Explosive Astrophysics: Supernovae

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Learning objectives

- The physics of core-collapse supernova explosion
- Gravitational waves and neutrino signals from core-collapse supernovae
- Supernova light curves
Core-collapse supernovae in numbers

- \( \sim (30 \text{yr})^{-1} \) in our galaxy
- O(few millions) to be discovered by LSST
- peak luminosity \( L_{\text{SN}} \sim 10^{10} L_\odot \)
- \( t_{\text{peak}} \sim \) few weeks \( \sim 10^6 \text{sec} \)

The radiated energy is

\[
E_{\text{EM}} \sim t_{\text{peak}} L \sim 10^{48} \text{erg}.
\]

- \( T_{\text{eff}} \sim 2 T_\odot \approx 12,000 \text{K} \)

For a black body we have

\[
L_{\text{SN}} = 4 \pi \sigma R_{\text{SN}}^2 T_{\text{eff}}^4 \implies R_{\text{SN}} \sim 10^{15} \text{cm} \gg R_*.
\]

The typical expansion velocity is

\[
v \sim \frac{R_{\text{SN}}}{t_{\text{peak}}} = \frac{10^{15} \text{cm}}{10^6 \text{s}} = 10^9 \text{cm s}^{-1} \implies K = \frac{1}{2} M v^2 \sim 10^{51} \left( \frac{M}{M_\odot} \right) \text{erg} \gg E_{\text{EM}}.
\]
The gravitational binding energy of a NS is

\[
U = -\frac{3}{5} \frac{G M^2}{R} = -\frac{3}{5} \frac{G M}{R c^2} M c^2 \sim 0.1 \, M_\odot c^2.
\]

**The SN problem:** can we transfer \( \sim 1\% \) of the binding to the envelope of the star?
Massive star evolution

Table 1 Evolution of a 15-solar-mass star.

<table>
<thead>
<tr>
<th>Stage</th>
<th>Timescale</th>
<th>Fuel or product</th>
<th>Ash or product</th>
<th>Temperature (10^9 K)</th>
<th>Density (gm cm^-3)</th>
<th>Luminosity (solar units)</th>
<th>Neutrino losses (solar units)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hydrogen</td>
<td>11 Myr</td>
<td>H</td>
<td>He</td>
<td>0.035</td>
<td>5.8</td>
<td>28,000</td>
<td>1,800</td>
</tr>
<tr>
<td>Helium</td>
<td>2.0 Myr</td>
<td>He</td>
<td>C, O</td>
<td>0.18</td>
<td>1,390</td>
<td>44,000</td>
<td>1,900</td>
</tr>
<tr>
<td>Carbon</td>
<td>2000 yr</td>
<td>C</td>
<td>Ne, Mg</td>
<td>0.81</td>
<td>2.8 × 10^5</td>
<td>72,000</td>
<td>3.7 × 10^5</td>
</tr>
<tr>
<td>Neon</td>
<td>0.7 yr</td>
<td>Ne</td>
<td>O, Mg</td>
<td>1.6</td>
<td>1.2 × 10^7</td>
<td>75,000</td>
<td>1.4 × 10^8</td>
</tr>
<tr>
<td>Oxygen</td>
<td>2.6 yr</td>
<td>O, Mg</td>
<td>Si, S, Ar, Ca</td>
<td>1.9</td>
<td>8.8 × 10^6</td>
<td>75,000</td>
<td>9.1 × 10^8</td>
</tr>
<tr>
<td>Silicon</td>
<td>18 d</td>
<td>Si, S, Ar, Ca</td>
<td>Fe, Ni, Cr, Ti, . . .</td>
<td>3.3</td>
<td>4.8 × 10^7</td>
<td>75,000</td>
<td>1.3 × 10^{11}</td>
</tr>
<tr>
<td>Iron core collapse*</td>
<td>~1 s</td>
<td>Fe, Ni, Cr, Ti, . . .</td>
<td>Neutron star</td>
<td>&gt; 7.1</td>
<td>&gt; 7.3 × 10^9</td>
<td>75,000</td>
<td>&gt; 3.6 × 10^{15}</td>
</tr>
</tbody>
</table>

* The pre-supernova star is defined by the time at which the contraction speed anywhere in the iron core reaches 1,000 km s^-1.

**Table.** From Woosley and Janka 2005.

**Figure.** From Janka et al 2012.

- Massive stars burn bright and fast
- A degenerate iron core is created at the center of the star
- Once the mass of the iron core goes above the Chandrasekhar mass the core collapses.
1. An iron core is formed in the star
2. The iron core becomes unstable and collapses under its own self-gravity
3. The inner-core bounces once it reaches nuclear density and launches a shock wave
4. The shock runs out of energy and stalls
5. Neutrinos revive the shock?
6. The proto-neutron star cools down over a minute

7. The shock wave reaches the surface in $\sim 12$ hours to a day
Neutrinos are trapped when $\rho \gtrsim 10^{12} \, \text{g/cm}^3 \implies$ core-collapse is close to adiabatic.

Inner core mass is $M_{IC} \sim 0.5 \, M_\odot$, independent on the progenitor, it is set by nuclear physics.

Core bounce is triggered by transition from electron-dominated $\Gamma = \frac{d \log p}{d \log \rho} = \frac{4}{3}$ to nuclear repulsion force $\Gamma = 2 - 3$. 

**Figure.** From E. Müller, Saas-Fee Advanced Course 27 (1997).
Post-bounce phase

Figure. $t - t_{\text{bounce}} = 0$. From Liebendörfer et al. (2001).
Figure. $t - t_{\text{bounce}} = 1\text{ ms}$. From Liebendörfer et al. (2001).
Figure. $t - t_{bounce} = 10\text{ms}$. From Liebendörfer et al. (2001).
Post-bounce phase

Figure. $t - t_{\text{bounce}} = 100\text{ms}$. From Liebendörfer et al. (2001).
Figure. $t - t_{\text{bounce}} = 500\text{ms}$. From Liebendörfer et al. (2001).
Why does the shock stall?

- $\sim 2B$ lost to the neutrino burst
- Energy lost to dissociation

$$\dot{E}_{\text{diss}} \sim 1.7 \dot{M} \_1 B$$

**Key neutrino processes**

$$e^- + p \leftrightarrow \nu_e + n$$
$$e^+ + n \leftrightarrow \bar{\nu}_e + p$$

Heating and cooling are:

$$Q^-_{\nu_e, \bar{\nu}_e} \approx 2.4 \left( \frac{T}{1 \text{ MeV}} \right)^6 Y_{n,p} \left[ \frac{\text{MeV}}{\text{s \cdot nucleon}} \right],$$
$$Q^+_{\nu_e, \bar{\nu}_e} \approx 110 \frac{L_{52}}{r_i^2} \frac{\langle \epsilon_{\nu}^2 \rangle}{(15 \text{ MeV})^2} Y_{p,n} \left[ \frac{\text{MeV}}{\text{s \cdot nucleon}} \right].$$
Accretion phase

\[
Q_{\nu_e, \bar{\nu}_e}^{-} \approx 2.4 \cdot \left( \frac{T}{1 \text{ MeV}} \right)^6 Y_{n,p} \left[ \frac{\text{MeV}}{\text{s \cdot nucleon}} \right], \quad Q_{\nu_e, \bar{\nu}_e}^{+} \approx 110 \cdot \frac{L_{52}}{r_7^2} \frac{\langle \epsilon_{\nu}^2 \rangle}{(15 \text{ MeV})^2} Y_{p,n} \left[ \frac{\text{MeV}}{\text{s \cdot nucleon}} \right].
\]

The pressure is dominated by radiation and relativistic \( e^- , e^+ \): \( p \propto \rho^{4/3} \propto T^4 \).

The subsonic accretion flow has \( v_r \propto r, \rho v_r r \sim \text{const} \), so \( \rho \propto r^{-3} \) and \( T \propto r^{-1} \). Then

\[
Q_{\nu_e, \bar{\nu}_e}^{-} \sim r^{-6}, \quad Q_{\nu_e, \bar{\nu}_e}^{+} \sim r^{-2},
\]

so there is a region where heating > cooling (gain layer)!

![Graph showing Net Gain vs. radius](image_url)

From Ott et al. (2008)
Critical luminosity

Figure. From Burrows and Ghosy 1993.
Modeling requirements

- Early 2000’s
- State of the art

Gravity
- Full-GR
- Conformally flat
- Newtonian + correction

Dimensionality
- 1D
- 2D
- 3D

Neutrino treatment
- Full-Boltzmann
- M1/ray-by-ray
- Idealized

Neutrino-radiation hydro
- Must solve Boltzmann equation in 7D
- Oscillations?
- MHD effects

Microphysics
- EOS of dense matter
- NSE, nuclear burning, electron capture rates

Multi-dimensional effects and turbulence
\[ t = 0.010 \text{s} \]
Figure. From Burrows and Vartanyan (2020).
Explosion energies

Theoretical Explosion Energies (this paper)

Morozova et al.
Martinez & Bersten
Pumo et al.
Utrobin & Chugai

**Figure.** From Burrows and Vartanyan (2020).
Compactness or ZAMS mass are not useful criteria for explodability.

The Si-O density drop is critical to trigger an explosion.

Explosion does not mean no black hole formation.

Figure. From Tsang et al. (2022)
Problem solved?

Figure. From Burrows and Vartanyan (2020).
Figure. From Li, Beacom, et al. (2023)
Supernova theory status

- What is missing?
  - Magnetic fields?
  - Better progenitor models and rotation?
  - Quantum effects in the neutrino sector?
  - BSM physics?

- Other open questions
  - Weird supernovae, GRB connection
  - Origin of pulsar magnetic fields
  - Connection with SN remnants morphology
  - Nucleosynthesis from CCSNe

From Mösta et al. (2014).
Figure. From Abdikamalov, Pagliaroli, & Radice (2020).
Rotating collapse

Figure. From Abdikamalov, Pagliaroli, & Radice (2020).
Low-\(T/|W|\) instability

**Figure.** From Shibagaki et al. (2020).
Low-$T/|W|$ instability

colorbar: $v_r/c$

- $nrot$,
  - $t-t_b=-8.89$ ms
- $rot_{ar075}$,
  - $t-t_b=-8.89$ ms
- $rot$,
  - $t-t_b=-9.27$ ms
Gravitational waves from nonrotating SNe

Signal is dominated by the oscillation modes of the PNS


Figure. From Radice et al. (2019).
Prospects for detection are not great with current gen observatories.

Next-generation detectors (ET, CE, NEMO) can detect nonrotating CCSNe in the galaxy at high-SNR, and fast rotating collapse to distances of a few Mpc.
On the blackboard: bolometric light curves

\[ \langle R \rangle = \langle r_1 + r_2 + r_3 + \cdots \rangle = 0 \]
\[ \langle R^2 \rangle = \langle (r_1 + r_2 + \cdots) \cdot (r_1 + r_2 + \cdots) \rangle \]
\[ = \langle r_1^2 \rangle + \langle r_2^2 \rangle + \cdots + 2 \langle r_1 \cdot r_2 \rangle = N \lambda \]

From Arnett, *Supernovae and Nucleosynthesis*, 1999

The peak timescale for the light curve can be estimated as

\[ t_p = \left( \frac{\kappa M}{\beta cv} \right)^{1/2}, \quad \beta \approx 13.8. \]
For Thompson scattering in an hydrogen rich, ionized gas \( \kappa = 0.4 \text{cm}^2 \text{g}^{-1} \), so

\[
t_p = 1.4 \times 10^6 \text{s} M_1^{1/2} \kappa_{0.4}^{1/2} v_9^{-1/2}
\]

For SN1987a \( v \sim 0.3 \times 10^9 \text{cm s}^{-1} \) and the peak time is \( t_p \approx 100 \text{days} \).

**Question.** What is \( R \) at peak? What is \( M_1 \)? What is the explosion energy?

**Answer.**
A “one-zone” model: Arnett’s law

We derive on the blackboard a one-zone model:

\[ \dot{E} + p \dot{V} = \dot{Q}_{\text{heat}} - L. \]

We show two important results

1. Most of the initial energy is lost to adiabatic expansion, so \( K \gg E_{EM} \).

2. The peak luminosity if \( \dot{Q}_{\text{heat}} \approx 0 \) is

\[ L \approx \frac{E_p}{\tau_{d,p}} = \frac{\beta R_p c}{\kappa M} E_p = 5.2 \cdot 10^{43} \text{ erg s}^{-1} E_{p,50} R_{p,15} M_1^{-1} \kappa_{0.4}^{-1}. \]

3. **Arnett’s law**: at peak \( \dot{Q}_{\text{heat}} \approx L \).
Figure. Bolometric light curve for SN1999em, a classical example of a Type II-P supernova. Data from Elmhamdi et al., MNRAS 338, 939-956 (2003), figure courtesy of Dr. Giryanskaya.
Figure. SN2008D optical (left) and spectra evolution (right). Credit: Gemini observatory.
Future directions

- Better supernova models
  - Inclusion of GR, magnetic fields, better neutrino transport, nuclear burning
  - More realistic “3D” stellar evolution models, binary stellar evolution
  - Extension to several seconds and then to breakout

- Observational data
  - Light curves from millions of supernovae, many spectra, progenitor images, etc.
  - Preparing for the next galactic CCSN

- Peculiar supernovae
  - Hypernovae, BH forming supernovae, LGRB
  - Ultra-striped supernovae, accretion-induced collapse of white dwarfs
  - White-dwarf neutron-star mergers

• Athem W. Alsabti, Paul Murdin, Handbook of Supernovae, Springer Nature 2017, doi:10.1007/978-3-319-20794-0

• Arnett, Supernovae and Nucleosynthesis, Princeton University Press 1999
