Probing New Physics with Gravitational Waves

Jan Schütte-Engel

N3AS meeting, UC Berkeley

based on
[Ringwald, **JSE**, Tamarit 20], [Ghiglieri, **JSE**, Speranza 22]

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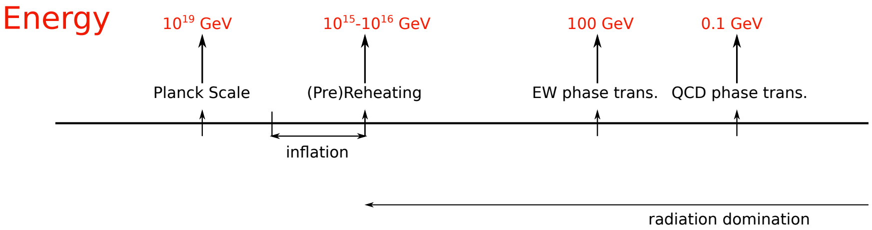
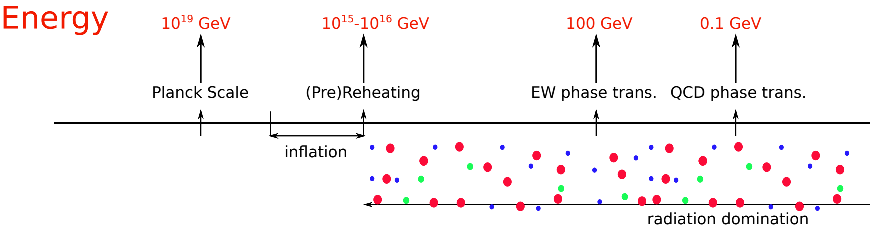
History of our universe



GWs from the thermal plasma



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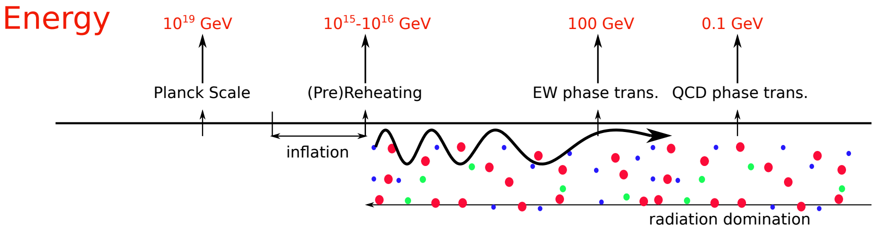
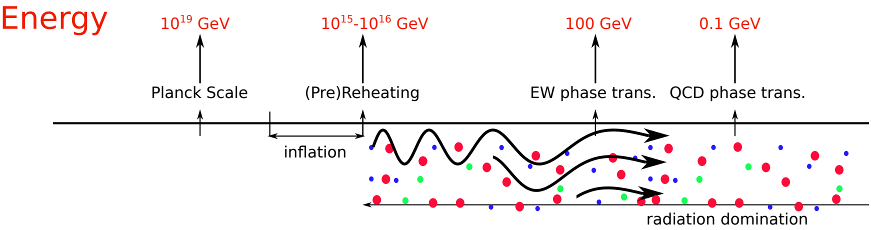


GWs from the thermal plasma



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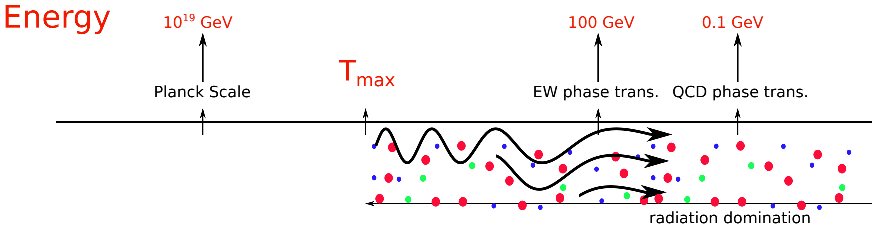
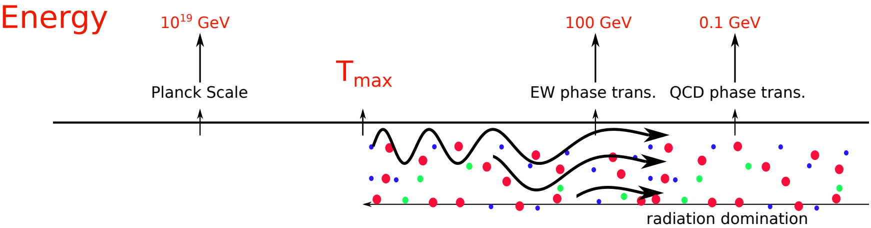


$$\omega_g^{\text{today}} = \left(\frac{g_{*s}(T_0)}{g_{*s}(T_{\max})} \right)^{\frac{1}{3}} \frac{k_{\max}}{T_{\max}} \, T_0$$

GWs from the thermal plasma



$$\omega_g^{\text{today}} = \left(\frac{g_{*s}(T_0)}{g_{*s}(T_{\text{max}})} \right)^{\frac{1}{3}} \frac{k_{\text{max}}}{T_{\text{max}}} T_0 \approx \left(\frac{g_{*s}(T_0)}{g_{*s}(T)} \right)^{\frac{1}{3}} \frac{k}{T} T_0$$

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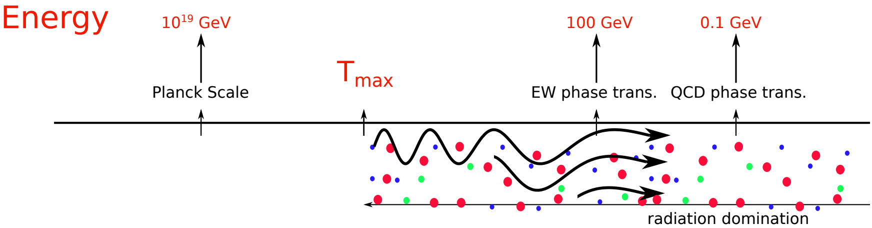

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Cosmic Gravitational Microwave Background (CGMB)

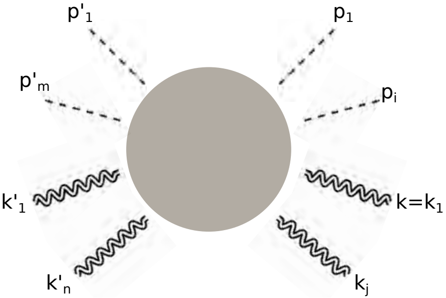
Distribution functions

$$f_\phi(t,k) \equiv \frac{\text{Number of } \phi\text{-states with momentum } k \text{ in } d^3k \text{ interval}}{V\, d^3k/(2\pi)^3},$$

$$f_h(t,k) \equiv \frac{\text{Number of gravitons with momentum } k \text{ in } d^3k \text{ interval}}{V\, d^3k/(2\pi)^3},$$

Evolution equations

$$\dot{f}_\phi(t,k) = G_\phi(t,k) - L_\phi(t,k),$$
$$\dot{f}_h(t,k) = G_h(t,k) - L_h(t,k),$$



$$G_h(t,k) = \frac{1}{4k} \sum_{\substack{\text{all processes } r \\ \text{with at least one} \\ \text{final state graviton}}} S_r \int d\Omega_r\, |\mathcal{M}_r|^2 \times f_\phi(p'_1)\cdots f_\phi(p'_m)\, f_h(k'_1)\cdots f_h(k'_n) \times$$
$$\times\, (1+f_\phi(p_1))\cdots(1+f_\phi(p_i))\, (1+f_h(k))\cdots(1+f_h(k_j))\,.$$

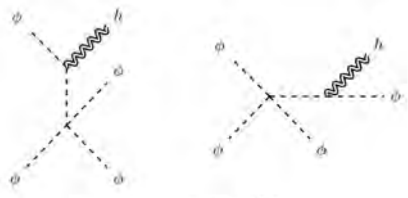
Lowest order processes

$$f_h(k) = 0 + f_h^{(2,2)}(k) + f_h^{(0,4)}(k),$$

Freeze-in scenario:

$$f_h \ll 1 \text{ and } |n_{\mathrm{B}} - f_\phi| \ll 1$$

Single graviton production



$$|\mathcal{M}|^2 \sim \lambda^2 \left(\frac{1}{m_p}\right)^2$$

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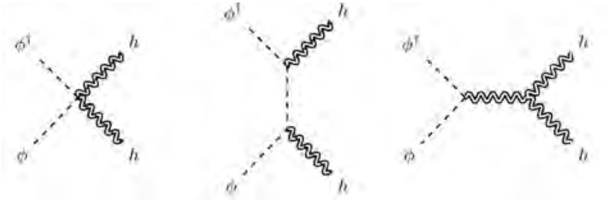
graviton pair production



$$|\mathcal{M}|^2 \sim \left(\frac{1}{m_p}\right)^4$$

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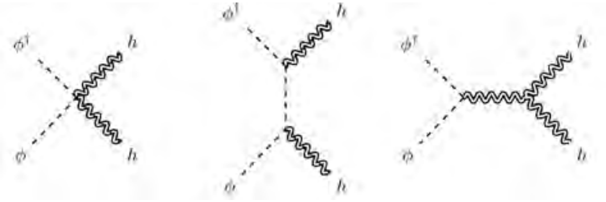
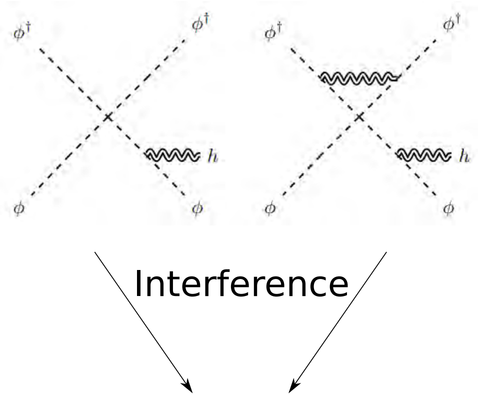


$$|\mathcal{M}|^2 \sim \left(\frac{T_{\max}}{m_p}\right)^4$$

An example:

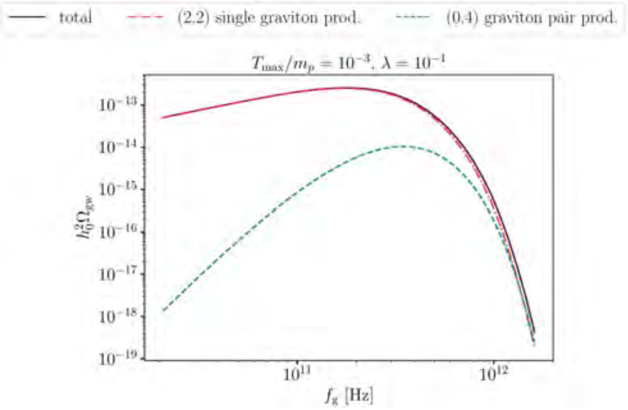


$$|\mathcal{M}|^2 \sim \lambda^2 \left(\frac{T_{\max}}{m_p} \right)^4$$

[Ghiglieri, **JSE**, Speranza, in progress]

$$h_0^2\Omega_{\rm gw}(f_{\rm g}) \sim \frac{T_{\rm max}}{m_{\rm p}}\, n_{\rm B}(y_{\rm max}) \left(\lambda^2 \psi^{(2,2)}(y_{\rm max}) + \frac{1}{3}\left(\frac{T_{\rm max}}{m_{\rm p}}\right)^2 \psi^{(0,4)}(y_{\rm max}) + \cdots \right)$$

with $y_{\rm max} \equiv \frac{2\pi f_{\rm g}}{T_{\rm today}} \left(\frac{g_{*s}(T_{\rm max})}{g_{*s}(T_{\rm today})}\right)^{1/3} = 0.14 \left(\frac{f_{\rm g}}{10^{10}\,{\rm Hz}}\right) \left(\frac{g_{*s}(T_{\rm max})}{2}\right)^{1/3}$



[Ghiglieri, **JSE**, Speranza 22]

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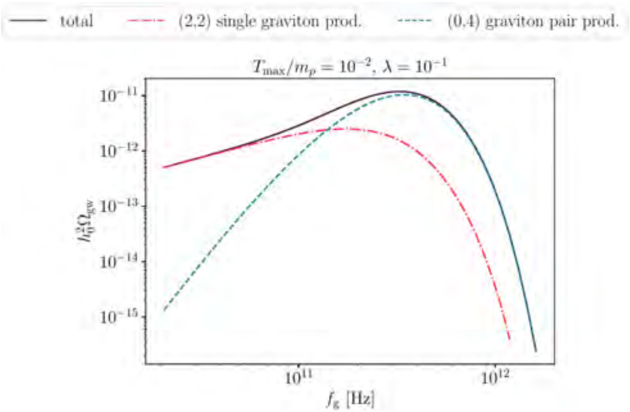

[Ghiglieri, **JSE**, Speranza 22]

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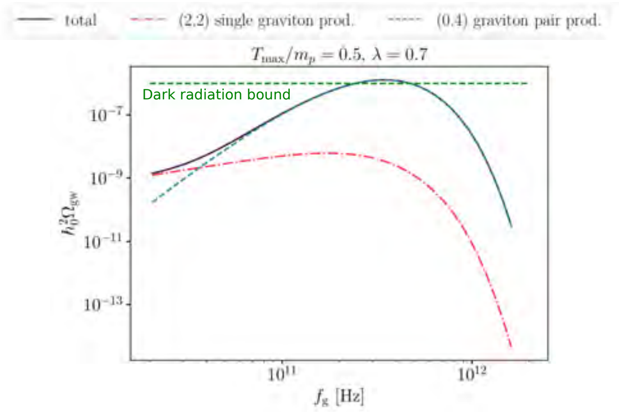
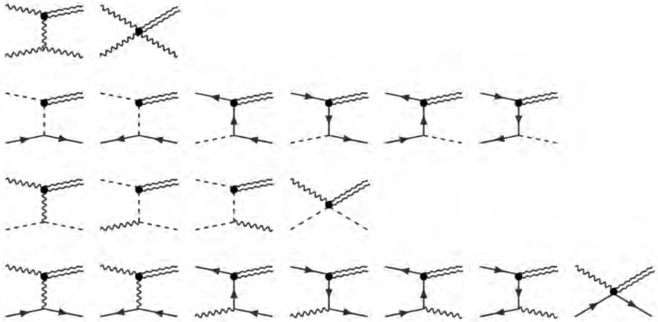


[Ghiglieri, **JSE**, Speranza 22]

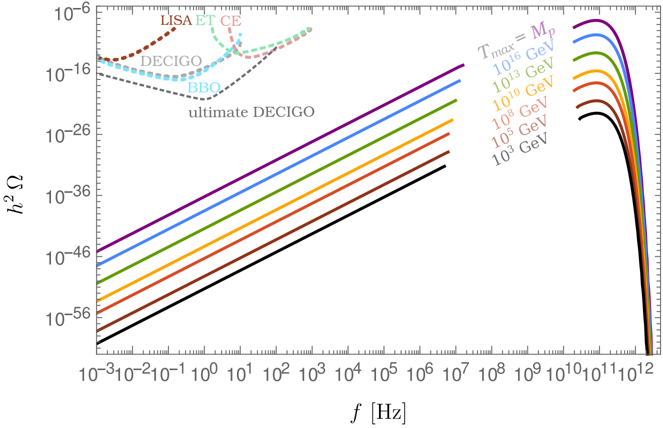
The CGMB spectrum in the Standard Model



[Ghiglieri, Jackson, Laine, Zhu 20]

(only single graviton production processes included)

The CGMB spectrum in the SM and beyond



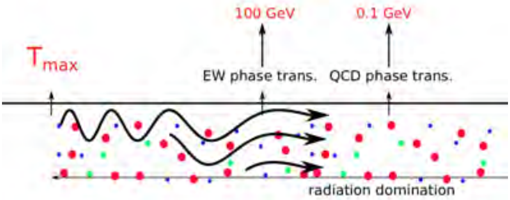
[Ghiglieri, Jackson, Laine, Zhu 20], [Ringwald, **JSE**, Tamarit 20]

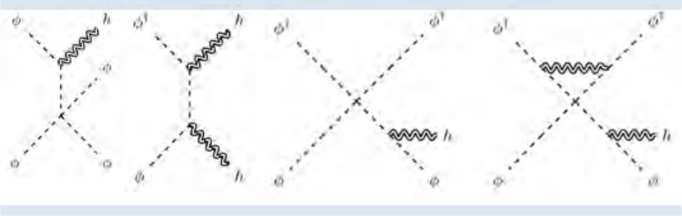
In BSM theories:

$$f_{\rm peak}(T_{\rm max}) \sim \left[\frac{106.75}{g_{*s}(T_{\rm max})}\right]^{\frac{1}{3}}$$

	SM	MSSM
$g_{*s}(T_{\rm max})$	106.75	228

Conclusions





$$f_{\text{peak}}(T_{\max}) \sim \left[\frac{106.75}{g_{*s}(T_{\max})}\right]^{\frac{1}{3}}$$

Backup

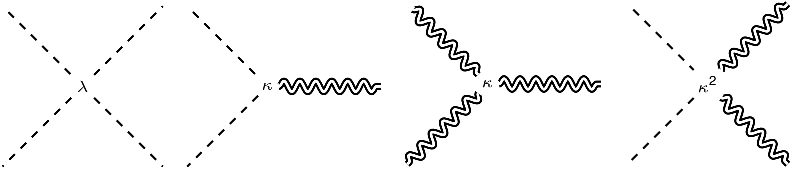
CGMB for complex scalar model

$$\mathcal{L}_\phi = \sqrt{-g}\left(-g^{\mu\nu}\left(\nabla_\mu\phi\right)^\dagger\nabla_\nu\phi + \frac{\lambda}{4}|\phi|^4\right),$$

$$\mathcal{L}_{\rm EH} = 2\kappa^{-2}\sqrt{-g}\,R, \quad \kappa \equiv \sqrt{32\pi G}, \quad G \equiv 1/m_p^2$$

Expansion around flat space-time:

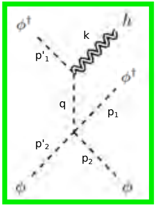
$$\begin{aligned}\mathcal{L} = \mathcal{L}_{\rm EH} + \mathcal{L}_\phi \quad &= \quad -\eta^{\mu\nu}\left(\partial_\mu\phi\right)^\dagger\partial_\nu\phi - U + \frac{1}{2}\partial_\mu h^{\sigma\nu}\partial^\mu h_{\sigma\nu} \\ &+\kappa\, h^{\mu\nu}\left(\partial_\mu\phi\right)^\dagger\partial_\nu\phi + \kappa\left(-\frac{1}{2}h^\alpha_{\ \beta}\partial_\alpha h^\mu_{\ \nu}\partial^\beta h^\nu_{\ \mu} - h^\alpha_{\ \beta}\partial_\mu h^\nu_{\ \alpha}\partial^\mu h^\beta_{\ \nu} + h^\beta_{\ \mu}\partial_\nu h^\alpha_{\ \beta}\partial^\mu h^\nu_{\ \alpha}\right) \\ &+\kappa^2\left[\left(-h^{\mu\lambda}h^\nu_{\ \lambda} + \frac{1}{4}\eta^{\mu\nu}h^\alpha_{\ \rho}h^\rho_{\ \alpha}\right)\left(\partial_\mu\phi\right)^\dagger\partial_\nu\phi + \frac{1}{4}h^\alpha_{\ \rho}h^\rho_{\ \alpha}\,U\right] + \mathcal{O}(h^3).\end{aligned}$$



Details on the Matrix elements

$$|\mathcal{M}|^2 = \frac{\lambda^2\kappa^2}{4} V_{\mu\nu} V_{\alpha\beta} \, \epsilon^{*\chi\alpha\beta}\epsilon^{\chi\mu\nu} \quad \text{(for process } \phi^\dagger\phi \to \phi^\dagger\phi h\text{)}$$

$$V_{\mu\nu} = -\frac{1}{q_1^2}\left(p'_{1\mu}q_{1\nu} + p'_{1\nu}q_{1\mu}\right) + \frac{1}{q_2^2}\left(p'_{2\mu}q_{2\nu} + p'_{2\nu}q_{2\mu}\right) - \frac{1}{q_3^2}\left(p_{1\mu}q_{3\nu} + p_{1\nu}q_{3\mu}\right) - \frac{1}{q_4^2}\left(p_{2\mu}q_{4\nu} + p_{2\nu}q_{4\mu}\right)$$



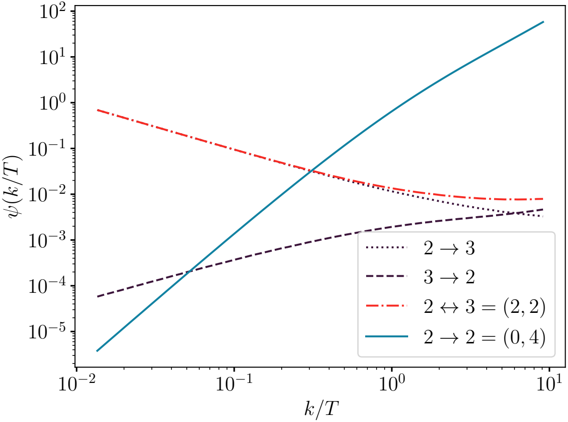
$$\sum_\chi |\mathcal{M}|^2 = \frac{\lambda^2\kappa^2}{4} V_{\mu\nu} V_{\alpha\beta} \, L^{\mu\nu\alpha\beta}$$

$$L_{\mu\nu\alpha\beta} = \frac{1}{2}\left(P_{\mu\alpha}P_{\nu\beta} + P_{\mu\beta}P_{\nu\alpha} - P_{\mu\nu}P_{\alpha\beta}\right), \quad P_{\mu\nu} = \eta_{\mu\nu} - \frac{k_\mu n_\nu + k_\nu n_\mu}{k\cdot n} - \frac{k_\mu k_\nu}{(k\cdot n)^2}$$

$$\sum_\chi |\mathcal{M}|^2 = \frac{\lambda^2 16\pi}{m_\mathrm{p}^2}\left(\frac{(p'_1\cdot p'_2)^2}{p'_1\cdot k\, p'_2\cdot k} + \frac{(p_1\cdot p_2)^2}{p_1\cdot k\, p_2\cdot k} - \frac{(p'_1\cdot p_1)^2}{p'_1\cdot k\, p_1\cdot k} - \frac{(p'_2\cdot p_1)^2}{p'_2\cdot k\, p_1\cdot k} - \frac{(p'_1\cdot p_2)^2}{p'_1\cdot k\, p_2\cdot k} - \frac{(p'_2\cdot p_2)^2}{p'_2\cdot k\, p_2\cdot k} - 2\right)$$

$$\dot{f}_h^{(i,j)}(k) = \frac{1}{2k}\, T^2 \left(\frac{T}{m_{\mathrm{p}}}\right)^j \lambda^i\, n_{\mathrm{B}}\left(\frac{k}{T}\right) \psi^{(i,j)}\left(\frac{k}{T}\right)$$

for $(i,j) = (2,2)$ and $(i,j) = (0,4)$



[Ghiglieri, **JSE**, Speranza]

GW spectrum

Reminder:

$$f_h(t,k) \equiv \frac{\text{Number of gravitons with momentum } k \text{ in } d^3k \text{ interval}}{V\, d^3k/(2\pi)^3},$$

$$\frac{d}{dt}\rho_{\rm gw}(t,k) = 2k\dot{f}_h(t,k)\,\frac{d^3k}{(2\pi)^3}$$

In expanding universe:

$$(\partial_t + 4H)\rho_{\rm gw}(t) = \int \frac{d^3k}{(2\pi)^3} R(t,k), \qquad R(t,k) \equiv 2\,k\,\dot{f}_h(t,k)$$

We can integrate this by using $\dot{s} + 3Hs = 0$:

$$\frac{\rho_{\rm gw}(t_1)}{s^{4/3}(t_1)} - \frac{\rho_{\rm gw}(t_0)}{s^{4/3}(t_0)} = \int_{t_0}^{t_1} dt\,\frac{1}{s^{4/3}(t)} \int \frac{d^3k}{(2\pi)^3} R(T(t),k)$$

Use to convert to temperature integral:

$$\frac{dT}{dt} = -\sqrt{\frac{4\pi^3}{45}} g_{*\rho}(T)^{\frac{1}{2}} \frac{g_{*s}(T)}{g_{*c}(T)} \frac{T^3}{m_{\rm p}}$$

Inflationary bound on $T_{\max}$

$$\rho_{\inf} = 3H_{\inf}^2 M_P^2 \approx \frac{3}{2}\pi^2 r A_S M_p^4$$

From CMB observations:

$$\rho_{\inf} < \left(1.6 \times 10^{16}\,\mathrm{GeV}\right)^4$$

And with

$$\rho_{\inf} = \frac{\pi^2}{30} g_{*\rho}(T_{\max})\, T_{\max}^4$$

$$T_{\max} < 6.6 \times 10^{15}\,\mathrm{GeV}\ \left[\frac{106.75}{g_{*\rho}(T_{\max})}\right]^{\frac{1}{4}}$$

Stochastic GWs: dark radiation constraints

$$\Omega_{\rm gw} = \frac{1}{\rho}\frac{d\rho_{\rm gw}}{d\ln f}, \qquad \text{with} \quad \rho = \frac{3H_0^2}{8\pi G}$$

$$\frac{\rho_{\rm gw}}{\rho} = \int_0^\infty \frac{df}{f}\Omega_{\rm gw} \approx \Omega_{\rm gw}(f_{\rm peak})$$

$$h_0^2\frac{\rho_{\rm gw}}{\rho} \leq h_0^2\frac{\Delta\rho_{\rm rad}}{\rho} = h_0^2\Omega_\gamma\frac{7}{8}\left(\frac{4}{11}\right)^{\frac{4}{3}}\Delta N_{\rm eff} = 5.6\times10^{-6}\,\Delta N_{\rm eff}$$

$$h_0^2\Omega_{\rm gw}(f_{\rm peak}) \leq 1.68\times10^{-6}\,\left(\frac{\Delta N_{\rm eff}}{0.3}\right)$$

$g_{*s}(T_0) = 3.90$, $g_{*s}(T) = 10.75$, $T_0 = 2.72\,\mathrm{K}$, $h^2\Omega_\gamma = 2.47\times10^{-5}$

Graviton pair production dominates if:

$$\lambda^2 \psi^{(2,2)}(y_{\max}) < \frac{1}{3}\left(\frac{T_{\max}}{m_{\rm p}}\right)^2 \psi^{(0,4)}(y_{\max})$$

$$10\,\frac{T_{\max}}{m_{\rm p}} \gtrsim \lambda$$

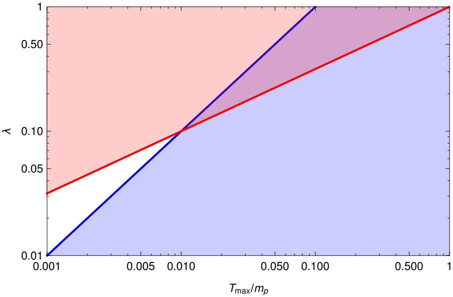
(result also applies for more general models)

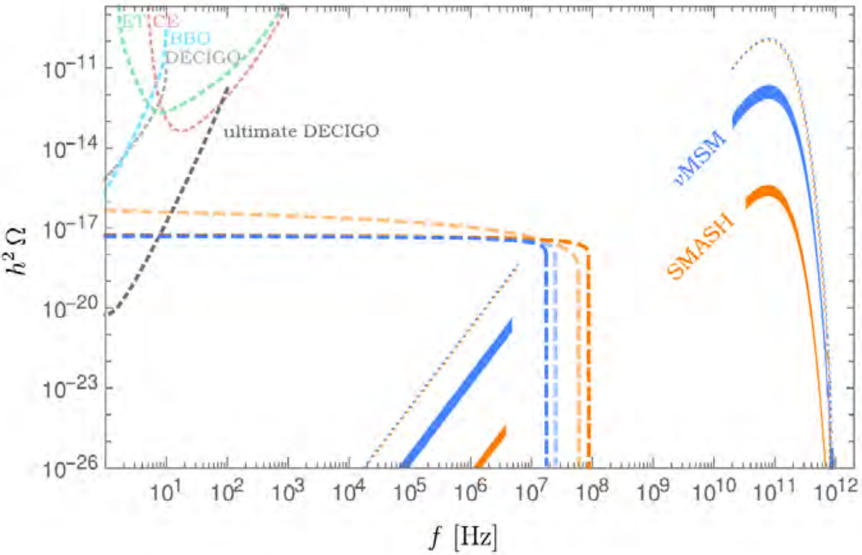
Equilibrium condition:

$$H < \dot{f}_\phi$$

with $H \sim \frac{T^2}{m_{\rm p}}$ and $\dot{f}_\phi \sim \lambda^2 T$

$$\lambda^2 \gtrsim \frac{T_{\max}}{m_{\rm p}}$$





[Ringwald, JSE, Tamarit 20]

SMASH: 8×10^9 GeV$<\ T_{\rm max} < 2 \times 10^9$ GeV

νMSM: 3×10^{13} GeV$<\ T_{\rm max} < 1 \times 10^{14}$ GeV

Detection prospects

GW photon mixing

$$\mathcal{L} \supset -\frac{1}{4}\eta^{\mu\alpha}\eta^{\nu\beta}F_{\mu\nu}F_{\alpha\beta}$$

In curved spacetime:

$$\mathcal{L} \supset -\sqrt{-g}\,\frac{1}{4}g^{\mu\alpha}g^{\nu\beta}F_{\mu\nu}F_{\alpha\beta}$$

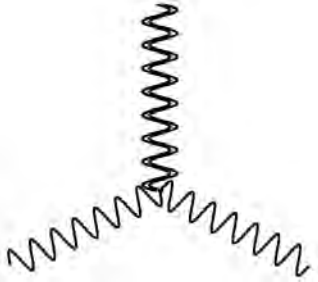
Linearized Gravity

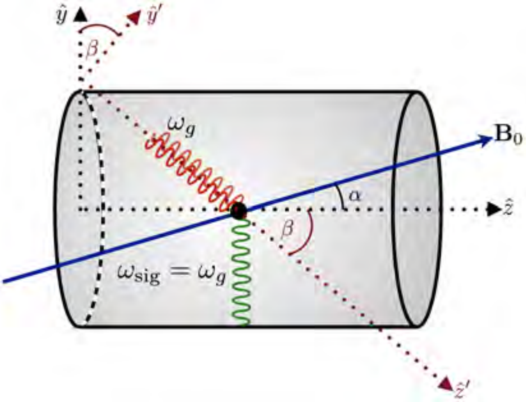
$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

Then Lagrangian contains terms of the form:

$$\mathcal{L} \supset -\frac{1}{4}\eta^{\mu\alpha}\,h^{\nu\beta}\,F_{\mu\nu}F_{\alpha\beta}$$

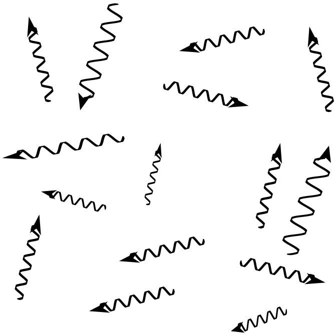
cf. axions $\mathcal{L} \supset -\frac{1}{4}g_{a\gamma}\,a\,F_{\mu\nu}\tilde{F}^{\mu\nu}$





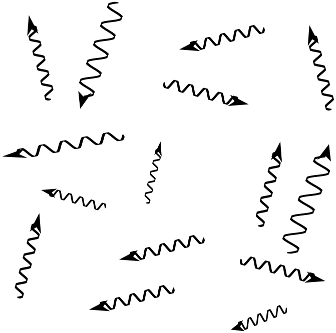
Resonance frequencies in GHz regime

Sensitivity to stochastic GW background



- $S_{\rm sig}(\omega) = \frac{\omega_n}{Q} \frac{(\omega\omega_n)^2}{(\omega^2-\omega_n^2)^2+(\omega\omega_n/Q)^2} \left|\eta\right|^2 B_0^2 V_{\rm cav} S_h(\omega)$
- Thermal noise $S_{\rm noise}(\omega) = \frac{4\pi T(\omega\omega_n/Q)^2}{(\omega^2-\omega_n)^2+(\omega\omega_n/Q)^2}$
- Non-coherent signal appears as an additional noise source in the detector

Sensitivity to stochastic GW background



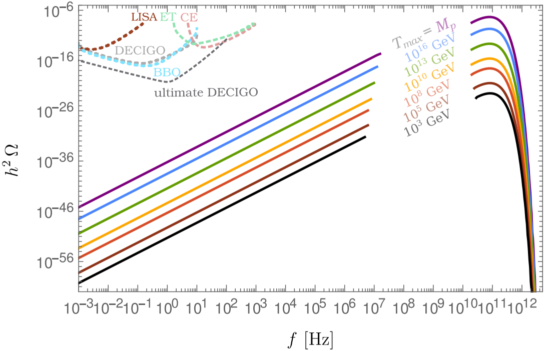
- $\text{SNR} = \frac{S_{\text{sig}}}{S_{\text{noise}}} = \frac{\omega_n Q}{4\pi T} |\eta|^2 B_0^2 V_{\text{cav}} S_h(\omega_g)$
- $\Omega_{\text{GW}}(\omega) = \frac{2}{3} \frac{\omega^3}{H_0^2} S_h(\omega)$

- $\Omega_{\text{GW}} = 8 \times 10^{10} \times \left(\frac{(0.2)^2}{|\eta|^2}\right) \left(\frac{10\,\text{T}}{B_0}\right)^2 \left(\frac{\omega_n}{1\,\text{GHz}}\right)^2 \left(\frac{1\,\text{m}^3}{V_{\text{cav}}}\right) \left(\frac{10^{12}}{Q}\right) \left(\frac{T_{\text{sys}}}{10\,\text{mK}}\right)$
- Cosmologically produced GW backgrounds $\Omega_{\text{GW}} < 10^{-6}$
- Without tricks the detection prospects are not great for stochastic GW backgrounds.

GW production in primordial plasma

$$\frac{1}{\rho_c^{(0)}}\frac{d\rho^0_{\rm CGMB}}{d\ln f} = \Omega_{\rm CGMB}(f)$$

$$\Omega_{\rm CGMB}(f) \simeq \frac{1440\sqrt{10}}{2\pi^2 M_P}\,\Omega_\gamma\,[g_{*s}(\text{fin})]^{1/3}\,\frac{f^3}{T_0^3}\int_{T_{\rm ewco}}^{T_{\max}} \mathrm{d}T\,\frac{g_{*c}(T)}{[g_{*s}(T)]^{4/3}\,[g_{*\rho}(T)]^{1/2}}\,\hat{\eta}\left(T, 2\pi\left[\frac{g_{*s}(T)}{g_{*s}(\text{fin})}\right]^{1/3}\frac{f}{T_0}\right)$$



(for SM)

GW production in primordial plasma

$$\Omega_{CGMB}(f_{\rm peak}) \approx 2.7 \times 10^{-8} \left(\frac{g_{*s}(T_{\rm max})}{106.75}\right)^{-11/6} \frac{T_{\rm max}}{M_p} \times (\text{model dep. factor})$$

$$f_{\rm peak}(T_{\rm max}) \approx 79.8\,\mathrm{GHz} \left[\frac{106.75}{g_{*s}(T_{\rm max})}\right]^{\frac{1}{3}} \times (\text{model dep. factor})$$

	SM	νMSM	SMASH	MSSM
$g_{*s}(T_{\rm max}) \approx$	106.75	109.75	124	228

characteristic amplitude:

$$h_c(f) = 1.26 \times 10^{-18} \left[\frac{\mathrm{Hz}}{f}\right] \times \sqrt{h^2\, \Omega^{(0)}_{\rm GW}(f)}\,.$$