Quantum Annealing for Many-Body Physics

Yu Hong Chan, Evan Rule

Department of Physics, University of California, Berkeley, CA, USA

Introduction

Many problems in atomic, condensed matter, and nuclear physics amount to the determination of the ground state of a system of many interacting fermions. While numerous methods exist to solve such problems, they generically suffer from the fact that the number of many-body basis states grows combinatorially in the number of fermions and the number of accessible single-particle states.

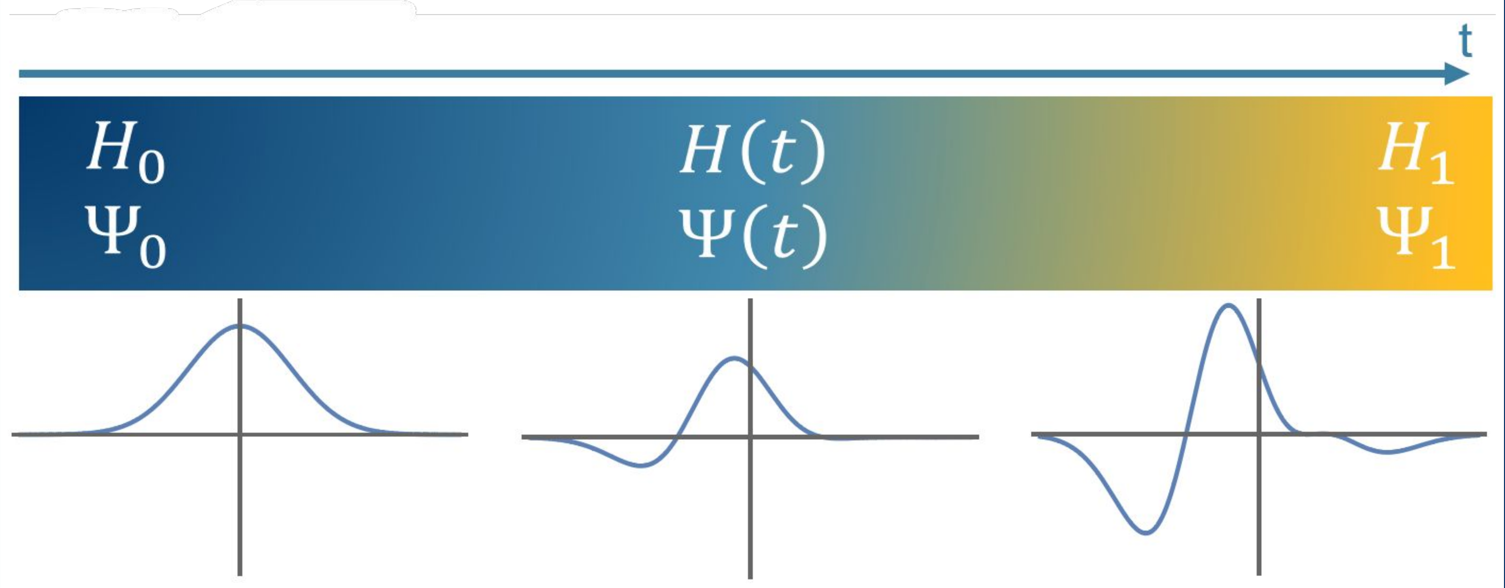
Quantum annealing is a form of quantum computing based on the adiabatic theorem:

1) Goal is to obtain ground state of "problem" Hamiltonian H_1
2) Prepare system in ground state of simple Hamiltonian H_0
3) Evolve the system under time-dependent Hamiltonian

$$H(t) = (1-t)H_0 + tH_1, \qquad 0 \leq t \leq 1$$

4) If change in $H(t)$ is *slow enough* and there is *no ground state energy crossings*, adiabatic theorem guarantees that the system will be in the ground state of H_1 at time t = 1.

Our aim is to study the use of quantum annealing to solve for the ground state of many-fermion systems.

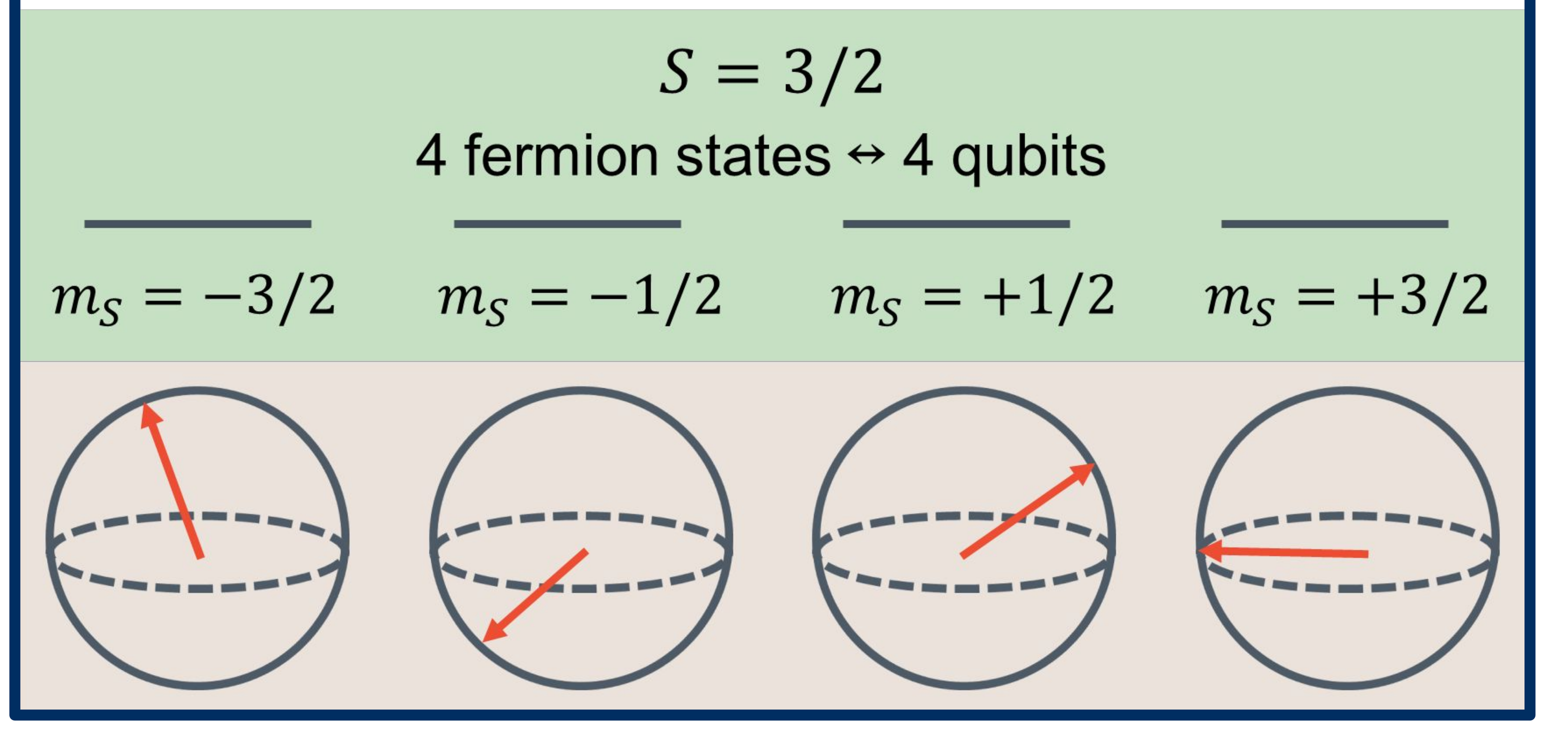


Mapping Fermions to Qubits

We map the second-quantized fermion system to qubits using the Jordan-Wigner transformation

$$\hat{a}_j \rightarrow \sigma_z^{\otimes(j-1)} \otimes \sigma_+ \otimes 1^{\otimes(N-j)}$$
$$\hat{a}_j^\dagger \rightarrow \sigma_z^{\otimes(j-1)} \otimes \sigma_- \otimes 1^{\otimes(N-j)}$$

- Fermions anticommute, but qubits commute
- Occupied fermion states ↔ spin-down qubits
- One qubit for each fermion single-particle state



Test Problem

The quantum hall effect describes electrons that are confined to 2 dimensions with a magnetic field perpendicular to the confining surface. We consider the case where N_e electrons are confined to the surface of a sphere with a radial magnetic field, characterized by its strength S (which must be integer or half-integer). Neglecting interactions, the electrons occupy Landau levels. The lowest Landau level, which we exclusively consider in this work, contains 2S+1 degenerate single-particle states, with each electron carrying total angular momentum S.

The fractional quantum Hall effect (FQHE) results from considering the Coulomb repulsion among the electrons. As the 2S+1 single-particle electron states of the lowest Landau level are degenerate, we can consider the "problem" Hamiltonian to be just the Coulomb interaction term

$$H_0 = -\frac{1}{2}\sum_i \sigma_x(i) \qquad H_1 = \sum_{i<j} \frac{\alpha\hbar c}{|\vec{r}_i - \vec{r}_j|}$$

FQHE is a good test problem:

1. Interaction is relatively simple, many-body physics is complicated
2. Lowest Landau level described by a single angular momentum subshell
3. Simple to scale the problem by adjusting two parameters S and N_e

Penalty Terms

The Hilbert space of the qubits contains states with all allowed numbers of fermions, from zero up to the number of available single-particle states (or number of qubits). In practice, we want to obtain the ground state of the FQHE Hamiltonian for a particular magnetic field strength S and a particular number of electrons N_e. We add a penalty term to the Hamiltonian singling out the states representing the desired number of electrons

$$H_{N_e} = E_{N_e}(N_e - \hat{N})^2$$

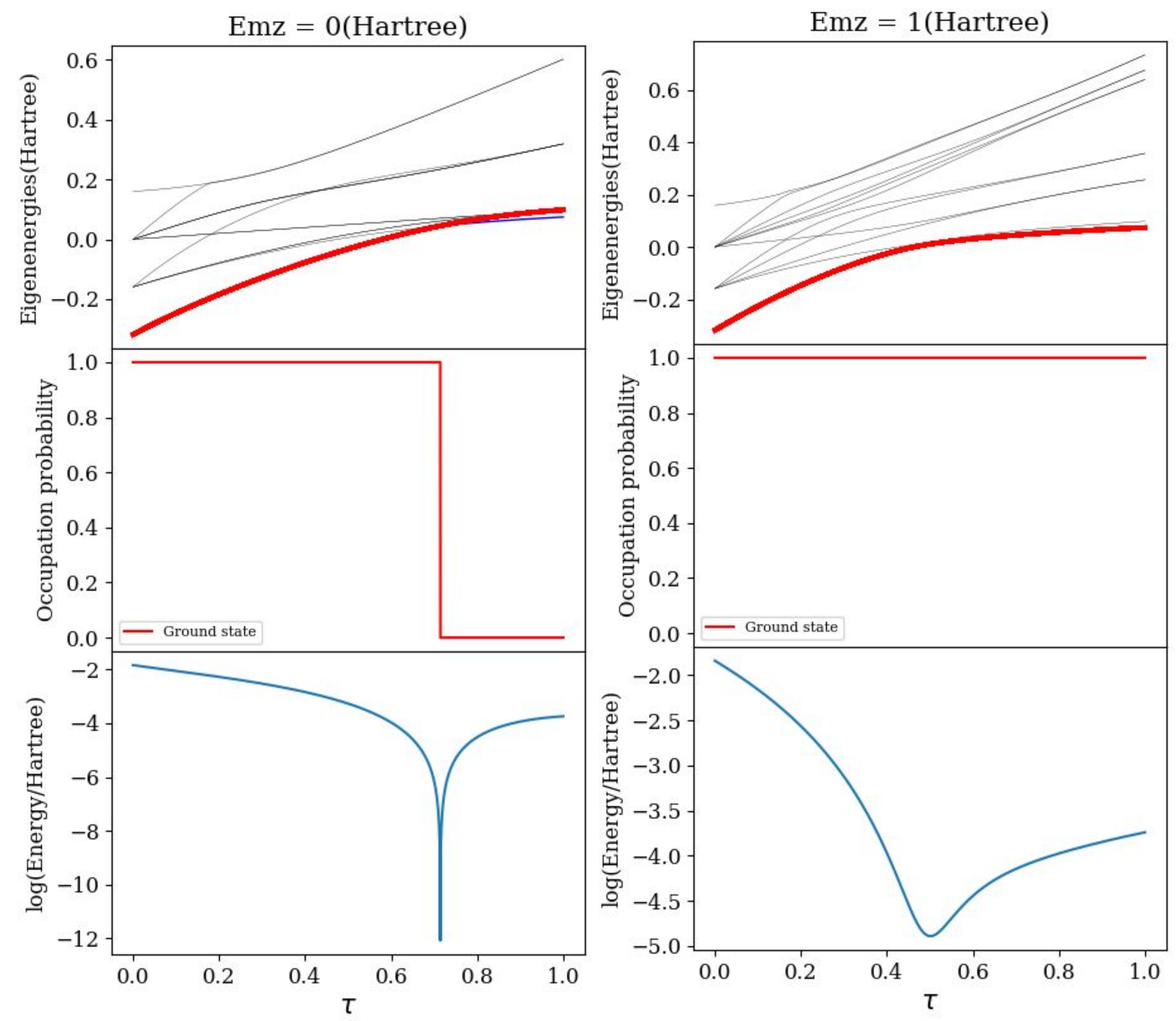
We also need to penalize states with different fermionic magnetic quantum numbers

$$H_{m_z} = E_{m_z}\Big(\sum_i m_z(i)\hat{a}_i^\dagger \hat{a}_i\Big)^2$$

Even when the target ground state is non-degenerate, this term is necessary to avoid degeneracies during the anneal.

Simulated Annealing Results

We study these systems in detail by performing classical simulations of the annealing process using the Python library QuTiP [1,2]. Here, we show that the magnetic penalty term H_{mz} is necessary to avoid a level-crossing during the anneal of the system with S = 3/2, N_e= 2.



Above: Comparison of a failed anneal (left) with magnetic penalty term E_{mz}=0 and a successful anneal (right) with E_{mz}=1 (Hartree units). From top to bottom, the first row shows the energy spectrum of the time-dependent hamiltonian $H(t)$; the second row shows the overlap of the annealed state with the ground state; the bottom row shows the energy gap between the ground and first excited states.

Conclusions and Future Work

Our simulations demonstrate that quantum annealing can be used to obtain the ground state of interacting, many-body fractional quantum hall effect systems. However, even if the problem Hamiltonian has a unique ground state, the anneal may still fail due to level crossings. We find that these crossings can be avoided by introducing an additional penalty term. Currently, we are focused on understanding the behavior of the system near these avoided level crossings as captured by measures of qubit entanglement and potential signatures of quantum phase transitions.

Acknowledgements

This work was supported in part by the National Science Foundation under cooperative agreement 2020275.

References

J. R. Johansson, P. D. Nation, and F. Nori: "QuTiP 2: A Python framework for the dynamics of open quantum systems.", Comp. Phys. Comm. **184**, 1234 (2013) [DOI: 10.1016/j.cpc.2012.11.019].

J. R. Johansson, P. D. Nation, and F. Nori: "QuTiP: An open-source Python framework for the dynamics of open quantum systems.", Comp. Phys. Comm. **183**, 1760–1772 (2012) [DOI: 10.1016/j.cpc.2012.02.021].

PHYSICS FRONTIER CENTER