frac{dn_B}{dt} = -n_B \lambda_{Bx} = -n_B Y_x \rho N_A \langle \sigma \nu \rangle \]

\[ \frac{dn_C}{dt} = +n_B \lambda_{Bx} \]

Now if a one-body decay produces B

\[ D \rightarrow B + z \]

\[ \frac{dn_B}{dt} = -n_B Y_x \rho N_A \langle \sigma \nu \rangle + n_D \lambda_D \]

\[ \frac{dn_D}{dt} = -n_D \lambda_D \]
A short intro to reaction networks

For the two-body reaction
\[ B + x \rightarrow C \]
\[ \frac{dn_B}{dt} = -n_B \lambda_B \]
\[ \frac{dn_C}{dt} = +n_B \lambda_B \]

Now if a one-body decay produces B
\[ D \rightarrow B + z \]
\[ \frac{dn_B}{dt} = -n_B Y_X \rho N_A \langle \sigma v \rangle \]
\[ \frac{dn_D}{dt} = -n_D \lambda_D \]
\[ \frac{dn_C}{dt} = +n_B \lambda_B \]

Thus network equations can be written as:
\[ \dot{Y}_i = \sum_j \xi_j \lambda_j Y_j + \sum_{j,k} \xi_{j,k}^i \rho N_A \langle \sigma v \rangle_{j,k} Y_j Y_k \]
\[ + \sum_{j,k,l} \xi_{j,k,l}^i \rho^2 N_A^2 \langle \sigma v \rangle_{j,k,l} Y_j Y_k Y_l \]

Where \( \xi \) is + when i created, - when i consumed, and corrects for overcounting in a reaction involving identical particles

*Coupled differential equations can be put into matrix form so networks use matrix solvers
A short intro to reaction networks

\[ Q = (M_B + M_x - M_c - M_D)c^2 \]

\[ n_B = \rho N_A \frac{X_B}{A_B} = \rho N_A Y_B \]

\[ \lambda_{Bx} = \frac{1}{1 + \delta_{Bx}} Y_B \rho N_A \langle \sigma v \rangle \]

Written out in a more schematic way:

\[ \dot{Y}_i = \sum (2\text{body reactions into } i) - \sum (2\text{body reactions out of } i) \]

& \[ + \sum (3\text{body reactions into } i) - \sum (3\text{body reactions out of } i) \]

& \[ + \sum (\text{decays into } i) - \sum (\text{decays out of } i) \]

& \[ + \sum (\text{fission into } i \text{ OR } \text{fission out of } i) \]

(ex: n capture, photodissociation, \(\alpha\alpha n, (n,2n)\), \(\beta\)-decay, \(\beta\)-delayed n emission, \(\alpha\)-decay, neutron-induced, \(\beta\)-delayed, spontaneous fission)

Thus network equations can be written as:

\[ \dot{Y}_i = \sum_j \xi_j \lambda_j Y_j + \sum_{j,k} \xi_{j,k} \rho N_A \langle \sigma v \rangle_{j,k} Y_j Y_k \]

\[ + \sum_{j,k,l} \xi_{j,k,l} \rho^2 N_A^2 \langle \sigma v \rangle_{j,k,l} Y_j Y_k Y_l \]

Where \(\xi\) is + when \(i\) created, - when \(i\) consumed, and corrects for overcounting in a reaction involving identical particles

*Coupled differential equations can be put into matrix form so networks use matrix solvers

See e.g. Hix&Meyer 06, Lippuner&Roberts 18 for discussions of solving network equations
Big Bang Nucleosynthesis network of reactions
Big Bang Nucleosynthesis network of reactions

Let’s write down some of the coupled differential equations:

\[
\frac{dY_p}{dt} = Y_n \lambda_{n\rightarrow p} - Y_p Y_n \rho N_A \langle \sigma v \rangle_{p(n,\gamma)}
\]

\[
\frac{dY_{Li}}{dt} = Y_{Be} Y_n \rho N_A \langle \sigma v \rangle_{Be(n,p)} + Y_T Y_\alpha \rho N_A \langle \sigma v \rangle_{T(\alpha,\gamma)} - Y_{Li} Y_p \rho N_A \langle \sigma v \rangle_{Li(p,\alpha)}
\]
• BBN primarily makes hydrogen (~75%) and helium (~25%)
• BBN abundances are a probe of new physics (ex sterile neutrinos, dark matter) in the early universe
• Ongoing work (ex: Fields+22) with BBN abundances: Li problem - abundances observed in metal poor stars lower than prediction
• Ongoing measurements of BBN reaction rates (ex: recently updated $^7\text{Be}(n,p)^7\text{Li}$ measurement Damone+18)
A (much) bigger network: rapid neutron capture and the heaviest elements

Movie by Vassh
Using simulation tracers

Networks permit nucleosynthesis calculations to account for the time evolution of the temperature and density of a particular mass element in an astrophysical environment (aka trajectory).
Using simulation tracers: Extrapolating trajectories and reheating

The density beyond the $\sim \text{ms}$ timescale considered in hydrodynamic simulations is typically extrapolated assuming "free expansion" (homologous expansion such that $r = vt$):

$$\rho(t) = \frac{\rho_0}{t}$$

Given $\rho(t)$, the composition, and the entropy $s_0$, the change in entropy can be calculated via the nuclear equation of state (EOS) which is then linked to temperature $\left( \Delta s = \frac{\Delta \rho}{\rho} \right)$ thus

$$T(t) = \text{EOS}[s_0, \rho(t), Y(t)]$$

This is called "reheating" or "self-heating" since the changes in the composition from nuclear reactions heat the system.
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Stellar fusion

- The Sun was first a cloud of gas that underwent gravitational collapse, causing the core to become hot and dense enough* for fusion to begin
  *have to overcome the Coulomb barrier (proton repulsion)
- The energy released by fusion provides an outward pressure, combating the gravitational inward pull

\[ {^2H} + {^3H} \rightarrow {^4He} + n, \quad Q = 17.6 \text{ MeV} \]

Energy released by fusion ~10-30 MeV
The pp chain

The Sun is mostly hydrogen and helium

Present day Solar composition (mass %)

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<th>this work</th>
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<td>all other elements</td>
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<td>total heavy elements (=Z)</td>
<td>1.41</td>
<td>1.22</td>
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</table>

Lodders+ 2009

\[
\begin{align*}
^{1}\text{H} + ^{1}\text{H} & \rightarrow ^{2}\text{H} + \text{e}^{+} + \nu_{\text{e}} \\
^{2}\text{H} + ^{1}\text{H} & \rightarrow ^{3}\text{He} + \gamma \\
^{3}\text{He} + ^{3}\text{He} & \rightarrow ^{4}\text{He} + ^{1}\text{H} + ^{1}\text{H} \\
^{3}\text{He} + ^{4}\text{He} & \rightarrow ^{7}\text{Be} + \gamma \\
^{7}\text{Be} + \text{e}^{-} & \rightarrow ^{7}\text{Li} + \nu_{\text{e}} \\
^{7}\text{Li} + ^{1}\text{H} & \rightarrow ^{4}\text{He} + ^{4}\text{He} \\
^{7}\text{Be} + ^{1}\text{H} & \rightarrow ^{8}\text{B} + \gamma \\
^{8}\text{B} & \rightarrow ^{8}\text{Be} + \text{e}^{+} + \nu_{\text{e}} \\
^{8}\text{Be} & \rightarrow ^{4}\text{He} + ^{4}\text{He} \\
\end{align*}
\]
The pp chain

*pp chain is the primary energy source for the Sun
CNO cycle

Uses carbon-12 as a catalyst to convert hydrogen ($p$) into helium ($\alpha$)

- With a core temperature $\sim$15 MK, CNO subdominant to pp in the Sun
- For stars heavier than the Sun with carbon-12 present, the CNO cycle becomes the dominant hydrogen burning process
CNO cycle

Uses carbon-12 as a catalyst to convert hydrogen (p) into helium (α).

![Diagram of the CNO cycle]

---

**Table 6.1: Effective Q-values**

<table>
<thead>
<tr>
<th>Process</th>
<th>$Q_{\text{eff}}$ (MeV)</th>
<th>% Solar energy</th>
</tr>
</thead>
<tbody>
<tr>
<td>PP-I</td>
<td>26.2</td>
<td>83.7</td>
</tr>
<tr>
<td>PP-II</td>
<td>25.7</td>
<td>14.7</td>
</tr>
<tr>
<td>PP-III</td>
<td>19.1</td>
<td>0.02</td>
</tr>
<tr>
<td>CNO</td>
<td>23.8</td>
<td>1.6</td>
</tr>
</tbody>
</table>

---

**Article**

*Experimental evidence of neutrinos produced in the CNO fusion cycle in the Sun*

*The Borexino Collaboration*

*Nature* 587, 577–582 (2020) | [Cite this article]

**Abstract**

For most of their existence, stars are fuelled by the fusion of hydrogen into helium. Fusion proceeds via two processes that are well understood theoretically: the proton–proton (pp) chain and the carbon–nitrogen–oxygen (CNO) cycle. Neutrinos that are emitted along such fusion processes in the solar core are the only direct probe of the deep interior of the Sun. A complete spectroscopic study of neutrinos from the pp chain, which produces about 99 per cent of the solar energy, has been performed previously; however, there has been no reported experimental evidence of the CNO cycle. Here we report the direct observation, with a high statistical significance, of neutrinos produced in the CNO cycle in the Sun. This experimental evidence was obtained using the highly radiopure, large-volume, liquid-scintillator detector of Borexino, an experiment located at the underground Laboratori Nazionali del Gran Sasso in Italy. The main experimental challenge was to identify the excess signal—only a few counts per day above the background per 100 tonnes of target—that is attributed to interactions of the CNO neutrinos. Advances in the thermal stabilization of the detector over the last five years enabled us to develop a method to constrain the rate of bismuth-210 contaminating the scintillator. In the CNO cycle, the fusion of hydrogen is catalysed by carbon, nitrogen and oxygen, and so its rate—as well as the flux of emitted CNO neutrinos—depends directly on the abundance of these elements in the solar core. This result therefore paves the way towards a direct measurement of the solar metallicity using CNO neutrinos. Our findings quantify the relative contribution of CNO fusion in the Sun to be of the order of 1 per cent; however, in massive stars, this is the dominant process of energy production. This work provides experimental evidence of the primary mechanism for the stellar conversion of hydrogen into helium in the Universe.
Outline for lecture I

• How can we study nuclear physics in astrophysics? Some observables [3-7]

• Some basic nuclear physics: masses, decays, reactions, reverse reaction rates, and equilibria [9-19]

• Reaction networks (BBN example and heavy element nucleosynthesis example) and using hydro simulations [21-30]

• Solar fusion [32-36]

• Stellar burning and stellar evolution [38-45]
Stellar fusion

- The Sun was first a cloud of gas that underwent gravitational collapse, causing the core to become hot and dense enough* for fusion to begin
  *have to overcome the Coulomb barrier (proton repulsion)
- The energy released by fusion provides an outward pressure, combating the gravitational inward pull

\[ ^2H + ^3H \rightarrow ^4He + n, \quad Q = 17.6 \text{ MeV} \]
Energy released by fusion \(\sim 10-30 \text{ MeV} \)
### Lifetime of stars

- It depends on metallicity, but it gives an idea.

### Pre-SN onion shell of massive stars

- Carbon burning, alpha, and other reactions.

### Timescale of stellar burning phases

- I can try to find values for the Sun if you want.

### Stellar Nucleosynthesis

#### Evolutionary Time Scales for a 15 M$_{\text{Sun}}$ Star

<table>
<thead>
<tr>
<th>Fused</th>
<th>Products</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>H</td>
<td>$^4\text{He}$</td>
<td>$10^7$ yrs.</td>
</tr>
<tr>
<td>$^4\text{He}$</td>
<td>$^{12}\text{C}$</td>
<td>Few X $10^6$ yrs</td>
</tr>
<tr>
<td>$^{12}\text{C}$</td>
<td>$^{16}\text{O}$, $^{20}\text{Ne}$, $^{24}\text{Mg}$, $^4\text{He}$</td>
<td>1000 yrs.</td>
</tr>
<tr>
<td>$^{20}\text{Ne}$ +</td>
<td>$^{16}\text{O}$, $^{24}\text{Mg}$</td>
<td>Few yrs.</td>
</tr>
<tr>
<td>$^{16}\text{O}$</td>
<td>$^{28}\text{Si}$, $^{32}\text{S}$</td>
<td>One year</td>
</tr>
<tr>
<td>$^{28}\text{Si}$ +</td>
<td>$^{56}\text{Fe}$</td>
<td>Days</td>
</tr>
<tr>
<td>$^{56}\text{Fe}$</td>
<td>Neutrons</td>
<td>$&lt; 1$ second</td>
</tr>
</tbody>
</table>

#### Diagram

- Core collapses at $M \sim 1.4M_{\text{Sun}}$
- Prominent constituents: H, He, 2% CNO, 0.1% Fe
Stellar Nucleosynthesis
Evolutionary Time Scales for a 15 $M_{\odot}$ Star

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<td>$^{12}$C</td>
<td>$^{16}$O, $^{20}$Ne, $^{24}$Mg, $^7$He</td>
<td>1000 yrs.</td>
</tr>
<tr>
<td>$^{30}$Ne +</td>
<td>$^{16}$O, $^{24}$Mg</td>
<td>Few yrs.</td>
</tr>
<tr>
<td>$^{16}$O</td>
<td>$^{28}$Si, $^{32}$S</td>
<td>One year</td>
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<td>$^{28}$Si +</td>
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<td>Days</td>
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<td>Neutrons</td>
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</tbody>
</table>

$^{56}$Fe + $\gamma$ $\rightarrow$ $^{134}$He + $4n$

$^{4}$He + $\gamma$ $\rightarrow$ 2p + 2n

$^{p} + e^{-} \rightarrow \nu_{e} + n$

Core collapse

$\rho_{c} = 3 \times 10^{14}$[g/c.c.]

UV/X-ray flash post shock $\sim 0.1$keV

Optical

Core collapses at $M \sim 1.4M_{\odot}$

Prominent constituents:
- H, He, 2% CNO, 0.1% Fe
- $^{16}$O, $^{24}$Mg, $^{22}$Na
- $^{16}$O, $^{20}$Ne, $^{23}$Na, $^{24}$Mg
- $^{12}$C, $^{16}$O, 2% $^{22}$Ne
- He, 2% $^{14}$N
Stellar evolution

Brown dwarf: Not able to burn H

C-O and O-Ne White dwarfs:
- Progenitor not able to proceed to fusion of heavier species, held up by electron degeneracy pressure
- ~200,000 times as dense as Earth with about same radius

Red dwarf:
- Most common in Solar neighborhood
- Burn H but can’t reach He burning

Brown dwarf: Not able to burn H

Neutron stars:
- Mostly neutrons, held up by neutron degeneracy pressure
- One teaspoon contains the mass of ~700 Great Pyramids
- ~10 mile diameter
Hertzsprung-Russell (HR) Diagrams
Hertzsprung-Russell (HR) Diagrams

Figure 1.3 Observational Hertzsprung–Russell diagrams, showing visual magnitude versus color index B–V. Each dot corresponds to a star. See the text for an explanation of the labels. (a) Sample of 5000 stars in the solar neighborhood with precisely known distances. The data were acquired by the Hipparcos astrometry satellite. The vast majority of stars occupy the main sequence, stretching diagonally from the hot (blue) and luminous upper left to the cool (red) and faint lower right. The cross hair indicates the position of the Sun. Certain categories of stars do not appear in the figure, for example, supergiants, which are rare in the solar neighborhood, and brown dwarfs, which are too faint for detection by Hipparcos. (b) Data for the globular cluster M 3. Apparent rather than absolute magnitude is displayed on the vertical axis since the stars have the same distance from the Earth. The RR Lyrae variable stars, located between the red (RHB) and blue (BHB) horizontal branches, are omitted. Data from Corwin and Carney (2001).

Stars in globular cluster M 3: these all formed around the same time and some have moved on to later stages, most still in hydrogen burning phase on main sequence.