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Neutron Stars

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WARNING

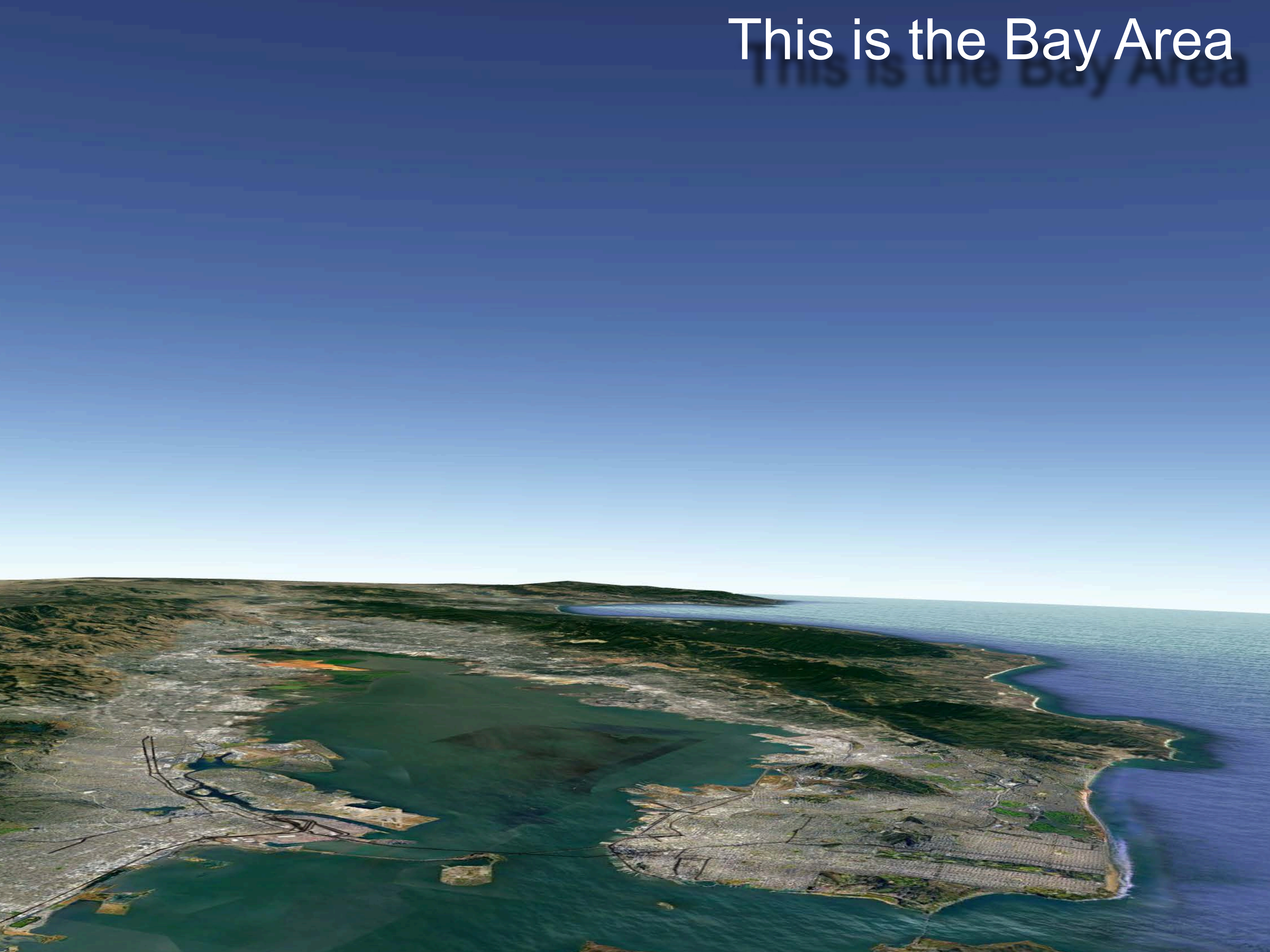
These are pedagogical lectures (I hope)
and have no intent to be complete.
I give key entries into the literature (I hope)
but make no claim at being comprehensive:
this is NOT a review work !

Neutron Stars

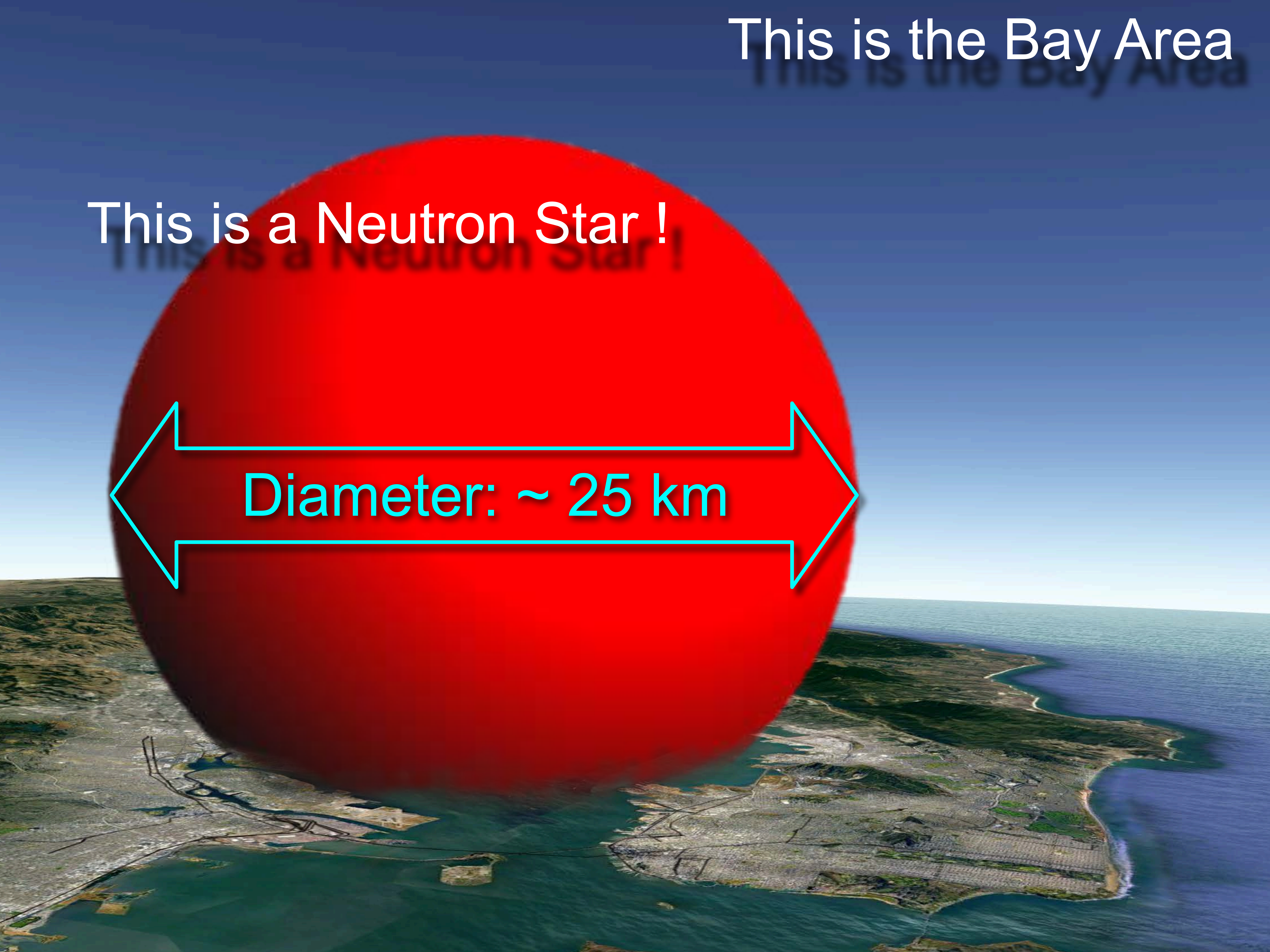
I - Structure

II - Evolution and Cooling

This is the Bay Area




This is the Bay Area
This is a Neutron Star !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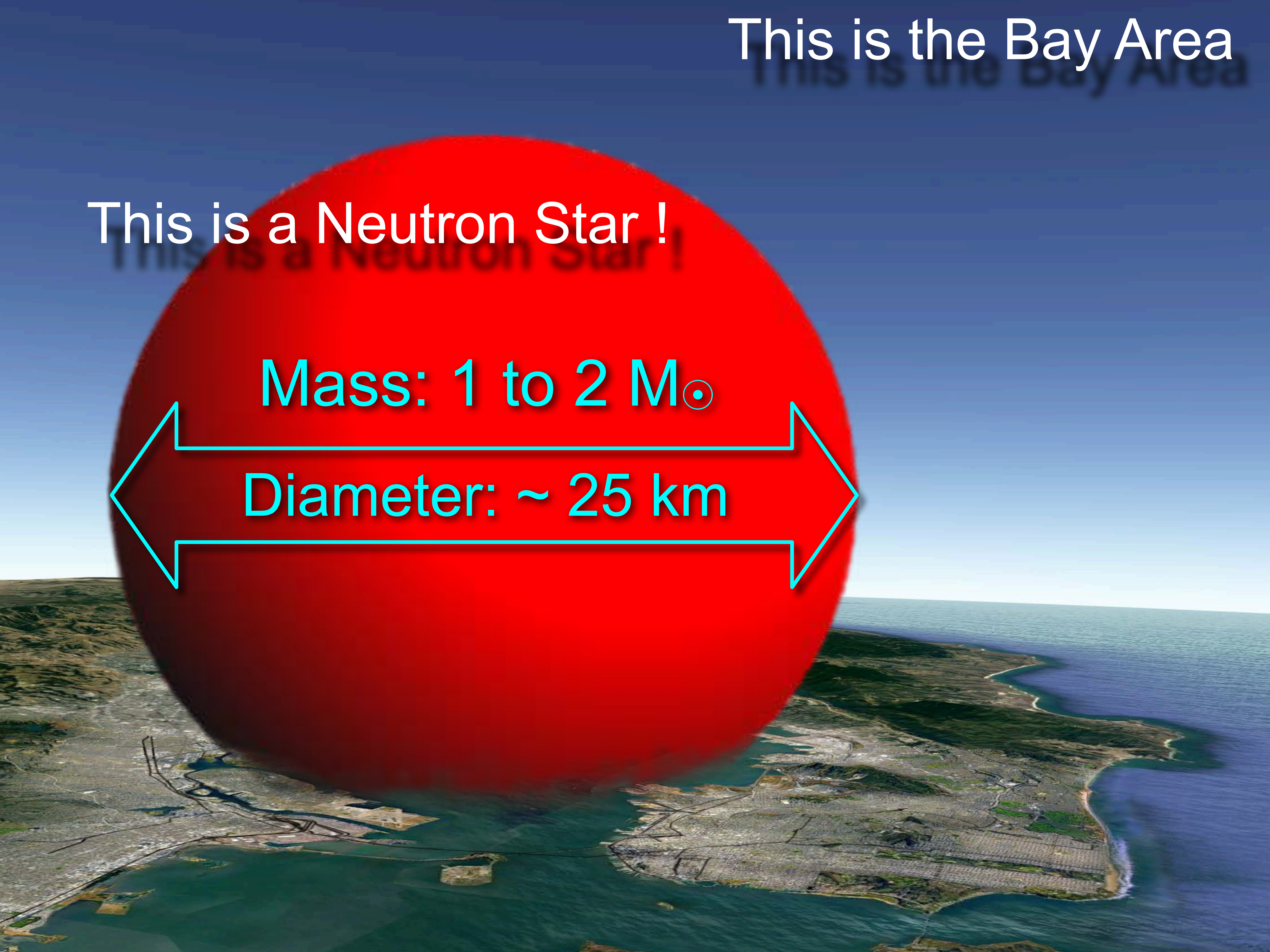
This is a Neutron Star !
Diameter: ~ 25 km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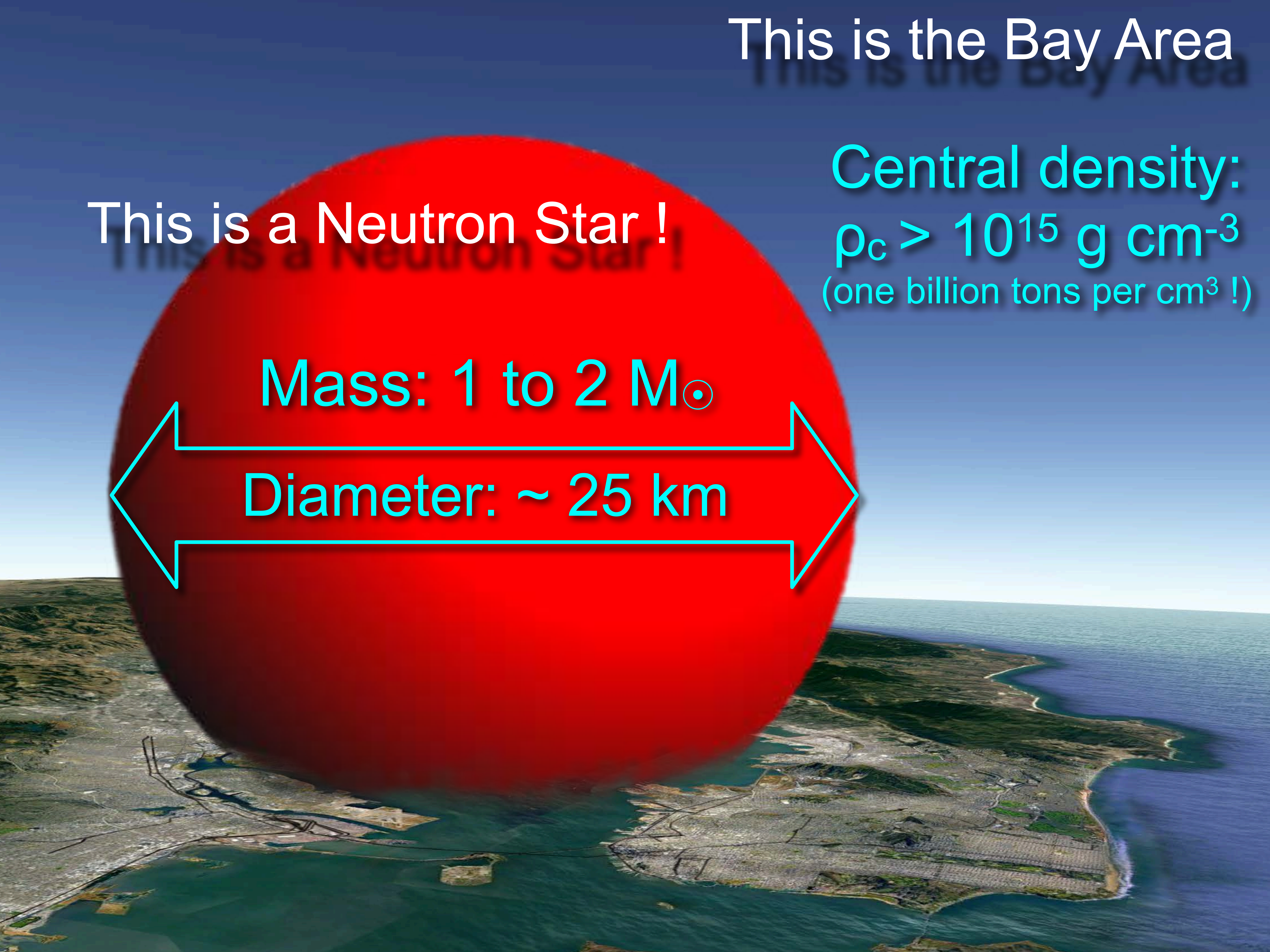
Mass: 1 to 2 $M_\odot$
Diameter: ~ 25 km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This is a Neutron Star !
Central density:
$\rho_c > 10^{15}$ g cm^{-3}
(one billion tons per cm^3 !)
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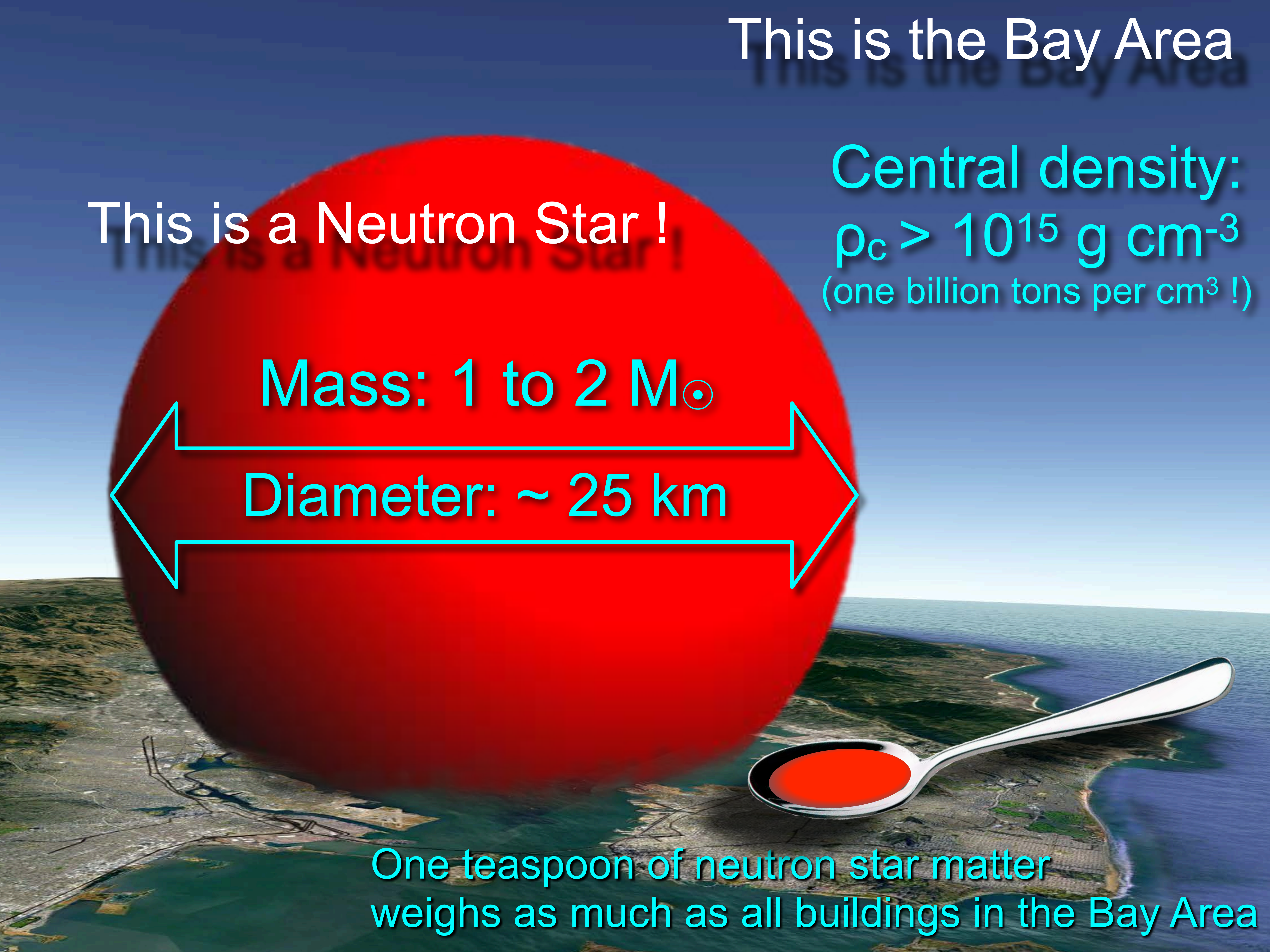
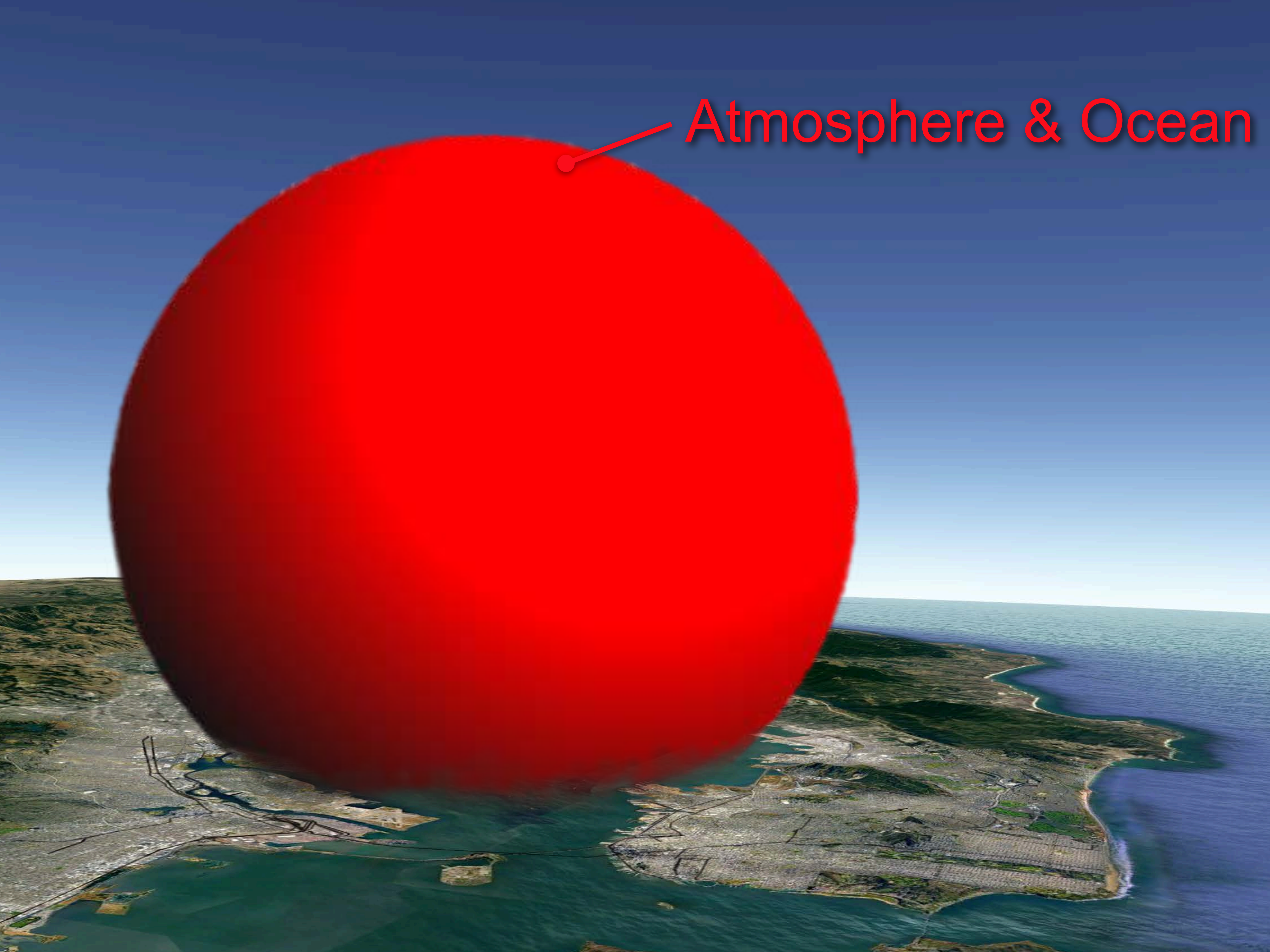
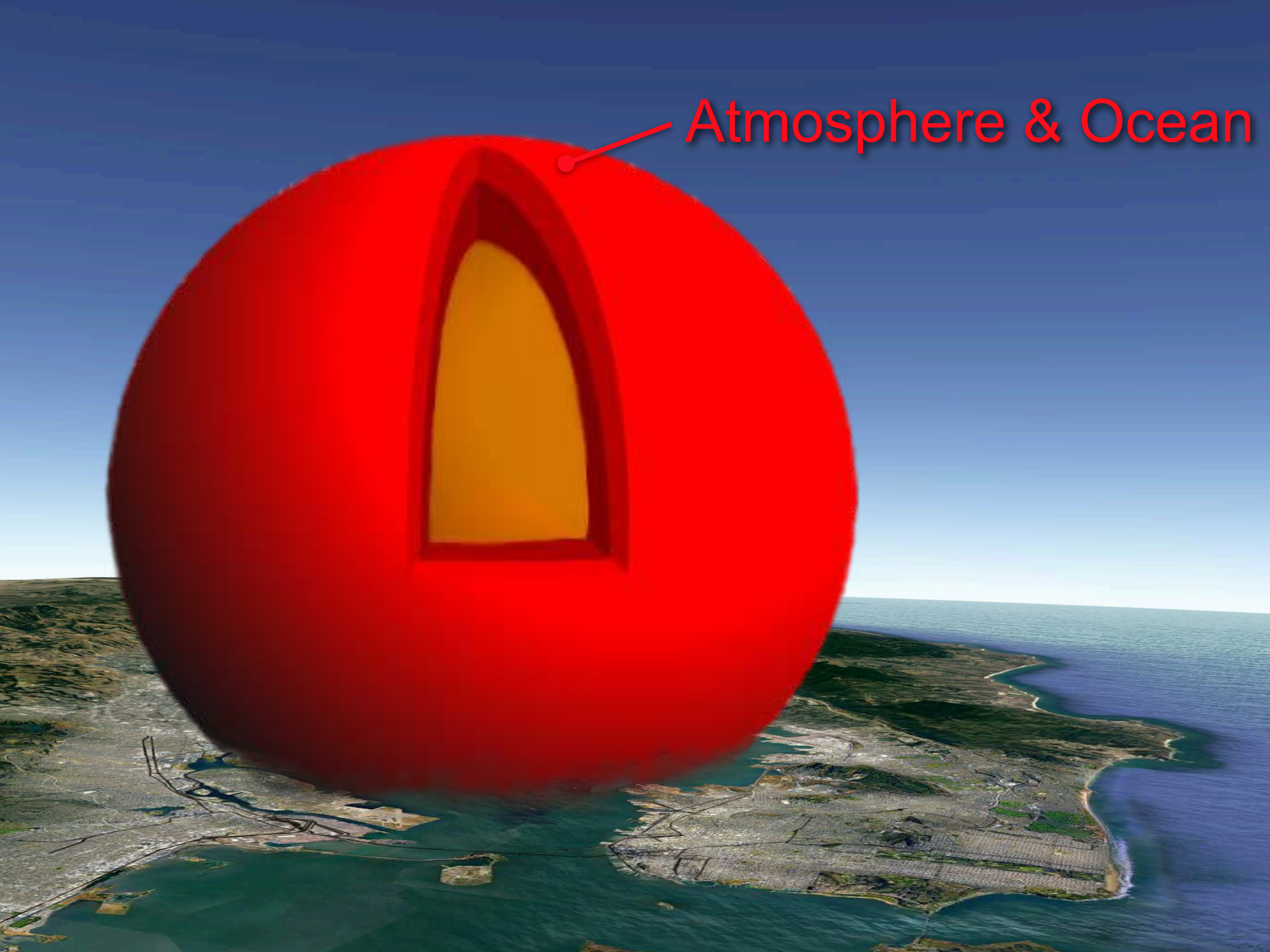

This is the Bay Area
Central density:
$\rho_c > 10^{15}$ g cm^{-3}
(one billion tons per cm^3 !)
This is a Neutron Star !
Mass: 1 to 2 $M_\odot$
Diameter: ~ 25 km
One teaspoon of neutron star matter
weighs as much as all buildings in the Bay Area


Atmosphere & Ocean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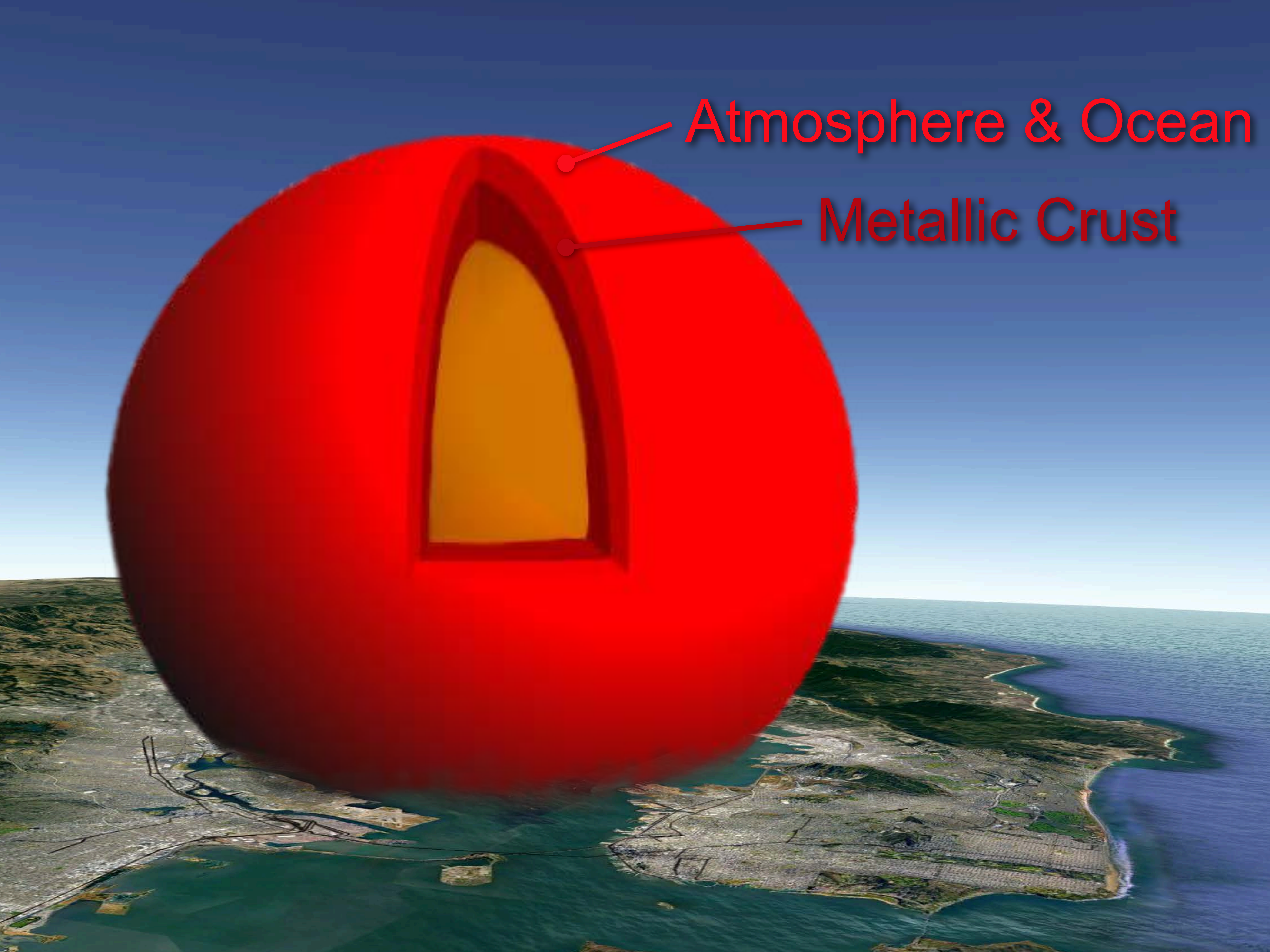
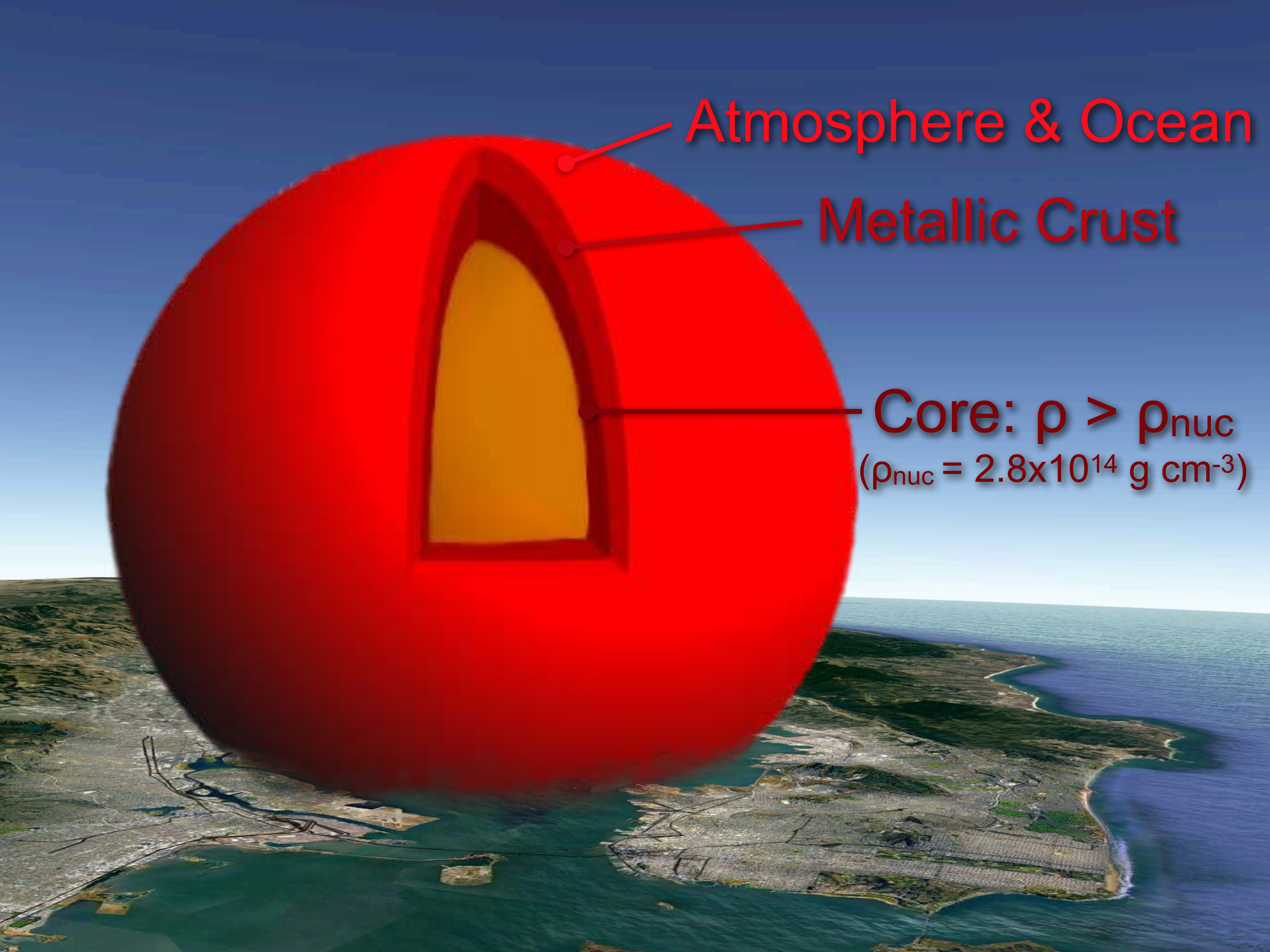
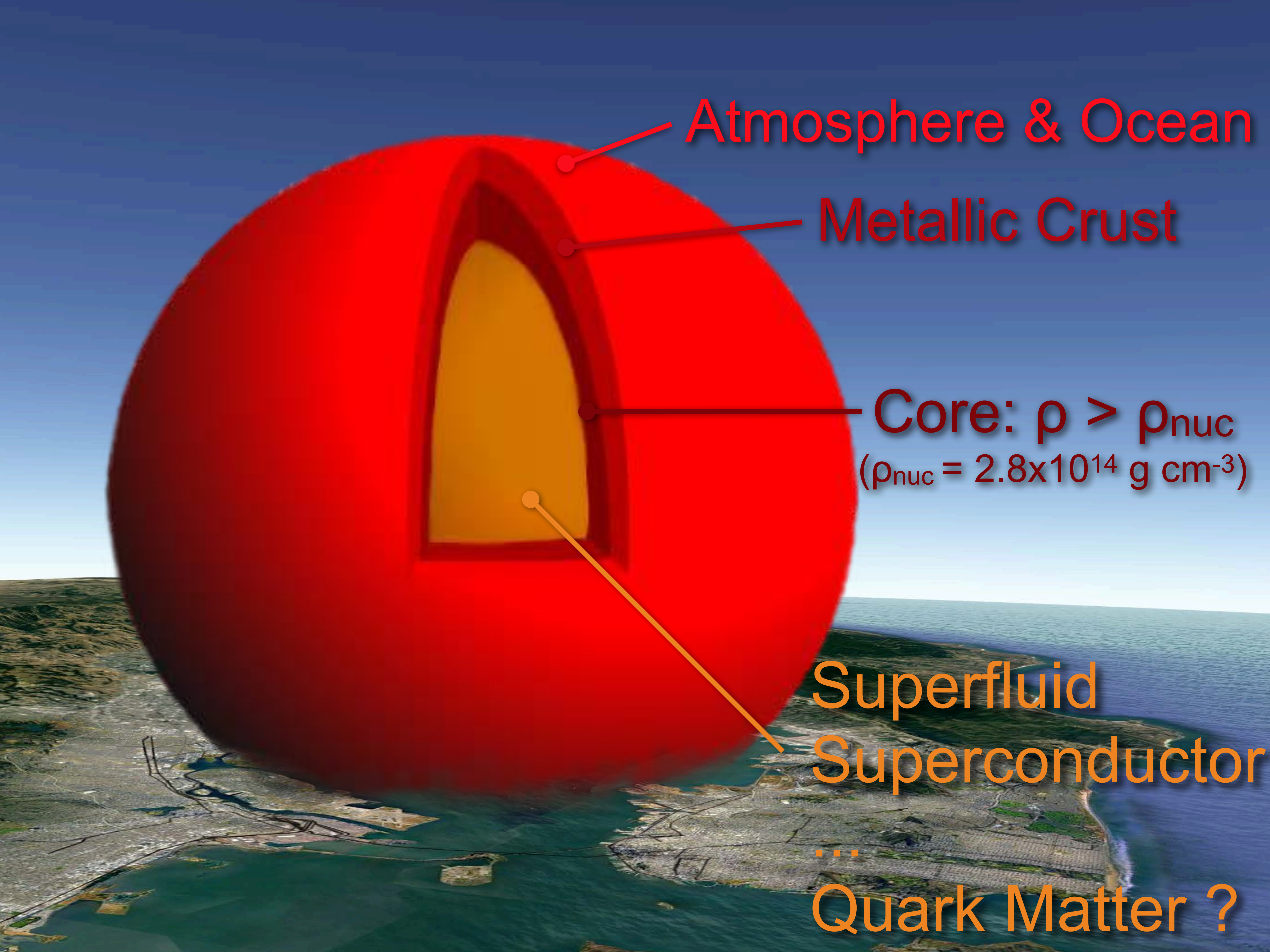

Atmosphere & Ocean
Metallic Crust



Atmosphere & Ocean
Metallic Crust
Core: $\rho > \rho_{nuc}$
($\rho_{nuc} = 2.8\text{x}10^{14}$ g cm^{-3})


Atmosphere & Ocean
Metallic Crust
Core: $\rho > \rho_{nuc}$
($\rho_{nuc} = 2.8\text{x}10^{14}$ g cm^{-3})
Superfluid
Superconductor
...
Quark Matter ?

Neutron Stars

Lecture 1: Structure

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Neutron Stars:
Yes, they exist !

“Neutron Stars” DO Exist

Consider the fastest known pulsar:
PSR J1748.2448aD in Terzan 5: rotational period **P=1.39 ms**

Velocity at equator < speed of light:

$$v_{\mathrm{ecuator}} = \Omega R = \frac{2\pi R}{P} < c \Longrightarrow R < \frac{cP}{2\pi} = 65\,\mathrm{km}$$

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Assume star is bound by gravity: $a_{\mathrm{gravity}} > a_{\mathrm{centrifugal}}$ at equator:

$$a_{\mathrm{gravity}} = \frac{GM}{R^2} > a_{\mathrm{centrifugal}} = \Omega^2 R = \frac{4\pi^2 R}{P^2} \quad \text{or} \quad \frac{M}{R^3} > \frac{4\pi^2}{GP^2}$$

$$\Longrightarrow \bar{\rho} = \frac{M}{\frac{4}{3}\pi R^3} > 8 \times 10^{13}\ \mathrm{g\,cm^{-3}}$$

“Neutron” Stars DO NOT Exist

“Neutron” stars as originally conceived by Landau, Baade & Zwicky and Oppenheimer & Volkoff DO NOT exist:

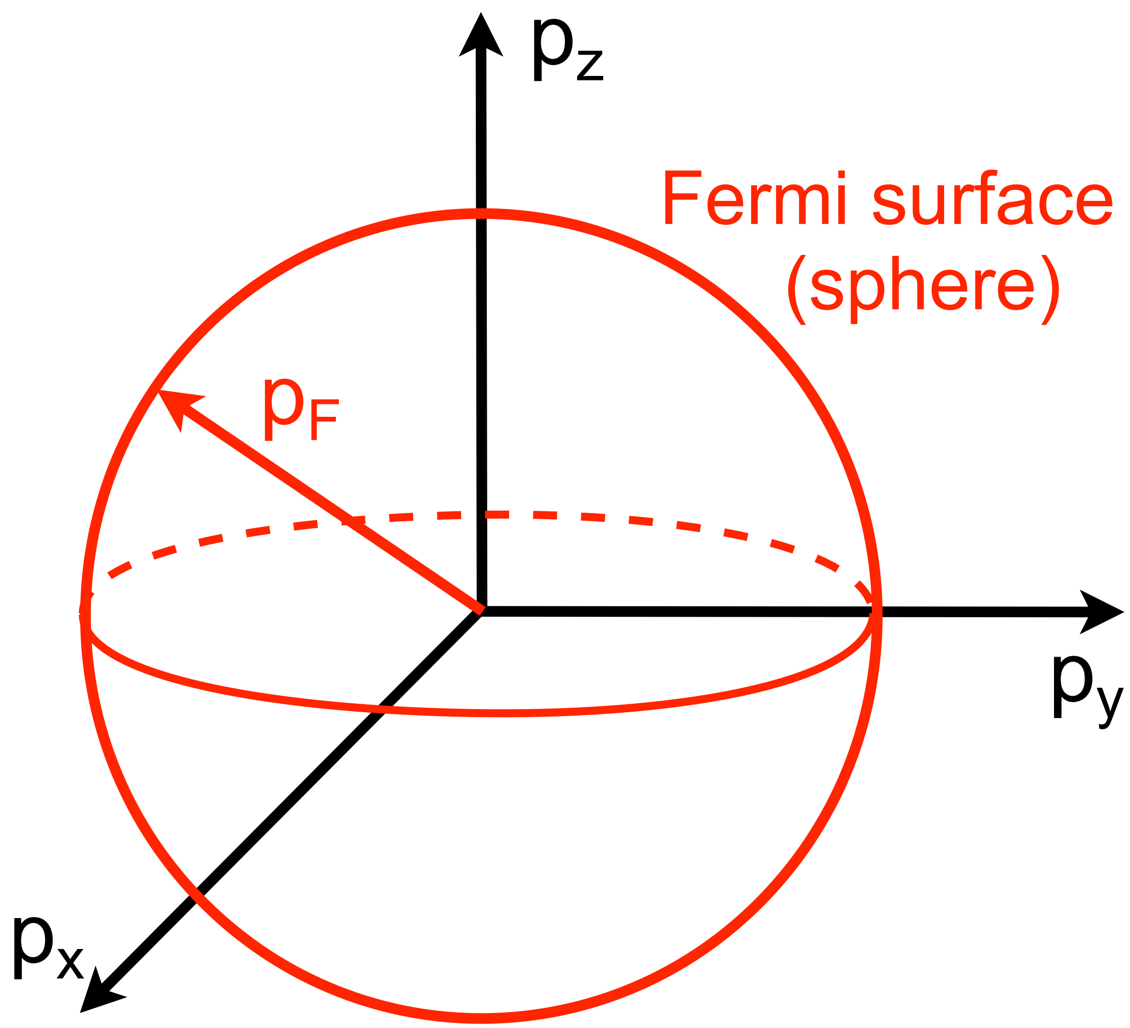
A ball of neutrons will immediately undergo decay into protons:

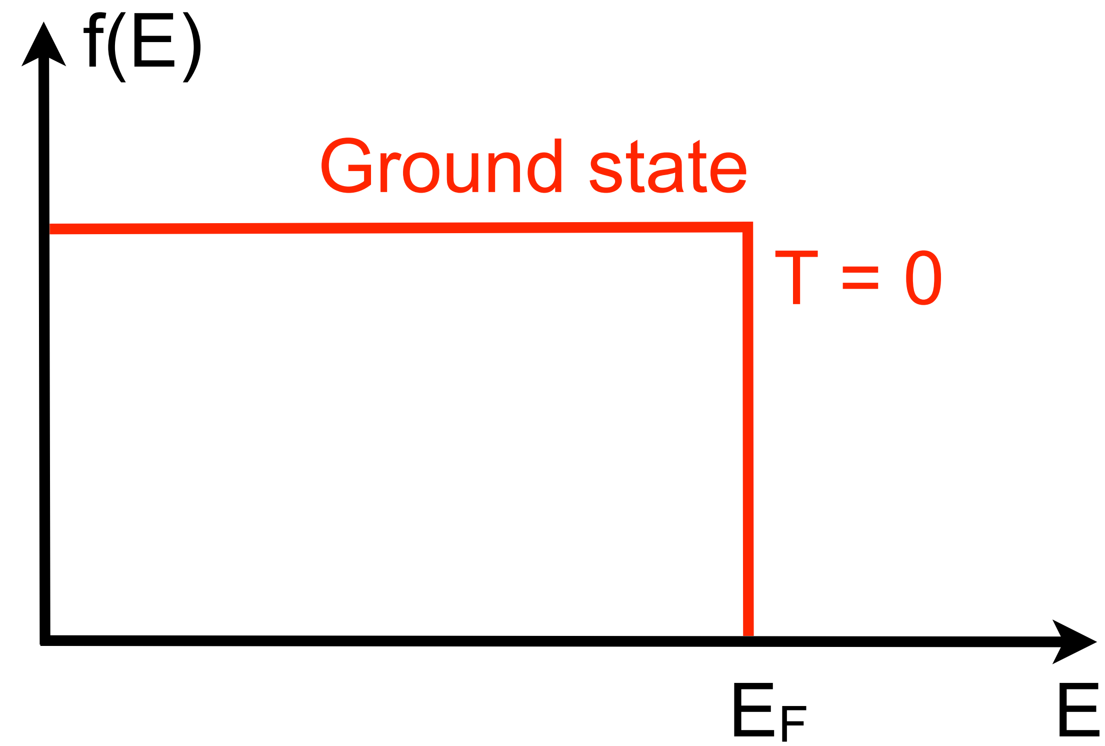
$$n \longrightarrow p + e^- + \overline{\nu}_e$$

since $M_n > M_p + m_e + m_\nu$

Degenerate Matter

A system of fermions (i.e., with spin 1/2, 3/2, ...) at T=0 is degenerate:
in momentum space particles fill up states with momenta $|\vec{p}| < p_F$





Energy at p_F = E_F = Fermi energy

Neutron Decay in Dense Matter

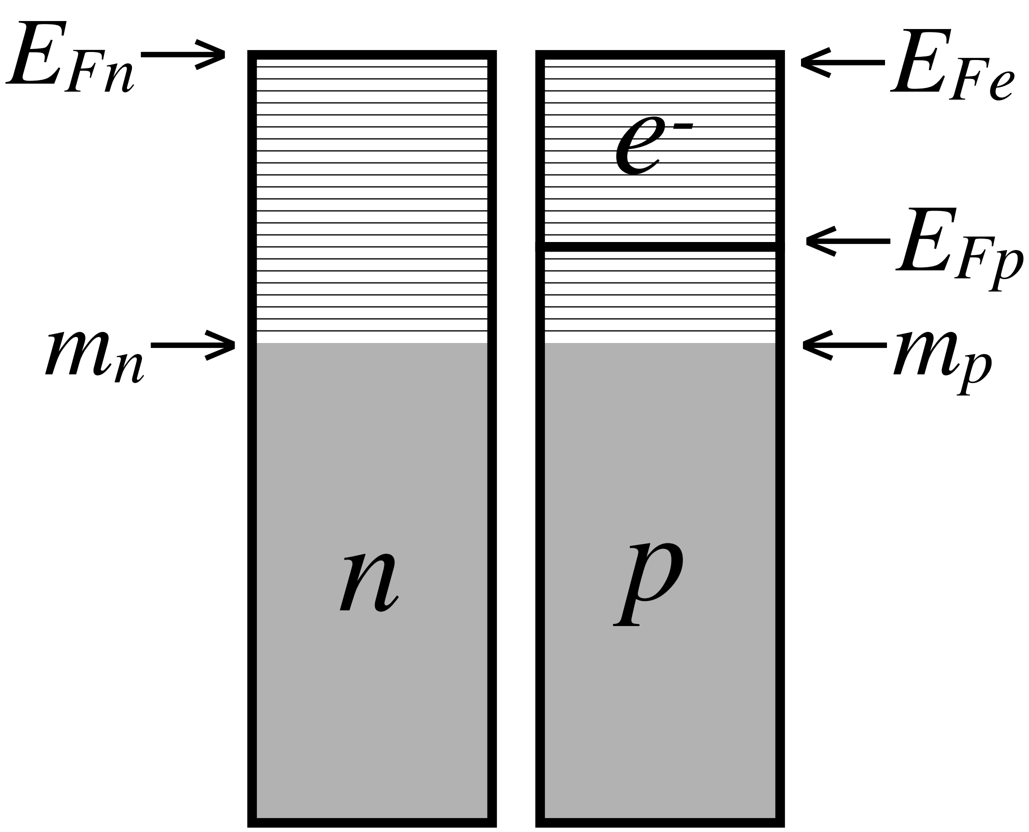
$$n \longrightarrow p + e^- + \overline{\nu}_e$$

Energies involved are the Fermi energies, not only the masses:

(assuming star is cold enough to be transparent to neutrinos: $E_{F\nu} = 0$)

Energy conservation:

$$E_{Fn} = E_{Fp} + E_{Fe}$$



For electrons: $E_F = p_F c$

For neutrons and protons:

$$E_F = mc^2 + p_F^2/2m + V$$

V = potential energy ⇒

Need a model for strong interactions.

⇒ Increasing uncertainty with increasing density

Neutron Decay in Dense Matter

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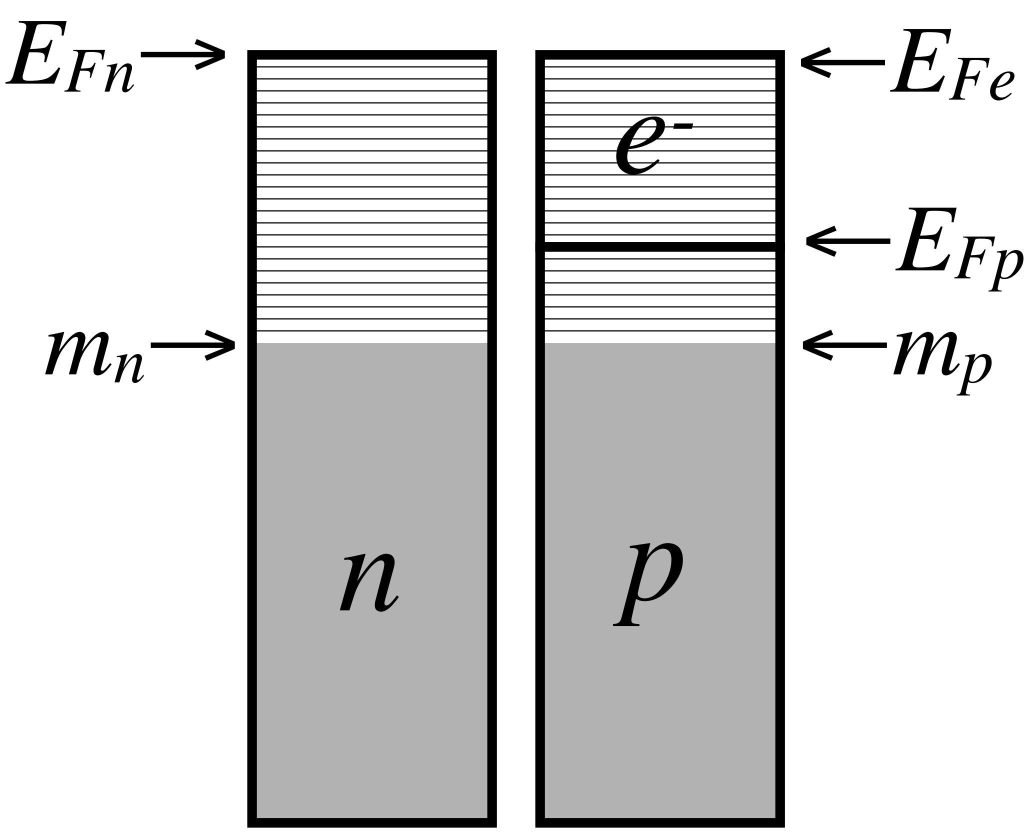

Then

$$p + e^- \longrightarrow n + \nu_e$$

is also possible, giving the DUrca cycle:

$$\begin{cases} n \longrightarrow p + e^- + \overline{\nu}_e \\ p + e^- \longrightarrow n + \nu_e \end{cases}$$

Pulsars

What is a pulsar ?

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A pulsar, “PSR”, is a NEUTRON STAR
with a *measured* rotational, “spin”, period, P.

Thats’ all.

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- Optical pulsar
- X-ray pulsar
- γ-ray pulsar

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- neutrino pulsar
- gravitational wave pulsar

yet to be detected !

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and it can be isolated or in a binary.
In a binary system it can be accreting or not accreting,
or sometimes accreting and other times not accreting, ...

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The immense majority of neutron stars are NOT pulsars.

The majority of known neutron stars are pulsars.

γ-ray pulsar

...

- neutrino pulsar
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A PSR can

γ-ray pulsar

...

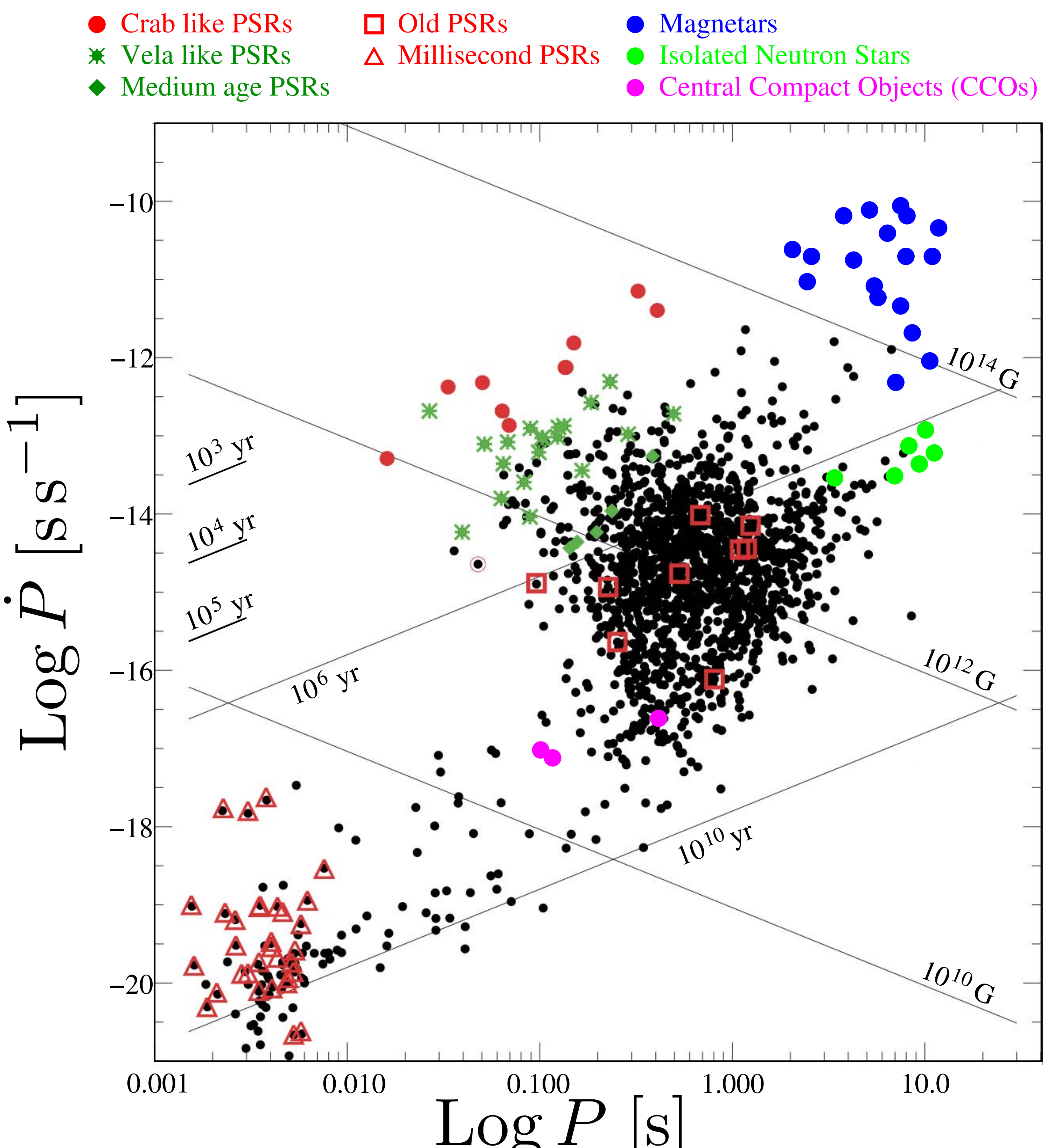
neutrino pulsar

The <u>immense</u> majority of neutron stars are NOT pulsars.

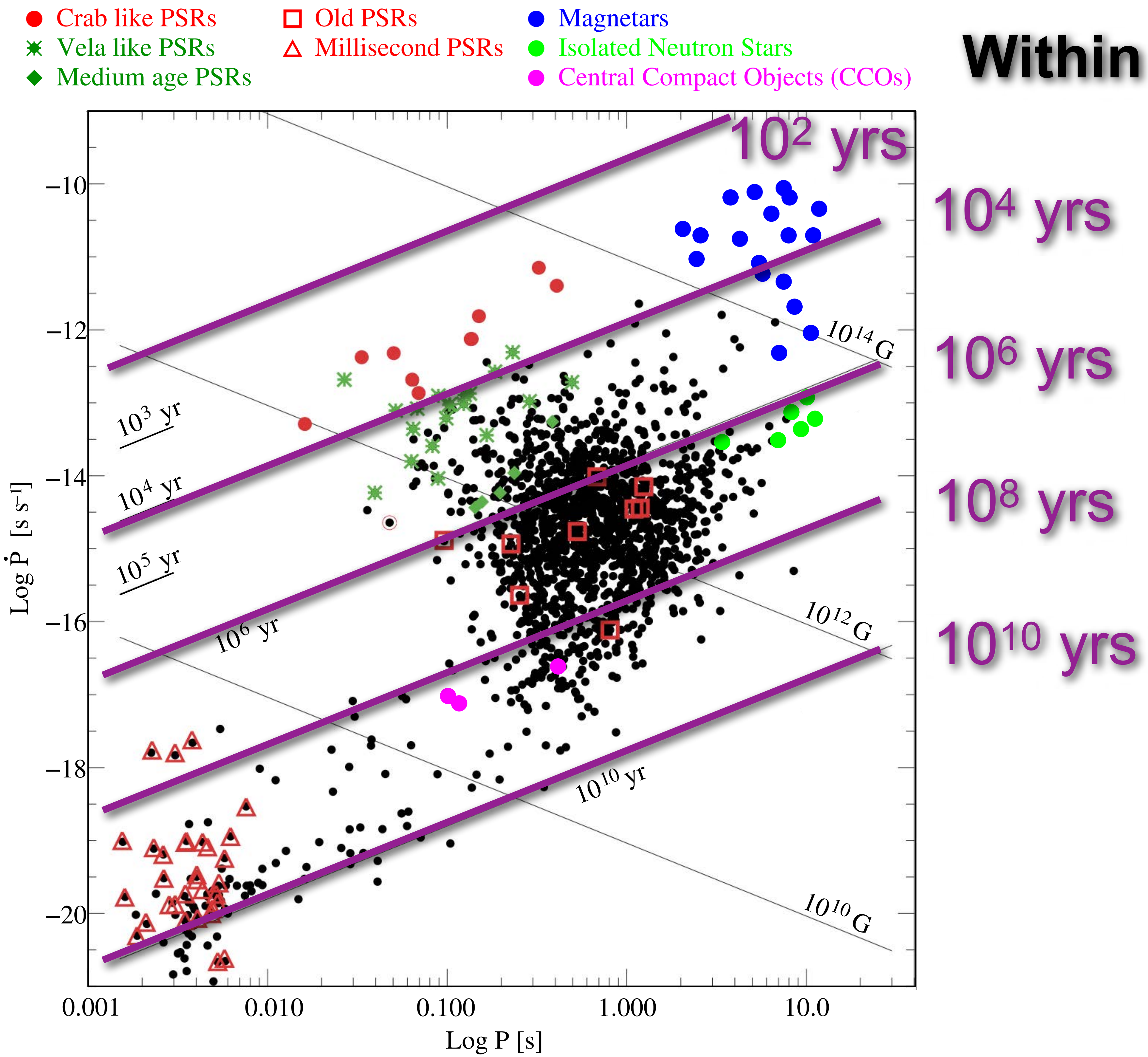
The majority of known neutron stars are pulsars.

There are about 10^{11} stars in the Milky Way and about 1% of them (10^{8}-10^{9}) are neutron stars of which about 0.1% of then (10^{4}-10^{5}) are expected to be pulsars.

The P-Ṗ Diagram



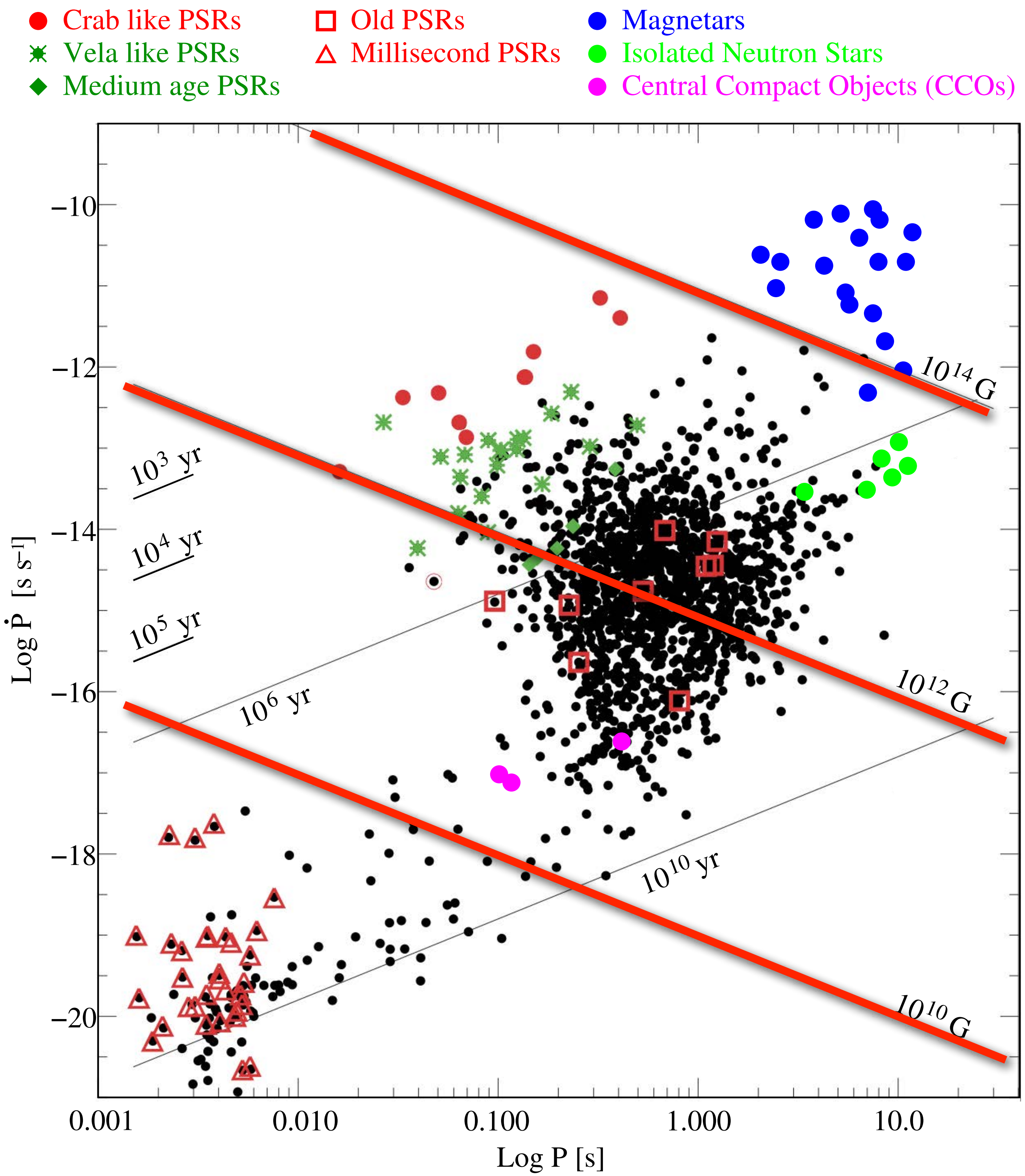
The P-Ṗ Diagram



Within a simple model:

Lines of constant τ_C

The P-Ṗ Diagram



Within a simple model:

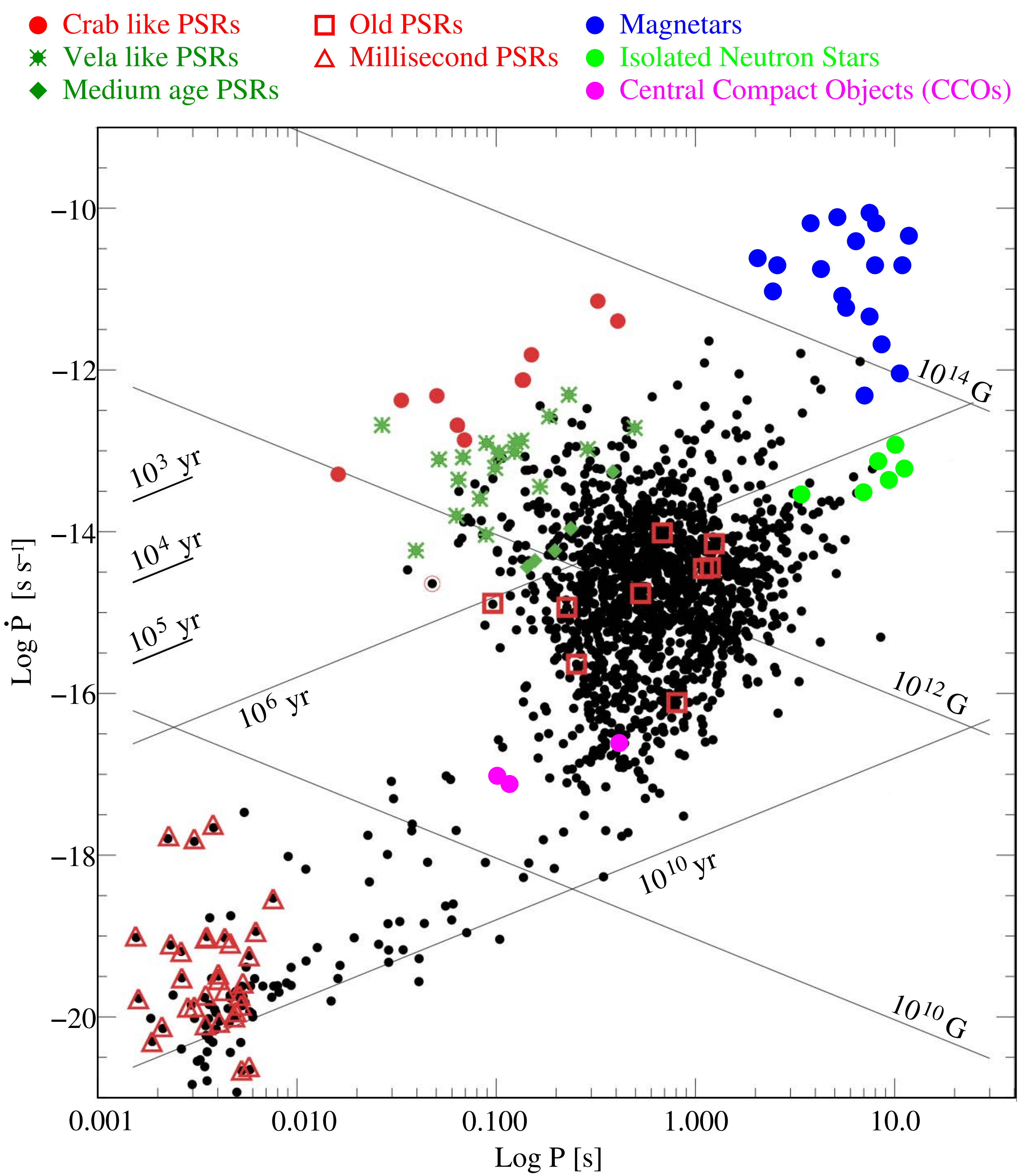
10^{14} G

10^{12} G

10^{10} G

Lines of constant B

The P-Ṗ Diagram



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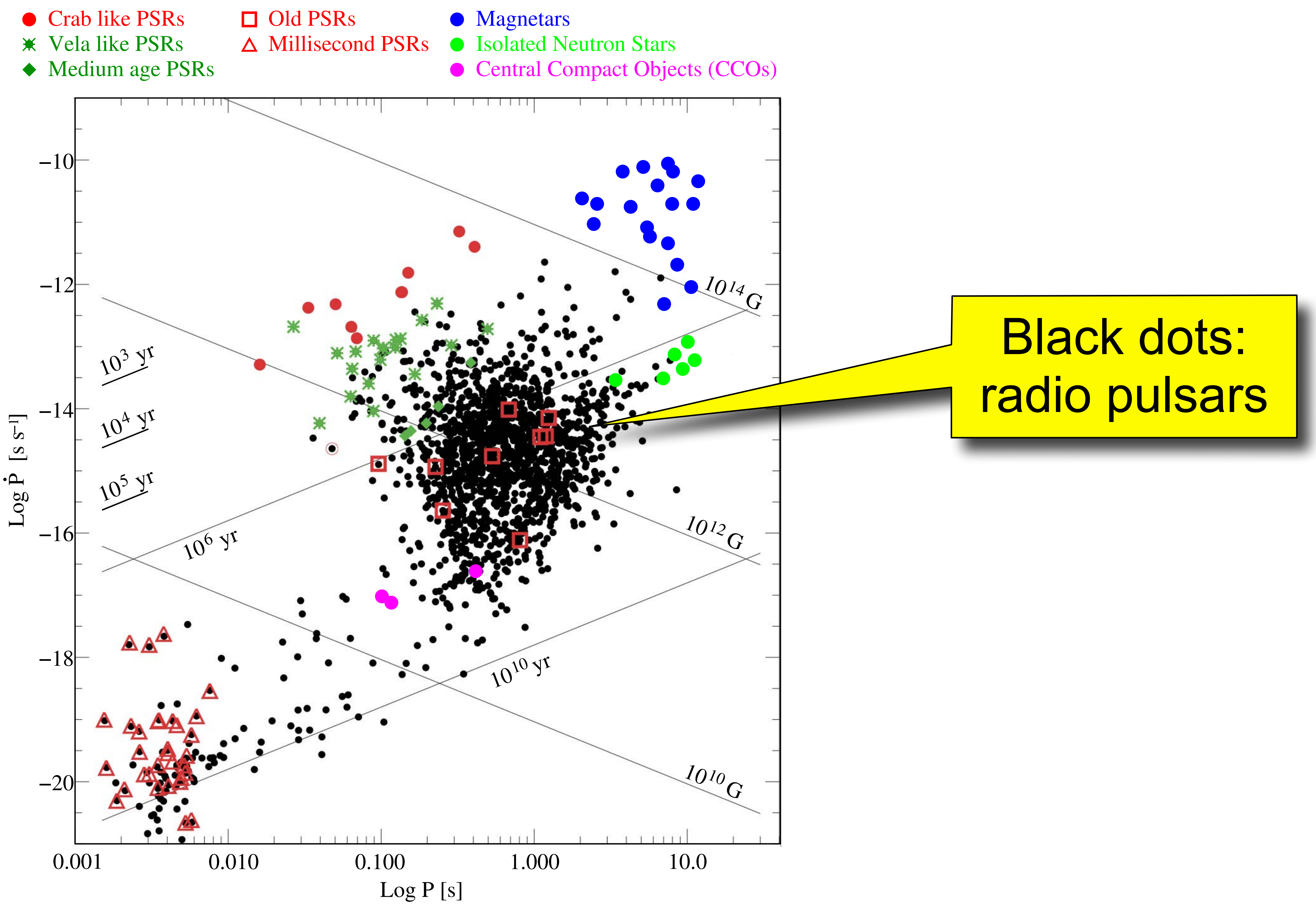

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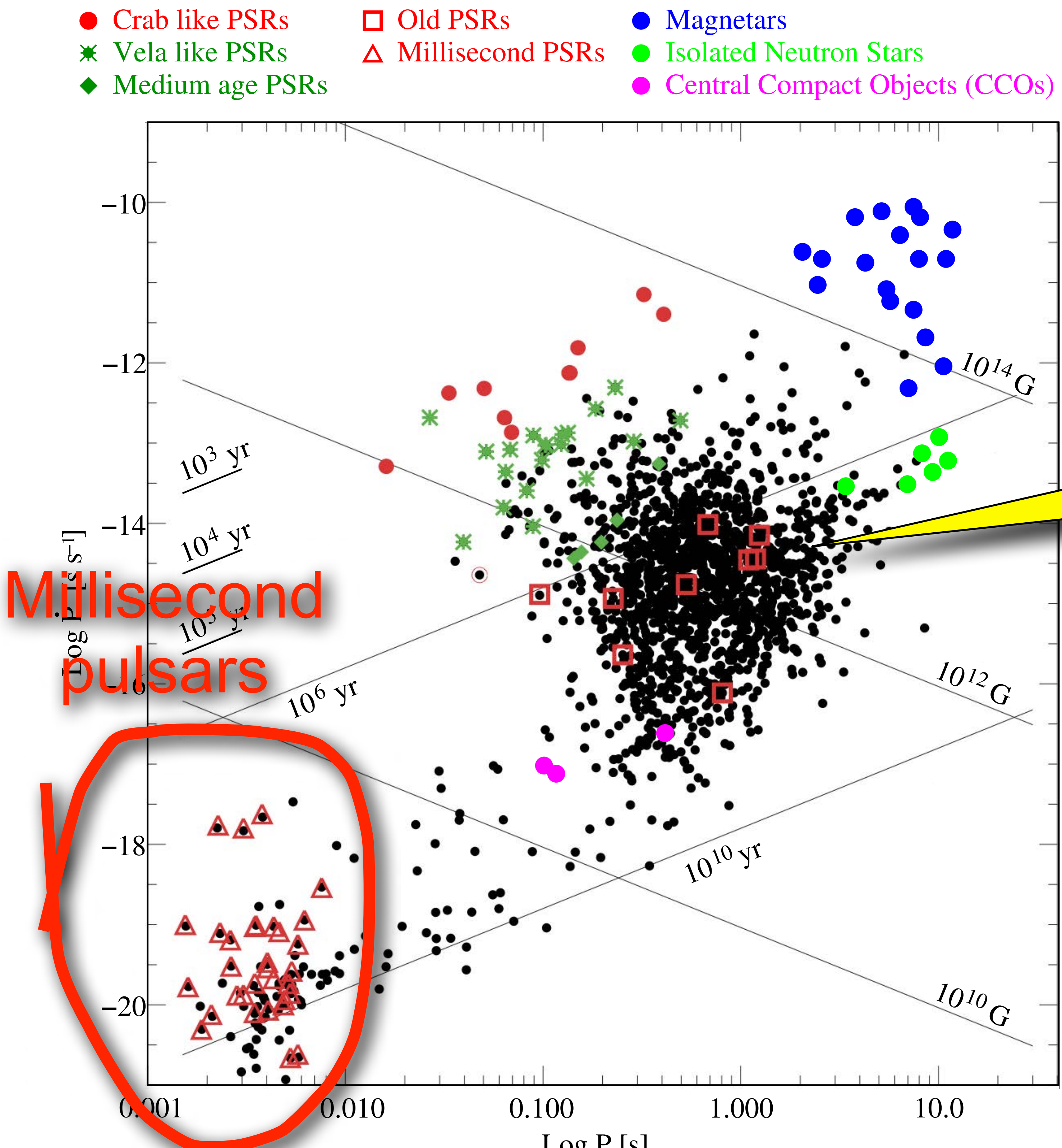
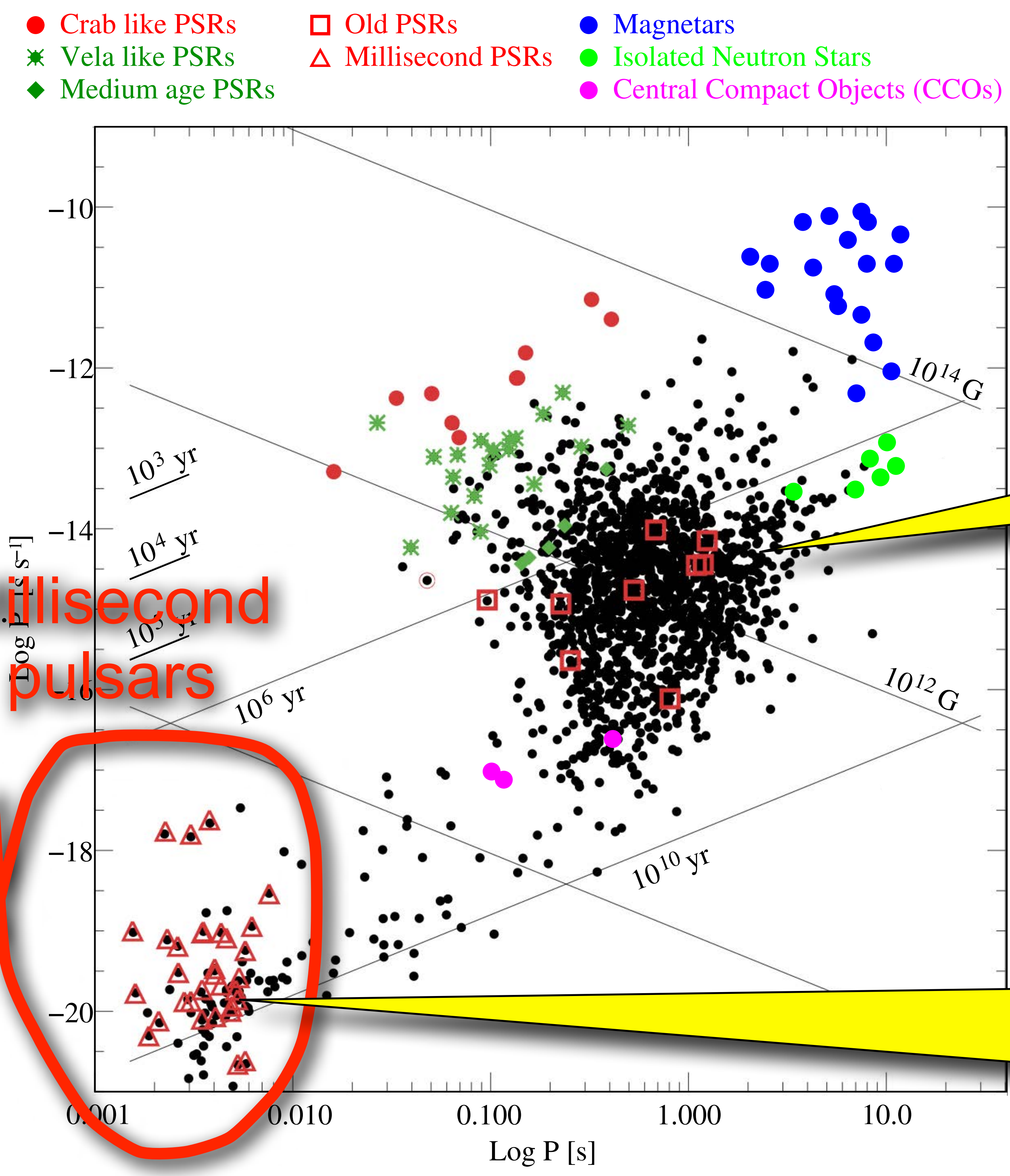


Black dots: radio pulsars

The P-Ṗ Diagram

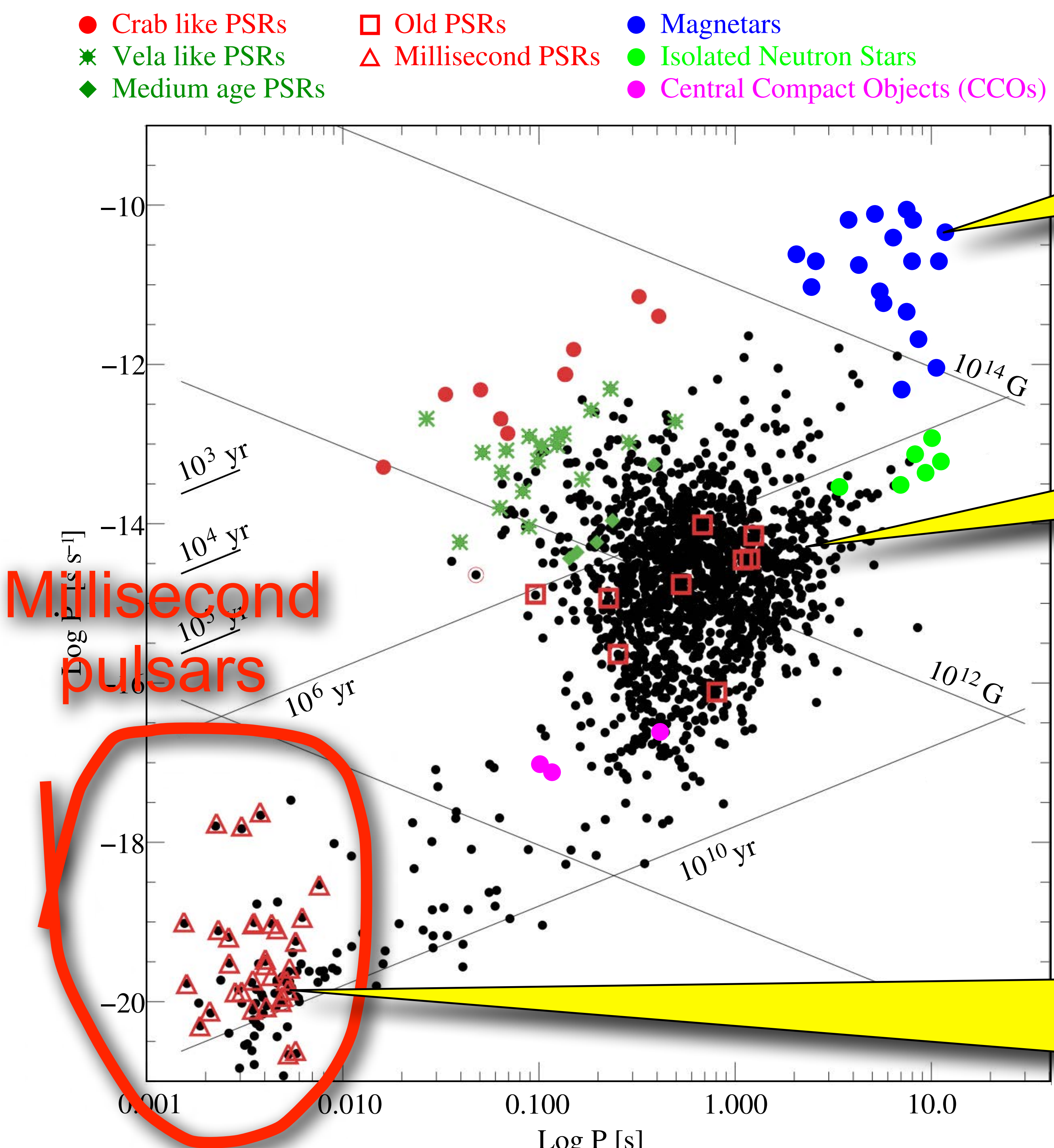


Millisecond pulsars

Black dots: radio pulsars

Red triangle: X-ray & radio ms pulsars

The P-$\dot{P}$ Diagram

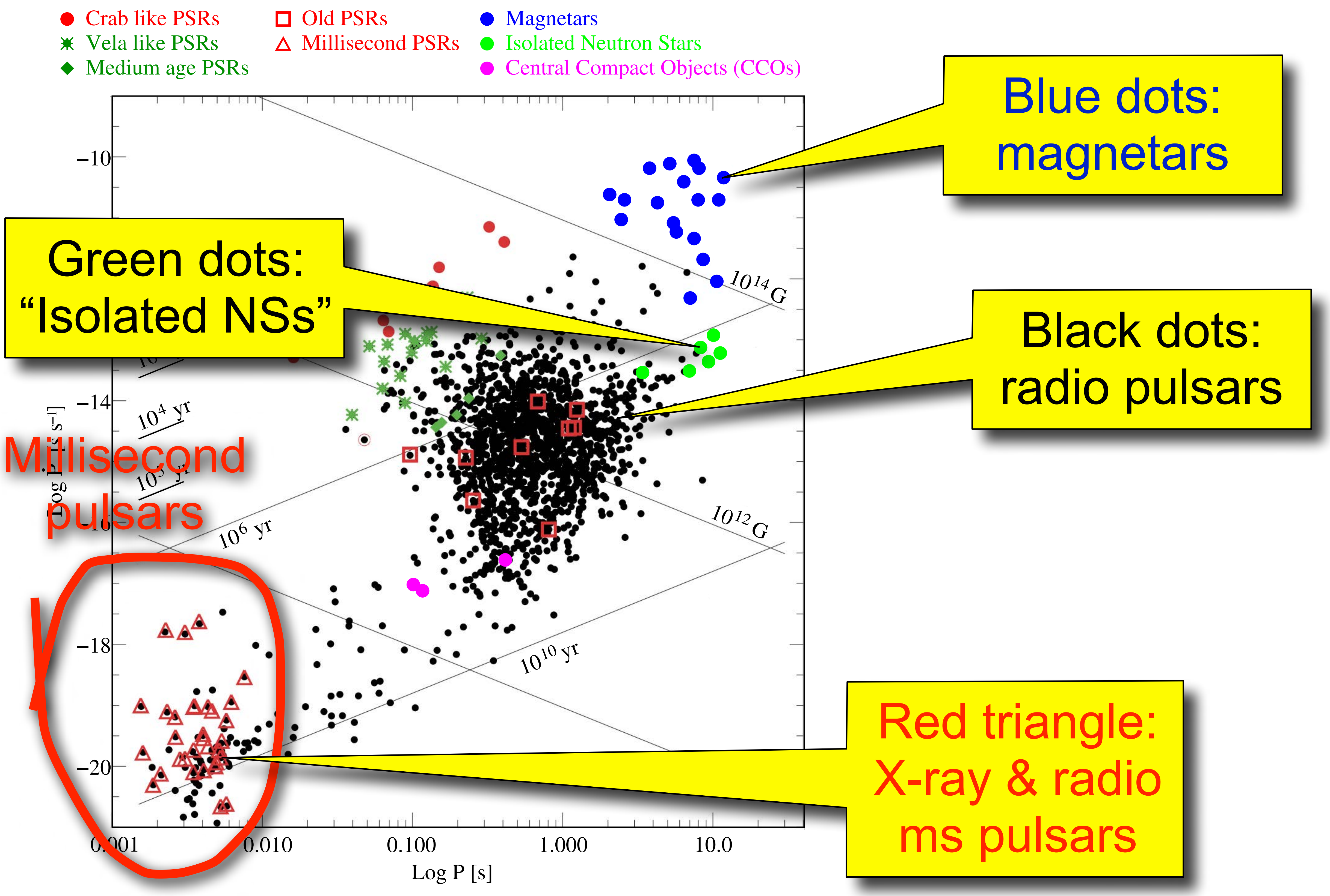


Blue dots: magnetars

Black dots: radio pulsars

Red triangle: X-ray & radio ms pulsars

The P-Ṗ Diagram



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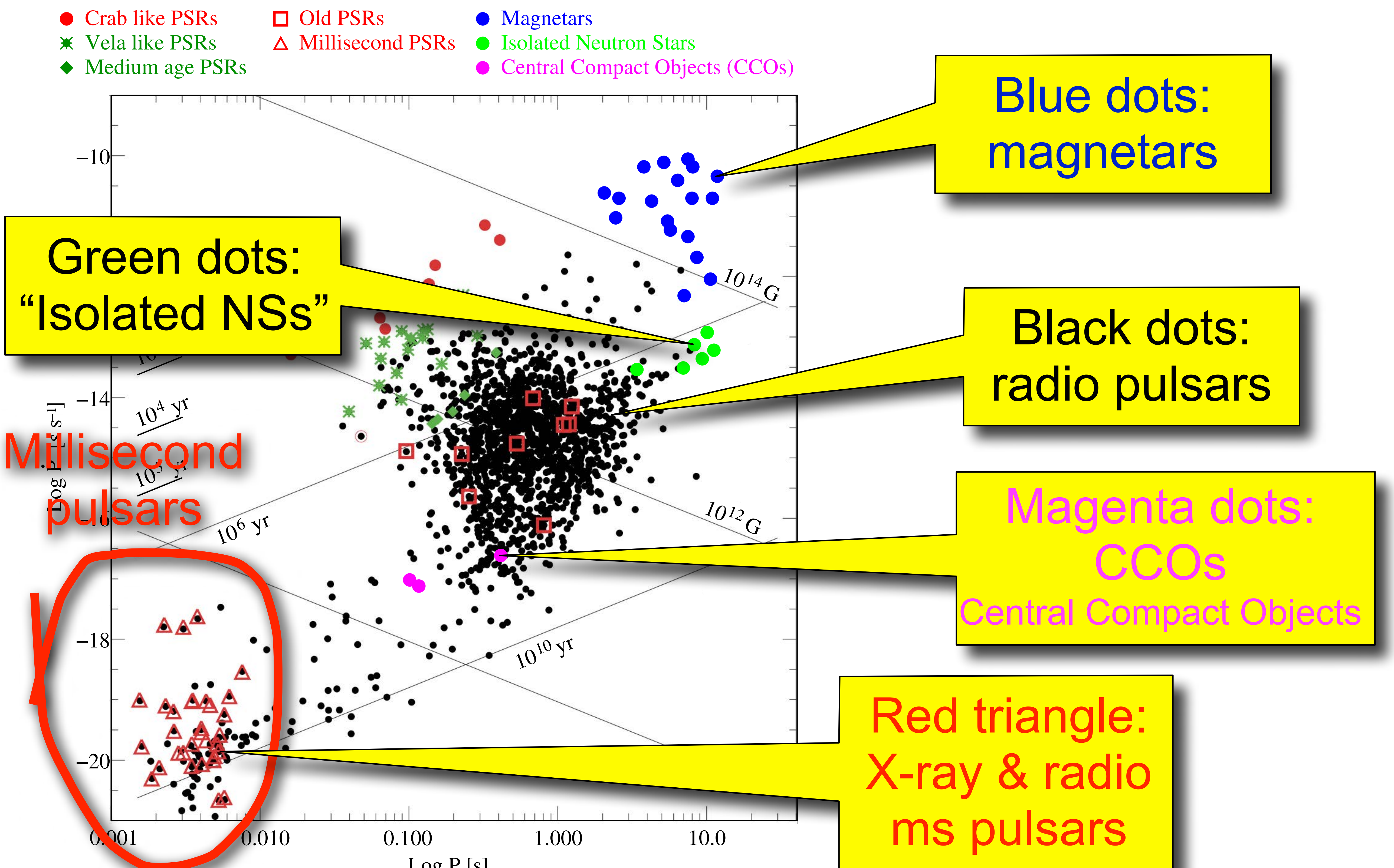
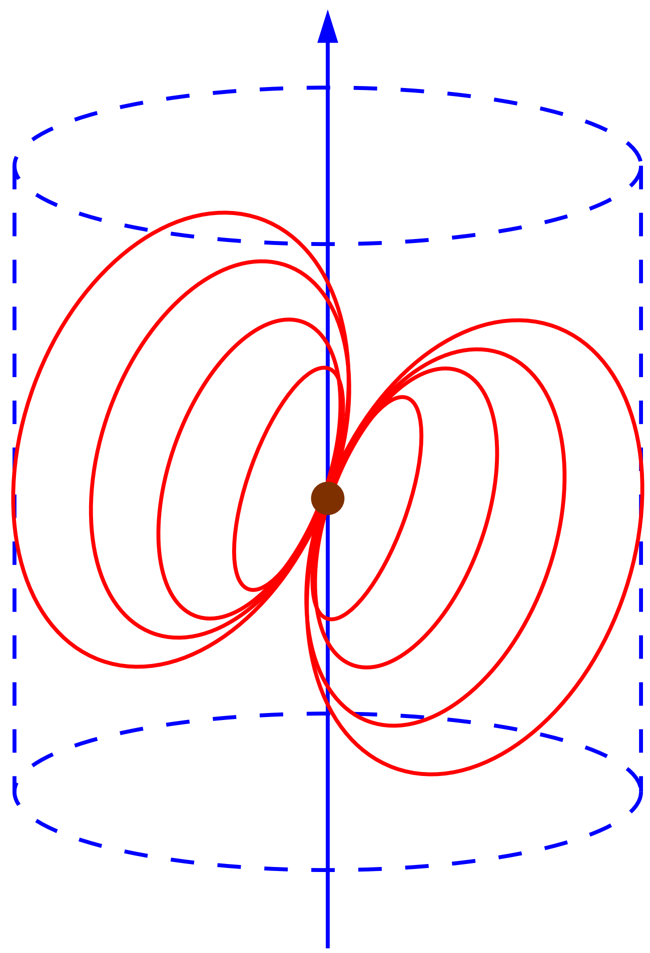


The Magnetic Dipole Model of Spin-Down



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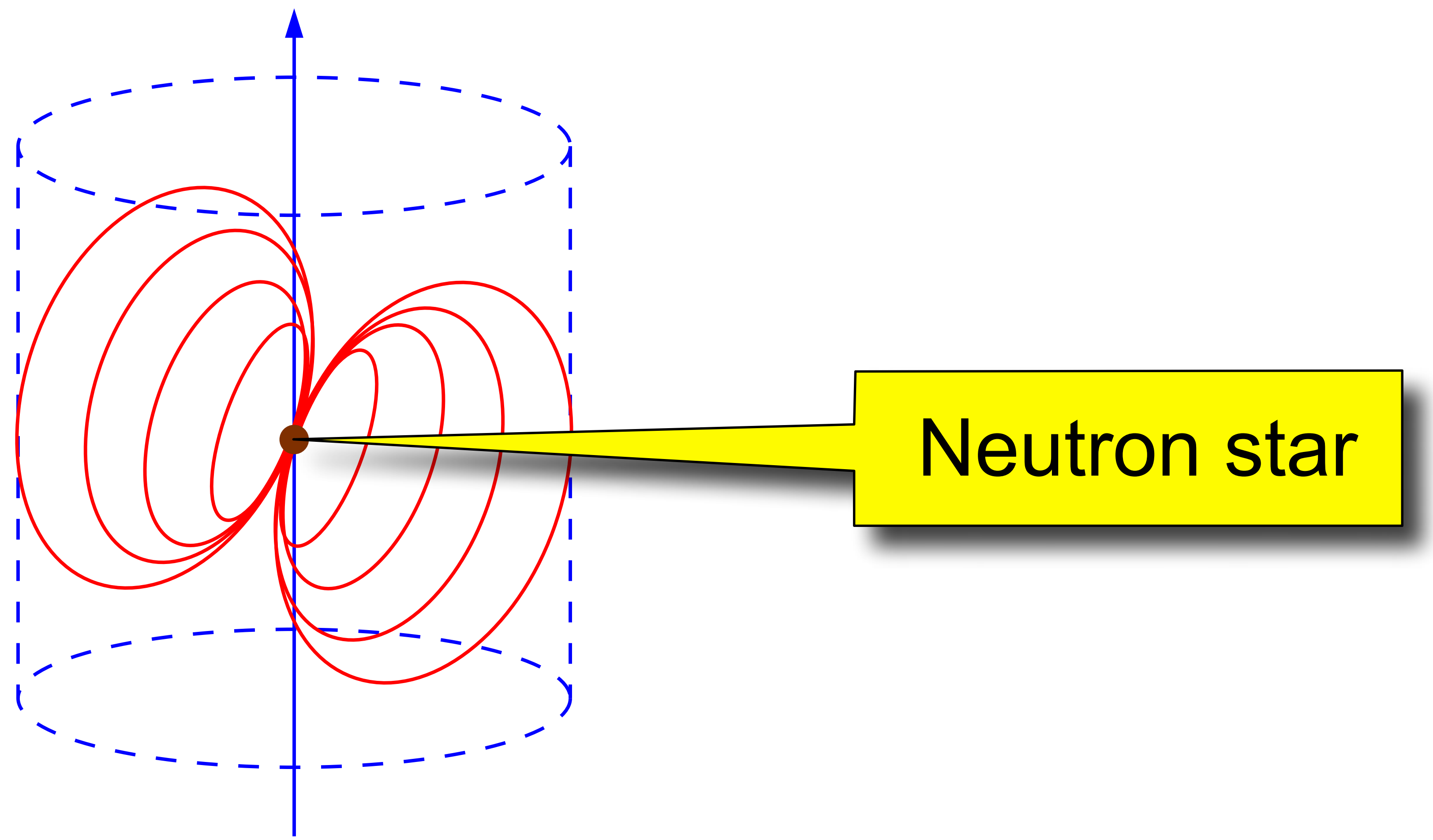

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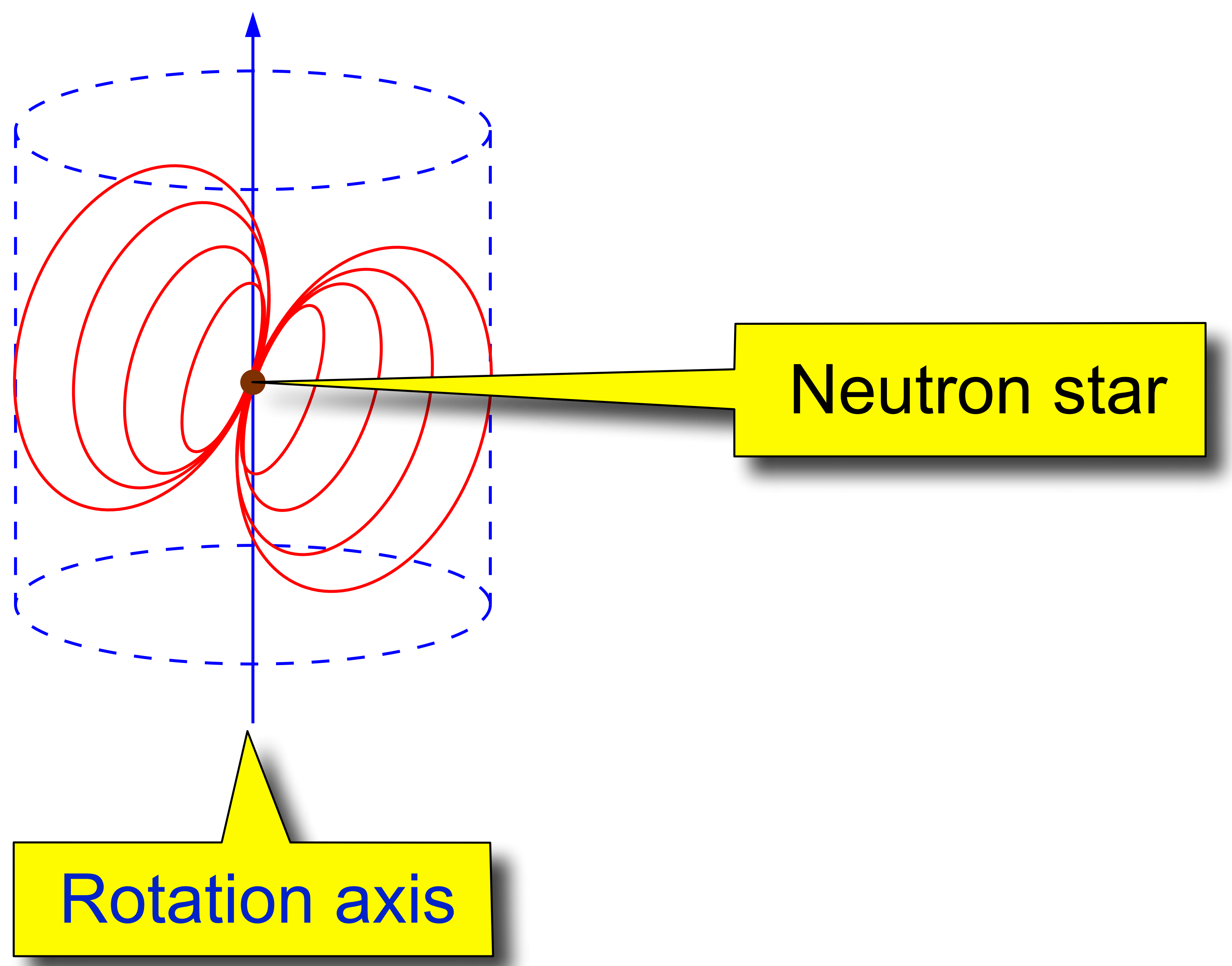

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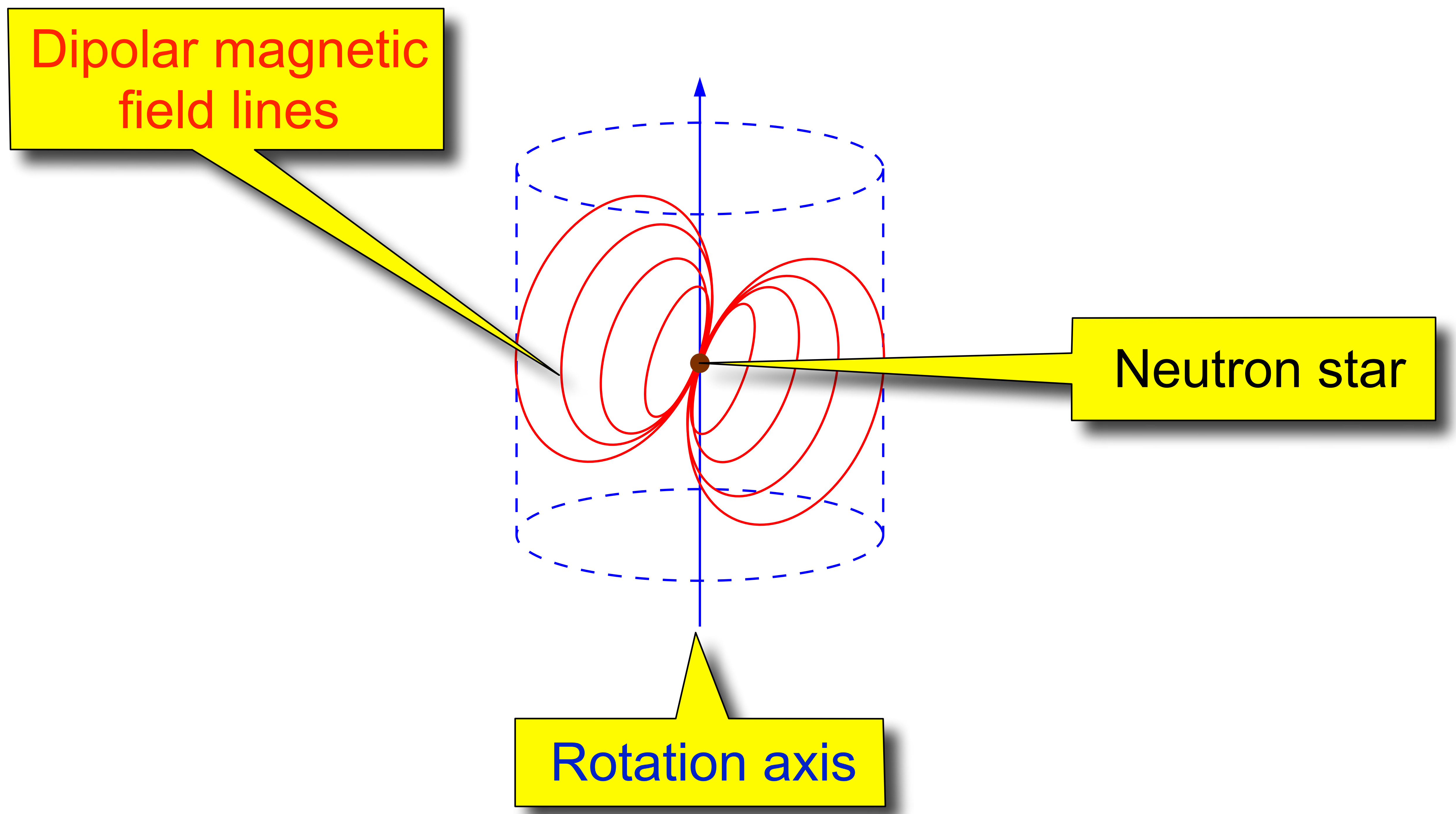

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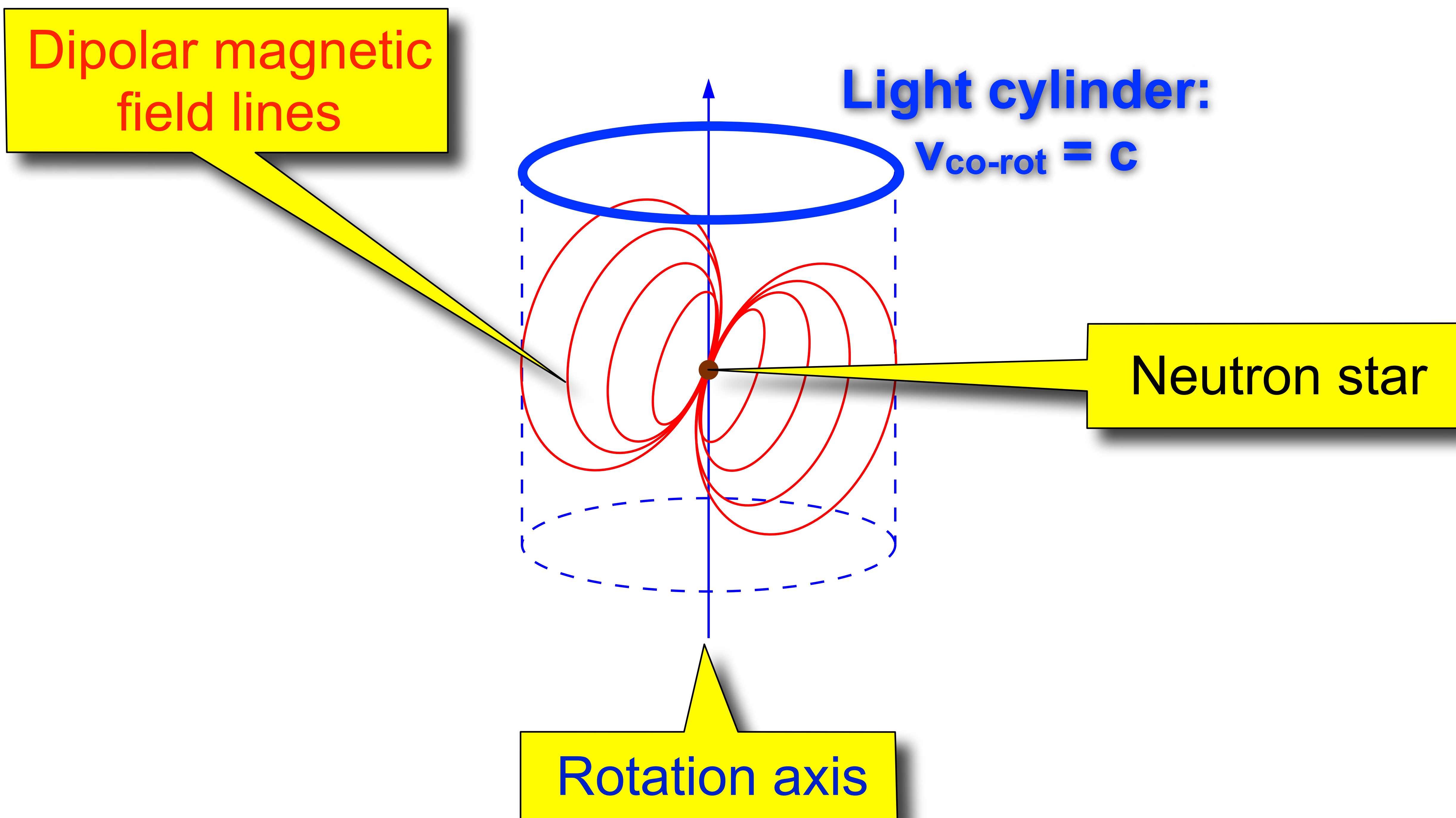

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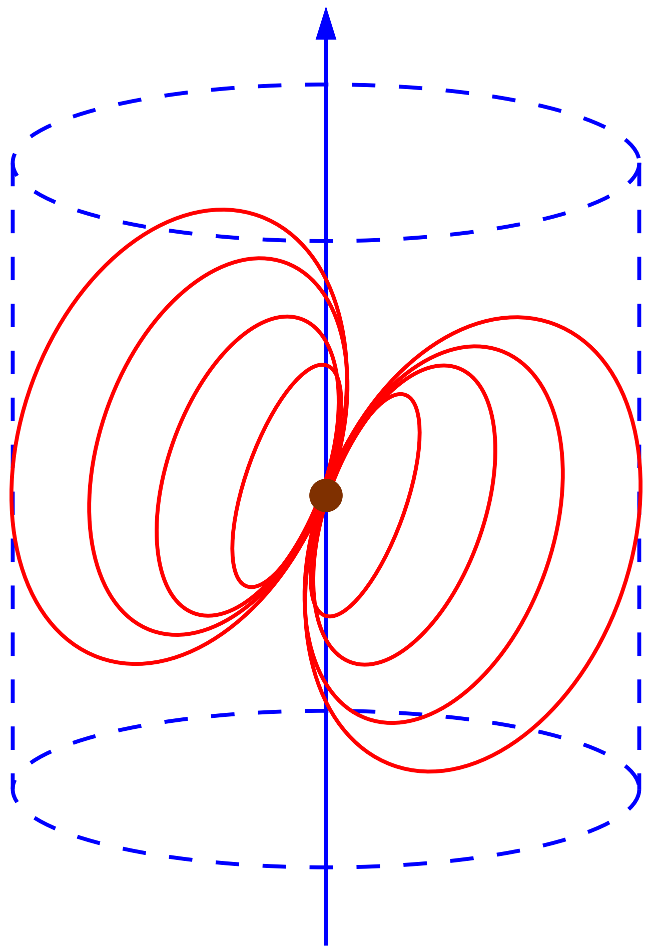


$$R_{lc} = \frac{c}{\Omega} = \frac{cP}{2\pi} \simeq 50\,\mathrm{km} \times P_{\mathrm{millisec.}}$$

$$B_{lc} \cong B_0 \left(\frac{R}{R_{lc}}\right)^3 = \frac{B_0 R^3}{c^3}\Omega^3$$

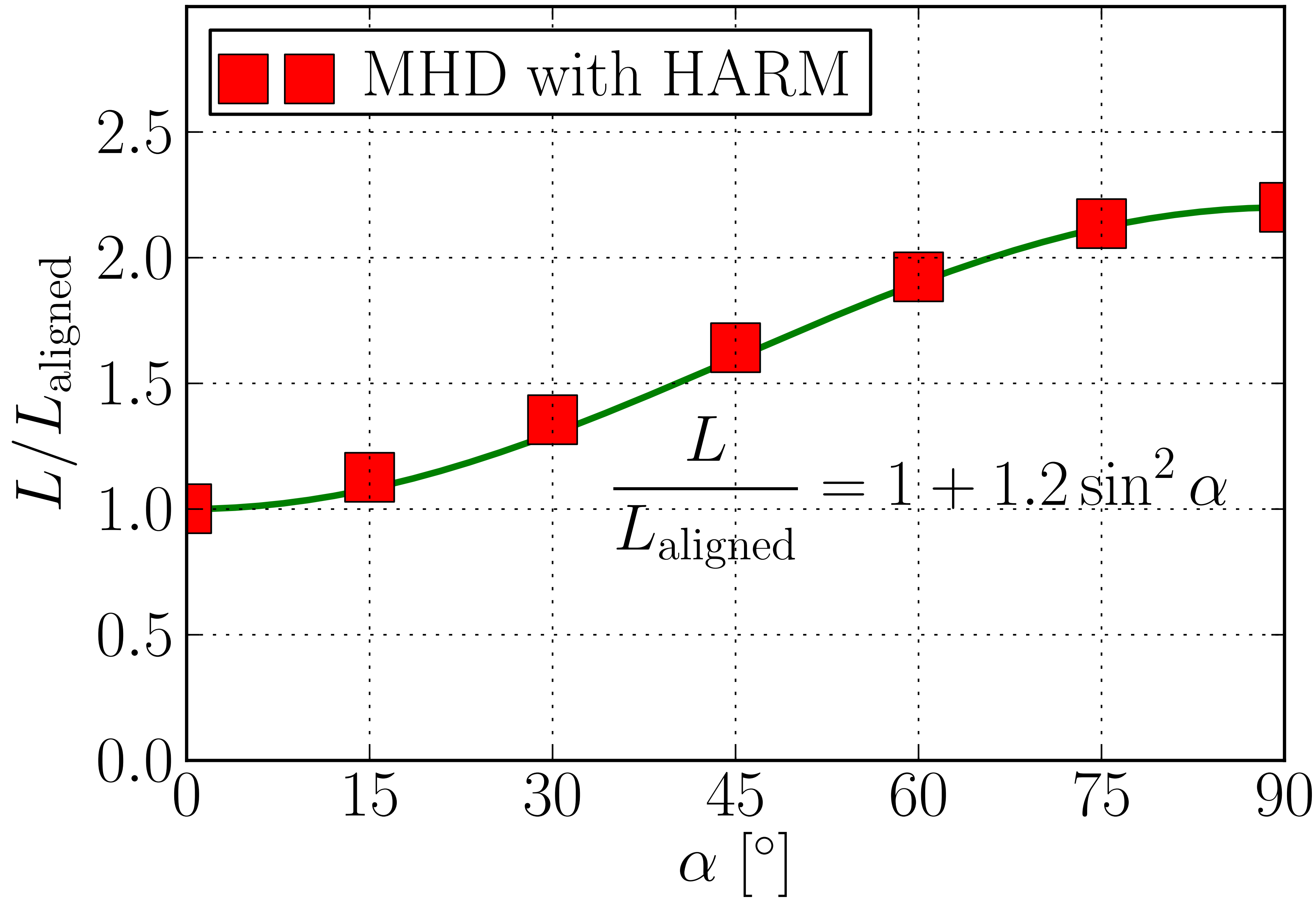
$$E_{m,\,lc} = \frac{B_{lc}^2}{4\pi} = \frac{B_0^2 R^6}{4\pi c^6}\Omega^6$$

$$\boxed{\dot{E}_{\mathrm{md}} \sim -A_{lc}\,E_{m,\,lc}\,c \simeq -\frac{B_0^2 R^6}{c^3}\Omega^4}$$ with $A_{lc} \simeq 4\pi R_{lc}^2$

Magneto-dipolar
radiation
in vacuum:

$$\dot{E}_{\mathrm{md}}^{\mathrm{vac.}} = -\frac{B_p^2 R^6 \Omega^4}{c^3} \times \frac{1}{6}\sin^2\alpha$$

Spitkovsky et al. Model



$$\dot{E}_{\mathrm{md}} = -k_1 \frac{B_p^2 R^6 \Omega^4}{c^3}(1 + k_2 \sin^2\alpha) \quad \text{with} \quad k_1 = 1 \pm 0.05 \;\; \text{and} \;\; k_2 = 1 \pm 0.05$$

Spitkovsky, A., Pulsar Magnetosphere: The Incredible Machine, AIP Conf. Proc. 983, 20 (2008)

The Magnetic Dipole Model of Spin-Down

Rotational kinetic energy: $E_{\rm rot} = \frac{1}{2} I \Omega^2 \implies \dot{E}_{\rm rot} = I \Omega \dot{\Omega}$

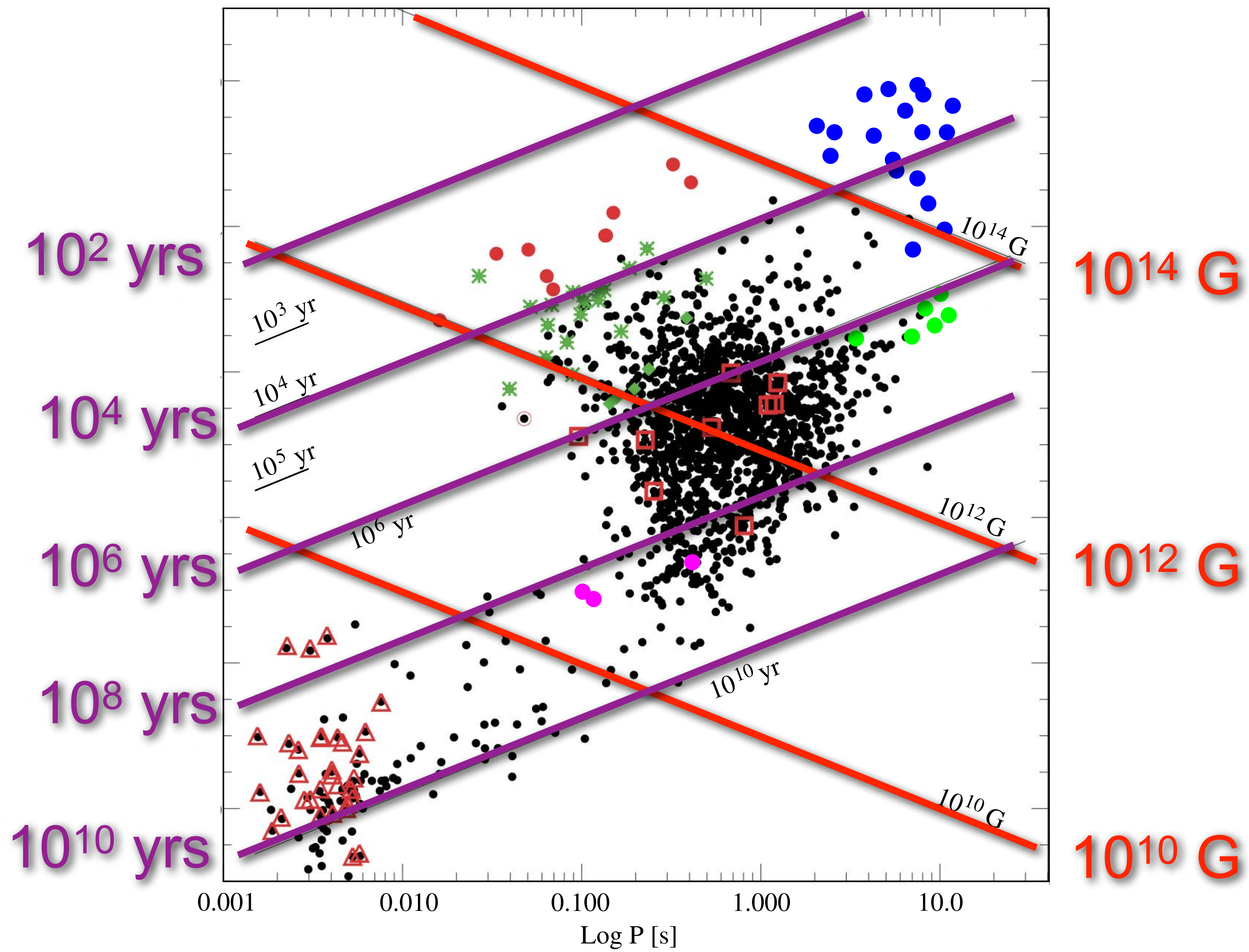
Magneto-dipolar spin-down equation: $\dot{E}_{\rm rot} = -\dot{E}_{\rm md}$

which gives: $\dot{\Omega} = -k \frac{B_0^2 R^6}{c^3 I} \Omega^3 \quad (k \simeq 1)$

Inferred surface magnetic field: $\boxed{B_p \simeq \sqrt{\frac{c^3 |\dot{\Omega}|}{\Omega^3 I R^6}}}$

$$\simeq 3.2 \times 10^{19} (P \dot{P})^{1/2} \ \mathrm{G} \simeq 1.5 \times 10^{12} \frac{P}{0.01\mathrm{s}} \left(\frac{1000 \, \mathrm{yr}}{\tau_c} \right)^{1/2} \ \mathrm{G}$$

The P-Ṗ Diagram



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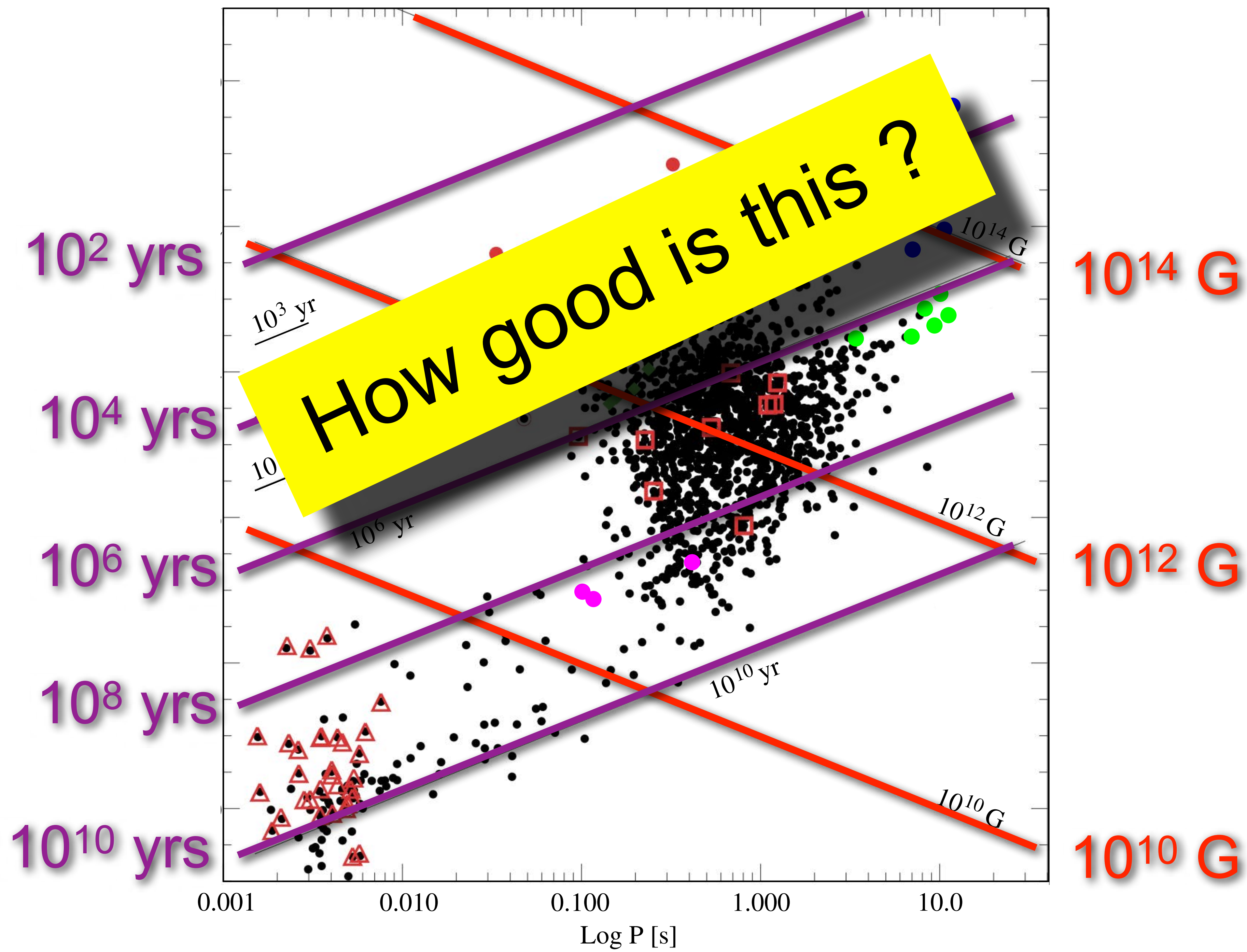

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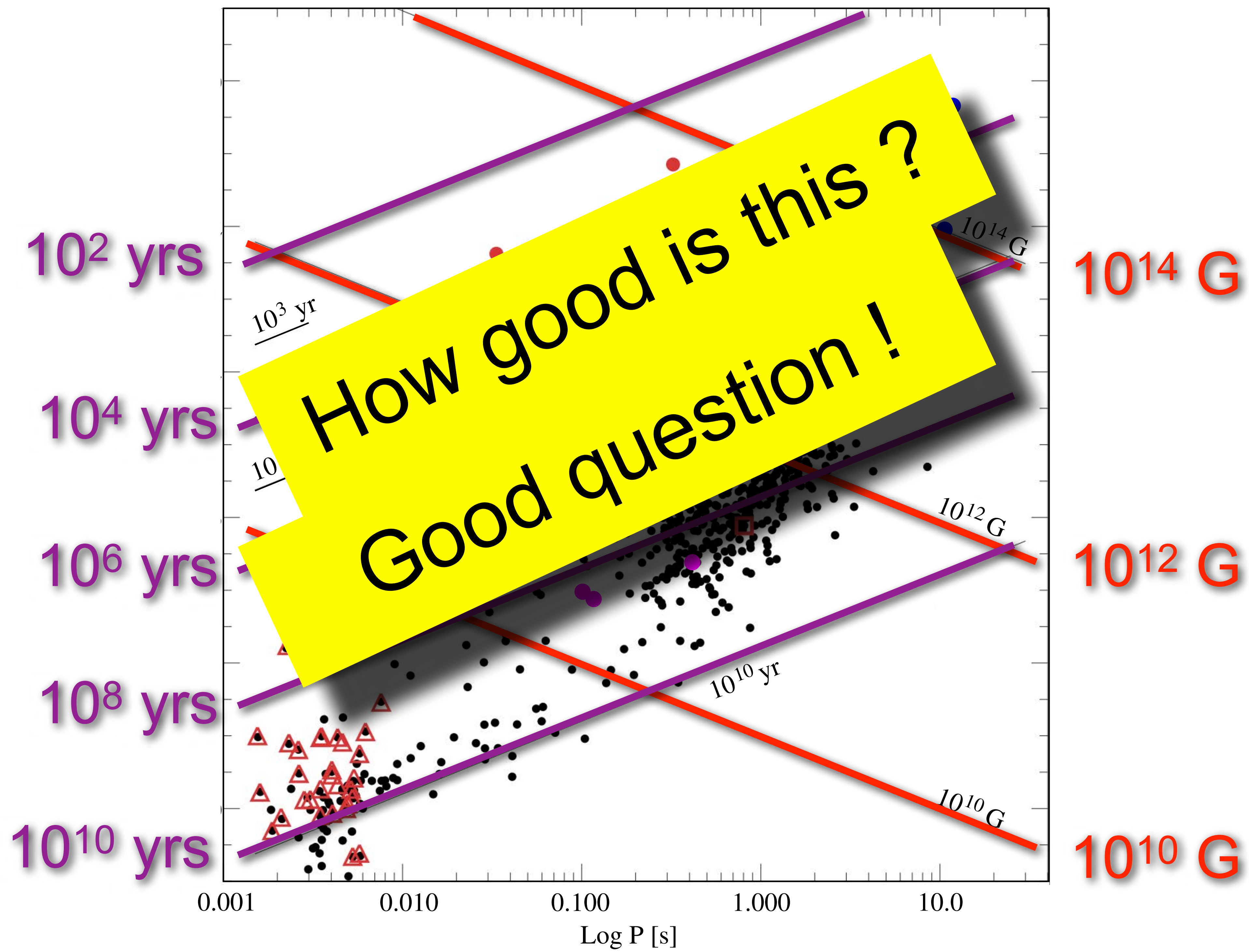

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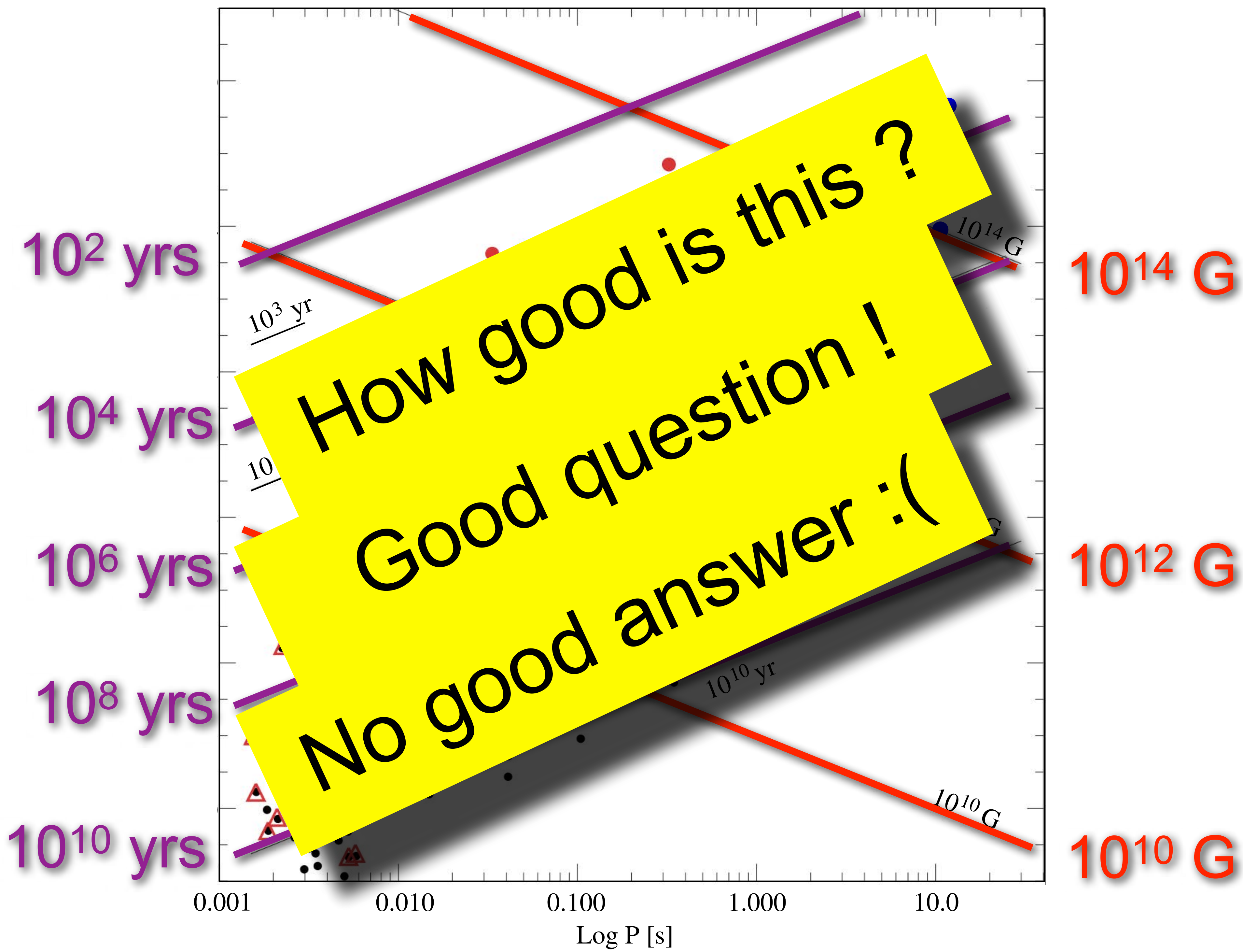


Hydrostatic Equilibrium Tolman-Oppenheimer-Volkoff Equations

Static Spherically Symmetric Metric

Spherically symmetric metric: $$\mathrm{d}s^2 = c^2\mathrm{d}t^2\mathrm{e}^{2\phi} - \mathrm{e}^{2\lambda}\mathrm{d}r^2 - r^2(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\phi)$$

Local observer (proper) metric: $$\mathrm{d}s^2 = c^2\mathrm{d}\tau^2 - \mathrm{d}l^2$$

Proper time and radial length: $$\mathrm{d}\tau = \mathrm{e}^{\phi}\,\mathrm{d}t < \mathrm{d}t \qquad \mathrm{d}l = \mathrm{e}^{\lambda}\,\mathrm{d}r > \mathrm{d}r$$

Metric of a 2D sphere (r & t fixed): $$\mathrm{d}s^2 = r^2(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\phi)$$

$\longrightarrow$ same metric as a sphere in Euclidean geometry
(r is known as the "areal coordinate")

Proper volume of a shell: $$\mathrm{d}v = 4\pi r^2\mathrm{d}l > 4\pi r^2\mathrm{d}r$$

Vacuum solution: the Schwarzschild metric:

$$\mathrm{d}s^2 = \left(1 - \frac{2GM}{rc^2}\right)c^2\mathrm{d}t^2 - \frac{\mathrm{d}r^2}{1 - \dfrac{2GM}{rc^2}} - r^2(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\phi)$$

t = proper time of an observer at infinity

Hydrostatic Equilibrium in Spherical Metric

Tolman - Oppenheimer - Volkoff equations:

$$\frac{\mathrm{d}m}{\mathrm{d}r} = 4\pi r^2 \rho$$

$$\frac{\mathrm{d}\phi}{\mathrm{d}r} = \frac{G}{c^2}\frac{m + 4\pi r^3 P/c^2}{r^2}\mathrm{e}^{2\lambda} \qquad \mathrm{e}^{-2\lambda} = 1 - \frac{2Gm}{rc^2}$$

$$\frac{\mathrm{d}P}{\mathrm{d}r} = -\frac{G(\rho + P/c^2)(m + 4\pi r^3 P/c^2)}{r^2}\mathrm{e}^{2\lambda} = -c^2(\rho + P/c^2)\frac{\mathrm{d}\phi}{\mathrm{d}r}$$

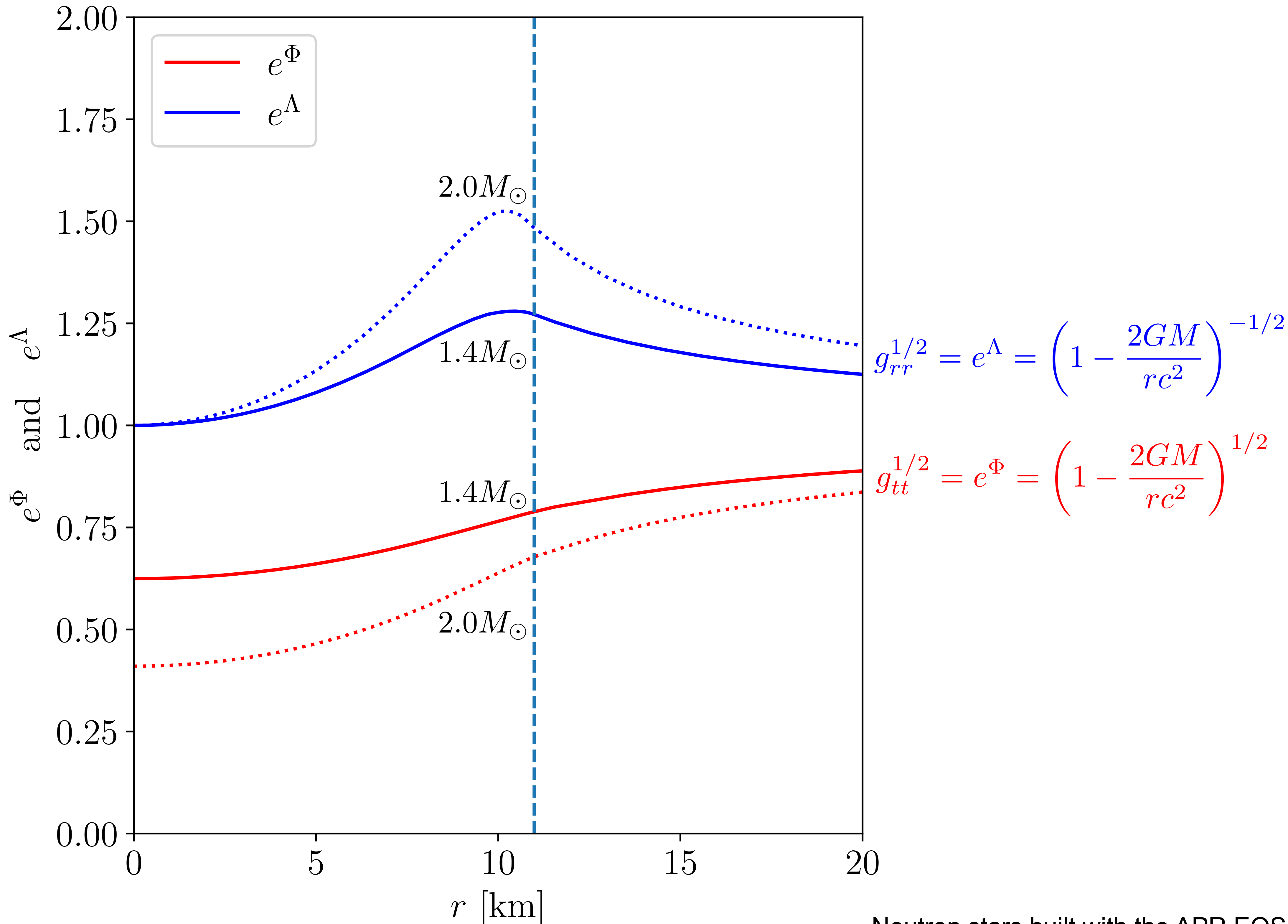
Equation of State (fully degenerate): $P = P(\rho)$ $\rho = \epsilon/c^2$

Solution: given the EOS, choose central pressure P_c or density ρ_c and integrate TOV equations outward till P=0 (surface).

Shift $\phi \longrightarrow \phi + \phi_0$ so that at the surface $\mathrm{e}^{\phi_s} = \mathrm{e}^{-\lambda_s} = \sqrt{1 - 2GM/Rc^2}$

to match the vacuum Schwarzschild solution

Metric Coefficients



Neutron stars built with the APR EOS

Various Concepts of Mass

Gravitational mass: $M = \int_0^R 4\pi\rho\, r^2 \mathrm{d}r$

it is the mass in the Schwarzschild metric and it determines planetary motions, binary motions, ... : all gravitational effects

Proper mass: $M_P = \int_0^R 4\pi\rho\, r^2 \mathrm{d}l$

Baryonic mass: $M_B = m_B N_B$ $\quad N_B = \int_0^R 4\pi n_B\, r^2 \mathrm{d}l =$ baryon number

n_B = baryon density

m_B = baryon mass

Gravitational mass = proper mass + gravitational binding energy:

$$M = \int_0^R 4\pi\rho\, r^2 \mathrm{d}r = \int_0^R 4\pi\rho\, r^2 \sqrt{1 - 2Gm/rc^2}\, \mathrm{d}l$$

$$\simeq \int_0^R 4\pi\rho\, r^2 (1 - Gm/rc^2)\, \mathrm{d}l = \int_0^R 4\pi\rho\, r^2 \mathrm{d}l - \frac{1}{c^2}\int_0^R \frac{Gm\rho}{r} 4\pi r^2\, \mathrm{d}l = M_P + \frac{E_G}{c^2}$$

A Sample of EOSs and M-R Curves

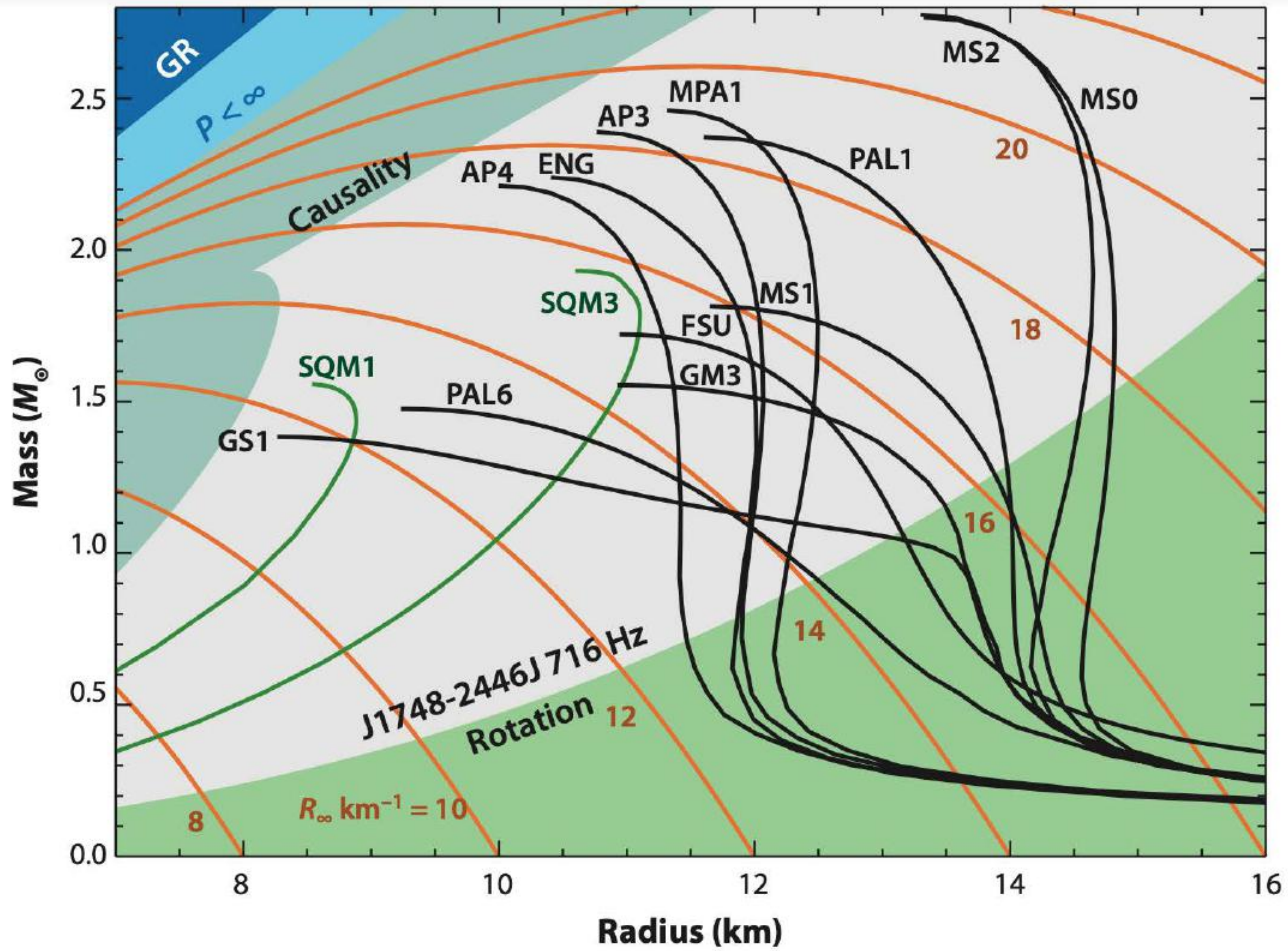
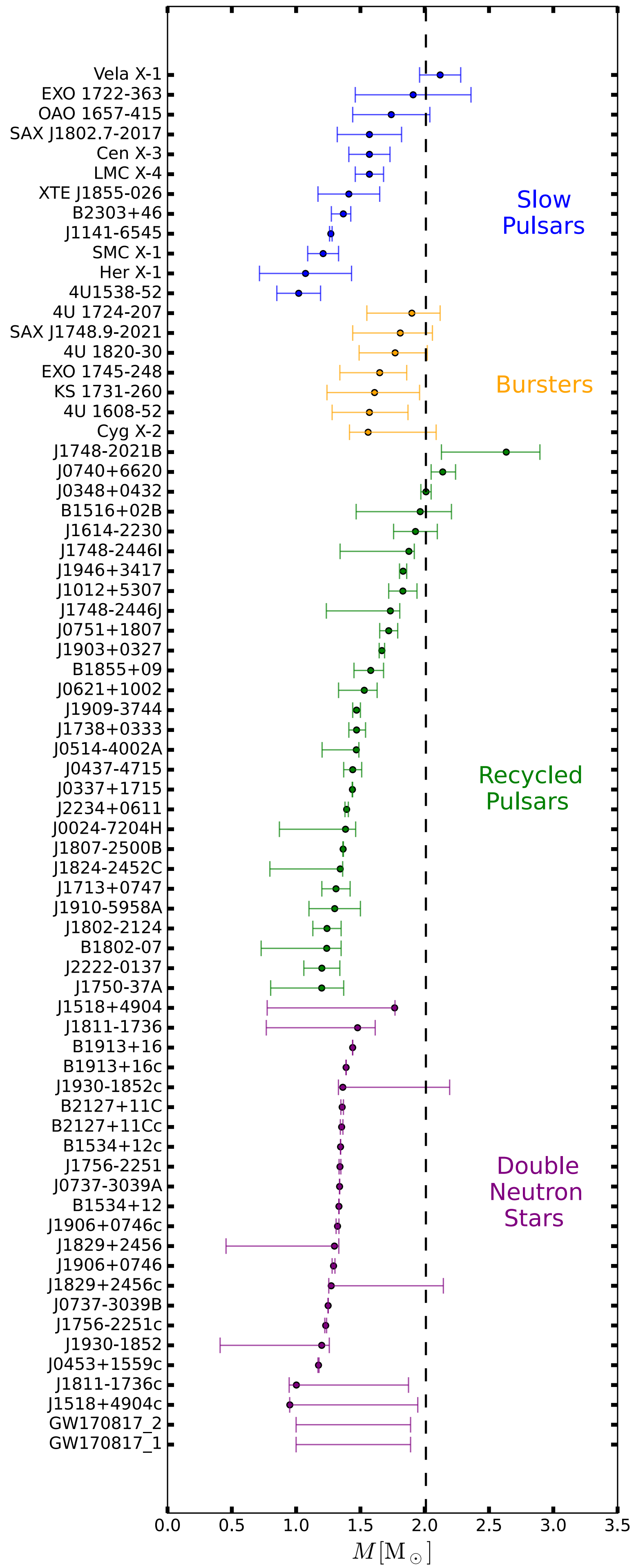


Figure 3

Typical M–R curves for hadronic equations of state (EOSs) (*black curves*) and strange quark matter (SQM) EOSs (*green curves*). The EOS names are given in Reference 13, and their P–n relations are displayed in **Figure 2**. Regions of the M–R plane excluded by general relativity (GR), finite pressure, and causality are indicated. The orange curves show contours of $R_\infty = R(1 - 2\,GM/Rc^2)^{-1/2}$. The region marked rotation is bounded by the realistic mass-shedding limit for the highest-known pulsar frequency, 716 Hz, for PSR J1748-2446J (14). Figure adapted from Reference 15.

The Nuclear Equation of State and Neutron Star Masses
Lattimer, James M.
2012ARNPS..62..485L

Neutron Star Masses



Neutron stars with strong $\vec{B}$ ("X-ray pulsars") in eclipsing binaries with a high mass companion. Optical observations of the companion (massive = bright star) gives complementary information about the orbit. Provide useful constraints but low precision.

Spectral studies of thermonuclear X-ray bursts provide measurements of their mass and radius: useful constraints but low precision.

Very old systems with a millisecond ("recycled") pulsar.

In a few cases with a white dwarf companion and an almost circular orbit accurate mass measurement have been obtained through the "Shapiro delay" effect. These have provided the highest mass measurements.

Binary systems with a pulsar and a second compact star (neutron star or white dwarf) companion. In the cases of very excentric and compact orbits many Post-Keplerian parameters can be measured with high accuracy and the orbit properties can be deduced with high precision. These provide the most accurate mass measurement. Typical example: the Hulse-Taylor system from which existence of gravitational waves was first obtained.

Masses, Radii, and the Equation of State of Neutron Stars
Özel, Feryal; Freire, Paulo
2016ARA&A..54..401O

http://xtreme.as.arizona.edu/neutronstars/

Mass Measurements from Binaries

Newtonian dynamics ("Kepler Laws") give the "mass function":

$$f = \frac{(M_c \sin i)^3}{M_T^2} = \frac{4\pi^2}{G M_\odot} \frac{x_{\rm PSR}^3}{P_b^2}$$

Wished for quantities

Observable quantities

P_b : binary (orbital) period

$x_{\rm PSR} = a_{\rm PSR} \sin i$

$a_{\rm PSR}$: orbit semi-major axis

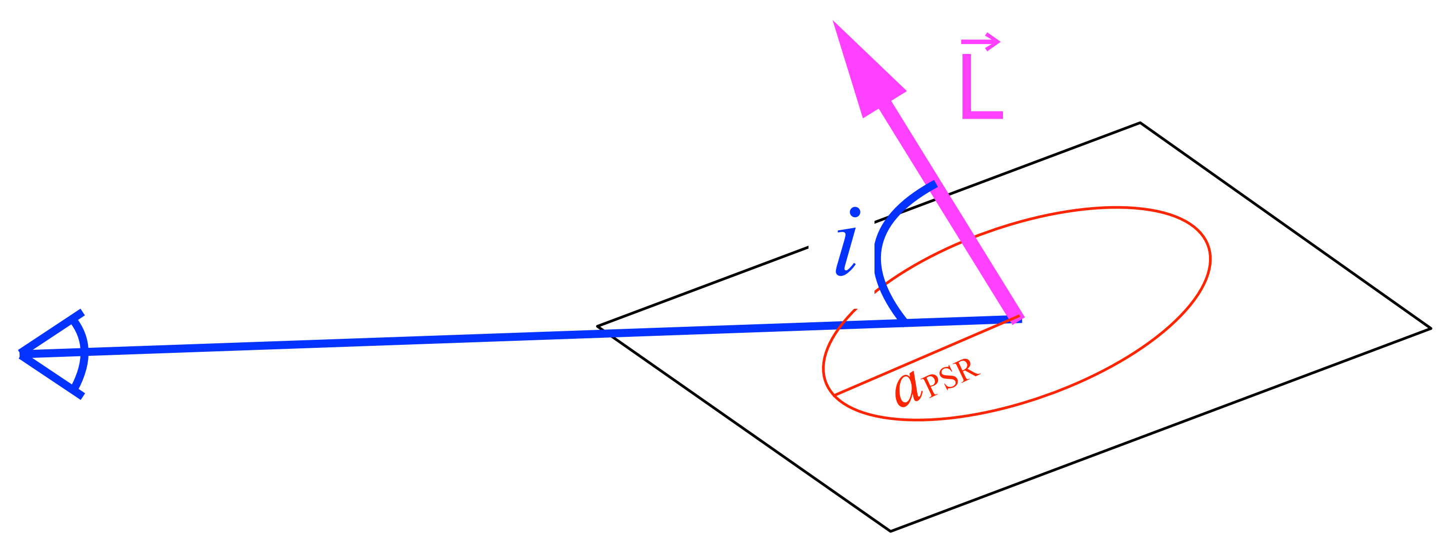
i: orbit inclination

$M_T = M_{\rm PSR} + M_c$

Pulsar timing give precise values for $x_{\rm PSR}$ and $P_{\rm b}$

Spectroscopy of the companion can give info about its type and $M_{\rm c}$

Eclipses can give info about i



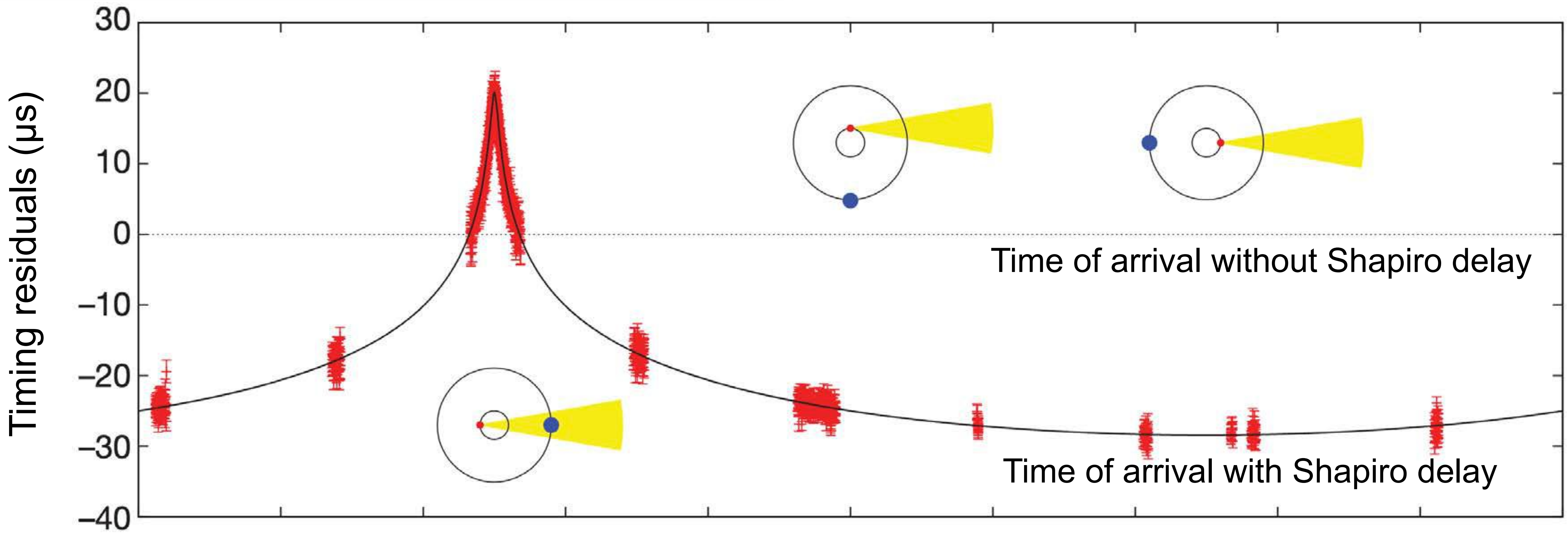
Mass Measurements with Shapiro Delay

General relativistic effects provide the complementary relation in the case of i ~ 90^o:

$$\Delta S = -2r \ln(1 - s\cos(\phi - \phi_0))$$

= delay in time arrival of pulses due to curvature of space-time by the pulsar companion (circular orbit case)

$$\begin{aligned} s &= \sin i && = \text{shape parameter} \\ r &= GM_c/c^2 && = \text{range parameter} \\ \phi & && = \text{orbital phase} \\ \phi_0 & && = \text{superior conjunction phase} \end{aligned}$$



Measuring ΔS gives r and s → i and M_c → M_{PSR} from the mass function !

A test of general relativity from the three-dimensional orbital geometry of a binary pulsar
van Straten, W.; Bailes, M.; Britton, M.; Kulkarni, S. R.; Anderson, S. B.; Manchester, R. N.; Sarkissian, J.
2001Natur.412..158V

A two-solar-mass neutron star measured using Shapiro delay
Demorest, P. B.; Pennucci, T.; Ransom, S. M.; Roberts, M. S. E.; Hessels, J. W. T.
2010Natur.467.1081D

Mass Measurements with Shapiro Delay

General relativistic effects provide the complementary relation in the case of i ~ 90º:

$$\Delta S = -2r \ln(1 - s\cos(\phi - \phi_0))$$

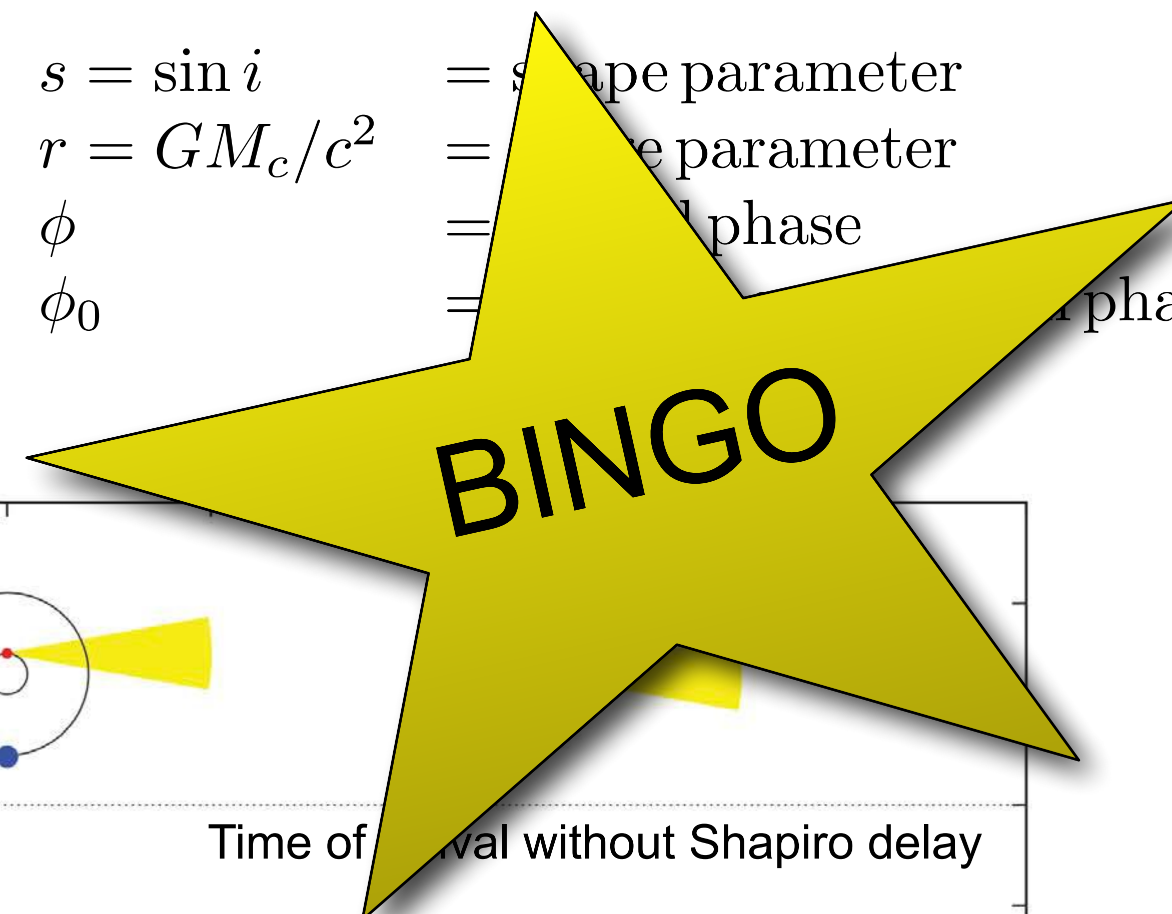
= delay in time arrival of pulses due to curvature of space-time by the pulsar companion (circular orbit case)

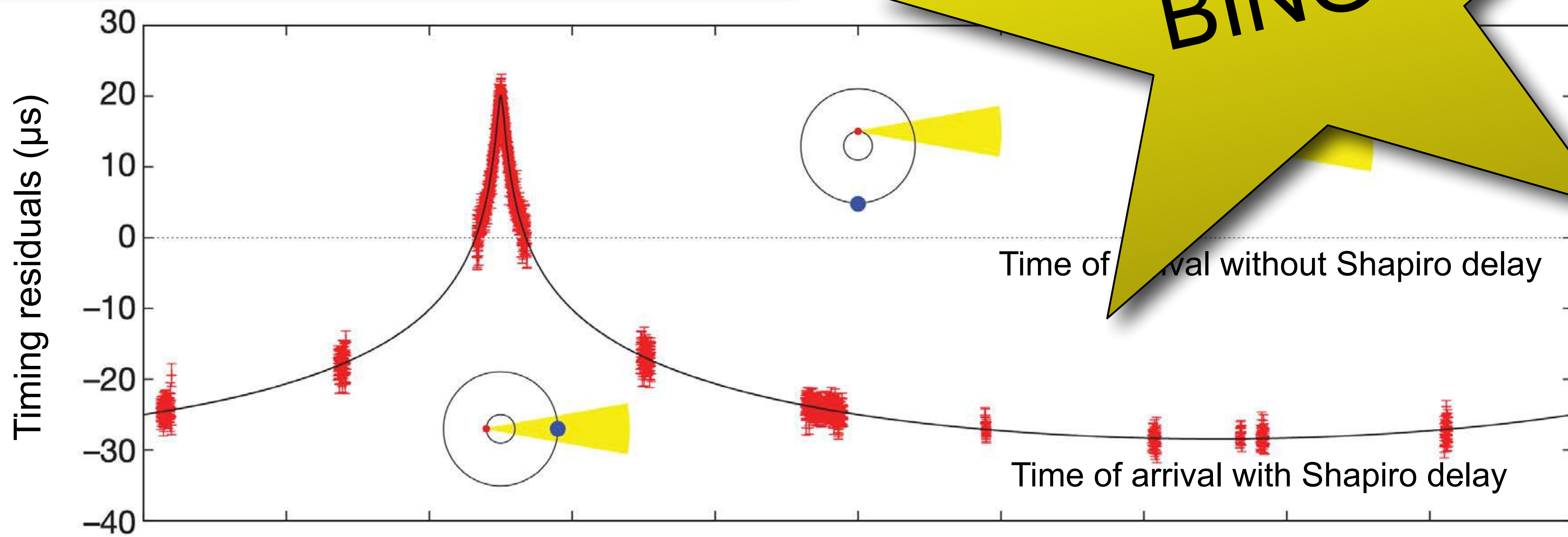
$s = \sin i$ = s ape parameter

$r = GM_c/c^2$ = e parameter

ϕ = l phase

ϕ_0 = phase





Measuring ΔS gives r and s → i and $M_c \rightarrow M_{PSR}$ from the mass function !

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van Straten, W.; Bailes, M.; Britton, M.; Kulkarni, S. R.; Anderson, S. B.; Manchester, R. N.; Sarkissian, J.
2001Natur.412..158V

A two-solar-mass neutron star measured using Shapiro delay
Demorest, P. B.; Pennucci, T.; Ransom, S. M.; Roberts, M. S. E.; Hessels, J. W. T.
2010Natur.467.1081D

2 M$_\odot$ Pulsars from Shapiro Delay

PSR J1614-2230: M = 1.97∓0.04 M$_\odot$

A two-solar-mass neutron star measured using Shapiro delay
Demorest, P. B.; Pennucci, T.; Ransom, S. M.; Roberts, M. S. E.; Hessels, J. W. T.
2010Natur.467.1081D

PSR J0348+0432: M = 2.01∓0.04 M$_\odot$

A Massive Pulsar in a Compact Relativistic Binary
Antoniadis, John; Freire, Paulo C. C.; Wex, Norbert; and 19 coauthors
2013Sci...340..448A

PSR J0740+6620: M = 2.08∓0.07 M$_\odot$

A very massive neutron star: relativistic Shapiro delay measurements of PSR J0740+6620
Thankful Cromartie, H.; Fonseca, Emmanuel; Ransom, Scott M.; Demorest, Paul B.; and 23 coauthors
2020NatAs...4...72C

Refined Mass and Geometric Measurements of the High-mass PSR J0740+6620
Fonseca, E.; Cromartie, H. T.; Pennucci, T. T. and 42 more
2021ApJ...915L..12F

Neutron Star Masses & Radii

M & R from Thermal Emission

$$L^\infty = 4\pi R_\infty^2 \sigma_{SB} {T_{\rm eff}^\infty}^4$$

From observed flux F
and
estimated distance D
$L = F\,D^2$

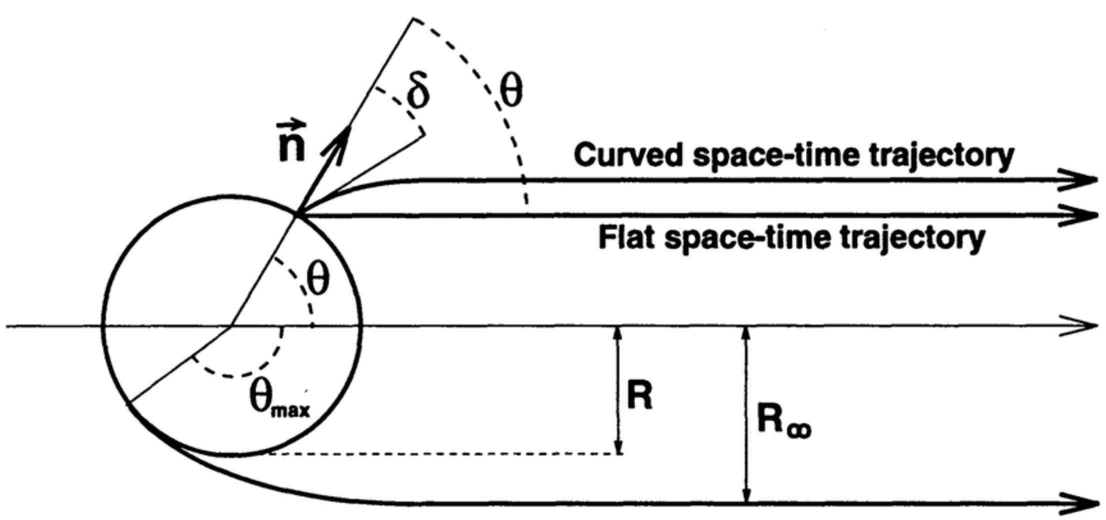
From spectral fit
that give T and
atmosphere that
depends on
surface gravity

$$g_s = \frac{GM}{R^2}\left(1 - 2GM/Rc^2\right)^{-1/2}$$

$$L^\infty = \mathrm{e}^{2\phi_s} L = \left(1 - 2GM/Rc^2\right) L$$

$$T_{\rm eff}^\infty = \mathrm{e}^{\phi_s} L = \left(1 - 2GM/Rc^2\right)^{1/2} T_{\rm eff}$$

$$R_\infty = \mathrm{e}^{\phi_s} L = \left(1 - 2GM/Rc^2\right)^{-1/2} R$$



M & R from Thermal Emission

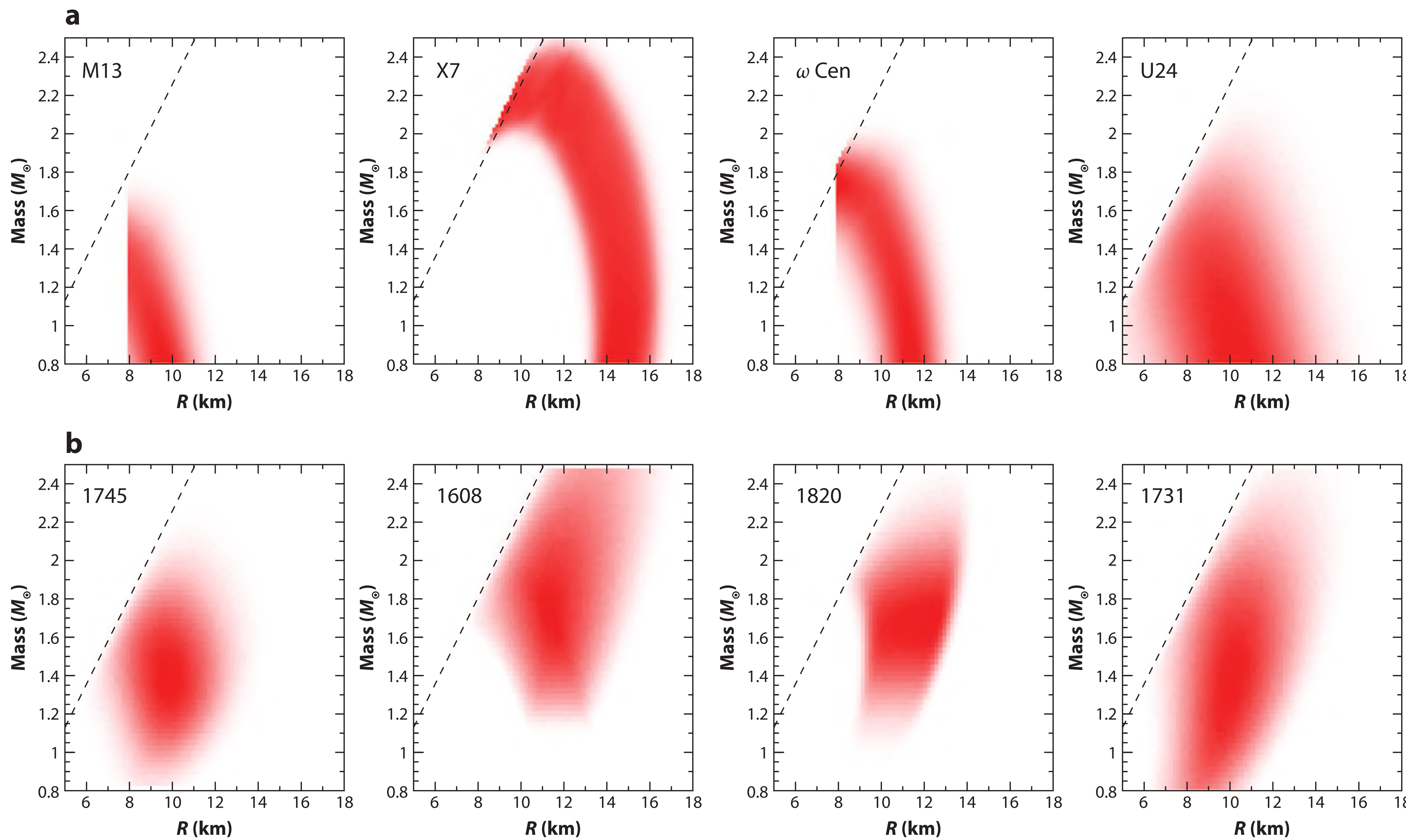


Figure 10

M–R probability densities of neutron stars from (*a*) four quiescent low-mass X-ray binaries in globular clusters and (*b*) four photospheric radius expansion bursts (incorporating the possibility that $R_{ph} > R$). The diagonal lines represent causality limits. Figure reproduced courtesy of A.W. Steiner.

[Masses, Radii, and Equation of State of Neutron Stars]
Ozel, Feryal; Freire, Paulo
2016ARA&A..54..401O

[The Nuclear Equation of State and Neutron Star Masses]
Lattimer, James M.
2012ARNPS..62..485L

M & R from Light-Curve Fitting (NICER)

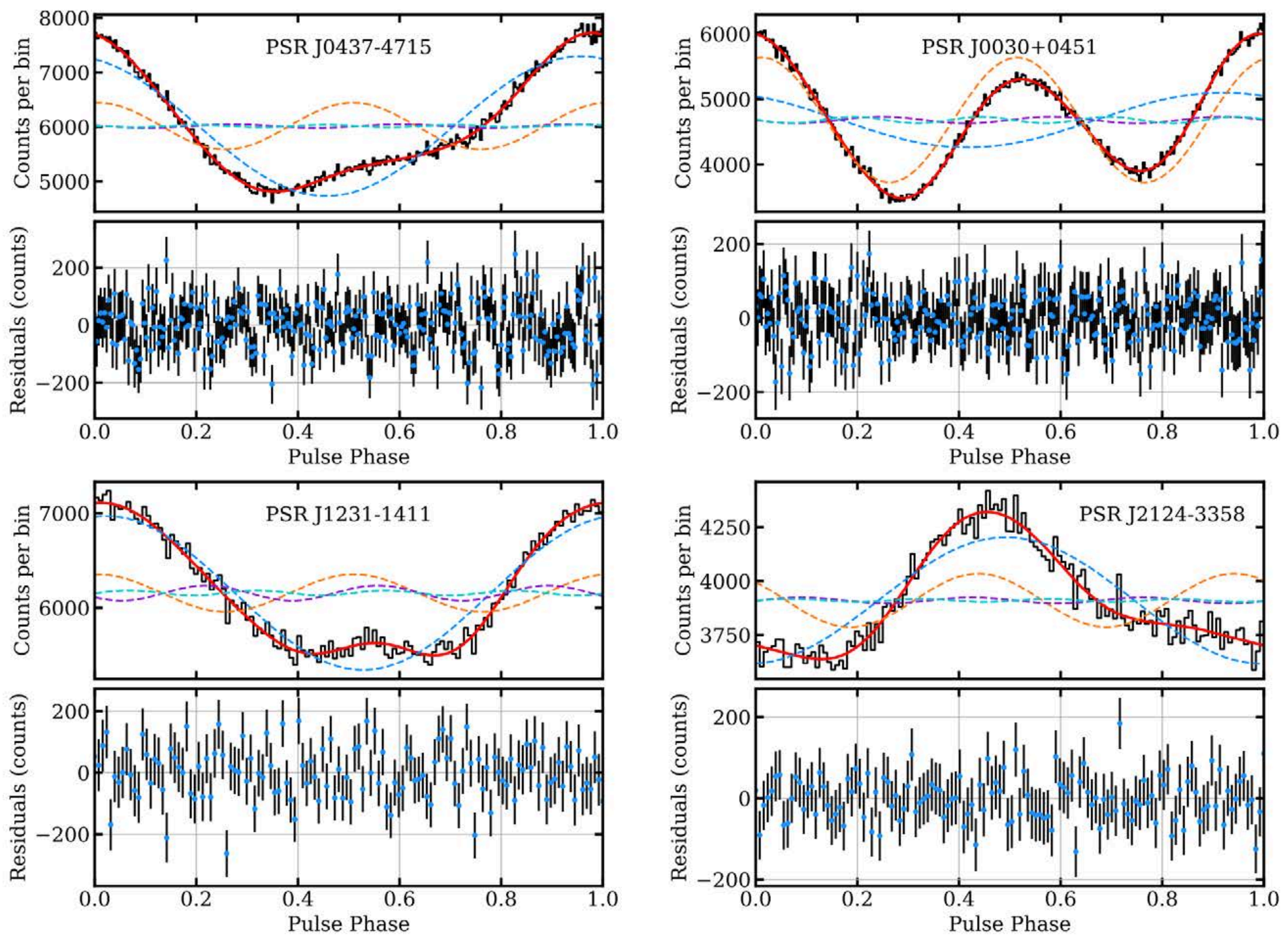


Figure 5. Folded *NICER* XTI profiles from Figure 2 but grouped in 256 (for PSRs J0437−4715 and J0030+0451) or 128 phase bins (for PSRs J1231−1411 and J2124−3358). The solid red lines show the best fit with a model of the profile constructed from the empirical Fourier coefficients, given by Equation (1), of the set of photon phases. The dashed lines show the sinusoids corresponding to the first four harmonic components of the fit (blue, orange, purple, and cyan for the first, second, third, and fourth harmonic, respectively). The bottom panel for each pulsar shows the residuals from the fit.

Constraining the Neutron Star Mass—Radius Relation and Dense Matter Equation of State with NICER. II. Emission from Hot Spots on a Rapidly Rotating Neutron Star
Bogdanov, Slavko; Lamb, Frederick K.; Mahmoodifar, Simin; Miller, M. Coleman; Morsink, Sharon M.; Riley, Thomas E.; Strohmayer, Tod E.; Tung, Albert K.; Watts, Anna L.; Dittmann, Alexander J.; Chakrabarty, Deepto; Guillot, Sebastien; Arzoumanian, Zaven; Gendreau, Keith C.
2019ApJ...887L..26B

Constraints from Gravitational Waves in Binary Merger(s)

The First BNS Merger: GW170817

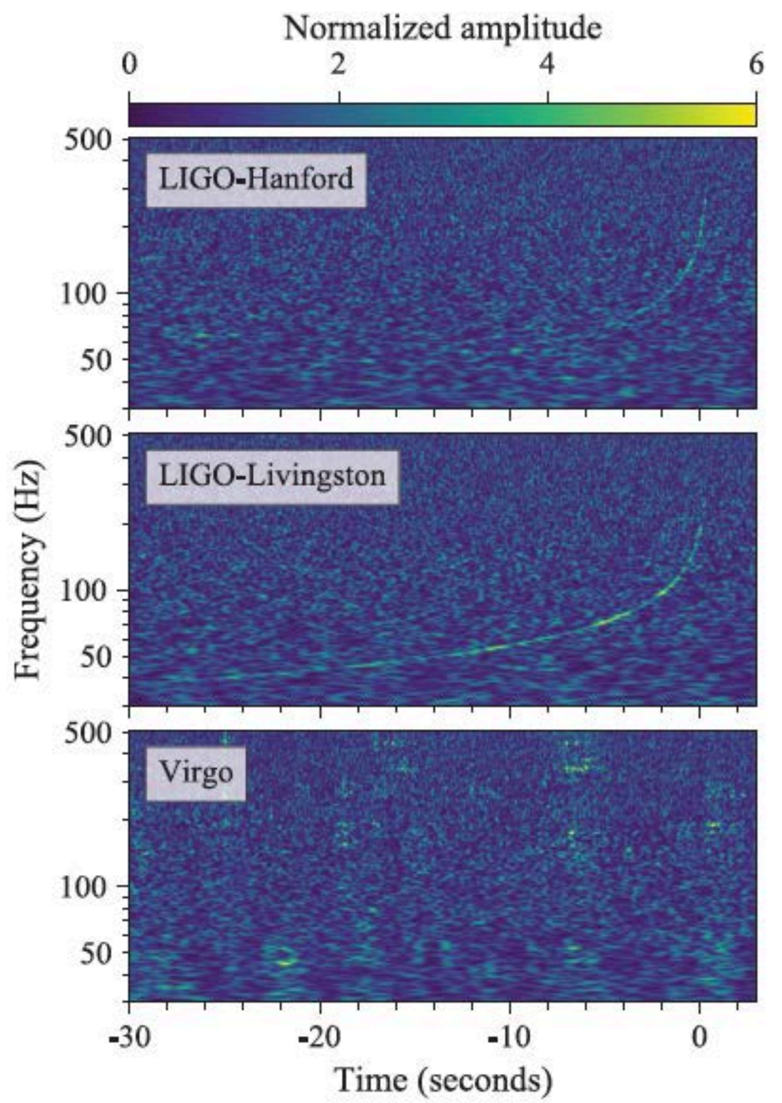


FIG. 1. Time-frequency representations [65] of data containing the gravitational-wave event GW170817, observed by the LIGO-Hanford (top), LIGO-Livingston (middle), and Virgo (bottom) detectors. Times are shown relative to August 17, 2017 12:41:04 UTC. The amplitude scale in each detector is normalized to that detector's noise amplitude spectral density. In the LIGO data, independently observable noise sources and a glitch that occurred in the LIGO-Livingston detector have been subtracted, as described in the text. This noise mitigation is the same as that used for the results presented in Sec. IV.

[GW170817: Observation of Gravitational Waves from a Binary Neutron Star Inspiral]
Abbott, B. P.; Abbott, R.; Abbott, T. D.; Acernese, F.; Ackley, K.; Adams, C.; Adams, T.; Addesso, P.; Adhikari, R. X.; Adya, V. B.; and 1116 coauthors
2017PhRvL.119p1101A

Gravitational Waves Emission: Point Masses in a Circular Orbit

Change of orbital energy E for a circular binary system

$$\frac{\mathrm{d}E}{\mathrm{d}t} = -\frac{32G}{5c^5}\mu^2 r^4 \omega^6$$

$$E \equiv \frac{1}{2}\mu r^2 \omega^2 - \frac{GM\mu}{r} = -\frac{GM\mu}{2r}$$

$$\omega = 2\pi f$$

Change of orbital frequency f for a circular binary system

$$\frac{\mathrm{d}f}{\mathrm{d}t} = \frac{96\pi}{5}\left(\frac{G\mathcal{M}\pi f}{c^3}\right)^{5/3} f^2 = \frac{96\pi}{5} x^{5/2} f^2$$

Chirp mass: $$\mathcal{M} \equiv \frac{(m_1 m_2)^{3/5}}{(m_1+m_2)^{1/5}} = \mu^{3/5} M^{2/5} = (\mu M^{2/3})^{3/5}$$

Post-Newtonian parameter: $$x \equiv \left(\frac{G\mathcal{M}\pi f}{c^3}\right)^{2/3}$$

$$M = m_1 + m_2$$

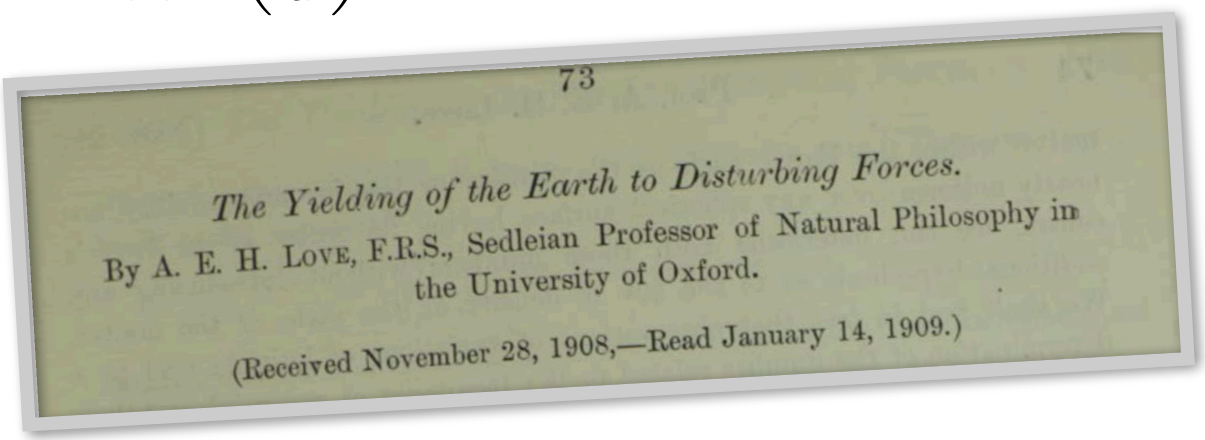
$$\mu = \frac{m_1 m_2}{(m_1+m_2)}$$

$$\eta = \frac{m_1 m_2}{(m_1+m_2)^2} = \frac{\mu}{M}$$

Tidal Deformation: Love Number

Love number k_2 determines deformation of surface.
Larger k_2 deforms more easily (e.g., ocean vs rock)

$$H = k_2 \left(\frac{m}{M}\right)\left(\frac{R}{a}\right)^3 R\, P_2(\cos\theta) \qquad P_2(x) = \frac{1}{2}(3x^2-1)$$



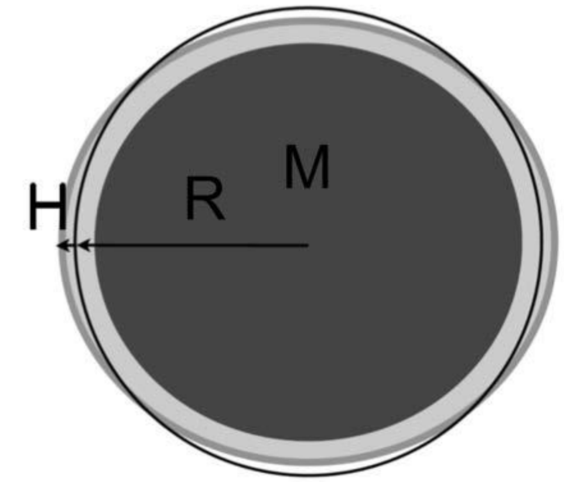
73

The Yielding of the Earth to Disturbing Forces.

By A. E. H. Love, F.R.S., Sedleian Professor of Natural Philosophy in the University of Oxford.

(Received November 28, 1908,—Read January 14, 1909.)

H = height of tide
R = Earth radius
M = Earth mass
m = Moon mass
a = Moon-Earth distance



The Yielding of the Earth to Disturbing Forces
Love, A. E. H.
1909RSPSA..82...73L

More generally: field ϕ of star deformed by a tidal field $\mathcal{E}_{ij}$:

$$\phi = -\frac{Gm}{r} - \frac{3GQ_{ij}}{2r^3}\left(n^i n^j - \frac{\delta^{ij}}{3}\right) + \frac{\mathcal{E}_{ij}}{2} x^i x^j + \ldots$$

$$Q_{ij} = \int \mathrm{d}^3x\, \delta\rho\,(x_i x_j - r^2\delta_{ij}/3)$$

$$n^i = x^i/r$$

Linear response: $Q_{ij} = -\lambda \mathcal{E}_{ij}$ $\quad \lambda = k_2 \frac{2}{3}\frac{R^5}{G}$ $\quad \Lambda = \frac{2}{3} k_2 \left(\frac{c^2 R}{GM}\right)^5 = \frac{2}{3}\frac{k_2}{\beta^5}$

k_2 and λ: Love numbers
Λ: tidal deformability or polarizability

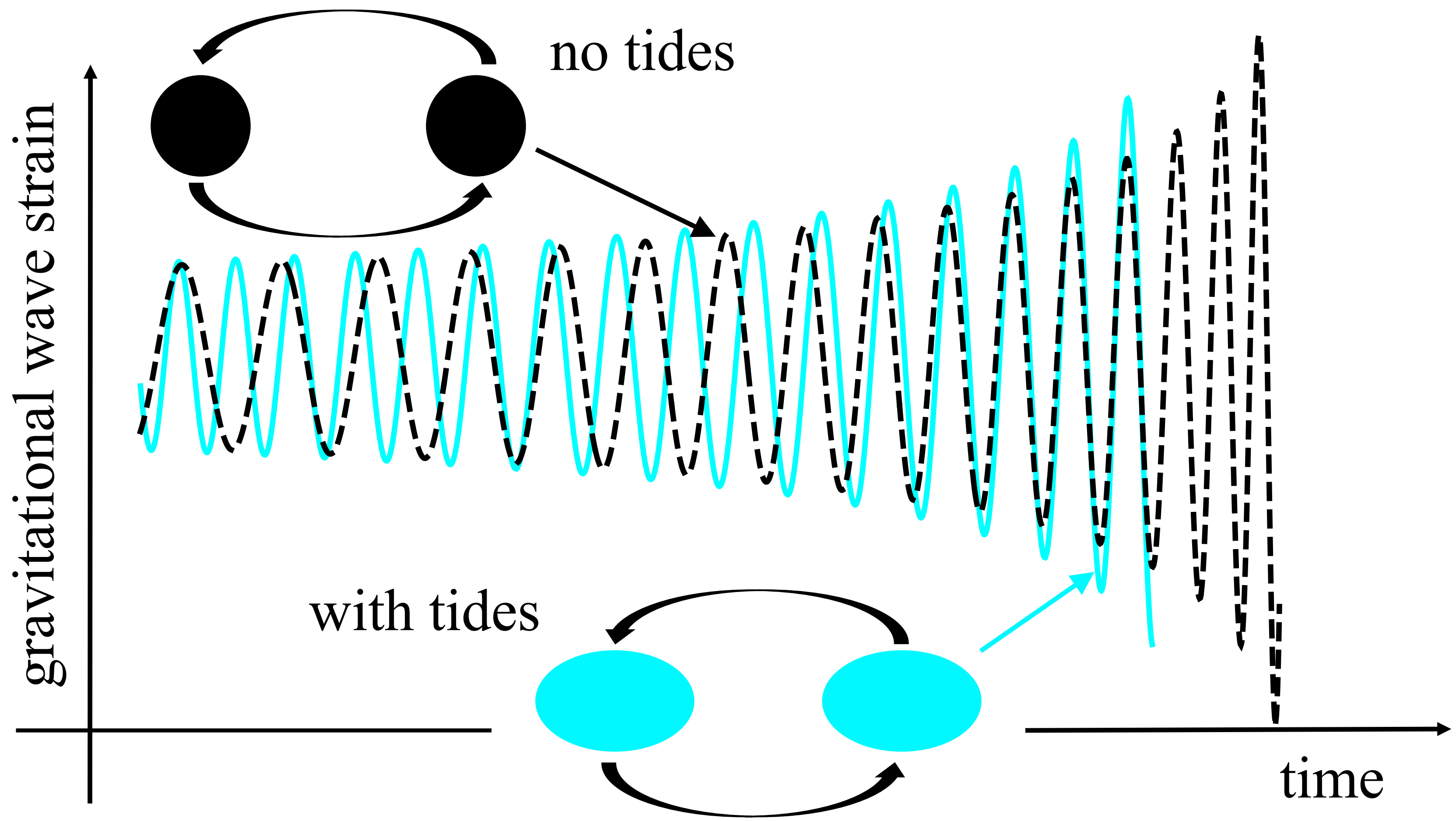
Constraining neutron star tidal Love numbers with gravitational-wave detectors
Flanagan, Éanna É.; Hinderer, Tanja
2008PhRvD..77b1502F

$[G\lambda]$ = Length^5, k_2 and Λ are dimensionless

Merger of Point Mass versus Stars

Change in the phase of the wave at frequency f:

$$\delta\Psi(f) = -\frac{9}{13}\frac{v^5}{\mu M^4}\tilde{\lambda}$$



$$\tilde{\lambda} = \frac{16}{13}[(m_1 + 12m_2)\lambda_1 + (m_2 + 12m_1)\lambda_2]$$

The Neutron Star Equation of State I. The Crust

Neutron Star Envelope

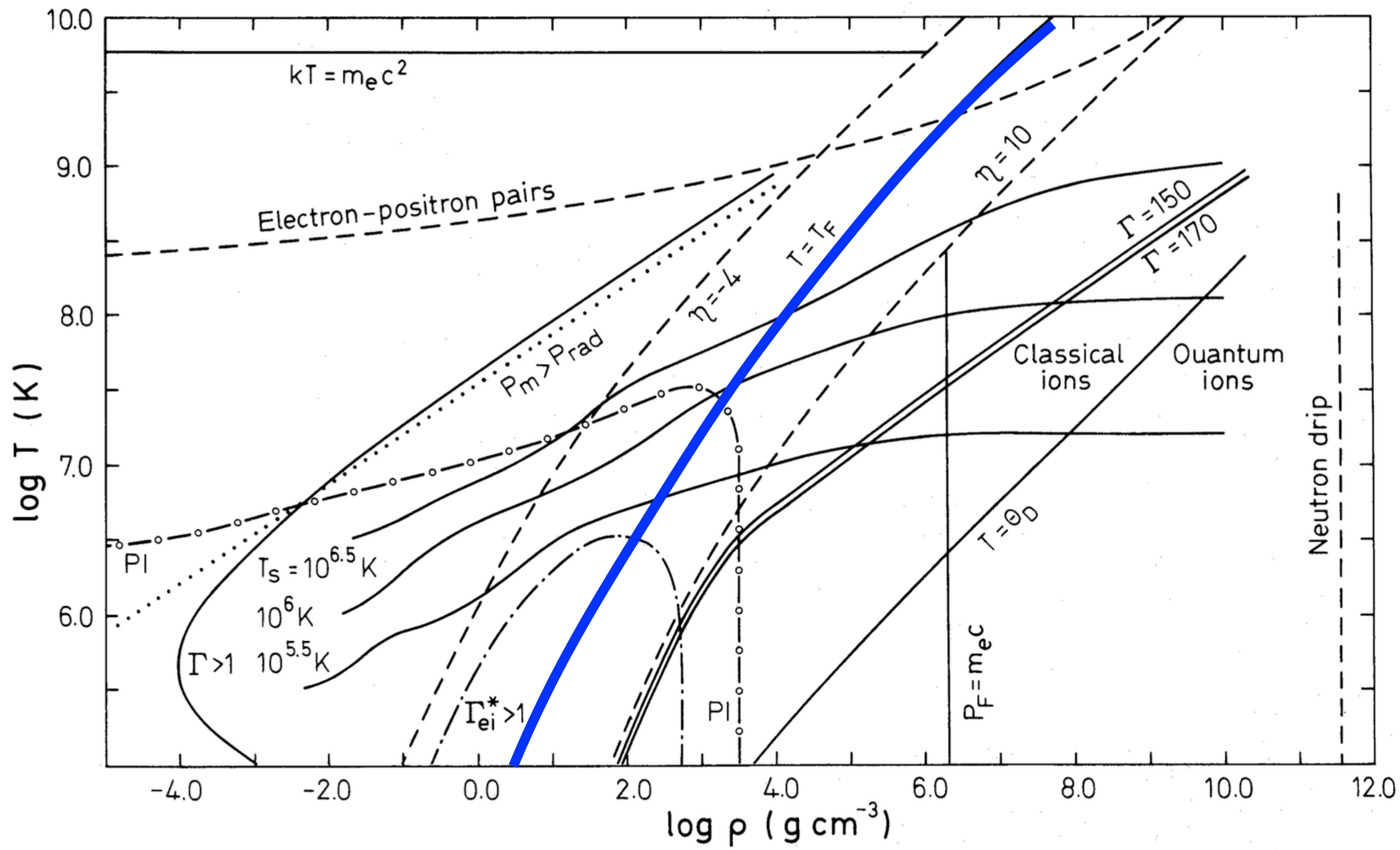


FIG. 1.—Physical conditions at densities and temperatures of interest in the study of neutron star envelopes. The various regions are identified in the text. Also shown are temperature-density profiles for envelopes for three values of the surface temperature and a surface gravity of 10^{14} cm s^{-2}.

Structure of neutron star envelopes
Gudmundsson, E. H.; Pethick, C. J.; Epstein, R. I.
1983ApJ...272..286G

Neutron Star Envelope

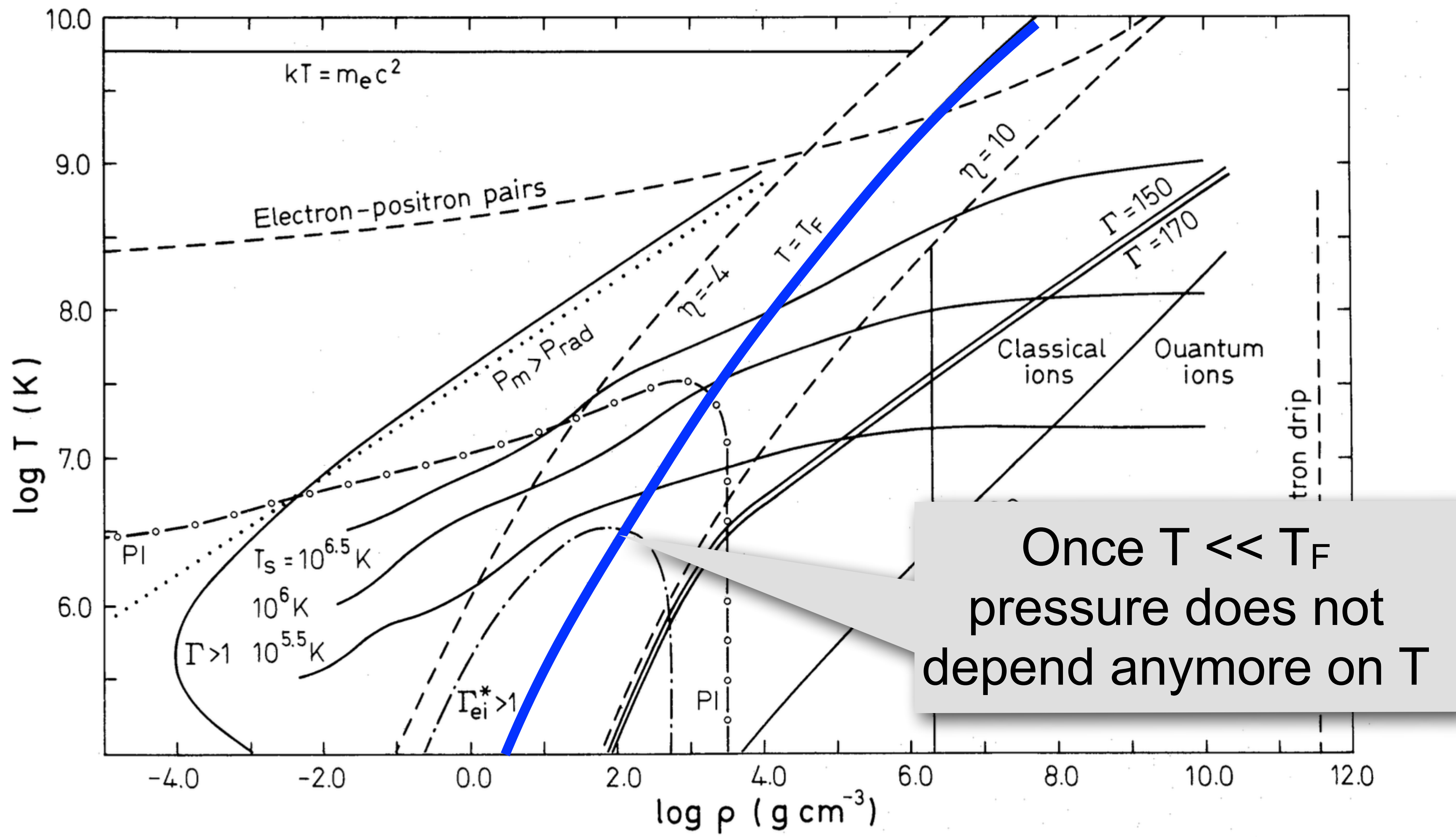


FIG. 1.—Physical conditions at densities and temperatures of interest in the study of neutron star envelopes. The various regions are identified in the text. Also shown are temperature-density profiles for envelopes for three values of the surface temperature and a surface gravity of 10^{14} cm s^{-2}.

Structure of neutron star envelopes
Gudmundsson, E. H.; Pethick, C. J.; Epstein, R. I.
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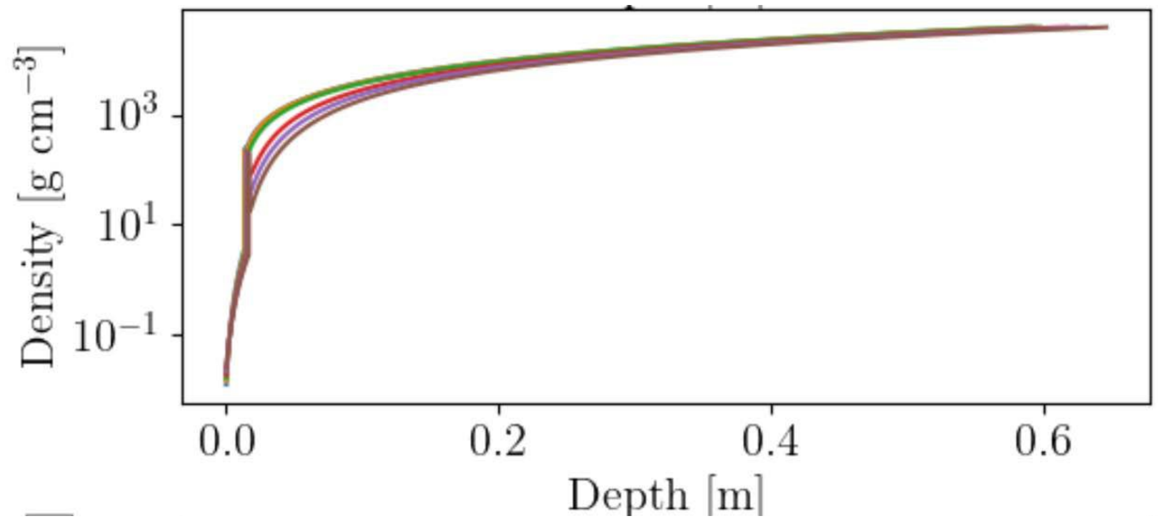
Neutron Star Envelope



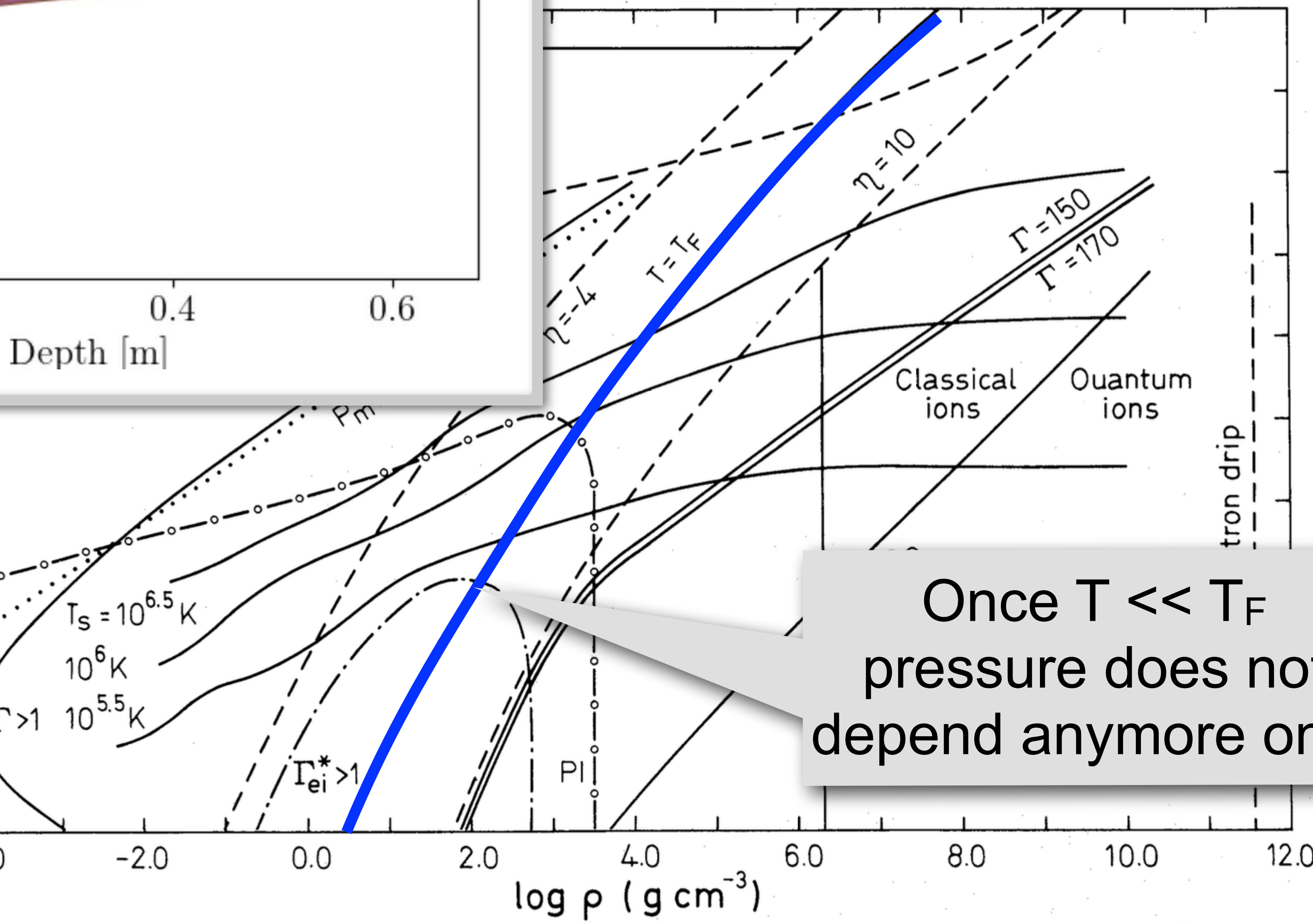
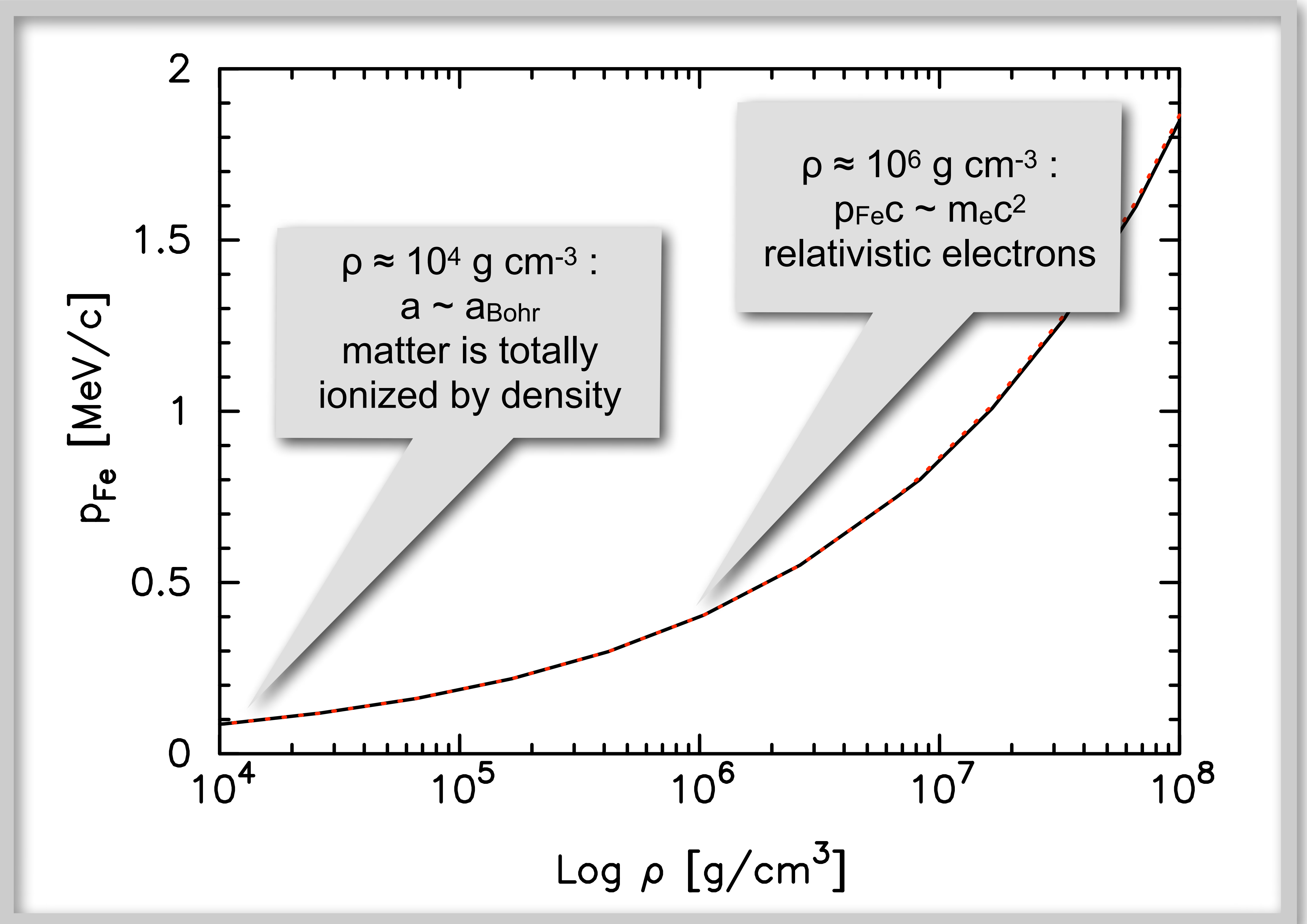


FIG. 1.—Physical conditions at densities and temperatures of interest in the study of neutron star envelopes. The various regions are identified in the text. Also shown are temperature-density profiles for envelopes for three values of the surface temperature and a surface gravity of 10^{14} cm s^{-2}.

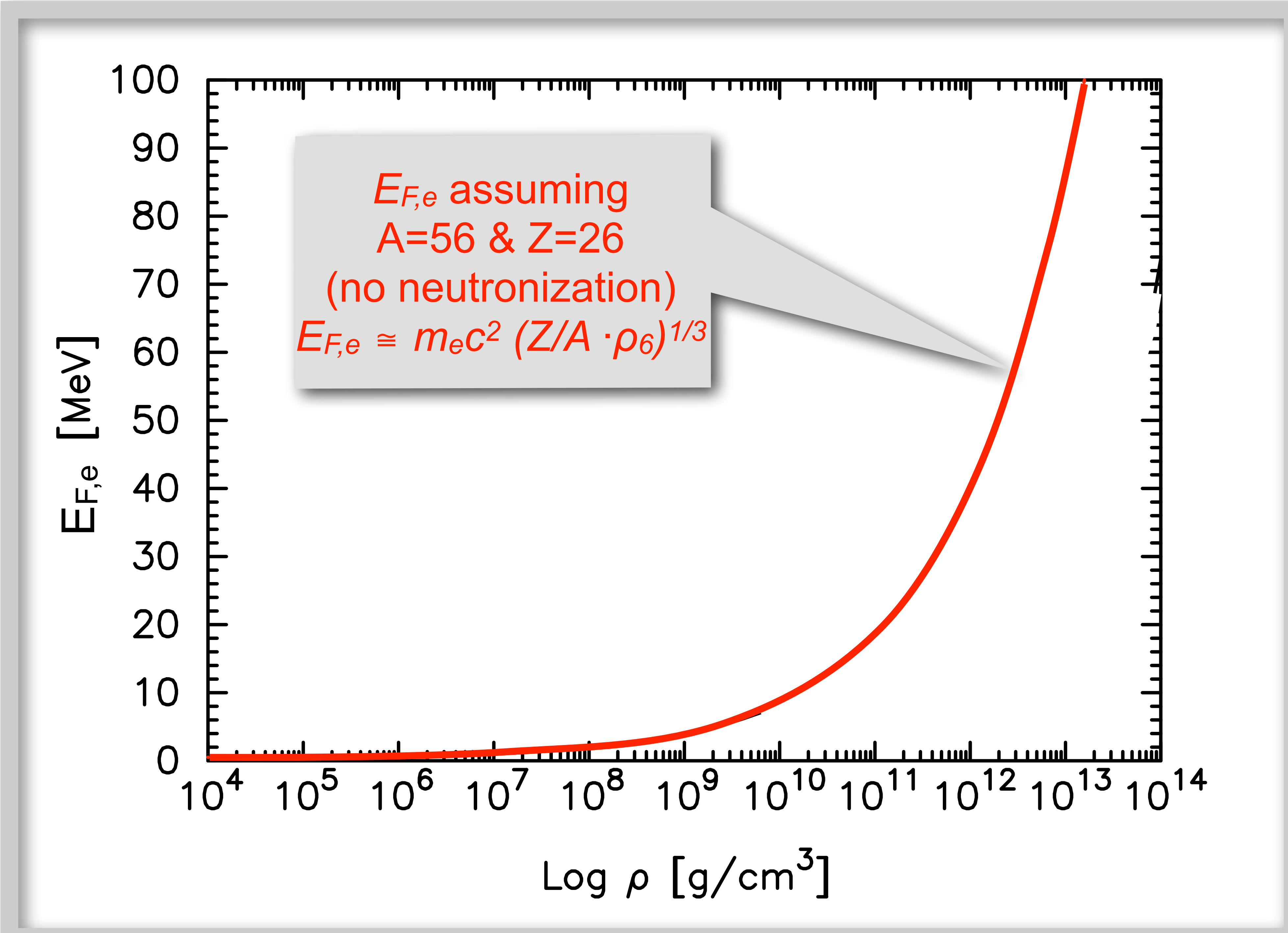
Once $T \ll T_F$ pressure does not depend anymore on T

[Structure of neutron star envelopes](#)
Gudmundsson, E. H.; Pethick, C. J.; Epstein, R. I.
[1983ApJ...272..286G](#)

Growth of $p_F(e)$ in the Crust



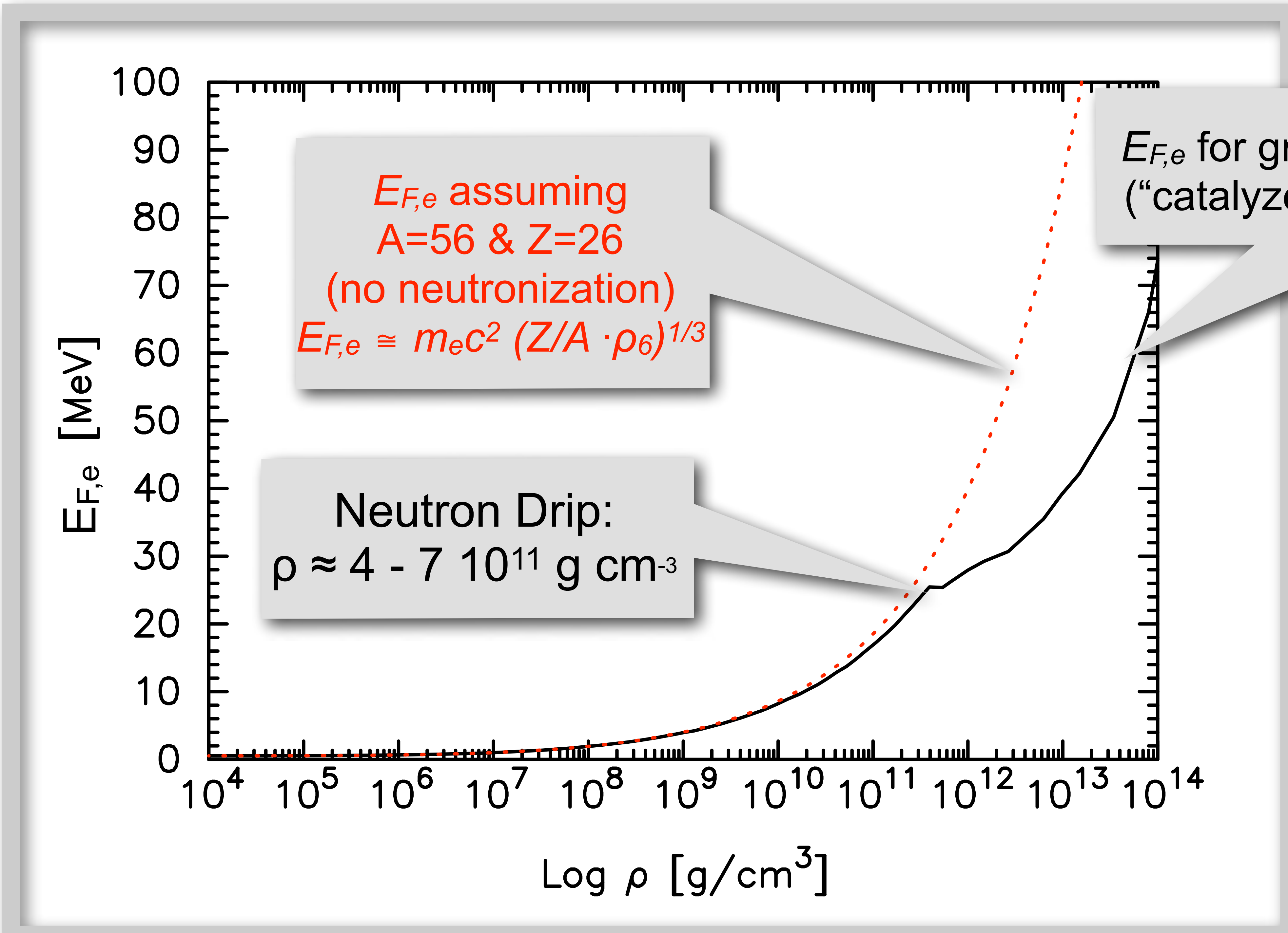
Neutronization of Matter



Dense matters tries to reduce energy density by absorbing electrons into protons:

$$e^- + (N,Z) \rightarrow \nu_e + (N+1,Z-1)$$

Neutronization of Matter



Dense matters tries to reduce energy density by absorbing electrons into protons:

$$e^- + (N,Z) \rightarrow \nu_e + (N+1,Z-1)$$

Cold Catalyzed Matter below Neutron Drip

Cold catalyzed matter is found by minimizing the Gibbs energy at a given pressure:

$$G_{\rm cell}(A,Z,n_{\rm Nuc}) = W_{\rm Nuc}(A,Z) + W_{\rm Lat}(Z,n_{\rm Nuc}) + [\varepsilon_{\rm e}(n_{\rm e}) + P]/n_{\rm Nuc}$$

$W_{\rm Nuc}(A,Z) =$ energy of nucleus (A,Z)

$W_{\rm Lat}(Z,n_{\rm Nuc}) =$ lattice energy por cell

$\varepsilon_{\rm e}(n_{\rm e}) =$ electron energy density

$P =$ total pressure

$n_{\rm Nuc} =$ nuclide density

$$P = P_{\rm e}(n_{\rm e}) + P_{\rm Lat}(Z,n_{\rm Nuc})$$

$$W_{\rm Lat}(Z,n_{\rm Nuc}) = 3P_{\rm Lat}(Z,n_{\rm Nuc}) = -\frac{9}{10}\frac{Z^2e^2}{a}$$

$$v_{\rm Nuc} = \frac{1}{n_{\rm Nuc}} = \frac{3}{4}\pi a^3$$

$$P_{\rm e}(n_{\rm e}) = \frac{1}{3}\varepsilon_{\rm e}(n_{\rm e}) \quad \text{relativistic Fermi-Dirac gas}$$

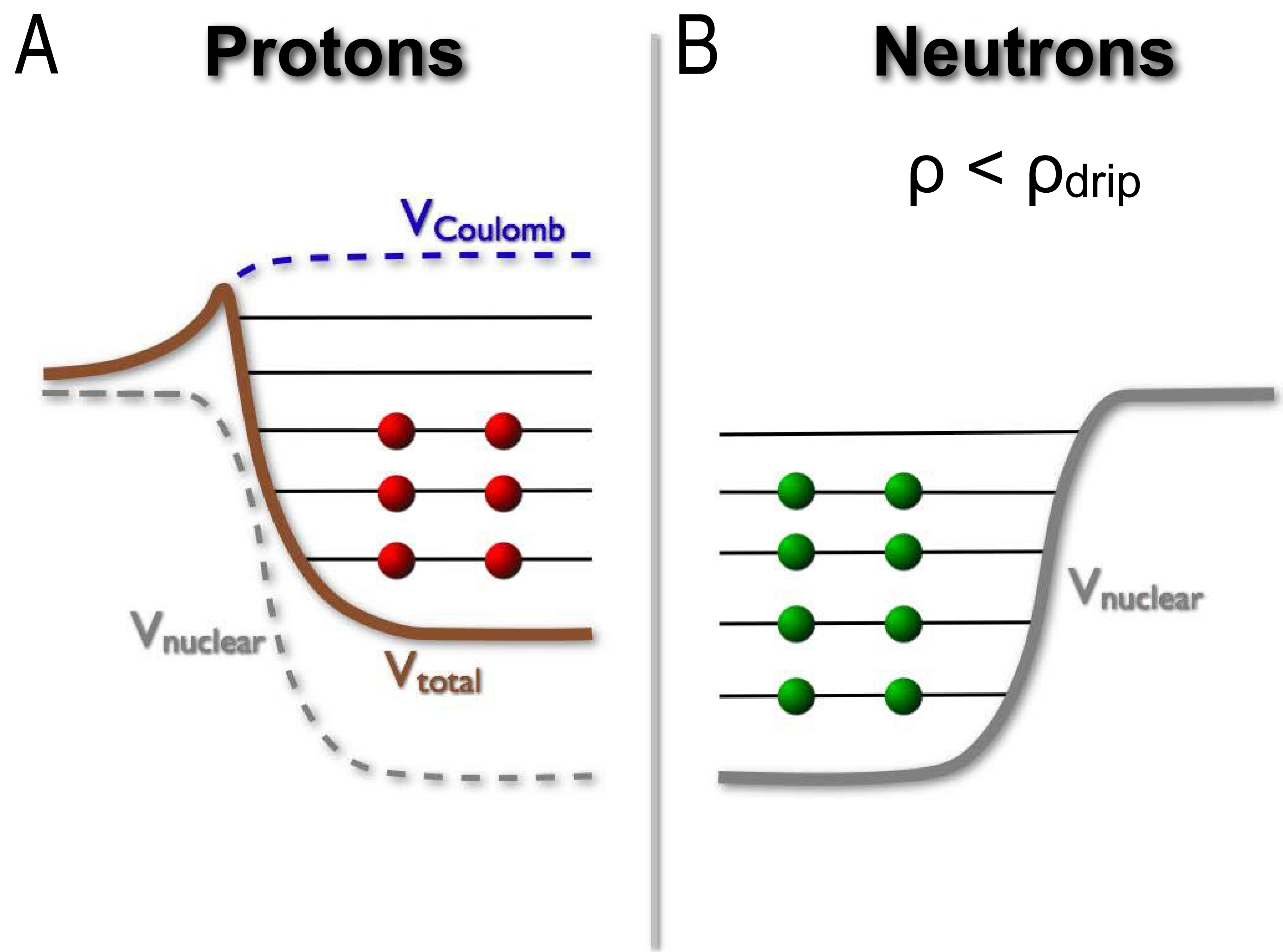
Table 1. Equilibrium nuclei below neutron drip. HFB calculation

Z	N	Z/A	ρ_{max} (g cm^{-3})	μ_e (MeV)	$\Delta\rho/\rho$ (%)
26	30	0.4643	$7.96\ 10^6$	0.95	2.9
28	34	0.4516	$2.70\ 10^8$	2.61	3.1
28	36	0.4375	$1.18\ 10^9$	4.17	6.2
28	40	0.4118	$5.88\ 10^9$	6.94	2.2
34	50	0.4048	$1.36\ 10^{10}$	9.12	0.7
28	42	0.4000	$1.40\ 10^{10}$	9.16	2.9
28	44	0.3889	$1.93\ 10^{10}$	10.06	1.3
30	48	0.3846	$2.43\ 10^{10}$	10.86	2.6
30	50	0.3750	$4.52\ 10^{10}$	13.24	1.6
28	48	0.3684	$4.97\ 10^{10}$	13.58	2.6
28	50	0.3590	$9.17\ 10^{10}$	16.51	2.6
28	52	0.3500	$1.23\ 10^{11}$	18.04	2.5
28	54	0.3415	$1.56\ 10^{11}$	19.38	2.4
28	56	0.3333	$2.07\ 10^{11}$	21.12	2.4
28	58	0.3256	$2.70\ 10^{11}$	22.89	2.3
28	60	0.3182	$3.33\ 10^{11}$	24.36	4.3
40	90	0.3077	$3.77\ 10^{11}$	25.10	1.5
40	92	0.3030	$4.245\ 10^{11}$	25.97	1.5
40	94	0.2985	$(4.45\ 10^{11})$	(26.26)	

Notice the decrease of Z/A as density grows

Composition and equation of state of cold catalyzed matter below neutron drip
Haensel, P.; Zdunik, J. L.; Dobaczewski, J. 1989A&A...222..353H

Proton/Neutron Nuclear Potential



Neutron & Proton Densities above Drip Density

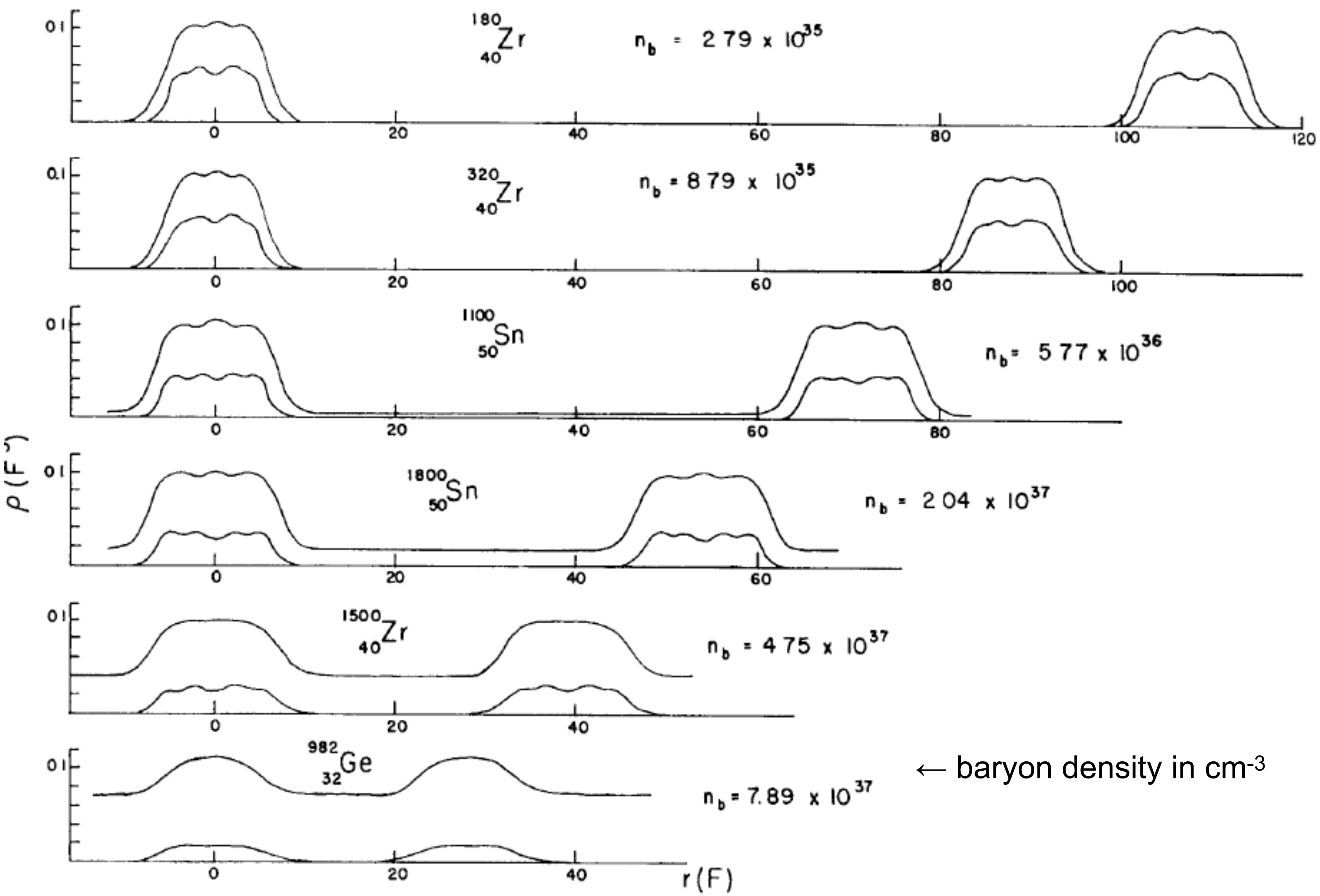
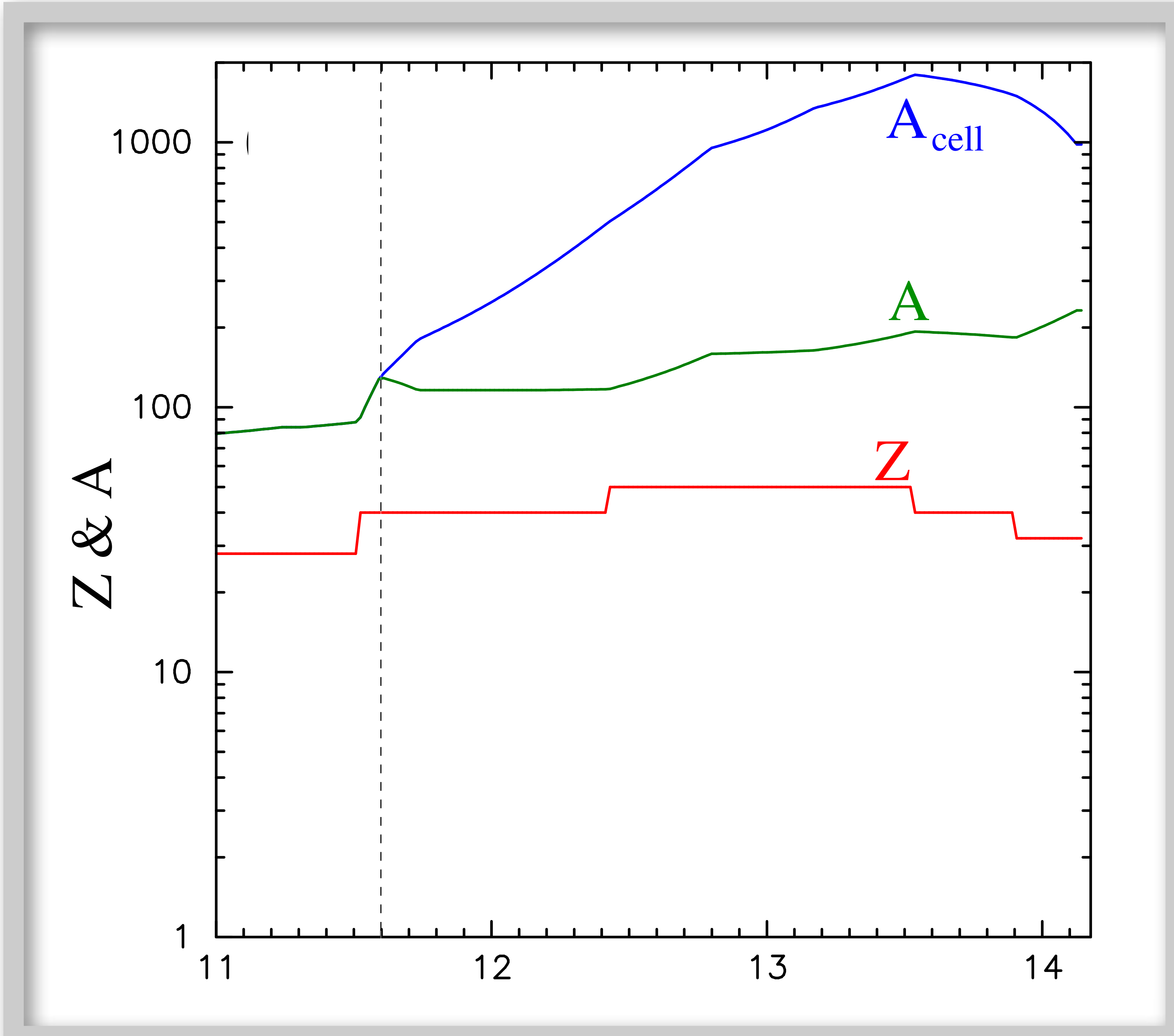


Fig. 3. Proton and neutron density distributions occurring along an axis joining the centers of two adjacent unit cells.

$$n_{\rm sat} \simeq 0.16\,{\rm fm}^{-3} = 1.6 \times 10^{38}\,{\rm cm}^{-3}$$

Neutron star matter at sub-nuclear densities
Negele, J. W.; Vautherin, D.
1973NuPhA.207..298N

Crust Chemical Composition



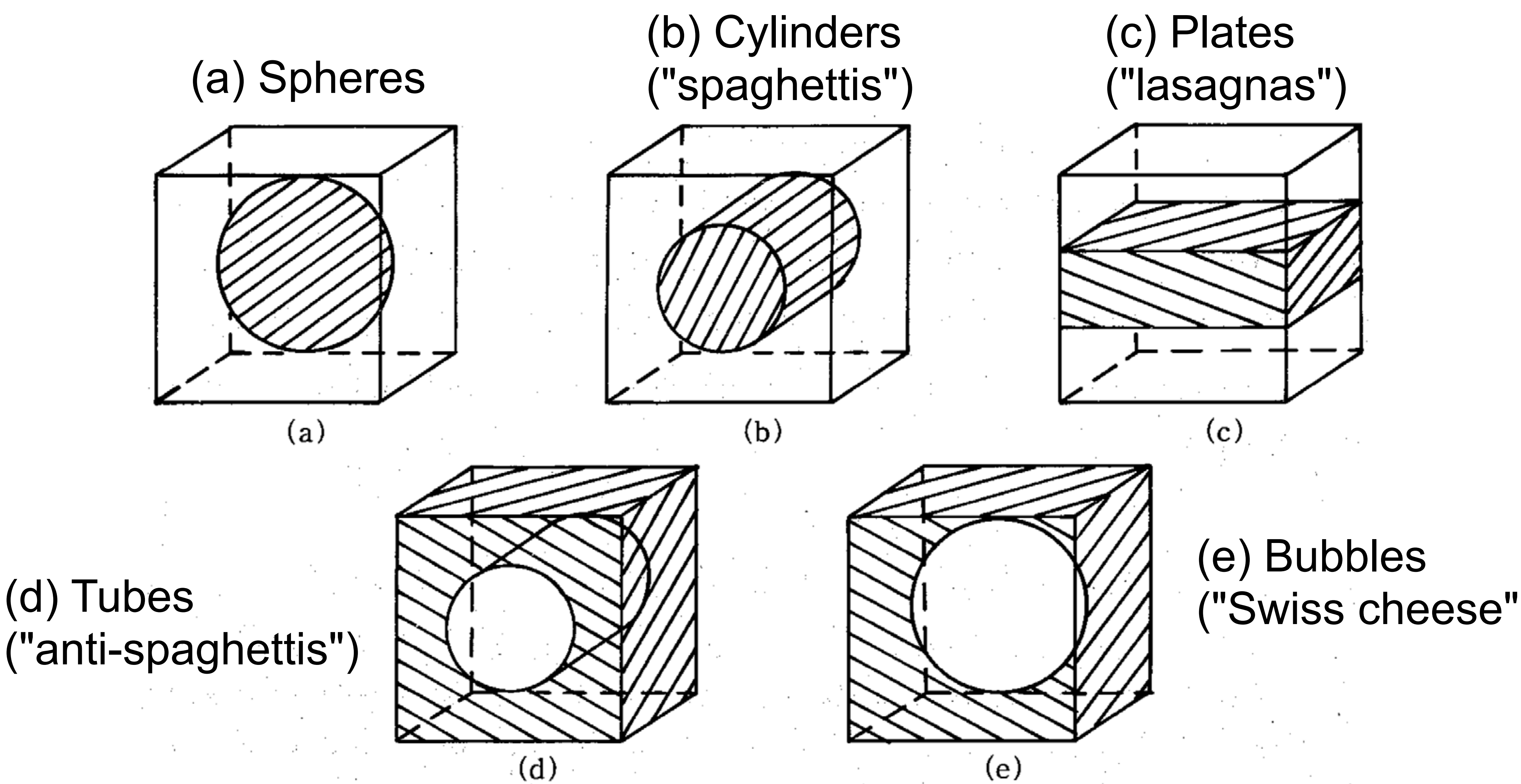
The ground state of matter at high densities: equation of state and stellar models
Baym, Gordon; Pethick, Christopher; Sutherland, Peter
1971ApJ...170..299B

Neutron star matter
Baym, G.; Bethe, H. A.; Pethick, C. J.
1971NuPhA.175..225B

Neutron star matter at sub-nuclear densities
Negele, J. W.; Vautherin, D.
1973NuPhA.207..298N

The Neutron Star Equation of State II. Pasta Phase and Crust-Core Transition

Pasta Phase



[Structure of matter below nuclear saturation density]()
DG Ravenhall, CJ Pethick, JR Wilson
[1983PhRvL..50.2066R]()

[Shape of nuclei in the crust of a neutron star]()
Hashimoto, M.; Seki, H.; Yamada, M
[1984PThPh..71..320H]()

Simple Liquid Drop Model Approach (1)

How to determine which shape of nuclear clusters appear and at which densities?

Energy, W, of a cell (= cube of size a) as sum of nuclear bulk, W_B, nuclear surface, W_S, and Coulomb, W_C, energies:

$$W = W_B + W_S + W_C + W_e = w_{B,in}(n_{\rm in}, x_p) u a^3 + w_{B,out}(n_{\rm out})(1-u)a^3 + w_e a^3$$
$$\sigma(n_{\rm in}, n_{\rm out}, x_p) g(u,s) a^2 + w_C(u,s)\frac{Z^2}{a}$$

$w_{B,in}(n_{\rm in}, x_p) =$ bulk nuclear energy density inside cluster

$w_{B,out}(n_{\rm out}) =$ bulk nuclear energy density outside (dripped neutrons) cluster

$\sigma(n_{\rm in}, n_{\rm out}, x_p) =$ nuclear surface energy density

$g(u,s) =$ area of nuclear cluster

$w_C(u,s) =$ relative Coulomb energy

$w_e(n_e) =$ electron energy density

$n_{\rm in} =$ nucleon (n and p) density inside cluster

$n_{\rm out} =$ nucleon (dripped neutrons) density outside cluster

$x_p =$ proton fraction inside cluster

$u =$ fractional volume of cluster

$s =$ shape of cluster

$n_e =$ electron density

Shape of nuclei in the crust of a neutron star
Hashimoto, M.; Seki, H.; Yamada, M
1984PThPh..71..320H

Simple Liquid Drop Model Approach (2)

How to determine which shape of nuclear clusters appear and at which densities?

Energy, W, of a cell (= cube of size a) as sum of nuclear bulk, W_B, nuclear surface, W_S, and Coulomb, W_C, energies:

$$W = W_B + W_S + W_C + W_e = w_{B,in}(n_{\rm in}, x_p) u a^3 + w_{B,out}(n_{\rm out})(1-u)a^3 + w_e a^3$$
$$\sigma(n_{\rm in}, n_{\rm out}, x_p) g(u,s) a^2 + w_C(u,s)\frac{Z^2}{a}$$

Minimize energy density: $$\frac{\partial}{\partial a}\left[\frac{W}{a^3}\right]_{n_{\rm in}, n_{\rm out}, x_p, u} = 0 \quad \Longrightarrow \quad W_S = 4 W_C$$

Beta decay stability: $$\mu_e = \mu_n^{(N)} - \mu_p^{(N)}$$

Minimize Charge neutrality: $$x_p u \, n_{\rm in} = n_e$$

Shape of nuclei in the crust of a neutron star
Hashimoto, M.; Seki, H.; Yamada, M
1984PThPh..71..320H

Simple Liquid Drop Model Approach (3)

Table I. Relative magnitude of the surface energy, $g(u)$.

shape of nucleus	surface area $g(u)$	range of u
sphere	$(6\pi^{1/2}u)^{2/3}$	$0 \leq u \leq \pi/6$
cylinder	$(4\pi u)^{1/2}$	$0 \leq u \leq \pi/4$
board	2	$0 \leq u \leq 1$
cylindrical hole	$(4\pi(1-u))^{1/2}$	$1-\pi/4 \leq u \leq 1$
spherical hole	$(6\pi^{1/2}(1-u))^{2/3}$	$1-\pi/6 \leq u \leq 1$

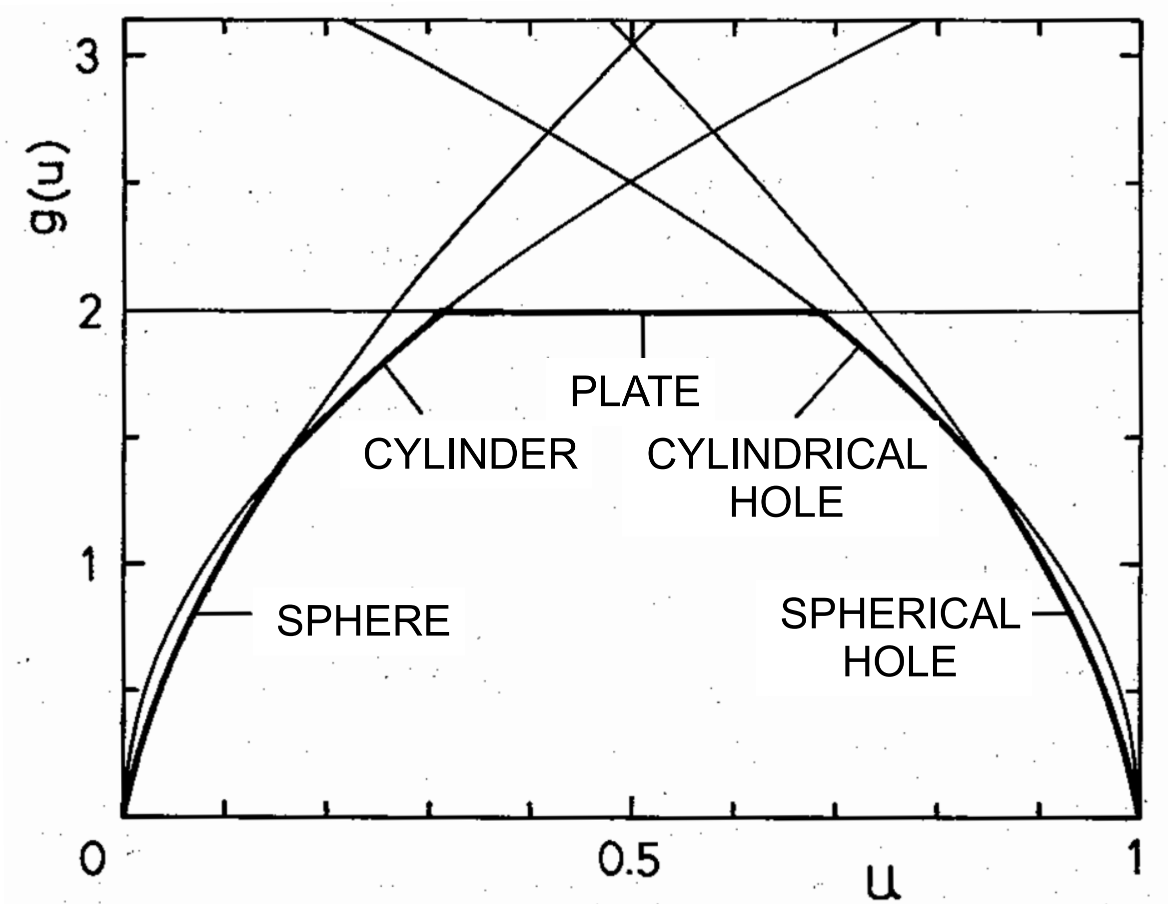


Fig. 2. Relative surface area $g(u)$. The parts giving the minimum area are drawn with thick lines.

Final solution:

compare W
at given density
for different
shapes

Shape of nuclei in the crust of a neutron star
Hashimoto, M.; Seki, H.; Yamada, M
1984PThPh..71..320H

Improved Liquid Drop Model Approach

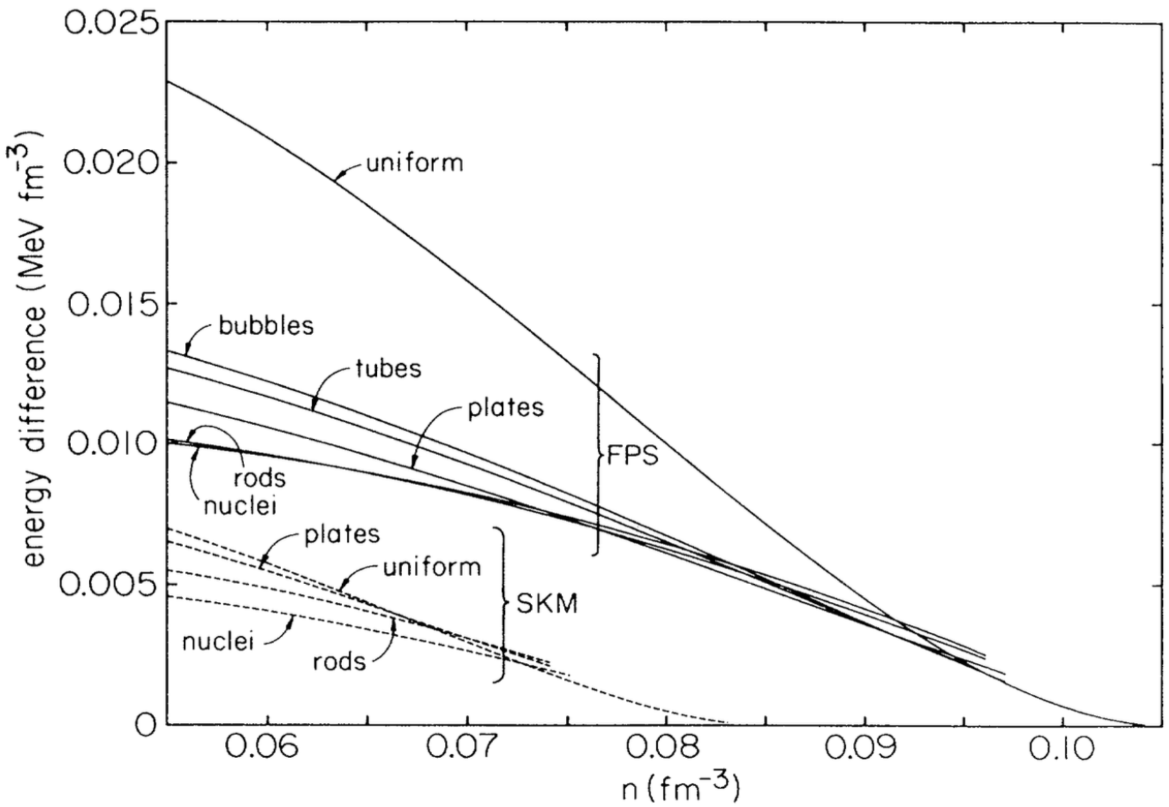


FIG. 1. Energies per unit volume as a function of density for the one-fluid phase, and the three-, two-, and one-dimensional nucleus phases, with (for FPS) the bubble (inverted structure) versions of the first two, after subtraction of the energy of the two-fluid phase, neglecting Coulomb and interface effects. The two nuclear interactions illustrated are SKM [6] and the version of FPS [8, 9] described in the text.

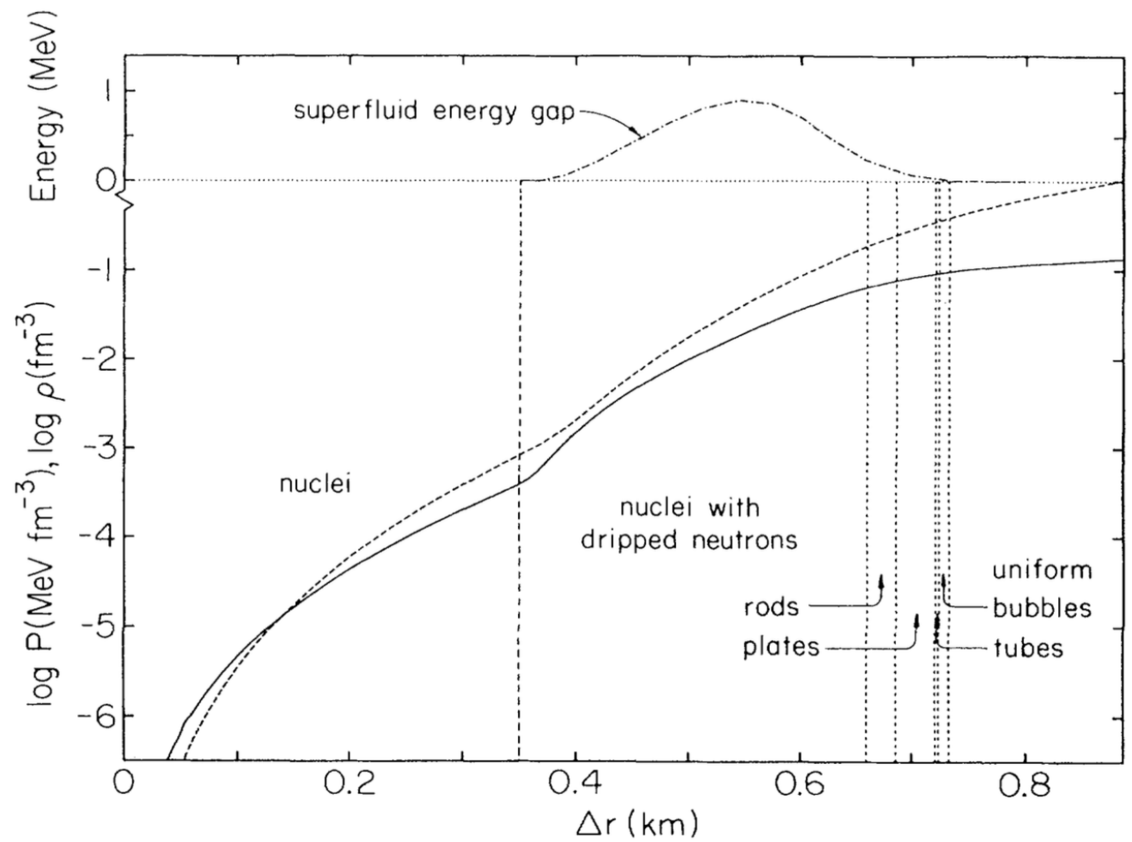


FIG. 2. Profile of a neutron star crust as given by FPS [8, 9]. The distance (in km) is measured from the surface. The solid line is density ρ/m_n, in fm^{-3}, and the dashed line is pressure, in MeV fm^{-3}, plotted logarithmically. Vertical lines indicate the phase boundaries described in the text. At the top is shown the superfluid energy gap [22].

Neutron star crusts
CP Lorentz, DG Ravenhall & CJ Pethick
1993PhRvL..70..379L

Molecular Dynamics Approach

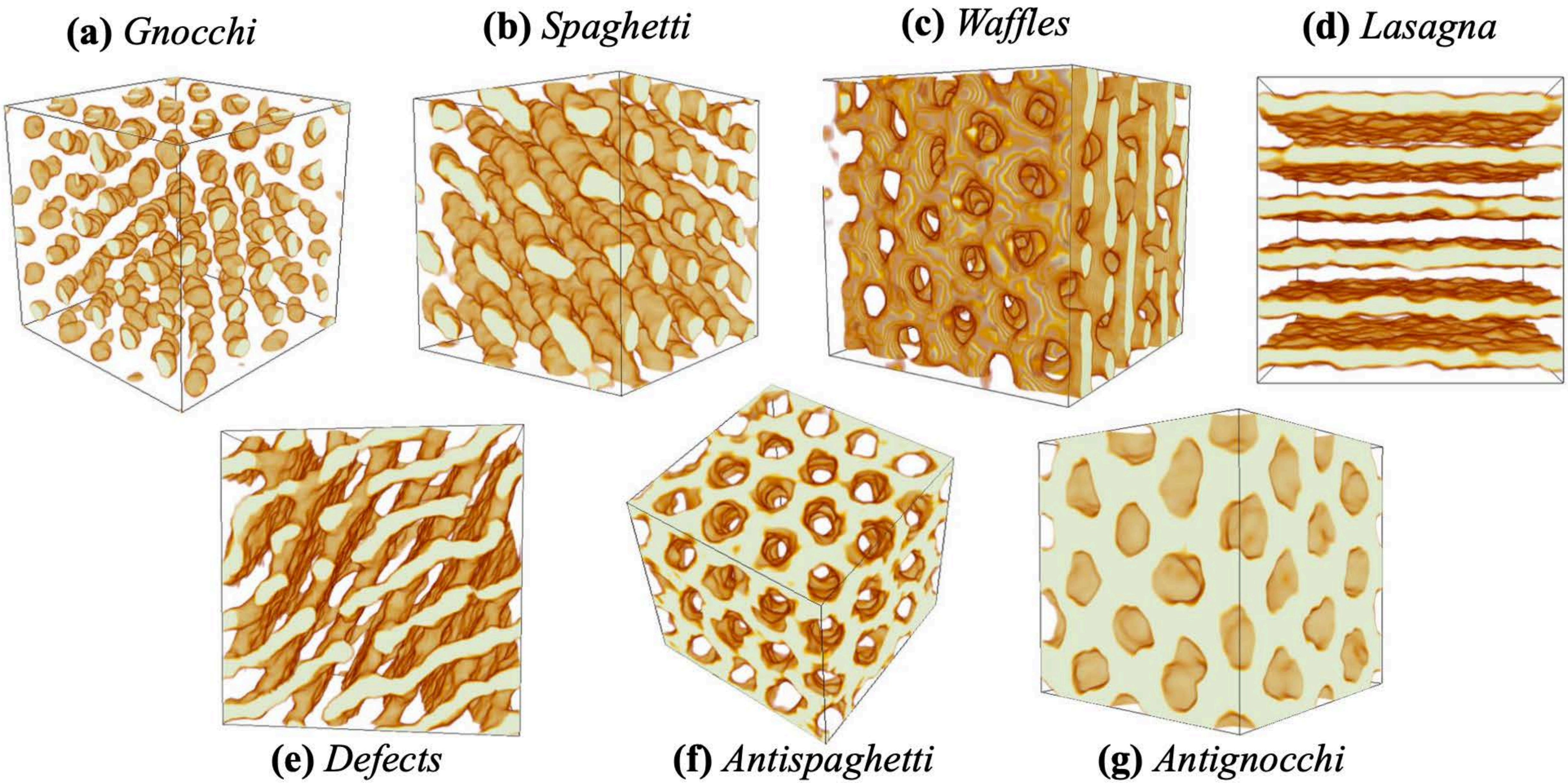
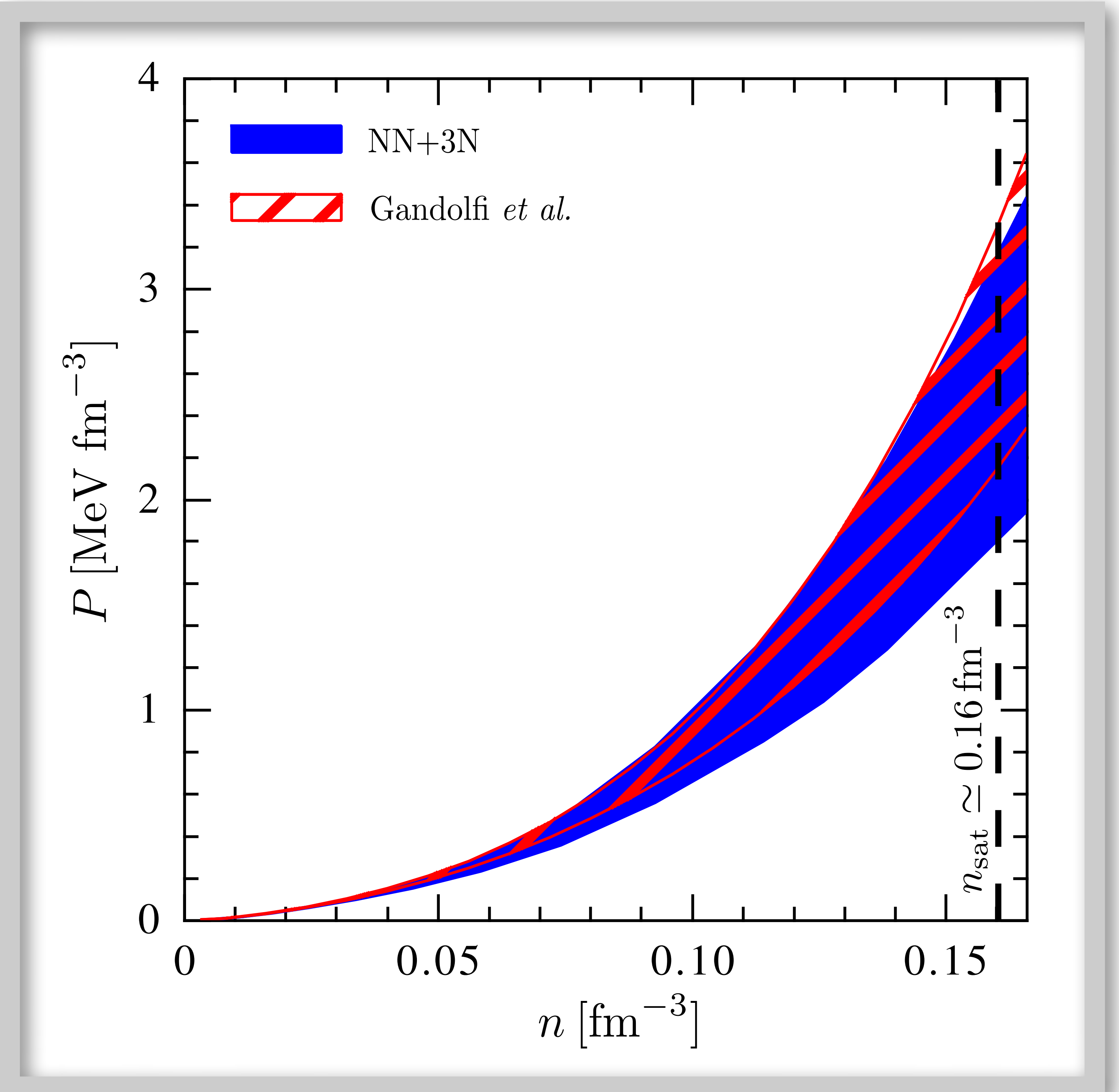


FIG. 3 Nuclear pasta configurations produced in our MD simulations with 51,200 nucleons (Horowitz *et al.*, 2015; Schneider *et al.*, 2014, 2013).

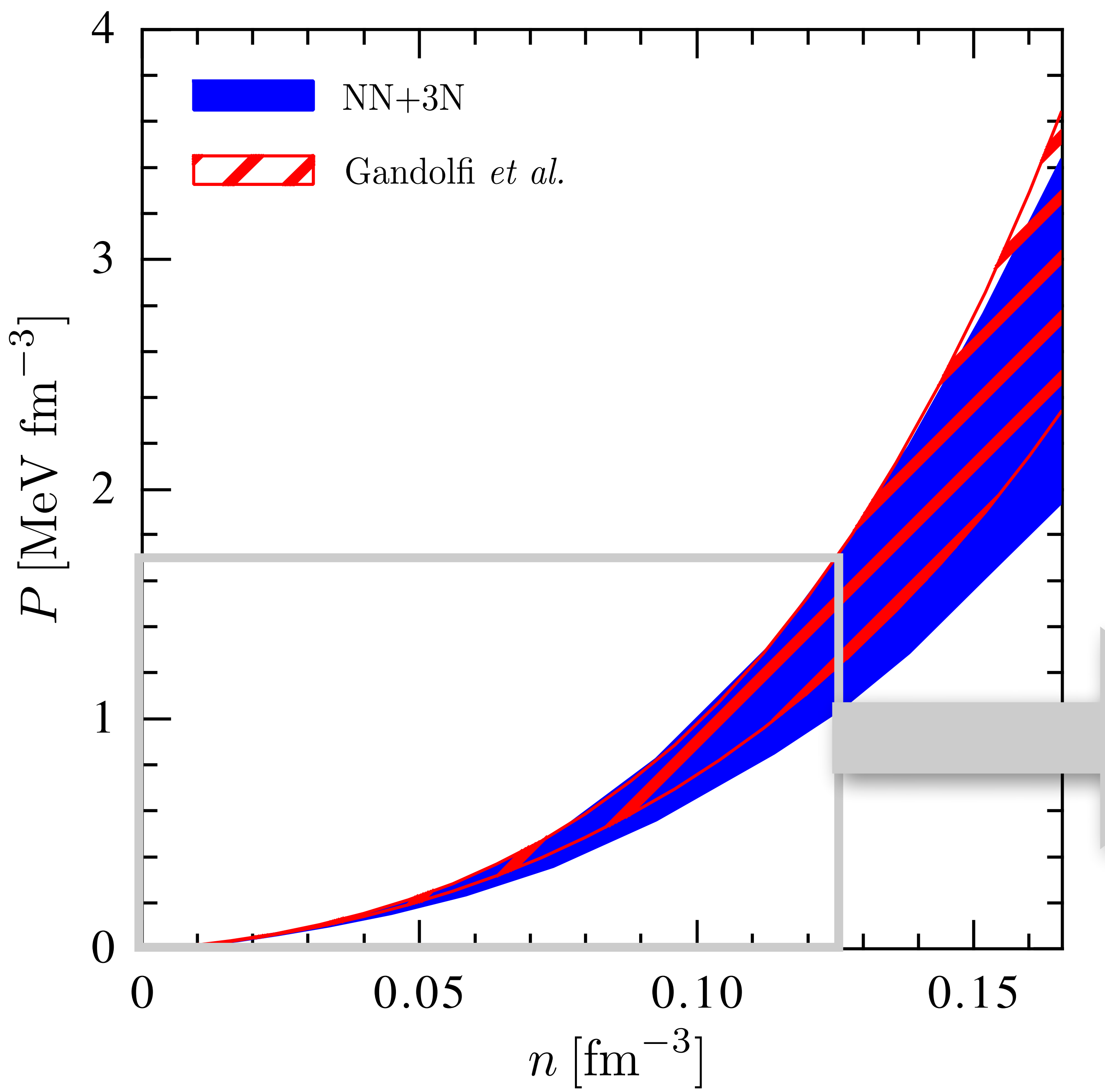
[Colloquium: Astromaterial Science and Nuclear Pasta](#)
Caplan, M. E.; Horowitz, C. J.
2017RvMP...89d1002C

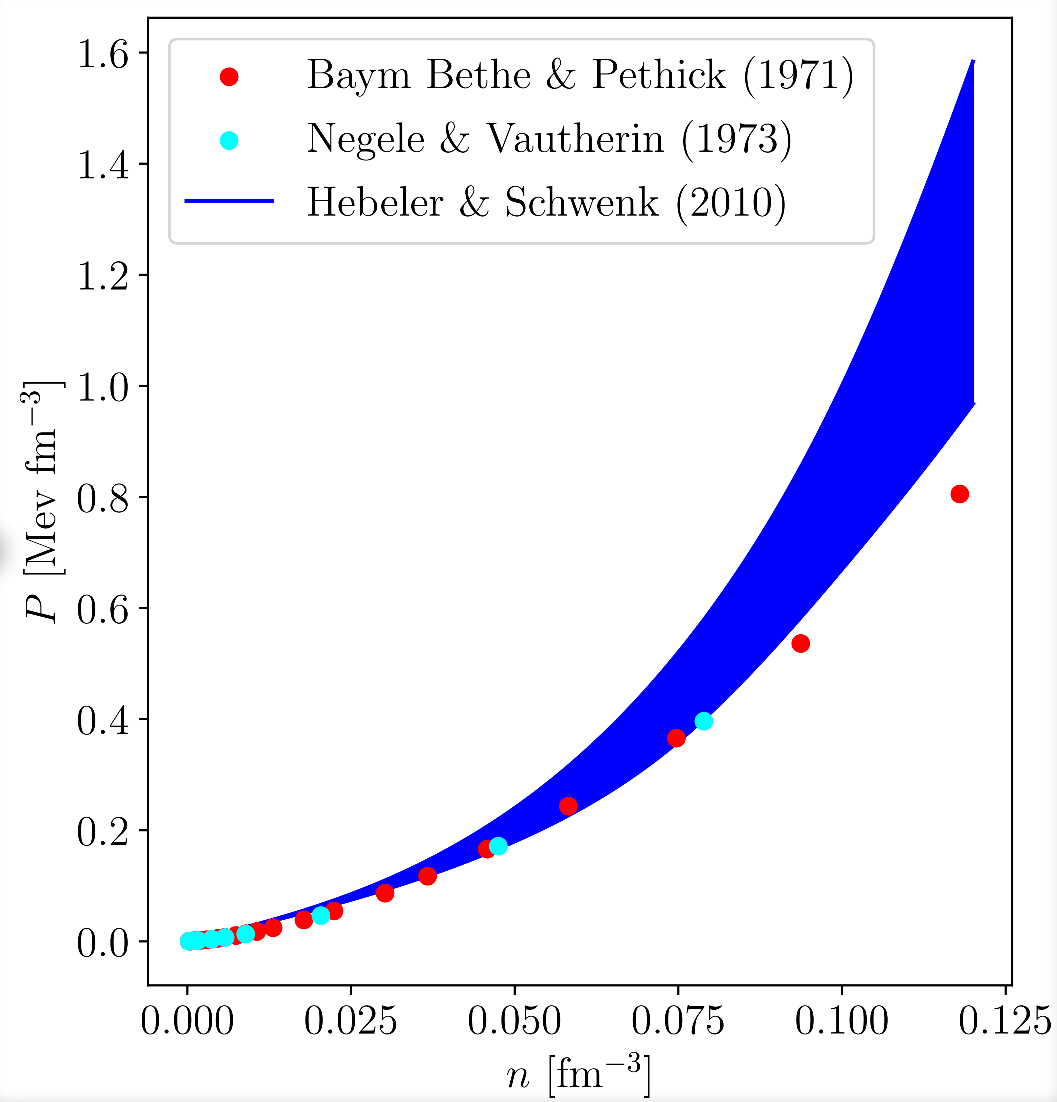
Deep Crust EOS



Equation of State and Neutron Star Properties Constrained by Nuclear Physics and Observation
Hebeler, K.; Lattimer, J. M.; Pethick, C. J.; Schwenk, A. 2013ApJ...773...11H

Deep Crust EOS

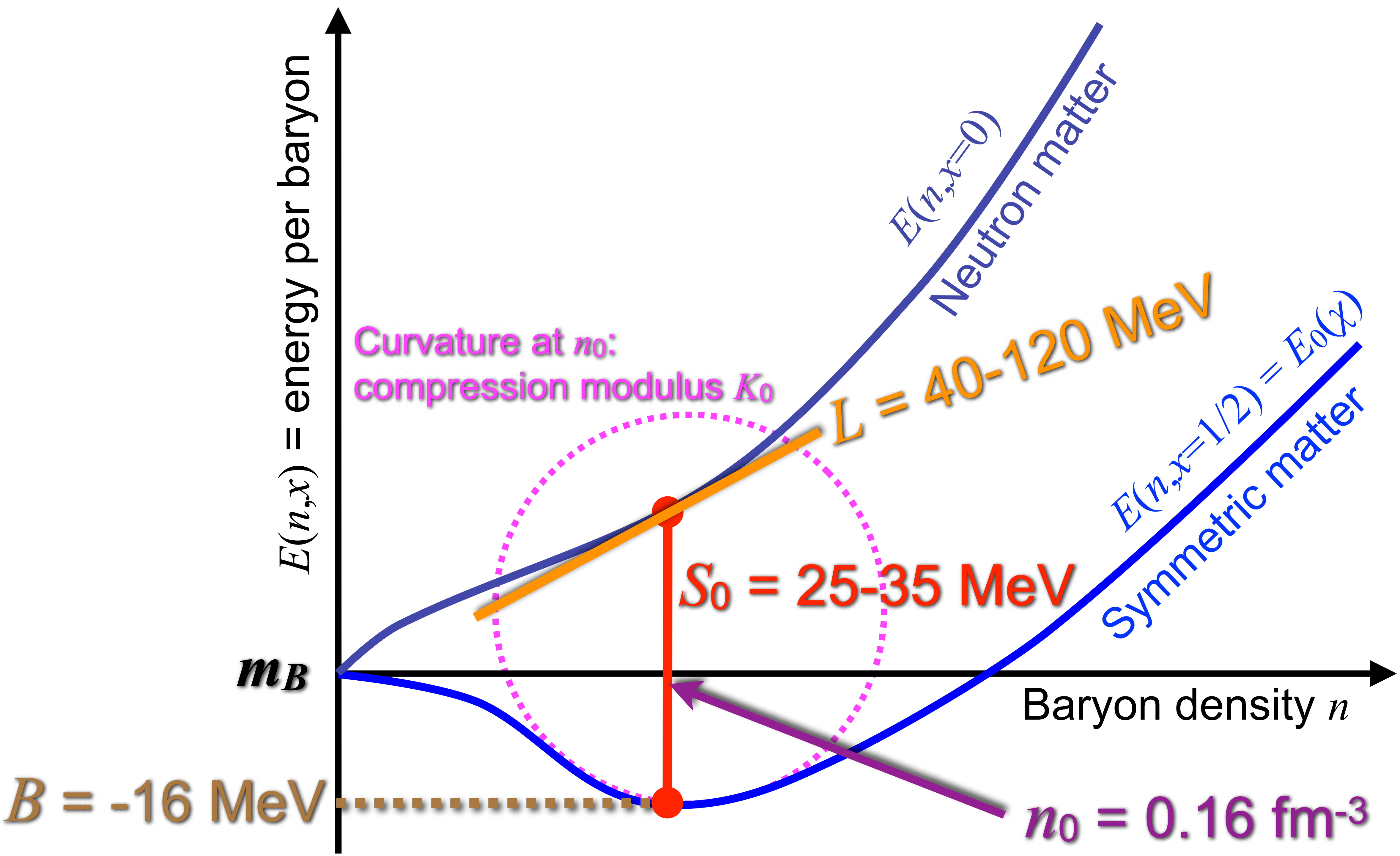




Equation of State and Neutron Star Properties Constrained by Nuclear Physics and Observation
Hebeler, K.; Lattimer, J. M.; Pethick, C. J.; Schwenk, A. 2013ApJ...773...11H

The Neutron Star Equation of State III. The Core

Symmetric/Neutron Matter: Symmetry Energy



$x = n_p/n$ = proton fraction = Z/A

A Simple Parametrization of $E(n,x)$ near $n=n_0$

n = baryon density

$n_0 \equiv n_{\rm sat} \simeq 0.16\,{\rm fm}^{-3}$

$$u = \frac{n}{n_0} \qquad \chi \equiv \frac{n-n_0}{3n_0} = \frac{1}{3}(u-1) \qquad \delta \equiv 1-2x \equiv \frac{n_n - n_p}{n} \qquad x \equiv \frac{x_p}{n}$$

Energy density: $\varepsilon = \varepsilon_N + \varepsilon_{\rm e}$ $\qquad \varepsilon_N = nE(n,x) = n\left[E_0(\chi) + S(\chi)\delta^2\right]$

Energy per baryon of symmetric matter: $E_0(\chi) = m_B + B + \frac{K_0}{18}(u-1)^2 = m_B + B + \frac{1}{2}K_0\chi^2$

Symmetry energy: $S(\chi) = S_0 + L\chi + \frac{1}{2}K_{\rm sym}\chi^2$

Nucleon pressure: $P_N = n^2\frac{dE}{dn} = \frac{n_0}{3}u^2[L\delta^2 + (K_0 + K_{\rm sym}\delta^2)\chi]$

for neutron matter, δ=1 : $P_N = \frac{n_0}{9}[Ku^3 + (3L-K)u^2] \quad \text{with} \quad K \equiv K_0 + K_{\rm sym}$

at saturation density $n = n_0$: $P_0 = \frac{1}{3}n_0 L$

P_0 is the dominant ingredient that determines R at 1.4 $M_\odot$

Nuclear Constraints on Models

1. Nuclear saturation density: $n_0 = 0.15 - 0.16 \text{ fm}^{-3}$
2. Binding energy per nucleon: $B = -16 \text{ MeV}$
3. Compression modulus: $K_0 \equiv \left.\frac{\partial^2 E}{\partial \chi^2}\right|_{\delta=0} = 9 \left.\frac{\partial^2 E}{\partial n^2}\right|_{x=1/2} = 240 \pm 20 \text{ MeV}$
4. Symmetry energy: $S_0 \equiv \frac{1}{2} \left.\frac{\partial^2 E}{\partial \delta^2}\right|_{\delta=0} = 2 \left.\frac{\partial^2 E}{\partial x^2}\right|_{x=1/2} = 25 - 35 \text{ MeV}$
5. Slope of symmetry energy: $L \equiv \frac{dS}{d\chi} = 40 - 120 \text{ MeV}$
6. Nucleon effective mass: $m^* = 0.8 m_B$

(Notice: L is in pure neutron matter, all other ones are in symmetric matter)

Analytical Fit to Microscopic Calculations

Energy density: $\varepsilon = \varepsilon_N + \varepsilon_\mathrm{e} \quad \varepsilon_N = nE(n,x) = n\left[T_F(n,x) + V_0(n) + (1-2x)^2 S(n)\right\}$

when going to high densities it is convenient to separate the potential and kinetic energies:

$$T_F(n,x) = \frac{1}{n}\left\{\frac{3}{5}\frac{p_{F,p}^2}{2m_p}n_p + \frac{3}{5}\frac{p_{F,n}^2}{2m_n}n_n\right\} = \frac{3}{5}\frac{\hbar^2}{2m}(3\pi^2 n)^{2/3}\left[x^{5/3} + (1-x)^{5/3}\right]$$

Beta equilibrium:

$$\left.\begin{array}{l} n \longrightarrow p + e^- + \overline{\nu}_e \\ p + e^- \longrightarrow n + \nu_e \\ \\ n \longrightarrow p + \mu^- + \overline{\nu}_\mu \\ p + \mu^- \longrightarrow n + \nu_\mu \end{array}\right\} \quad \mu_n - \mu_p = \mu_e = \mu_\mu$$

$$\mu_n - \mu_p = \frac{\partial\varepsilon}{\partial n_n} - \frac{\partial\varepsilon}{\partial n_p} = \frac{1}{n}\frac{\partial\varepsilon}{\partial x} = \frac{\partial E}{\partial x} = \frac{\hbar^2}{2m}(3\pi^2 n)^{2/3}\left[(1-x)^{2/3} - x^{2/3}\right] + 4(1-2x)S_2(n)$$

$$\mu_e = p_{F,e}c = \hbar(3\pi^2 n x_e)^{1/3} \qquad \mu_\mu = (p_{F,\mu}^2 c^2 + m_\mu^2 c^4)^{1/2} = \left[\hbar^2(3\pi^2 n x_\mu)^{2/3} + m_\mu^2 c^4\right]^{1/2}$$

Charge neutrality: $n_e + n_\mu = n_p \quad \text{or} \quad x_e + x_\mu = x_p$

$$(n,x) \longleftrightarrow (n_p, n_n) \qquad \frac{\partial}{\partial n_n} = \frac{\partial}{\partial n} - \frac{x}{n}\frac{\partial}{\partial x} \qquad \frac{\partial}{\partial n_p} = \frac{\partial}{\partial n} + \frac{1-x}{n}\frac{\partial}{\partial x}$$

Equation of state for dense nucleon matter
Wiringa, R. B.; Fiks, V.; Fabrocini, A.
1988PhRvC..38.1010W

$E(n)$ for APR EOS

SNM = symmetric nuclear matter

TABLE VI. The $E(\rho)$ of SNM in MeV.

ρ	A18	A18+ δv	A18+UIX	A18+ δv +UIX*	corrected
0.04	−4.28	−4.08	−4.39	−4.31	−6.48
0.08	−8.72	−8.07	−8.06	−7.97	−12.13
0.12			−10.52	−10.54	−15.04
0.16	−14.59	−12.54	−11.85	−12.16	−16.00
0.20			−11.28	−12.21	−15.09
0.24	−17.61	−13.69	−8.99	−10.89	−12.88
0.32	−18.13	−11.87	0.84	−4.21	−5.03
0.40	−16.37	−7.70	12.23	2.42	2.13
0.48	−12.21	−1.01	32.18	15.56	15.46
0.56	−5.79	8.16	59.99	34.42	34.39
0.64	2.76	19.54	95.05	58.36	58.35
0.80	25.01	45.24	188.51	121.25	121.25
0.96	56.51	82.63	313.46	204.02	204.02

PNM = pure neutron matter

TABLE VII. The $E(\rho)$ of PNM in MeV.

ρ	A18	A18+ δv	A18+UIX	A18+ δv +UIX*
0.02			4.35	4.45
0.04	6.06	6.32	6.23	6.45
0.08	8.53	9.26	9.21	9.65
0.12			12.71	13.29
0.16	12.33	14.51	17.38	17.94
0.20			23.47	22.92
0.24	16.69	20.76	28.85	27.49
0.32	22.19	28.59	43.28	38.82
0.40	29.41	38.10	63.79	54.95
0.48	38.91	50.35	90.46	75.13
0.56	49.08	66.00	123.93	99.75
0.64	59.37	81.15	165.40	127.58
0.80	88.27	119.46	273.37	205.34
0.96	125.29	167.02	412.30	305.87

A18 = Argonne v_{18} 2-body interaction

δv = relativistic "boost" correction

UIX = Urbana IX model of 3-body interaction and a modified version: UIX*

Equation of state of nucleon matter and neutron star structure
Akmal, A.; Pandharipande, V. R.; Ravenhall, D. G.
1998PhRvC..58.1804A

$E(n)$ for WFF EOS

Symmetric nuclear matter

TABLE II. Energy of nuclear matter in MeV/particle as function of number density for five Hamiltonians.

ρ (fm^{-3})	AV14	AV14 plus UVII	UV14	UV14 plus UVII	UV14 plus TNI
0.07	−7.34	−7.73	−7.46	−7.53	−11.23
0.08	−8.03	−8.43	−8.22	−8.22	−12.41
0.10	−9.32	−9.64	−9.64	−9.42	−14.34
0.125	−10.76	−10.85	−11.25	−10.56	−15.88
0.15	−11.99	−11.74	−12.66	−11.25	−16.52
0.175	−13.03	−12.24	−13.87	−11.49	−16.39
0.20	−13.88	−12.37	−14.87	−11.26	−15.61
0.25	−14.99	−11.43	−16.29	−9.42	−12.59
0.30	−15.55	−9.06	−17.01	−5.70	−8.02
0.35	−15.52	−5.68	−17.06	−0.47	−2.41
0.40	−14.94	−1.39	−16.50	6.04	4.28
0.50	−12.36	11.11	−13.72	24.50	22.97
0.60		29.2		50.4	46.5
0.70		53.4		84.2	76.0
0.80		90.2		126.7	114.8
1.00		189.0		244.3	201.7
1.25		366.0		452.0	321.0
1.50		605.0		717.0	452.0

Pure neutron matter

TABLE III. Energy of neutron matter in MeV/particle as function of number density for five Hamiltonians.

ρ (fm^{-3})	AV14	AV14 plus UVII	UV14	UV14 plus UVII	UV14 plus TNI
0.07	8.65	8.49	8.88	9.46	7.93
0.08	9.19	9.34	9.45	10.26	8.46
0.10	10.16	10.90	10.50	11.88	9.58
0.125	11.22	12.80	11.68	14.03	11.21
0.15	12.17	14.75	12.86	16.41	13.05
0.175	13.07	16.86	13.94	19.06	15.17
0.20	13.92	19.14	15.06	21.99	17.57
0.25	15.55	22.94	17.39	28.71	23.07
0.30	17.16	26.56	19.92	35.94	29.35
0.35	18.80	31.31	22.65	44.14	36.34
0.40	20.47	37.21	25.65	54.44	44.12
0.50	24.03	52.94	32.50	79.63	62.42
0.60		77.4		112.2	84.0
0.70		109.0		154.5	108.8
0.80		148.5		204.2	136.0
1.00		248.7		328.3	200.9
1.25		420.0		524.0	294.0
1.50		637.0		756.0	393.0

[Equation of state for dense nucleon matter](#)
Wiringa, R. B.; Fiks, V.; Fabrocini, A.
[1988PhRvC..38.1010W](#)

$E(n)$ for WFF & APR EOSs

WFF

APR

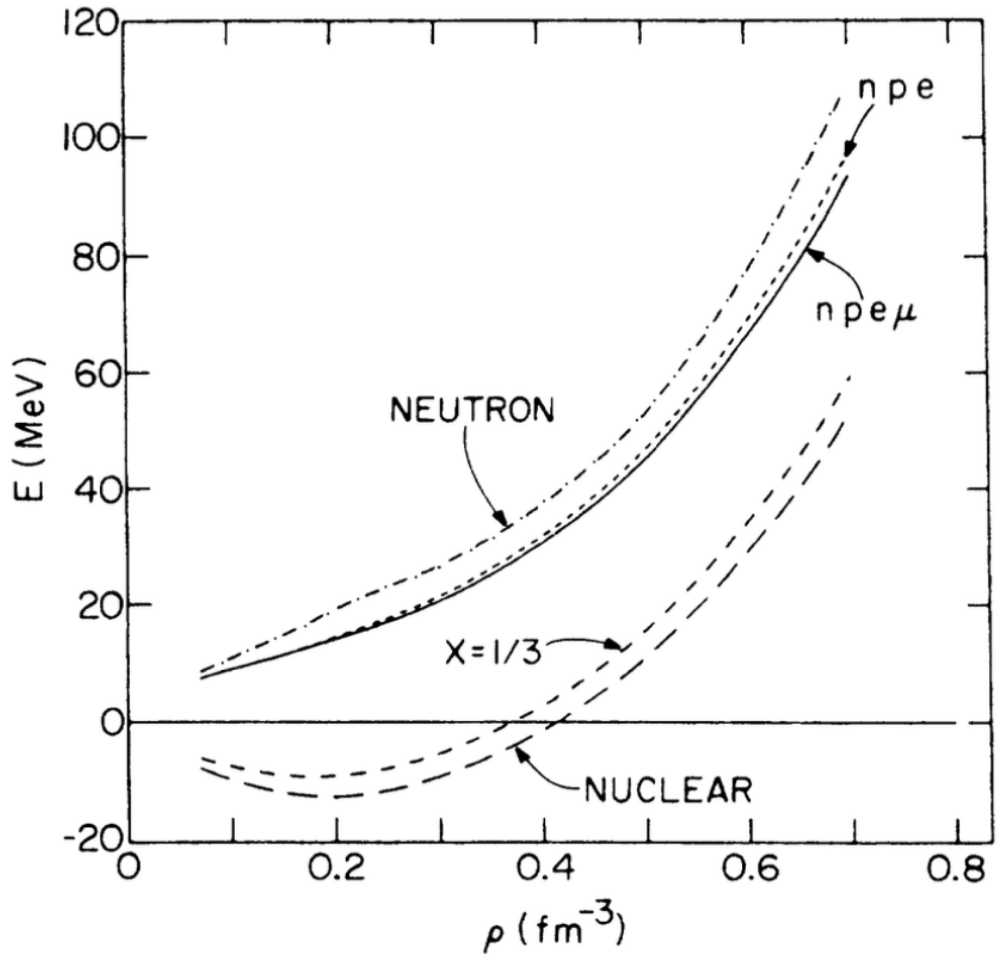


FIG. 9. $E(\rho)$ for AV14 plus UVII is shown for nuclear matter (long-dashed line), neutron matter (dash-dotted line), asymmetric matter with $x=\frac{1}{3}$ (dashed line), beta-stable matter with electrons only (short-dashed line), and beta-stable matter with electrons and muons (solid line).

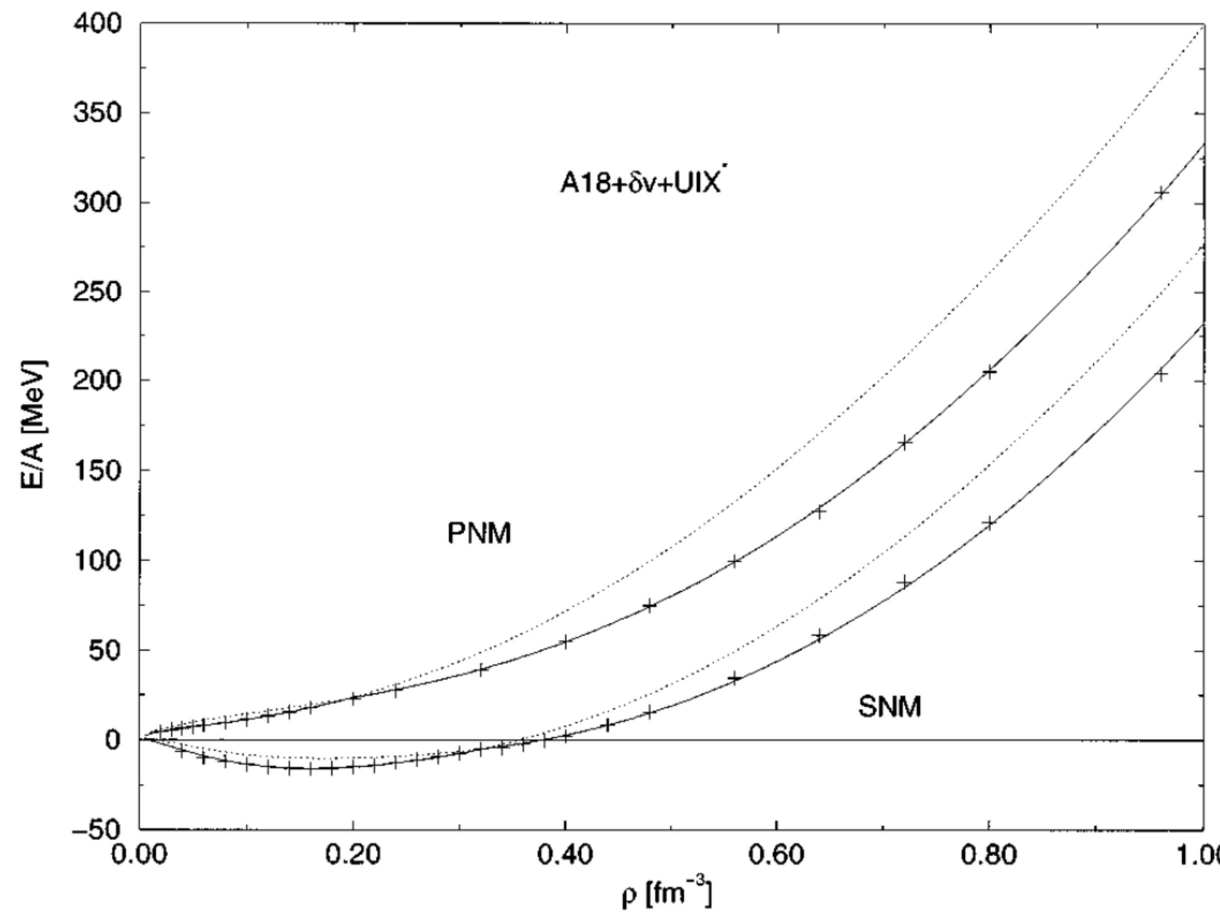


FIG. 6. The PNM and SNM energies for the A18+ δv + UIX* model, and the fits to them using an effective interaction. The full lines represent the stable phases, and the dotted lines are their extrapolations.

A Sample of EOSs and M-R Curves

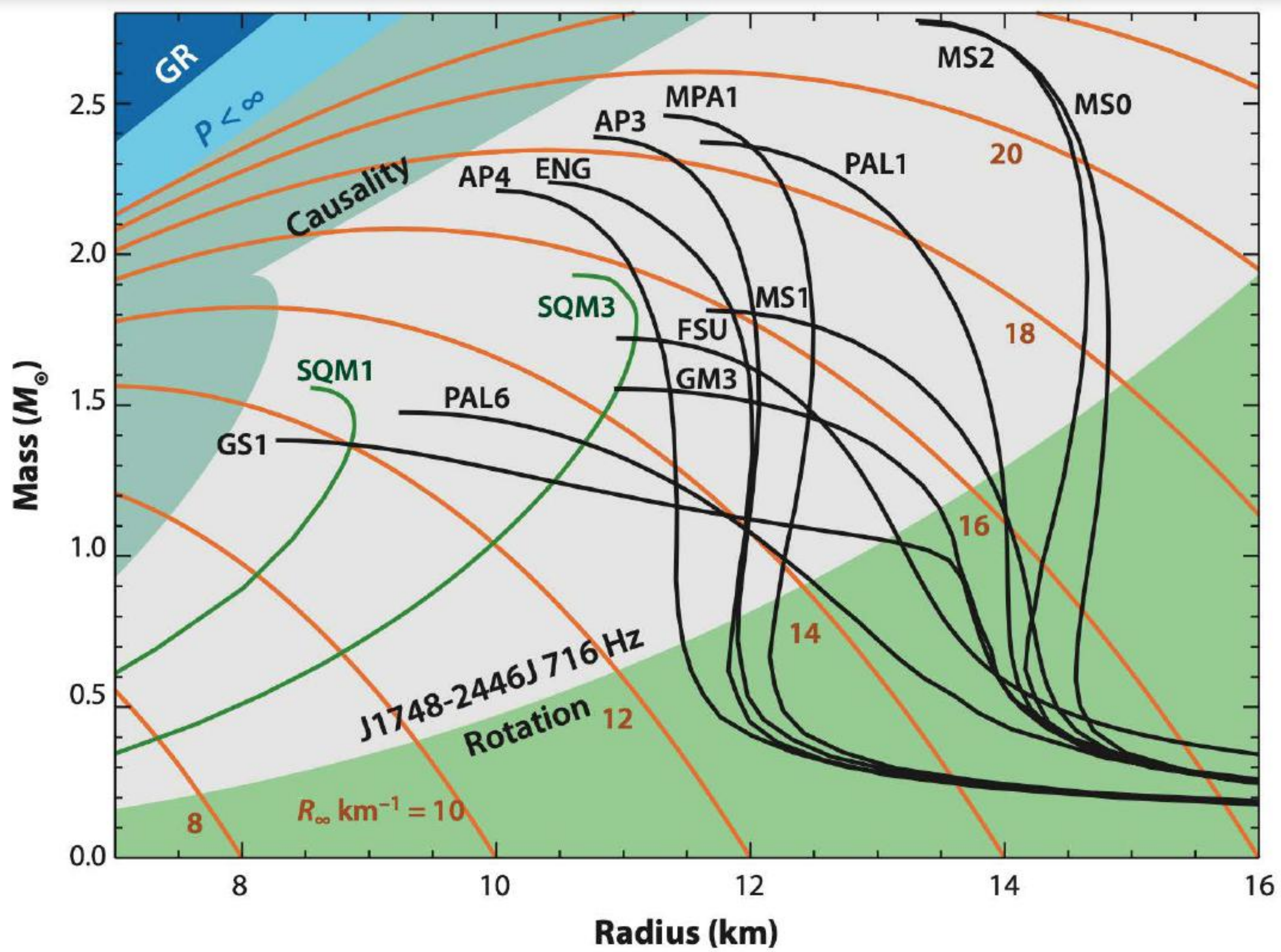
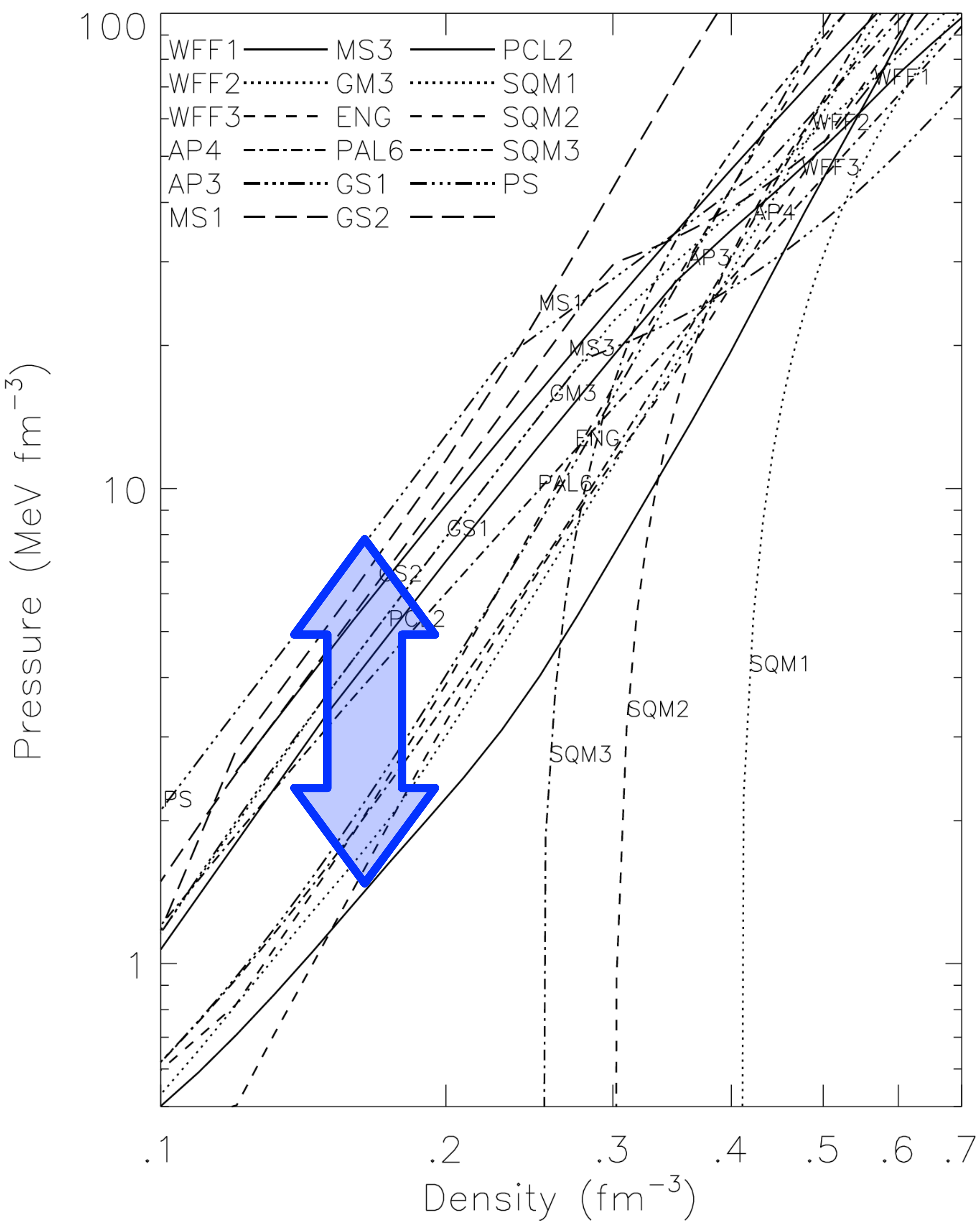


Figure 3

Typical M–R curves for hadronic equations of state (EOSs) (*black curves*) and strange quark matter (SQM) EOSs (*green curves*). The EOS names are given in Reference 13, and their P–n relations are displayed in **Figure 2**. Regions of the M–R plane excluded by general relativity (GR), finite pressure, and causality are indicated. The orange curves show contours of $R_\infty = R(1 - 2\,GM/Rc^2)^{-1/2}$. The region marked rotation is bounded by the realistic mass-shedding limit for the highest-known pulsar frequency, 716 Hz, for PSR J1748-2446J (14). Figure adapted from Reference 15.

The Nuclear Equation of State and Neutron Star Masses
Lattimer, James M.
2012ARNPS..62..485L

EOS Comparison at Low Densities



In 2001 there was a factor SEVEN uncertainty in the pressure at saturation density for neutron matter !

[Neutron Star Structure and the Equation of State](#)
Lattimer, J. M.; Prakash, M.
[2001ApJ...550..426L](#)

FIG. 1.—Pressure-density relation for a selected set of EOSs contained in Table 1. The pressure is in units of MeV fm^{-3}, and the density is in units of baryons fm^{-3}. The nuclear saturation density is approximately 0.16 fm^{-3}.

A Sample of EOSs and M-R Curves

Maximum mass is determined by P at the highest densities

Uncertainty on the neutron star radius around 1.4 M☉ directly correlate with uncertainty on P at 1-2 n_0 !

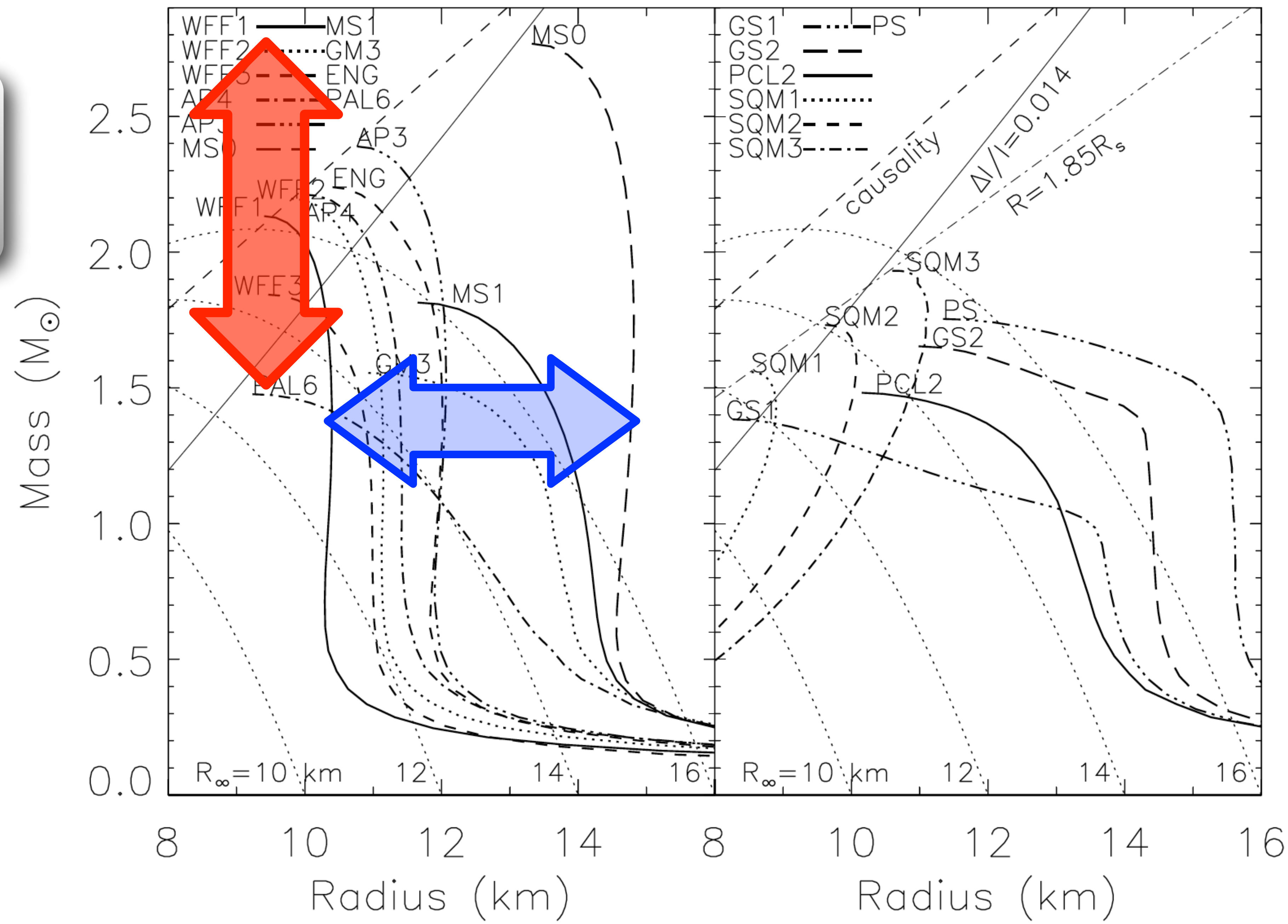


FIG. 2.—Mass-radius curves for several EOSs listed in Table 1. The left-hand panel is for stars containing nucleons and, in some cases, hyperons. The right-hand panel is for stars containing more exotic components, such as mixed phases with kaon condensates or strange quark matter, or pure strange quark matter stars. In both panels, the lower limit causality places on R is shown as a dashed line, a constraint derived from glitches in the Vela pulsar is shown as the solid line labeled $\Delta I/I = 0.014$, and contours of constant $R_\infty = R/(1 - 2GM/Rc^2)^{1/2}$ are shown as dotted curves. In the right-hand panel, the theoretical trajectory of maximum masses and radii for pure strange quark matter stars is marked by the dot-dashed curve labeled $R = 1.85R_s$.

Neutron Star Structure and the Equation of State
Lattimer, J. M.; Prakash, M.
2001ApJ...550..426L

Relativistic Mean Field Models: Toy Models

$$\begin{aligned}\mathcal{L}_N = \quad & \overline{\Psi}_N(i\gamma^\mu\partial_\mu - m_N^* - g_{\omega N}\gamma^\mu\omega_\mu - g_{\rho N}\gamma^\mu\vec{\tau}_N\cdot\vec{\rho}_\mu)\Psi_N \\ & + \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - \frac{1}{2}m_\sigma^2\sigma^2 - U(\sigma) \\ & - \frac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu} + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu - \frac{1}{4}\vec{\rho}_{\mu\nu}\cdot\vec{\rho}^{\mu\nu} + \frac{1}{2}m_\rho^2\vec{\rho}_\mu\cdot\vec{\rho}^\mu\end{aligned}$$

$\Psi_N = \begin{pmatrix}\psi_p \\ \psi_n\end{pmatrix}$ and $\psi_p,\, \psi_n$: nucleon field (4D Dirac spinor x 2D isospin (p,n)).

σ : scalar field, simulates 2-pion exchange: medium range attraction.

ω_μ : vector field: short range repulsion (known particle).

ρ_μ^a : vector-isovector field: iso-spin dependence (distinguish n from p, known particle).

Dirac effective mass:

$m_N^* = m_N - g_{\sigma N}\sigma$

$$U(\sigma) = b m_N (g_{\sigma N}\sigma)^3 + c(g_{\sigma N}\sigma)^4$$

$$\omega_{\mu\nu} = \partial_\mu\omega_\nu - \partial_\nu\omega_\mu$$

$$\vec{\rho}_{\mu\nu} = \partial_\mu\vec{\rho}_\nu - \partial_\nu\vec{\rho}_\mu$$

5 parameters: $g_\sigma,\, g_\omega,\, g_\rho,\, b$ and c allow to fit 5 observable: n_0, B, K_0, S_0, m^*.

Adding more meson interactions in U (as ω^4, ρ^4, $\sigma^2\rho^2$, $\omega^2\rho^2$, ...) provides richer models, in particular they allow to reproduce results from serious microscopic calculations (APR, ...):

$$U(\sigma,\omega,\vec{\rho}) = b\, m\, (g_\sigma\sigma)^3 + c\,(g_\sigma\sigma)^4 - d\,\left(g_\omega^2\,\omega_\mu\omega^\mu\right)^2 - e\,\left(g_\rho^2\,\vec{\rho}_\mu\cdot\vec{\rho}^\mu\right)^2 - \left[f\,\left(g_\sigma^2\,\sigma^2\right) + h\,\left(g_\omega^2\,\omega_\mu\omega^\mu\right)\right]\left(g_\rho^2\,\vec{\rho}_\mu\cdot\vec{\rho}^\mu\right)$$

Compact Stars, Nuclear Physics, Particle Physics, and general relativity
N.K. Glendenning, 2nd edition (Springer Verlagm New York, 2000)
2000csnp.conf.....G

Dense Matter in Compact Stars: Theoretical Developments and Observational Constraints
Page, Dany; Reddy, Sanjay
2006ARNPS..56..327P

Note on Units in the Lagrangian

$$\mathcal{L}_N = \quad \overline{\Psi}_N(i\gamma^\mu\partial_\mu - m_N^* - g_{\omega N}\gamma^\mu\omega_\mu - g_{\rho N}\gamma^\mu\vec{\tau}_N\cdot\vec{\rho}_\mu)\Psi_N$$
$$+\frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - \frac{1}{2}m_\sigma^2\sigma^2 - U(\sigma)$$
$$-\frac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu} + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu - \frac{1}{4}\vec{\rho}_{\mu\nu}\cdot\vec{\rho}^{\mu\nu} + \frac{1}{2}m_\rho^2\vec{\rho}_\mu\cdot\vec{\rho}^\mu$$

$$U(\sigma,\omega,\vec{\rho}) = b\,m\,(g_\sigma\sigma)^3 + c\,(g_\sigma\sigma)^4 - d\,\left(g_\omega^2\,\omega_\mu\omega^\mu\right)^2 - e\,\left(g_\rho^2\,\vec{\rho}_\mu\cdot\vec{\rho}^\mu\right)^2 - \left[f\,\left(g_\sigma^2\,\sigma^2\right) + h\,\left(g_\omega^2\,\omega_\mu\omega^\mu\right)\right]\left(g_\rho^2\,\vec{\rho}_\mu\cdot\vec{\rho}^\mu\right)$$

- $\hbar = c = 1$ and we measure everything in MeV, x^μ is in MeV^{-1}.
- $\mathcal{L}$ has units of MeV^4 so that $S = \int \mathrm{d}x^4\mathcal{L}$ has units of action, i.e., $\hbar = 1$.
- γ_μ and $\vec{\tau}$ are just dimensionless matrices.
- m and all masses are in MeV, ∂_μ or ∂^μ are in MeV.
- Ψ is in MeV$^{3/2}$.
- σ, ω and ρ are in MeV.
- g_σ, g_ω, and g_ρ are dimensionless.
- b, c, d, e, f, and h are dimensionless.

Relativistic Mean Field Models: Euler-Lagrange Equations

Euler-Lagrange equations: $$\frac{\partial \mathcal{L}}{\partial \phi(x)} = \partial_\mu \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi(x))}$$

For mesons with the non-linear meson interactions:

$$U(\sigma,\omega,\vec{\rho}) = b\, m\, (g_\sigma \sigma)^3 + c\, (g_\sigma \sigma)^4 - d\, \left(g_\omega^2\, \omega_\mu \omega^\mu\right)^2 - e\, \left(g_\rho^2\, \vec{\rho}_\mu \cdot \vec{\rho}^\mu\right)^2 - \left[f\, \left(g_\sigma^2\, \sigma^2\right) + h\, \left(g_\omega^2\, \omega_\mu \omega^\mu\right)\right] \left(g_\rho^2\, \vec{\rho}_\mu \cdot \vec{\rho}^\mu\right)$$

$$\left(\Box + m_\sigma^2\right) \sigma(x) + 3b\, m g_\sigma^3 \sigma^2(x) + 4c\, g_\sigma^4 \sigma^3(x) - 2f g_\sigma^2 g_\rho^2\, \vec{\rho}_\mu(x) \cdot \vec{\rho}^\mu(x)\, \sigma(x) = g_\sigma \overline{\Psi}_N(x) \Psi_N(x)$$

$$\left(\Box + m_\omega^2\right) \omega_\mu(x) + 4d\, g_\omega^4\, \omega_\nu(x) \omega^\nu(x)\, \omega_\mu(x) + 2h g_\omega^2 g_\rho^2\, \vec{\rho}_\mu(x) \cdot \vec{\rho}^\mu(x)\, \omega_\mu(x) = g_\omega \overline{\Psi}_N(x) \gamma_\mu \Psi_N(x)$$

$$\left(\Box + m_\rho^2\right) \vec{\rho}_\mu(x) + \partial_\mu \partial^\nu \vec{\rho}_\nu(x) + 4e g_\rho^4 (\vec{\rho}_\nu(x) \cdot \vec{\rho}^\nu(x)) \vec{\rho}_\mu(x) + 2(f g_\sigma^2 \sigma^2 + h g_\omega^2\, \omega_\nu \omega^\nu) g_\rho^2 \vec{\rho}_\mu(x) = \frac{1}{2} g_\rho \overline{\Psi}_N(x) \gamma_\mu \vec{\tau}\, \Psi_N(x)$$

For nucleons and leptons:

$$\left[i\gamma_\mu \partial^\mu - m + g_\sigma \sigma(x) - g_\omega \gamma_\mu \omega^\mu(x) - \frac{1}{2} g_\rho \gamma_\mu \vec{\tau} \cdot \vec{\rho}^\mu(x)\right] \Psi_N(x) = 0$$

$$\left[i\gamma_\mu \partial^\mu - m_\lambda\right] \psi_\lambda(x) = 0$$

Relativistic Mean Field Models: The Mean Field Approximation

Replace all meson fields by their space-time averages = $\langle \dots \rangle$

$$\sigma(x) \longrightarrow \langle \sigma(x) \rangle \equiv \sigma$$
$$\omega_\mu(x) \longrightarrow \langle \omega_\mu(x) \rangle \equiv \omega_\mu$$
$$\rho_\mu^a(x) \longrightarrow \langle \rho_\mu^a(x) \rangle \equiv \rho_\mu^a$$

Rotational symmetry: no spacial components of vector fields:

$$\omega_\mu \longrightarrow (\omega_0, \vec{0})$$
$$\rho_\mu^a \longrightarrow (\rho_0^a, \vec{0})$$

In the ground state, Fermi seas of neutrons and protons have definite charge and isospin (determined by beta equilibrium) $\longrightarrow \quad \rho_0^1 = \rho_0^2 = 0$

When calculating ground-state properties of dense matter only 3 of the meson field components survive:

$$\sigma, \ \omega_0 \equiv \omega, \ \text{and} \ \rho_0^3 \equiv \rho$$

At a given density σ, ω, and ρ are just numbers

Relativistic Mean Field Models: Baryon Sector

Look for a plane wave solution $\Psi(x) = e^{-ik^\mu x_\mu}\Phi(k)$ and let's write

$$K^\mu \equiv k^\mu - g_\omega \omega^\mu - \frac{g_\rho}{2}\vec{\tau}\cdot\vec{\rho}^\mu \quad \text{and} \quad m^* \equiv m - g_\sigma \sigma \quad \text{so that} \quad [\gamma_\mu K^\mu - m^*]\Phi = 0$$

then

$$K_\mu K^\mu = m^{*2} \quad \text{and} \quad E(\vec{K}) \equiv K^0 = \sqrt{K_i K^i + m^{*2}}$$

and the baryons' energy is

$$e(\vec{k}) \equiv k_0 = E(\vec{K}) + g_\omega \omega^0 + \frac{g_\rho}{2}\vec{\tau}\cdot\vec{\rho}_0 = E(\vec{K}) + g_\omega \omega + g_\rho \rho I_3$$

with $I_3 = +1/2$ for protons and $= -1/2$ for neutrons.

Nucleons Fermi energies and chemical potentials:

$$E_{Fp} = \mu_p(T=0) = \sqrt{k_F^2 + m^{*2}} + g_\omega \omega + \frac{1}{2} g_\rho \rho$$

$$E_{Fn} = \mu_n(T=0) = \sqrt{k_F^2 + m^{*2}} + g_\omega \omega - \frac{1}{2} g_\rho \rho$$

Relativistic Mean Field Models: Mean Field Meson Sector

Take the meson equations and replace meson fields by their mean values, evaluate the rhs by filling the nucleon Fermi sees to get:

$$m_\sigma^2 \sigma + 3b\, m g_\sigma^3 \sigma^2 + 4c\, g_\sigma^4 \sigma^3 - 2f g_\sigma^2 g_\rho^2 \sigma \rho^2 = g_\sigma \left\langle \overline{\Psi}_N(x) \Psi_N(x) \right\rangle = g_\sigma \sum_B n_S(B) \qquad (1)$$

$$m_\omega^2 \omega + 4d\, g_\omega^4\, \omega^3 + 2h g_\rho^2 g_\omega^2 \rho^2 \omega = g_\omega \left\langle \overline{\Psi}_N(x) \gamma_0 \Psi_N(x) \right\rangle = g_\omega \sum_B n_B \qquad (2)$$

$$m_\rho^2 \rho + 4e\, g_\rho^4 \rho^3 + 2\left[f g_\sigma^2 \sigma^2 + h g_\omega^2 \omega^2 \right] g_\rho^2 \rho = \frac{1}{2} g_\rho \left\langle \overline{\Psi}_N(x) \gamma_0 \tau_3\, \Psi_N(x) \right\rangle = \sum_B I_B n_B \qquad (3)$$

$$\sum_B n_S(B) \equiv \sum_B \frac{g_B m_B^{*\,3}}{4\pi^2} \left[x_B \sqrt{1+x_B^2} - \ln\left(x_B + \sqrt{1+x_B^2} \right) \right] \qquad \sum_B n_B = n_p + n_n = n \qquad \sum_B I_B n_B = \frac{1}{2}(n_p - n_n)$$

Relativistic Mean Field Models: Final Set of Equations

Take the meson equations and replace meson fields by their mean values, evaluate the rhs by filling the nucleon Fermi sees to get:

$$m_\sigma^2 \sigma + 3b\, m g_\sigma^3 \sigma^2 + 4c\, g_\sigma^4 \sigma^3 - 2f g_\sigma^2 g_\rho^2 \sigma \rho^2 = g_\sigma \left\langle \overline{\Psi}_N(x) \Psi_N(x) \right\rangle = g_\sigma \sum_B n_S(B) \quad (1)$$

$$m_\omega^2 \omega + 4d\, g_\omega^4\, \omega^3 + 2h g_\rho^2 g_\omega^2 \rho^2 \omega = g_\omega \left\langle \overline{\Psi}_N(x) \gamma_0 \Psi_N(x) \right\rangle = g_\omega \sum_B n_B \quad (2)$$

$$m_\rho^2 \rho + 4e\, g_\rho^4 \rho^3 + 2\left[f g_\sigma^2 \sigma^2 + h g_\omega^2 \omega^2 \right] g_\rho^2 \rho = \frac{1}{2} g_\rho \left\langle \overline{\Psi}_N(x) \gamma_0 \tau_3\, \Psi_N(x) \right\rangle = \sum_B I_B n_B \quad (3)$$

$$\sum_B n_S(B) \equiv \sum_B \frac{g_B m_B^{*\,3}}{4\pi^2} \left[x_B \sqrt{1 + x_B^2} - \ln\left(x_B + \sqrt{1 + x_B^2} \right) \right] \qquad \sum_B n_B = n_p + n_n = n \qquad \sum_B I_B n_B = \frac{1}{2}(n_p - n_n)$$

Also solve for baryon number charge neutrality and beta equilibrium

$$\begin{cases} n_p + n_n = n & (4) \\ n_e + n_\mu = n_p & (5) \\ \mu_n = \mu_p + \mu_e & (6) \\ \mu_\mu = \mu_e & (7) \end{cases}$$

7 equations for 7 unknowns:
$\sigma,\ \omega,\ \rho,\ n_p,\ n_n,\ n_e,\ n_\mu$
depending on one input:
baryon density n.

Iterate on a series of baryon densities to get an EOS (after calculating ε and P)

Relativistic Mean Field Models: The Equation of State (EOS)

Energy momentum tensor: $$T^{\mu\nu} \equiv \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi_i)} \partial^\nu \phi_i - \eta^{\mu\nu}\mathcal{L} = \mathrm{diag}(\epsilon, P, P, P)$$

Mean field ε and P: $$\epsilon = -\langle \mathcal{L} \rangle + \left\langle \overline{\Psi}_N \gamma_0 k_0 \Psi_N \right\rangle \qquad P = \langle \mathcal{L} \rangle + \frac{1}{3}\left\langle \overline{\Psi}_N \gamma_i k_i \Psi_N \right\rangle$$

Mean field Lagrangian: $$\langle \mathcal{L} \rangle = -\frac{1}{2} m_\sigma^2 \sigma^2 + \frac{1}{2} m_\omega^2 \omega_0^2 + \frac{1}{2} m_\rho^2 \rho_{30}^2 - \langle U_{\mathrm{eff}} \rangle$$

$$\langle U_{\mathrm{eff}} \rangle = b\, m g_\sigma^3 \sigma^3 + c\, g_\sigma^4 \sigma^4 - d\, g_\omega^4 \omega^4 - e\, g_\rho^4 \rho^4 - \left[f(g_\sigma \sigma)^2 + h(g_\omega \omega)^2 \right] (g_\rho \rho)^2$$

$$\begin{aligned}
\epsilon =& \frac{1}{2} m_\sigma^2 \sigma^2 + \frac{1}{2} m_\omega^2 \omega^2 + \frac{1}{2} m_\rho^2 \rho^2 \\
&+ b\, m g_\sigma^3 \sigma^3 + c\, g_\sigma^4 \sigma^4 + 3d\, g_\omega^4 \omega^4 + 3e\, g_\rho^4 \rho^4 \\
&+ \left[f(g_\sigma \sigma)^2 + h(g_\omega \omega)^2 \right] (g_\rho \rho)^2 \\
&+ \sum_B \frac{g_B}{2} \frac{m_B^{*4}}{8\pi^2} \left[x_B \sqrt{1+x_B^2}(1+2x_B^2) - \ln\left(x_B + \sqrt{1+x_B^2} \right) \right] \\
&+ \sum_L \frac{g_L}{2} \frac{m_L^{*4}}{8\pi^2} \left[x_L \sqrt{1+x_L^2}(1+2x_L^2) - \ln\left(x_L + \sqrt{1+x_L^2} \right) \right]
\end{aligned}$$

$$\begin{aligned}
P =& -\frac{1}{2} m_\sigma^2 \sigma^2 + \frac{1}{2} m_\omega^2 \omega^2 + \frac{1}{2} m_\rho^2 \rho^2 \\
&- b\, m g_\sigma^3 \sigma^3 - c\, g_\sigma^4 \sigma^4 + d\, g_\omega^4 \omega^4 + e\, g_\rho^4 \rho^4 \\
&+ \left[f(g_\sigma \sigma)^2 + h(g_\omega \omega)^2 \right] (g_\rho \rho)^2 \\
&+ \sum_B \frac{g_B}{2} \frac{m_B^{*4}}{8\pi^2} \left[x_B \sqrt{1+x_B^2} \left(\frac{2}{3} x_B^2 - 1 \right) + \ln\left(x_B + \sqrt{1+x_B^2} \right) \right] \\
&+ \sum_L \frac{g_B}{2} \frac{m_L^{*4}}{8\pi^2} \left[x_L \sqrt{1+x_L^2} \left(\frac{2}{3} x_L^2 - 1 \right) + \ln\left(x_L + \sqrt{1+x_L^2} \right) \right]
\end{aligned}$$

Hyperons
in
Neutron Stars

Hyperons in Neutron Stars ?

$$\mu_{B^0} = \mu_n$$
$$\mu_{B^-} = \mu_n + \mu_e$$
$$\mu_{B^+} = \mu_n - \mu_e$$

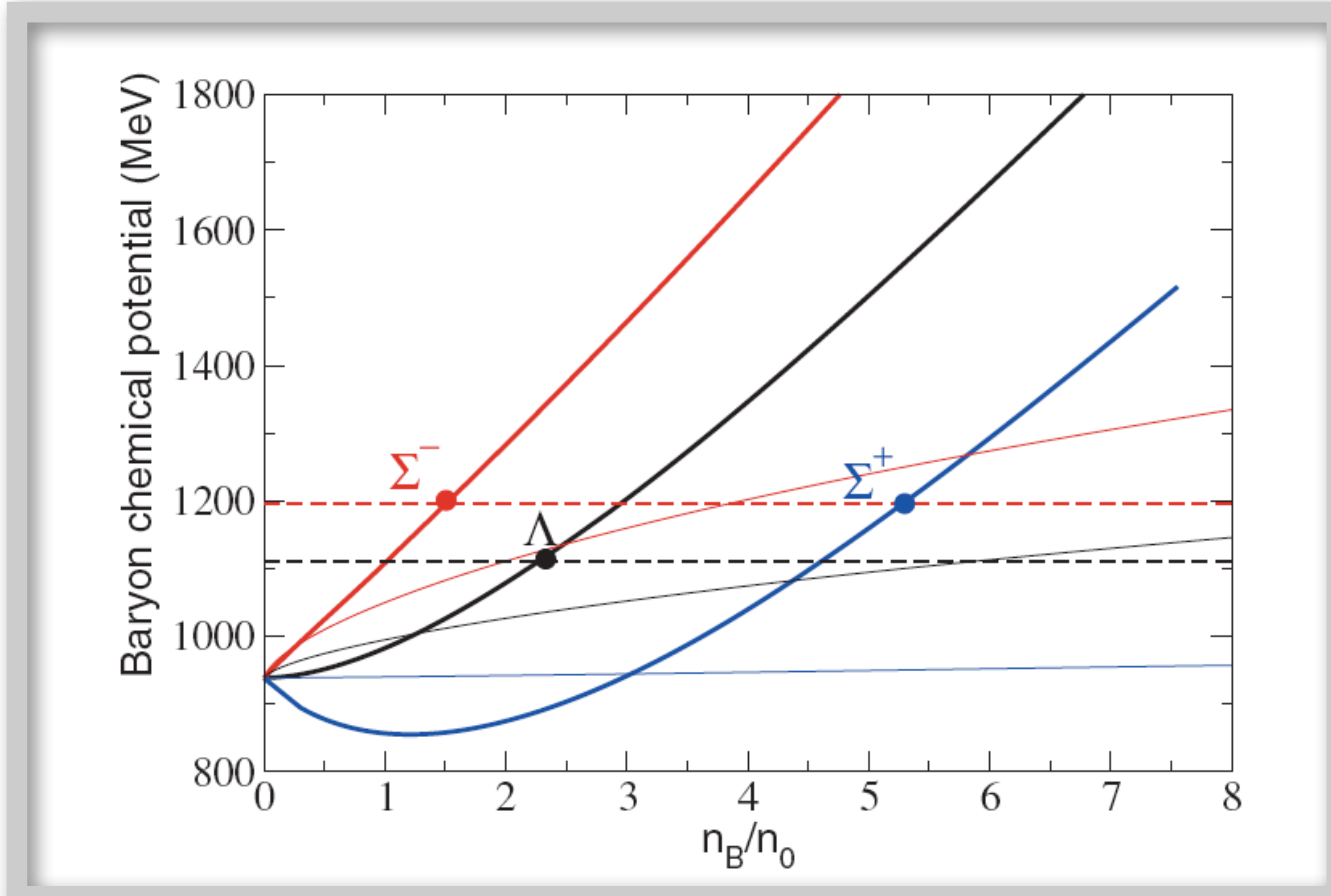


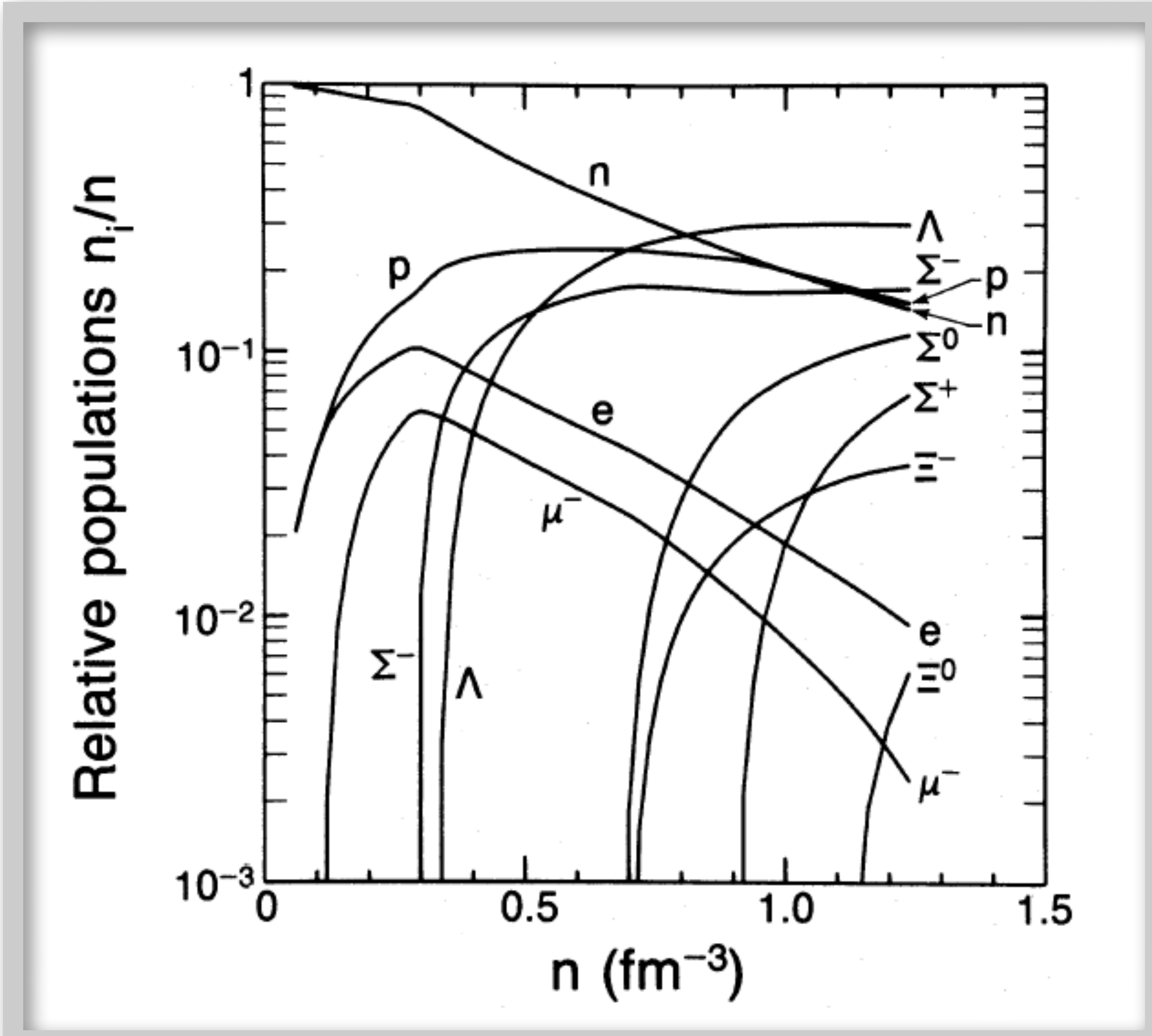
Figure 10 Baryon chemical potentials in dense stellar matter.

Thick lines: Walecka type model

Thin lines: free gases

Dense Matter in Compact Stars: Theoretical Developments and Observational Constraints,
Page D. & Reddy S., 2006, Annu. Rev. Nucl. Part. Sci. 56, 327

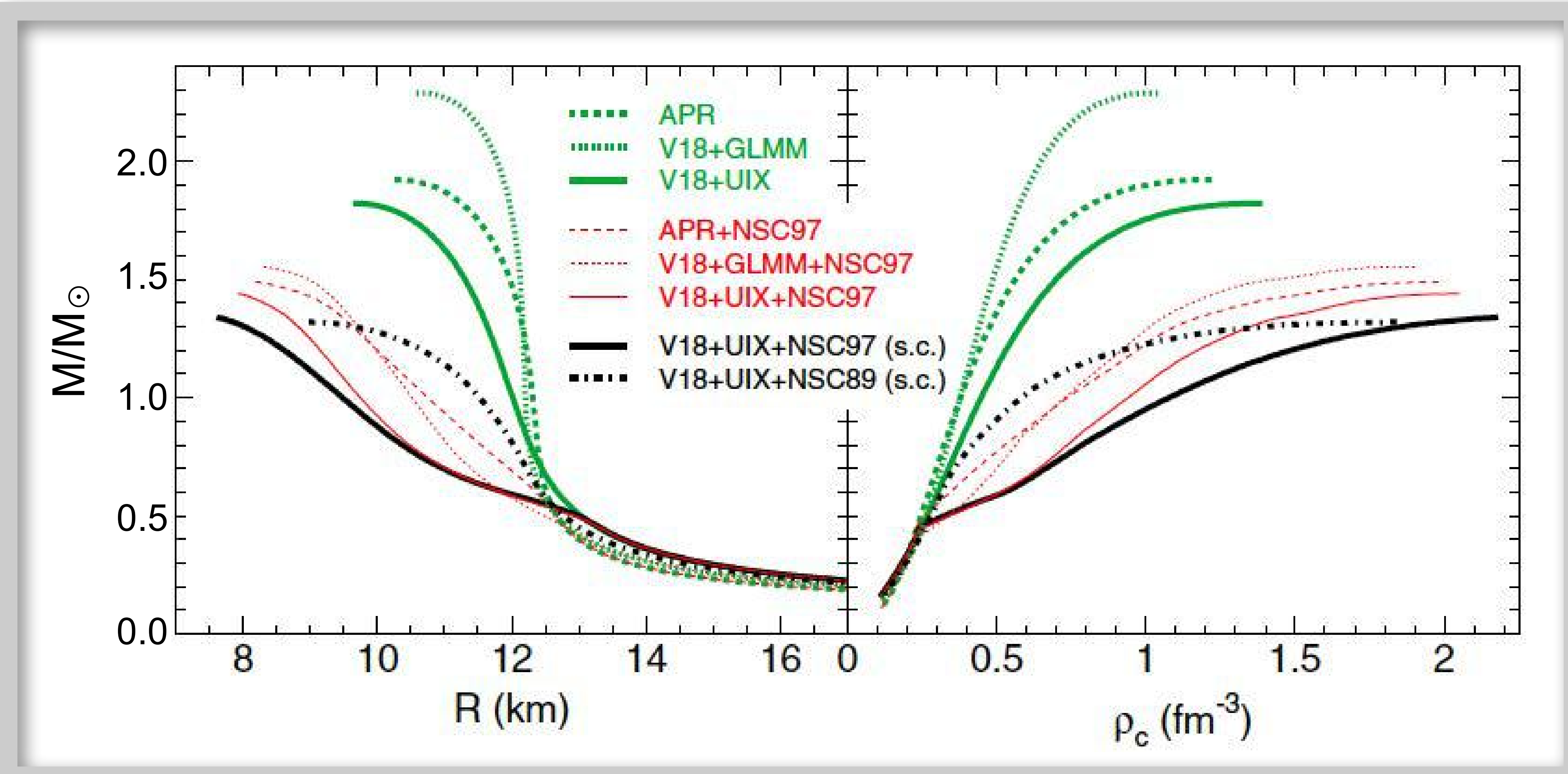
Hyperons in Neutron Stars ?



Neutron stars are giant hypernuclei ?

N.K. Glendenning, ApJ 293, 470 (1985)

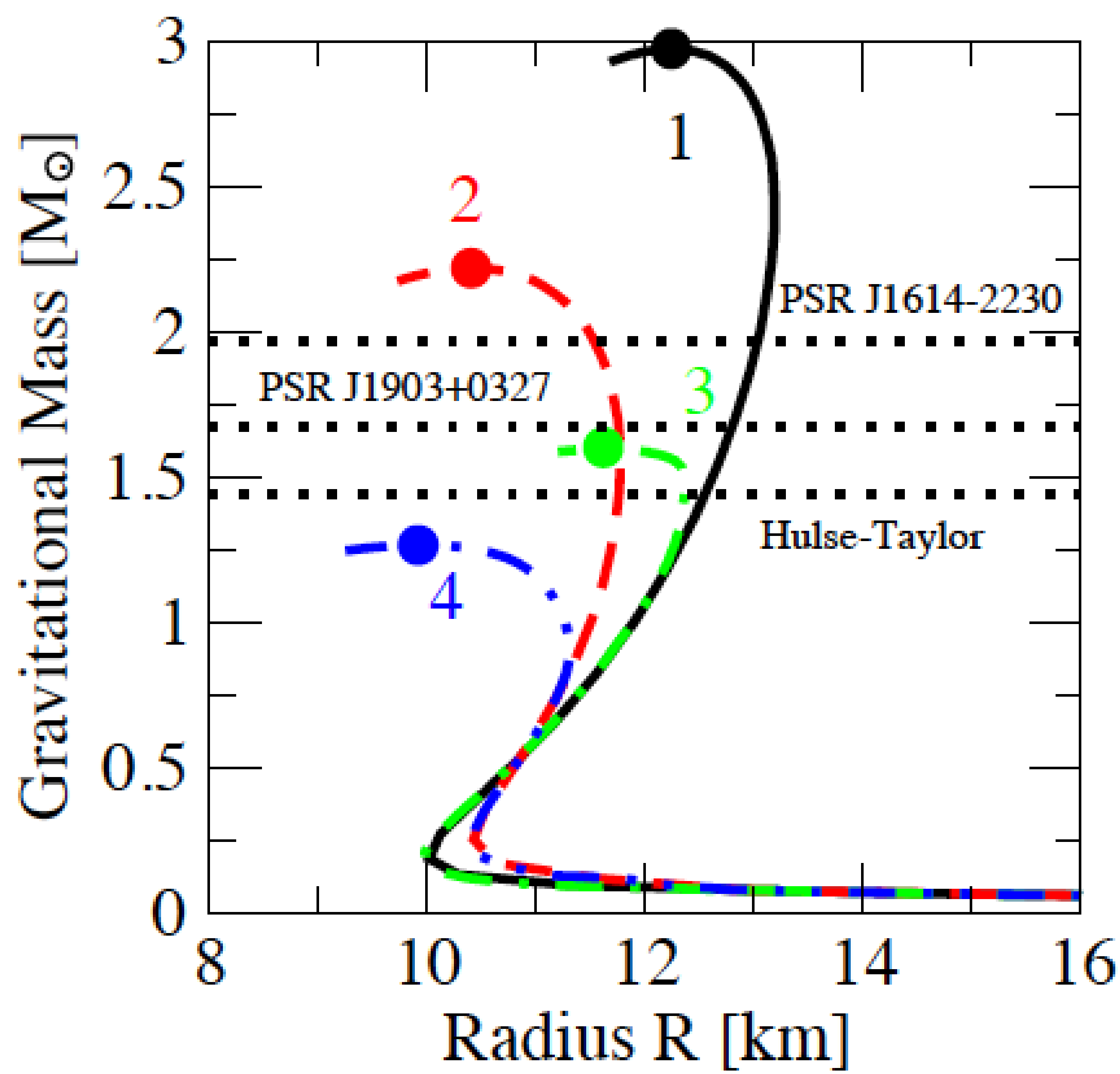
Maximum Mass with Hyperons (1)



Brückner-Hartree-Fock calculations:
Argonne V_{18} : NN interaction
GLMM or UIX NN : three-body forces
NSC89 (NN+NY) and NSC97 (NN+NY+YY) : Nijmegen soft-core

[Maximum mass of neutron stars](#)
Schulze, H.-J.; Polls, A.; Ramos, A.; Vidaña, I.
[2006PhRvC..73e8801S](#)

Maximum Mass with Hyperons (2)



Simple model (Skyrme-like) model of three-body forces.

1 and **2** : no hyperons, 2 models of three-body forces.
4: with hyperons and NNN TBF.
3: with hyperons and TBF: NNN NNY,NYY, YYY

Estimation of the effect of hyperonic three-body forces on the maximum mass of neutron stars
Vidaña, I.; Logoteta, D.; Providência, C.; Polls, A.; Bombaci, I. 2011EL.....9411002V

Maximum Mass with Hyperons (3): A Successful Model

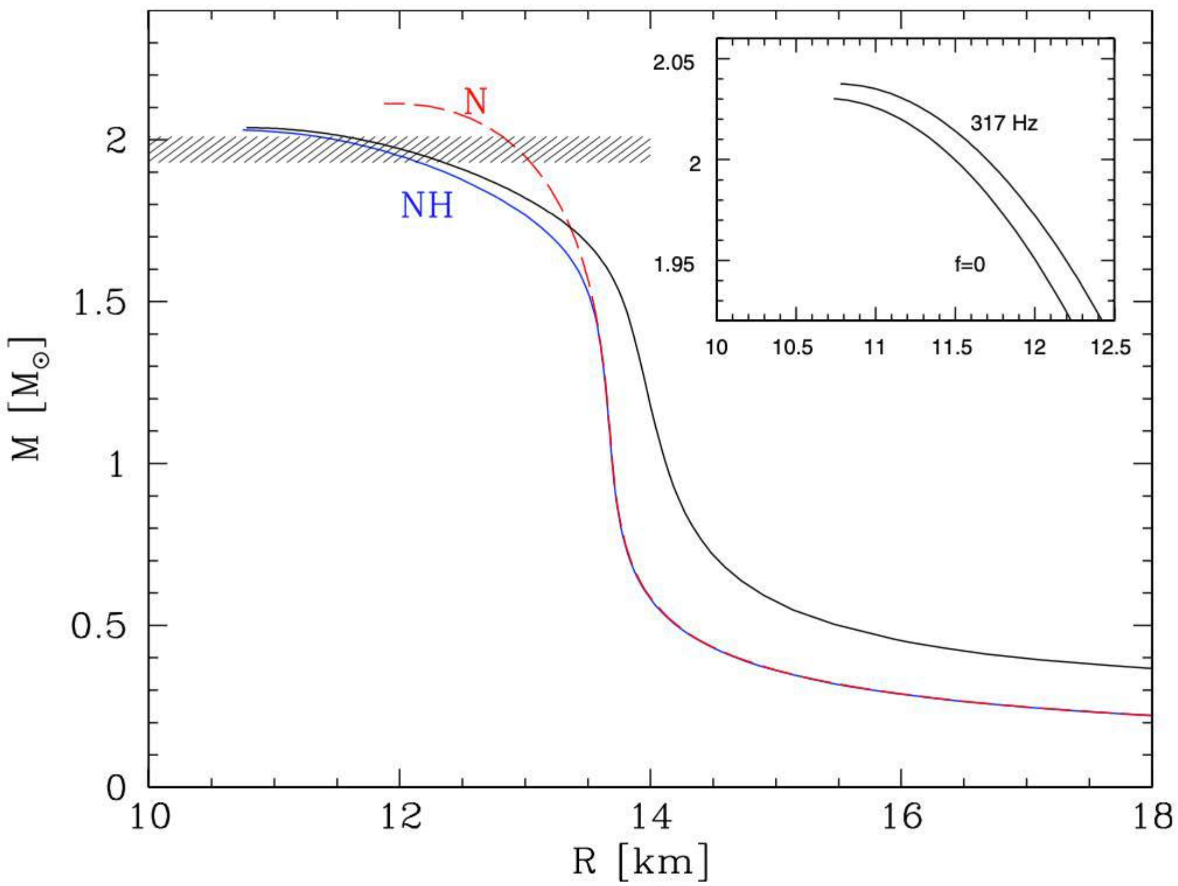


Fig. 5. Gravitational stellar mass, M, versus circumferential radius, R, calculated for the EOS.N and EOS.NH. Only stable configurations are displayed. Inset: effect of rotation at $f = 317$ Hz on the M – equatorial circumferential radius curve near $M_{\max}$.

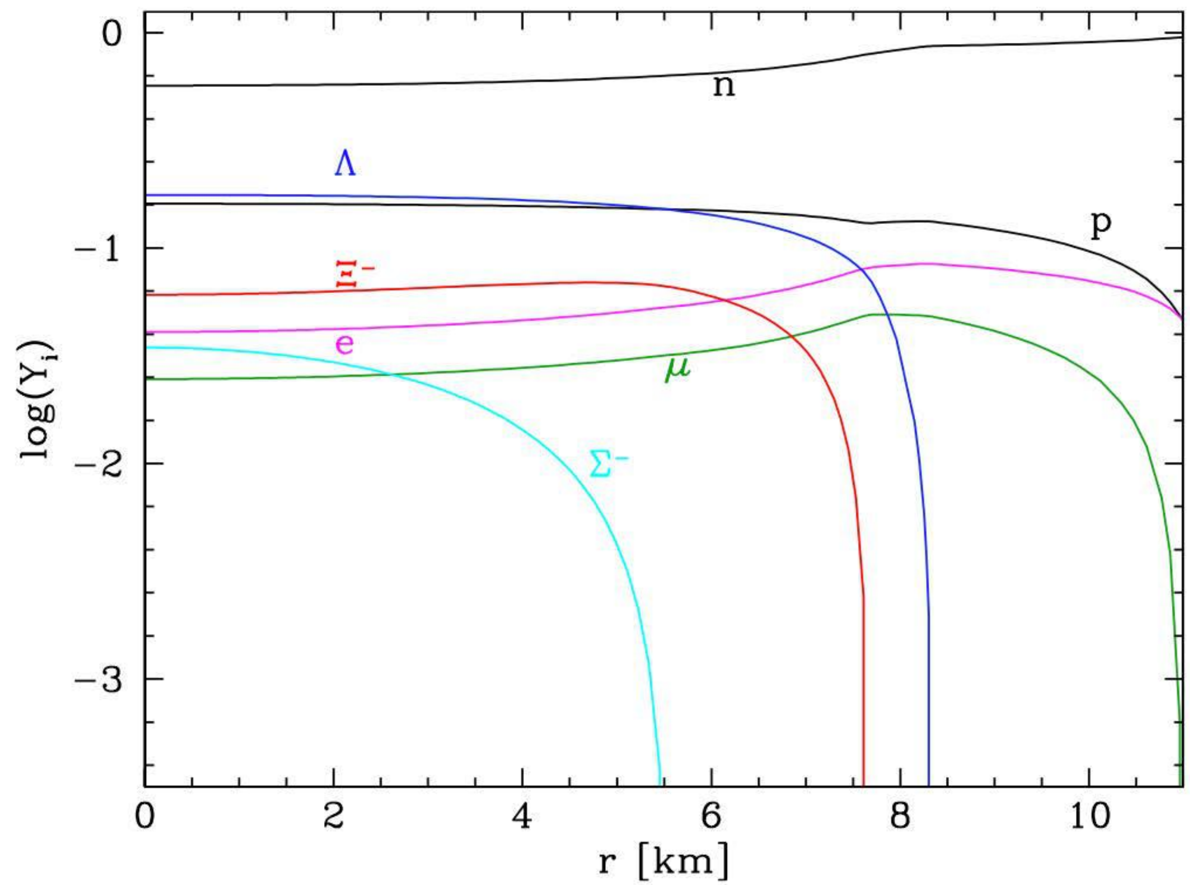


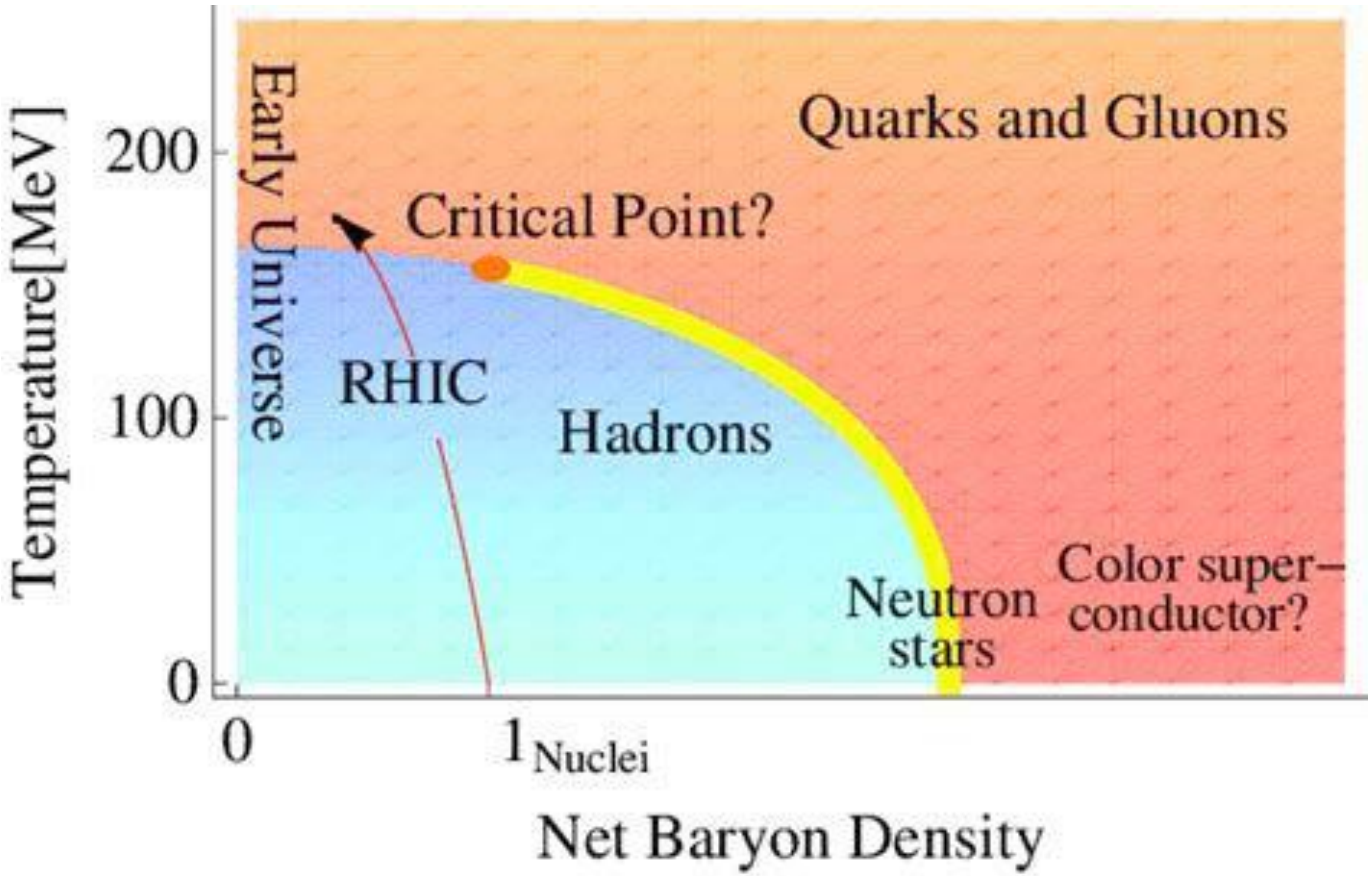
Fig. 4. The logarithm of the number fractions of the constituents of dense matter, $\log_{10}(Y_i)$, versus circumferential radius, in the liquid core of a 1.97 $M_\odot$ star model based on the BM165 EOS.

Recipe for success:
- Non-linear self-interactions for mesons (4-th degree)
- Include strange mesons that do not couple to nucleons

Hyperons in neutron-star cores and a 2 M_{sun} pulsar
Bednarek, I.; Haensel, P.; Zdunik, J. L.; Bejger, M.; Mańka, R.
2012A&A...543A.157B

Quark Matter
in
Neutron Stars

The QCD Phase diagram



Pressure: Quark Matter vs Nucleons

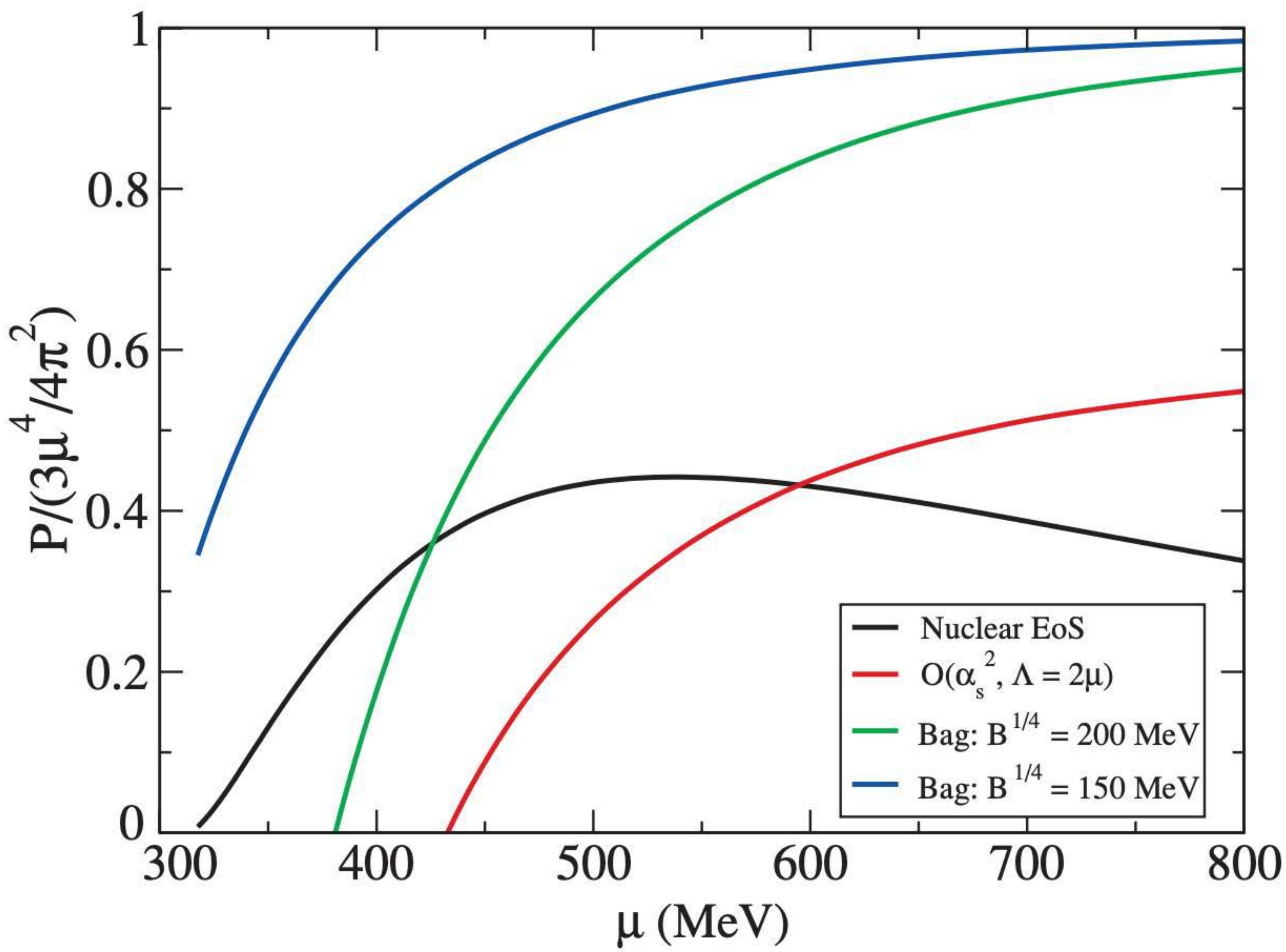
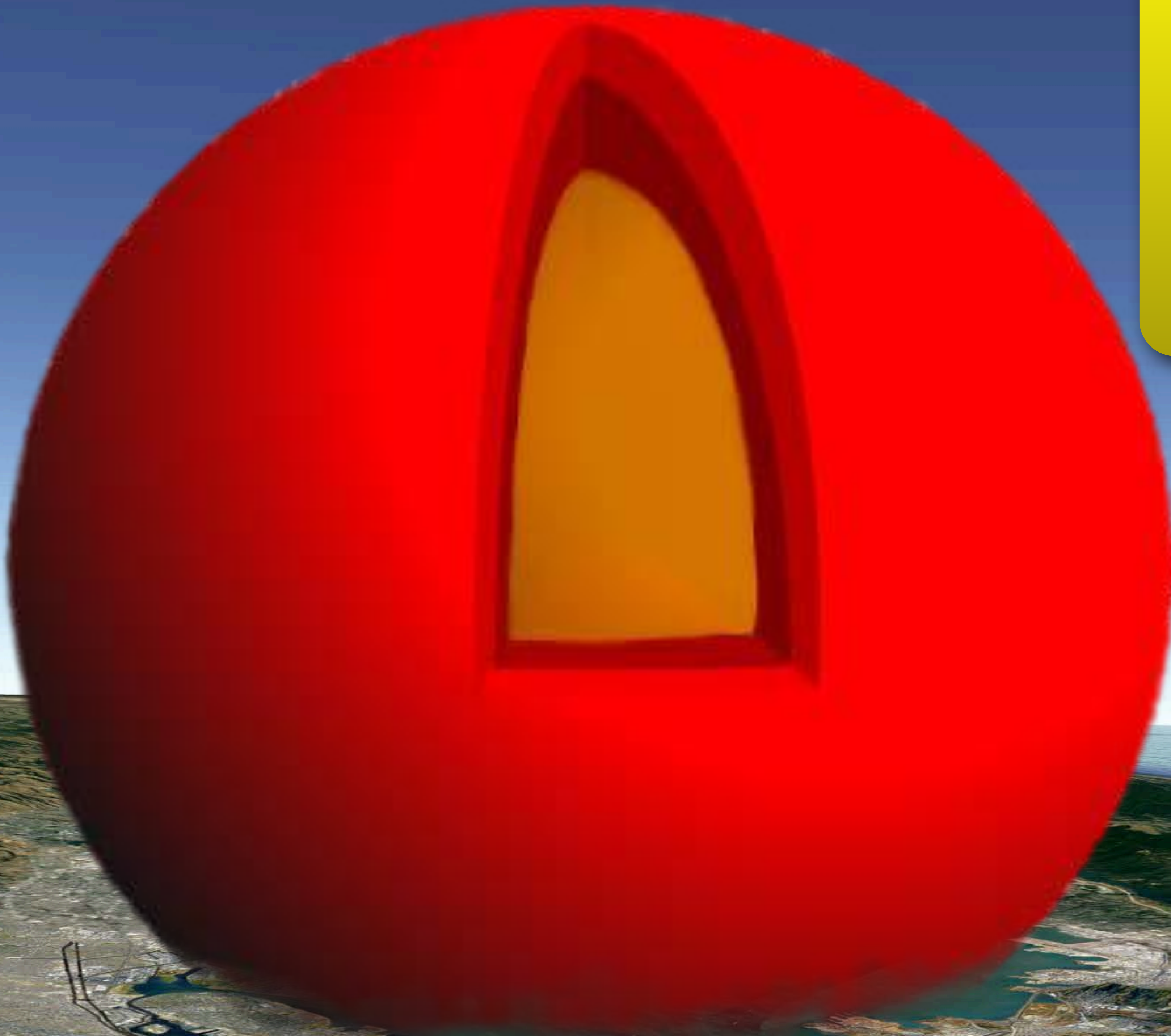


Figure 12 Pressure versus baryon chemical potential for several quark matter equation of state models. For reference, the pressure in the nuclear model is also shown.



The Interior of
Neutron Stars
is one of the
Best Kept Secrets
of the Universe

Tomorrow's
Lecture:
not all of this
theory is
Fairy Tale


"That's all Folks!"