Bayesian Analysis of the Detection of Astrophysical Neutrinos

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INTRODUCTION

- When a nearby core collapse supernova occurs, terrestrial detectors should be able to detect thousands of neutrinos, providing us with information on their energy spectrum and flavor composition. These detections are expected to have uncertainties associated with the experimental setup that need to be included in the analysis.
- By utilizing optimization techniques, we include measurement uncertainties in the inference of unknown parameters of the flavor evolution model.
- We characterize the measurement uncertainties under the Bayesian statistics framework by assuming a rather non-informative prior.

NEUTRINO FLAVOR OSCILLATIONS

In a dense neutrino environment, neutrino oscillations exhibit a synchronized behavior known as collective neutrino oscillation creating a many-body problem. The equation of motion of neutrino flavor oscillations in this setting is given by

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{P}_p = (\omega_p \mathbf{B} + V(t)\hat{z} + \mu(t)\mathbf{P}) \times \mathbf{P}_p \quad (1)$$

- $\omega_p = \delta m^2/(2E_p)$ is the vacuum oscillation frequency where δm^2 is the mass squared difference between two energy eigenstates and E_p is the associated energy.
- **B** is a unit vector describing the flavor mixing between the flavor and mass basis as a function of the mixing angle α.
- $\mathbf{P}_p$ describes the flavor composition of neutrinos as a net difference between the electron flavor and x flavor, a superposition of the muon and tau neutrinos.
- $V(t)$ is the matter potential modeled by $V(t) = \frac{V_0}{t^3}$ [1].
- $\mu(t)$ is the neutrino-neutrino potential modeled by $\mu(t) = \frac{\mu_0}{t^4}$ [1].

In our problem, we will be using two neutrinos and their corresponding antineutrinos: ν_e, ν_x, $\bar{\nu}_e$, $\bar{\nu}_x$. By using the optimal values for ω_p, V_0, μ_0, and α, we simulate detector measurements that exhibit a complete flavor transformation as shown in Fig. 1.

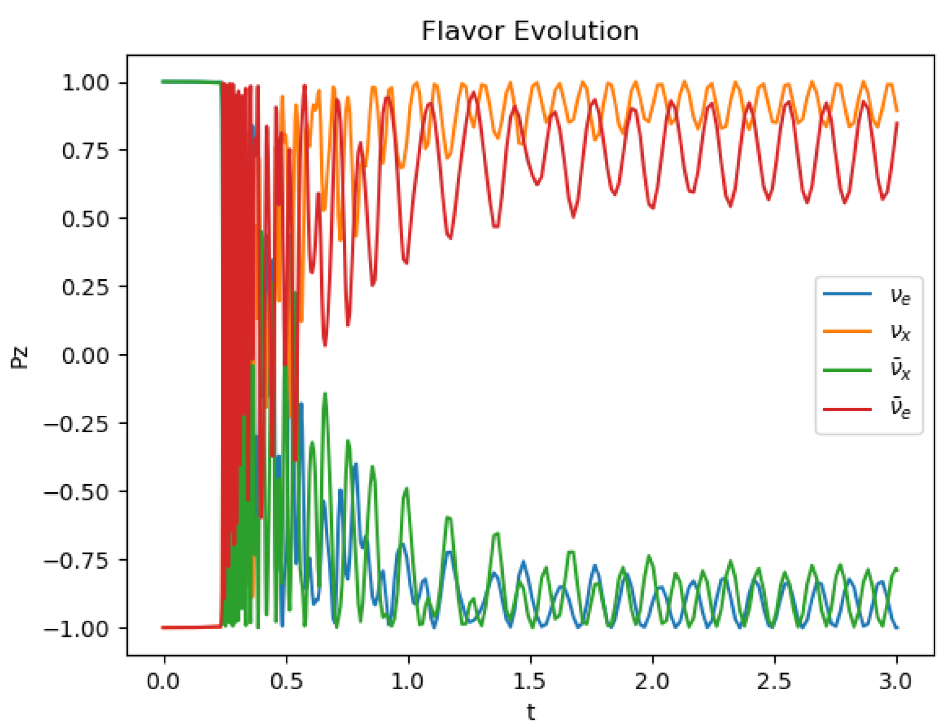

Fig. 1: Flavor evolution of two neutrinos, ν_e and ν_x, and two antineutrinos, $\bar{\nu}_e$ and $\bar{\nu}_x$, using optimal parameters.

OPTIMIZATION

Given a detector measurement shown in Fig. 1, we estimate the model parameters responsible for such a measurement by employing an inference procedure known as statistical data assimilation (SDA). We formulated SDA as an optimization problem wherein the cost function is minimized [2]. For our purposes, the cost function is defined as the difference between the model prediction and measured data

$$\mathrm{Cost}(\boldsymbol{\theta}) = [u_D(R,\theta) - u_\theta(R)]^2 \quad (2)$$

where u_{D} is a measurement made at a location R and $u_\theta(R)$ is the prediction from our model (see Eq. 1) [1]. θ is an unknown parameter we seek to optimize which, in our case, is the matter potential V_0. This optimization process is shown in Fig. 2.

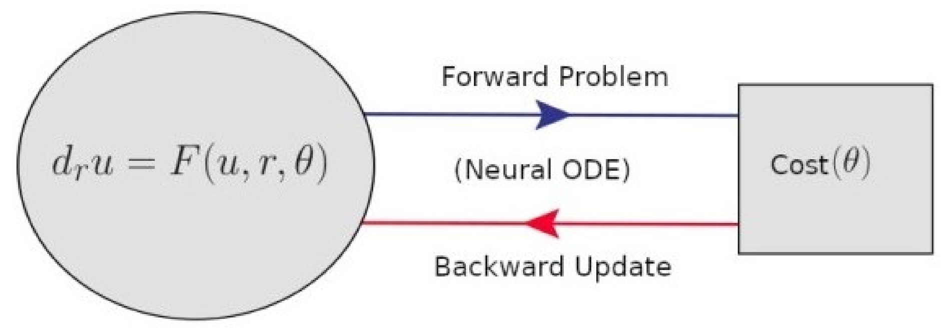

Fig. 2: The unknown parameter is denoted by θ. We want to find the optimal value for θ that minimizes Cost(θ) [1].

- In the forward problem, we generate the model prediction by using neural ODEs. The forward code is run for various values of V_0 and compared to the true value ($V_0^* = 10$) that is used to generate Fig. 1.
- In the backward update, the minimization of the cost is performed through an optimization algorithm.
- By minimizing the cost function, we can determine the unknown parameter $\boldsymbol{\theta}$ that best matches the measurements.

To represent a more robust system, we include measurement uncertainties to the energy, represented by ω_p in Eq. 1. With this set-up, we compare the performances of various optimization algorithms.. We found that the Adaptive Particle Swarm Algorithm (APS) performed the best out of all of them by identifying a global minimum at V_0^* even with various measurement uncertainties as shown in Fig. 3.

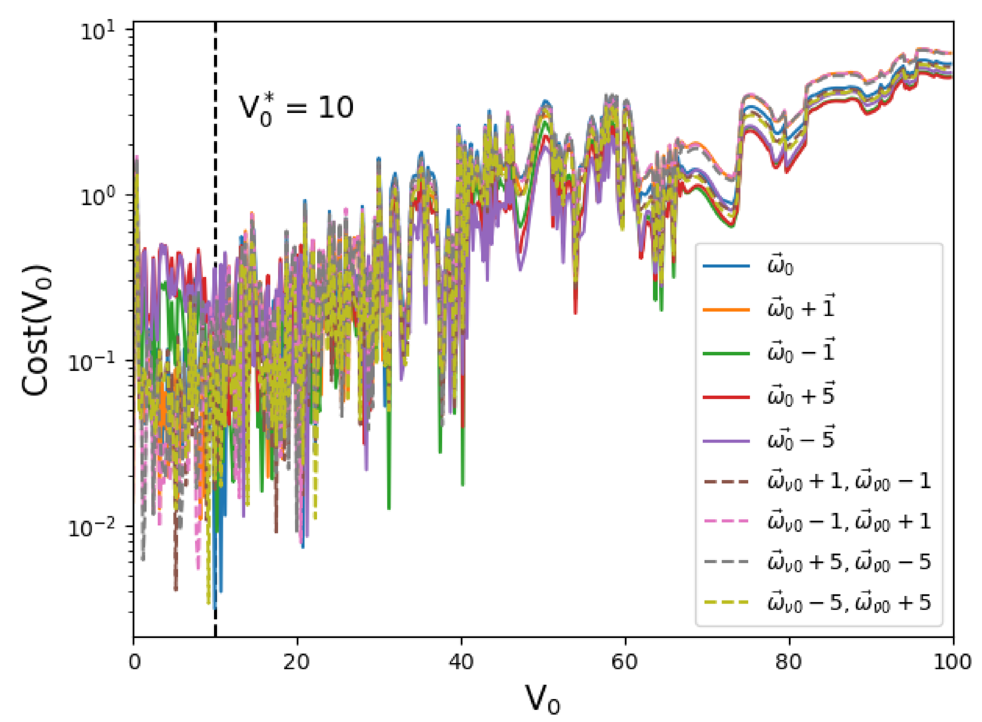

Fig. 3: Cost function dependent on unknown parameter V_0 for various frequencies. The global minimum is indicated by the black dotted line, $\mathrm{V_0}^* = 10$.

BAYESIAN ANALYSIS

Fig. 3 assumes a simple framework to examine measurement uncertainty. A more thorough analysis on the effects of neutrino energy measurement uncertainty in matter potential estimation can be performed with Bayesian statistics. We assume a rather non-informative prior to estimate a posterior distribution by treating measurements as samples from either a uniform or Gaussian distribution. With Markov Chain Monte Carlo algorithms, we generate 40,000 samples from the assumed distributions. A few conclusions can be drawn from Fig. 4:

- Compared to the uniform distribution, the Gaussian distribution produced poor estimations of the ideal frequency.
- The posterior variance for the Gaussian prior is larger than the posterior variance from the uniform distribution implying a larger uncertainty in parameter estimation.
- We can conclude the Gaussian priors were less informative than the uniform priors.

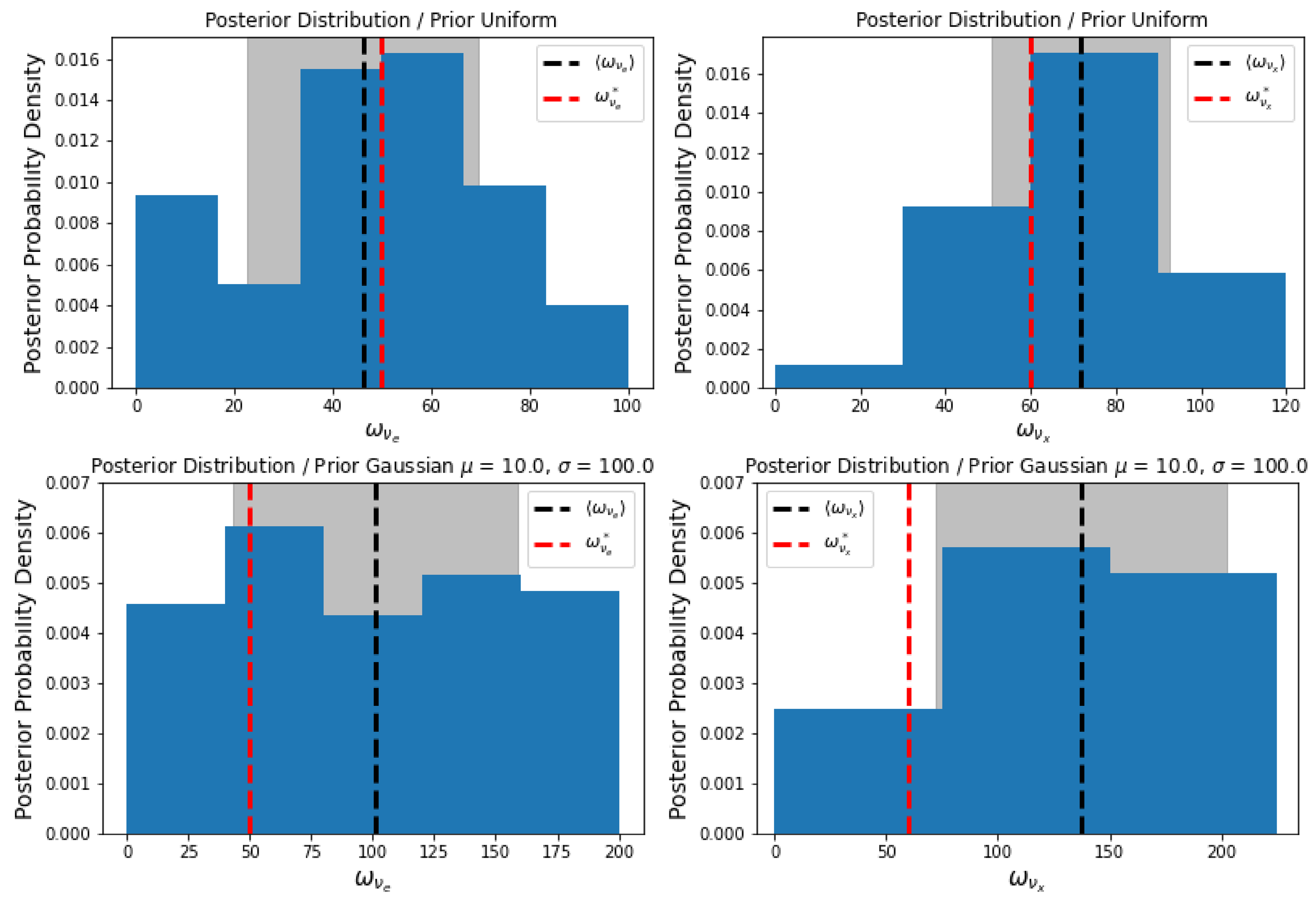

Fig 4: Posterior distribution for a uniform or Gaussian prior of the neutrino frequencies. The uniform distribution was sampled from 0.0 to 100.0. The black dotted line is the posterior mean and the red dotted line is the ideal frequency value. The shaded region is one sigma deviation from the posterior mean.

CONCLUSION

- By combining neural networks with differential equation solvers, we solved the neutrino flavor oscillation equation as a function of the unknown parameter and used various optimization procedures to infer the parameter range of V_0.
- Using this, we employed a Bayesian inference framework to assign probabilities to the frequency, describing the associated uncertainties. We assumed non-informative priors in the form of a uniform or normal distribution to estimate the posterior distributions of the parameters.
- As the sample sizes were constant, the more informative priors produced the best parameter estimations in the posterior distributions. A larger sample size coupled with an informative prior might provide a better parameter estimation. Furthermore, we can create a more realistic framework by studying a larger system of neutrinos.

ACKNOWLEDGEMENTS & REFERENCES

Iman Fahmy acknowledges N3AS and NSF Physics Frontier Center Award #2020275 for supporting this work. Special thanks to Ermal Rrapaj for guiding me throughout this work.

[1] Ermal Rrapaj et al. "Inference of neutrino flavor evolution through data assimilation and neural differential equations". In: *Physical Review D* 103.4 (Feb. 2021). ISSN: 2470-0029. DOI: 10.1103/physrevd.103.043006. URL: http://dx.doi.org/10.1103/PhysRevD.103.043006.

[2] Eve Armstrong et al. "Inference offers a metric to constrain dynamical models of neutrino flavor transformation". In: *Physical Review D* 102.4 (2020), p. 043013.