Neutrinos in Medium

Medium-Enhanced/Suppressed Neutrino Flavor Transformation

Photons acquire an index of refraction from forward scattering on the atoms (electrons) in, for example, glass or air. The propagation speed of light in materials like these is reduced from the vacuum propagation speed. It is as if the photons acquire an "*effective mass*" in medium.

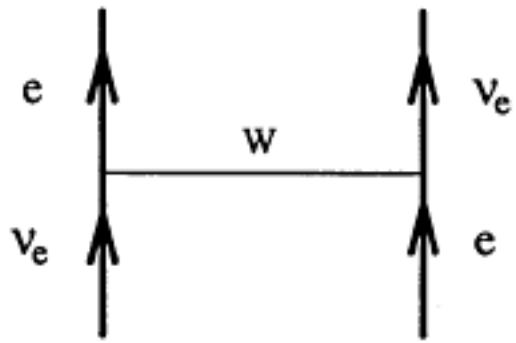
Likewise, a neutrino can acquire an index of refraction by forward scattering on any particles that carry weak charge (e.g., electrons, positrons, quarks, or other neutrinos). Again, we can think of this as an "effective mass" acquired by the neutrino in medium. Flavor mixing properties altered as well.

In optically birefringent media like calcite crystals, the index of refraction depends on the polarization state of the photon.

All media of interest in the cosmos are "optically birefringent" for neutrinos. That is, the effective mass acquired by a neutrino depends on its flavor state.

Coherently propagating
active neutrinos acquire effective masses
(like index of refraction) via forward scattering
on particles that carry weak charge.

Consider neutrino-electron charged current forward exchange scattering as a source of neutrino index of refraction, effective mass:



A schematic view of neutrino effective masses

The current-current Lagrangian for neutrino-electron scattering

$$L_{total} = \overline{\Psi}_\nu (i\partial\!\!\!/ - m_\nu)\Psi_\nu + \overline{\Psi}_e (i\partial\!\!\!/ - m_e)\Psi_e - \frac{G_F}{\sqrt{2}}(\overline{\Psi}_\nu \gamma^\mu (1-\gamma_5)\Psi_\nu)(\overline{\Psi}_e \gamma_\mu (1-\gamma_5)\Psi_e)$$

From this we can define a potential stemming from the electron background:

$$A^\mu \equiv \frac{G_F}{\sqrt{2}}\left[\overline{\Psi}_e \gamma^\mu (1-\gamma_5)\Psi_e\right] = (\varphi, \mathbf{A})$$

The neutrino Lagrangian is then:

$$L_\nu = \overline{\Psi}_\nu \left(i\partial\!\!\!/ - A\!\!\!/(1-\gamma_5) - m_\nu\right)\Psi_\nu$$

The equation of motion (Dirac equation) corresponding to this is . . .

$$\underbrace{\left(i\frac{\partial}{\partial t} - \varphi\right)\Psi_\nu = \left[\boldsymbol{\alpha} \bullet (\tfrac{1}{i}\nabla - \mathbf{A}) + \beta m_\nu\right]\Psi_\nu}$$

and the dispersion relation is . . .

$$(E_\nu - \varphi)^2 = (\mathbf{p}_\nu - \mathbf{A})^2 + m_\nu^2$$

$$E_\nu^2 = \mathbf{p}_\nu^2 + \left[m_\nu^2 + 2E_\nu\varphi - 2\mathbf{p}_\nu \bullet \mathbf{A} + \left(\mathbf{A}^2 - \varphi^2\right)\right]$$

$$2E_\nu \frac{G_F}{\sqrt{2}}\left(\overline{\Psi}_e \gamma^0 (1-\gamma_5)\Psi_e\right) \approx 1.5\times 10^{-7}\,\mathrm{eV}^2 \left(\frac{\rho N_a Y_e}{1\mathrm{g\ cm}^{-3}}\right)\left(\frac{E_\nu}{\mathrm{MeV}}\right)$$

$$E_\nu^2 = \mathbf{p}_\nu^2 + \left[m_\nu^2 + 2E_\nu\langle\varphi\rangle - 2\langle\mathbf{p}_\nu \bullet \mathbf{A}\rangle + \left(\langle\mathbf{A}^2\rangle - \langle\varphi^2\rangle\right)\right]$$

$$\sim G_F^2$$

$$2\langle p_\nu A\cos\theta\rangle \approx 2E_\nu\langle\varphi\rangle\langle\cos\theta\rangle$$

zero if electron distribution is isotropic