Oscillating Neutrinos as Open Quantum Systems

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MOTIVATION

- Formalizing neutrino interaction dynamics in extremely dense astrophysical environments can provide insight into the early universe, supernovae, and neutron-star mergers.
- In dense astrophysical environments flavor instabilities can alter the kinetics[1][2] and it becomes important to consider neutrinos as open quantum systems.
- This work applies the formal foundation established by Ref.[3] to examine the spatiotemporal evolution of flavor in dense environments by utilizing the computational advantages of the Lindblad master equation and its solution.

THE NEUTRINO MASTER EQUATION

The Lindblad master equation describes the evolution of a two flavor mixed state interacting with an environment. It takes the form,

$$i\frac{d\rho}{dt} = \hat{\mathcal{L}}\rho = [H,\rho] + i\mathcal{D}\sum_j \left[A_j\rho A_j^\dagger - \frac{1}{2}\{A_j^\dagger A_j, \rho\}\right] \quad (1)$$

- H is the neutrino vacuum oscillation Hamiltonian.
- $\mathcal{D}$ is the decoherence rate.
- A_j are Lindblad operators that dictate the nature of the dissipation.
- $\hat{\mathcal{L}}$ is a positive and trace preserving super-operator.
- The terms $\{A_j^\dagger A_j, \rho\}$ describe the non-unitary dissipation.
- The terms $A_j\rho A_j^\dagger$ describe quantum jumps[4].

All together, this expression can be represented as a linear system of the Liouvillian super-operator $\hat{\mathcal{L}}$ acting on a density vector,

$$\hat{\mathcal{L}}\rho_k = \lambda_k\rho_k. \quad (2)$$

The solution, $\rho(t)$, is entirely dependent on the eivenvalues and eigenmatrices of the Liouvillian,

$$\rho(t) = \sum_k c_k\rho_k e^{\lambda_k t}. \quad (3)$$

Here, c_k is the product of the left eigenmatrices and the initial density state $\rho(t=0)$. When negative, $Re[\lambda_k]$ contributes a damping effect while a nonzero $Im[\lambda_k]$ contributes an oscillatory behavior to the flavor evolution. Moreover, any zero eigenvalues govern the steady state [5].

EXCEPTIONAL POINTS

- In 1998 Carl Bender and Stefan Boettcher showed that $\mathcal{PT}$-symmetric Hamiltonians can exhibit real eigenvalues [6] and can therefore be used to describe observables.
- **Exceptional points** are defined as a particular form of spectral degeneracy characterized by the simultaneous coalescence of eigenvalues and eigenvectors.
- For oscillatory systems, Exceptional points identify the parameters of critical damping and occur when the discriminant of the eigenvalues vanish.

RESULTS

To examine the dissipation effects of flavor measurement, we plot the eigenvalues of the Liouvillian super-operator, choosing the Lindblad operator to be $A = \sigma_z$, the Pauli matrix. This operator induces decoherence of the density matrix.

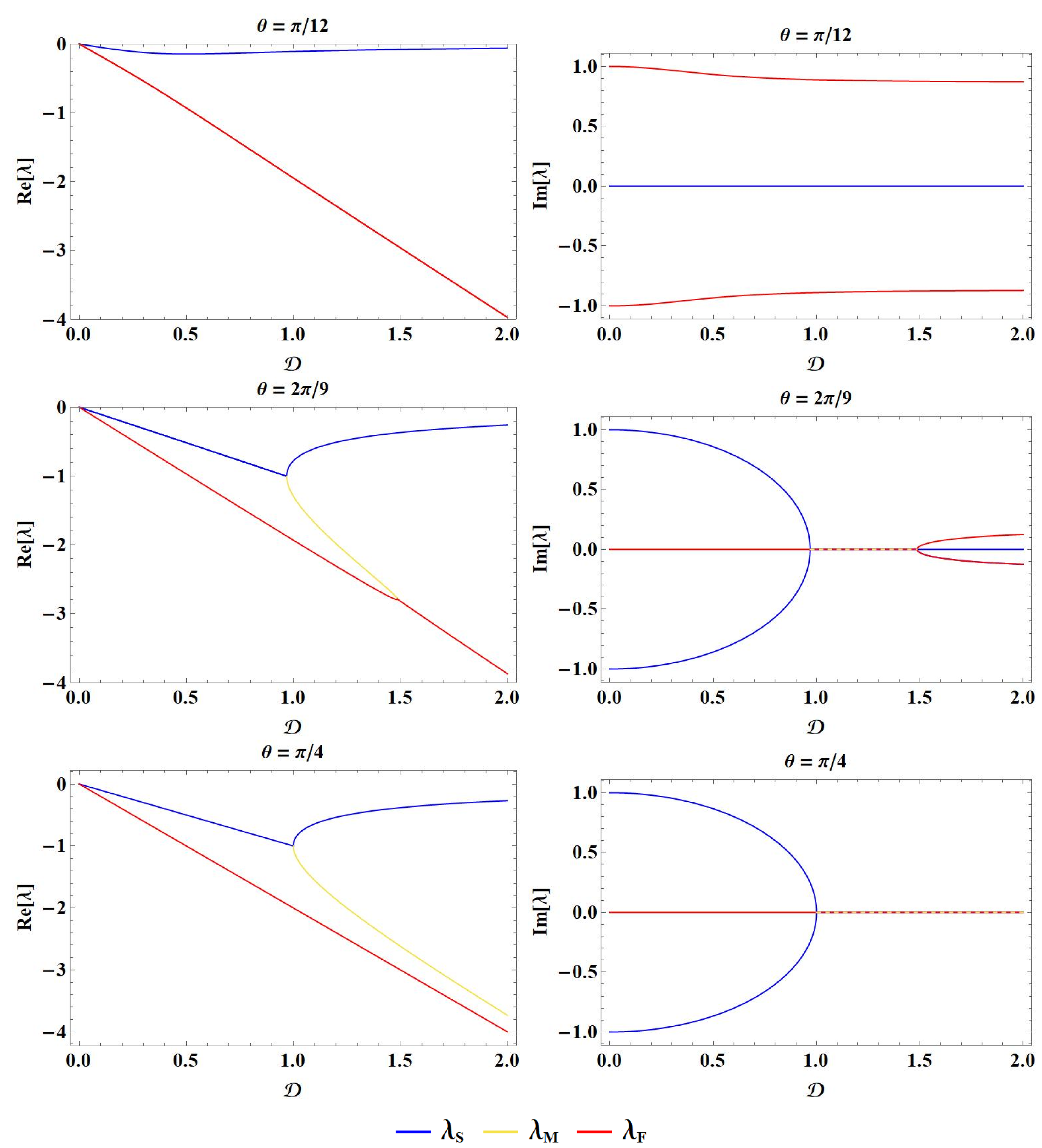

Figure 1: Eigenvalues of $\hat{\mathcal{L}}$ as a function of the decoherence rate when $A_j = \sigma_z$ to model flavor measurements. The blue, yellow, and red curves represent slow, medium, and fast damping rates, respectively. Exceptional points are observed when two eigenvalues are equal. The quantum Zeno effect emerges from the EP.

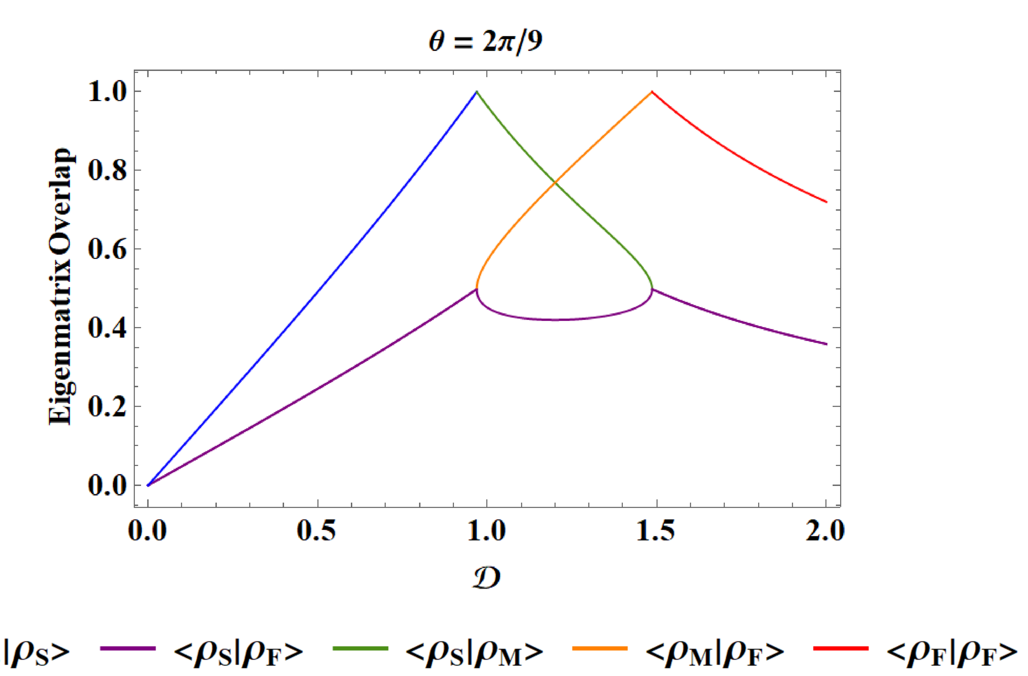

Figure 2: The inner product of the eigenmatrices when $A_j = \sigma_z$. Exceptional points occur when eigenmatrices become parallel, i.e. when the overlap is 1.

ANALYSIS

- **Small mixing:** When $\mathcal{D}$ increases, the fast eigenvalue, λ_F, becomes large and negative while the slow eigenvalue, λ_S, asymptotically approaches zero.
- **Near maximal mixing:** Two Exceptional points emerge. For $0 < \mathcal{D} < 0.97$, all eigenvalues damp $\rho(t)$ such that the steady state is reached relatively quickly. For $1.49 < \mathcal{D} < \infty$ the slow eigenvalue asymptotically approaches zero, creating a quasi-steady state as $\mathcal{D} \to \infty$. In these regions, the system exhibits oscillatory decay. In the region between EPs, the eigenvalues are real and so the system exhibits decay without oscillation. The abrupt emergence of the slow eigenvalue's asymptotic behavior from the Exceptional point is interpreted as the **quantum Zeno effect**.
- **At maximal mixing:** The EP between λ_S and λ_M occurs at $\mathcal{D} = 1$, while the EP between λ_M and λ_F goes to infinity. For these parameters, the effective evolution does not significantly differ from the case of near maximal mixing, however, the EP moving off to infinity is formally interesting and is likely connected to the fact that the discriminant is doubly degenerate here,

REMARKS

- When the dissipation from the environment is caused by flavor measurements, the quantum Zeno effect emerges from Exceptional points in the Liouvillian spectrum. This does not occur when the dissipation is caused by flavor transitions (when $A = \sigma_{+,-}$).
- In the 2D parameter space of the flavor mixing angle and the decoherence rate, the Exceptional points are extended to 'Exceptional lines'.
- This work can be extended to create an elegant method of calculating many-body astrophysical neutrinos within an environment by making an appropriate modification to the Von-Neumann Hamiltonian in lieu of the vacuum Hamiltonian [7].

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