Gravitational-Wave Physics and Astronomy

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California State University Fullerton
2023 N3AS Summer School in Santa Cruz
Session 11: July 22 and 23rd

Image: LIGO Livingston Observatory
Credit: LIGO

GW PAC NICHOLAS AND LEE BEGOVICH Center for Gravitational-Wave Physics and Astronomy

CALIFORNIA STATE UNIVERSITY FULLERTON

- Established 2012 with mission: research, education, and outreach in gravitational-wave science
- 4 professors, 1 software developer, 2 postdocs, 1 admin, 29 students, 60 alumni

We are grateful to acknowledge: the National Science Foundation, the Research Corporation for Science Advancement, Dan Black and Family and Nancy Goodhue-McWilliams

NSF

RESEARCH CORPORATION for SCIENCE ADVANCEMENT

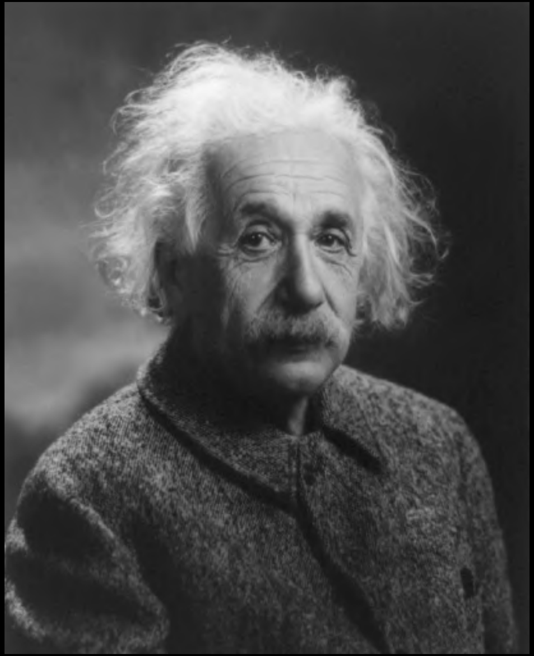
Goals

- Day 1:
  - What are gravitational waves?
  - How can we detect them?
  - Why do the detectors need to use such cool technology?
- Day 2:
  - What has been observed so far?
  - How are LIGO and Virgo being improved?
  - What will the next generation of gravitational wave detectors do?

Gravitational Waves

Albert Einstein, 1915

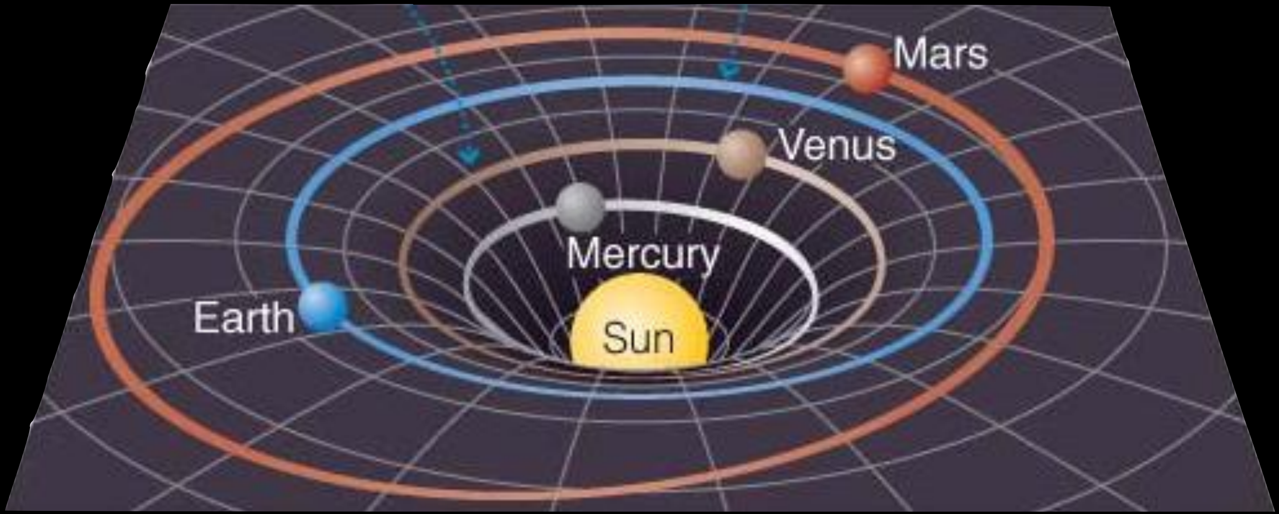
- The Theory of General Relativity is Einstein's theory of gravity
- More accurate than Newton's view of gravity as a force
    - Especially for strong gravity and high velocity
- Key idea is that gravity is an effect of the curvature of space and time



Credit: The Library of Congress

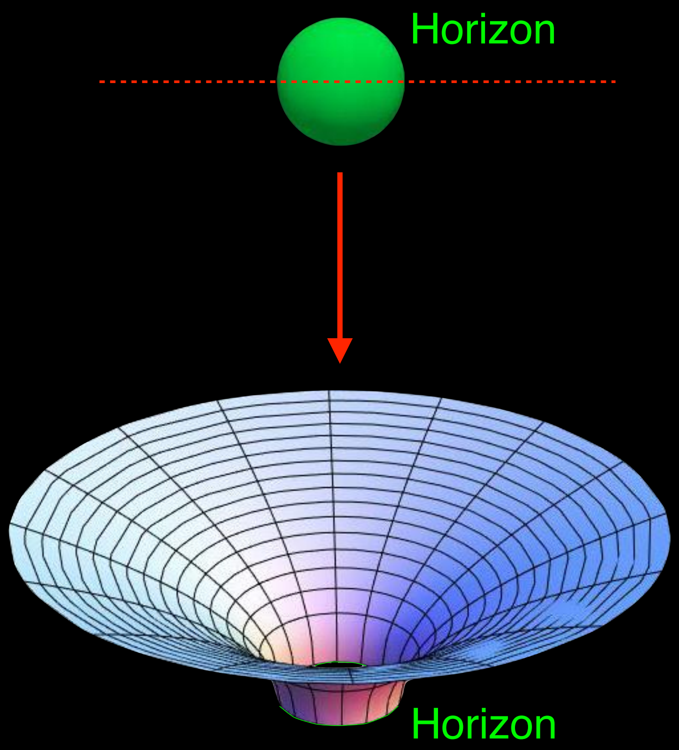
Curved spacetime

"Matter tells spacetime how to curve and space-time tells matter how to move."
- John Wheeler



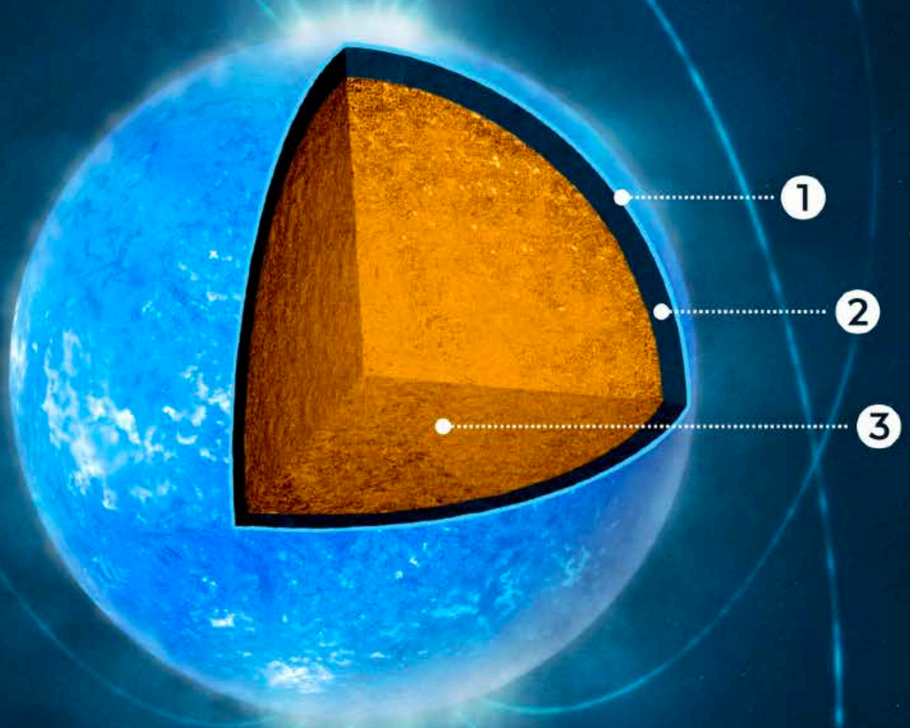
Extreme curvature: black holes

- Gravity so strong...
  - Nothing (even light) can escape from inside hole's horizon (surface)
  - Singularity inside horizon: *infinitely* strong gravity
- Formed when the most massive stars die



Credit: Geoffrey Lovelace

Neutron Stars: The Densest Matter



1 | OUTER CRUST (0.1 km)

NUCLEI
ELECTRONS

2 | INNER CRUST (0.5 km)

NUCLEI
ELECTRONS
SUPERFLUID NEUTRONS

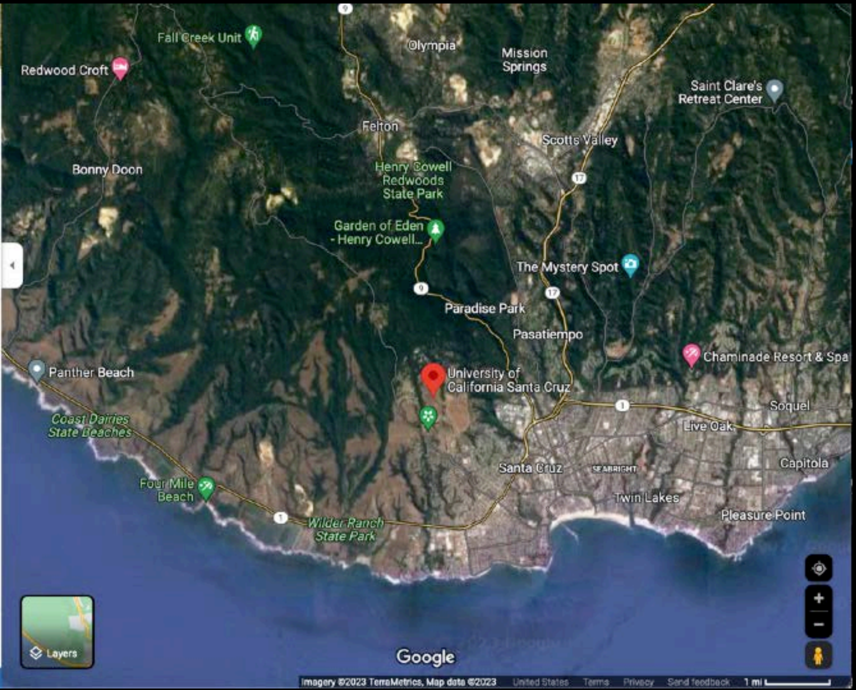
3 | CORE (10-13 km?)

SUPERFLUID NEUTRONS
SUPERCONDUCTING PROTONS
HYPERONS?
DECONFINED QUARKS?
COLOR SUPERCONDUCTOR?

Watts et al Rev. Mod. Phys. 88, 021001 (2016)

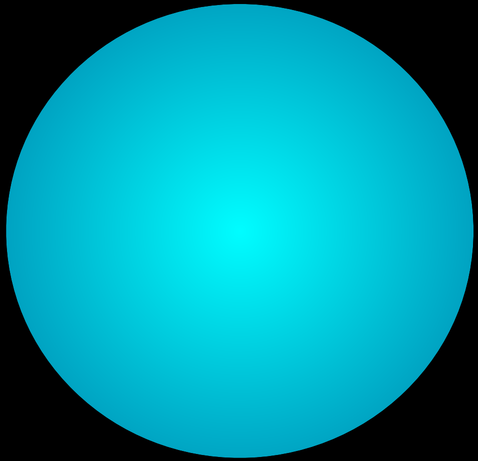
Black holes and neutron stars

UCSC Area



About 15 km

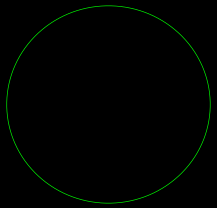
Neutron star



Mass = 1.5 ☀

Radius = 10-13 km

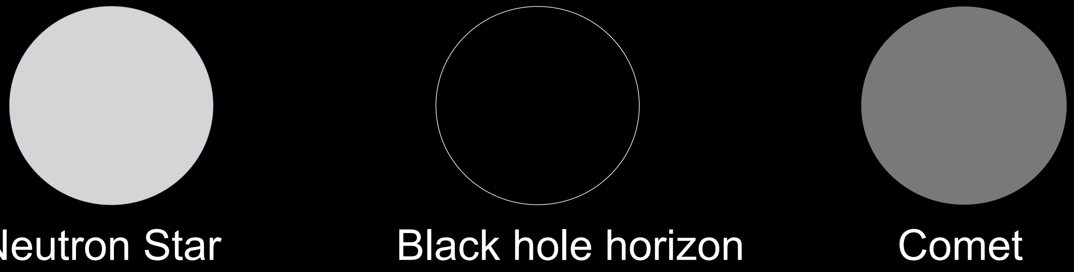
Black hole



Mass = 1.5 ☀

Radius = 4.5km

Assume that the following objects have the same surface area:



Which object would have the largest mass?

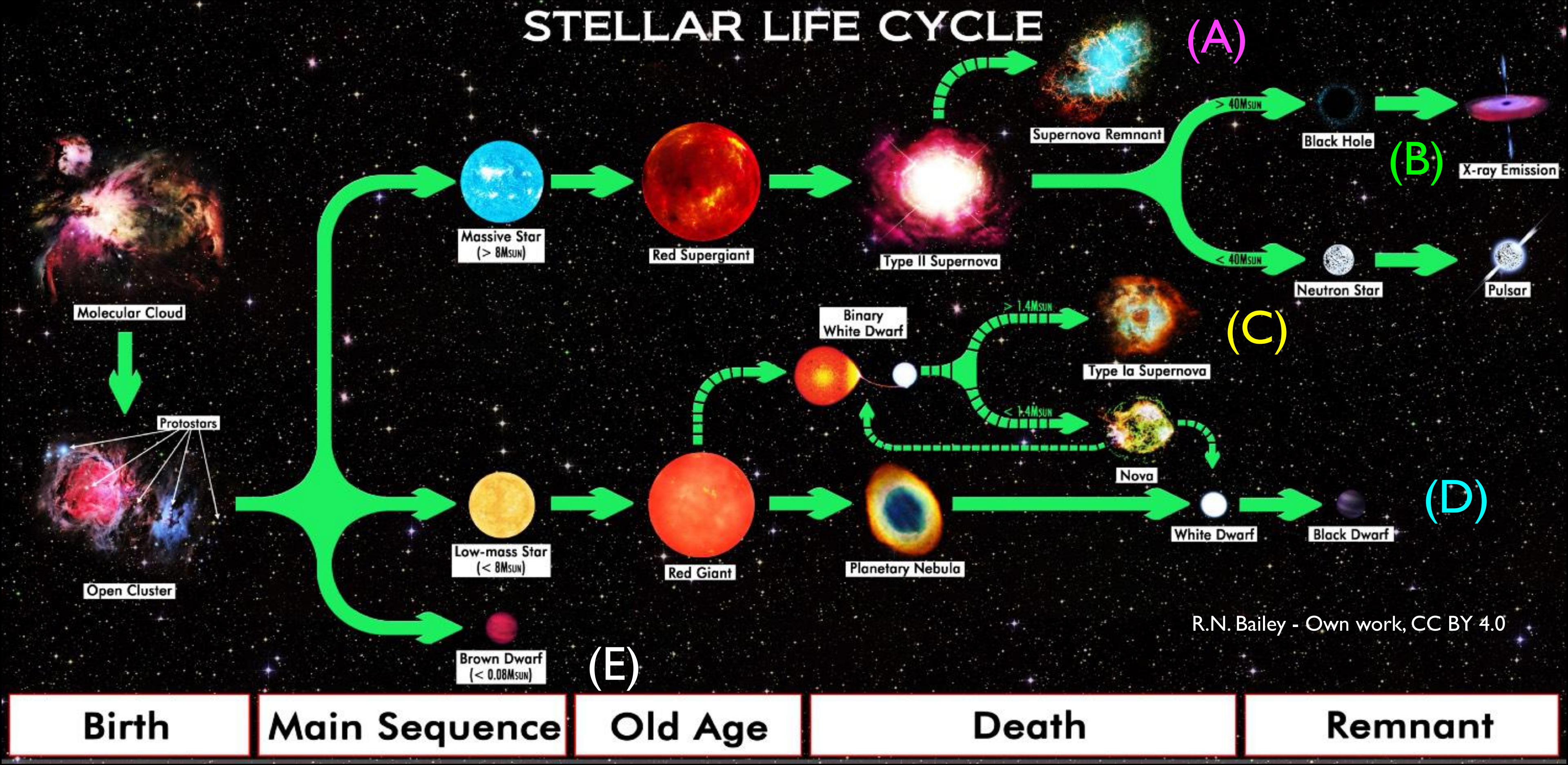
A. Neutron Star

B. Black hole

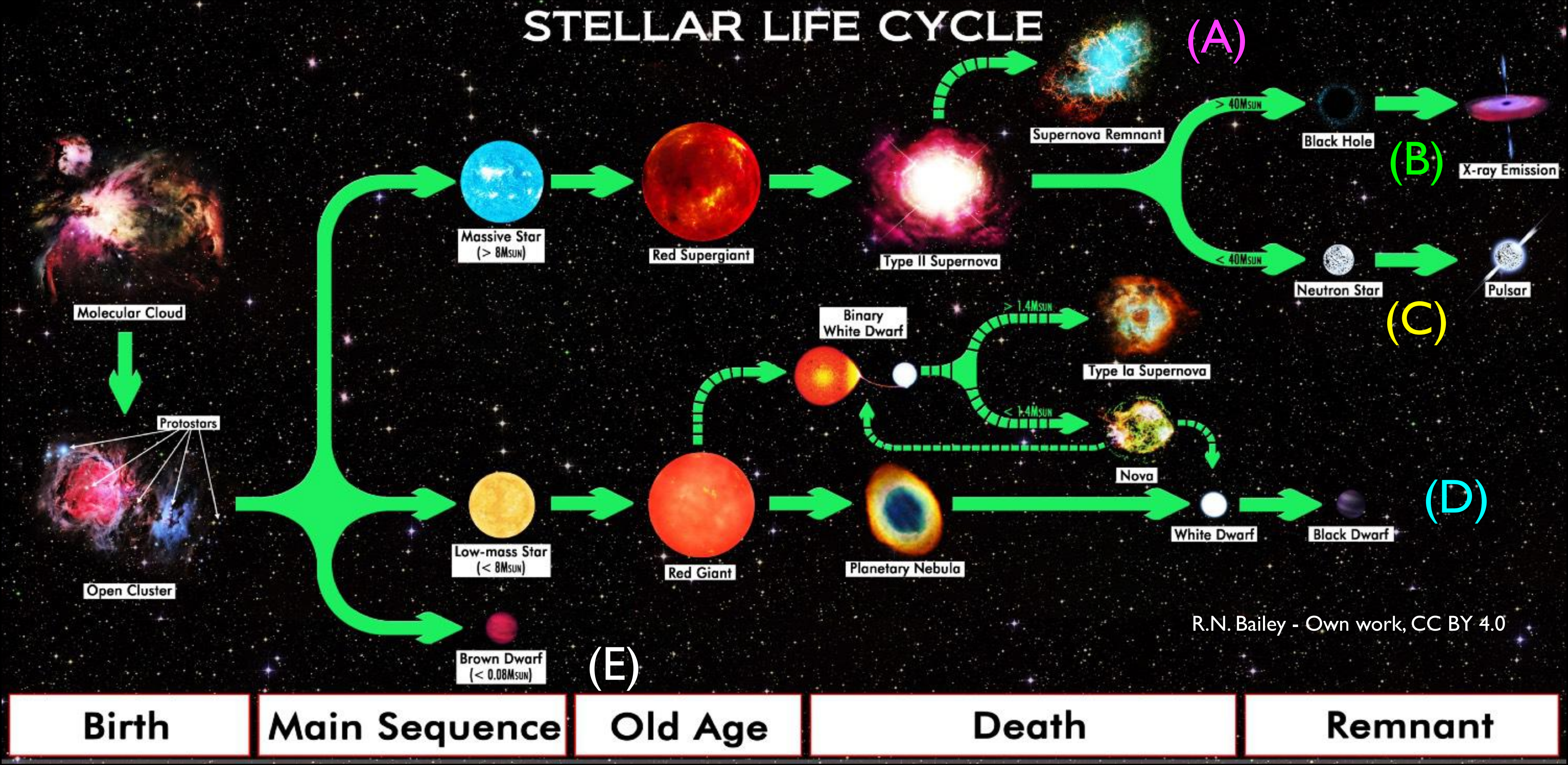
C. Comet

D. All would have the same mass

The following image shows the possible life cycles of various stars. Which letter marks the end of the Sun's life cycle?

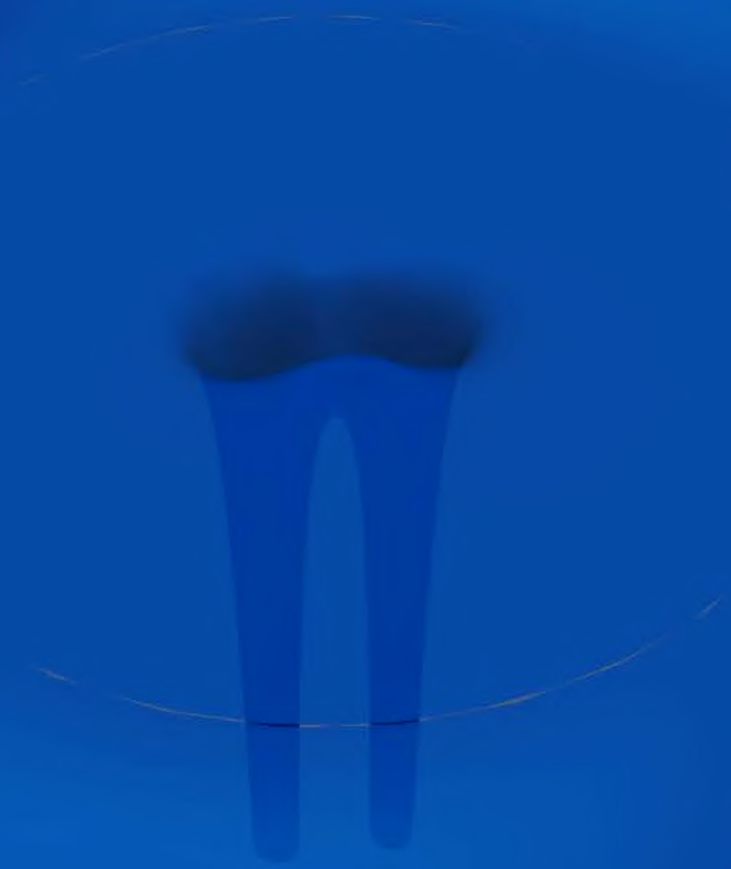


The following image shows the possible life cycles of various stars. Which letter marks the end of a main sequence star that with 10 solar masses?



Larry Kiwano, Astrocamp, Idyllwild CA, 2013

What causes the marbles to spiral into the large ball?



By CSUF Undergrad
Nick Demos
(now MIT PhD student)

SXS Collaboration: “Calculation of warped spacetime consistent with GW170104 (zoomed)”

Binary black holes

- Binary black holes obey equations of general relativity
    - Nonlinear: sum of solutions *not* a solution, can only calculate solutions numerically
        - Single black holes: stationary
          Schwarzschild (1915)
          Kerr (1963)
        - Merging black holes: dynamic
          First: Pretorius, PRL **95**, 121101 (2005)
          Many research groups (2005 – today)
            - Solutions include emitted gravitational waves
            - But how to detect these waves?

Single black hole

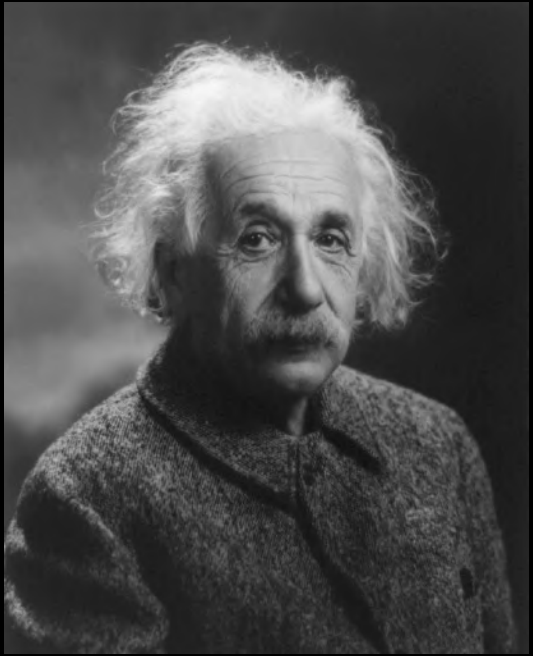
Colliding black holes

Images and movies courtesy Kip Thorne, CSUF undergraduate Nick Demos, SXS Collaboration

Slide by Geoffrey Lovelace

A new prediction

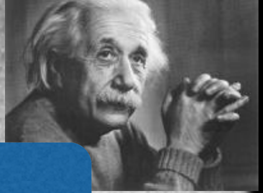
- Relativity: no signals can travel faster than light.
- But in Newtonian gravity: if you shake a mass, its gravitational field changes *instantaneously* throughout the universe.
- Hence: Gravitational waves must exist, playing the same role for the gravitational field as EM waves do for the electric field.
- The theory was so subtle, Einstein was never sure that this prediction was correct.
- Could the waves be just a math mistake, with no physical reality? Einstein didn't live long enough to learn the answer.
- (Paraphrased from original slides by Peter Saulson)



Credit: The Library of Congress

Gravitational waves

- Ripples in space-time that travel at light speed
- Generated by co-orbiting, or spinning, or exploding asymmetric objects
- Travel through the universe without being blocked by dust, galaxies, etc.
- Allow us to “see” astronomy inaccessible to light; universe’s most violent violent systems
- An entirely new spectrum in which to view the universe; Complimentary to light



One obtains the radiated energy of the system per unit time… sees that it must have in all conceivable situations a practically vanishing value.

Von A. Einstein.

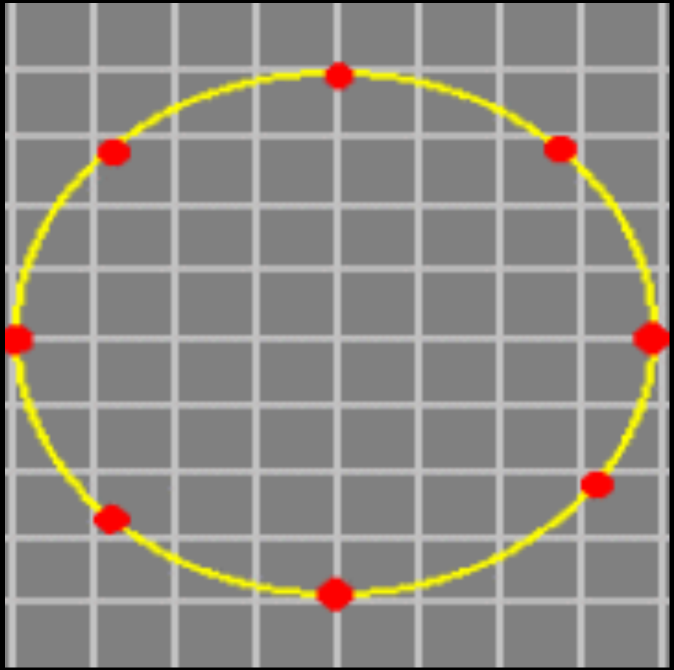
ein. Man erhält aus ihm also die Ausstrahlung A des Systems pro Zeiteinheit durch Multiplikation mit $4\pi R^2$:

$$A = \frac{\varkappa}{24\pi} \sum_{\alpha\beta} \left(\frac{\partial^3 J_{\alpha\beta}}{\partial t^3} \right)^2. \tag{21}$$

Würde man die Zeit in Sekunden, die Energie in Erg messen, so würde zu diesem Ausdruck der Zahlenfaktor $\frac{1}{c^4}$ hinzutreten. Berücksichtigt man außerdem, daß $\varkappa = 1.87 \cdot 10^{-27}$, so sieht man, daß A in allen nur denkbaren Fällen einen praktisch verschwindenden Wert haben muß.

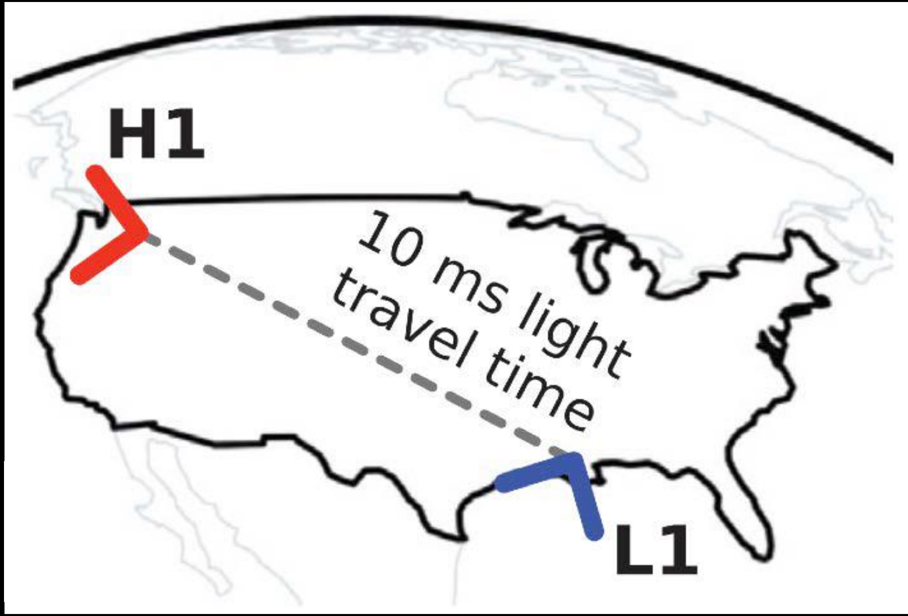
Effects of gravitational waves

- Cause the distance between objects to change as *strain*, $h(t) \sim \Delta L(t)/L$
- Fractional change shown 10% ($h \sim 10^{-1}$)
- Fractional change from gravitational waves arriving at Earth is $h \sim 10^{-21}$
- Suppose circle radius = 4 km, position change is 10^{-18} m ($\approx r_{\text{proton}}/1000$)

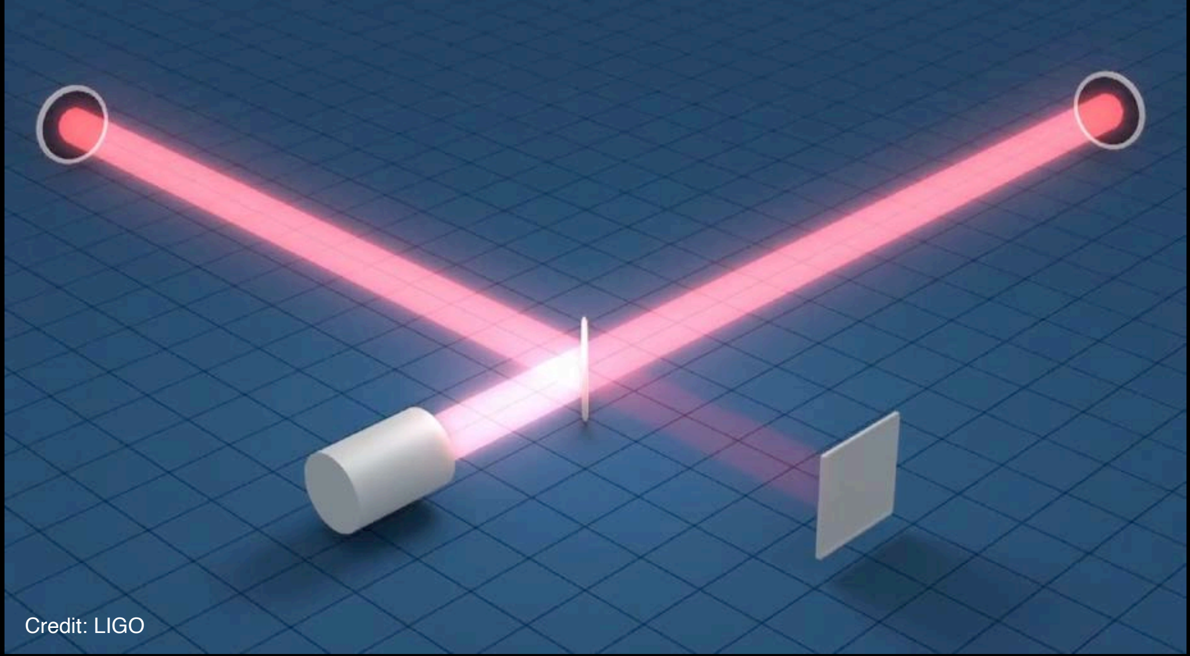


Freely falling massless test particles

The Detectors



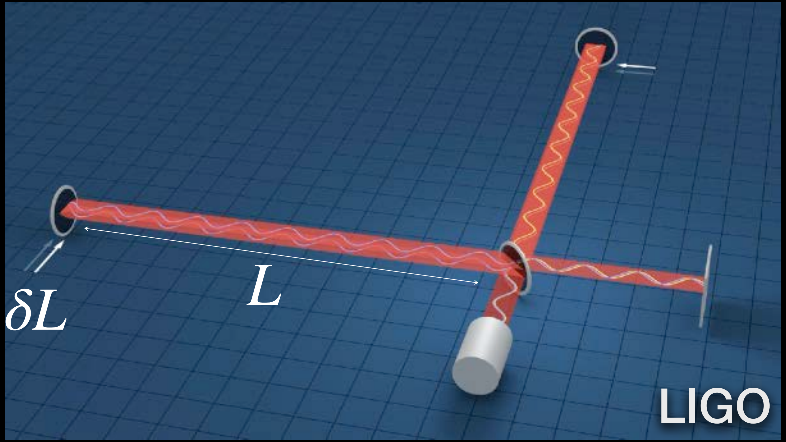
Phys. Rev. Lett.
116, 061102



- Transforms changes in arm-length difference $L_x - L_y$ (eg., from GWs) to changes of optical power P
  - Insensitive to laser frequency noise (common-mode, $L_x + L_y$)
  - Signal proportional to input power, strain, and length, $\delta L = hL$
- Michelson 1887: 10^{-8} m sensitivity (filtered sunlight, table floated on mercury)
- LIGO 2019: 10^{-20} m sensitivity (high laser power, optical cavities, vacuum, vibration isolation…)

Measuring gravitational radiation

$$h = 2\frac{\delta L}{L} \sim \frac{G}{c^2}\frac{M}{D_L}\frac{v_\perp^2}{c^2}$$



Gravitational waves come from oscillatory sources where mass is changing across your line of sight

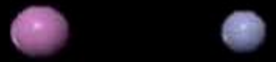
Amplitude falls off with distance from source

Slide by Jocelyn Read

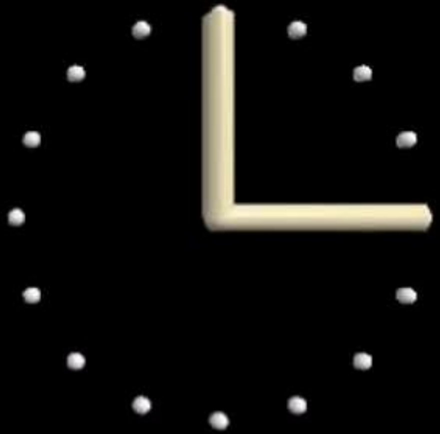
Keplerian orbits at separation a,

$$h \sim 10^{-21} \frac{100\ \mathrm{Mpc}}{D_L} \frac{M}{1.4\, M_\odot} \frac{R_S}{a}$$

Size of BH with mass M
$R_S = 2GM/c^2$



Time-varying
mass quadrupole

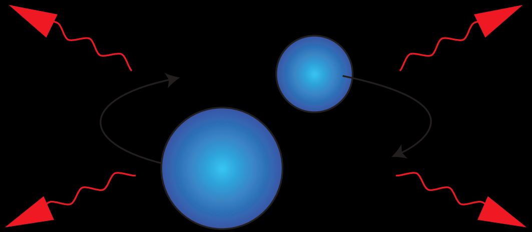


Stretch and squeeze
distances perpendicular
to direction of propagation

Movie by
Megan Loh, CSUF
https://www.youtube.com/watch?v=loZlghbMoW8

Slide by Jocelyn Read

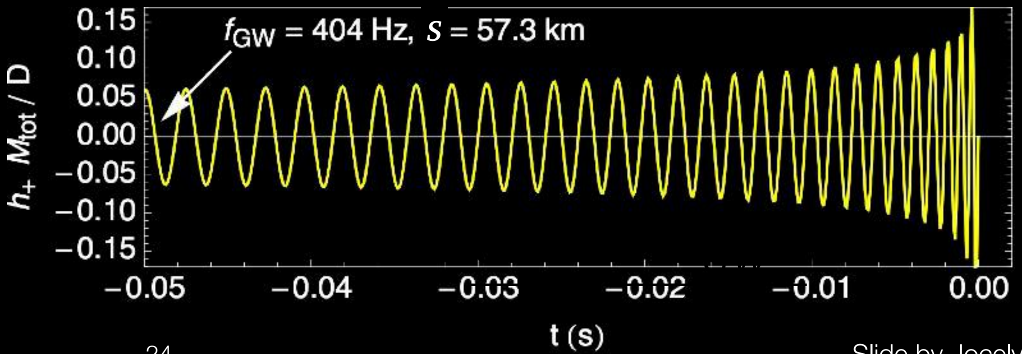
Binary luminosity



$$\frac{dE}{dt} = -\mathscr{L}_{GW} \sim 10^{59}\ \text{erg s}^{-1} \frac{R_S}{\boxed{a}}$$

The “chirp”

$$\frac{da}{dt} = \frac{-\mathscr{L}_{GW}}{dE_{orb}(a)/da}$$



Slide by Jocelyn Read

If we wanted to measure a 1,000Hz gravitational wave using a Michelson interferometer, what would be the best choice for L?

A. 4 km

B. 75 km

C. 1.5 Mkm

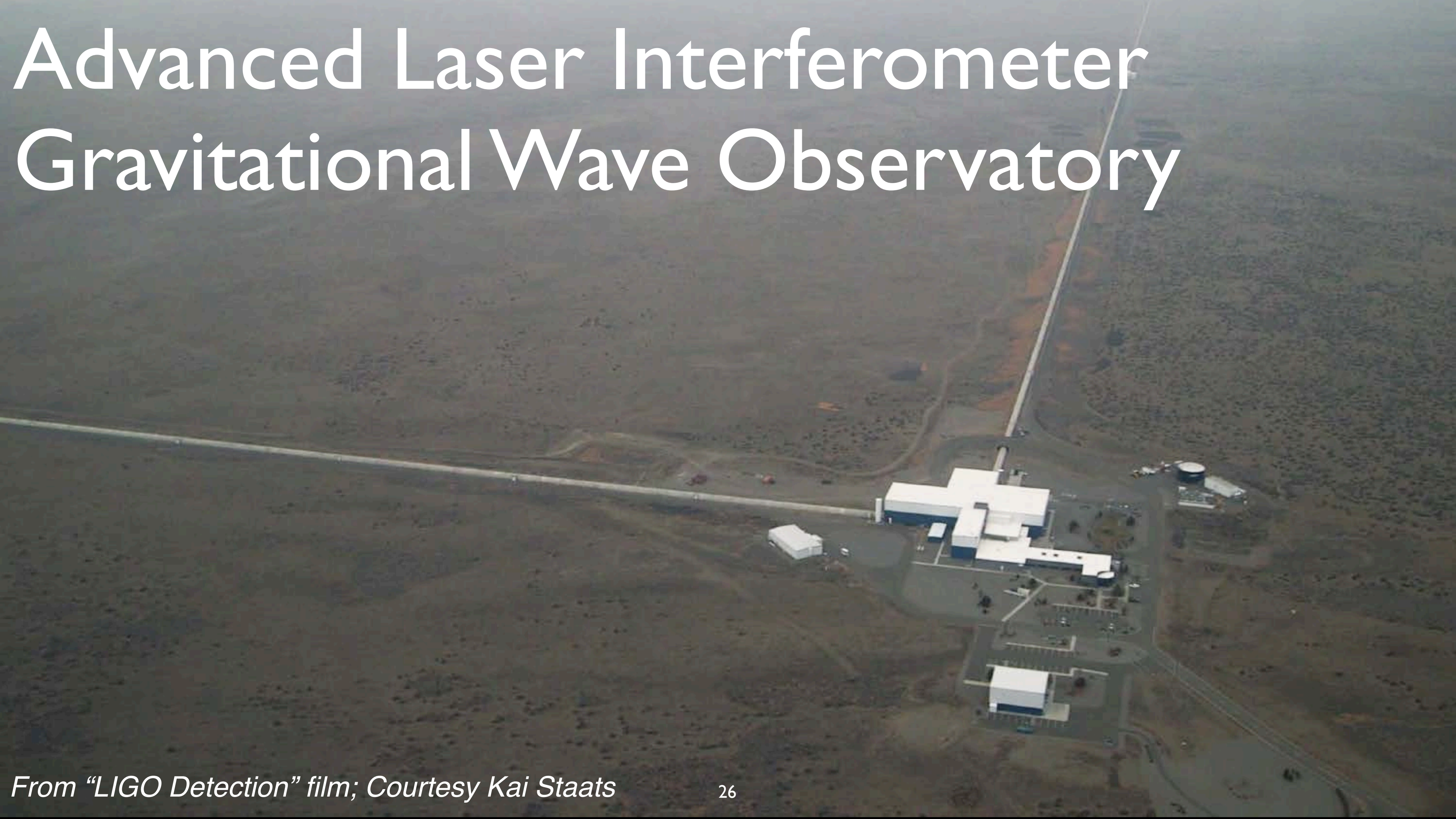
$$c = 3.0 \times 10^8\,\mathrm{m}$$

$$f = 1000\,\mathrm{Hz}$$

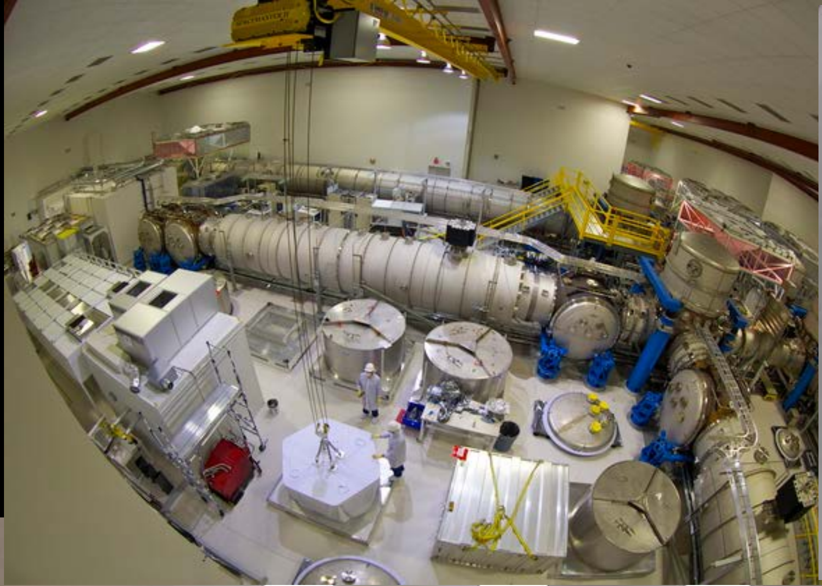
$$\lambda = c/f = 300{,}000\,\mathrm{m}$$

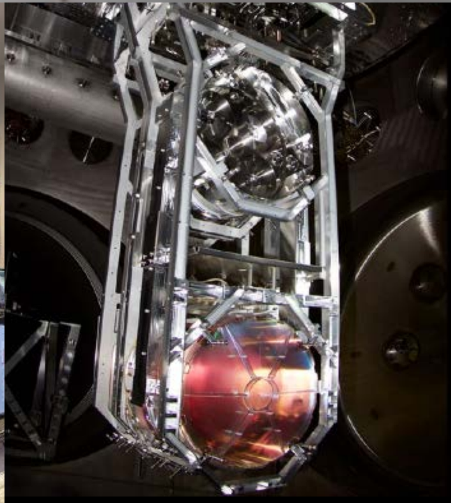
$$L_{opt} = \lambda/4 = 75{,}000\,\mathrm{m}$$

Advanced Laser Interferometer Gravitational Wave Observatory



From “LIGO Detection” film; Courtesy Kai Staats

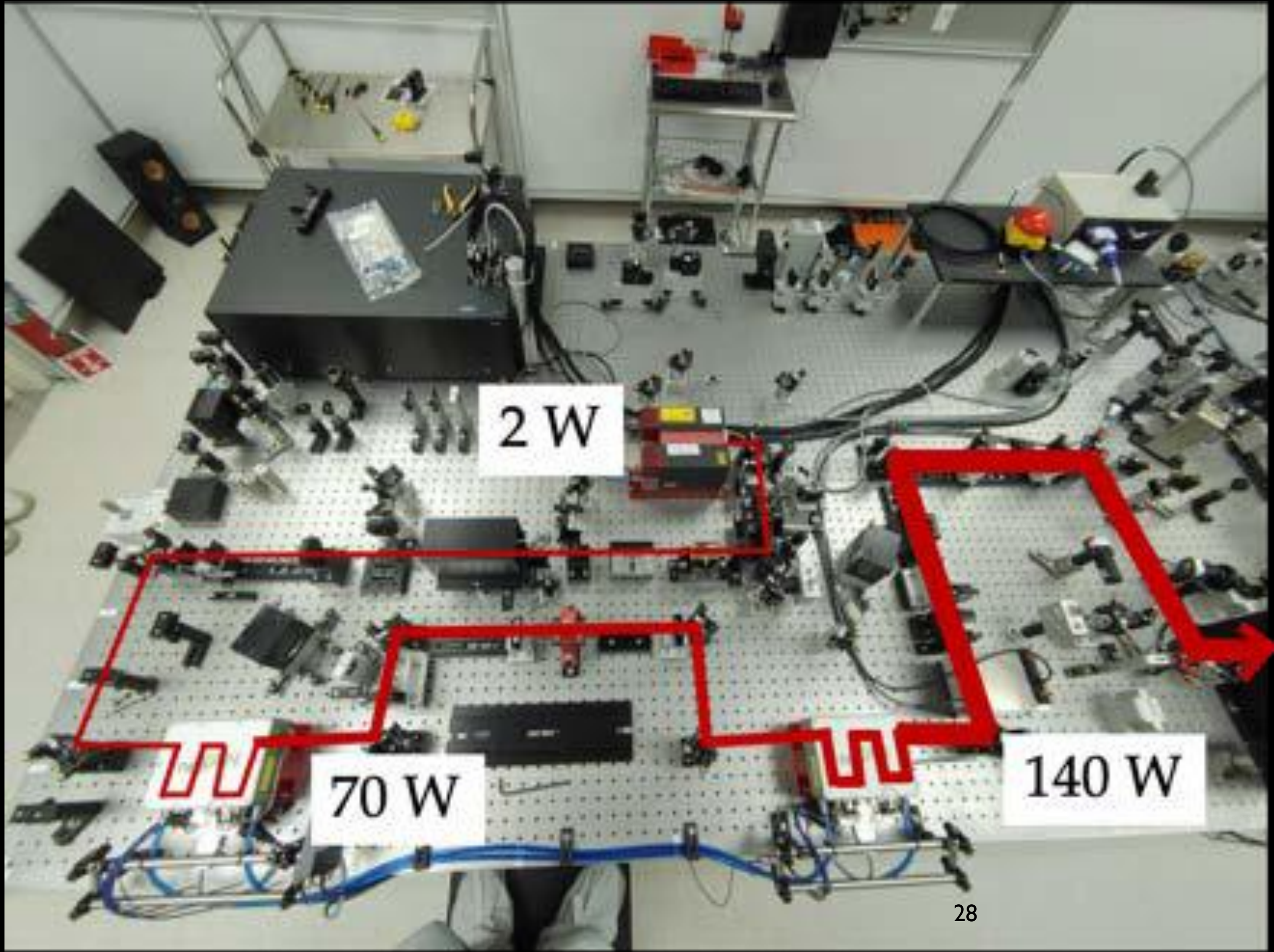


Credit: LIGO

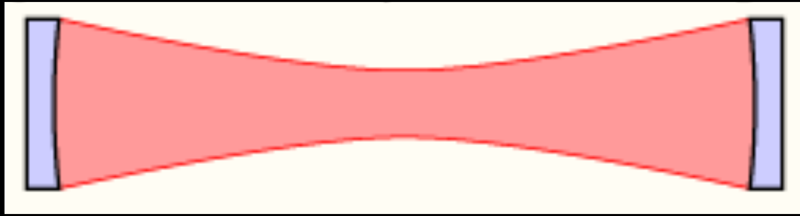
Pre-Stabilized Laser

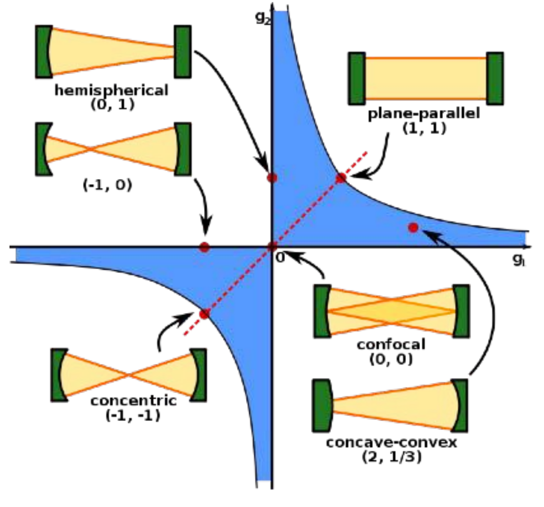
- 2W NPRO seed laser, 2 neoVAN-4S-HP amplification stages
- Delivers 110W to vacuum system



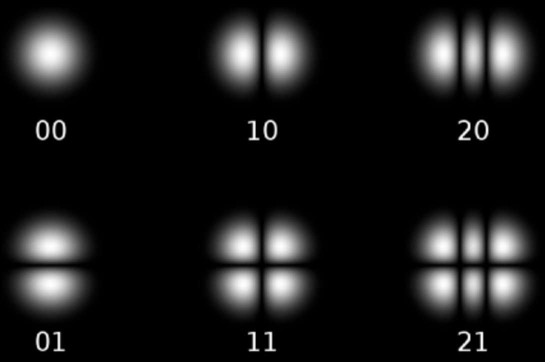
Credit: Elenna Capote and LIGO

Optical Cavity





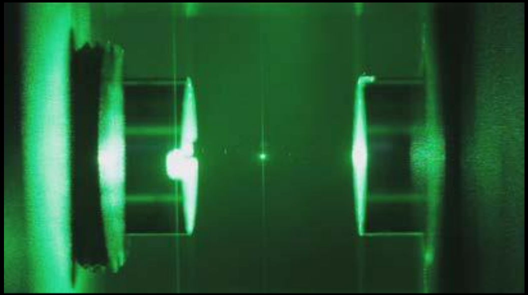
- Light bounces many times, building up power
- The longitudinal mode supported by the cavity is a standing wave
- The transverse modes that our cavities can support are TEM(mn) patterns



Images from Wikipedia, Optical Cavity

Optical Cavity Stability (General)

- If after traversing a cavity some number of times a light ray returns to its original location and orientation the cavity is said to be stable
- Stability criterion (calculable with Ray Transfer Matrices), where L is length of cavity, R_1 and R_2 are mirror curvatures:
- Stability parameter, g, for each mirror:



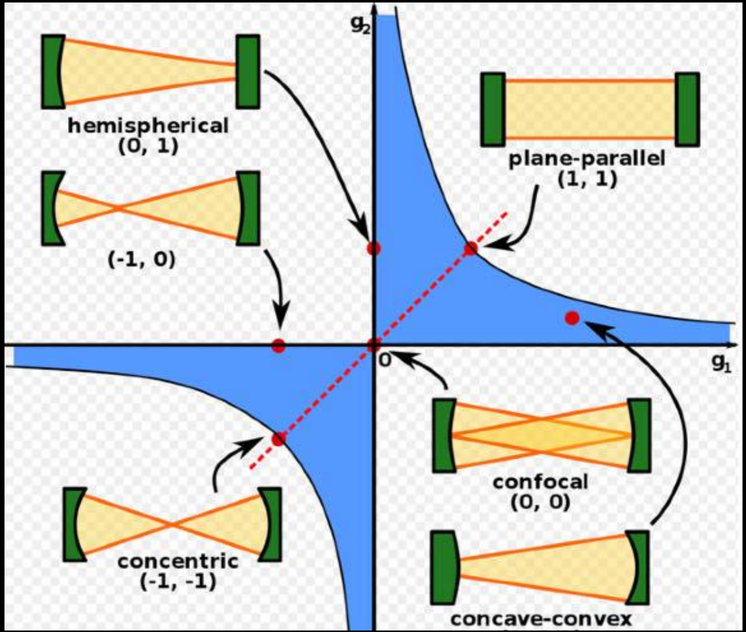
James Millen (UCL Physics & Astronomy)

$$0 \leqslant \left(1 - \frac{L}{R_1}\right)\left(1 - \frac{L}{R_2}\right) \leqslant 1.$$

$$g_1 = 1 - \frac{L}{R_1}, \qquad g_2 = 1 - \frac{L}{R_2}$$

$$0 \leq g_1 g_2 \leq 1$$

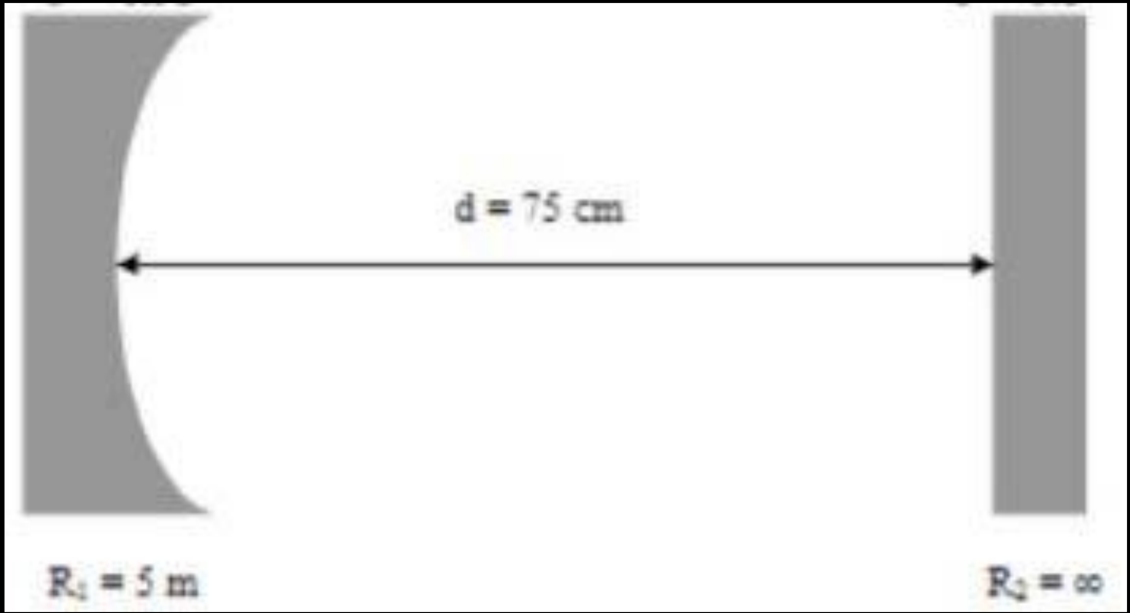
Plotting g_1 against g_2: blue areas are stable cavities, on the boundary are marginally stable



https://en.wikipedia.org/wiki/Optical_cavity

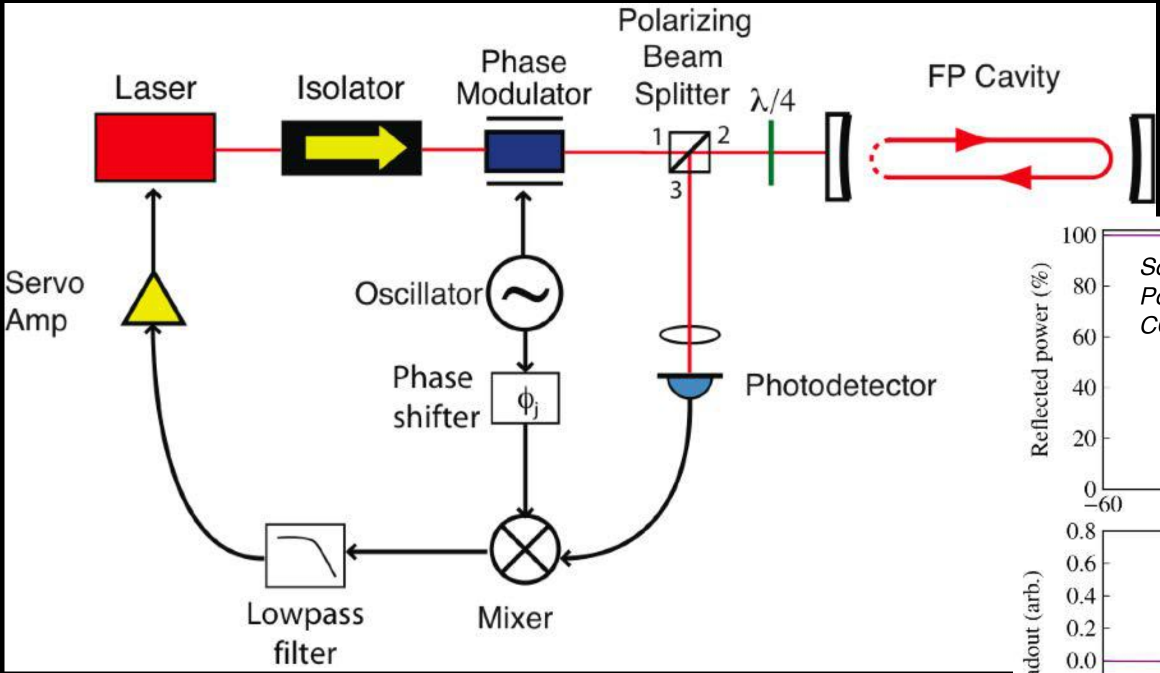
You are designing an optical cavity for a Nd:YAG laser operating at $\lambda_0 = 1.06\mu m$. Is the following cavity stable?

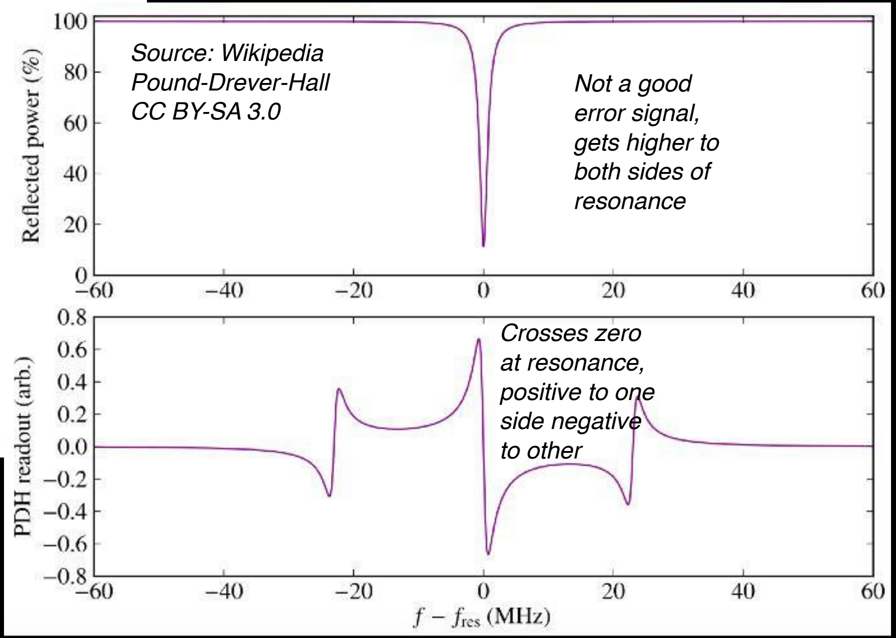
- A. Yes stable
- B. Marginally stable
- C. No unstable
- D. Not enough information was given
- E.



$$0 \leqslant \left(1 - \frac{L}{R_1}\right)\left(1 - \frac{L}{R_2}\right) \leqslant 1.$$

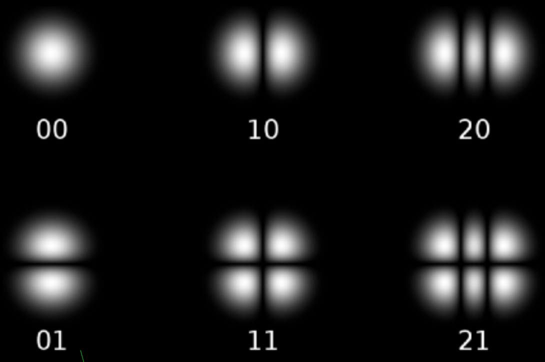
Pound-Drever-Hall cavity control: reflection locking technique



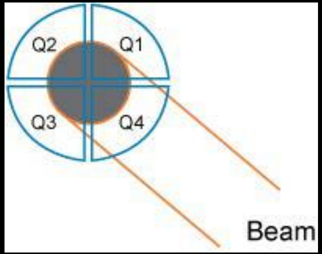


Alignment control

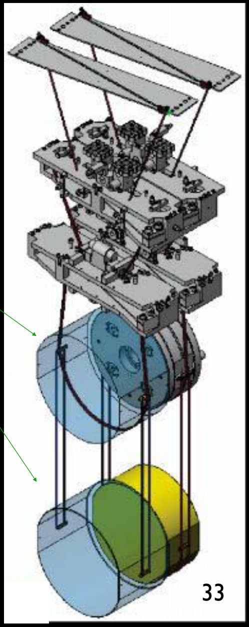
- The transverse modes that our cavities can support are TEM(mn) patterns
- Want to keep the cavities aligned to support the TEM00 mode and suppress other modes
- Use the differential wavefront sensing technique to measure alignment deviations
- The alignment error signal is turned into a control signal and sent to the mirrors



Control

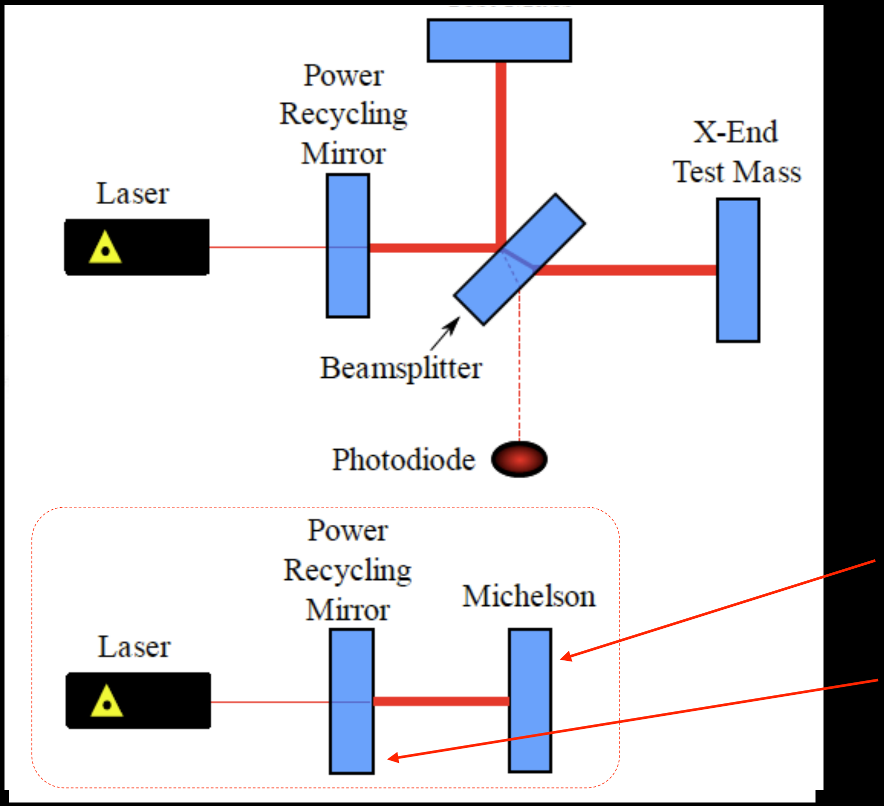


Credit: Thorlabs



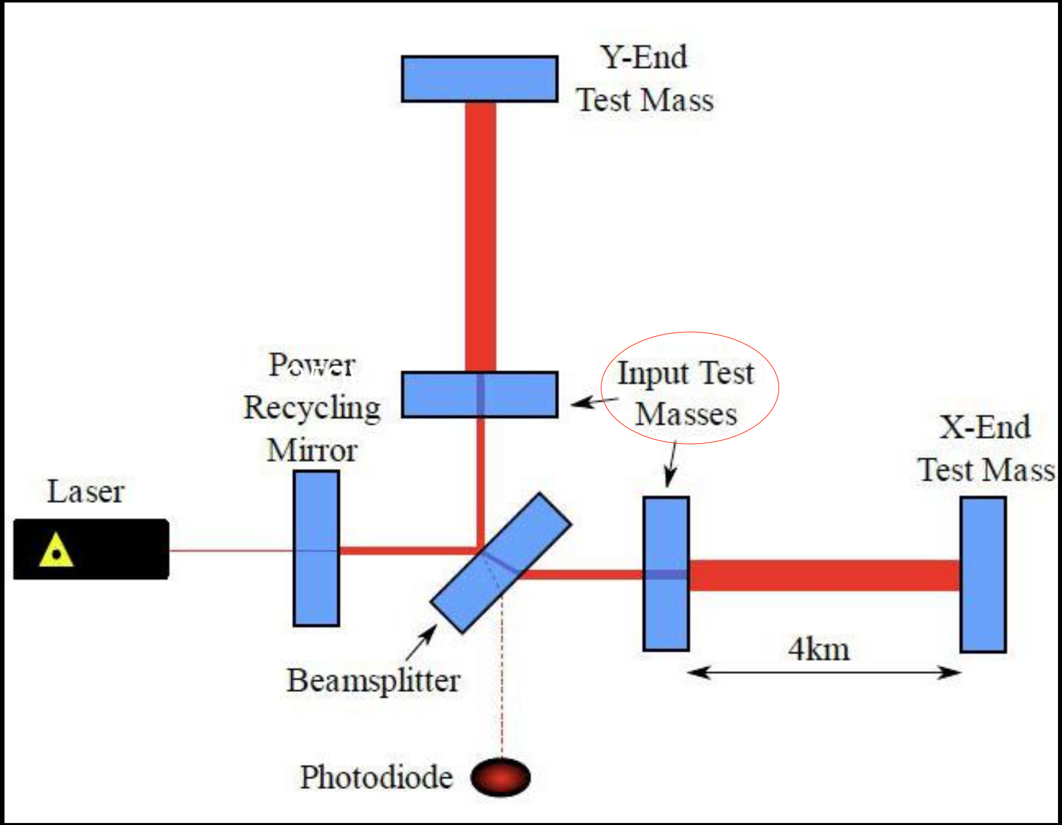
Credit: LIGO

Power Recycling



- Dark fringe: light returning from arms interferes destructively toward output port, constructively toward laser
- Shot noise scales as $P^{1/2}$, GW signal scales as P
- Michelson acts like highly reflective mirror for incoming light
- Adding another mirror builds up effective input power (40x)

Arm Cavities



- Optimal arm length: $\lambda_{GW}/4$ (so light spends half a period in an arm)
- Arm cavities make the 4km arms effectively longer (in some ways)
- They give ~ 200x optical power buildup
- With 60W input, 40x power recycling and 200x arm cavities, reach **~400kW in the arms!**

Wavefront Control

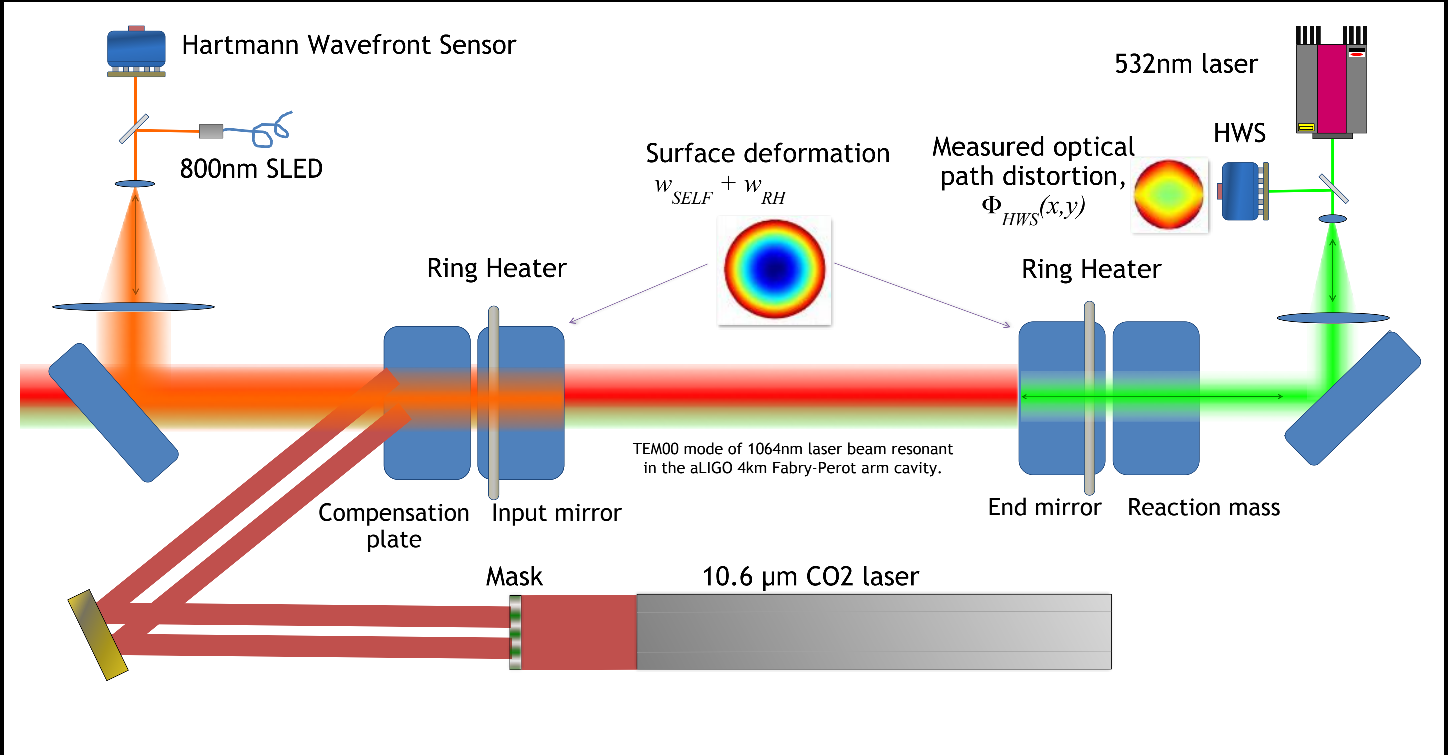
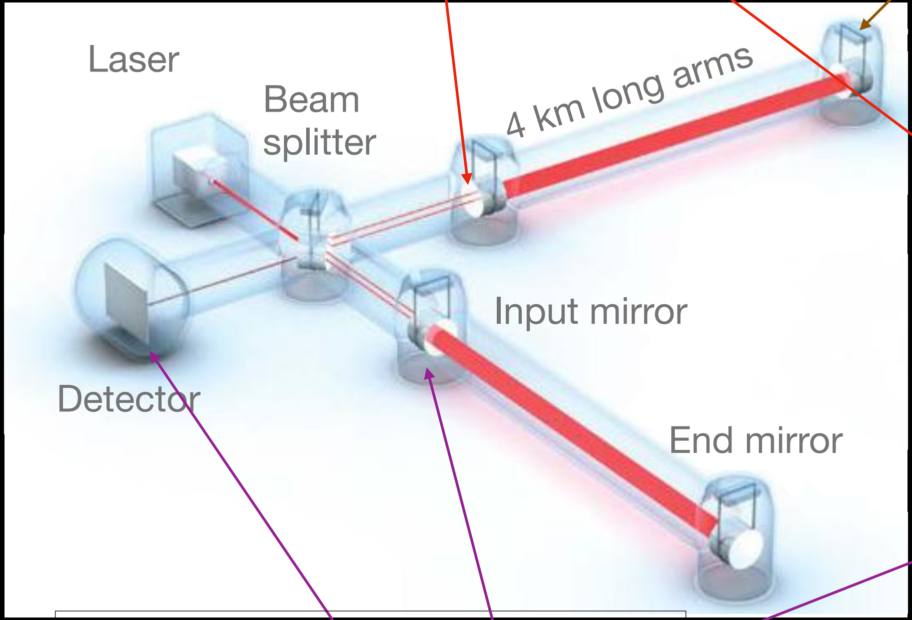


Diagram by Aidan Brooks, LIGO Caltech

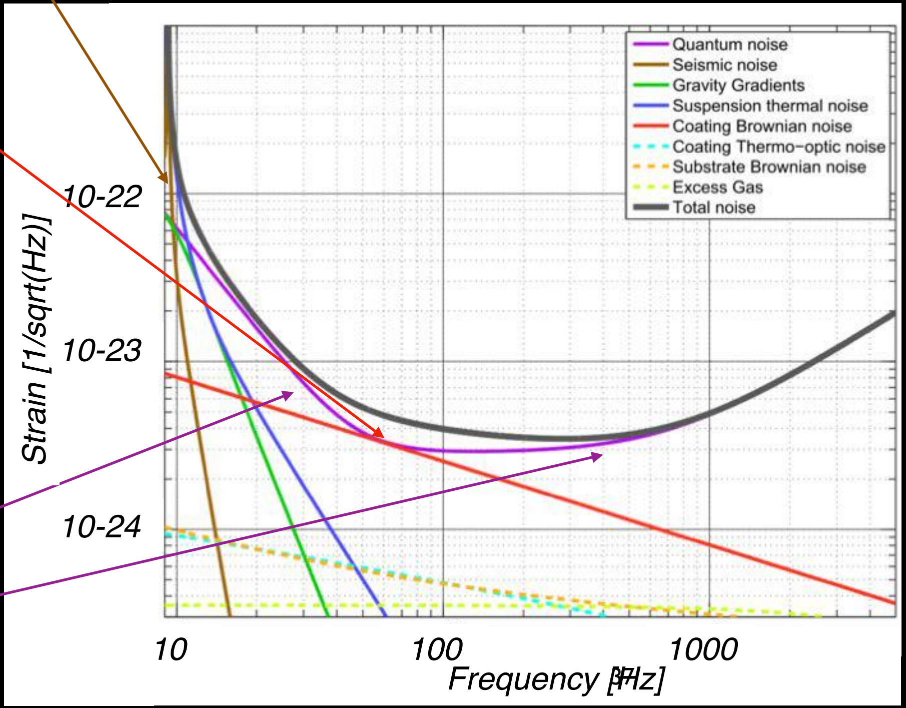
What limits Advanced LIGO?

Thermal motion of the mirror coatings

Seismic motion shakes mirrors

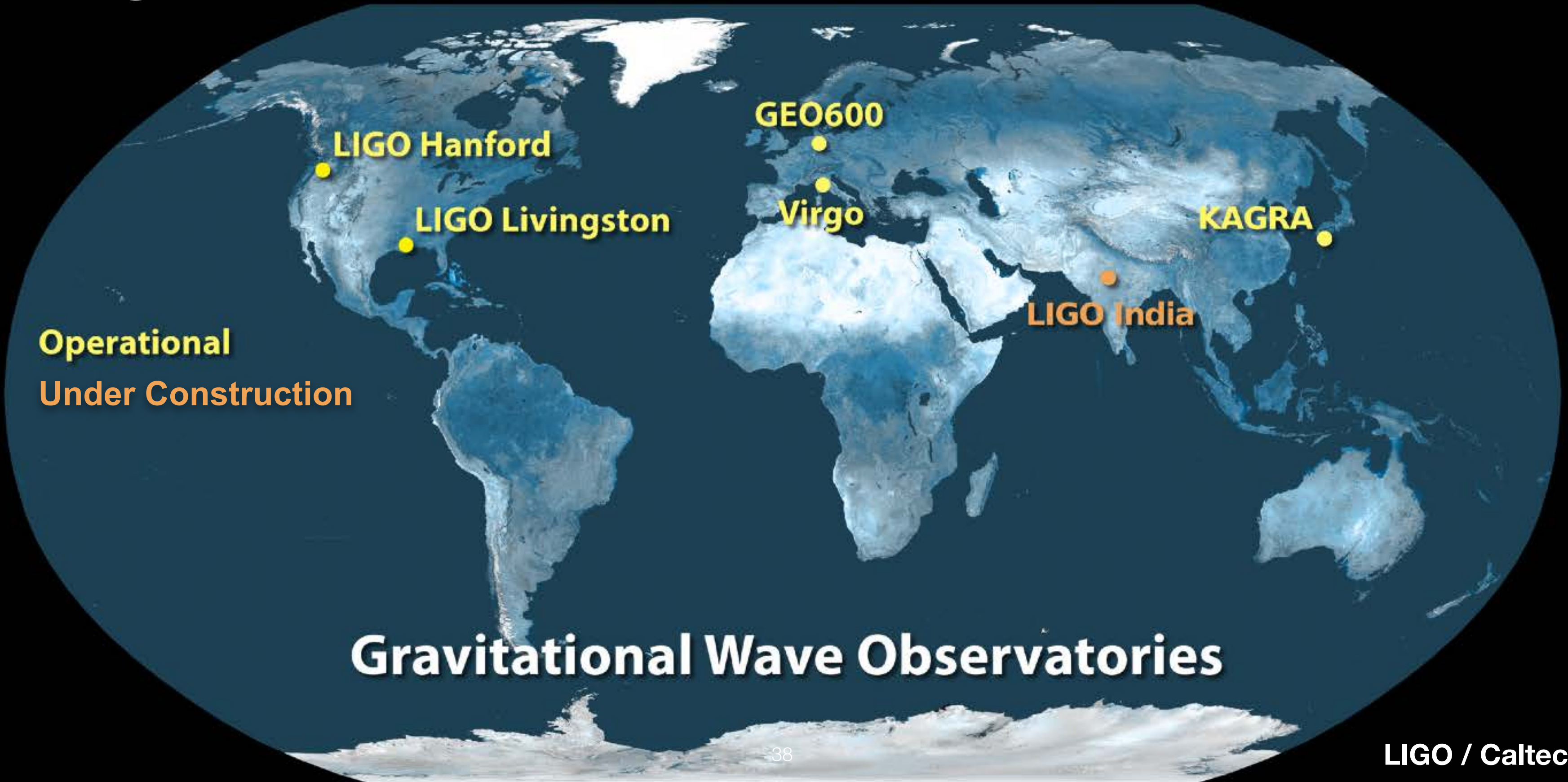


Class. Quantum Grav. 32 (2015) 074001



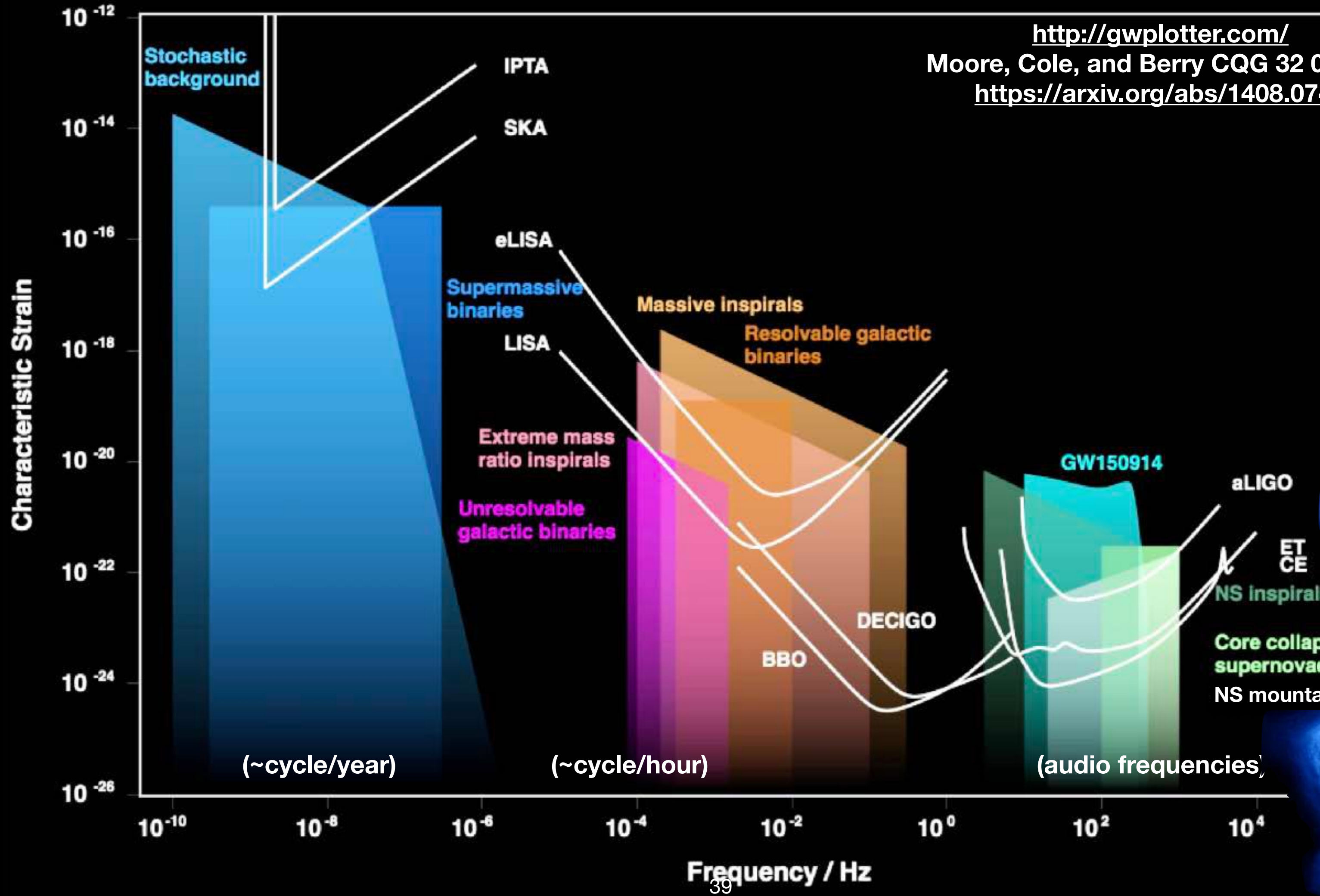
Quantum radiation pressure and shot noise

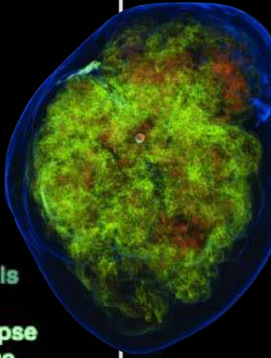
The current generation of gravitational-wave observatories

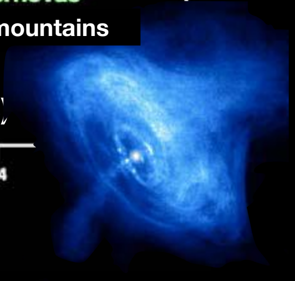


LIGO / Caltech

http://gwplotter.com/
Moore, Cole, and Berry CQG 32 015014
https://arxiv.org/abs/1408.0740



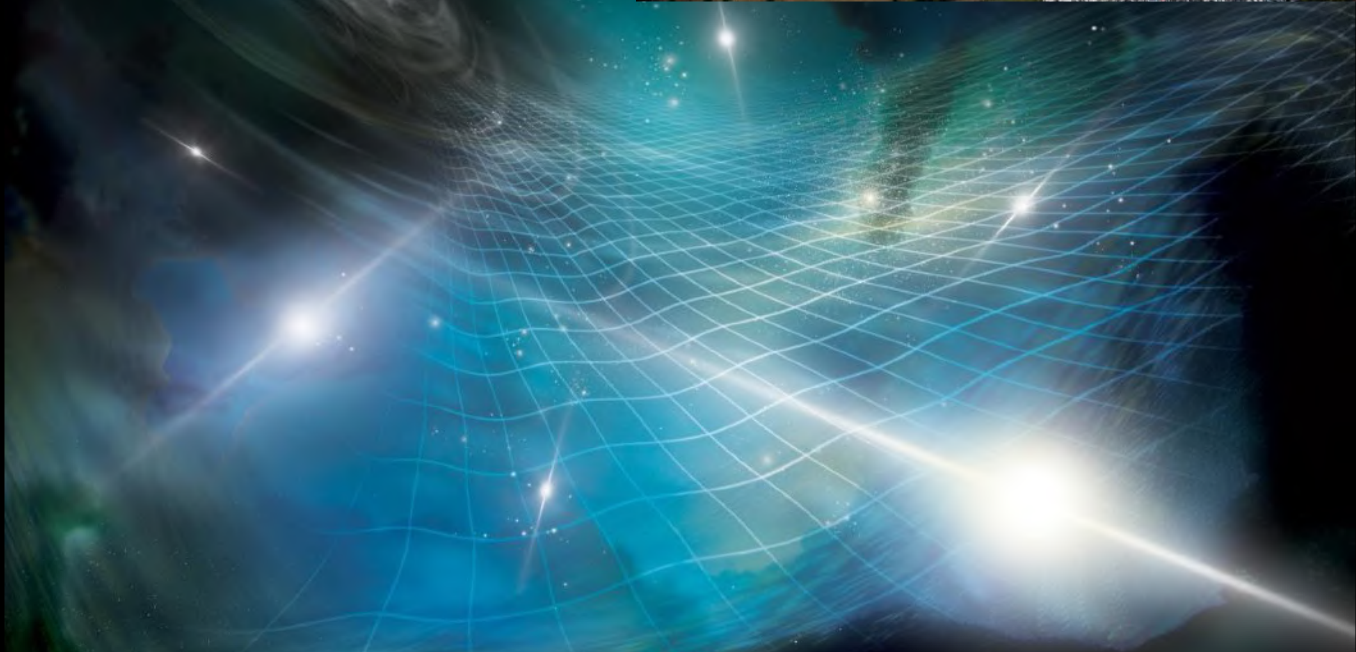




June 28, 2023: Pulsar Timing Discovery

- Radio telescopes
- Time the arrival of pulses
- Reconstruct gravitational-wave properties
- Revealed "Hum from Cosmic Symphony"

Photograph Credit: *Jay Young for Green Bank Observatory*



https://nanograv.org/news/15yrRelease

Credit: Aurore Simonnet for the NANOGrav Collaboration

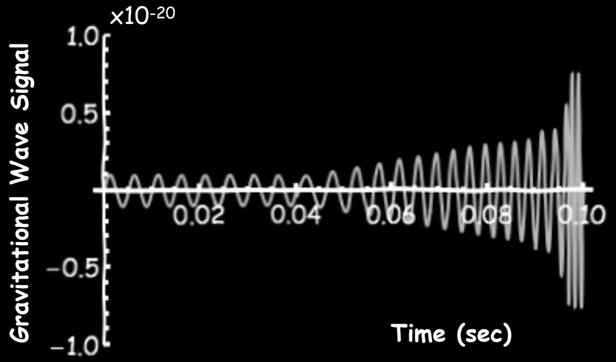
Goals

- Day 1:
  - What are gravitational waves?
  - How can we detect them?
  - Why do the detectors need to use such cool technology?
- Day 2:
  - What has been observed so far?
  - How are LIGO and Virgo being improved?
  - What will the next generation of gravitational wave detectors do?

If a gravitational wave and an electromagnetic wave were both emitted at the same time from a distant galaxy, which wave would get to Earth first?

A. Gravitational Wave
B. Electromagnetic Wave
C. Both would reach Earth at the same time

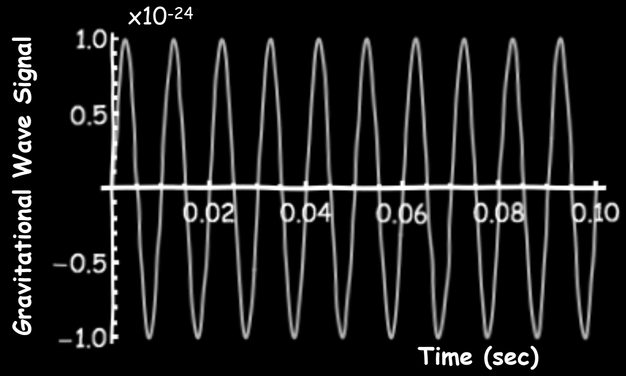
LIGO measures the following waveform:



What is the most likely source?

A. A nearby red giant going supernova
B. The hot early universe
C. A non-spherical pulsar
D. A neutron star and a black hole spiraling into each other
E. None of the above

LIGO measures the following waveform:



What is the most likely source?

A. A nearby red giant going supernova
B. The hot early universe
C. A non-spherical pulsar
D. A neutron star and a black hole spiraling into each other
E. None of the above

Which of the following sources of gravitational waves could LIGO conceivably detect?

A. Two supermassive black holes orbiting each other

B. Two 10-solar-mass black holes orbiting very close to each other

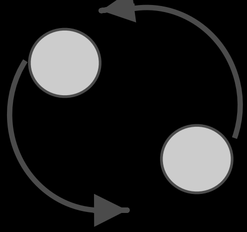
C. Two neutron stars orbiting very close to each other

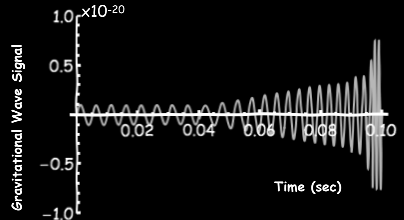
D. The Moon orbiting the Earth

E. Both B and C

Orchestra of gravitational waves

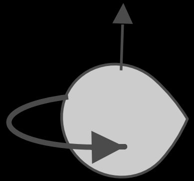
Colliding neutron stars & black holes

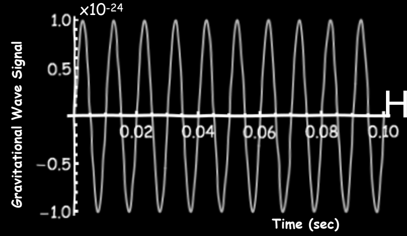




Chirp

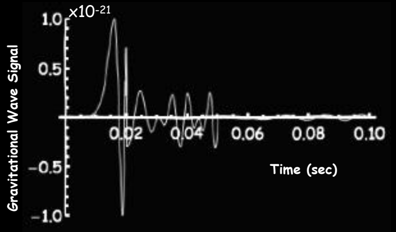
Spinning neutron star with a mountain (image not to scale)





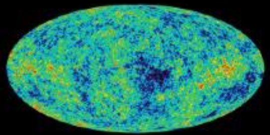
Harmonic wave

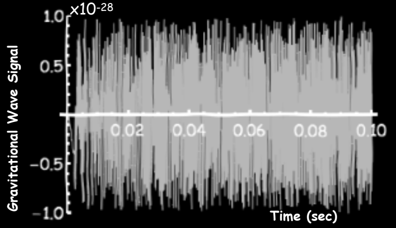
Non-spherical Supernova



Burst

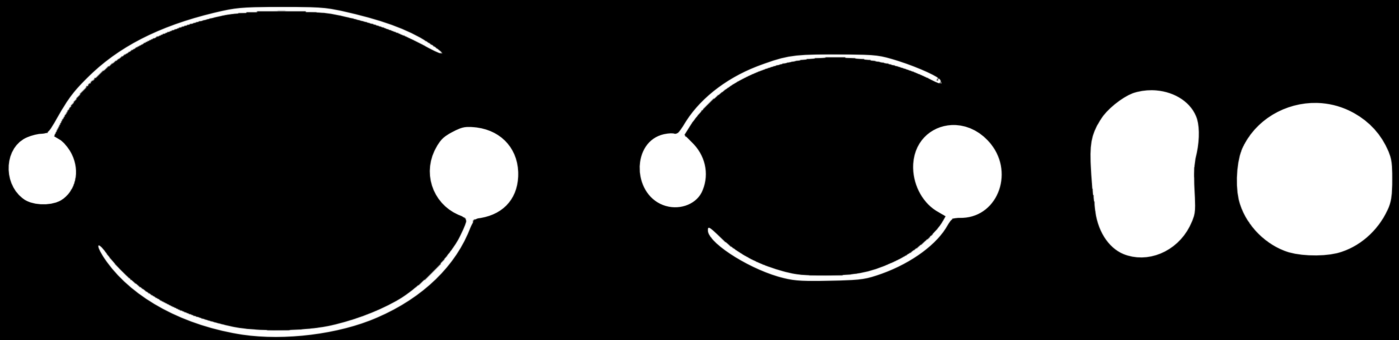
Cosmic/Astrophysical Gravitational wave background



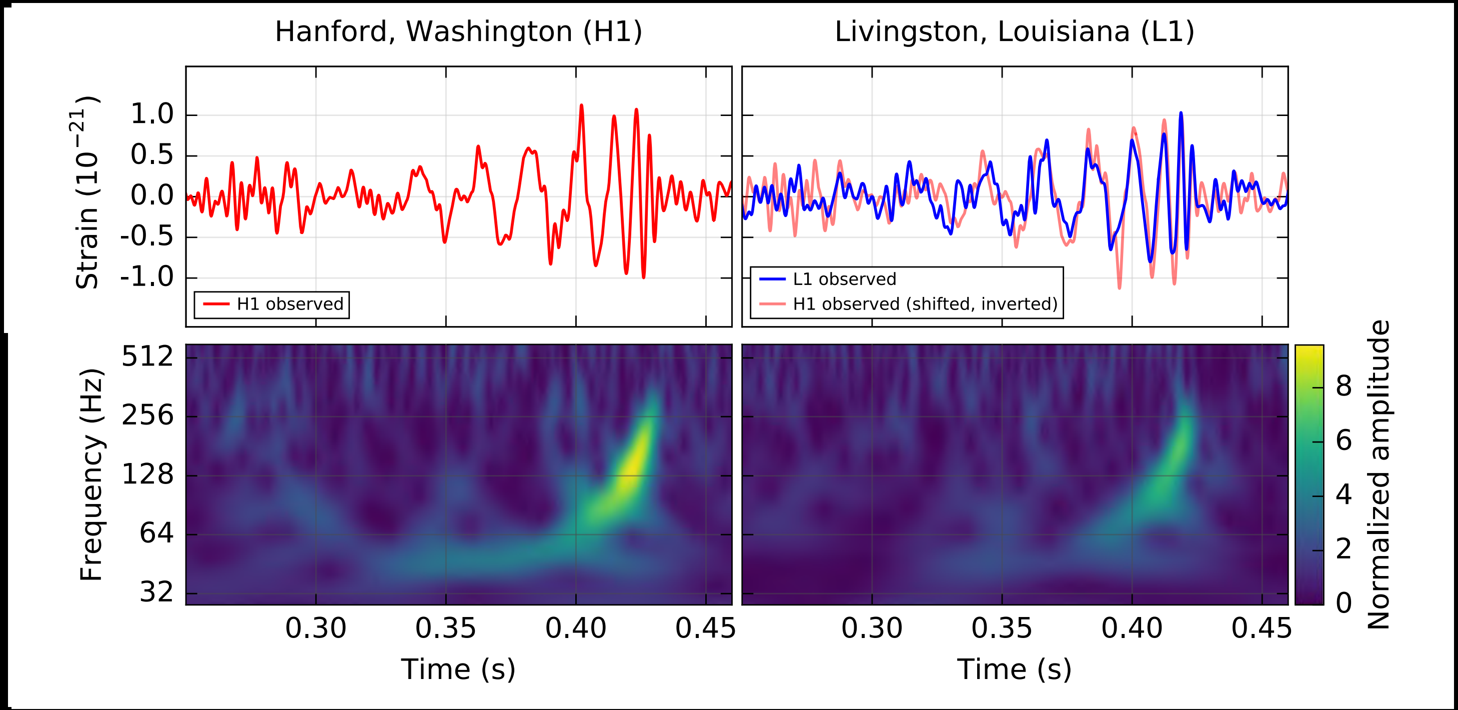


Random noise

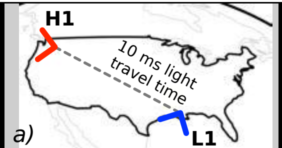
Observations



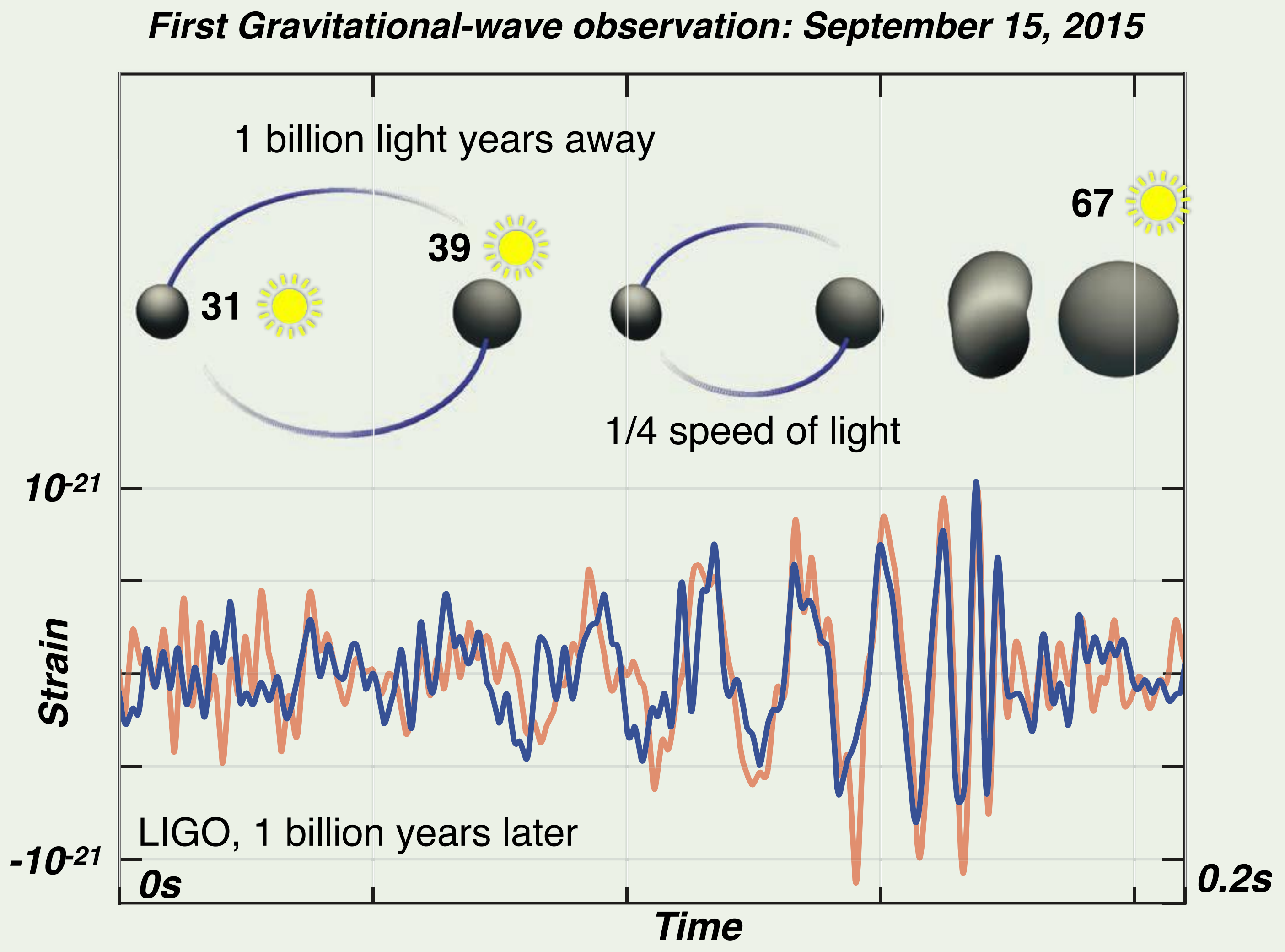
Observation of Gravitational Waves from a Binary Black Hole Merger



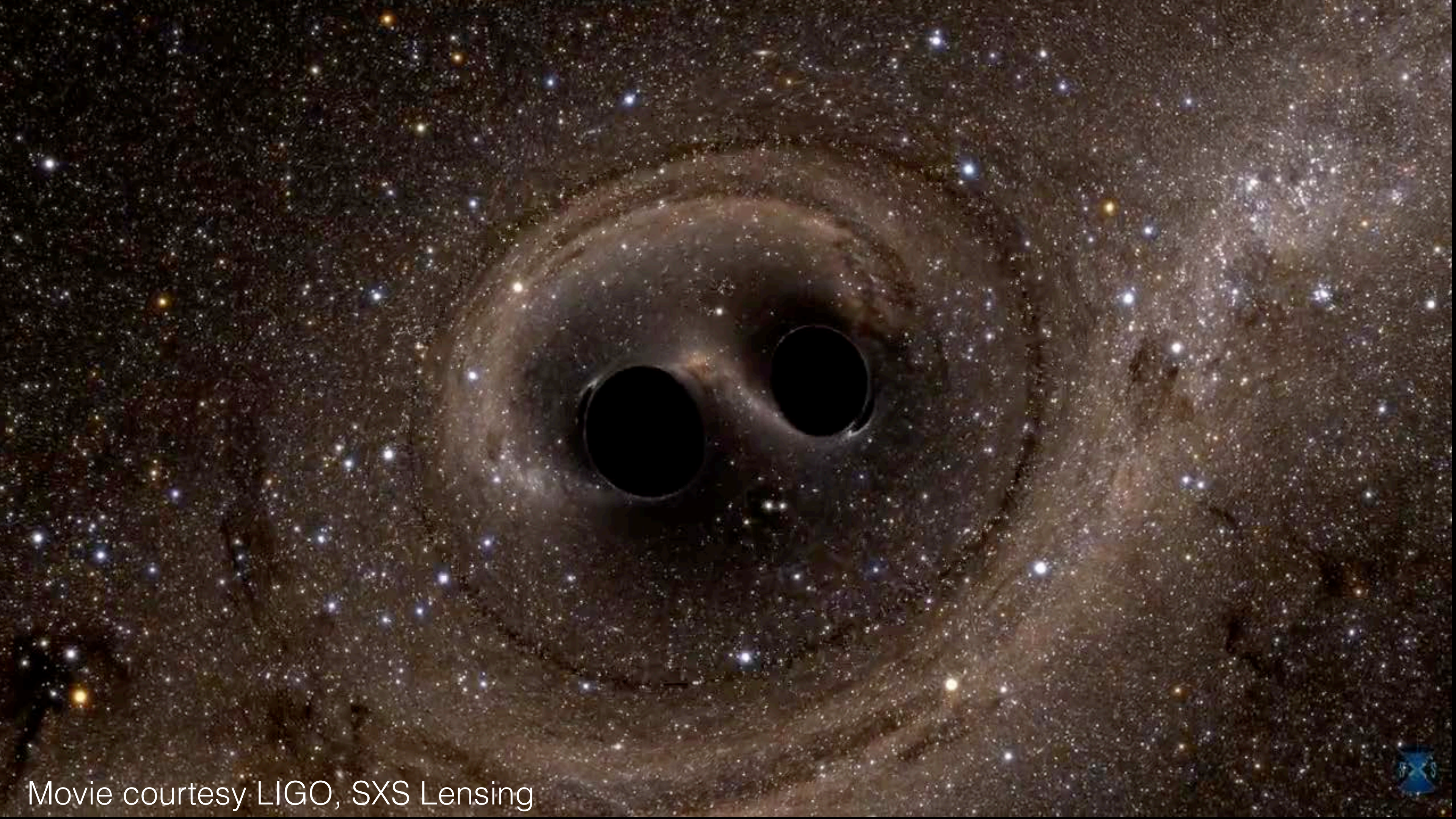
September 14, 2015 at 02:50:45 PDT



Physical Review Letters 116, 061102 (2016)



Phys. Rev. Lett. 116, 061102



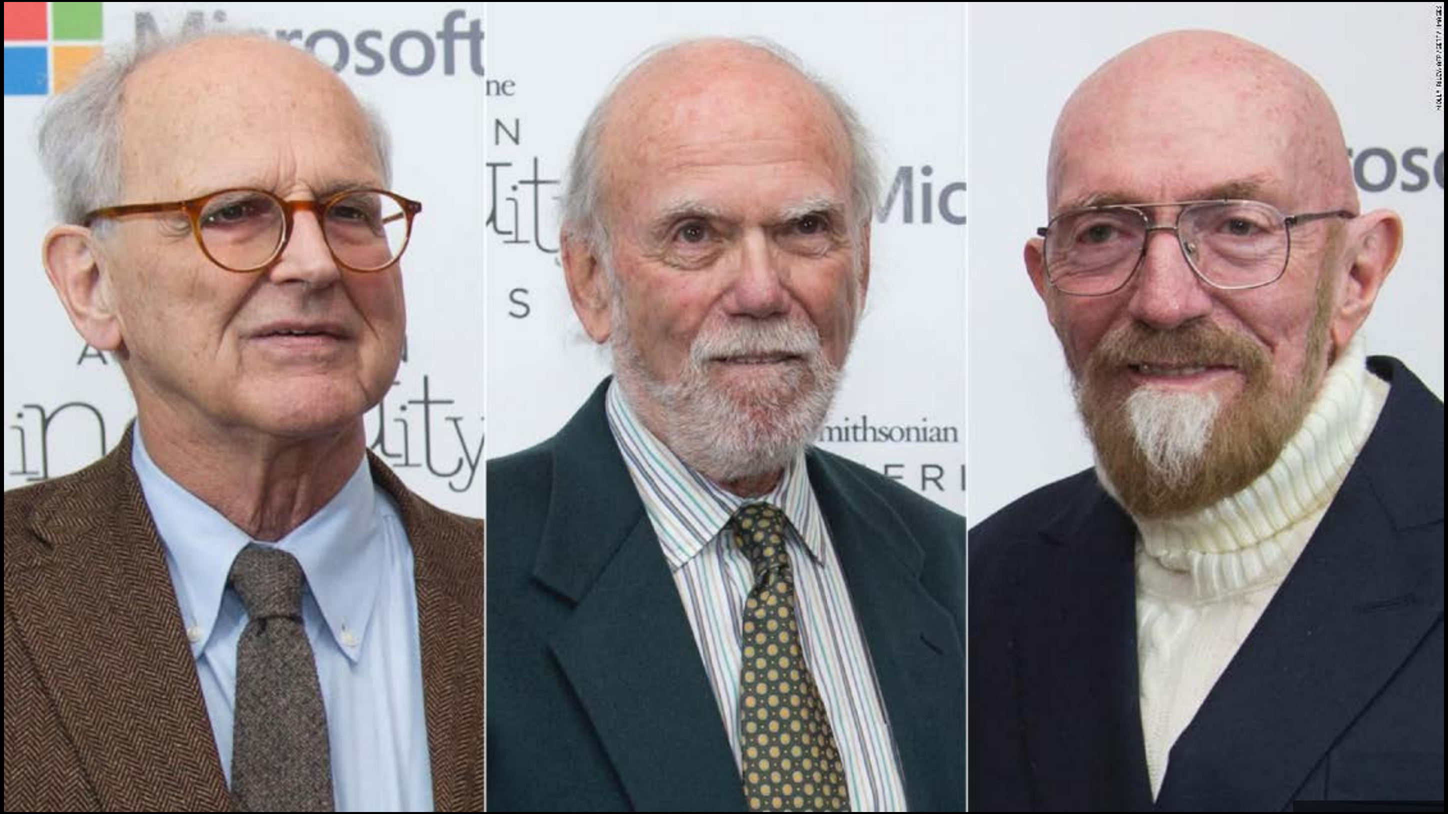
Movie courtesy LIGO, SXS Lensing

First Black Hole Merger

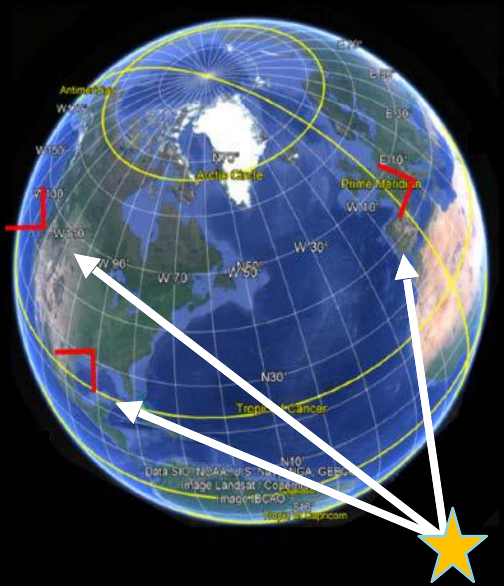
- First detection of gravitational-wave strain
  - Opens a new field: gravitational-wave astronomy
- First observation of black holes of this size (30 ☉)
- First observation of two black holes merging
- Tests and agrees with Einstein's predictions

The 2017 Nobel Prize in Physics

Rainer Weiss, Kip Thorne, and Barry Barish “for decisive contributions to the LIGO detector and the observation of gravitational waves".



The VIRGO detector, located in Italy, joined LIGO in August 2017 – and we observed a Binary Neutron Star coalescence a few weeks later

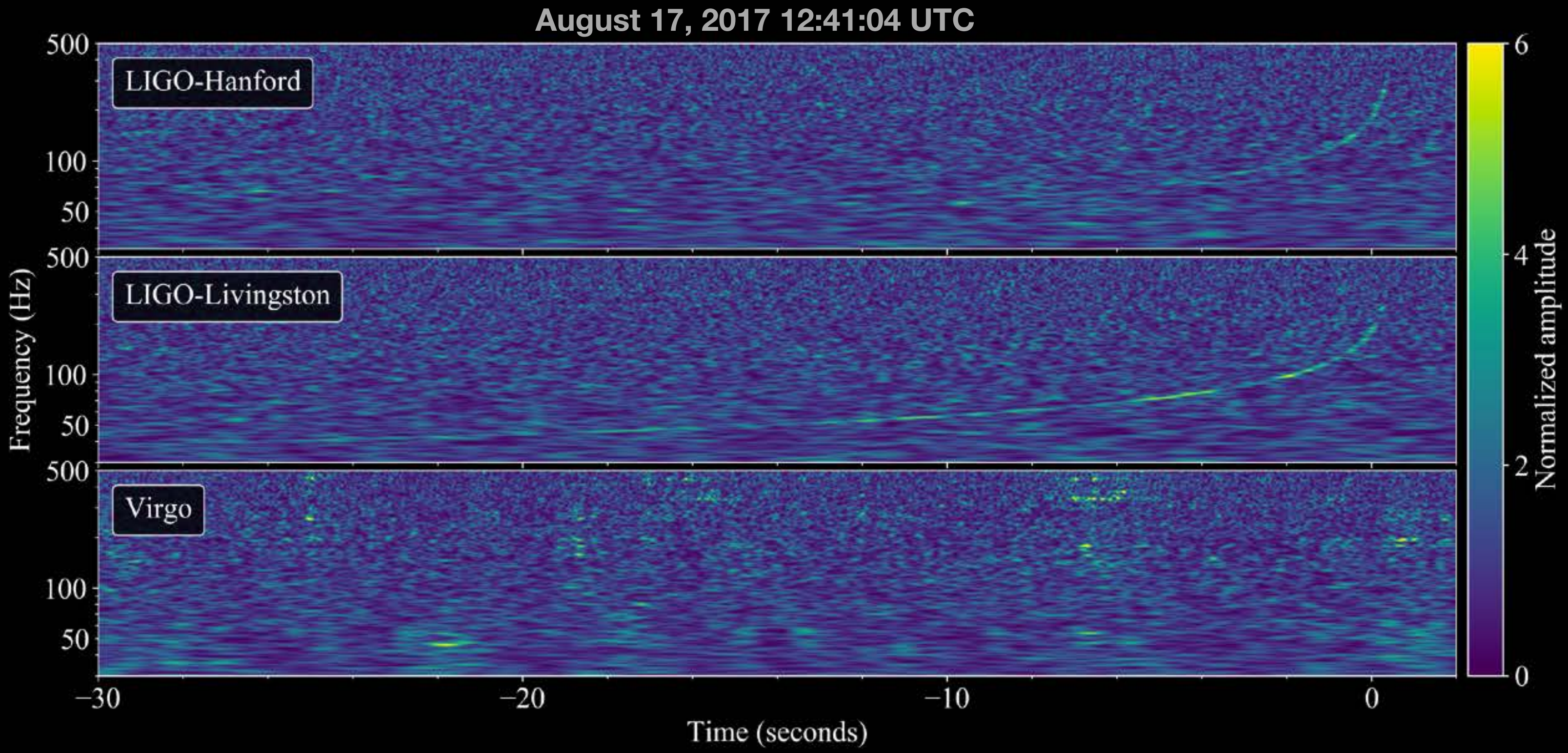


GW170817: Observation of Gravitational Waves from a Binary Neutron Star Inspiral. LVC (2017) PRL 119, 161101

- First joint GW-EM source observation
- Linked short GRBs, binary neutron stars, kilonovae
- Independently measured the local Hubble constant
- Measured speed of gravitational-wave propagation
- Made initial constraints on neutron star equation of state
- Constrained rate of binary neutron star mergers in the local Universe (and thus their production of heavy metals)

Slide adapted from David Shoemaker's

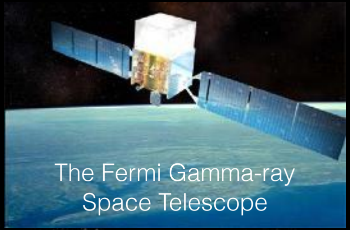
GW170817: a neutron star merger



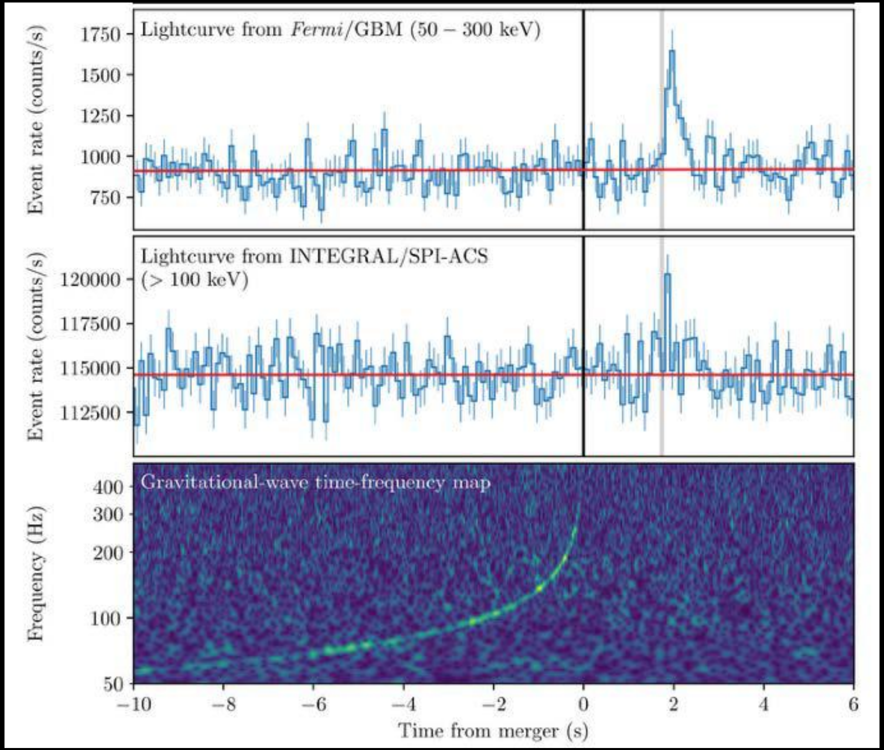
LIGO/Virgo/Lovelace, Brown, Macleod, McIver, Nitz

No significant signal in Virgo, but that's actually very helpful!

A gamma ray burst (GRB170817A)



1.7 seconds later, gamma rays!



- *Solved*: Neutron star mergers cause short hard GRBs!
- Gravitational waves and gamma rays traveled 130 Million years, arrived 2 seconds apart
- *Measured*: speed of gravity equals speed of light (to part in 10^{15})

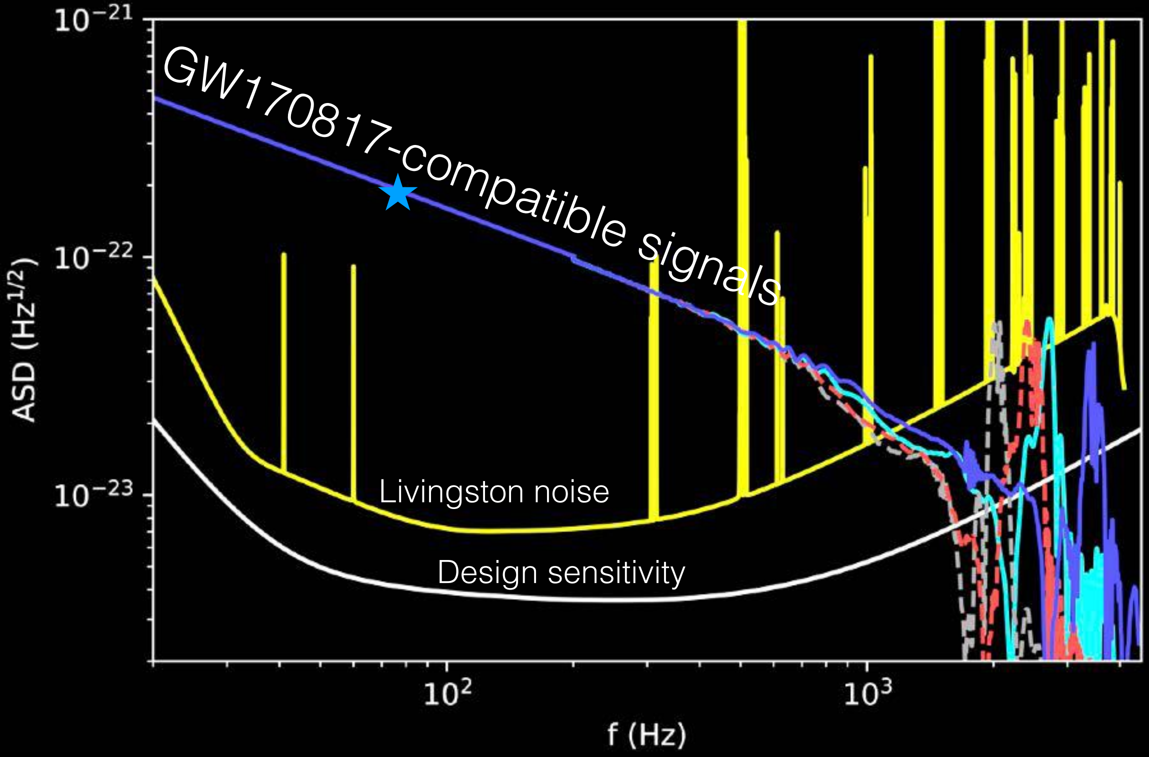
Abbott et al. ApJL 848 2 (2017)

Binary neutron star discovery

- First joint gravitational-wave and light-wave observation
  - Pinpointed the host galaxy: NGC 4993 — 130 million light years away
  - Broad spectrum astronomy, continues today
- Measured that gravity travels at speed of light
- Identified neutron star mergers as cause of mysterious (short) gamma ray bursts
- Explored matter in its densest state, ruling out some nuclear models
- Independent measurement of the universe’s expansion (Hubble constant)

Observing neutron star mergers

5 seconds before merger
orbital distance ~190 km
GW frequency ~70 Hz



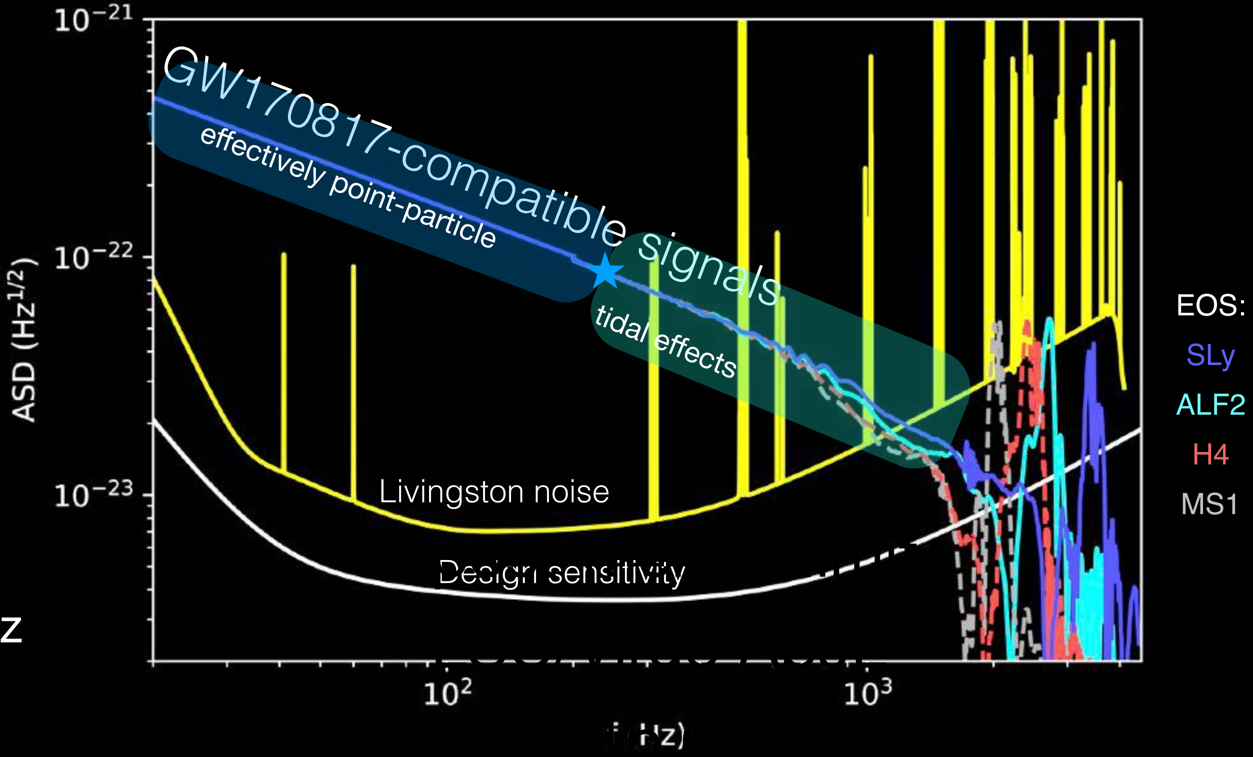
EOS:
SLy
ALF2
H4
MS1

Movie by
Megan Loh, CSUF

E. Leon/LIGO/Virgo. Noise curves from LIGO-P1800061-v11. Effective distance from GraceDB.
Numerical simulation data (above ~500 Hz) courtesy Tim Dietrich (AEI/FSU/BAM Collaboration)
Simulations published in Phys. Rev. D95(12):124006 and Phys. Rev. D95(2):024029

Observing neutron star mergers

Final 0.25 seconds
orbital distance 90 – 24 km
GW frequency 210 – 1600 Hz



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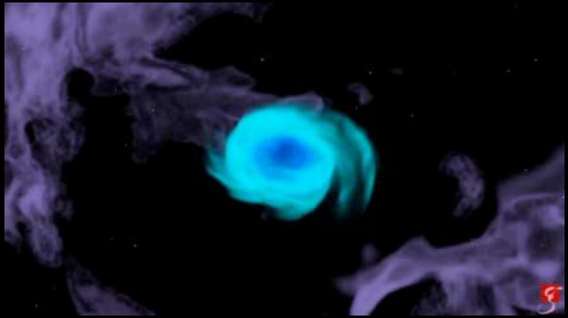
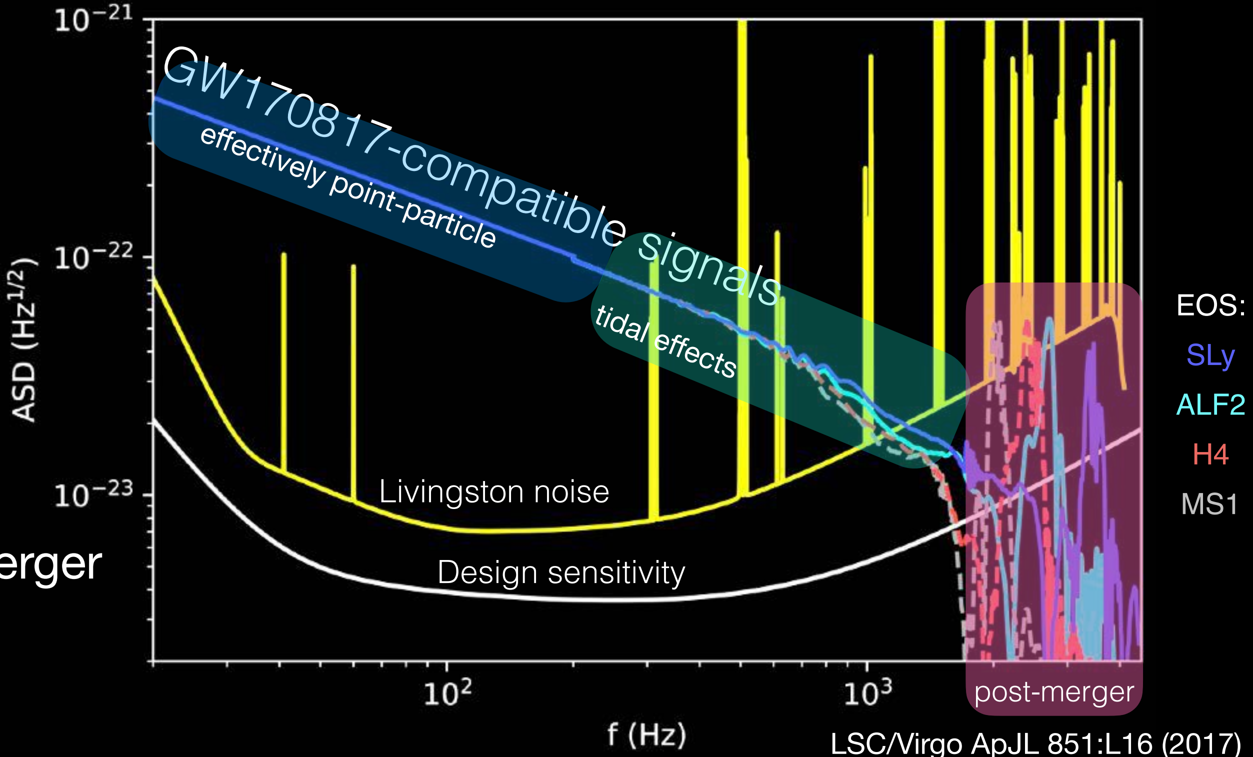
Observing neutron star mergers



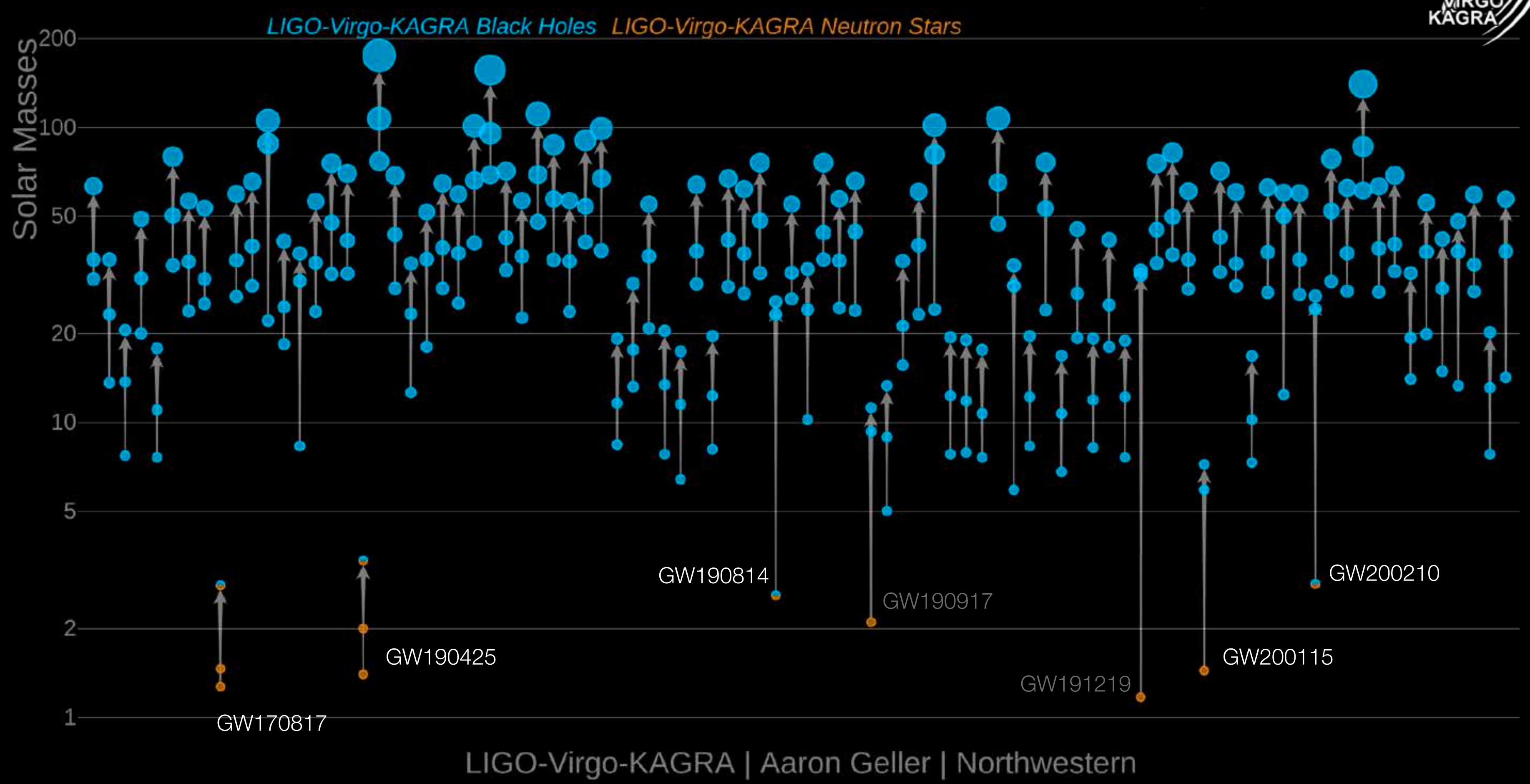


LSC/Virgo ApJL 851:L16 (2017)

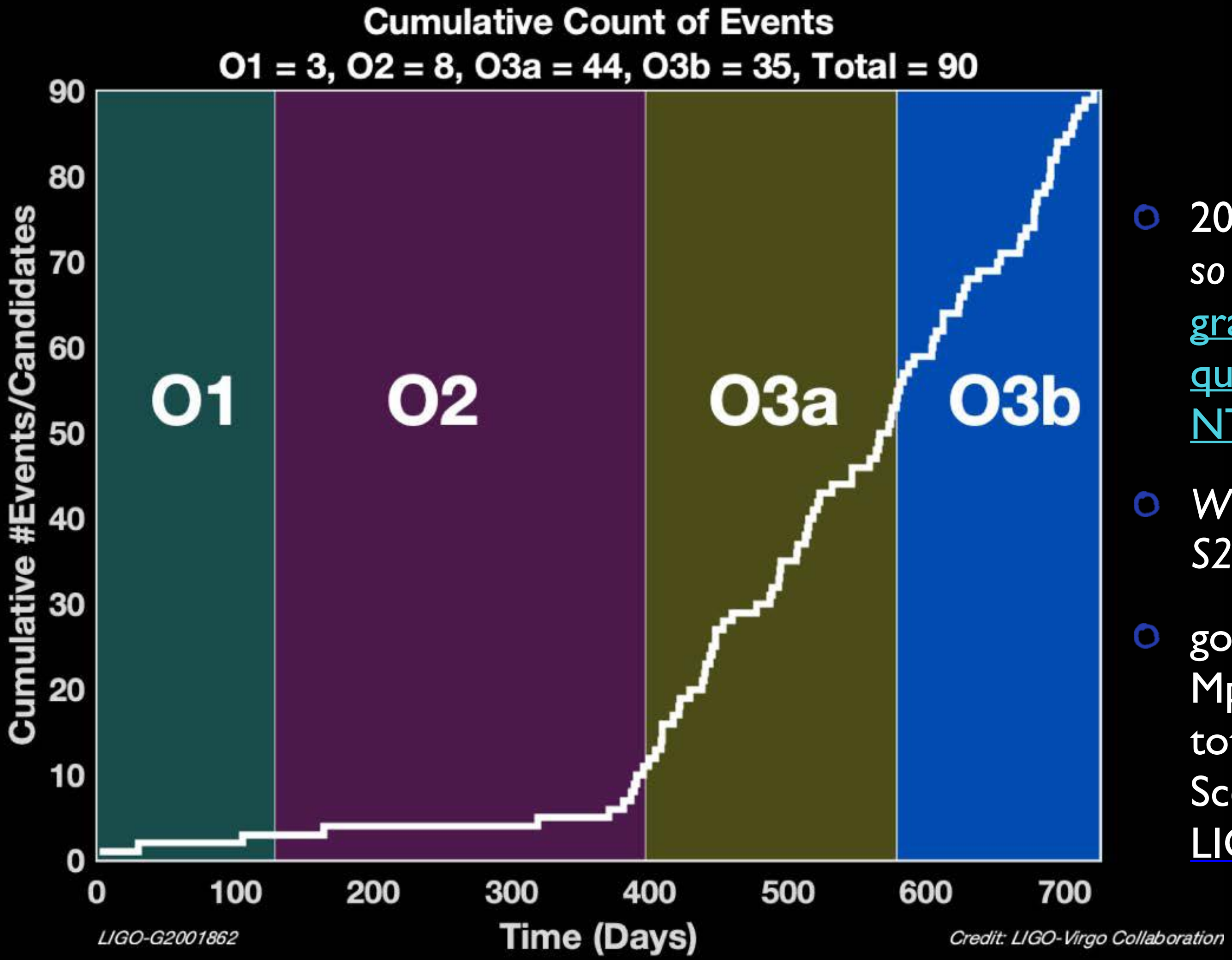
Final 40 milliseconds
of inspiral, tens of ms post merger
orbital distance $\lesssim$ 30 km
GW frequency >1000 Hz

Neutron-star merger simulation:
T. Dietrich, S. Ossokine,
H. Pfeiffer, A. Buonanno (AEI)

E. Leon/LIGO/Virgo. Noise curves from LIGO-P1800061-v11. Effective distance from GraceDB.
Numerical simulation data (above ~500 Hz) courtesy Tim Dietrich (AEI/FSU/BAM Collaboration)
Simulations published in Phys. Rev. D95(12):124006 and Phys. Rev. D95(2):024029



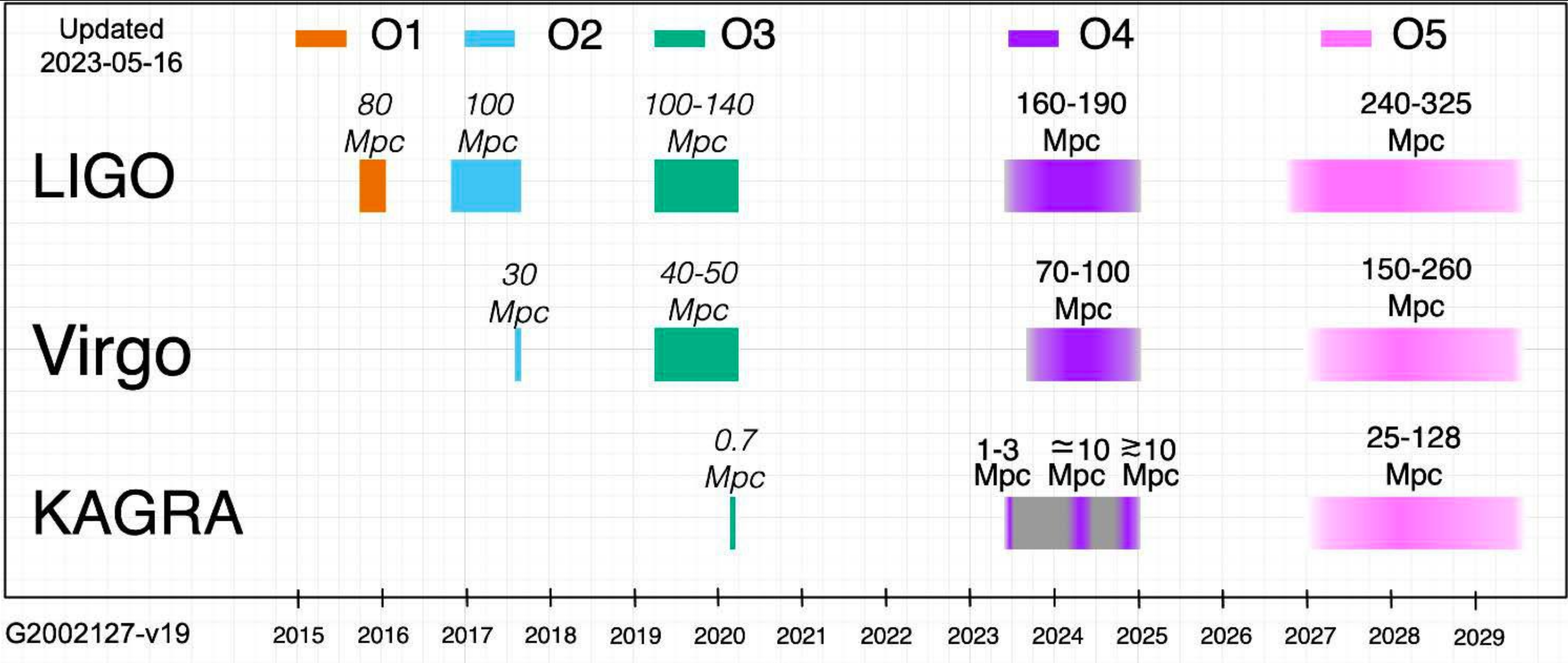
LIGO-Virgo-Kagra GWTC-3 Catalog LIGO-P2000318-v8,



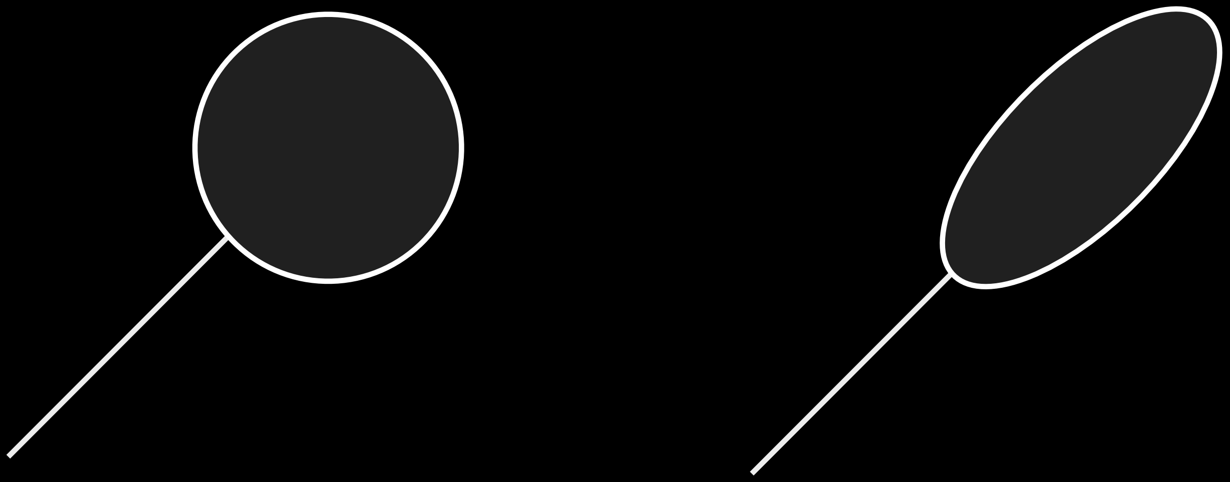
- *20 significant alerts in O4 so far (https://gracedb.ligo.org/latest/?query=GCN_PRELIM_SENT choose supervene)*
- *Which likely have a NS? S230518h, S230529ay*
- goal BNS range 160-190 Mpc (4x previous volume total) LVK Observing Scenarios Document, LIGO-P1200087

https://observing.docs.ligo.org/plan/

LVK Observing Plan



How can we improve today’s detectors, such as LIGO?



Thermal Motions

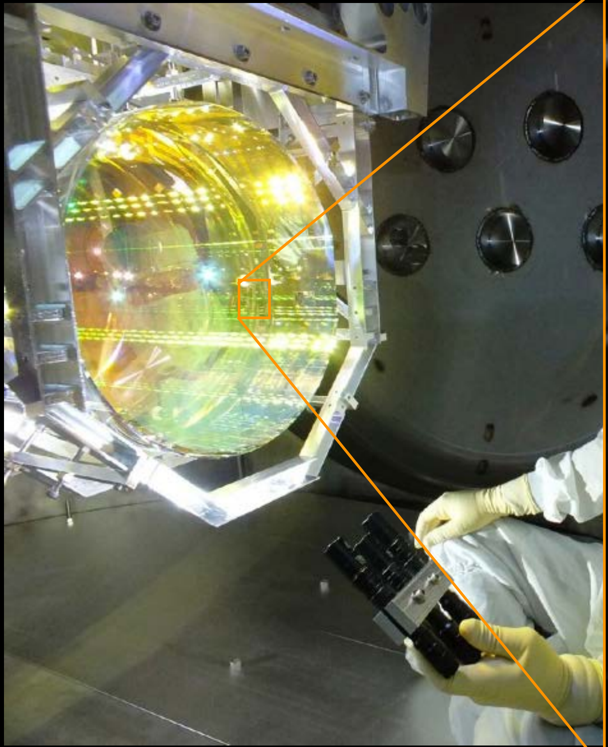
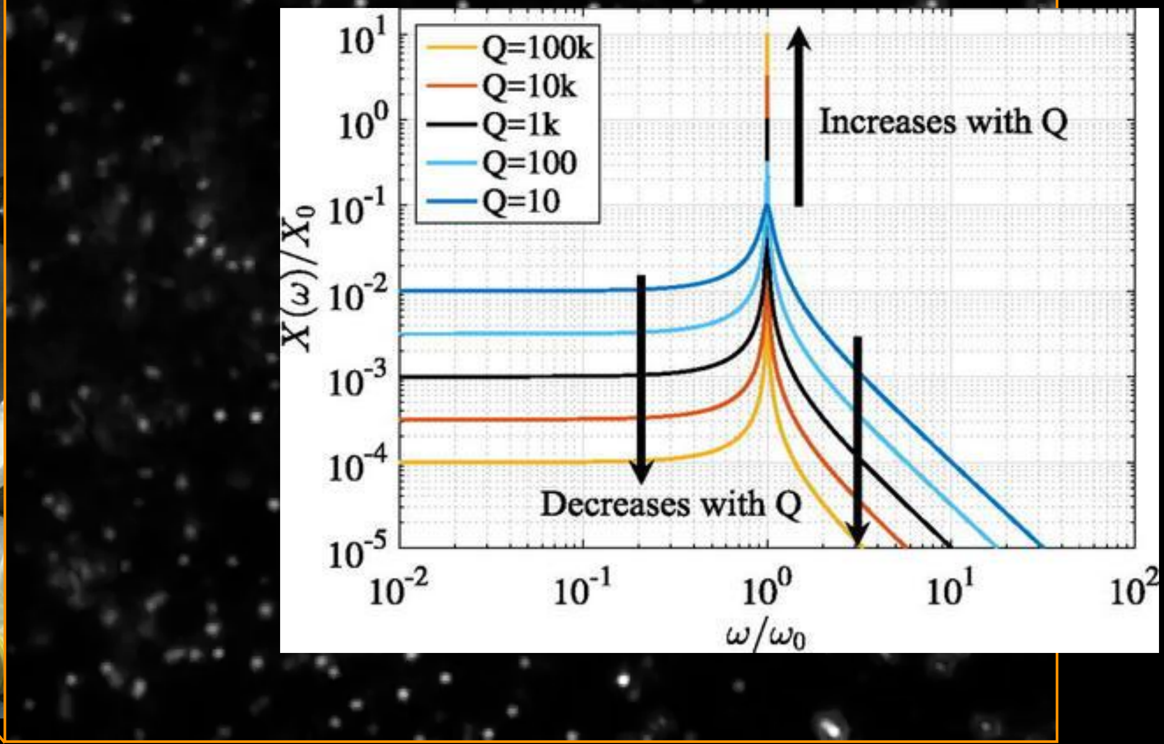


Image credit LIGO

Miller et al., Applied Physics Reviews 5, 041307 (2018)

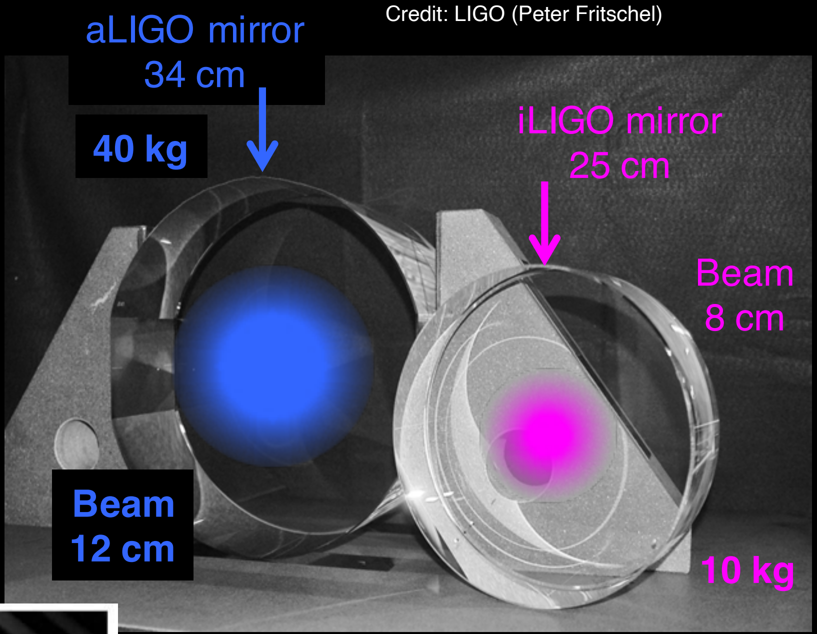


Rutger Saly, YouTube, "We filmed Brownian motion, random movement of particles in water…"

Reducing coating thermal motions

- Larger beam spots on mirrors
- Longer arms (requires new facilities)
- Improved thermal noise optical coatings, e.g.
  - TiO2:SiO2 [Tait+, Phys. Rev. Lett. 125, 011102]
  - TiO2:GeO2 [Vajente+, Phys. Rev. Lett. 127, 071101]
  - SiN [Granata+, Applied Optics 59, 5, A229]
  - GaAs/AlGaAs [Cole+ Nature Photonics 7, 644]
- Cryogenically cooled mirrors
  - Pioneered in Japan's KAGRA
  - LIGO Voyager concept

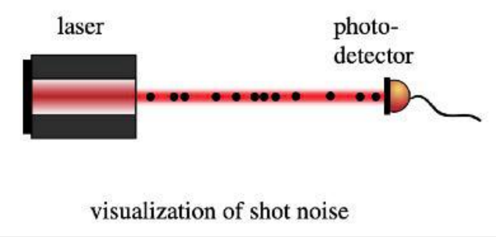
Credit: LIGO (Peter Fritschel)

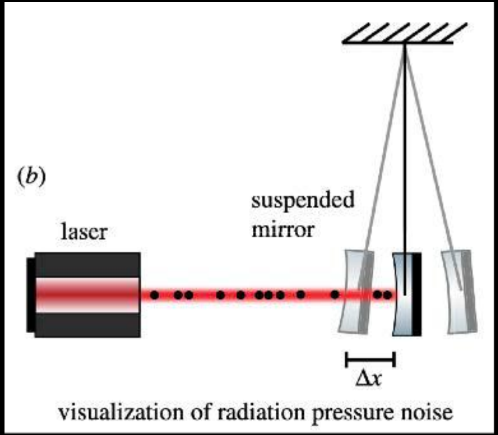


$$h_{\mathrm{CTN}} \propto \sqrt{\frac{T}{1}}\sqrt{\frac{\phi_{\mathrm{eff}}}{1}}\left(\frac{1}{r_{\mathrm{beam}}}\right)\left(\frac{1}{L_{\mathrm{arm}}}\right)$$

Reducing quantum noise

- "Classical" (optics) methods
  - Increase laser power in arms to improve the shot noise limited sensitivity which scales as $h_{\text{shot}} \propto 1/\sqrt{P_{\text{arm}}}$
    - However, the radiation pressure noise scales as $h_{\text{RPN}} \propto \sqrt{P_{\text{arm}}}$ and more power creates "thermal lensing" and "parametric instabilities"
  - Increase mirror mass so they're shaken less by radiation pressure, $h_{\text{RPN}} \propto 1/m_{\text{mirror}}$
- Quantum (optics) methods
  - Injection of "squeezed states" of light

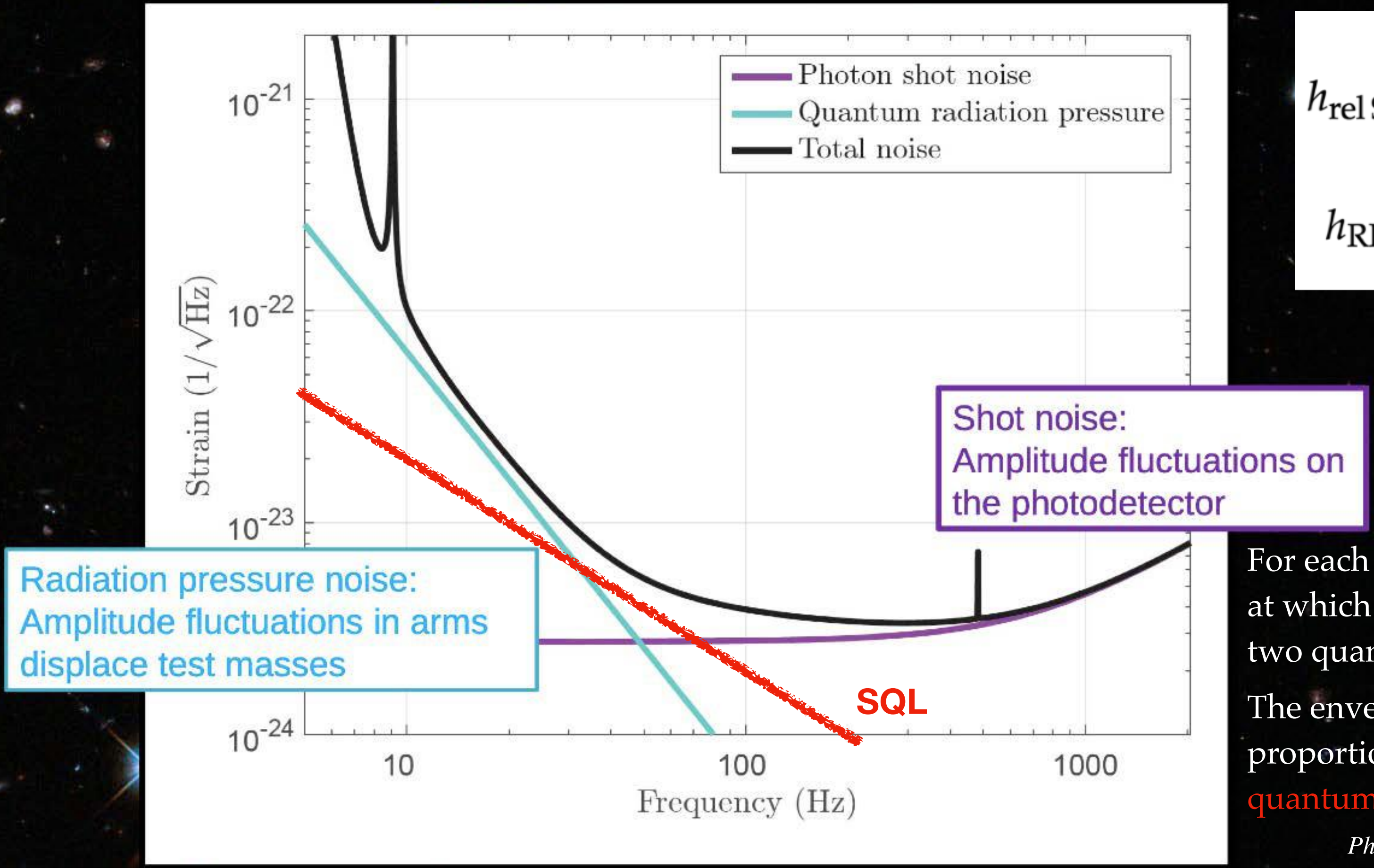




Phil. Trans. R. Soc. A **376**: 20170289

Quantum noise and Standard Quantum Limit (SQL)

Hartmut Grote



$$h_{\mathrm{rel\,SN}}(f) = \frac{1}{L}\sqrt{\frac{\hbar c \lambda}{2\pi P}}$$

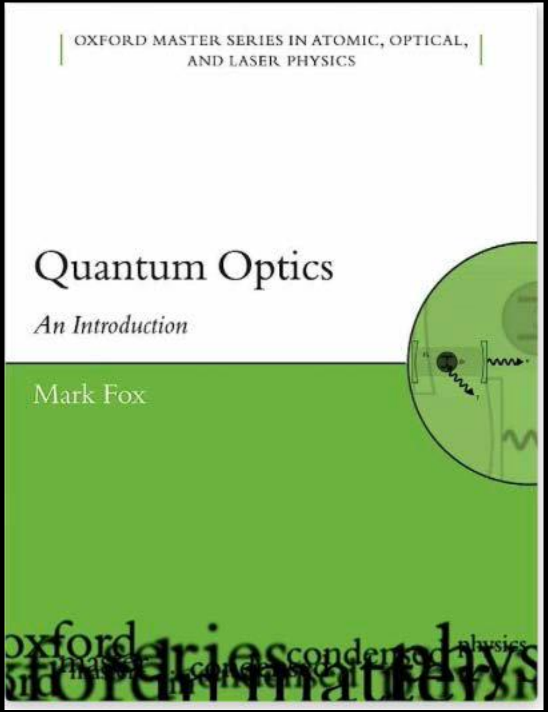
$$h_{\mathrm{RPN}}(f) = \frac{1}{m f^2 L}\sqrt{\frac{\hbar P}{2\pi^3 c \lambda}}$$

For each power there's one frequency at which the geometric sum of the two quantum noise terms is minimal.

The envelope of these minima is proportional to $1/f$ — the standard quantum limit

Phil. Trans. R. Soc. A **376**: 20170289

Squeezed light (algebra version), based on Fox, Quantum Optics



Mark Fox, Quantum Optics

Coherent States and Squeezed Light (Fox 7)

- To describe quantum states of light we must first quantize the electromagnetic field
- Draw a connection between light and the quantum harmonic oscillator, we see:
    - the vacuum field that corresponds to the zero-point fluctuations of the quantized light field
    - Coherent states which are the QM equivalent to classical EM waves
- Write uncertainty principle as number-phase uncertainty and understand shot noise
- Squeezed states and how to generate them

Light Waves as Classical Harmonic Oscillators I (Fox 7.1)

- Wave phenomena (like light) can be related to harmonic oscillators
- A harmonic oscillator of mass m and angular frequency ω described by:
    - $p_x = m\dot{x}$ and $m\ddot{x} = \dot{p}_x = -m\omega^2 x$ where x is displacement, p_x is linear momentum and the frequency is $\omega = \sqrt{k/m}$.
    - Solutions can be: $x(t) = x_0 \sin \omega t$ and $p(t) = p_0 \cos \omega t$, where $p_0 = m\omega x_0$
    - Energy is $E_{\rm SHO} = \dfrac{p_x^2}{2m} + \dfrac{1}{2}m\omega^2 x^2$
- Let's look at the equations for light wave and show they are similar

Light Waves as Classical Harmonic Oscillators II (Fox 7.1)

- Consider a linearly polarized electromagnetic wave with wavelength λ, polarized with electric field along x-direction and wave traveling along z-axis, that's enclosed in an empty cavity (box) with dimensions L
  - Electric field part is $E_x(z,t) = E_0 \sin kz \sin \omega t$ where E_0 is amplitude, $k = 2\pi/\lambda$ is wave vector, ω is angular frequency
- Which direction will magnetic field be oscillating?
  - Along y-axis, and using Maxwell Equation $-\frac{\partial B_y}{\partial_z} = \epsilon_0 \mu_0 \frac{\partial E_x}{\partial_t}$ get
  - Magnetic field $B_y(z,t) = B_0 \cos kz \cos \omega t$ with amplitude $B_0 = E_0/c$ (since $\omega = ck$)
- Electric and magnetic fields are 90° out of phase with each other just like $x(t)$ and $p(t)$ in the mechanical oscillator

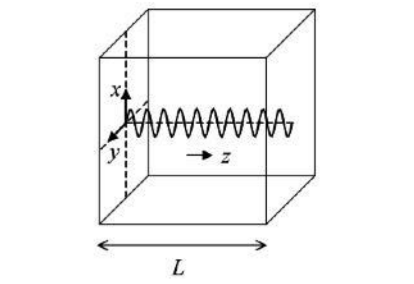


Fig. 7.1 Electric field of an electromagnetic wave polarized in the x-direction enclosed within an empty cavity of dimension L.

Mark Fox, Quantum Optics

Light Waves as Classical Harmonic Oscillators III (Fox 7.1)

- Find energy of the wave in the cavity with dimensions L by integrating the energy density
  - $U = \frac{1}{2}\left(\epsilon_0 E^2 + \frac{1}{\mu_0} B^2\right)$ over the volume V, assuming the light mode area A
- $E_{\text{electric}} = \frac{1}{2}\epsilon_0 A \int_0^L E_0^2 \sin^2 kz \sin^2 \omega t dz = \frac{1}{4}\epsilon_0 A E_0^2 \sin^2 \omega t \int_0^L (1 - \cos 2kz) dz = \frac{1}{4}\epsilon_0 V E_0^2 \sin^2 \omega t$
  - Using $2\sin^2\theta = 1 - \cos 2\theta$ and $AL = V$ and because standing wave in the cavity has nodes at 0 and L, $\sin kl = 0$, so $\int_0^L \cos 2kz dz = \sin 2kL/2k = \sin kL \cos kL/k = 0$
- $E_{\text{magnetic}} = \frac{1}{2\mu_0} A \int_0^L B_0^2 \cos^2 kz \cos^2 \omega t dz = \frac{1}{4\mu_0} A B_0^2 \cos^2 \omega t \int_0^L (1 + \cos 2kz) dz = \frac{1}{4\mu_0} V B_0^2 \cos^2 \omega t$
- So the total energy is $E = \frac{V}{4}\left(\epsilon_0 E_0^2 \sin^2 \omega t + \frac{B_0^2}{\mu_0}\cos^2 \omega t\right)$
- (can show energy is evenly distributed between electric and magnetic fields using $B_0 = E_0/c$ and $c = 1/\sqrt{\mu_0 \epsilon_0}$)

Light Waves as Classical Harmonic Oscillators IV (Fox 7.1)

- Total energy is $E = \frac{V}{4}\left(\epsilon_0 E_0^2 \sin^2 \omega t + \frac{B_0^2}{\mu_0} \cos^2 \omega t\right)$
- We don't want to introduce a mass m, so instead we'll introduce generalized coordinates $q(t)$ and $p(t)$
  - $q(t) = \sqrt{\frac{\epsilon_0 V}{2\omega^2}} E_0 \sin \omega t$ and $p(t) = \sqrt{\frac{V}{2\mu_0}} B_0 \cos \omega t = \sqrt{\frac{\epsilon_0 V}{2}} E_0 \cos \omega t$
- $q(t)$ and $p(t)$ play the role in EM HO of position and momentum in QHO
  - $p = \dot{q}$ and $\dot{p} = -\omega^2 q$
  - substituting these back into E above gives $E = \frac{1}{2}(p^2 + \omega^2 q^2)$
  - These equations with substitutions $q(t) = \sqrt{m}\,x(t)$ and $p(t) = (1/\sqrt{m})p_x(t)$ are identical to the QHO equations

$$p_x = m\dot{x} \quad \dot{p}_x = -m\omega^2 x$$

$$E_{\text{SHO}} = \frac{p_x^2}{2m} + \frac{1}{2}m\omega^2 x^2$$

Phasor Diagrams and Quadratures I (Fox 7.2)

- Plane-polarized wave with arbitrary phase $E_x(z,t) = E_0 \sin kz \sin(\omega t + \phi)$
- Using $\sin \alpha + \beta = \sin \alpha \cos \beta + \cos \alpha \sin \beta$ can rewrite wave as $E_x(z,t) = E_0 \sin kz (\cos \phi \sin \omega t + \sin \phi \cos \omega t)$
- Then setting $E_1 = E_0 \sin kz \cos \phi$ and $E_2 = E_0 \sin kz \sin \phi$ this gives $E_x(z,t) = E_1 \sin \omega t + E_2 \cos \omega t$
- Amplitudes E_1 and E_2 are called field quadratures. They correspond to two oscillating electric fields 90° out of phase

Phasor Diagrams and Quadratures II (Fox 7.2)

- Field quadratures $E_1 = E_0 \sin kz \cos \phi$ and $E_2 = E_0 \sin kz \sin \phi$ and field $E_x(z,t) = E_1 \sin \omega t + E_2 \cos \omega t$
- Using complex numbers can rewrite field amplitude at a point in space as
  - $E(z) = E_0(z) e^{i\omega t} = E_0(z) \cos \phi + i E_0(z) \sin \phi = E_1(z) + i E_2(z)$ where $E_0(z) = E_0 \sin kz$
- Can represent the real part of E on the x-axis and imaginary part on the y-axis making a phasor diagram where field is represented by vector length E_0 at angle ϕ wrt x-axis

Fig. 7.2 (a) Phasor diagram for a classical wave of amplitude $\mathcal{E}_0$ and phase ϕ. (b) Equivalent phasor diagram in dimensionless quadrature field units. (c) Time dependence of the X_1 field quadrature. The quadrature amplitude X_{10} is related to the electric field amplitude $\mathcal{E}_0$ through $(\epsilon_0 V/4\hbar\omega)^{1/2}\mathcal{E}_0$.

(a)

Mark Fox, Quantum Optics (c)

Phasor Diagrams and Quadratures III (Fox 7.2)

- Quantum optics convention to use dimensionless units for the field
  - Redraw vector as length $\sqrt{\dfrac{\epsilon_0 V}{2\hbar\omega}}E_0$ and label axes X_1 and X_2, where these two field quadratures are directly related to position and momentum coordinates
    - $X_1(t) = \sqrt{\dfrac{\omega}{2\hbar}}q(t)$, and $X_2(t) = \sqrt{\dfrac{1}{2\hbar\omega}}p(t)$

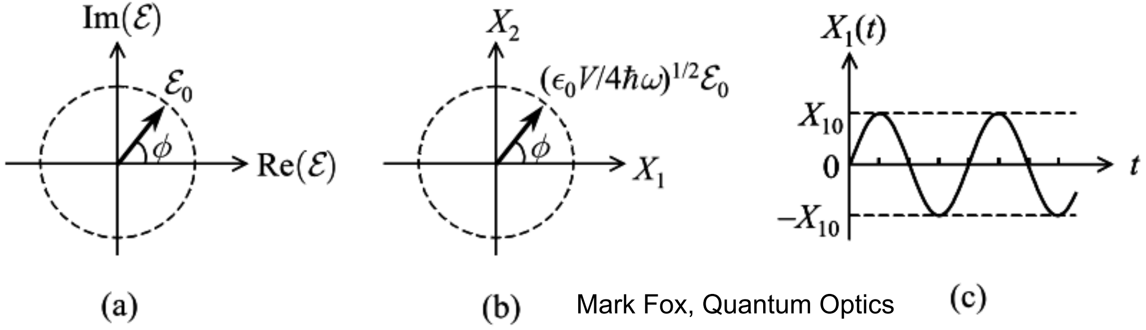
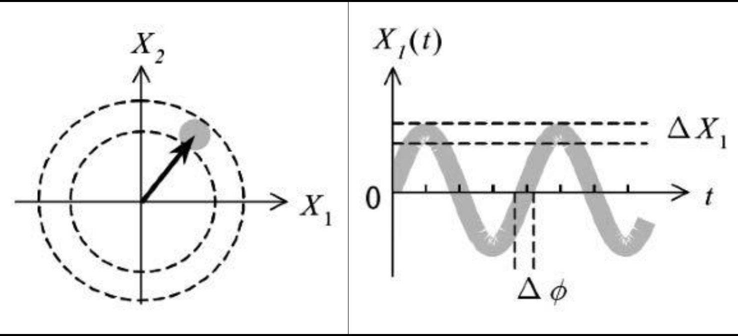


Fig. 7.2 (a) Phasor diagram for a classical wave of amplitude $\mathcal{E}_0$ and phase ϕ. (b) Equivalent phasor diagram in dimensionless quadrature field units. (c) Time dependence of the X_1 field quadrature. The quadrature amplitude X_{10} is related to the electric field amplitude $\mathcal{E}_0$ through $(\epsilon_0 V/4\hbar\omega)^{1/2}\mathcal{E}_0$.

Mark Fox, Quantum Optics

Light as Quantum Harmonic Oscillator I (Fox 7.3)

- Apply knowledge of the quantized harmonic oscillator to quantized EM field (and zero-point energy)
  - QHO: Energy quantized $E_n = \left(n + \frac{1}{2}\right)\hbar\omega$, and x and p_x satisfy uncertainty principle $\Delta x \Delta p \geq \frac{\hbar}{2}$
  - Electromagnetic mode at ω in cavity volume V:
    - Field quadratures $X_1(t)$ and $X_2(t)$
    - $\Delta X_1 \Delta X_2 = \sqrt{\frac{\omega}{2\hbar}}\Delta q \sqrt{\frac{1}{2\hbar\omega}}\Delta p = \frac{1}{2\hbar}\Delta q \Delta p$
    - Relating these back to x and p_x: $\Delta X_1 \Delta X_2 = \frac{1}{2\hbar}\frac{\Delta x}{\sqrt{m}}\sqrt{m}\Delta p_x = \frac{1}{2\hbar}\Delta x \Delta p_x$
    - Then using, $\Delta x \Delta p \geq \frac{\hbar}{2}$ gives, $\Delta X_1 \Delta X_2 \geq \frac{1}{4}$



- The field quadratures are subject to analogous quantum uncertainty to that of of x and p_x for the QHO; Quantum theory says intrinsic uncertainty in light amplitude and phase!

Coherent State I (Fox 7.5)

- Quantum mechanical equivalent to classical monochromatic EM wave is coherent state $|\alpha>$
- For linearly polarized mode in cavity volume V, $\alpha = X_1 + iX_2$
- Can separate into amplitude and phase $\alpha = |\alpha| e^{i\phi}$ with $|\alpha| = \sqrt{X_1^2 + X_2^2}$ and $X_1 = |\alpha| \cos\phi$ and $X_2 = |\alpha| \sin\phi$, α can be represented with phasor length $|\alpha|$, angle ϕ
- Coherent state is a minimum uncertainty state so $\Delta X_1 = \Delta X_2 = \dfrac{1}{2}$ (shaded circle)
- Relating with $E_{\text{classical}} = \bar{n}\hbar\omega$, find $|\alpha| = \sqrt{n}$

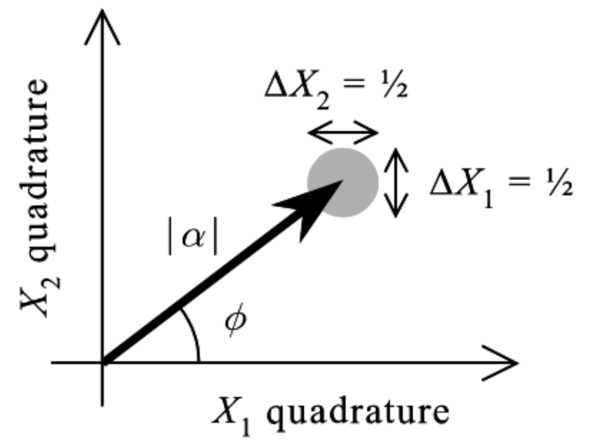


Fig. 7.5 Phasor diagram for the coherent state $|\alpha\rangle$. The length of the phasor is equal to $|\alpha|$, and the angle from the X_1-axis is the optical phase ϕ. The quantum uncertainty is shown by a circle of diameter 1/2 at the end of the phasor.

Mark Fox, Quantum Optics

Shot Noise (Fox 7.6)

- Average phasor length α, circle diameter is $1/2$, length of phasor uncertain between $(\alpha + 1/4)$ and $(\alpha - 1/4)$
- $\Delta n = (|\alpha| + 1/4)^2 - (|\alpha| - 1/4)^2 = |\alpha| = \sqrt{\bar{n}}$
  - coherent states have Poissonian photon statistics and shot noise (from light's quantum uncertainty)
- For large $|\alpha|$, phase uncertainty is $\Delta\phi = \dfrac{1/2}{\sqrt{\bar{n}}}$
- These give number-phase uncertainty of light, $\Delta n \Delta \phi \geq \dfrac{1}{2}$

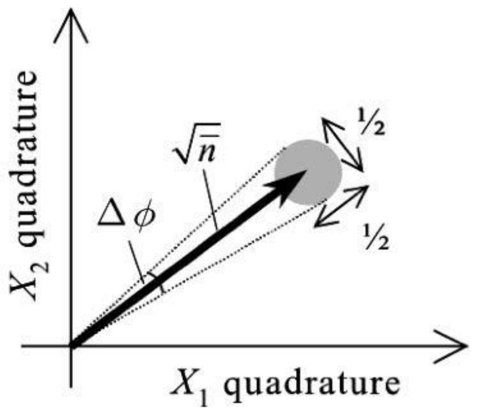


Fig. 7.6 The uncertainty circle of a coherent state $|\alpha\rangle$ introduces both photon number and phase uncertainty. Note that the phase uncertainty $\Delta\phi$ is only well-defined when $|\alpha| = \sqrt{\bar{n}} \gg 1$.

Mark Fox, Quantum Optics

Vacuum Field I (Fox 7.4)

- Zero-point energy of QHO is $(1/2)\hbar\omega$
- Quantum optics: this energy from randomly fluctuating field called vacuum field (present everywhere)
- Equating zero-point energy of QHO to time-averaged energy of E and B fields gives, $E_{\text{vac}} = \sqrt{\dfrac{\hbar\omega}{2\epsilon_0 V}}$
- Uncertainties in the two quadratures equal the minimum allowed, minimum uncertainty state $\Delta X_1^{\text{vac}} = \Delta X_2^{\text{vac}} = \dfrac{1}{2}$

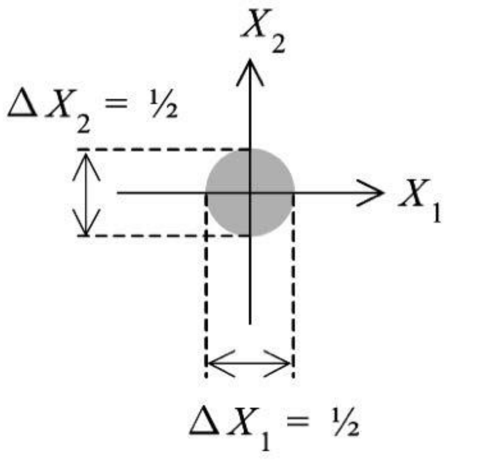
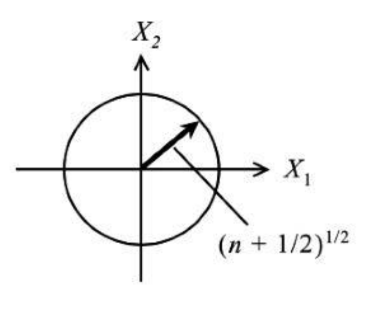


Fig. 7.4 Phasor diagram for the vacuum state. The uncertainties in the two field quadratures are identical, with $\Delta X_1 = \Delta X_2 = 1/2$. Note that this figure is essentially the same as Fig. 7.3(a) except that the uncertainty circle is displaced to the origin to account for the zero classical field of the vacuum.

Mark Fox, Quantum Optics

Squeezed States (Fox 7.7)

- Can squeeze the uncertainty circle of vacuum or coherent state into ellipse of same area: quadrature-squeezed state
- Phase squeezed light allows interferometric measurements with greater precision
- Amplitude squeezed light allows lower amplitude noise
    - has sub-Poissonian statistics
- Could squeeze along any angle
- Could also make photon number state where $\Delta n = 0$ and phase is completely undefined!



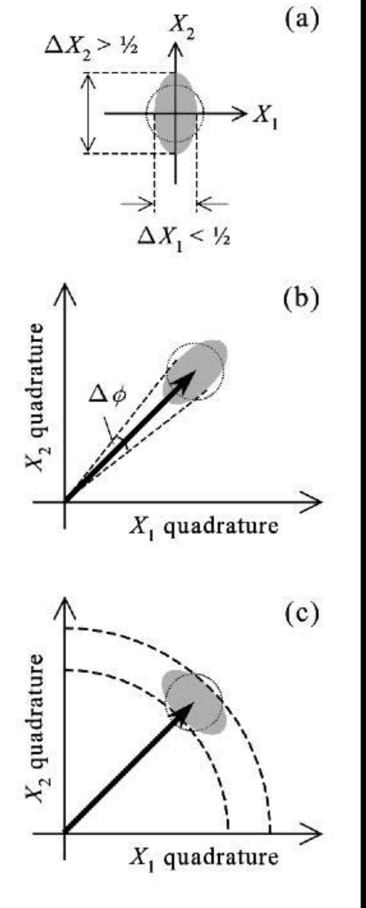
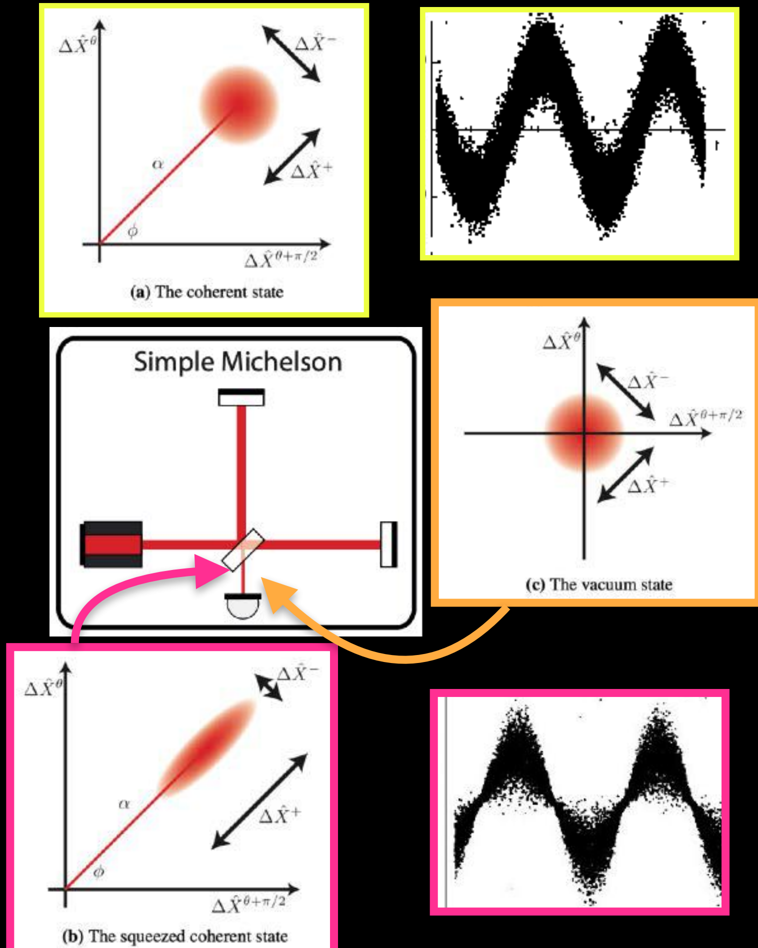


Fig. 7.8 Quadrature squeezed states. (a) Squeezed vacuum. (b) Phase-squeezed light. (c) Amplitude-squeezed light. The dotted circle in each of the diagrams shows the quadrature uncertainty of the vacuum/coherent states with $\Delta X_1 = \Delta X_2 = 1/2$.

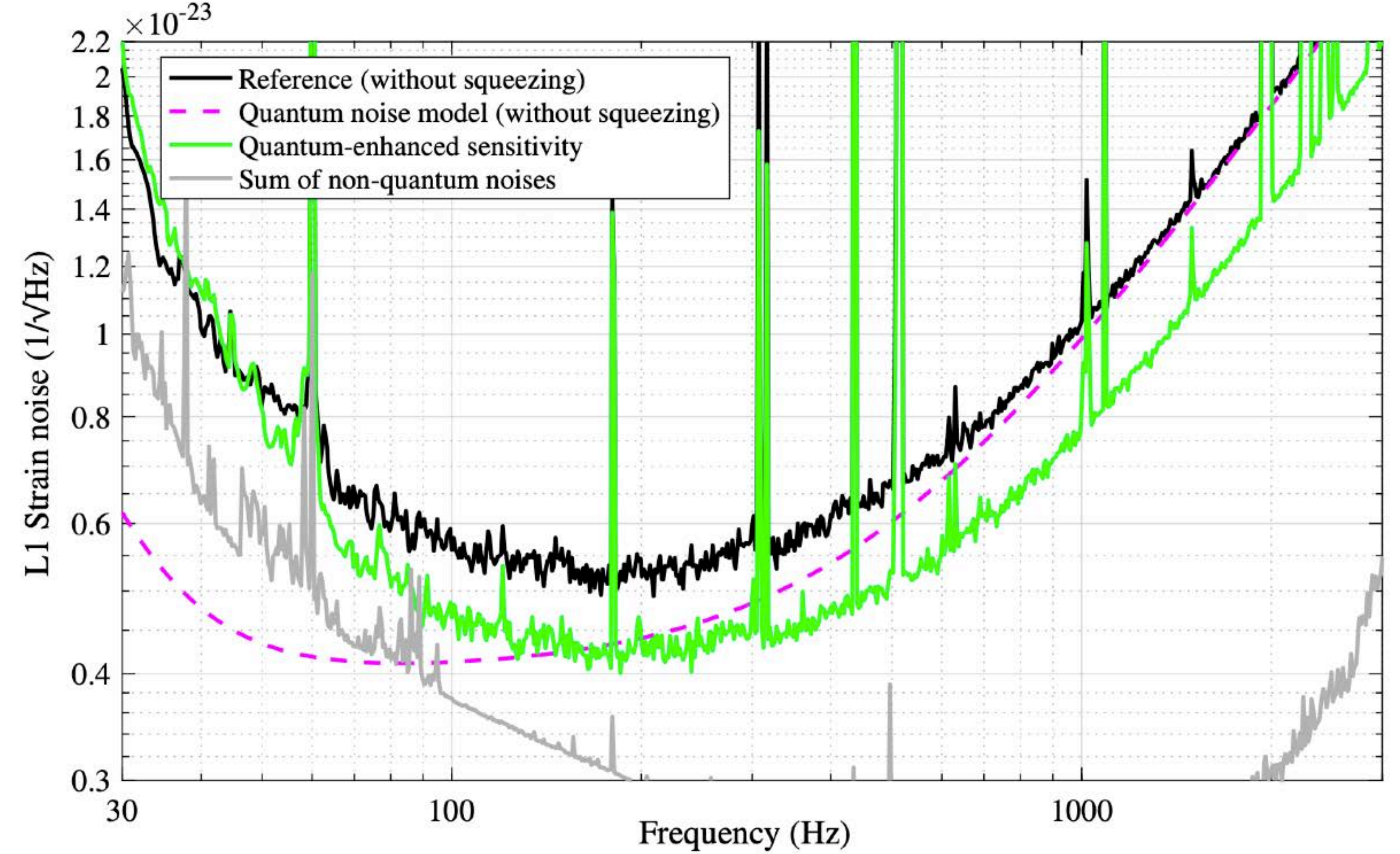
Squeezing to make interferometers more sensitive

- Light has uncertainty in amplitude and phase related by an uncertainty principle $\Delta n \Delta \phi \geq \frac{1}{2}$
  - coherent state: balanced minimum uncertainty, Poisson photon statistics $\Delta n = \sqrt{\bar{n}}$
- Vacuum also has uncertainty: zero-point fluctuations
- At the beamsplitter, the laser light is mixed with zero-point fluctuations → quantum noise
- Replace vacuum field with squeezed light/vacuum that has reduced uncertainty in amplitude or phase
  - Correlate photons using nonlinear optics
  - Sub-Poissonian statistics $\Delta n < \sqrt{\bar{n}}$



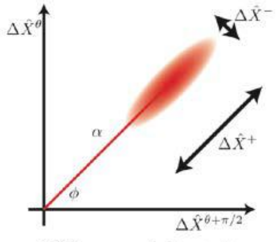
Adapted from Georgia Mansell

PHYSICAL REVIEW LETTERS **123**, 231107 (2019)

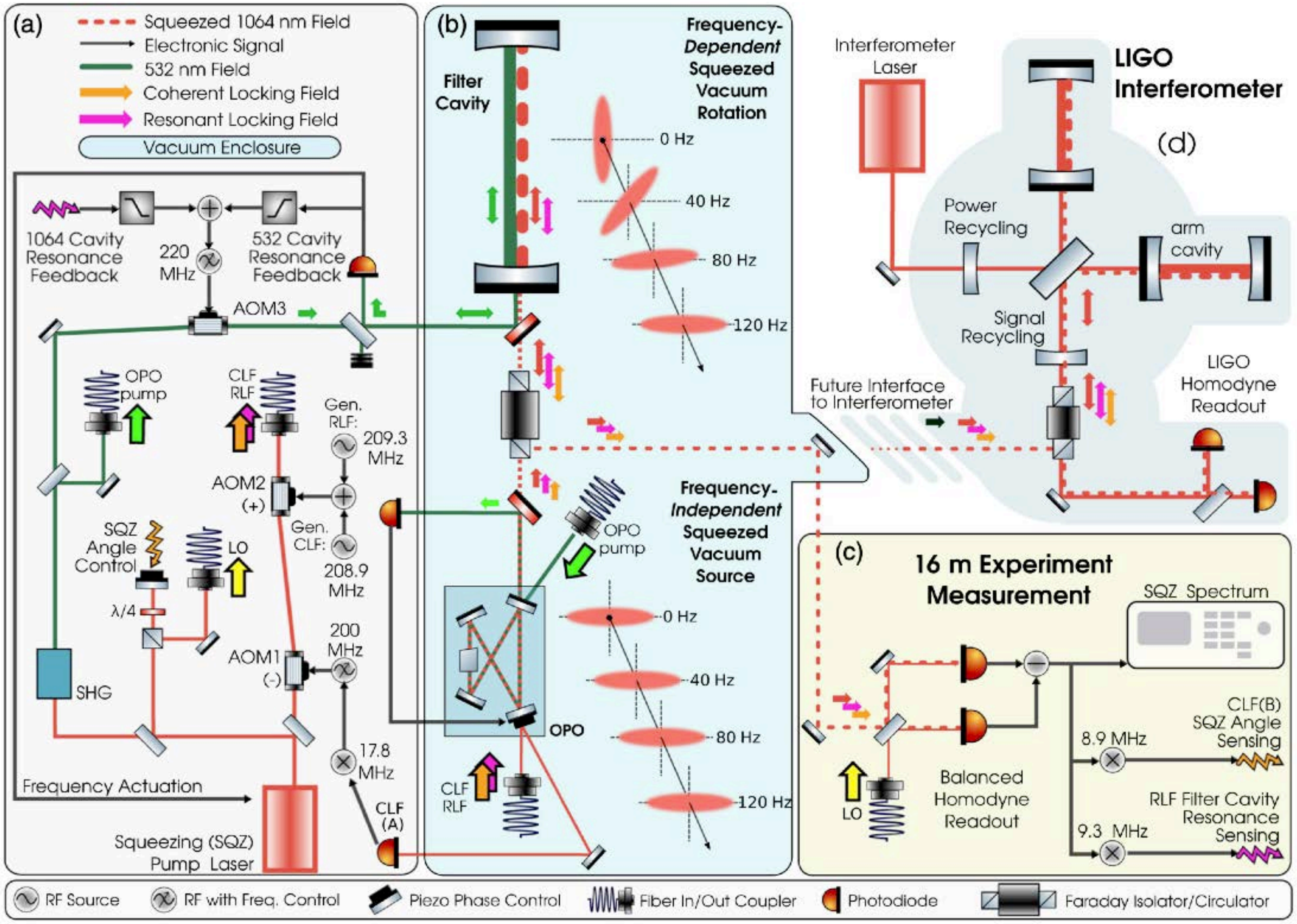


Squeezing in LIGO

Phase-squeezed



(b) The squeezed coherent state



LIGO Frequency-dependent Squeezing

Image credit LIGO

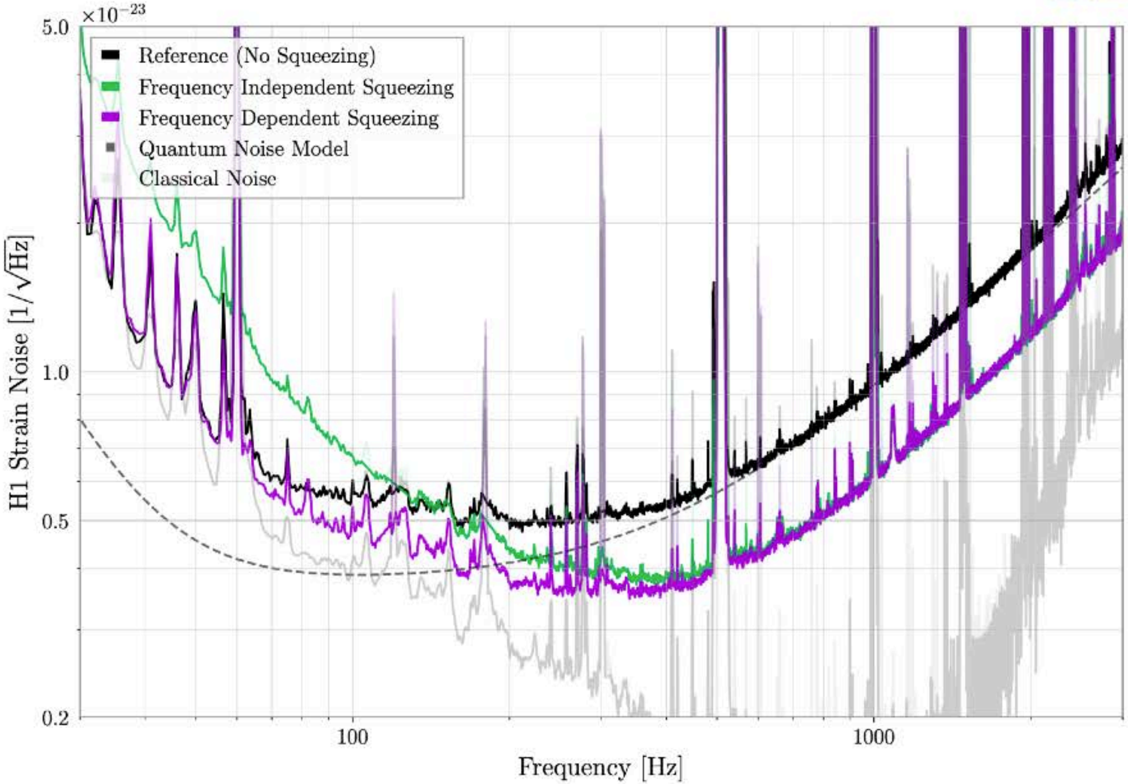
Squeezing

Filter cavity commissioned

- Frequency dependent squeezing injected

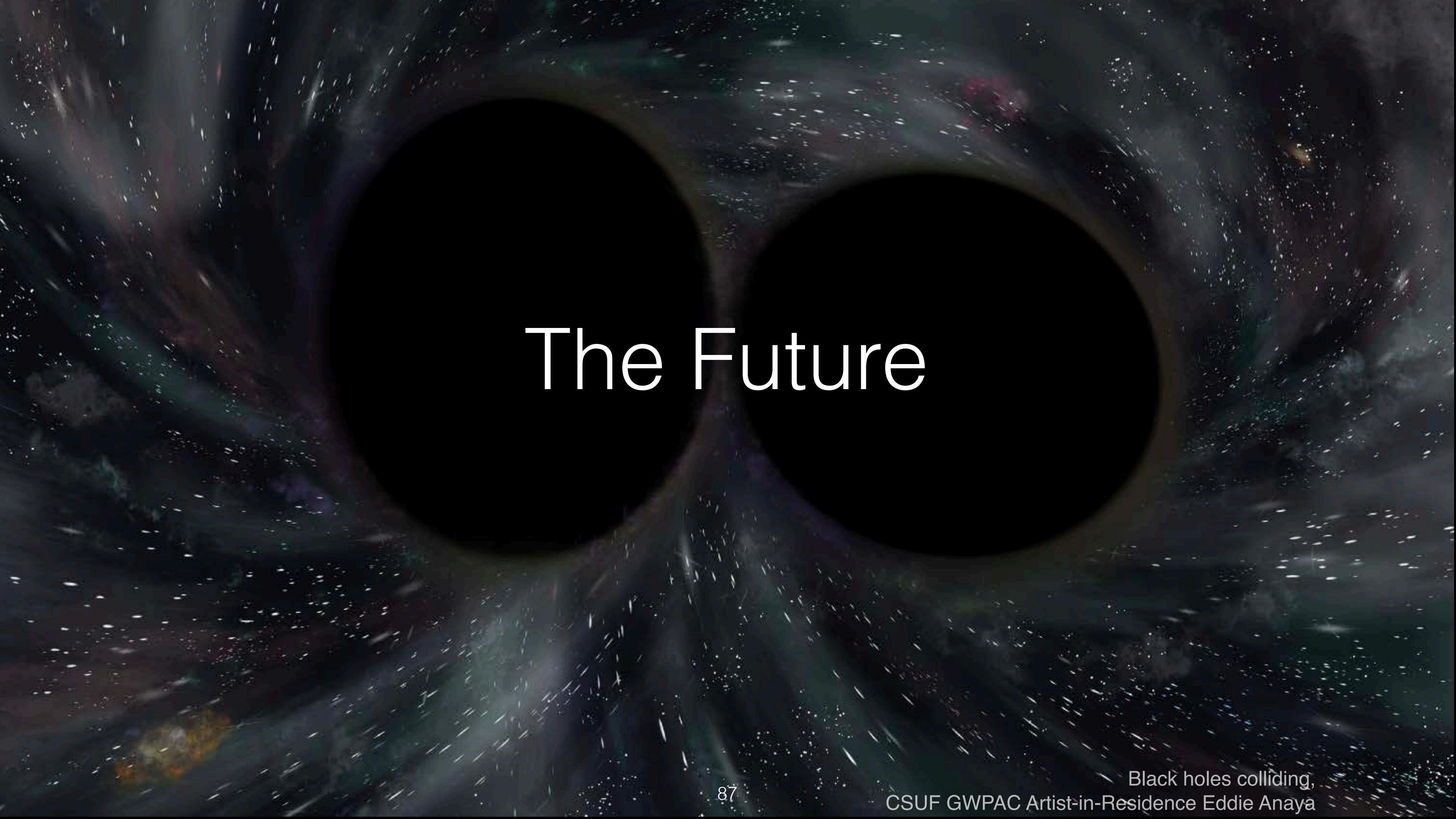
4 dB of squeezing achieved

- Increases range by 20 Mpc



Credit: D. Ganapathy

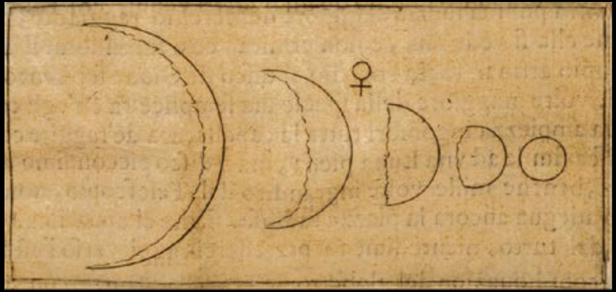
The Future



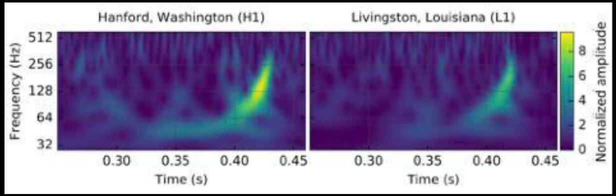
Black holes colliding,
CSUF GWPAC Artist-in-Residence Eddie Anaya

Toward the next generation

- Gravitational waves have given us an entirely new way to observe the universe
- Current gravitational-wave detectors are ***just sensitive enough to detect gravitational waves***
- This parallels Galileo's telescope and its first discoveries and the potential for growth
- Now developing next-generation detectors that will see gravitational waves from remnants of the first stars and with incredible precision



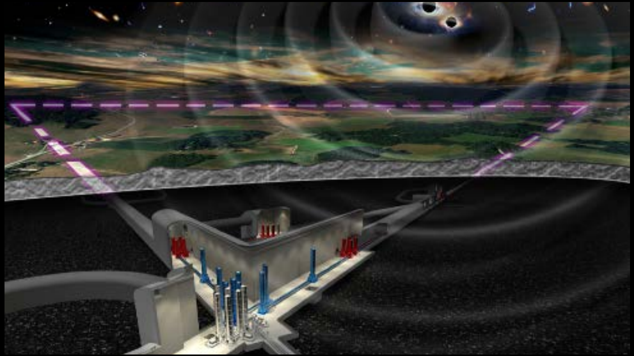
Galileo, Ca. 1600



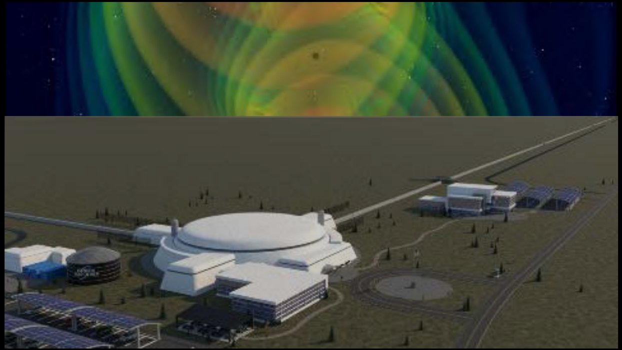
LIGO, 2015

Next-generation gravitational-wave observatories

LISA



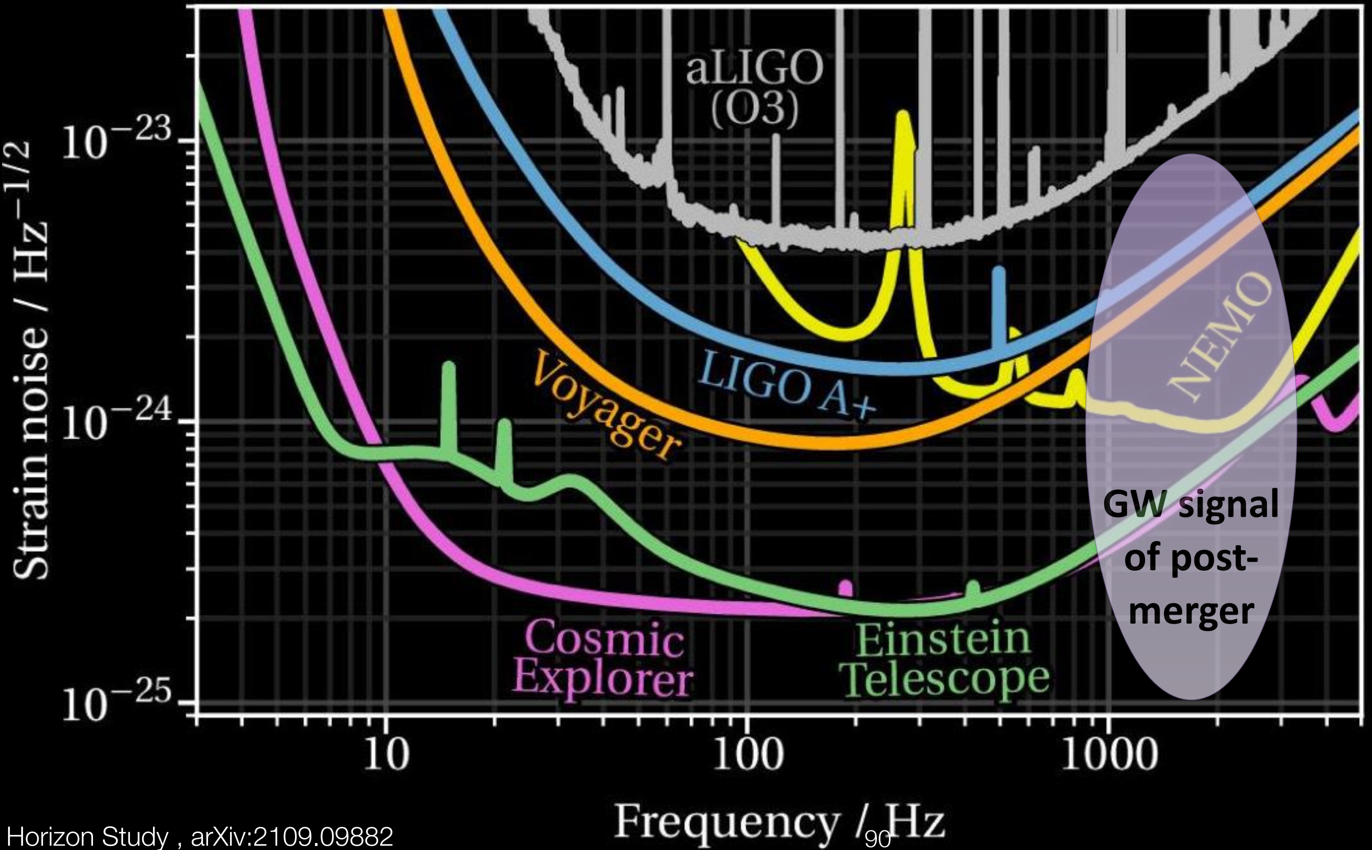
Einstein Telescope (ET)



Cosmic Explorer (CE)
(this talk)

Images courtesy Einstein Telescope, LISA, Cosmic Explorer, Eddie Anaya, Nils Vu, SXS Collaboration

GW Landscape

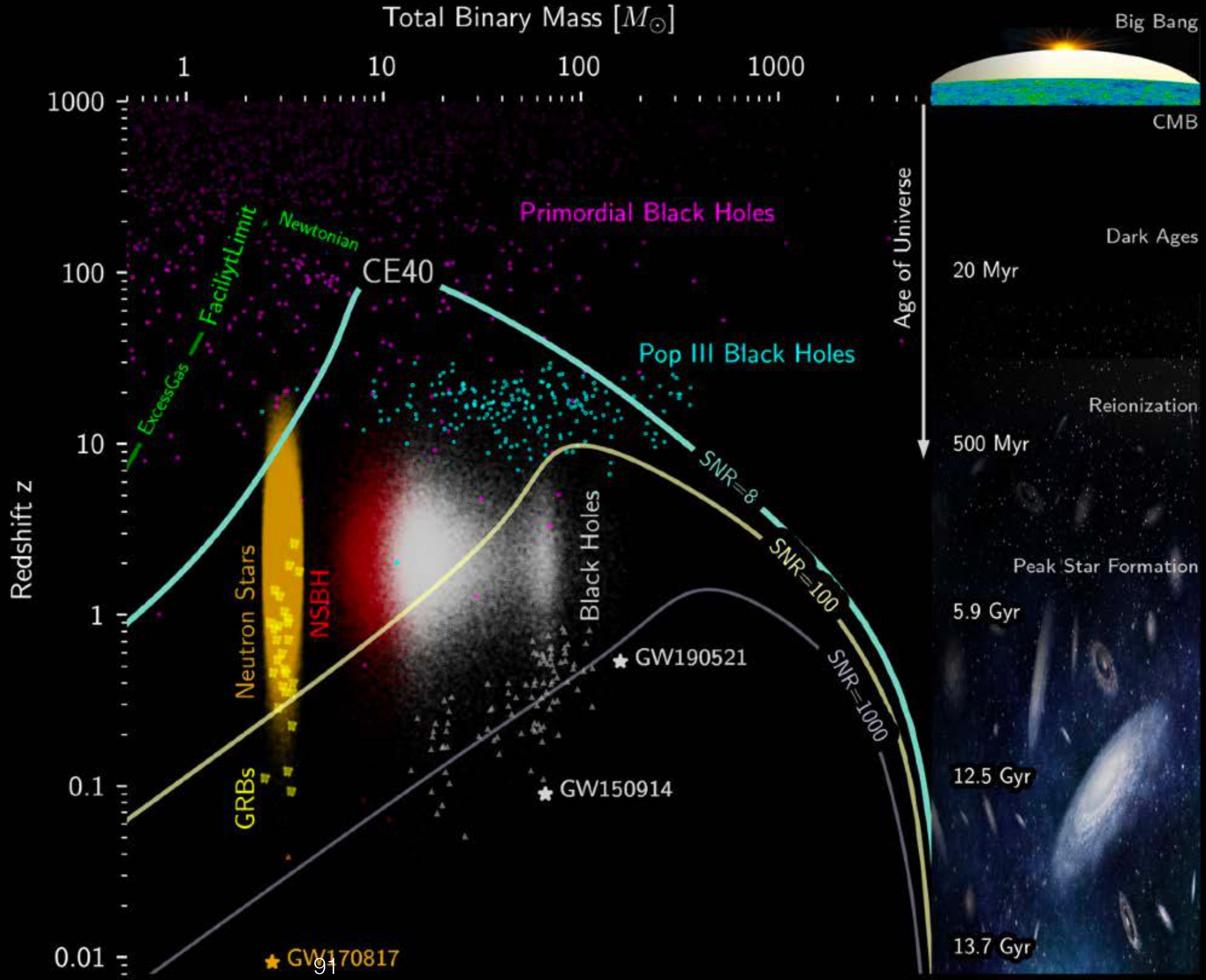


2020

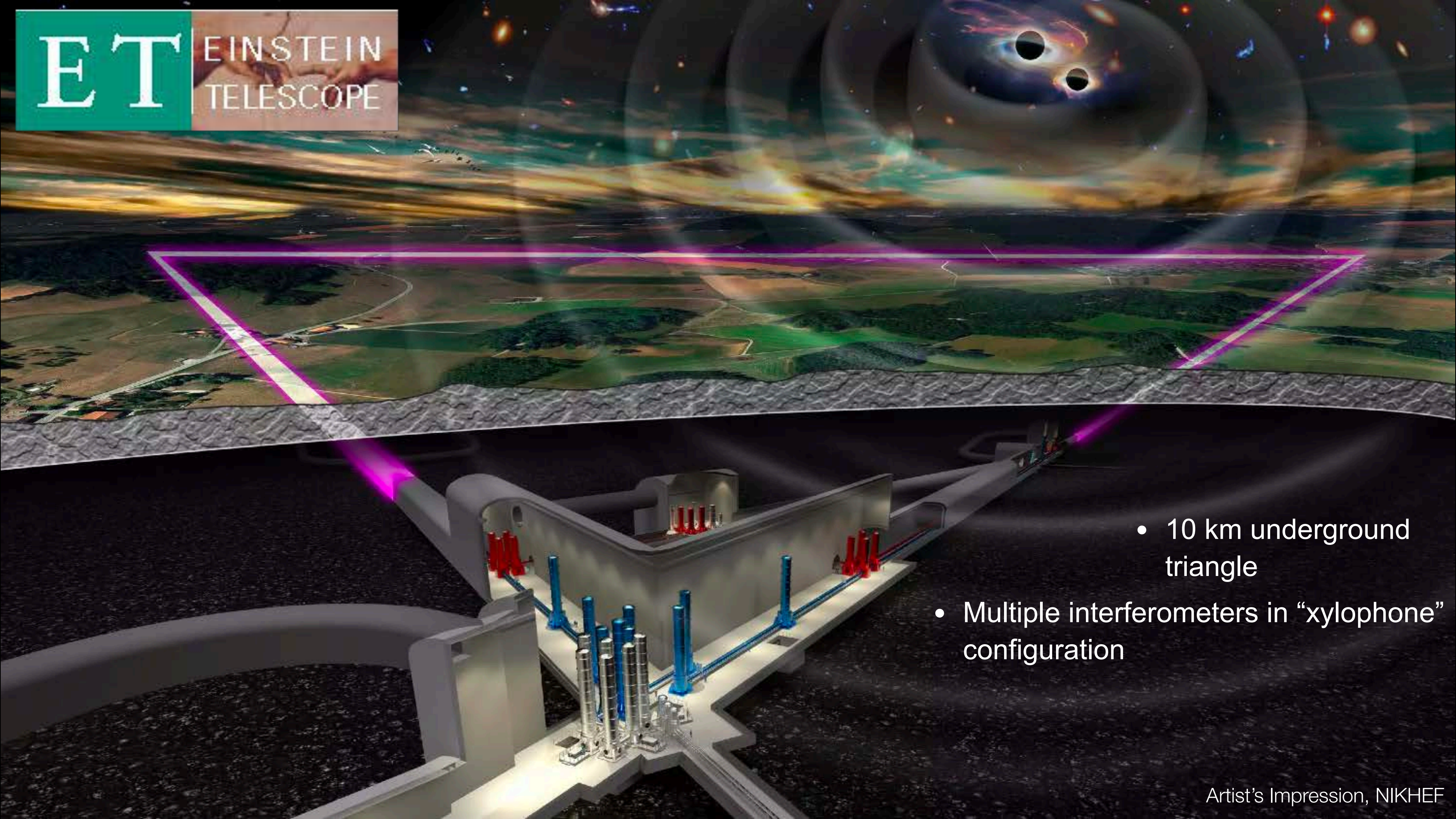
Next-generation (~2035)

CE Horizon Study , arXiv:2109.09882

ngGW Universe

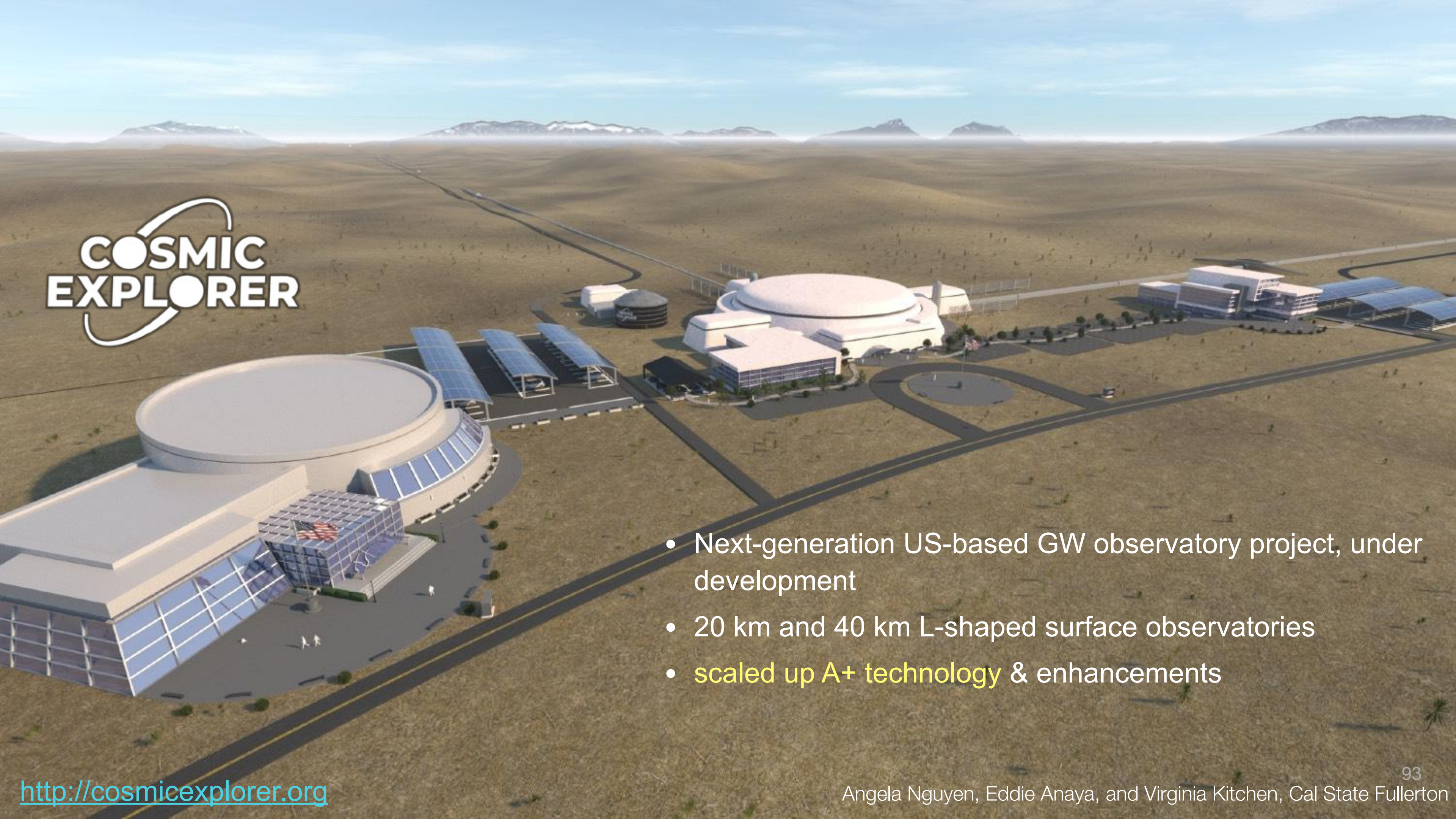


CE White Paper for NSF MSCAC ngGW , arXiv:2109.09882



- 10 km underground triangle
- Multiple interferometers in “xylophone” configuration

Artist’s Impression, NIKHEF



- Next-generation US-based GW observatory project, under development
- 20 km and 40 km L-shaped surface observatories
- scaled up A+ technology & enhancements

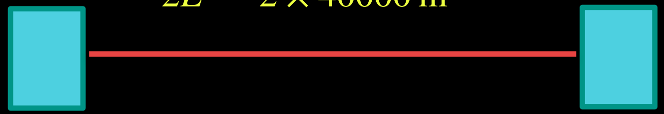
http://cosmicexplorer.org

Angela Nguyen, Eddie Anaya, and Virginia Kitchen, Cal State Fullerton

Challenges of a long detector

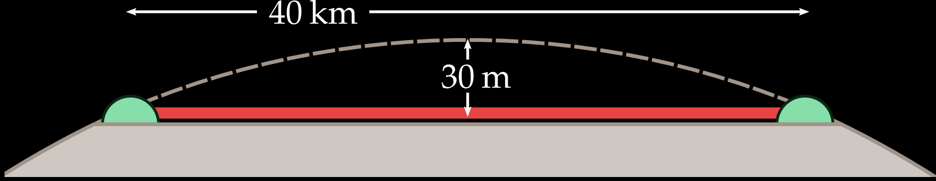
- If an arm cavity is too long, higher frequency gravitational waves will fall outside its resonant bandwidth

$$FSR = \frac{c}{2L} = \frac{3 \times 10^8 \,\mathrm{m/s}}{2 \times 40000 \,\mathrm{m}} = 3750 \,\mathrm{Hz}$$



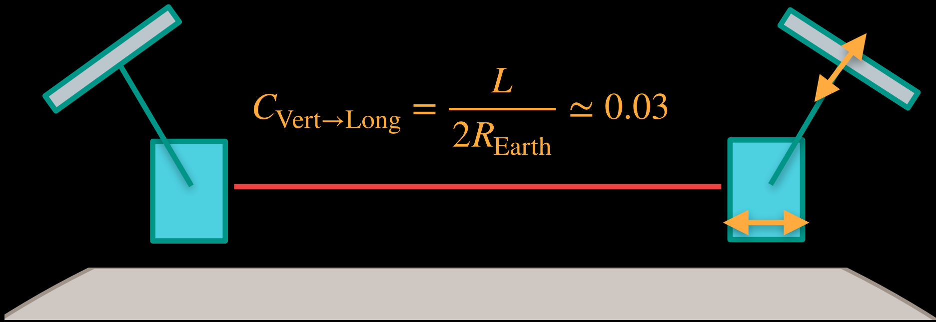
- The Earth is curved so:
  - Building on a site that follows the Earth's geode would require lots of soil redistribution ($\approx 10^6 \mathrm{m}^3$)

$$H = \frac{L}{8R_{\mathrm{Earth}}} \simeq \frac{(40000 \,\mathrm{m})^2}{8 \times 6400000 \,\mathrm{m}} \simeq 30 \,\mathrm{m}$$



  - The pendulums hang toward the earth's center, not perpendicular to the laser beam, coupling vertical motion to longitudinal

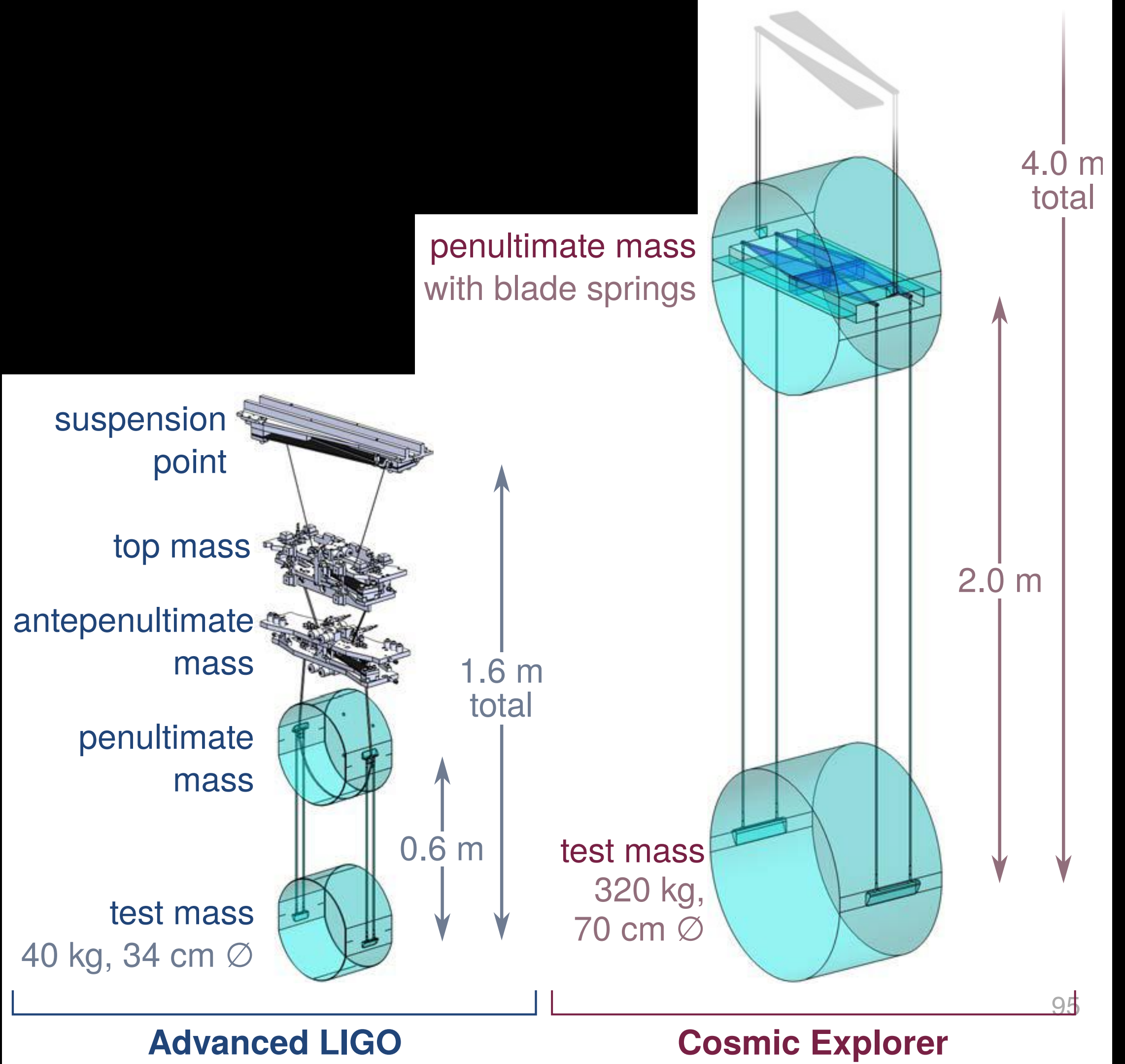
$$C_{\mathrm{Vert} \rightarrow \mathrm{Long}} = \frac{L}{2R_{\mathrm{Earth}}} \simeq 0.03$$



Vibration reduction

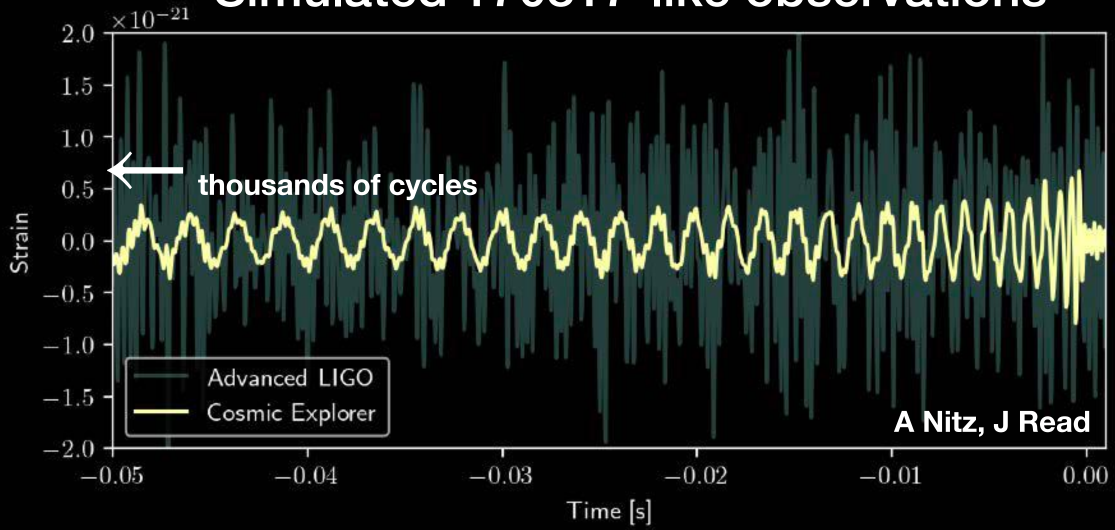
- Quadruple pendulums
- Filter vibrations above 5 Hz
- Add minimal noise from thermodynamic fluctuations

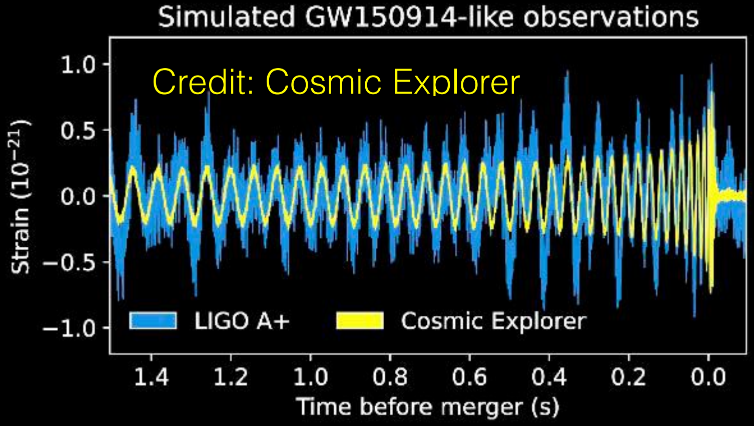
  => monolithic glass lower stages
- Support 320 kg test mass



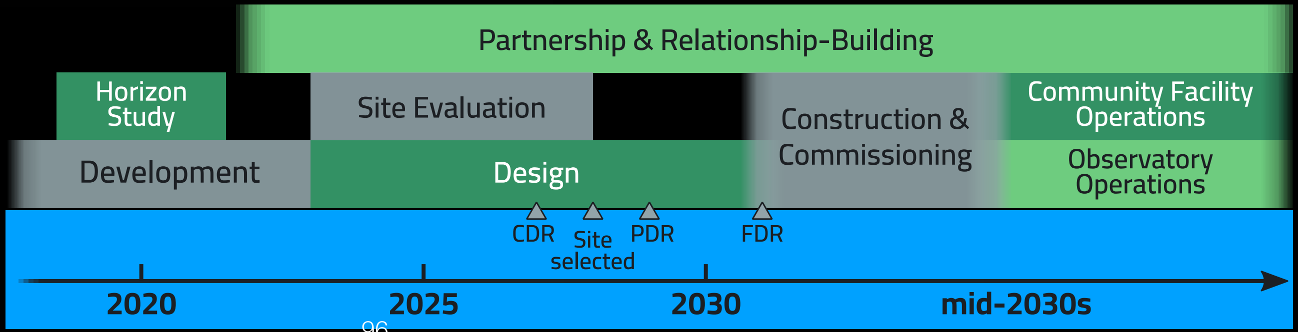
Slide: Evan Hall, MIT   Hall+ *PRD* **103** 122004 (2021)

Simulated 170817-like observations





Timeline



White Paper for NSF MSCAC ngGW , arXiv:2109.09882

Thank you

Credit: Eddie Anaya, Cal State Fullerton