Neutrino mixing and mass
Part 1: mixing

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Mass generation mechanism

Large scale structure formation

Dark Matter

Cosmic messengers

Matter Anti-matter asymmetry
Questions for today

- How do we know neutrinos exist?
- How do we know neutrinos have a mass?
- Which properties of neutrinos are still unknown?
- Are there more than three neutrinos?
Imagine we’d be a physicist in 1930...

\[ n \rightarrow p + e \]
Imagine we’d be a physicist in 1930...

\[ n \to p + e \]

Electron energy
Imagine we’d be a physicist in 1930...
1930: Postulation of the neutrino
1930: Postulation of the neutrino
1930: Postulation of the neutrino

"Today I did something terrible, something no theoretical physicist should ever do. I proposed something that can never be verified experimentally."
Neutrinos from a reactor

\[ n \rightarrow p + e + \bar{\nu}_e \]

On average six anti-neutrinos per fission
Neutrino detection

\[ \bar{\nu}_e + p \rightarrow n + e^+ \]
1956: Discovery of Neutrino

1995: Nobel price in physics
(Frederick Reines)
More neutrinos

Muon neutrinos (1962)

Tau neutrinos (2000)
Neutrinos in the Standard Model

- 3 Flavours
- Spin $\frac{1}{2}$
- Electrically neutral
- Only interact weakly
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  - Experimental discovery in 1956 (reactor neutrinos, inverse beta decay)
"How can I detect neutrinos from the sun?"
Nuclear fusion in the sun
Only electron neutrinos are created
60 Billion $\nu$’s/cm$^2$/s on earth
Is this possible?

\[ \nu_e + p \rightarrow n + e^+ \]
This is possible:

\[ \nu_e + n \rightarrow p + e^- \]
Radiochemical Detection

1 neutrino every 2 days

$\nu_e + ^{37}\text{Cl} \rightarrow ^{37}\text{Ar} + e$

$T_{1/2} (^{37}\text{Ar}) = 35\text{ days}$

e + ^{37}\text{Ar} \rightarrow \nu_e + ^{37}\text{Cl}$

615 t perchloroethylene
Solar Neutrino Problem

• ...All experiments measure less neutrinos than expected
• What is wrong? The expectation? The measurement?
Flavours und Masses

PMNS matrix

\[
\begin{pmatrix}
\nu_e \\
\nu_\mu \\
\nu_\tau
\end{pmatrix}
= 
\begin{pmatrix}
U_{e1} & U_{e2} & U_{e3} \\
U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\
U_{\tau 1} & U_{\tau 2} & U_{\tau 3}
\end{pmatrix}
\begin{pmatrix}
\nu_1 \\
\nu_2 \\
\nu_3
\end{pmatrix}
\]
Neutrino Propagation

Neutrino creation

Neutrino propagation
Neutrino Propagation

\[
|v_e\rangle = U_{e1} \cdot |v_1\rangle + U_{e2} \cdot |v_2\rangle + U_{e3} \cdot |v_3\rangle
\]

\[
e^{-i\hat{H}t/\hbar} |v_e\rangle = U_{e1} \cdot e^{-i\hat{H}t/\hbar} |v_1\rangle + U_{e2} \cdot e^{-i\hat{H}t/\hbar} |v_2\rangle + U_{e3} \cdot e^{-i\hat{H}t/\hbar} |v_3\rangle
\]

\[
= U_{e1} \cdot e^{-iE_1t/\hbar} |v_1\rangle + U_{e2} \cdot e^{-iE_2t/\hbar} |v_2\rangle + U_{e3} \cdot e^{-iE_3t/\hbar} |v_3\rangle
\]
Neutrino Oscillations (for 2 flavours)

\[ P(\nu_\mu \rightarrow \nu_e) = \sin^2 2\theta \cdot \sin^2 (\Delta m^2 \cdot L / E) \]

Amplitude $\sin^2 2\theta$
Frequency $\sin^2 (\Delta m^2 \cdot L / E)$
Neutrino Oscillations (for 3 flavours)

\[ \sin^2 2\theta_{13} \]

\[ \Delta m_{23}^2 \]

\[ \Delta m_{12}^2 \]
SNO Experiment in Canada

Bowl filled with heavy water = Deuterium

How can we test that the neutrinos change their flavour?
The Idea!

charged current (CC) $\rightarrow$ only electron flavour

neutral current (NC) $\rightarrow$ all flavours
SNO Phase 1: only heavy water
SNO Phase 2: with 2t NaCl (Salt)

- Chlor captures neutron
- Deexcitation of Cl* is measured via Cherenkov signal
...and what did SNO find?

So my calculations were right...

... and my measurements, too

2001 (30 years later)...
...and what did SNO find?

Takaaki Kajita and Arthur B. McDonald: “for the discovery of neutrino oscillations, which shows that neutrinos have mass”
\( \theta_{12} = 33^\circ (\theta_{\text{sol}}) \)

- SNO, Kanada (Sun)
- KamLAND, Japan (Reactor)

\( \theta_{23} = 45^\circ (\theta_{\text{atm}}) \)

- Super Kamokande, Japan (Atmosphere)
- T2K, (Accelerator)
- Minos, USA (Accelerator)

\( \theta_{13} = 9^\circ \)

- Double Chooz, FR (Reactor)
- Daya Bay, China (Reactor)
- RENO, Korea (Reactor)

\[ \Delta m_{21}^2 = 8 \cdot 10^{-5} \text{eV}^2 (\Delta m_{\text{sol}}^2) \]

\[ \Delta m_{32}^2 \approx \Delta m_{31}^2 = (\pm) 2.5 \cdot 10^{-3} \text{eV}^2 (\Delta m_{\text{atm}}^2) \]
Questions for today

How do we know neutrinos exist?
- Postulation by Pauli in 1930 (continuous beta decay spectrum)
- Experimental discovery in 1956 (reactor neutrinos, inverse beta decay)

How do we know neutrinos have a mass?
- Solar (+atmospheric) neutrino problem
- Discovery of neutrino oscillations
- Sensitive to mass squared difference

Which properties of neutrinos are still unknown?

Are there more than three neutrinos?
What we know, and don’t know...

- At least two neutrinos have a mass
- $\Delta m^2_{21} = 8 \cdot 10^{-5} \text{ eV}^2$, $\Delta m^2_{31} = 2.5 \cdot 10^{-3} \text{eV}^2$
- $\nu_1$ is lighter than $\nu_2$ (matter effects)

- How are the neutrinos masses ordered?
- Do neutrinos violate CP?
- Are there more than three neutrinos?
- What is the mass of the lightest neutrino?
- Is the neutrino its own antiparticle?
What we know, and don’t know...

- At least two neutrinos have a mass
  - $\Delta m^2_{21} = 8 \cdot 10^{-5} \, \text{eV}^2$, $\Delta m^2_{31} = 2.5 \cdot 10^{-3} \, \text{eV}^2$
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Two ways to resolve the ordering

**Precision**

- **Normal ordering**
  - $m^2$
  - $\Delta m_{13}^2$
  - $\nu_e$, $\nu_\mu$, $\nu_\tau$
  - $\nu_3$, $\nu_2$, $\nu_1$

- **Inverted ordering**
  - $m^2$
  - $\Delta m_{13}^2$
  - $\nu_e$, $\nu_\mu$, $\nu_\tau$
  - $\nu_3$, $\nu_2$, $\nu_1$

**Matter effects**

- **Normal ordering**
  - $m^2$
  - $\Delta m_{13}^2$
  - $\nu_e$, $\nu_\mu$, $\nu_\tau$
  - $\nu_3$, $\nu_2$, $\nu_1$

- **Inverted ordering**
  - $m^2$
  - $\Delta m_{13}^2$
  - $\nu_e$, $\nu_\mu$, $\nu_\tau$
  - $\nu_3$, $\nu_2$, $\nu_1$
Neutrino ordering

\[
\begin{align*}
P_{ee} & = 1 - P_{21} - P_{31} - P_{32} \\
P_{21} & = \cos^4 \theta_{13} \sin^2 2 \theta_{12} \sin^2 \Delta_{21} \\
P_{31} & = \cos^2 \theta_{12} \sin^2 2 \theta_{13} \sin^2 \Delta_{31} \\
P_{32} & = \sin^2 \theta_{12} \sin^2 2 \theta_{13} \sin^2 \Delta_{32}
\end{align*}
\]

- Reactor neutrino oscillation \(\rightarrow\) simplification: \(\Delta m^2_{13} \approx \Delta m^2_{23}\)
- But in reality: The oscillation frequency is slightly different for normal and inverted ordering

JUNO

neutrino source: only \(\bar{\nu}_e\) created

55km baseline
JUNO

- Experiment in China
- Detection of reactor neutrinos with large underground detector
- 35 m diameter, 20 000 tons of liquid scintillator, 15 000 PMTs
- Starts data taking soon
Two ways to resolve the hierarchy

**Precision**

- Normal ordering
- Inverted ordering

**Matter effects**

- Normal ordering
- Inverted ordering

\[
m^2
\]

\[
\Delta m^2_{13}
\]

\[
\nu_e \quad \nu_\mu \quad \nu_\tau
\]

\[
\nu_1 \quad \nu_2 \quad \nu_3
\]
Matter effects

\[ P(\nu_\mu \rightarrow \nu_e) = \sin^2 2\theta \cdot \sin^2 \left( \frac{\Delta m^2 \cdot L}{E} \right) \]
Matter effects

Small energies (1 MeV):
- The oscillation length is shorter than the mean free path
- Neutrino changes its flavor before it interacts
- Matter effects don’t matter

Large energies (10 MeV):
- Oscillation length larger than mean free path
- Neutrino interacts before it oscillates
- Neutrino stays in electron flavor eigenstate
  (electron flavor eigenstate = effective mass eigenstate)
- Matter effects matter
Neutrino ordering – matter effects

normal ordering

\[ m^2 \]

\[ \begin{align*}
\nu_e & \quad (\text{green}) \\
\nu_\mu & \quad (\text{red}) \\
\nu_\tau & \quad (\text{blue}) \\
\nu_3 & \quad (\text{red})
\end{align*} \]

inverted ordering

\[ m^2 \]

\[ \begin{align*}
\nu_e & \quad (\text{green}) \\
\nu_\mu & \quad (\text{red}) \\
\nu_\tau & \quad (\text{blue}) \\
\nu_1 & \quad (\text{red})
\end{align*} \]
Signature of mass hierarchy

\[ P(\nu_\mu \rightarrow \nu_e) = \sin^2 2\theta \cdot \sin^2 \left( \frac{\Delta m^2 \cdot L}{E} \right) \]

- Why is the signal more prominent for longer baseline?
Signature of mass hierarchy

\[ P(\nu_\mu \rightarrow \nu_e) = \sin^2 2\theta \cdot \sin^2 \left( \frac{\Delta m^2 \cdot L}{E_v} \right) \]

- Longer baseline \rightarrow larger energy (to be at the oscillation maximum) \rightarrow larger matter effect!
The DUNE experiment

- Primary goal: detection of CP violation
- Liquid Argon TPC
- 4 x 17 kt fiducial volume
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  - Solar (+atmospheric) neutrino problem
  - Discovery of neutrino oscillations
  - Sensitive to mass squared difference

- Which properties of neutrinos are still unknown?
  - Mass ordering
  - CP violation
  - Important experiments: DUNE, JUNO, Hyper-K, IceCube

- Are there more than three neutrinos?
Sterile neutrinos

This is what we call “sterile” neutrino
Active neutrinos: 3x3 PMNS matrix

\[
\begin{pmatrix}
\nu_1 \\
\nu_2 \\
\nu_3
\end{pmatrix}
= 
\begin{pmatrix}
U_{1e} & U_{1\mu} & U_{1\tau} \\
U_{2e} & U_{2\mu} & U_{2\tau} \\
U_{3e} & U_{3\mu} & U_{3\tau}
\end{pmatrix}
\begin{pmatrix}
\nu_e \\
\nu_\mu \\
\nu_\tau
\end{pmatrix}
\]
Sterile neutrinos: 4x4 PMNS matrix

\[ \begin{pmatrix} 
\nu_1 \\
\nu_2 \\
\nu_3 \\
\nu_4 
\end{pmatrix} = 
\begin{pmatrix} 
U_{1e} & U_{1\mu} & U_{1\tau} & U_{1s} \\
U_{2e} & U_{2\mu} & U_{2\tau} & U_{2s} \\
U_{3e} & U_{3\mu} & U_{3\tau} & U_{3s} \\
U_{4e} & U_{4\mu} & U_{4\tau} & U_{4s} 
\end{pmatrix} \begin{pmatrix} 
\nu_e \\
\nu_\mu \\
\nu_\tau \\
\nu_s 
\end{pmatrix} \]

Large
Small
Right-handed neutrino

The Higgs transforms left- to right-handed chirality
No neutrino mass in the SM
Adding a right-handed neutrino

Higgs Field

Not in the SM
Dirac neutrino mass

The Yukawa coupling to the Higgs would have to be tiny to explain the smallness of the neutrino mass.
Majorana neutrino mass

- A mass term for $\nu_r$ is allowed without Higgs mechanism
- New mass eigenstates = “sterile” neutrino

See Saw Type 1:
Heavy right-handed $\nu$

- $m_1 \approx \frac{m_D^2}{m_M}$ (small)
- $m_2 \approx m_M$ (large)
Sterile Neutrinos

**Heavy sterile neutrinos (> GeV)**
- Lightness of neutrinos
  + Matter/Anti-matter asymmetry

**Light sterile neutrinos (~1 eV)**
- Short-baseline neutrino oscillation anomalies

**KeV-scale sterile neutrinos (~ 1 - 50 keV)**
- Dark matter candidate
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  - Solar (+atmospheric) neutrino problem
  - Discovery of neutrino oscillations
  - Sensitive to mass squared difference

- Which properties of neutrinos are still unknown?
  - Mass ordering
  - CP violation
  - Important experiments: DUNE, JUNO, and others

- Are there more than three neutrinos?
  - Maybe.
  - New neutrino mass eigenstates = almost sterile
  - eV = oscillation anomalies
  - keV = dark matter
  - very heavy = see saw
Thank you for your attention

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