Neutrino Astrophysics I

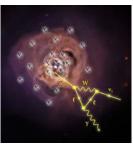
Neutrino Physics, Entropy, Gravitation: The early universe, nucleosynthesis, the CvB



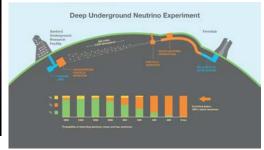
CMB: e.g., Simons Observatory



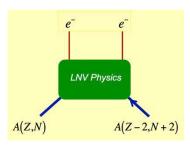
30m telescope/NAOJ



X-Ray Astronomy Image credit: Science/Abazajian



Long Baseline Oscillation experiments, e.g., DUNE, https://www.dunescience.org/



Lepton Number Violation: $0\nu\beta\beta$ Lepton degeneracy

N3AS Summer School in Multi-Messenger Astrophysics July 22 and 23, 2023



George M. Fuller

Department of Physics
University of California, San Diego









Weak Nuclear Force

Much about this aspect of nature remains mysterious. Consequently, it is the subject of frontier research in elementary particle physics, nuclear physics, and astrophysics.

Despite its well deserved moniker, "weak", this force is the origin of all of humanity's energy sources (fossil fuels, solar, nuclear fission & fusion), save one (tidal).

An insidious conspiracy of this weak nuclear force and an even weaker force, gravitation, sets the conditions in the early universe, murders massive stars by causing their collapse and explosion, but brings about the creation of the elements in both of these venues!

Astrophysical Neutrino "Laboratories"

Early Universe, Weak Decoupling/BBN

Gravitation dictates a **slow expansion**, allowing very weakly interacting particles to affect the physics. **Large entropy-per-baryon**, $S/k_{\rm b} \sim 10^{10}$, simplifying the nuclear physics. **Low lepton numbers**, implying very small $\nu - \bar{\nu}$ asymmetry. n/p, deuterium (D), helium, $N_{\rm eff}$ sensitive to any BSM physics that alters the time/temperature/scale factor relationship.

very tightly constrained

by CMB (soon Stage-4) observables and 30m-class telescope-determined D/H.



Stellar Collapse, supernovae, binary compact object mergers

Weak interaction dictates all aspects of evolution. Very large electron lepton number, so evolution is exquisitely sensitive to lepton number violation.

Low-to-high entropy, $S/k_b \sim 1$ to ~ 100 ; primary site for intermediate and heavy nucleus nucleosynthesis; many aspects can be sensitive to neutrino flavor transformation and BSM physics.

Manufactures neutron stars and black holes.

Not well constrained



$$\nu_e + n \rightleftharpoons p + e^-$$

$$\bar{\nu}_e + p \rightleftharpoons n + e^+$$

$$n \rightleftharpoons p + e^- + \bar{\nu}_e$$

Gravitation is really, really feeble compared to the strong, electromagnetic, and even the weak interaction

$$\Gamma = (nv) \cdot \sigma$$

scattering rate $(s^{-1}) = \text{flux } (cm^{-2} s^{-1}) \cdot \text{cross section } (cm^2)$

 $\sigma_{\rm strong} = \text{geometric area of particle}$

$$\sigma_{\rm elec} \approx \sigma_{\rm T} \approx 6.65246 \times 10^{-25} \, {\rm cm}^2 \approx 10^{-24} \, {\rm cm}^2 = 1 \, {\rm barn}$$

$$\sigma_{\rm weak} \sim G_{\rm F}^2 E_{\nu}^2 \approx 10^{-44} \, {\rm cm}^2 = 10^{-20} \, {\rm barn}$$

$$\sigma_{\rm grav} \sim G^2 \, m_{\rm nuc}^2 \sim 10^{-80} \, {\rm barn}$$

Dimensionless comparison of the forces

$$\alpha_{\rm strong} = 1$$

$$\alpha_{\rm elec} = \frac{1}{4\pi\,\epsilon_0} \cdot \frac{e^2}{\hbar c} \approx \frac{1}{137.036} \approx 7.297 \times 10^{-3}$$
 $\alpha_{\rm grav} = G\,m_{\rm e}^2 = \left(\frac{m_{\rm e}}{m_{\rm pl}}\right)^2 \approx 1.751 \times 10^{-45}$

$$\alpha_{\rm weak} = G_{\rm F} \, m_{\rm e}^2 \approx 3.045 \times 10^{-12}$$

$$\alpha_{\rm grav} = G m_{\rm e}^2 = \left(\frac{m_{\rm e}}{m_{\rm pl}}\right)^2 \approx 1.751 \times 10^{-45}$$

The Periodic Table of Elementary Particles and Forces

The elementary particles:

These particles are the building blocks of the **Standard Model**.

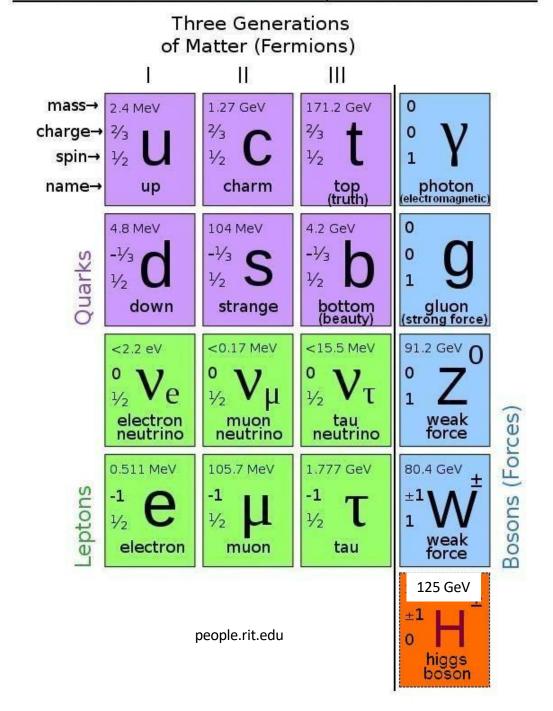
Neutrinos, like the charged leptons and quarks, are spin-1/2 but, unlike those particles, neutrinos have no electric charge.

Each particle has an antiparticle, so there are six *known* neutrinos:

$$u_e, \overline{\nu}_e, \nu_\mu, \overline{\nu}_\mu, \nu_ au, \overline{\nu}_ au$$
electron muon tau flavor flavor

Fermions: half-integral spin

Bosons: *integral spin*



much remains mysterious about neutrinos and the weak interaction

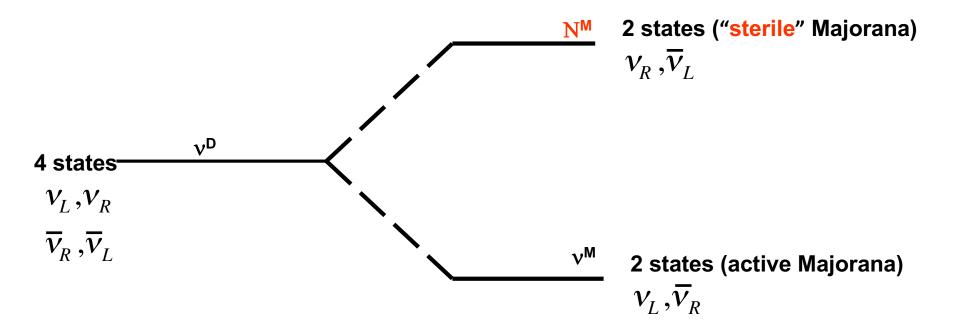
- origin of neutrino mass (another Higgs? See-Saw? Heavy sterile states?)
- Majorana or Dirac?
- baryon/lepton numbers (related to neutrino mass physics, e.g., vMSM?)
- Dark Matter/Dark Sector (Non-standard v interactions? v dark sector?)
- v & other BSM physics may affect relic neutrinos/cosmology
 as well as compact object physics and nucleosynthesis

We have: mass-squared differences/mixing angles.

We need: absolute masses; mass-hierarchy (normal?); Majorana/Dirac; CP-violating phase(s); unknown unknowns!

Why Are Neutrinos so light? - See-Saw?

Dirac Neutrinos
$$v \neq \overline{v}$$
 $v + \overline{v} = 4$ states Majorana Neutrinos $v = \overline{v}$ $v + \overline{v} = 2$ states



See-Saw Relation for the Product of Neutrino Masses: $(m_N)(m_v) \sim (Really Big Mass Scale)^2$ Unification Scale?

Gell-Mann, Ramond, Slansky; Yanagida; Mohapatra & Senjanovic

(after a slide by Boris Kayser)

Neutrino Properties

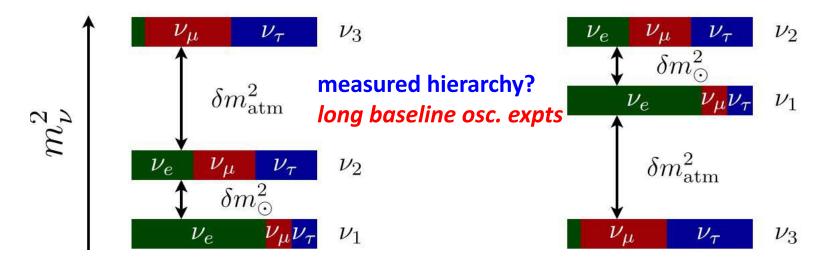
we know the mass-squared differences: $\begin{cases} \delta m_{\odot}^2 \approx 7.6 \times 10^{-5} \, \mathrm{eV}^2 \\ \delta m_{\odot}^2 \approx 2.4 \times 10^{-3} \, \mathrm{eV}^2 \end{cases}$

$$e.g., \ \delta m_{21}^2 \equiv m_2^2 - m_1^2$$

$$\begin{cases} \delta m_{\odot}^2 \approx 7.6 \times 10^{-5} \,\text{eV}^2 \\ \delta m_{\text{atm}}^2 \approx 2.4 \times 10^{-3} \,\text{eV}^2 \end{cases}$$

we do not know the absolute masses, but likely we soon will know the *mass hierarchy*:

normal mass hierarchy inverted mass hierarchy



A key development that will *increase* the discovery potential of the CMB-Neutrino connection and and compact object/nucleosynthesis/neutrino signal physics:

DUNE/long baseline experiments:

Will pin down the neutrino mass hierarchy at 5σ within 6 years Good hints within a few years(?)

$$\begin{vmatrix} \nu_e \rangle \\ |\nu_\mu \rangle \\ |\nu_\tau \rangle \end{pmatrix} = U_m \begin{pmatrix} |\nu_1 \rangle \\ |\nu_2 \rangle \\ |\nu_3 \rangle \end{pmatrix}$$

Pontecorvo-Maki-Nakagawa-Sakata matrix

$$U_m = U_{23} \, U_{13} \, U_{12}$$

If Neutrinos are Majorana . . .

$$U_{23} \equiv \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \theta_{23} & \sin \theta_{23} \\ 0 & -\sin \theta_{23} & \cos \theta_{23} \end{pmatrix}$$

$$U_{13} \equiv \begin{pmatrix} \cos \theta_{13} & 0 & e^{i\delta} \sin \theta_{13} \\ 0 & 1 & 0 \\ -e^{-i\delta} \sin \theta_{13} & 0 & \cos \theta_{13} \end{pmatrix}$$

$$U_{12} \equiv \begin{pmatrix} \cos \theta_{12} & \sin \theta_{12} & 0 \\ -\sin \theta_{12} & \cos \theta_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad \theta_{12} \approx 0.59^{+0.02}_{-0.015} \\ \theta_{23} \approx 0.785^{+0.124}_{-0.124} \approx \frac{\pi}{4} \\ \theta_{13} \approx 0.154^{+0.065}_{-0.065}$$

$$M = \begin{bmatrix} 1 & 0 & 0 \\ 0 & e^{-i\alpha_1/2} & 0 \\ 0 & 0 & e^{-i\alpha_2/2} \end{bmatrix}$$

unknown:

Majorana phases α_1 , α_2

Hints in $0v\beta\beta$ or supernovae?

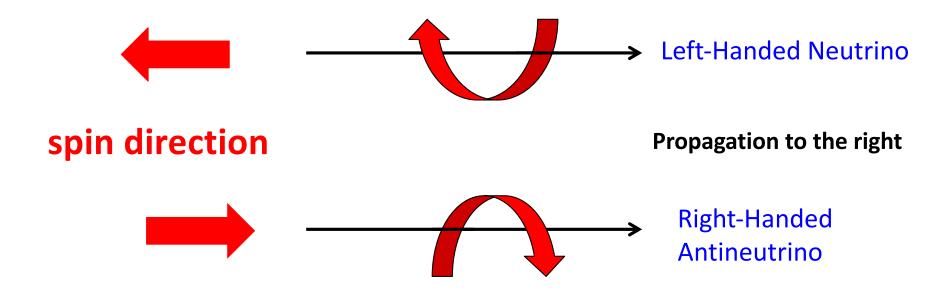
4 parameters

$$\theta_{12}, \theta_{23}, \theta_{13}, \delta$$

$$\theta_{12} \approx 0.59^{+0.02}_{-0.015}$$
 $\theta_{23} \approx 0.785^{+0.124}_{-0.124} \approx \frac{\pi}{4}$
 $\theta_{13} \approx 0.154^{+0.065}_{-0.065}$

 $\delta = CP$ violating phase =?

neutrino "spin"



neutrino spin flip, i.e., converting a left-handed neutrino to a right-handed antineutrino is always $\propto m_{\nu}/E_{\nu}$

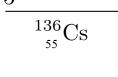
The surprise is that it can be done coherently and possibly collectively in environments with anisotropic neutrino distributions

neutrinoless double beta decay $(0v\beta\beta)$

an observation of $0\nu\beta\beta$ implies that neutrinos are Majorana and lepton number is violated

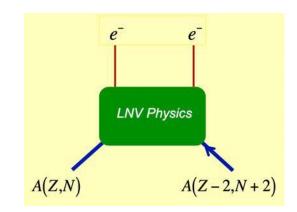
 $\frac{0^{+}}{136}Xe$

Two neutrons decay to two protons. Example: $^{136}\mathrm{Xe} \rightarrow ^{136}\mathrm{Ba}$



$$\frac{0^{+}}{{}^{136}_{56} \text{Ba}}$$

 $\begin{array}{cccc}
e^{-} & e^{-} \\
W^{-} & W^{-}
\end{array}$ A(Z,N) & A(Z-2,N+2)



Discovery of **Lepton Number Violation** (LNV) would have profound implications for particle physics and astrophysics

Neutrino mass-mediated spin flip

 $0\nu\beta\beta$ decay rate: $\lambda = G^0(E,Z) |m_{\alpha\beta}|^2 \cdot 1$

 $\lambda = G^0(E, Z) |m_{\beta\beta}|^2 \cdot |M_{\text{NUC}}|^2$ with half-life $\tau_{1/2} = \ln 2/\lambda$

$$|m_{\beta\beta}| = |m_1 c_{12}^2 c_{13}^2 + m_2 s_{12}^2 c_{13}^2 e^{2i\alpha} + m_3 s_{13}^2 e^{2i\beta}|$$

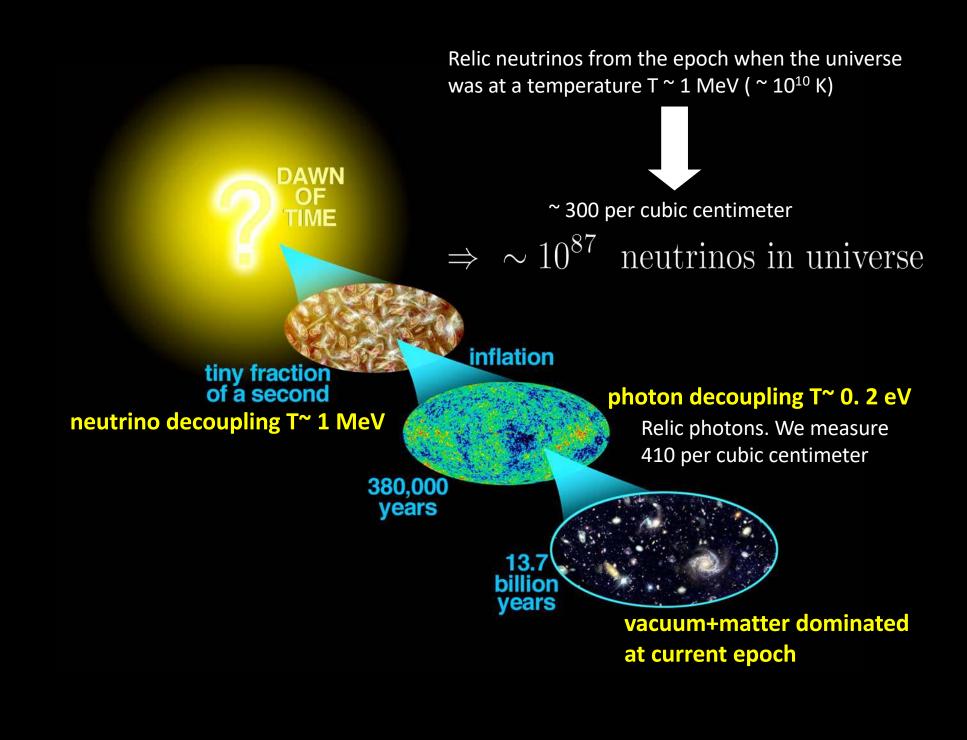
In addition to the conventional neutrino mass-mediated spin flip channel, many BSM physics possibilities could also enable 0νββ.

The alternative BSM scenarios include right-handed currents, heavy neutrino states, R-parity-violating incarnations of supersymmetry, etc.

But even if facilitated by these BSM channels, an observation of $0\nu\beta\beta$ implies that neutrinos are Majorana and lepton number is violated.

Observing 0vββ in different nuclei could help ferret out what standard or BSM channel is involved, and this is why the US DOE is backing several different detector ideas.

The Early Universe



Gravitation is weak

Gravitation drives the expansion of the universe

Consequently, the expansion rate is *SLOW*, and this allows very weakly interacting particles to affect the physics of the early universe

The *infinite raisin bread* model of the universe . . .



Image: Pleasant Hill Grain

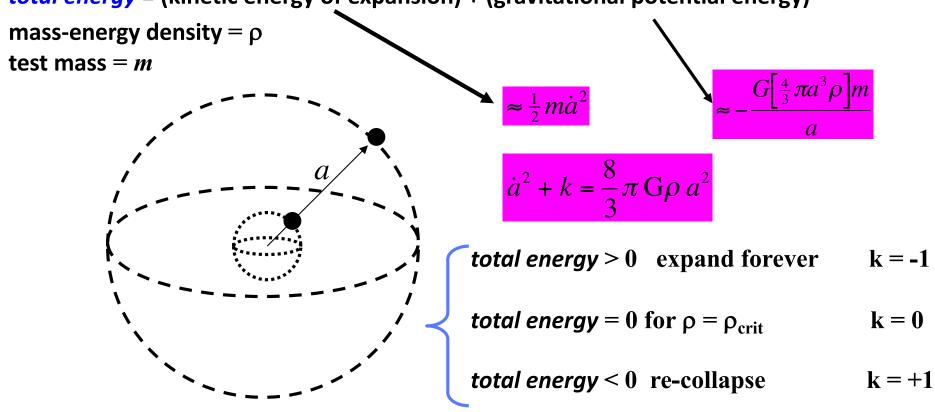
. . . as it bakes it expands uniformly everywhere

the expansion is slow because it is driven by gravitation, which is very weak, and it is uniform everywhere -- there is no center. This is not "an explosion into vacuum".

Homogeneity and isotropy of the universe:

implies that total energy inside a co-moving spherical surface is constant with time.

total energy = (kinetic energy of expansion) + (gravitational potential energy)

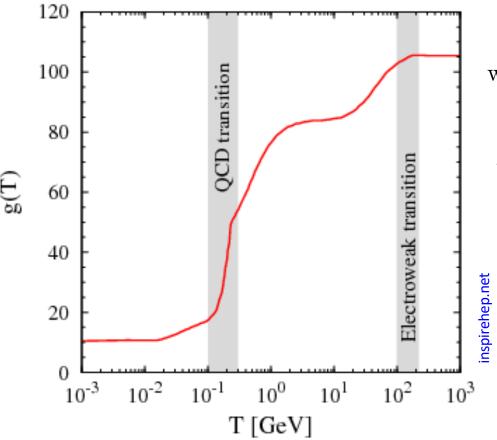


$$\Omega = \rho/\rho_{crit} = \Omega_{\gamma} + \Omega_{\nu} + \Omega_{baryon} + \Omega_{dark matter} + \Omega_{vacuum} \approx 1$$
 (k=0) ≈ 0.27

Depending on temperature, particles could be in thermal equilibrium in the "soup" of the early universe.



particle i, j, contribution to energy density $\rho_{i,j} = \frac{g}{2\pi^2} \cdot \int_{m_{i,j}}^{\infty} \frac{E^2 \left(E^2 - m_{i,j}^2\right)^{1/2} dE}{\exp\left[E/T - \eta_{i,j}\right] \pm 1}$



where degeneracy parameter is $\eta_{i,j} \equiv \frac{\mu_{i,j}}{T}$

where
$$g = \sum_{i} g_{i}^{b} \left(\frac{T_{i}}{T}\right)^{4} + \frac{7}{8} \sum_{j} g_{j}^{f} \left(\frac{T_{j}}{T}\right)^{4}$$

in the ultra relativistic limit, $\mu_{i,j} \to 0$ and $m_{i,j}/T \to 0$, we have $\rho = g \frac{\pi^2}{30} T^4$

With expansion, temperature T falls below rest mass $m_{i,j}$, and particle i, j will no longer contribute significantly to the statistical weight g.

Equation of state and entropy for particles with relativistic kinematics

energy density
$$\rho = g \, \frac{\pi^2}{30} \, T^4 \qquad \text{where} \quad g = \sum_i g_i^{\rm b} \left(\frac{T_i}{T}\right)^4 + \frac{7}{8} \, \sum_j g_j^{\rm f} \left(\frac{T_j}{T}\right)^4$$

example:
$$T = 1 \text{ MeV} \implies g = 2_{\gamma} + \frac{7}{8} (2_{e^{-}} + 2_{e^{+}} + [2 + 2 + 2]_{6 \text{ neutrinos}}) = 10.75$$

entropy per unit proper volume
$$S = \frac{4}{3} \cdot \frac{\pi^2}{30} g_s T^3 = \frac{2\pi^2}{45} g_s T^3$$

where $g = \sum_i g_{si}^b \left(\frac{T_i}{T}\right)^3 + \frac{7}{8} \sum_i g_{sj}^f \left(\frac{T_j}{T}\right)^3$

entropy – per – baryon
$$s=\frac{S}{n_{\rm b}}$$

baryon – to – photon ratio $\eta\equiv\frac{n_{\rm b}}{n_{\gamma}}\approx 6.11\times 10^{-10}$

relation between entropy-per-baryon, a co-moving invariant at low T, and the $s=\frac{\pi^4}{45\,\zeta\left(3\right)}\cdot g_s\cdot \frac{1}{\eta} \approx \left[5.89\times 10^9~k_{\mathrm{b}}\,\mathrm{per\ baryon}\right]\left(\frac{g_s}{2}\right)\left(\frac{6.11\times 10^{-11}}{\eta}\right)$ baryon-to-photon ratio (not invariant)

The temperature in the weak decoupling/BBN epoch is

$$T \sim 1 \, \mathrm{MeV}$$

The time-temperature relationship there is roughly

$$t \sim \frac{1 \text{ second}}{(T/\text{MeV})^2}$$

baryon number of universe

From CMB acoustic peaks, and/or observationally-inferred primordial D/H:

$$\eta = 6.11 \times 10^{-10}$$

three lepton numbers

$$\begin{cases} L_{\nu_e} \approx \frac{n_{\nu_e} - n_{\bar{\nu}_e}}{n_{\gamma}} \\ L_{\nu_{\mu}} = \frac{n_{\nu_{\mu}} - n_{\bar{\nu}_{\mu}}}{n_{\gamma}} \end{cases}$$

$$L_{\nu_{\tau}} = \frac{n_{\nu_{\tau}} - n_{\bar{\nu}_{\tau}}}{n_{\gamma}}$$

$$L_{
u_{ au}} = rac{n_{
u_{ au}} - n_{ar{
u}_{ au}}}{n_{\gamma}}$$

From observationally-inferred ⁴He and large scale structure and using collective (synchronized) active-active neutrino oscillations (Abazajian, Beacom, Bell 03; Dolgov et al. 03):

$$|L_{\nu_{\mu,\tau}}| \sim L_{\nu_e} < 0.15$$

Conditions in the universe when it is $\sim 1\,\mathrm{s}$ in age

Temperature $T \sim 1 \, \mathrm{MeV},$ a thousand times the temperature at the center of the sun.

We will discuss a temperature range $10\,\mathrm{MeV} > T > 10\,\mathrm{keV}$, a range which subsumes the weak decoupling/BBN epoch.

Baryon rest mass density $\rho_b \sim 10^{-3} \, \mathrm{g \, cm^{-3}}$, roughly the density of air.

But, effective inertia of the intense radiation field at this epoch $\Rightarrow \rho_{\rm rad} \sim 10^4 \, {\rm g \, cm^{-3}}$

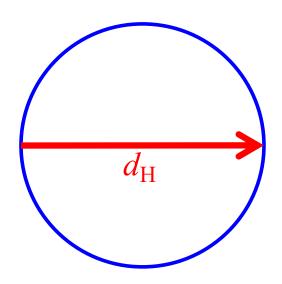
Dynamic viscosity of this fluid ranges from $\sim 10^{21} \,\mathrm{MeV^3}$, roughly the same as the earth's mantle rock (!!!), to near zero later as neutrinos decouple.

Why does this time in the early history of the universe hold promise to be a laboratory for probing *Beyond-Standard-Model* (BSM) physics?

Mass-energy inside horizon: $M_{\rm H} = \frac{4}{3} \pi \left(\frac{d_{\rm H}}{2}\right)^3 \rho$

radiation-dominated case: $M_{\rm H} = \frac{4}{3} \pi \left(t^3\right) \left[g \frac{\pi^2}{30} T^4\right];$ with $t \approx \frac{1}{2} \left(\frac{90}{8 \pi^3}\right)^{\frac{1}{2}} g^{-\frac{1}{2}} \frac{m_{\rm pl}}{T^2}$, implying $\Rightarrow M_{\rm H} \approx \left(\frac{45}{2^{10} \pi^3}\right)^{\frac{1}{2}} g^{-\frac{1}{2}} \frac{m_{\rm pl}^3}{T^2} \approx 2 \times 10^4 \,\mathrm{M}_{\odot} \left[\frac{10.75}{g}\right]^{\frac{1}{2}} \left[\frac{\mathrm{MeV}}{T}\right]^2.$

radiation-dominated horizon length: $d_{\rm H} \approx \left(\frac{90}{8\,\pi^3}\right)^{\frac{1}{2}} g^{-\frac{1}{2}} \frac{m_{\rm pl}}{T^2} \approx 4.43 \times 10^{10} \, {\rm cm} \left[\frac{10.75}{g}\right]^{\frac{1}{2}} \left[\frac{{\rm MeV}}{T}\right]^2$



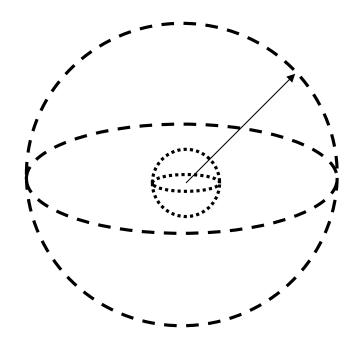
some significant events/epochs in the early universe

Epoch	Т	g _{eff}	Horizon Length	Mass-Energy (solar masses)	Baryon Mass (solar masses)
Electroweak phase transition	100 GeV	~100	~ 1 cm	~ 10 ⁻⁶ (~ earth mass)	~ 10 ⁻¹⁸
QCD	100 MeV	51 - 62	20 km	~ 1	~ 10 ⁻⁹
weak decoupling	2 MeV	10.75	~ 10 ¹⁰ cm	~ 104	~ 10 ⁻³
weak freeze out	0.7 MeV	10.75	~ 10 ¹¹ cm	~ 10 ⁵	~ 10-2
BBN	100 keV	10.75	~ 10 ¹³ cm (~ 1 A.U.)	~ 10 ⁶	~ 1
e ⁻ /e ⁺ annihilation	~ 20 keV	3.36	~ 10 ¹⁴ cm	~ 108	~ 100
photon decoupling	0. 2 eV	-	~ 350 kpc	~ 10 ¹⁸ dark matter	~ 10 ¹⁷

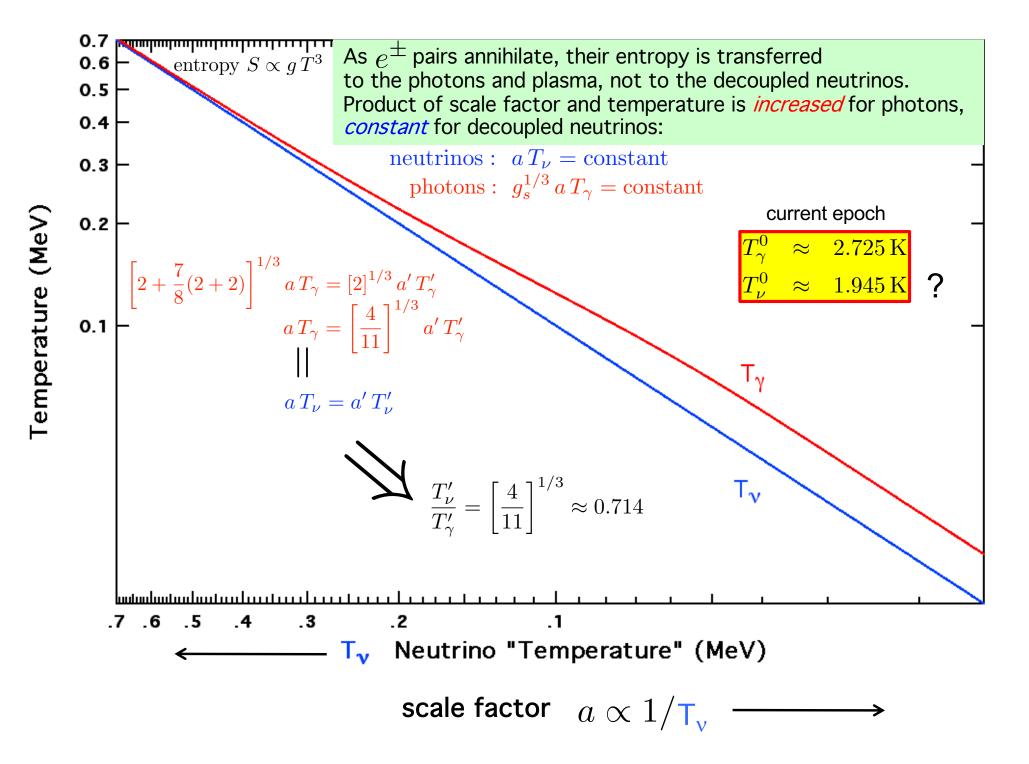
Symmetry is (nearly) everything in General Relativity

homogeneity and isotropy of this FLRW spacetime dictates that there be no spacelike heat flow or non-uniform heat sources: evolution is adiabatic

entropy in a co-moving volume is conserved*



^{*}Symmetry does not preclude *timelike* heat flows, e.g., from decaying particles or from non-equilibrium processes like decoupling v's scattering on e^-/e^+



Is Co-Moving Entropy *always* constant?

What happens when particles have dropped out of equilibrium, and then decay into particles that are coupled, that do interact?

This results in entropy generation.

If these decoupled particles are distributed homogeneously and isotropically, then the decay-generated entropy represents a *timelike* heat flow. This is allowed by symmetry!

There could be (timelike) sources of entropy:

These must be uniform, homogeneous and isotropic

Example-1: out-of-equilibrium processes

e.g., neutrino decoupling

Example-2: particle decay where the particles are (reasonably) homogenously distributed on a spacelike hypersurface *t*=constant

e.g., inflaton decay

Weak Decoupling, where neutrinos cease to scatter frequently enough to maintain thermal equilibrium with the electron/positron/photon plasma, and Weak Freeze Out, where weak interactions with nucleons are not fast enough to maintain chemical equilibrium, occur *concurrently* over a protracted time duration, ~ 100 Hubble times!

This is in stark contrast with photon (CMB) decoupling which occurs abruptly, in a small fraction of a Hubble time, Driven by the (exponential) process of the formation of atomic bound states for the electrons at temperature scale $T \sim 0.2 \text{ eV}$

HIGH ENTROPY

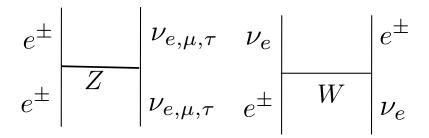
Radiation Dominated!

The entropy is high, $s \sim 10^{10}$ units of Boltzmann's constant per baryon, and that means $\sim 10^{10}$ photons per baryon, and $\sim 10^{10}$ neutrinos per baryon!

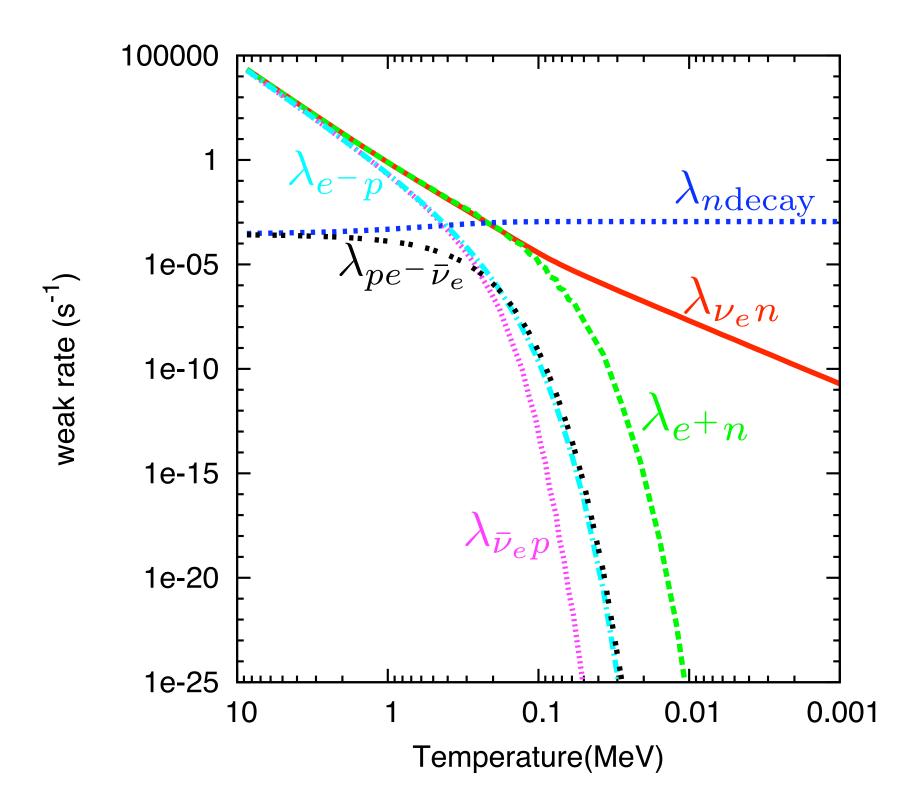
Even down to temperatures $\sim 10\,\mathrm{keV}$ there will be plenty of photons on the tail of the Planck distribution with enough energy to make e^{\pm} -pairs. We have a pair-dominated plasma all the way through weak decoupling/freeze-out and BBN!

... and lots of neutrinos per baryon, so even when the neutrinos have decoupled thermally, they still "have purchase" on neutron-to-proton inter-conversion!

neutral- and (and for electron flavor) charged-current neutrino scattering on electrons and positrons couples the neutrinos to the electron/positron/photon plasma



+ neutral current neutrino-neutrino scattering



Follow the Boltzmann evolution of the (binned) energy distribution functions f of the of the active neutrinos

$$\frac{df(p,t)}{dt} = C[f(p,t)] \qquad \qquad \left(\frac{\partial}{\partial t} - H p \frac{\partial}{\partial p}\right) f(p,t) = C(f)$$

f(p,t) =occupation probabilities for a neutrino with momentum p at FLRW time coordinate t.

If all neutrino/antineutrino scattering rates fast compared to the expansion rate of the universe H, then we attain thermal, chemical equilibrium

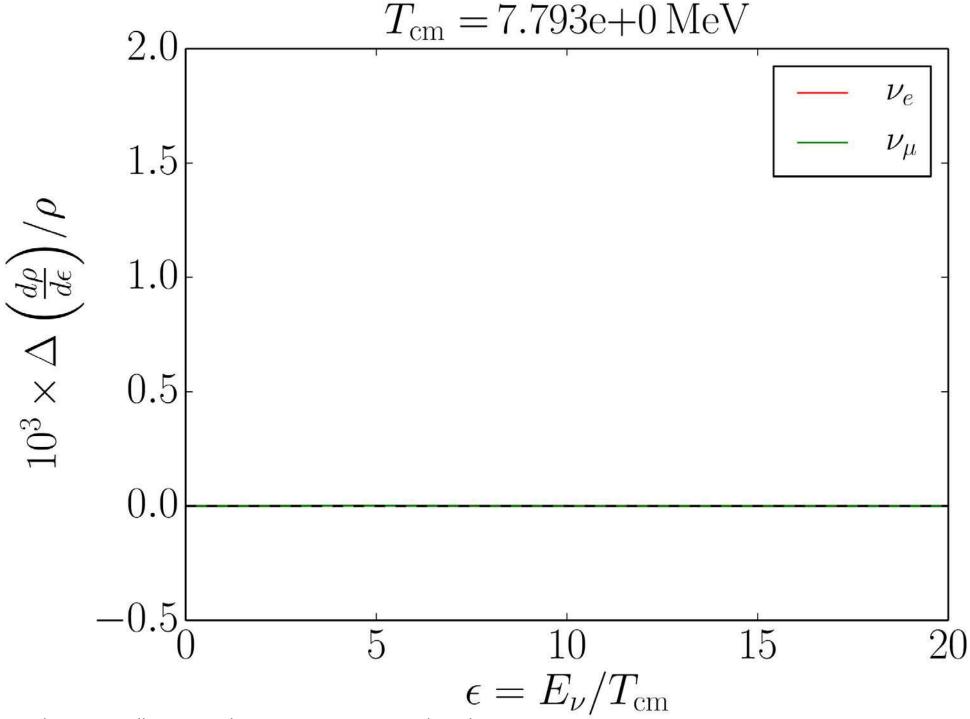
$$\Rightarrow f(p,t) = \frac{1}{e^{p/T-\eta}+1}$$

- A. Dolgov, S. Hansen, D. V. Semikoz, NP B 503, 426 (1997) arXiv:9712199
- S. Esposito et al. NP 590, 539 (2000), arXiv:0005973

"Neutrino energy transport in weak decoupling and big bang nucleosynthesis" E. Grohs, G. M. Fuller, C. T. Kishimoto, M. W. Paris, A. Vlasenko arXiv:1512.02205

Plasma/QED corrections critical:

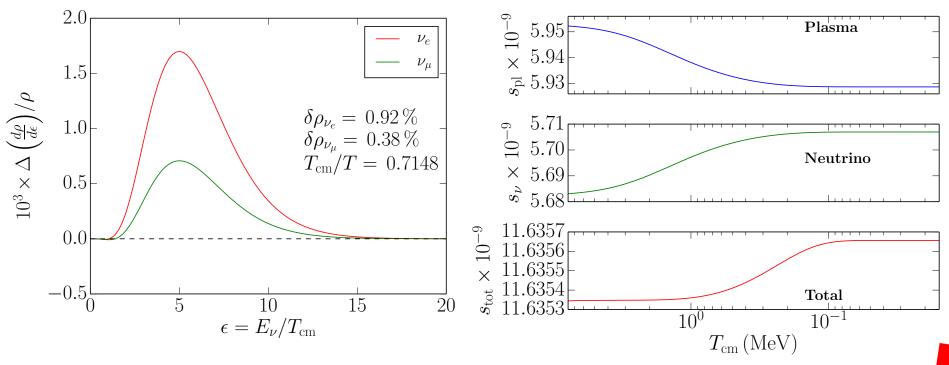
- D. Dicus, E. W. Kolb et al, PRD 26, 2649 (1982)
- S. Dodelson & M. S. Turner PRD 46, 3372 (1992)
- R. Lopez & M. S. Turner, PRD 59, 103502 (1999)



E. Grohs, G. M. Fuller, C. T. Kishimoto, M. W. Paris, A. Vlasenko arXiv:1512.02205

out-of-equilibrium neutrino scattering leads to neutrino spectral distortions and timelike entropy flow/generation

$$\nu - \nu/e^{\pm}$$
 scattering



"Neutrino energy transport in weak decoupling and big bang nucleosynthesis". **E.Grohs**, G. M. Fuller, C. T. Kishimoto, M. W. Paris, A. Vlasenko Phys. Rev. D **93**, 083522 (2016)

$$\nu_e + n \rightleftharpoons p + e^-$$

$$\bar{\nu}_e + p \rightleftharpoons n + e^+$$

$$n \rightleftharpoons p + e^- + \bar{\nu}_e$$

integrated history of the rates of these charged current processes and entropy flow determine n/p which is key to nucleosynthesis $Z p + N n \Longrightarrow A(Z, N) + \gamma$



D, ⁴He

N_{eff}