The Flavor Pendulum in Collective Neutrino Oscillations

And The Role of Collisions in Flavor Instability

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Introduction to Flavor Oscillation

Neutrinos come in three different flavors and masses. Each of these **masses** is in a different superposition of the 3 **flavor** states, and vice-versa. Due to the mass difference, the wave packets of the mass states evolve out of phase. This causes an **oscillation** in the expectation value of the flavor state of the (anti)neutrinos, as discovered by Bruno Pontecorvo in 1957.

The **mixing** between any two of the three flavors is given by the 3 mixing angles $\theta_{i,j}$. We can simplify to a 2 flavor system assuming that most conversion happens between those 2 flavors. Then, only one mixing angle is required, which allows us to make the 2-flavor **pendulum** analogy. Figure 1 shows the flavor and mass in the two neutrino case.

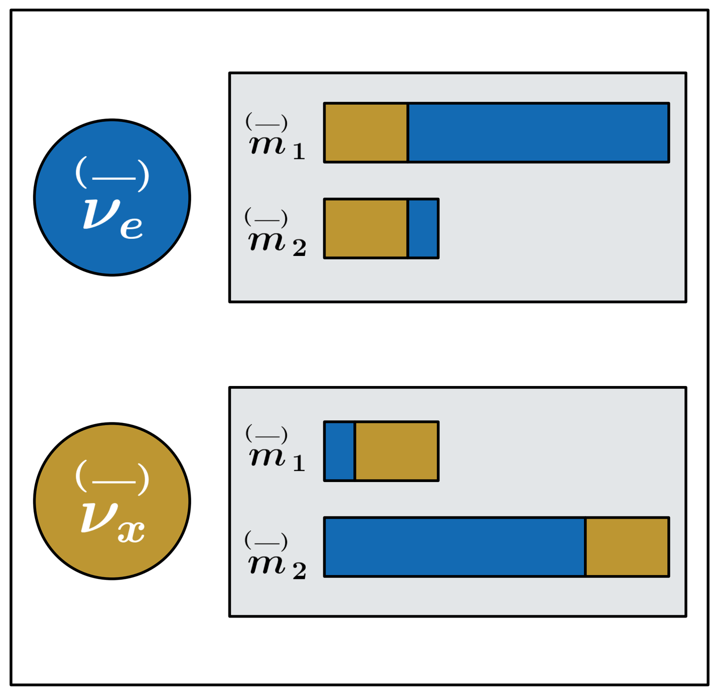

Figure 1. Two neutrinos flavors and their admixture of masses (bins) and flavor expectation (colors) combined. Bar in parenthesis for antineutrino case. [1-3]

Expanding the Pendulum Analogy

2-Flavor (Anti)Neutrino as a Spring-Coupled Oscillator

$\Delta m^2 \equiv m_2^2 - m_1^2$ Electron Flavor X Flavor Spring String

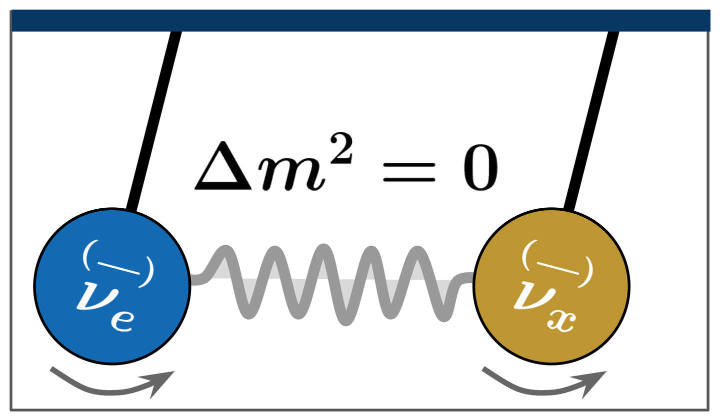

Figure 2. Low Frequency Normal Mode

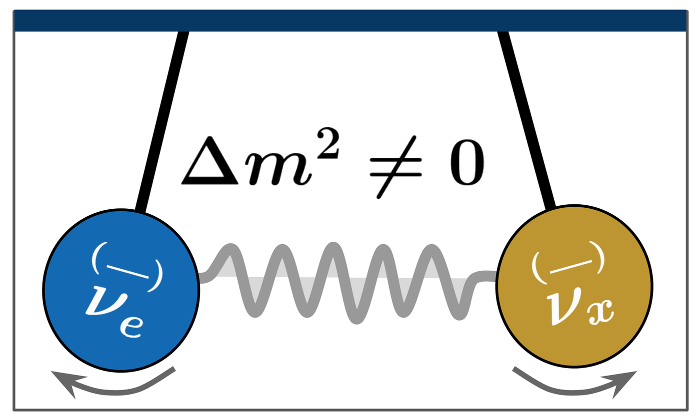

Figure 3. High Frequency Normal Mode

Collective Flavor Oscillation

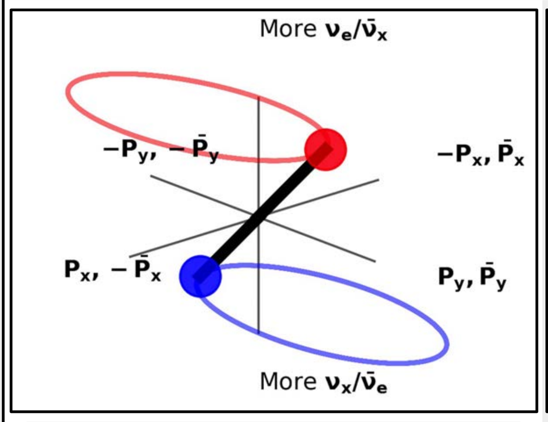

Figure 4. No Flavor Conversion

$\mu(n_{\nu_e} - n_{\nu_x}) \ll \omega$

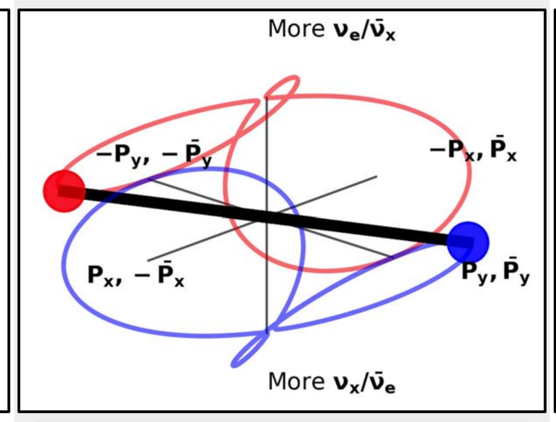

Figure 5. Flavor Depolarization

$\mu(n_{\nu_e} - n_{\nu_x}) = \omega$

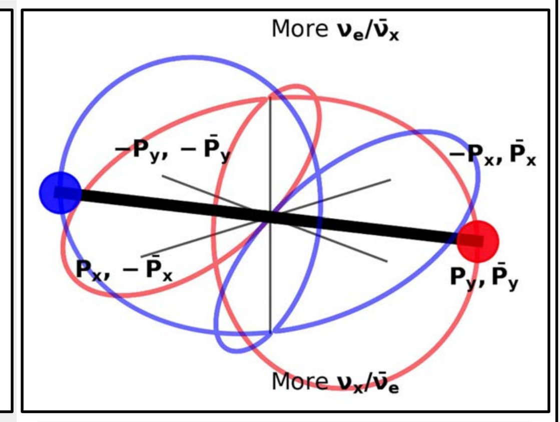

Figure 6. Flavor Inversion

$\mu(n_{\nu_e} - n_{\nu_x}) \gg \omega$

Quantum Kinetic Equations

The basics of **collective neutrino oscillations** and the **flavor pendulum** were established in previous works [3-7], so we start from the 1-D equation for neutrino transport in Johns (2021) [8]:

$$i(\partial_t + \mathbf{v}\cdot\partial_\mathbf{x} + \dot{\mathbf{p}}\cdot\partial_\mathbf{p}) = [H,\rho] + iC \qquad (1)$$

The hamiltonian term includes the **vacuum oscillations** and the **neutrino-neutrino scattering**: $H = (\pm)\omega\mathbf{B} + \mu(\mathbf{P} - \bar{\mathbf{P}})$. Where $\mathbf{B} = (sin2\theta, 0, -cos2\theta)$, $\omega = \Delta m^2/2E_\nu$, and $\mu = \sqrt{2}G_F$.

We consider a **2-flavor ensemble of (anti)neutrinos** with energy $E_\nu = 20MeV$, $\Delta m^2 = 2.4\cdot 10^{-3}eV^{-2}$ and $\theta = \pi/2 - 10^{-2}$

Defining ρ and the initial neutrino density of the system:

$$\rho = (P_0 + \boldsymbol{P}\cdot\boldsymbol{\sigma})/2 \qquad P_0(0) = (n_{\nu_e} + n_{\nu_x})/2$$

$$\mathbf{P}(0) = (0, 0, |P(0)|) = P_z(0) = (n_{\nu_e} - n_{\nu_x})/2$$

The **collisional term** iC accounts for the rate of charged-current scattering ($\Gamma^{CC}_{e,x}$) and absorption/emission ($\Gamma^{AE}_{e,x}$) interactions. Neutral-current interactions are excluded since they are not flavor resolving in an isotropic setting. Using $\Gamma^p_\pm = (\Gamma^p_e \pm \Gamma^p_x)/2$, the equations of motion are:

$$\dot{\mathbf{P}} = \omega\mathbf{B}\times\mathbf{P} + \mu(\mathbf{P} - \bar{\mathbf{P}})\times\mathbf{P} - \Gamma^{CC}_{+}\mathbf{P}_T + \Gamma^{AE}_{+}(\mathbf{P}^{AE} - \mathbf{P}) + \Gamma^{AE}_{-}(P^{AE}_0 - P_0)_\mathbf{z} \qquad (2)$$

$$\dot{P}_0 = \Gamma^{AE}_{+}(P^{AE}_0 - P_0) + \Gamma^{AE}_{-}(P^{AE}_z - P_z) \qquad (3)$$

Mechanisms of The Flavor Pendulum

The flavor pendulum can be better studied using the **sum and difference vectors**. We plot them, along with their dot product $\mathbf{S}\cdot\mathbf{D}$, normalized to $\mathbf{S}(0)$.

$$\mathbf{S} \equiv \mathbf{P} + \bar{\mathbf{P}} \qquad \mathbf{D} \equiv \mathbf{P} - \bar{\mathbf{P}}$$

Following a recent surge in research in the flavor pendulum dynamics and their unstable modes [8-12], we explore the different limits of the system. The pendulum can oscillate in synchronized mode (spinning top and precession in the off-diagonal plane x-y, Fig. 9) and bipolar mode (complete periodic flavor inversion, Fig. 8) without decohering. When $\mu P_z \gg \omega$, the bipolar mode dominates and $\mathbf{S}$ is conserved. The polarization vector is then constrained to a sphere, and we get a spherical pendulum. We inverse the sign of the antineutrino polarization to visualize the symmetry of this spherical pendulum. At $\mathbf{D}(0) = 0$, the pendulum is constrained to oscillate in a circle (Fig. 7). The conditions for bipolar oscillations to occur are given by:

$$\omega \leqslant \mu < 4\omega\frac{(1+a)}{(1-a)^2} \qquad a = \frac{\bar{P}_z(0)}{P_z(0)}$$

Finally, because $\Gamma \ll \mu P_z$, collisions can enhance flavor mixing or decohere the system in proportion to $1/\Gamma$.

Numerical Simulation Examples

When the length of the polarization vector of neutrinos and antineutrinos is equal, we get:

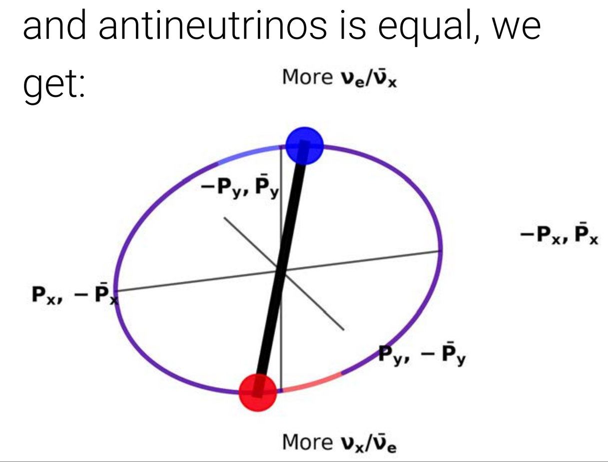

Figure 7a. Circular Pendulum

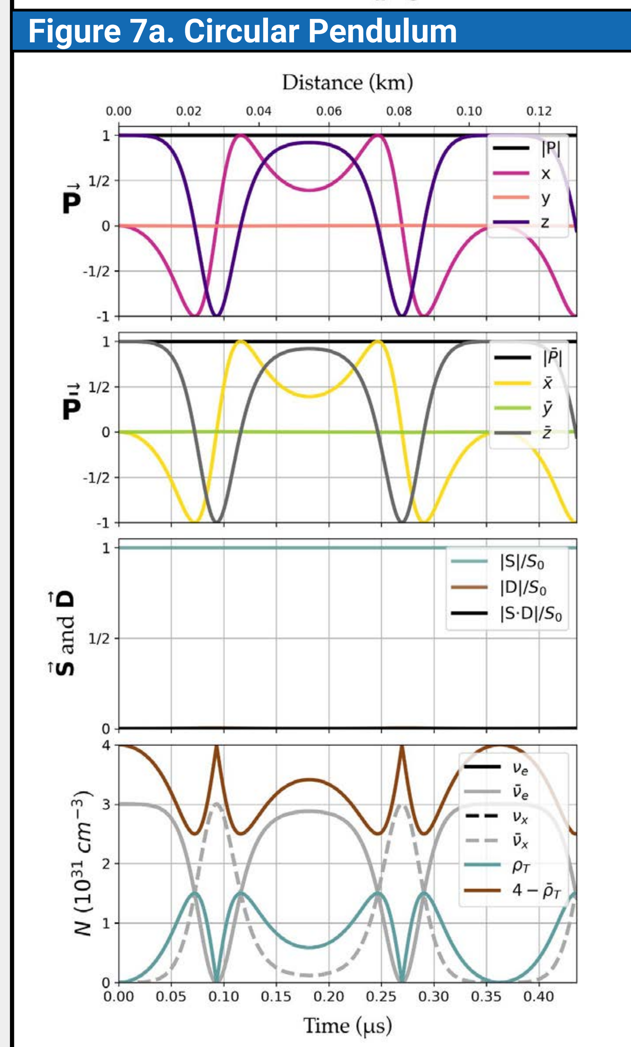

Figure 7b. Plot

An asymmetric polarization ($\mathbf{D} \neq 0$), introduces precession around **Z**:

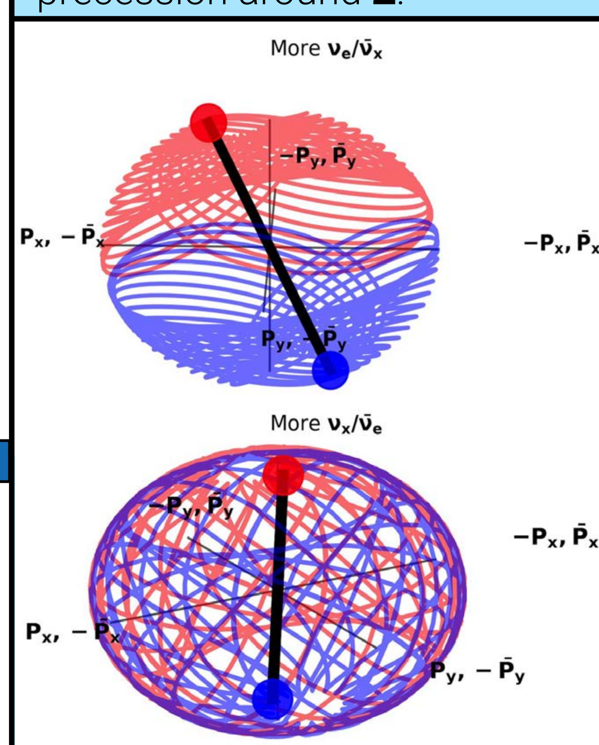

Figure 8. Spherical Pendulum

Lowering μ and increasing $\mathbf{D}$, enters the synchronized regime. $\mathbf{P}$ and $\bar{\mathbf{P}}$ are asymmetrically engaging the bipolar and synchronized modes, which introduces nutation.

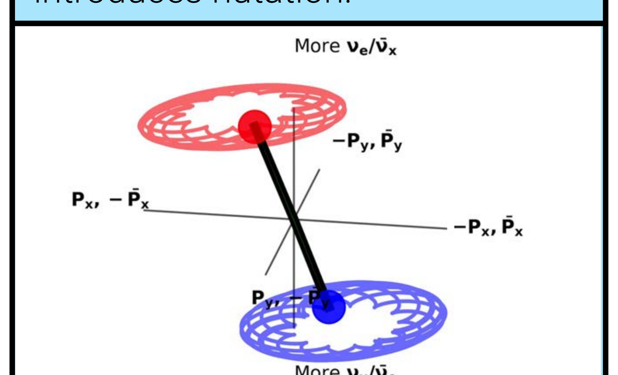

Figure 9. Synchronized mode

Collisional Instabilities quickstart the flavor mixing in proportion to $\sqrt{\Gamma\mu P_z}$, and make $\mathbf{S}$ decay in proportion to Γ.

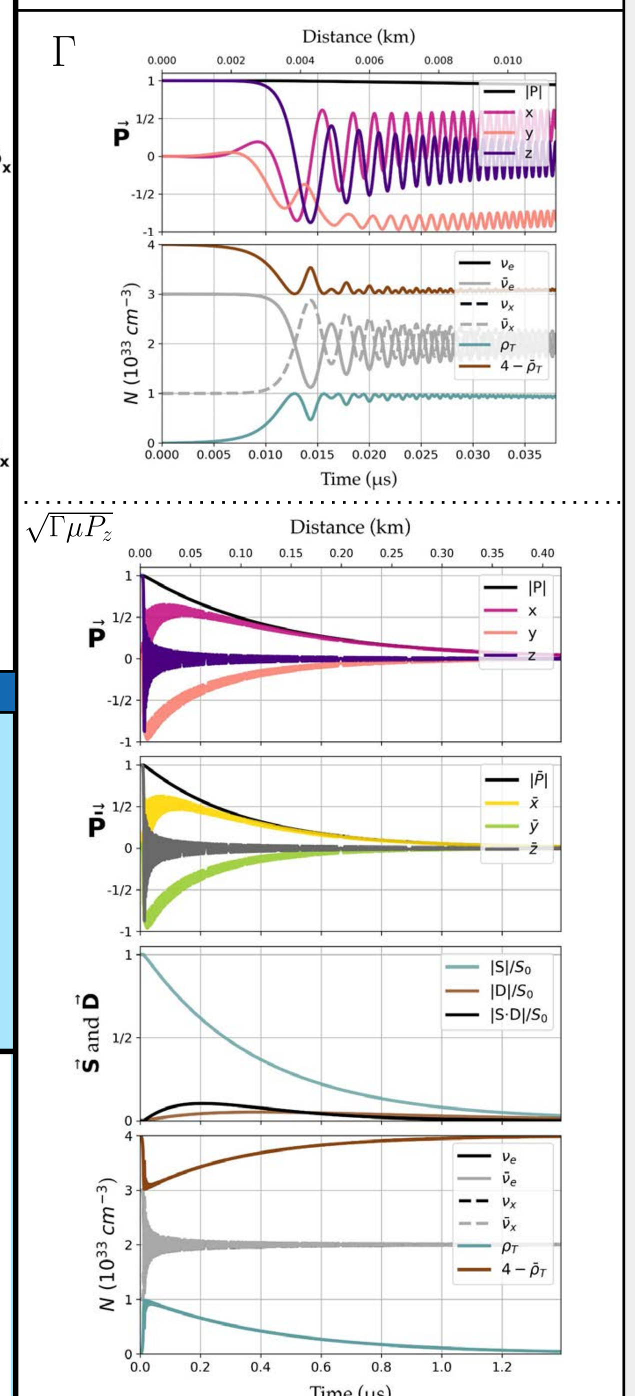

Figure 10. Collisional instability plot

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