The Equation of State of Neutron Star Matter from Considerations of Theories, Experiments, and Observations

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Static Neutron Star Properties

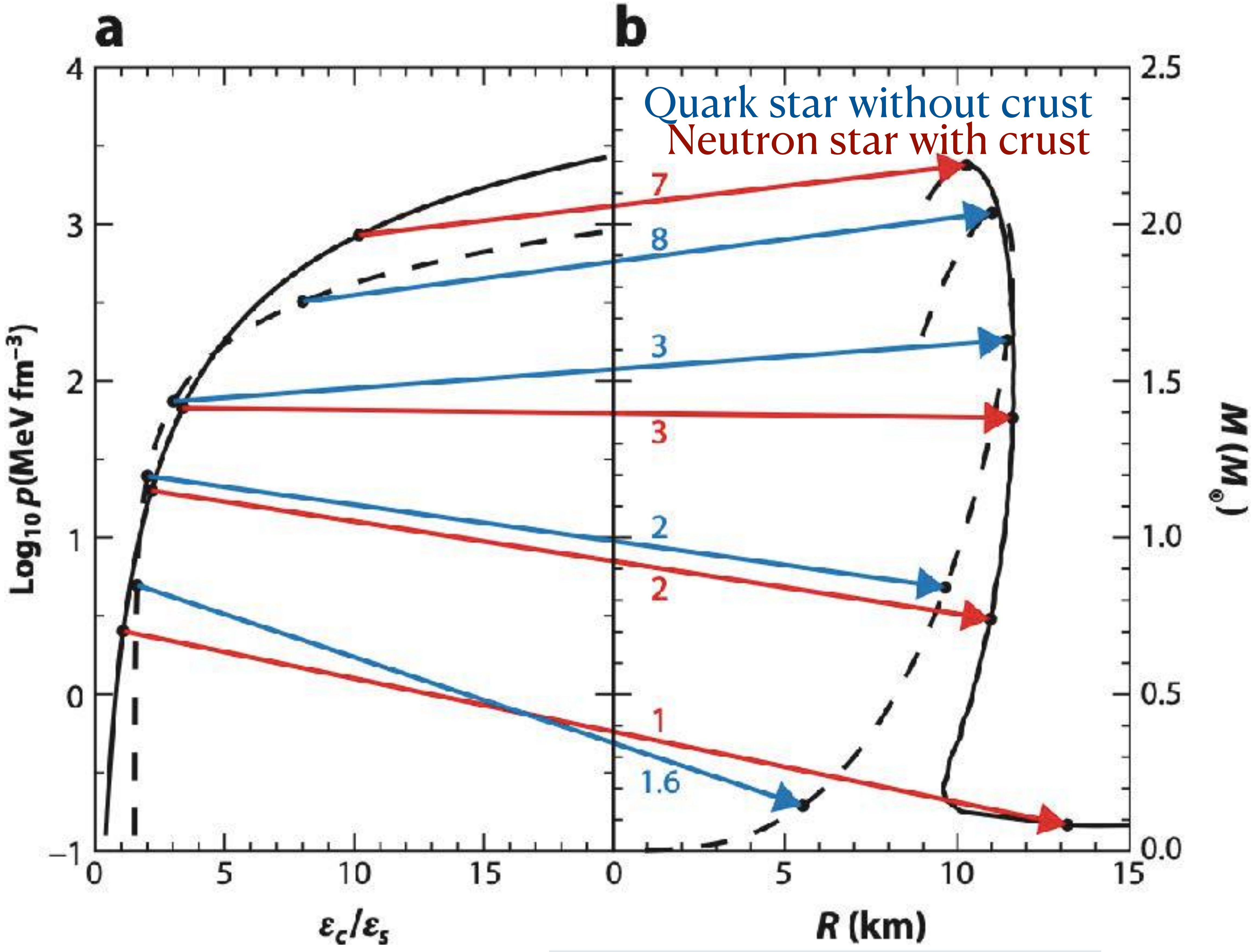
- Tolman–Oppenheimer–Volkoff(TOV) equations:

$$\frac{dp}{dr} = -(\varepsilon + p)\frac{d\Phi}{dr}$$

$$\frac{d\Phi}{dr} = -\frac{G(mc^2 + 4\pi r^3 p)}{r^2 c^4 (1 - \frac{2Gm}{rc^2})}$$

$$\frac{dm}{dr} = 4\pi r^2 \frac{\varepsilon}{c^2}$$

$\boxed{p(\varepsilon) \text{ the EOS}}$ → NS mass M and radius R

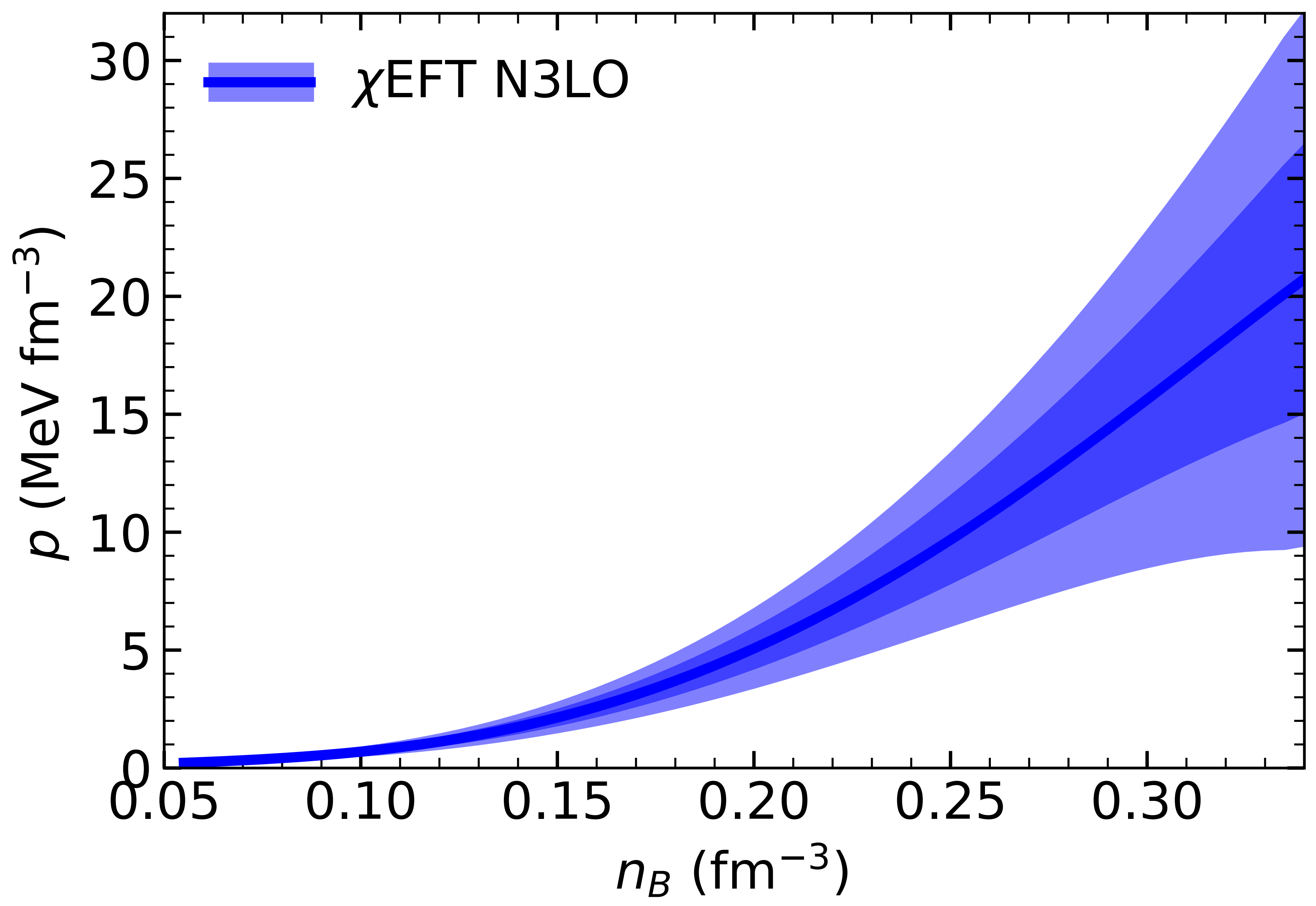


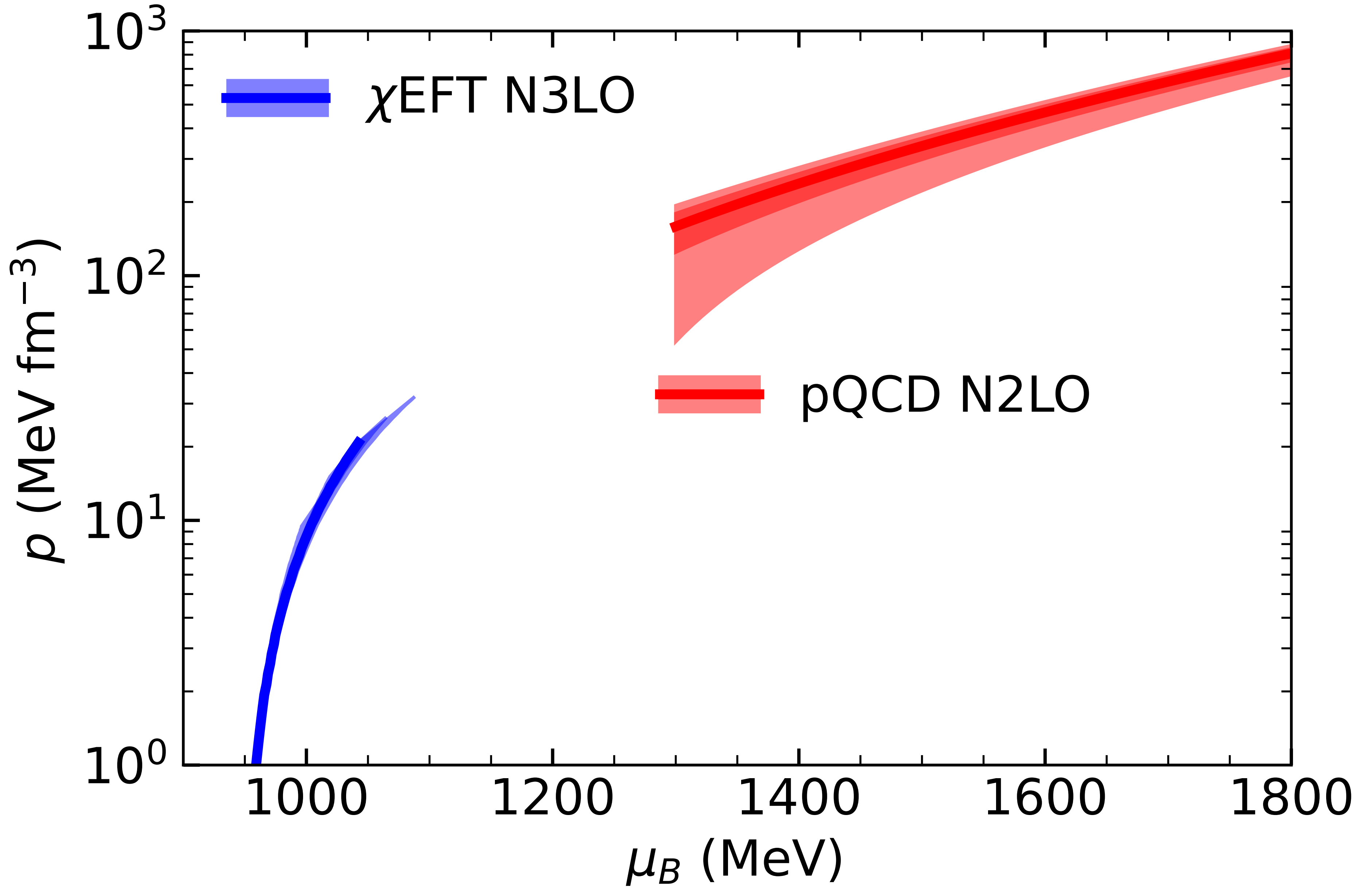
Credit: figure from Lattimer 2012

- Other Properties:
  Binding energy
  Moment of Inertia
  Tidal deformability
  Oscillating frequency
  …

Theories

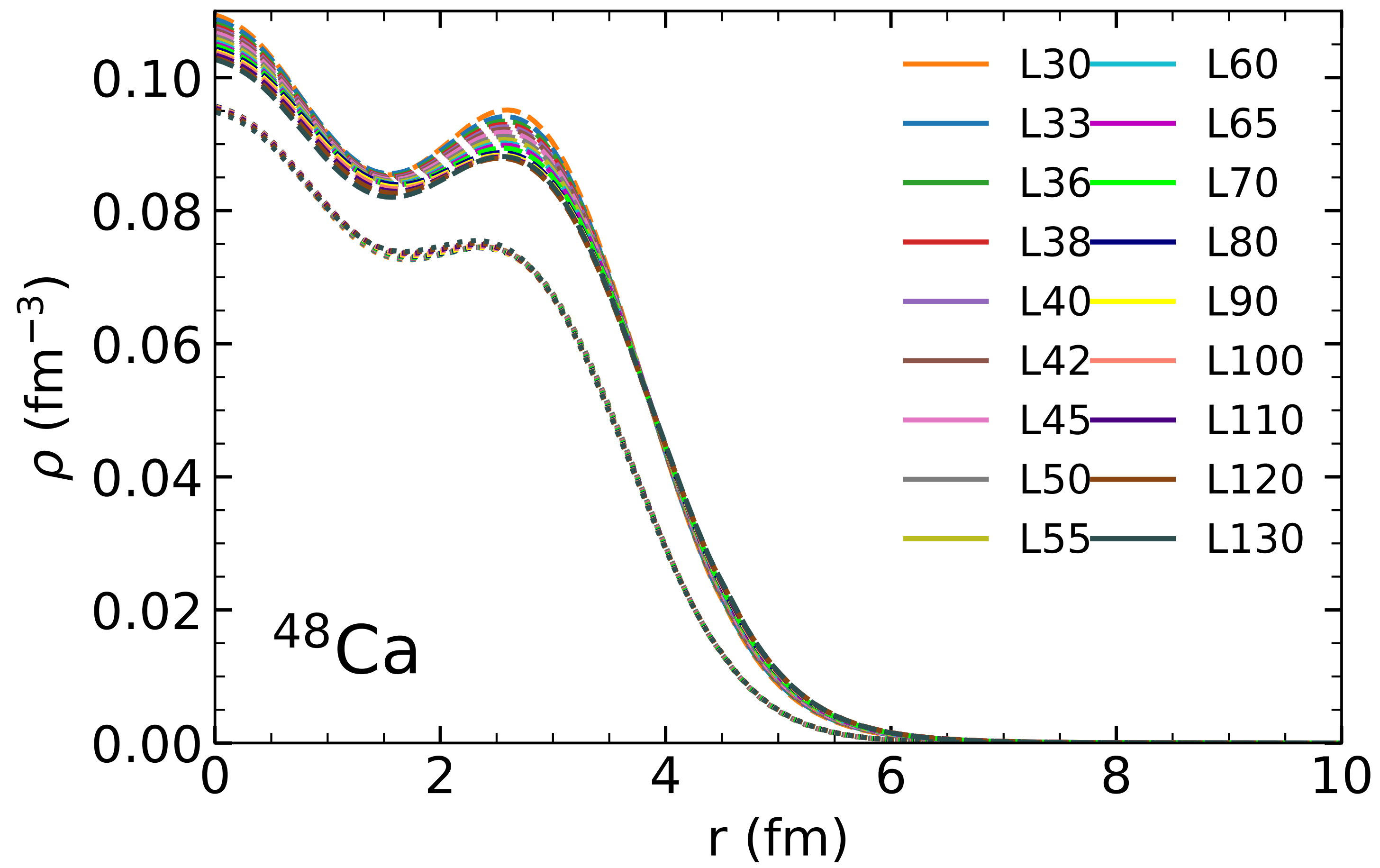
- Ab-initio:
  Inter-nucleon Hamiltonian (χEFT, ...)
  + Many-body method (Perturbation, Monto Carlo, ..)
- Energy density functional:
  Mean-field Hamiltonian
  + Single-body method (Hartree-Fock, ...)
  Schrödinger equation (Skyrme models)
  Dirac equation (Relativistic mean field models)
- Perturbative QCD:
  QCD Lagrangian (quark-gluon coupling)
  + Analytical method (vacuum and ring diagram)

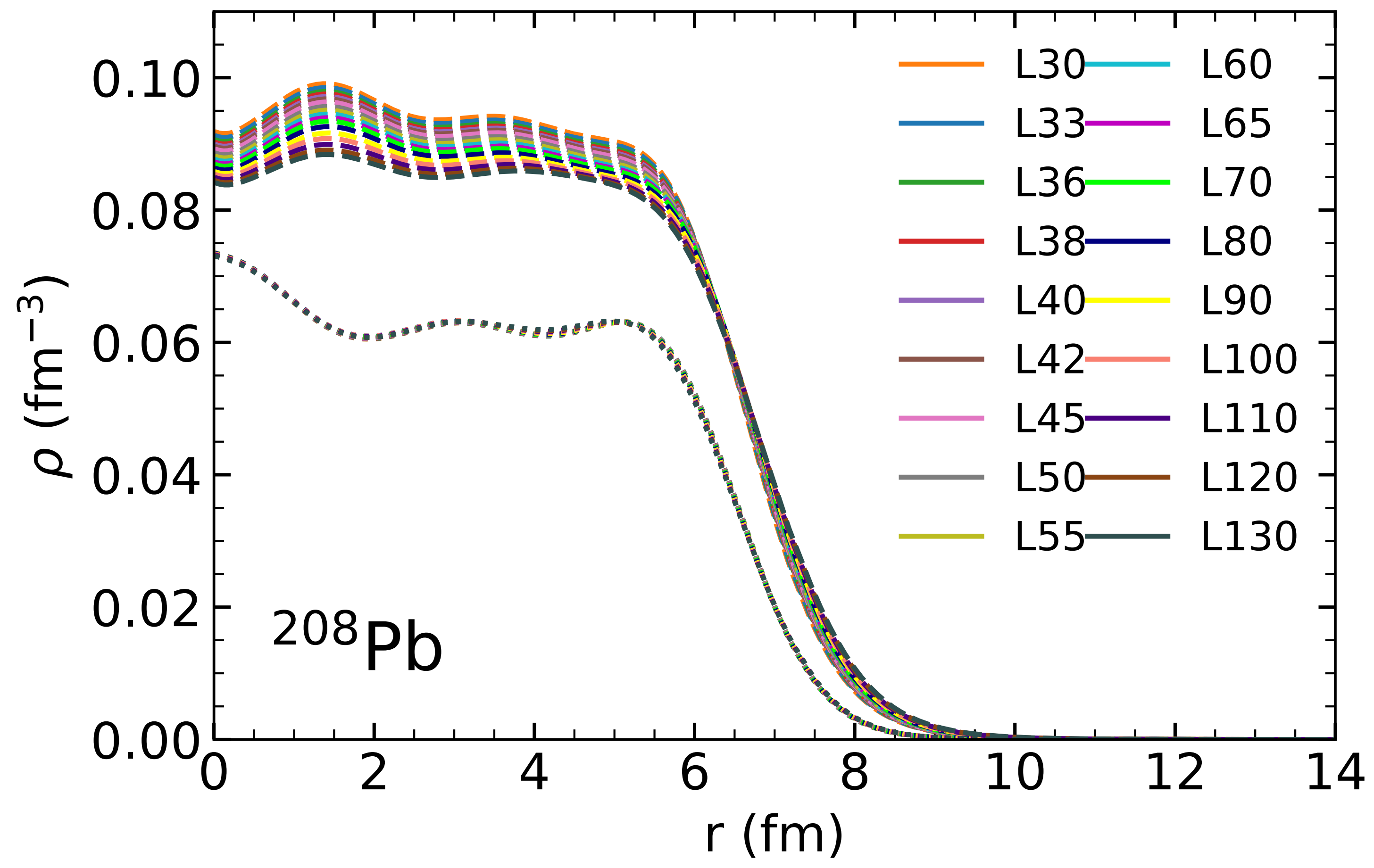


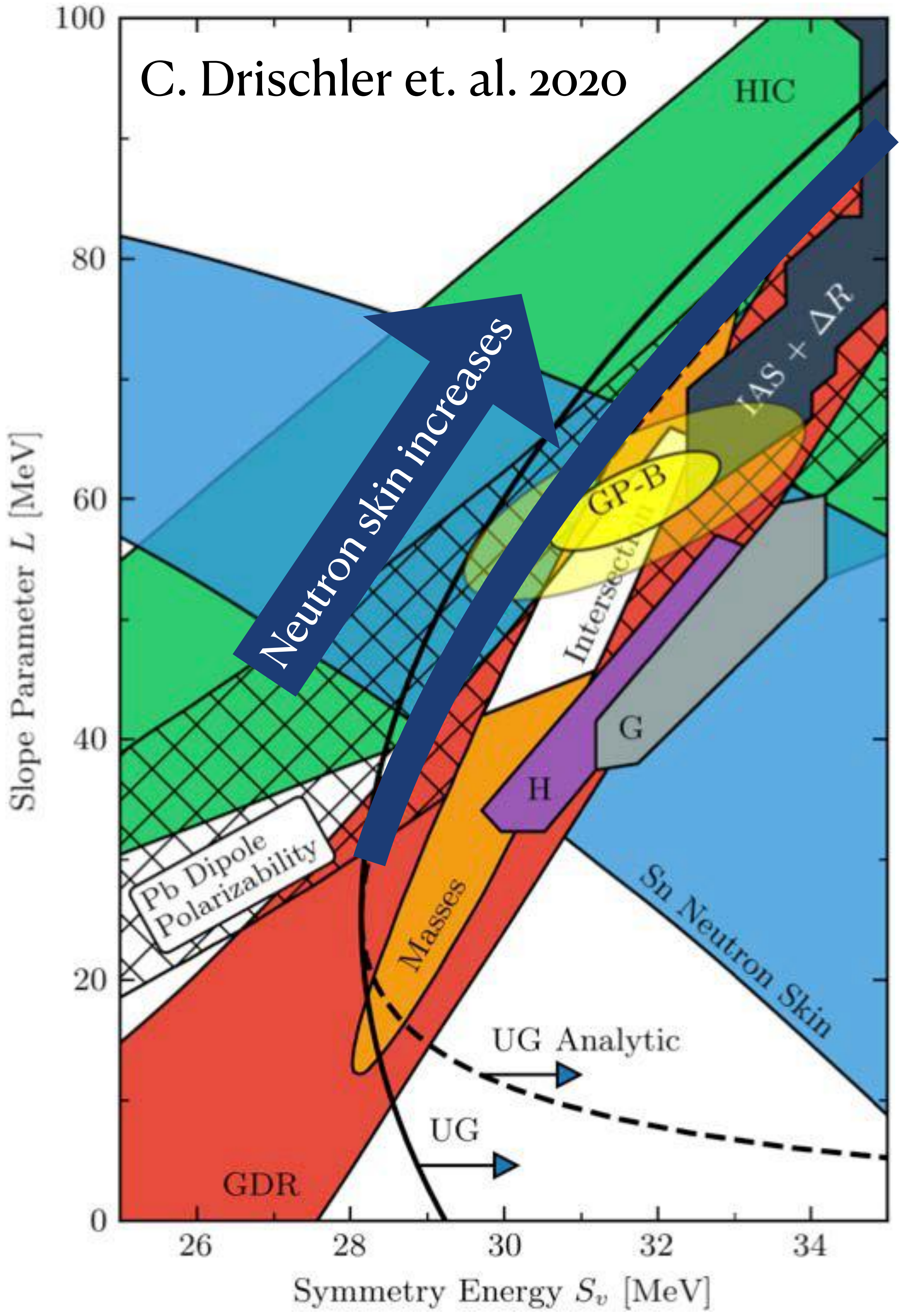


Symmetry energy $E_{SYM}(u = n_B/n_s, x = n_p/n_B)$

Neutron star matter ≈ Pure neutron matter = Symmetric nuclear matter + Symmetry energy







$$E(n_B, x) \approx$$

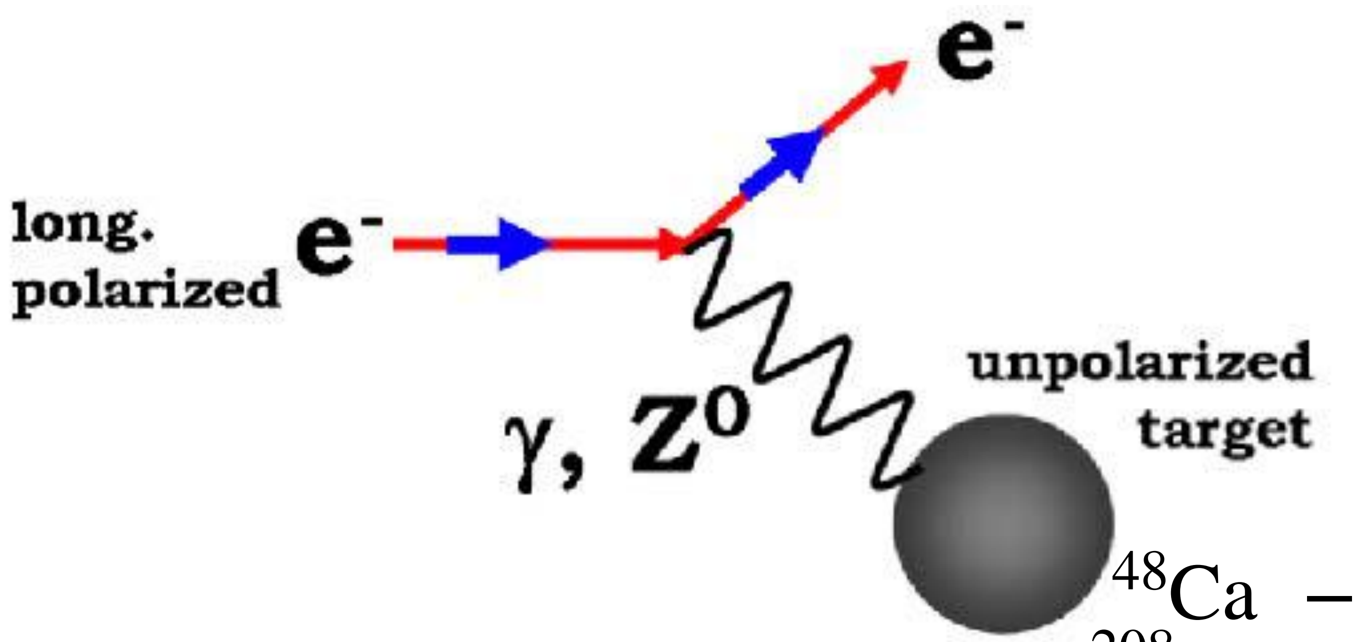
$$E_{SNM}(u) + E_{SYM}(u)\,(1-2x)^2 + \dots$$

$$BE + \frac{K}{18}(u-1)^2 + \dots$$

$$S_v + \frac{L}{3}(u-1) + \frac{K_{SYM}}{18}(u-1)^2 + \dots$$

Neutron Skin $\Delta R = R_n - R_p$
is “**perpendicular**” to others

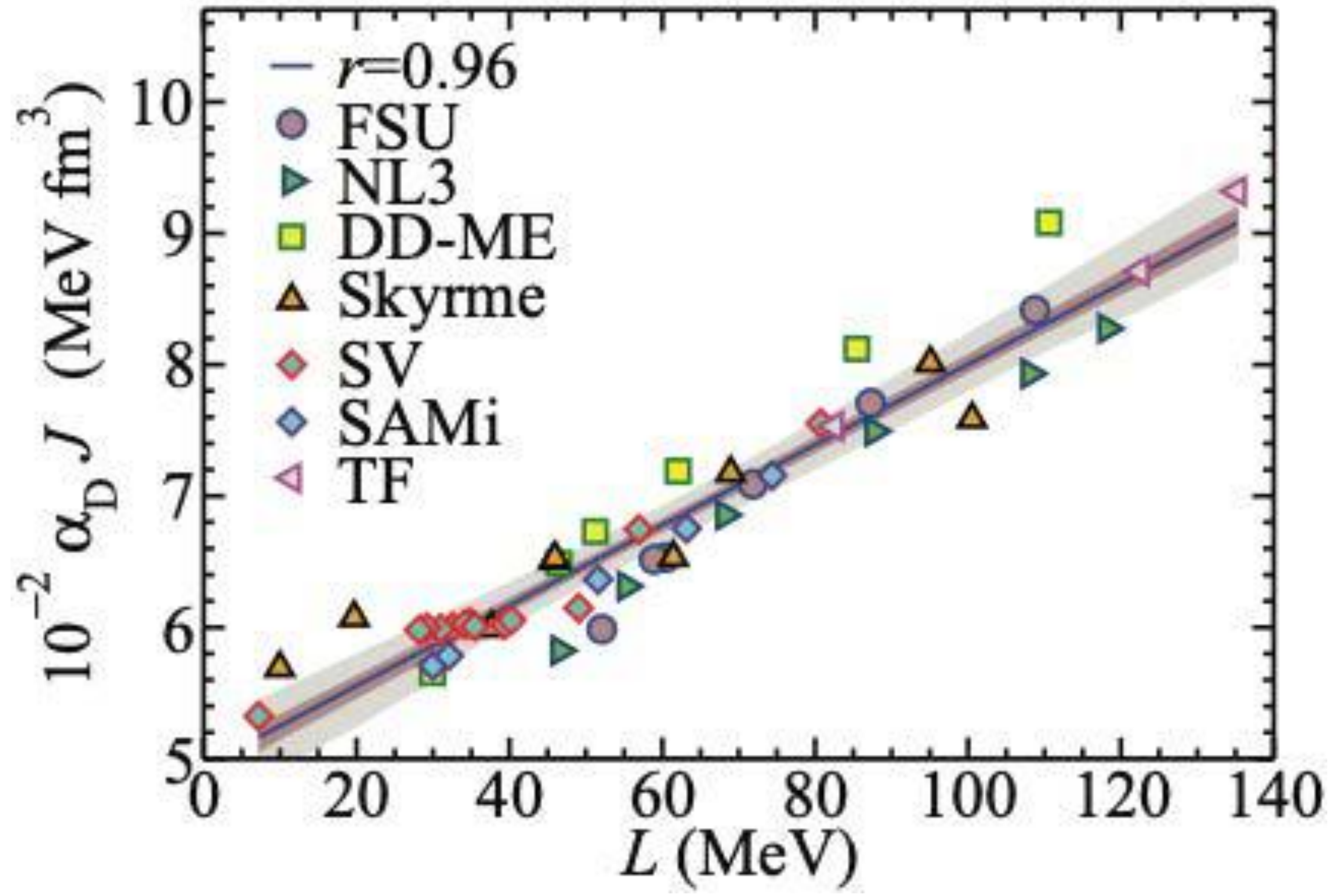
$L = 30 - 90$ MeV
$\Delta R_{^{208}Pb} = 0.11 - 0.25$ fm



^{48}Ca – CREX
^{208}Pb – PREX

Bayesian Analysis

Posterior of Sv and L



Moca-Maza et. al. 2013

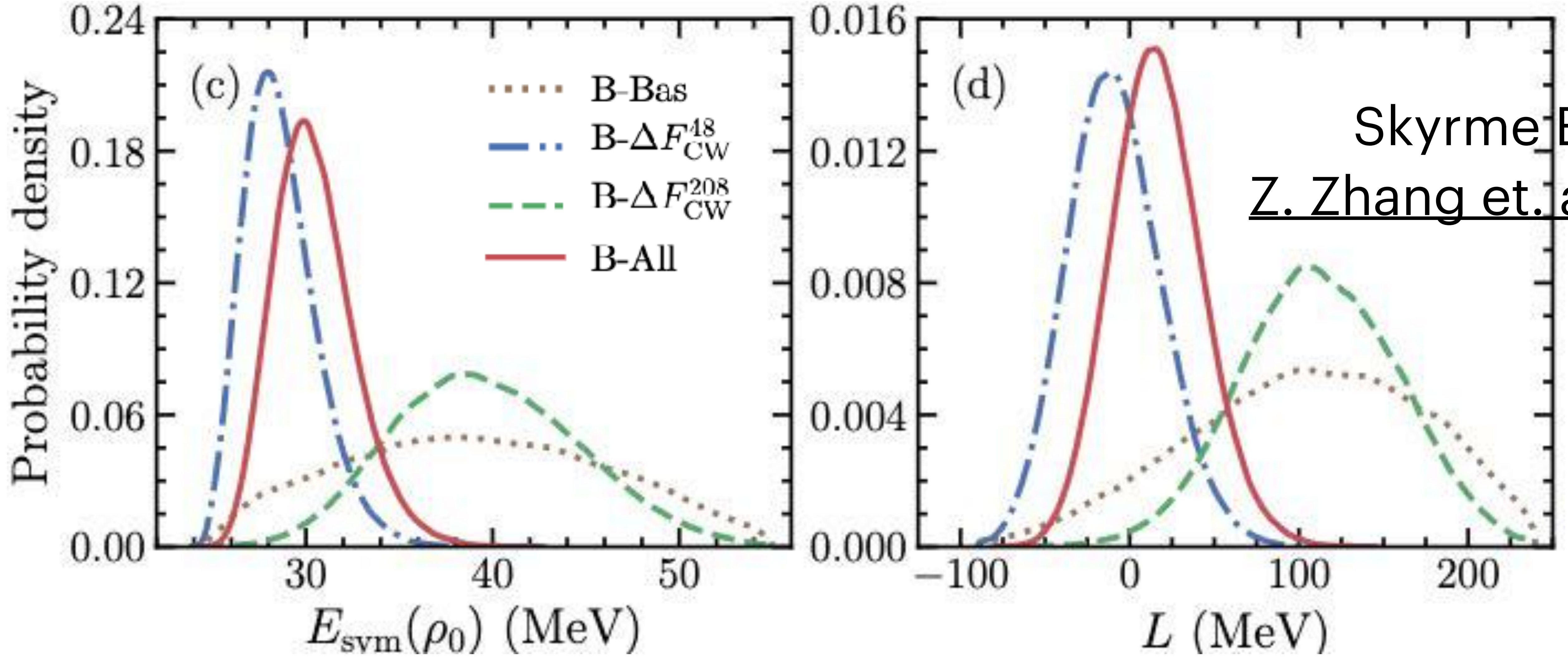
- The weak form factor of PREX+CREX:

$$S_V = 33.2^{+5.7}_{-7.33}\text{MeV}, L = 41.8^{+15.5}_{-16.7}\text{MeV}$$

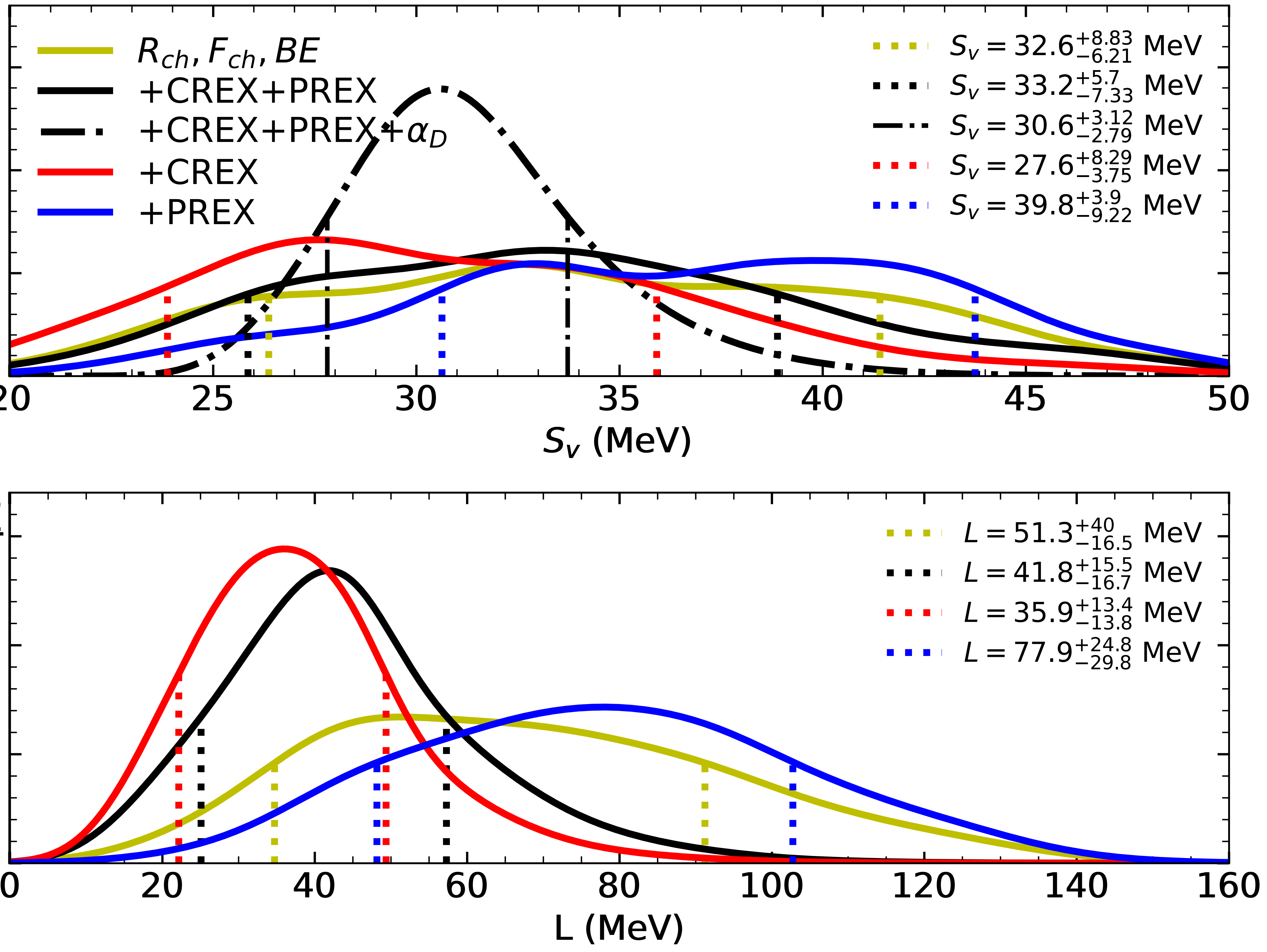
- Dipole polarizability of ^{208}Pb:

$$L = [6.11 \pm 0.316]\, S_v - [146 \pm 1] \;\; \text{MeV}$$

$$S_V = 30.6^{+3.12}_{-2.79} \text{ MeV}$$

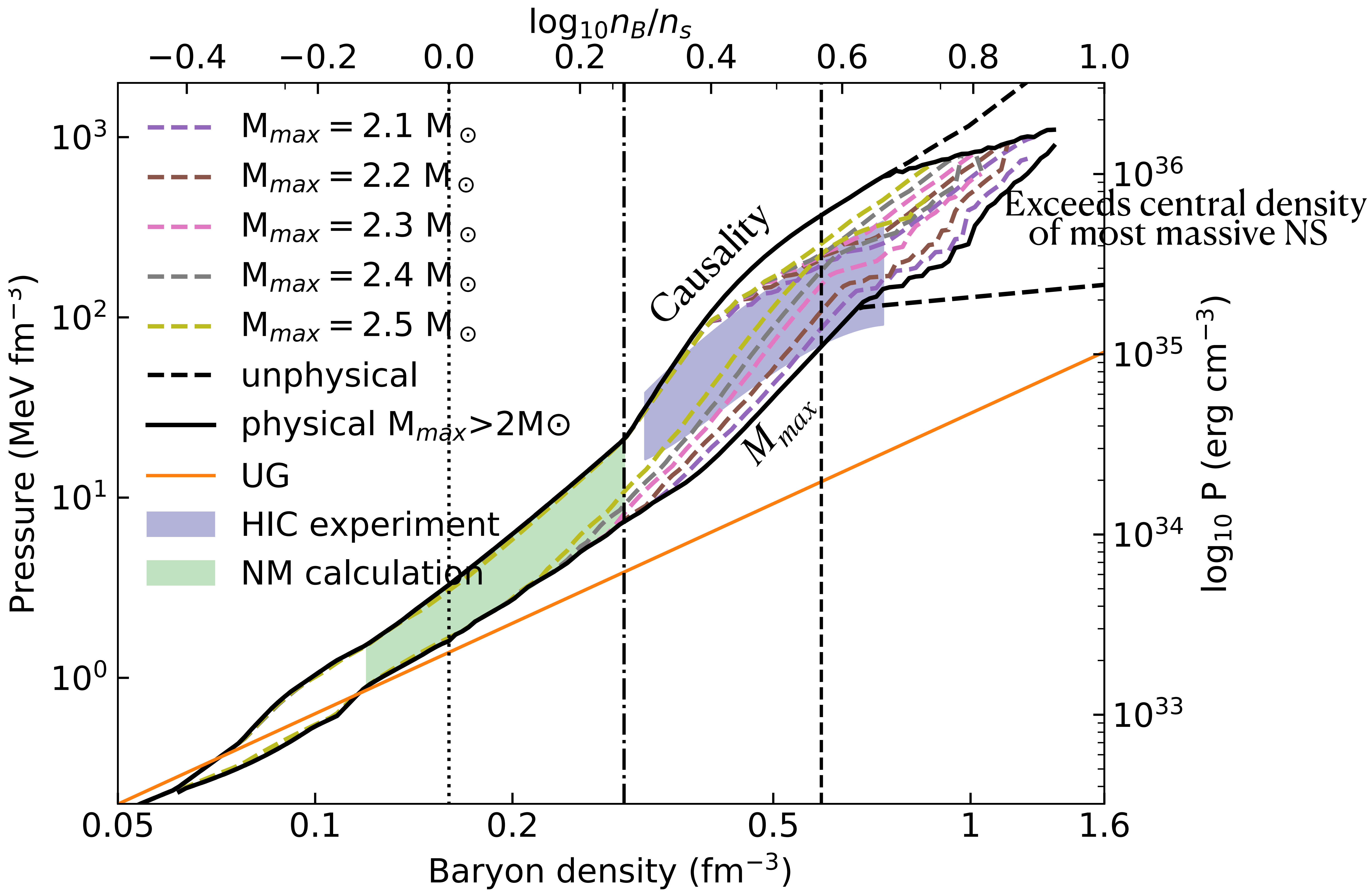


Skyrme EDF
Z. Zhang et. al. 2022



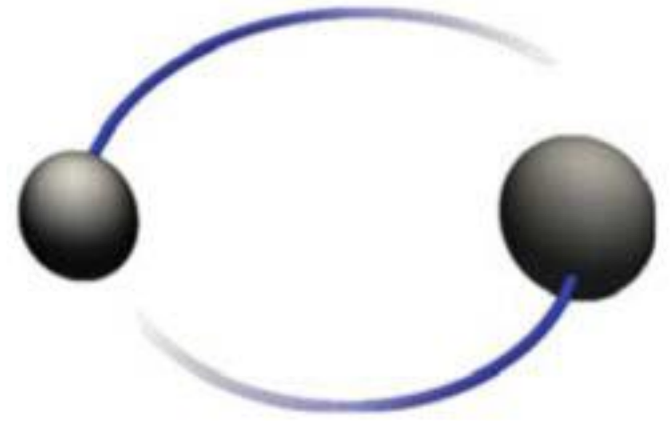
Maximum Mass of NS

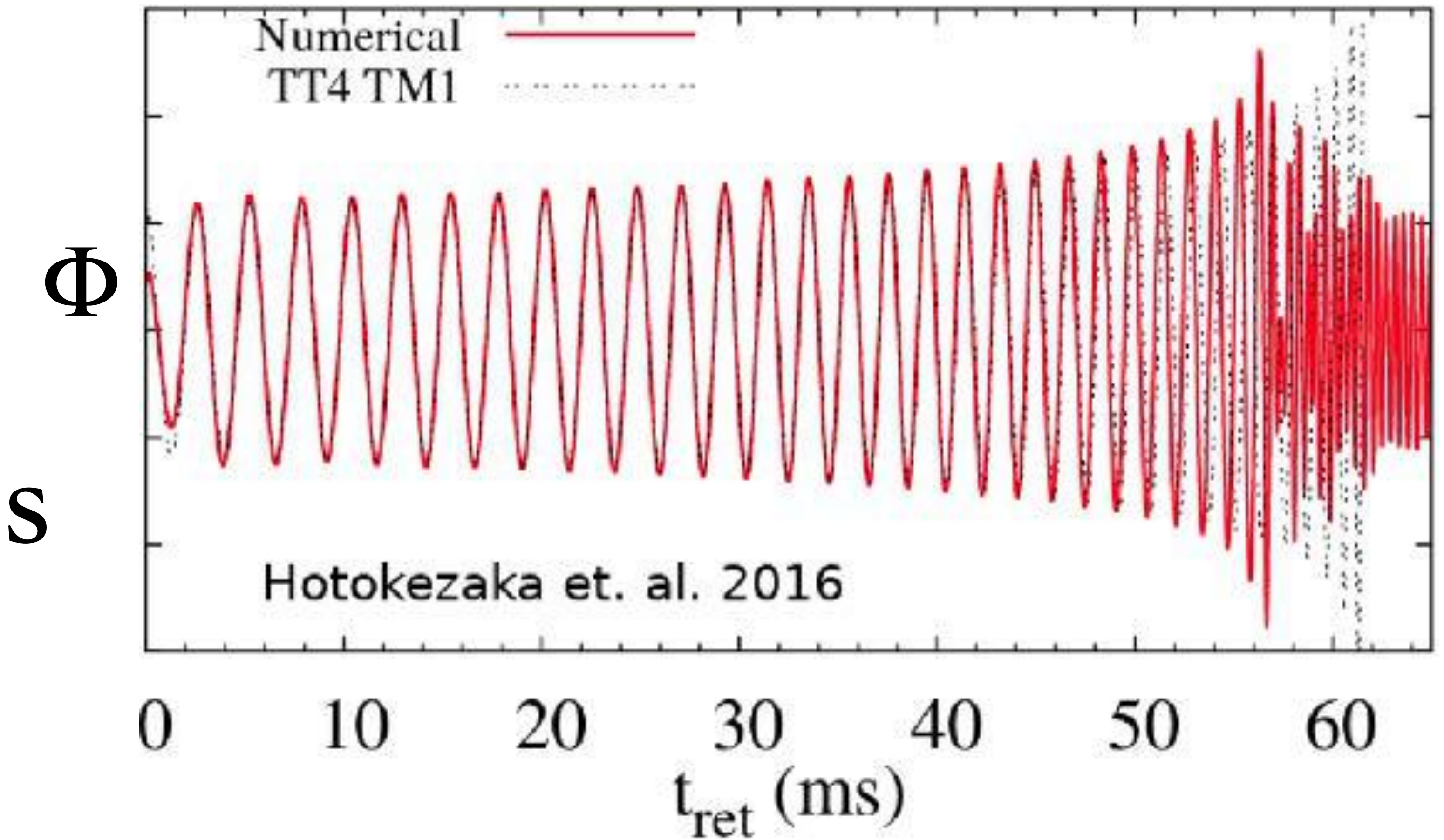
EOS bounds



EOS bounds with neutron matter constraints(fixed maximum mass)

Tidal deformability from NS merger





Hotokezaka et. al. 2016

- Tidal fields → Quadrupole moments

$$-\lambda\varepsilon_{ij}(t) = Q_{ij}(t)$$

- Oscillating Quadruple moments → Gravitational waves

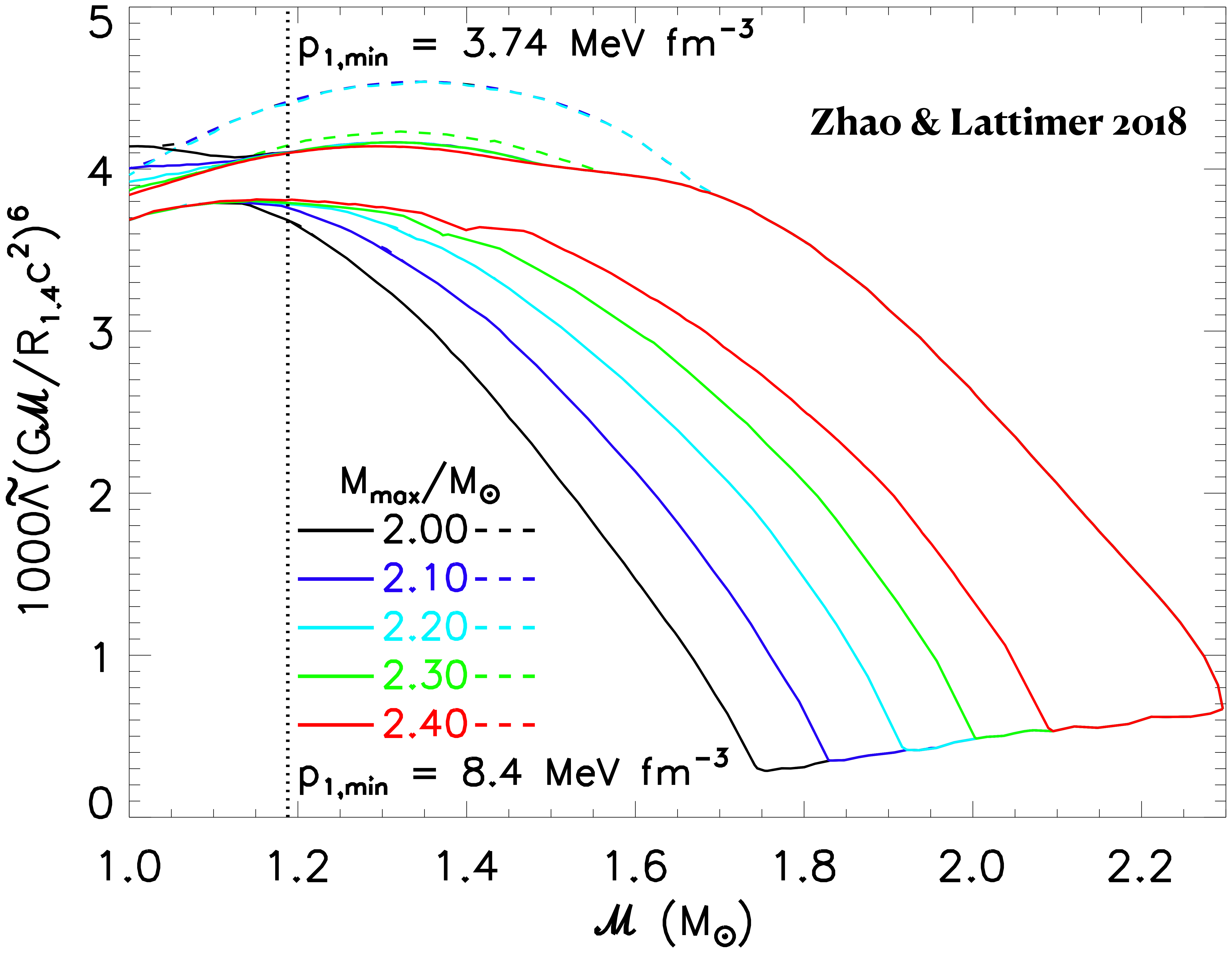
$$\dot{E}(\omega) = -\frac{1}{5} < \dddot{Q}^T_{ij}\dddot{Q}^T_{ij} > = -\frac{32}{5}\underset{\propto \boxed{\mathscr{M}^5/a^5}}{\boxed{M^{4/3}\mu^2\omega^{10/3}}}[1+g(\omega,\Lambda,\omega_f)]$$

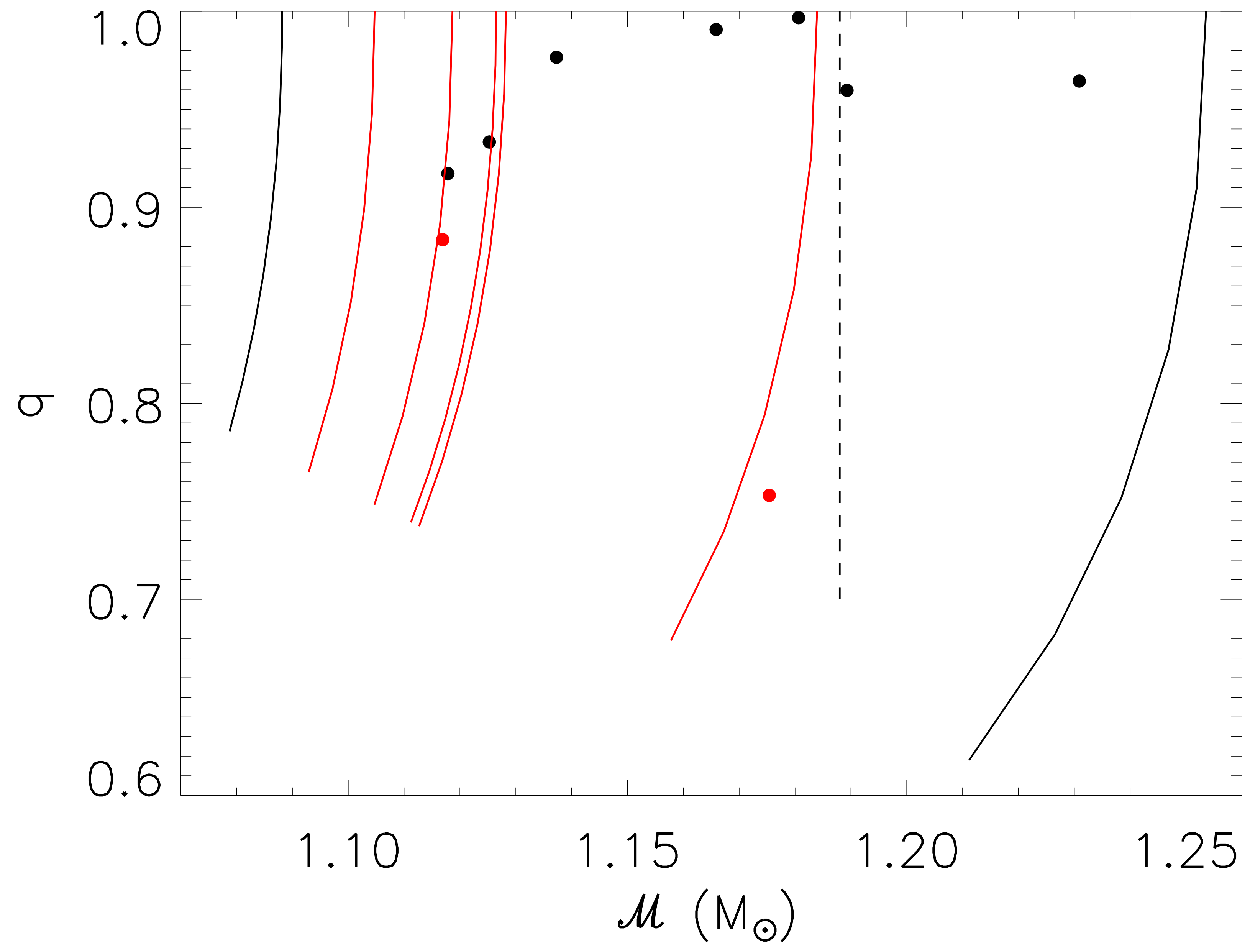
- Energy of stable orbit: $E(\omega) = -M^{1/3}\omega^{-2/3}[1+f(\omega,\Lambda,\omega_f)]$

- Orbital phase: $\frac{d\Phi}{dt} = 2\omega$ → phase advance $\frac{d\Phi}{d\omega} = 2\omega\frac{\frac{dE(\omega)}{d\omega}}{\dot{E}(\omega)}$

- **Tidal effect contribution** $\propto$ $\tilde{\Lambda} = \frac{16}{13}\frac{(12q+1)\Lambda_1 + (12+q)q^4\Lambda_2}{(1+q)^5}$

$\tilde{\Lambda} \propto R_{1.4}^6$ for Precise NS Radius





Binary NS with Shapiro time delay

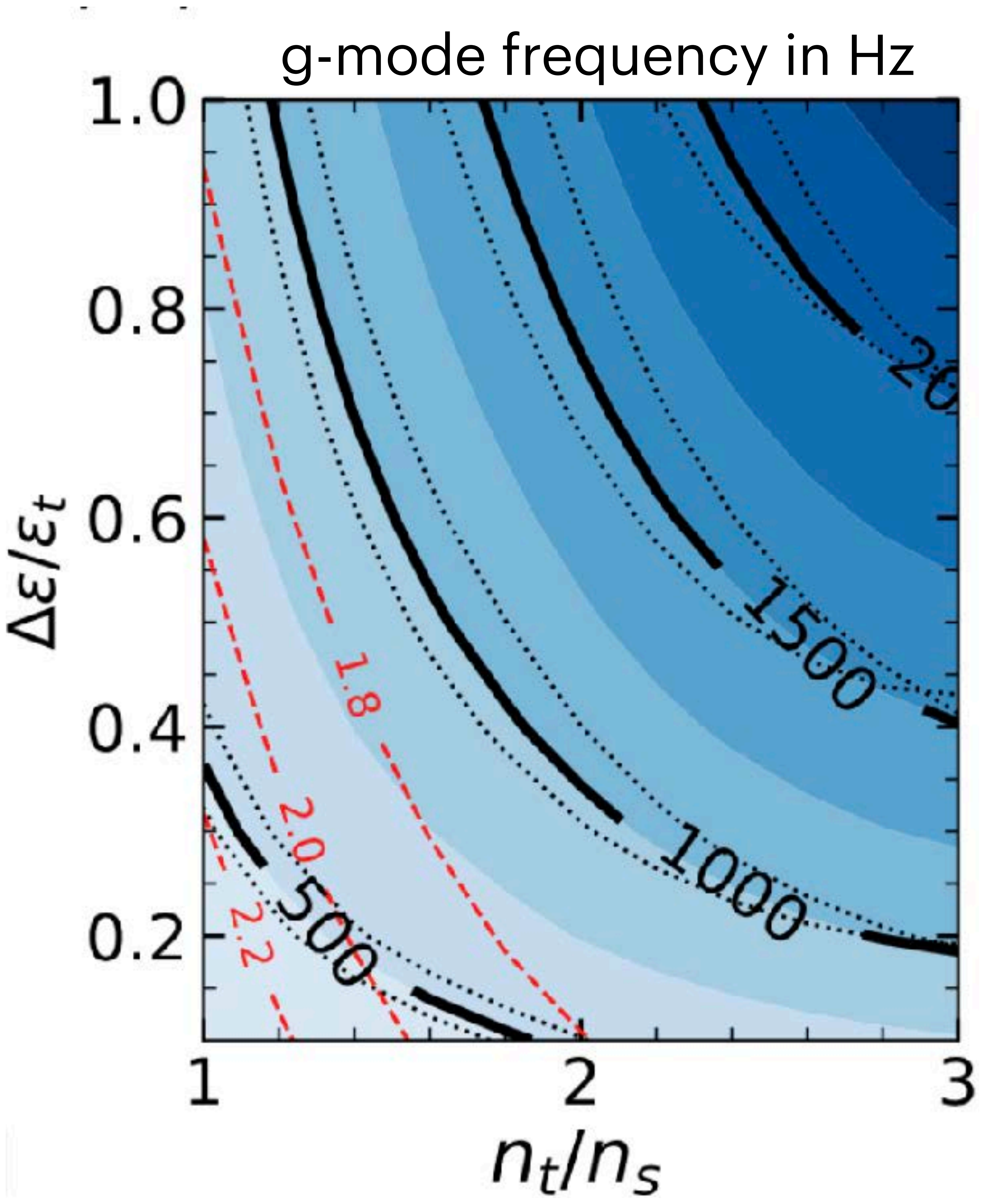
$$R_{1.4} = (11.5 \pm 0.3)\frac{\mathcal{M}}{M_\odot}(\frac{\tilde{\Lambda}}{800})^{\frac{1}{6}}\text{km}$$

$$\mathcal{M}_{GW170817} = 1.186M_\odot$$

$$R_{1.4} = (13.4 \pm 0.1)(\frac{\tilde{\Lambda}}{800})^{\frac{1}{6}}\text{km}$$

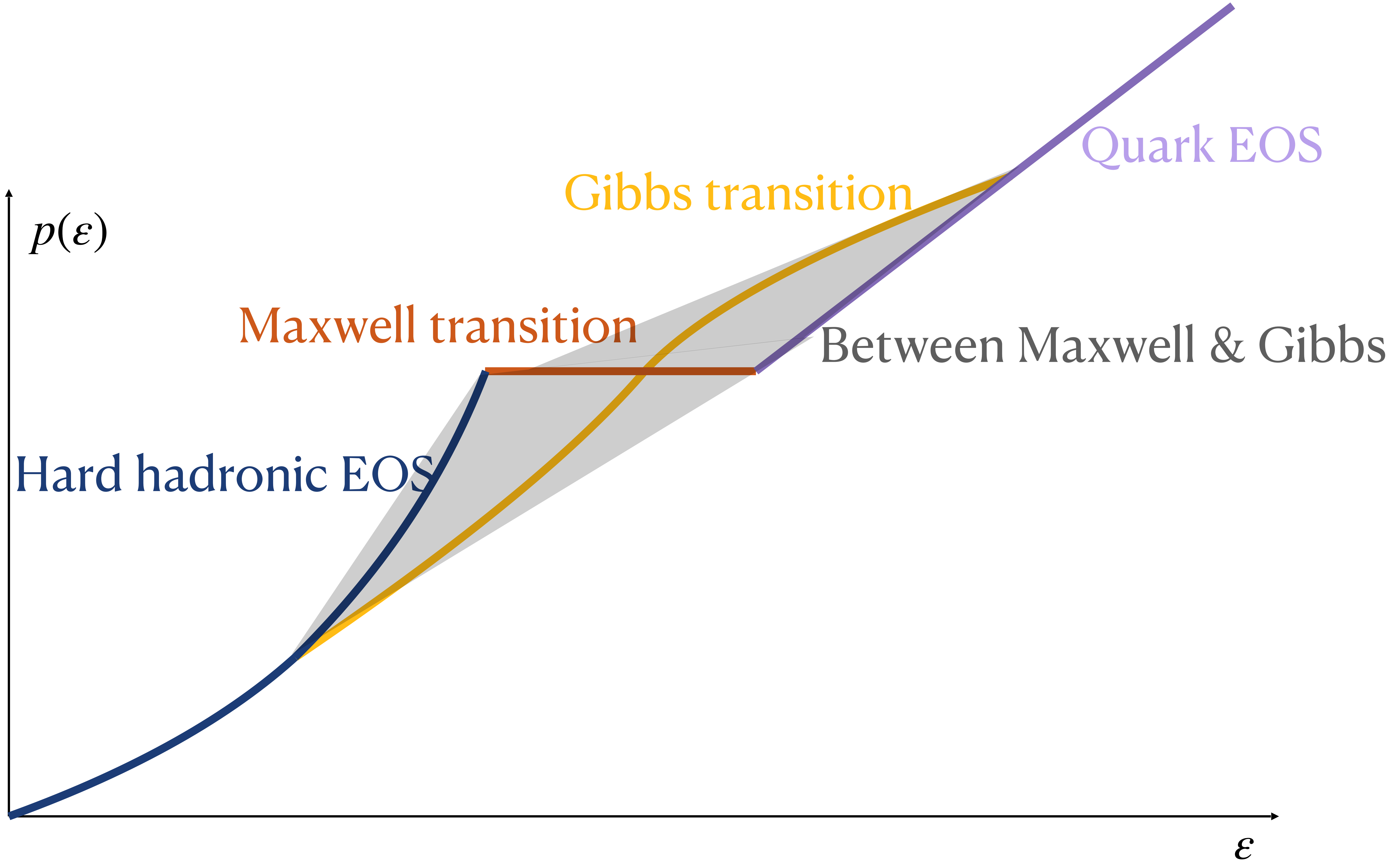
EOS Sensitivity to NS Oscillations

- p-mode are sensitive to pressure at saturation density and twice saturation density.
- f-mode are sensitive to pressure at twice the saturation density.
- Composition g-mode is sensitive to the chemical composition of dense matter, e.g. lepton and quark fractions, Y_{lep} and Y_{qak}
- Discontinuity g-mode is sensitive to phase transition properties in the core of NSs



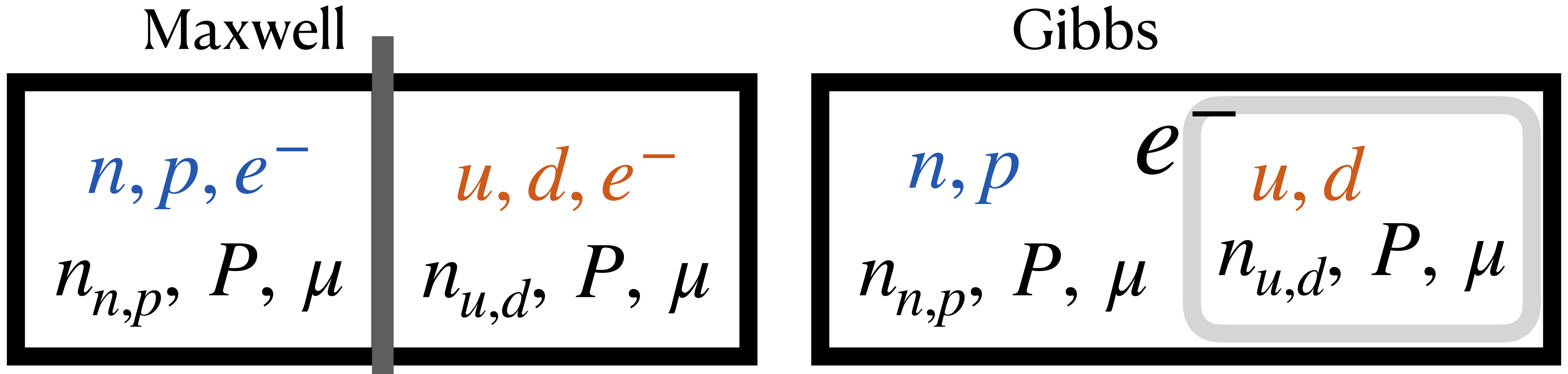
Zhao & Lattimer 2022

Hadron-quark Transition in Neutron Star Core



Between Maxwell & Gibbs

Partially local & partially global



Maxwell

n, p, e^-

$n_{n,p},\ P,\ \mu$

u, d, e^-

$n_{u,d},\ P,\ \mu$

Local neutral lepton fraction
g

Gibbs

n, p

e^-

u, d

$n_{n,p},\ P,\ \mu$

$n_{u,d},\ P,\ \mu$

Global lepton fraction
$1 - g$

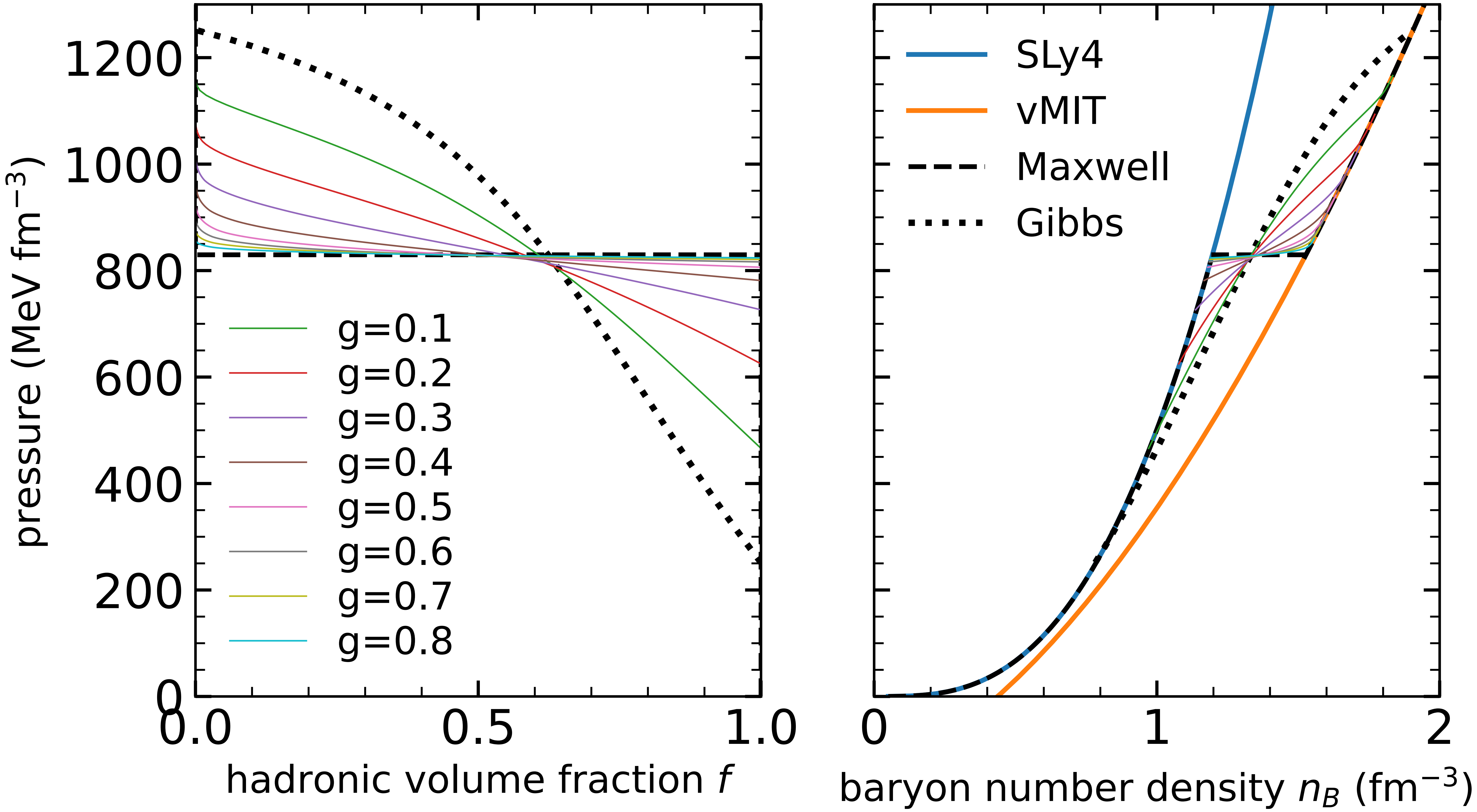
- Locally neutral lepton densities:

$$n_{e,N} = n_p,\ n_{e,Q} = \frac{2}{3}n_u - \frac{1}{3}n_d$$

- Global lepton density, $n_{e,G}$
- Total lepton density:

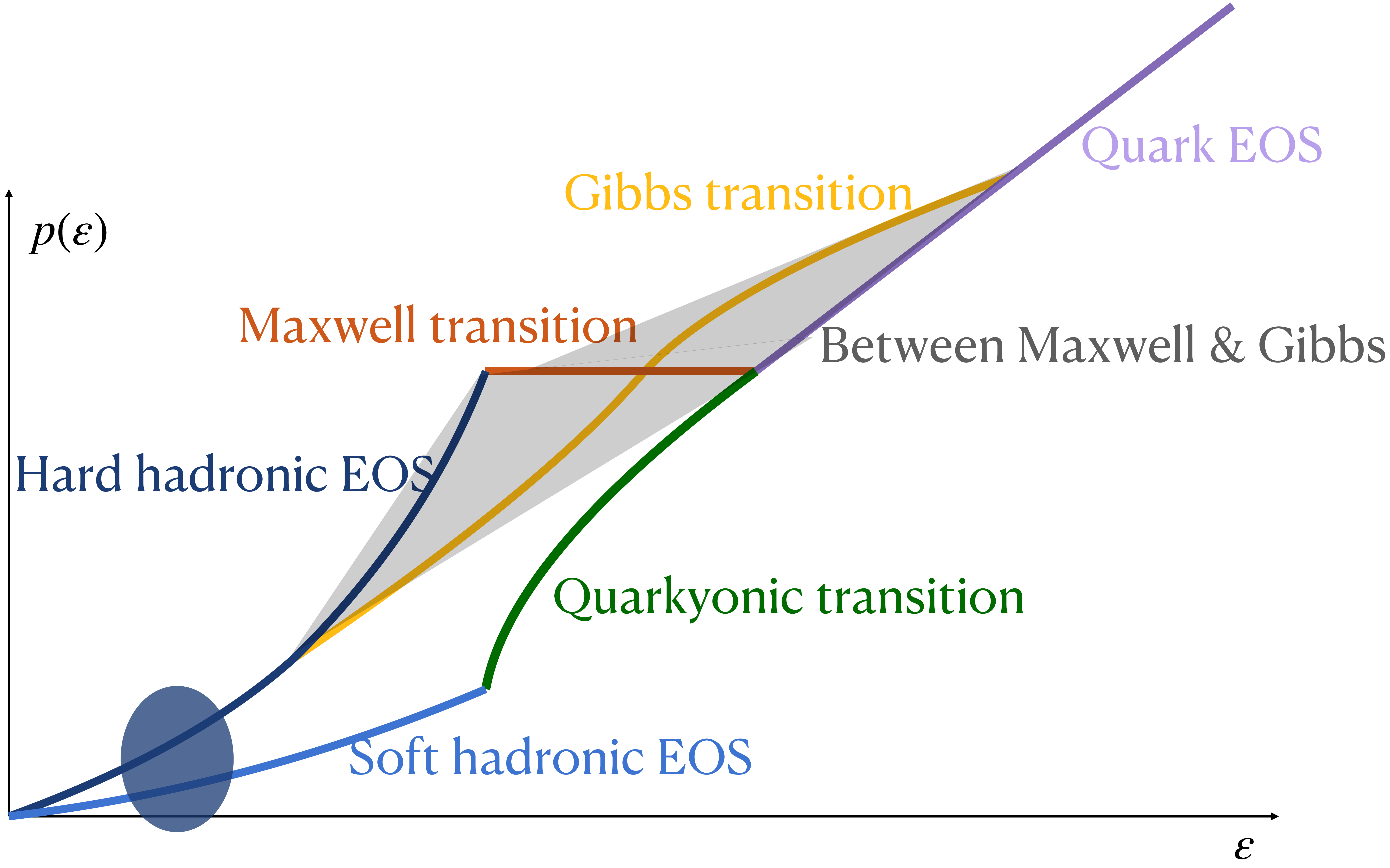
$$n_e = g(fn_{e,N} + (1-f)n_{e,Q}) + (1-g)n_{e,G}$$

- $g = 0$ → Gibbs transition
  $g = 1$ → Maxwell transition
- g could be related to Surface & Coulomb energy.



C. Constantinous et. al. 2023

Hadron-quark Transition in Neutron Star Core



Soft hadronic EOSs is flavored by ab-initio calculation,
neutron skin experiments & neutron star merger observation.

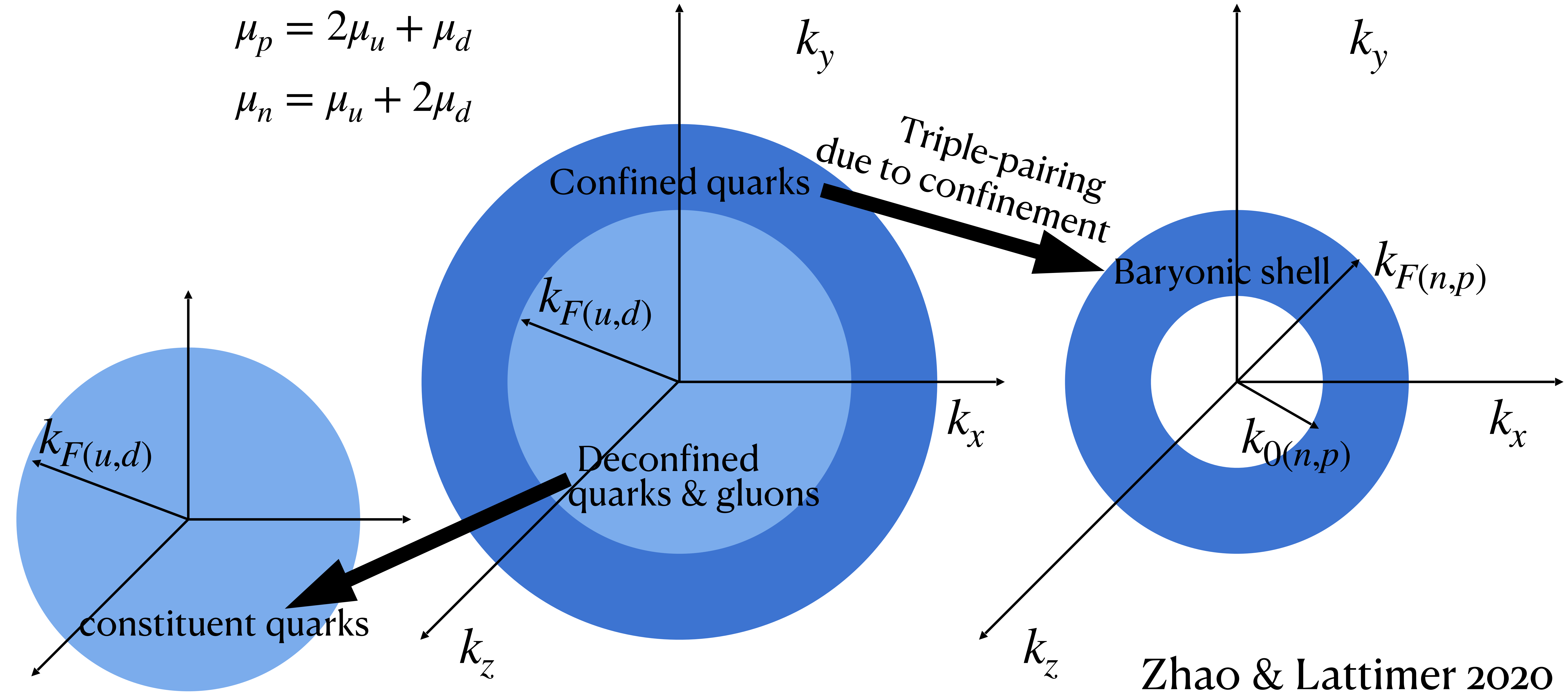
Quarkyonic Matter EOS

- Perturbative quarks = quarks deep inside Fermi sphere
- Baryons = triple-pair of quarks near Fermi surface
- Quarks ⟷ baryons

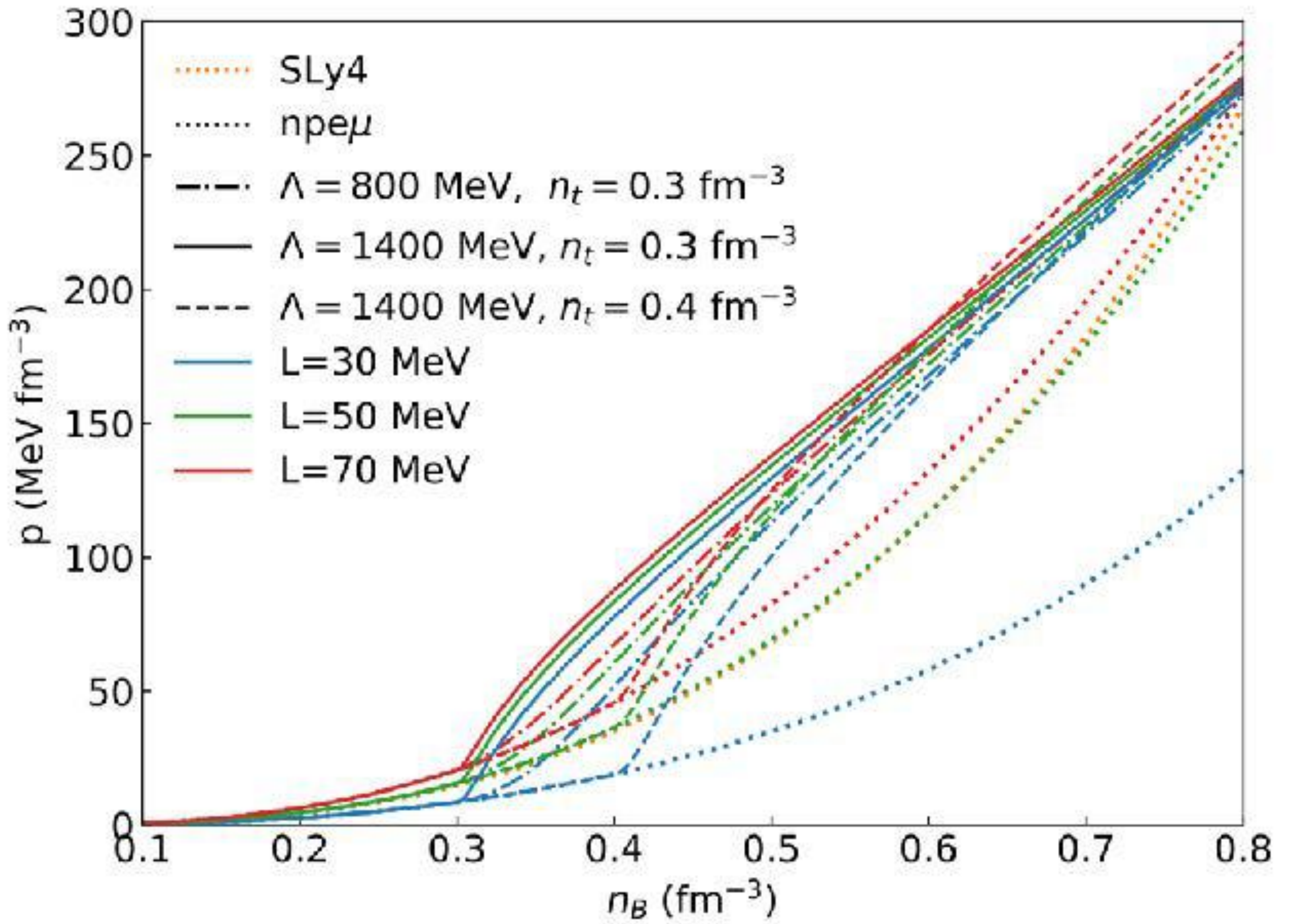
chemical equilibrium

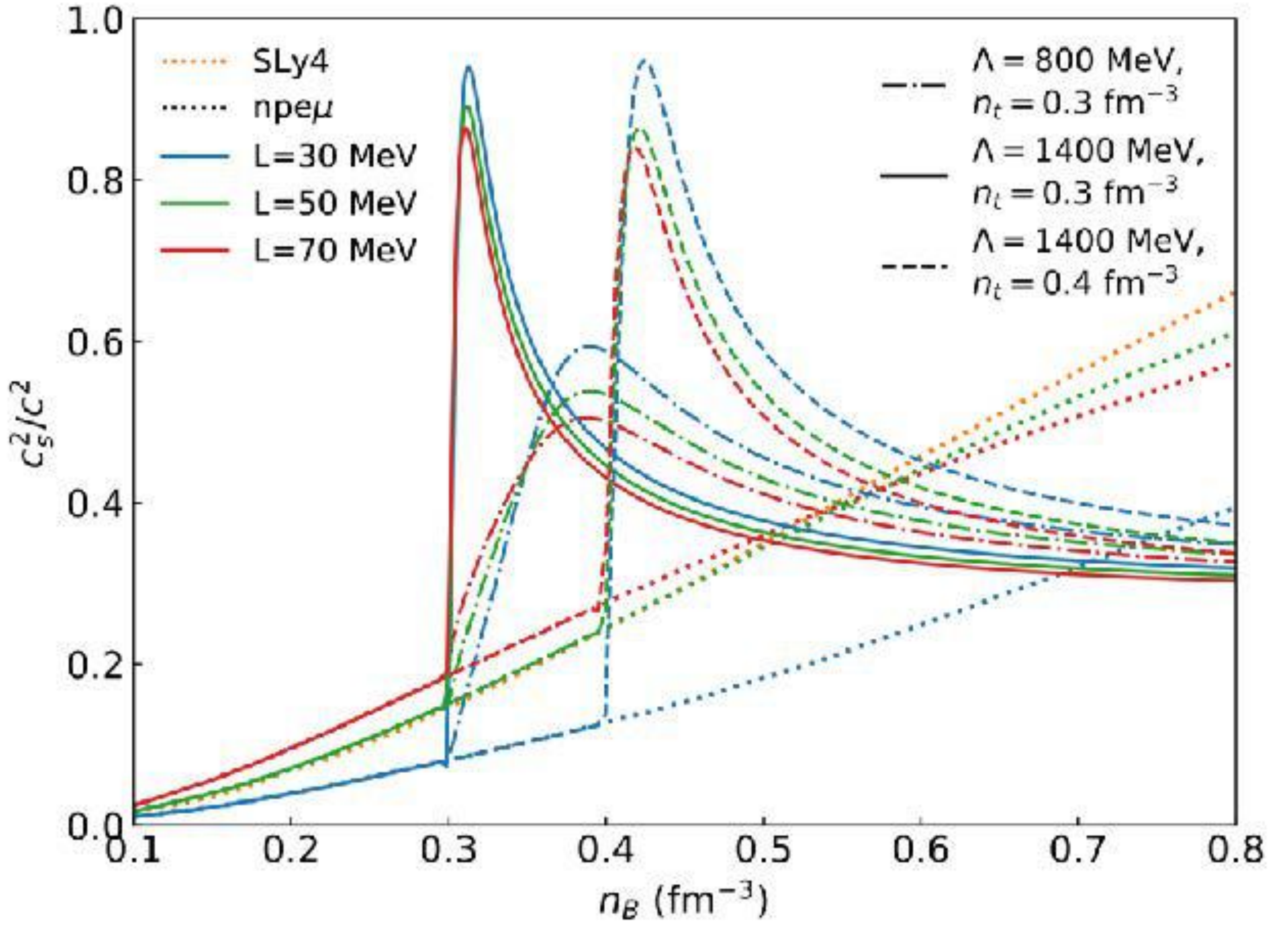
$$\mu_p = 2\mu_u + \mu_d$$

$$\mu_n = \mu_u + 2\mu_d$$



Zhao & Lattimer 2020



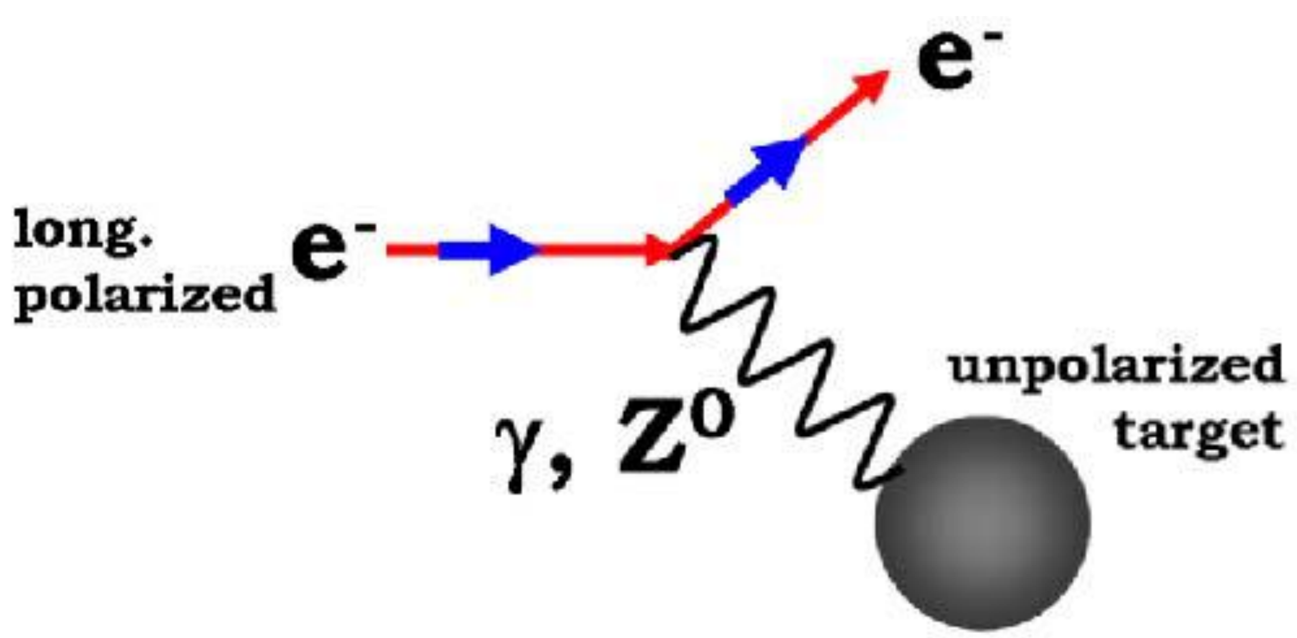


Summary

- EOS constraints around saturation:
  ab-initio > experiment > observation (but improves fast)
- Maximum mass constraints EOS around four times saturation.
- Low-density extension of pQCD is marginally relevant.
- Soft-to-stiff transition is flavored.
- First-order transition is possible but needs fine-tuning.
- Quarkyonic-like models are more natural to explain all.
- Thank you!

Backup Slides

Why electron-weak probe?



- $\mathscr{L} = \ldots + \frac{g_W}{\cos(\Theta_W)} J_Z^{\mu} Z_{\mu} - \frac{M_Z^2}{2} Z^{\mu} Z_{\mu} + \ldots$ $\quad G_F = \frac{g_W^2}{4\sqrt{2}M_W^2}$

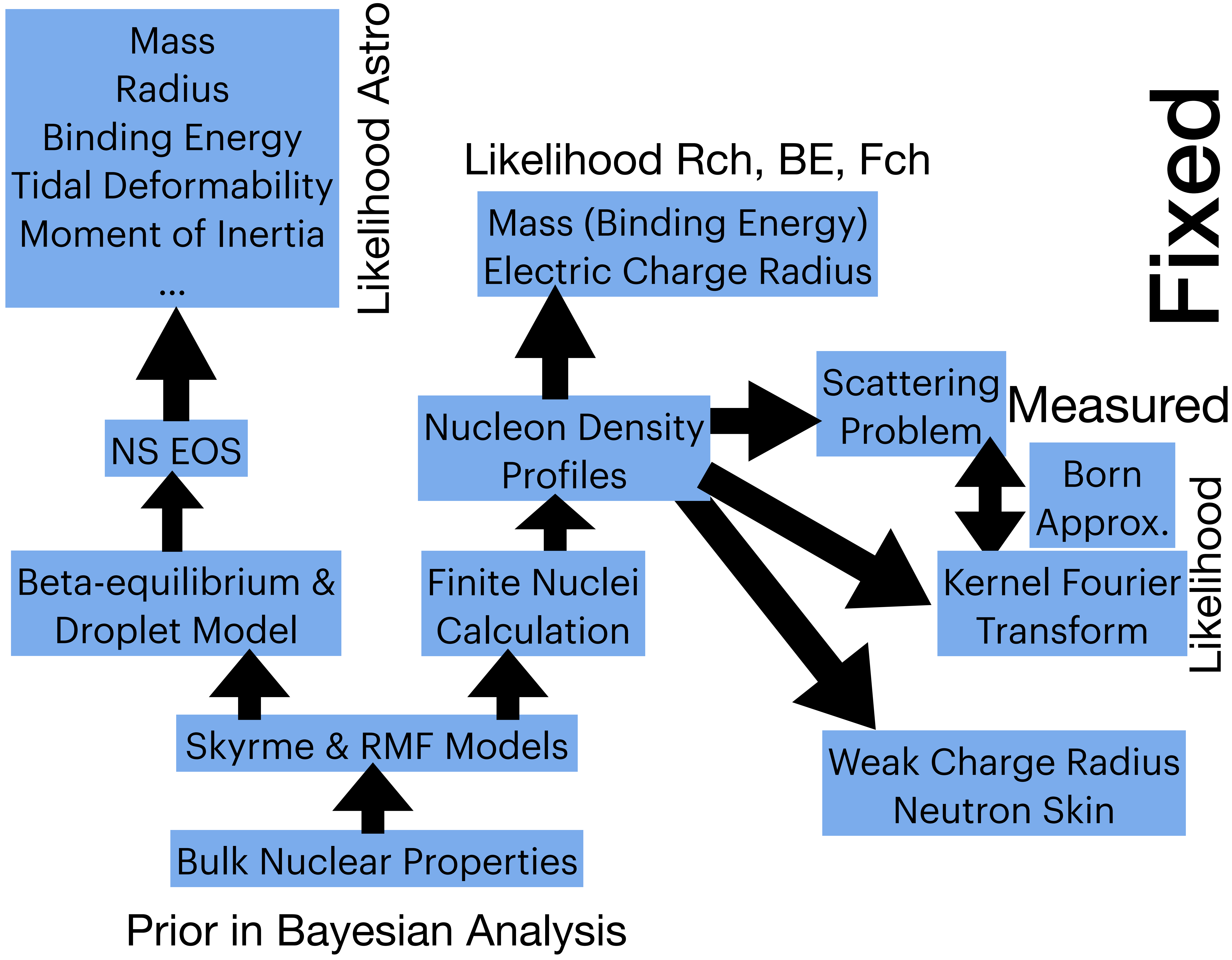
  $\sin^2(\Theta_W) = 0.223$
- The only source of parity violation:

  $$J_Z^{\mu} = -\frac{1}{2}\bar{\psi}_L\gamma^{\mu}\psi_L - \sin^2(\Theta)\bar{\psi}\gamma^{\mu}\psi = -\frac{1}{4}\bar{\psi}\left[1 - 4\sin^2(\Theta_W) - \gamma^5\right]\psi$$
- Approximately zero-range, since $M_Z \approx 500 \text{ fm}^{-1}$:

  $$\Phi_W(r) = \int \frac{\rho_W(r')e^{-M_Z|r-r'|}}{4\pi|r-r'|}dr'^3 \approx \frac{\rho_W(r')}{M_Z^2}$$
- Weak compared with Coulomb interaction: $\Phi_W << \Phi_E$
- $Q_p = 1 - 4\sin^2(\Theta_W)$ << $Q_n = -1$

  (0.0721 with correction) (–0.9878 with radiative correction)

Flowchart of Applying PREX and CREX Data



Mass
Radius
Binding Energy
Tidal Deformability
Moment of Inertia
…

Likelihood Astro

Likelihood Rch, BE, Fch

Mass (Binding Energy)
Electric Charge Radius

NS EOS

Nucleon Density Profiles

Scattering Problem

Measured

Born Approx.

Beta-equilibrium & Droplet Model

Finite Nuclei Calculation

Kernel Fourier Transform

Likelihood

Skyrme & RMF Models

Weak Charge Radius
Neutron Skin

Bulk Nuclear Properties

Prior in Bayesian Analysis

Fixed

PREX & CREX

	CREX	PREX
(N,Z)	(28,20) Ca	(126,82) Pb
q (fm-1)	0.8733	0.3977
Fch, Rch(fm)	0.1581, 3.481	0.409, 5.503
Apv	2668±106(stat) ±40(syst)	550±16(stat) ±8(syst)
Fw	0.1304±0.0052(stat) ±0.002(syst)	0.368±0.013(exp) ±0.001(theo)
Fch-Fw	0.0277±0.0052(stat) ±0.002(syst)	0.041±0.013(exp) ±0.001(theo)
Rw	3.64±0.026(exp) ±0.023(theo)	5.8±0.075(tot)
Rw-Rch	0.159±0.026(exp) ±0.023(theo)	0.297±0.075(tot)
Rn-Rp	0.121±0.026(exp) ±0.024(theo)	0.283±0.071(tot)

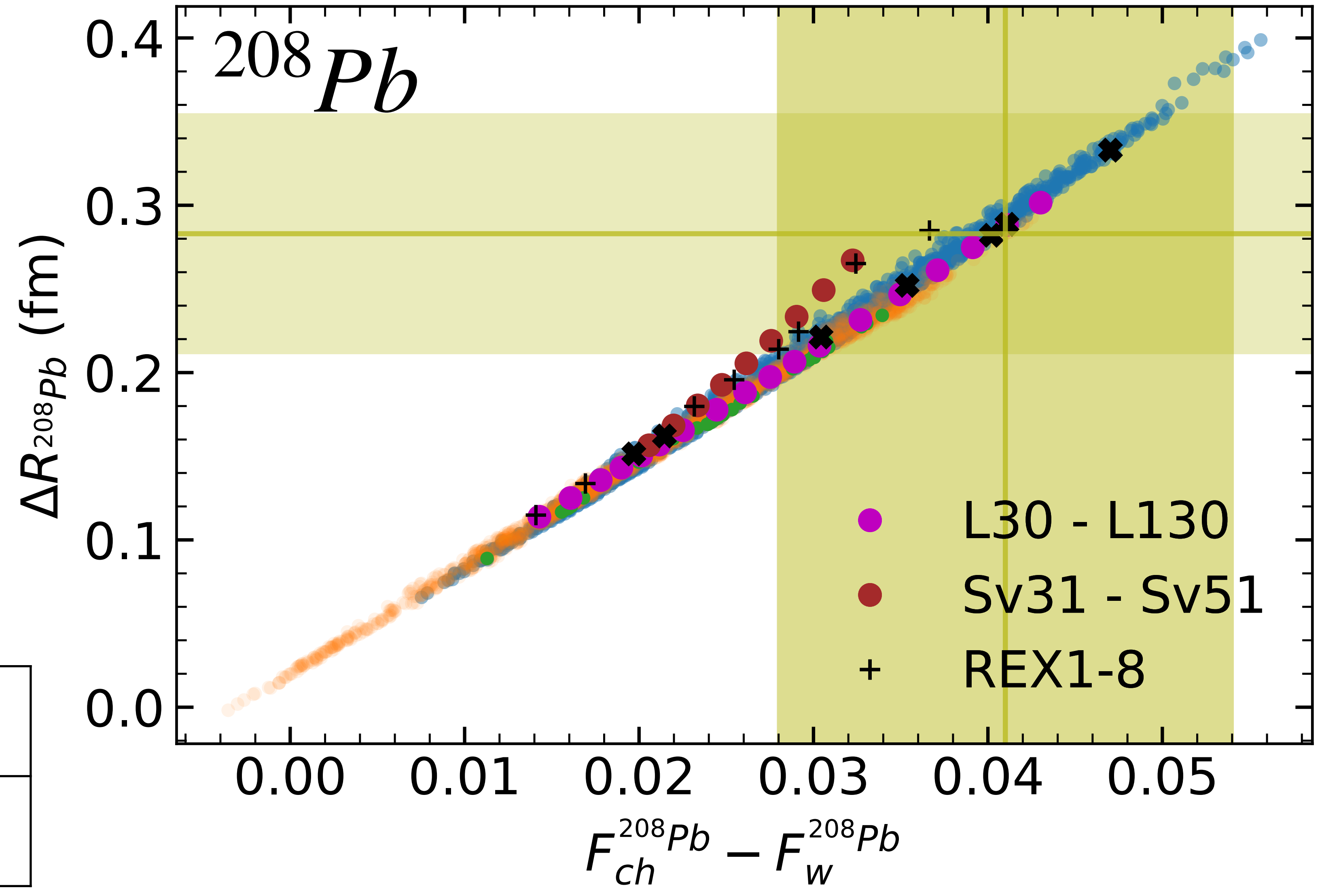
CREX 2022 PREX I 2012 PREX II 2021

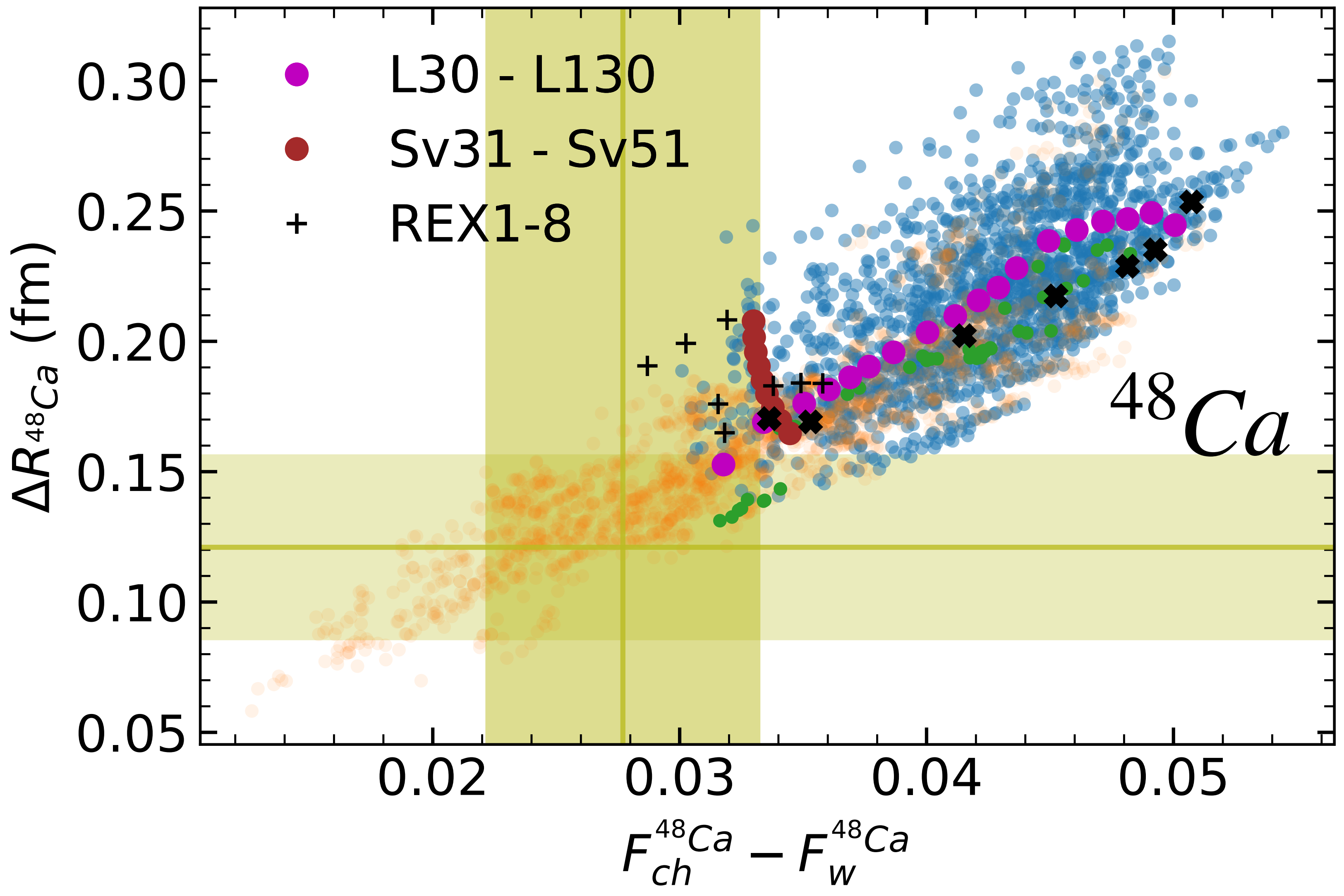
Can CREX Measure Neutron skin?

- RMS radius: $< r^2 > = \frac{1}{Q}\int r^2\rho(\mathbf{r})d^3r$
- Form Factor: $F(\mathbf{q}) = \frac{1}{Q}\int e^{i\mathbf{q}\cdot\mathbf{r}}\rho(\mathbf{r})d^3r = 1 - \frac{1}{6}q^2 < r^2 > + \dots$
- $\lim\limits_{q<<1/\sqrt{< r^2 >}} < r^2 > = \frac{6[1 - F(q)]}{q^2}$

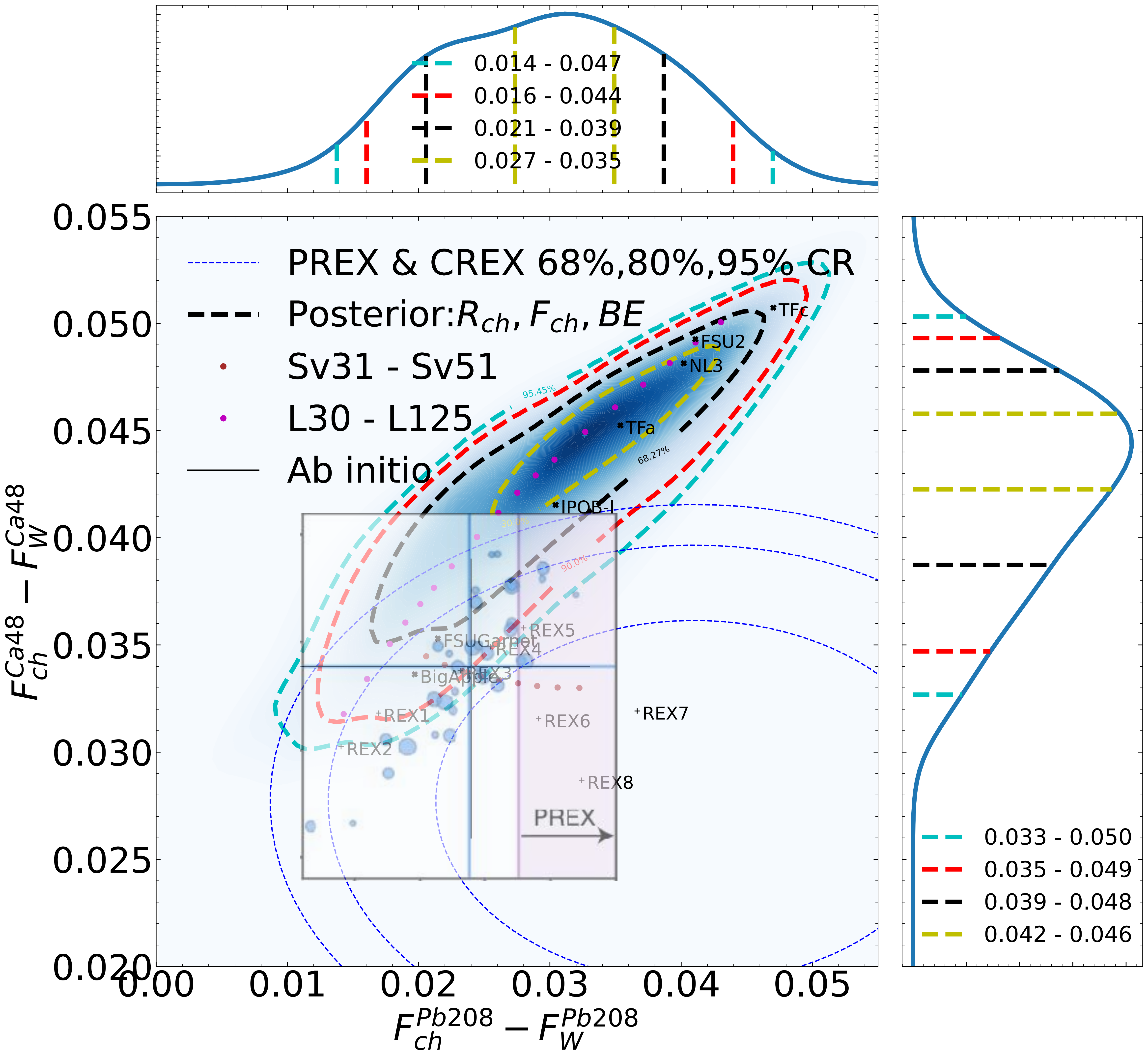
208Pb	1	-0.8	0.19
48Ca	1	-1.56	0.73

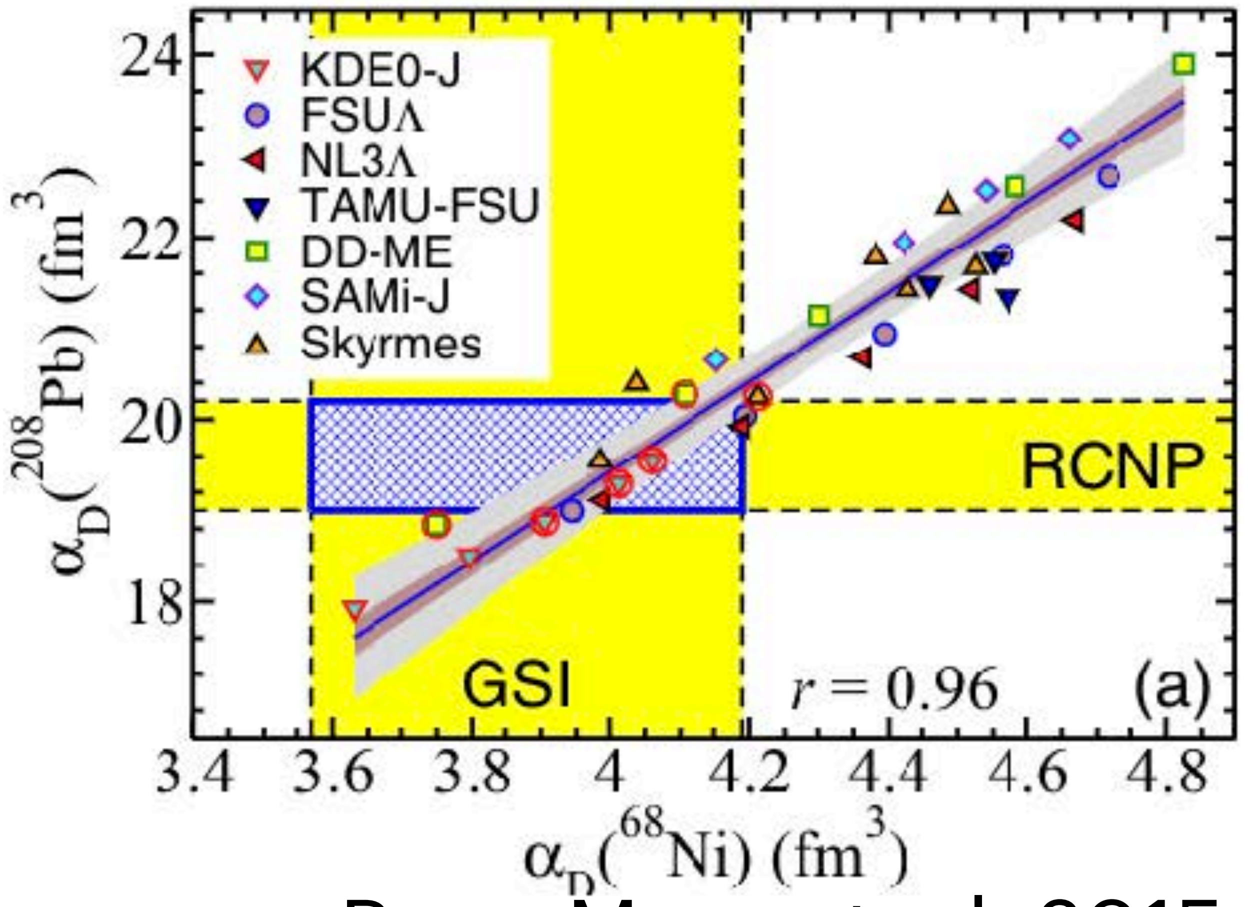
- Form factor to radius mapping is much less accurate for CREX:
  1. Momentum transfer q is a bit too large for CREX.
  2. MFT uncertainty increase for lighter nuclei.
- Better use form factor to constrain nuclear model.





How Much Tension?





Roca-Maza et. al. 2015

- Confidence discrepancy:

$$P(\Delta \vec{x}) = \int P_{data}(\vec{x}) P_{posterior}(\vec{x} + \Delta \vec{x}) d^n \vec{x}$$

$$P_{disagree} = \int_{P(\Delta \vec{x}) > P(0)} P(\Delta \vec{x}) d^n \Delta \vec{x}$$

Sigma Tension	Rch, BE, Fch	+PREX
Rch, BE, Fch	1.98 (95.2%)	2.12 (96.6%)
+CREX	1.59 (88.8%)	1.52 (87.2%)

- Consistent with Ab-initio calculation:

$$z = M(\theta) + \varepsilon_{\mathrm{exp}} + \varepsilon_{\mathrm{em}} + \varepsilon_{\mathrm{method}} + \varepsilon_{\mathrm{model}},$$

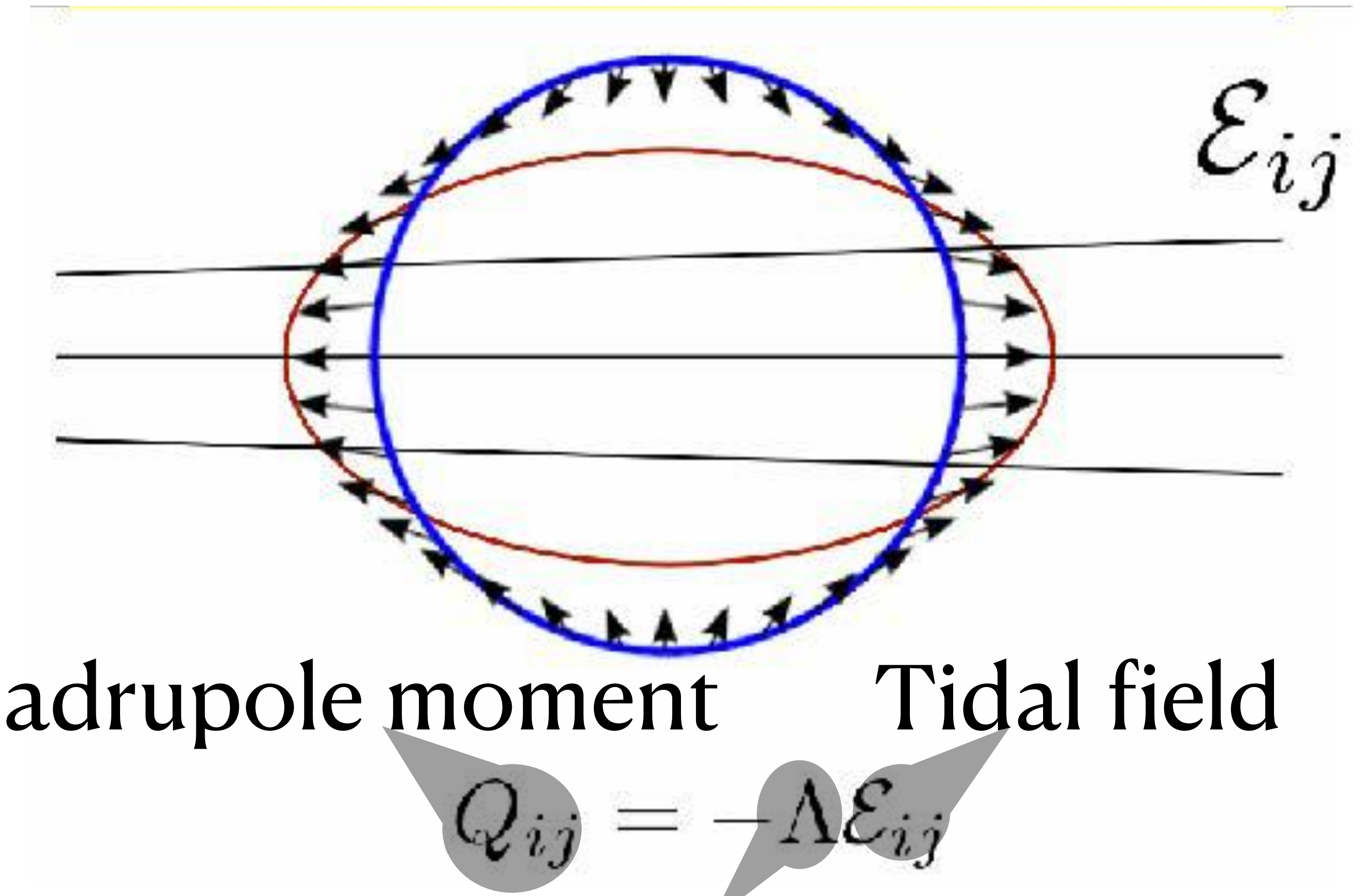
IMSRG, MBPT and CC calculation of 48Ca and 208Pb.

Hu et. al. 2022

Regge-Wheeler metric

Perturbation on Schwarzschild metric

- $$\begin{aligned} ds^2 = &- e^{2\Phi(r)}(1 + H_0(r)e^{i\omega t}Y_{lm}(\phi, \theta))\cancel{c^2}dt^2 \\ &+e^{2\Lambda(r)}(1 - H_0(r)e^{i\omega t}Y_{lm}(\phi, \cancel{\theta}))dr^2 \\ &-2i\omega r^{l+1}H_1(r)e^{i\omega t}dtdr \\ &+(1 - K(r)e^{i\omega t}Y_{lm}(\phi, \cancel{\theta}))r^2d\Omega^2 \end{aligned}$$



Quadrupole moment Tidal field

$Q_{ij} = -\Lambda\mathcal{E}_{ij}$

Tidal deformability

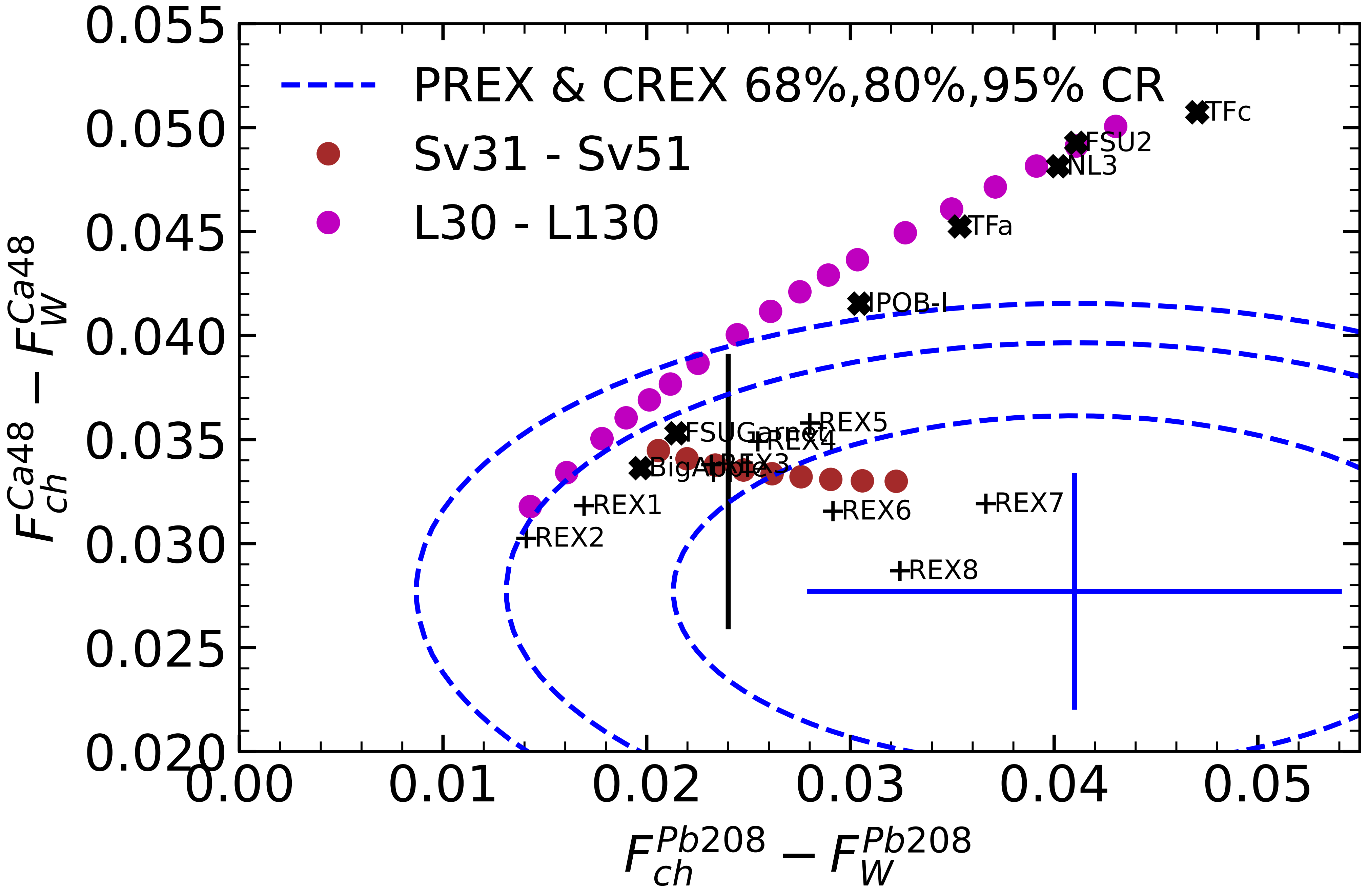
- ODE for Static tidal deformation : $\omega = 0,\ l = 2,\ m = 0$
  use g_{00} term to match Newtonian limit at large r,
  $$-\frac{1 + g_{00}}{2} = \frac{1}{2}\varepsilon_{ij}x^ix^j - \frac{M}{r} - \frac{3}{2}\frac{Q_{ij}x^ix^j}{r^5} + \sum C_nr^n \quad (n \neq 2, -1, -3)$$

Newtonian potential of deformed star

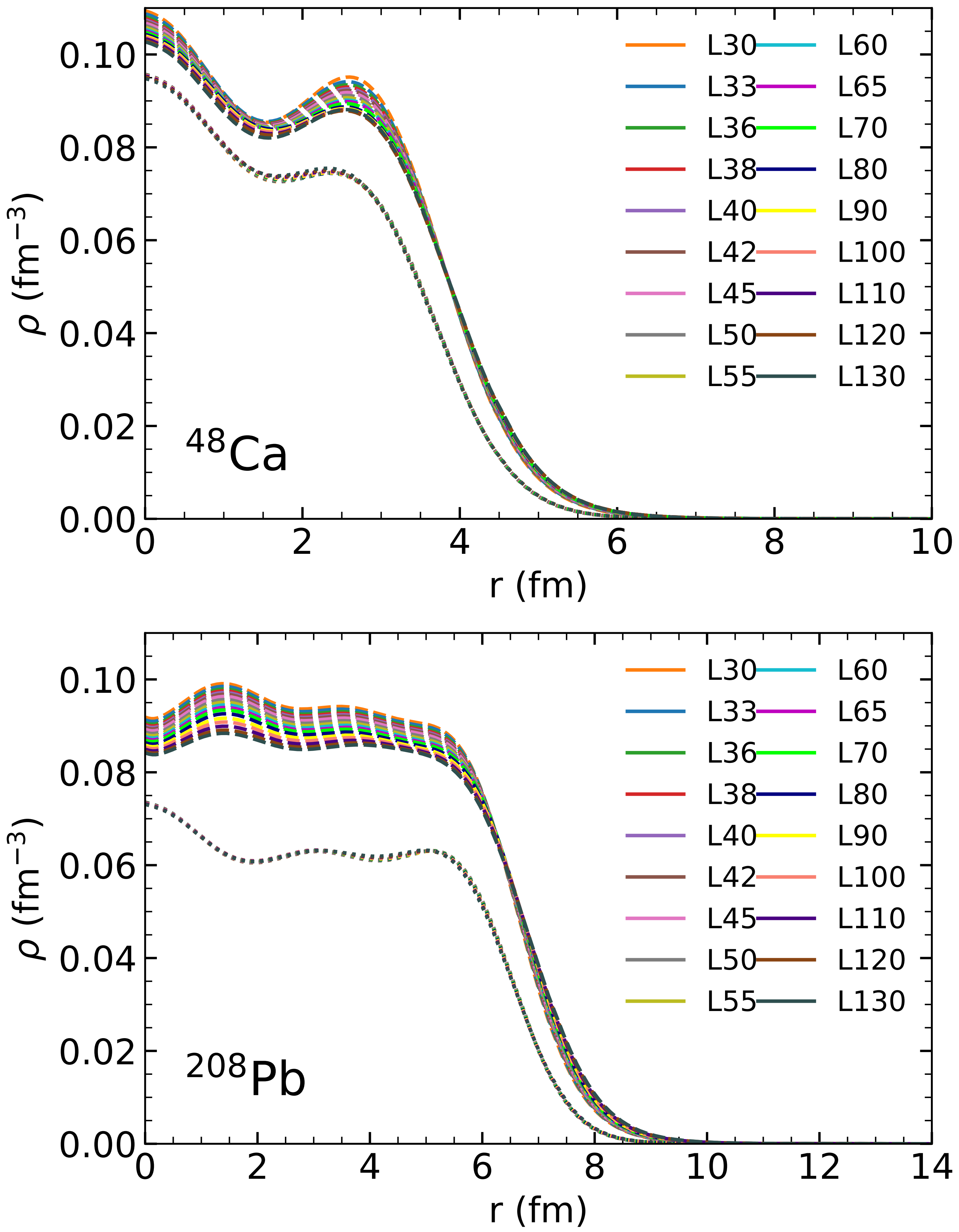
Attempts to Solve the Tension

Density dependence of symmetry energy:

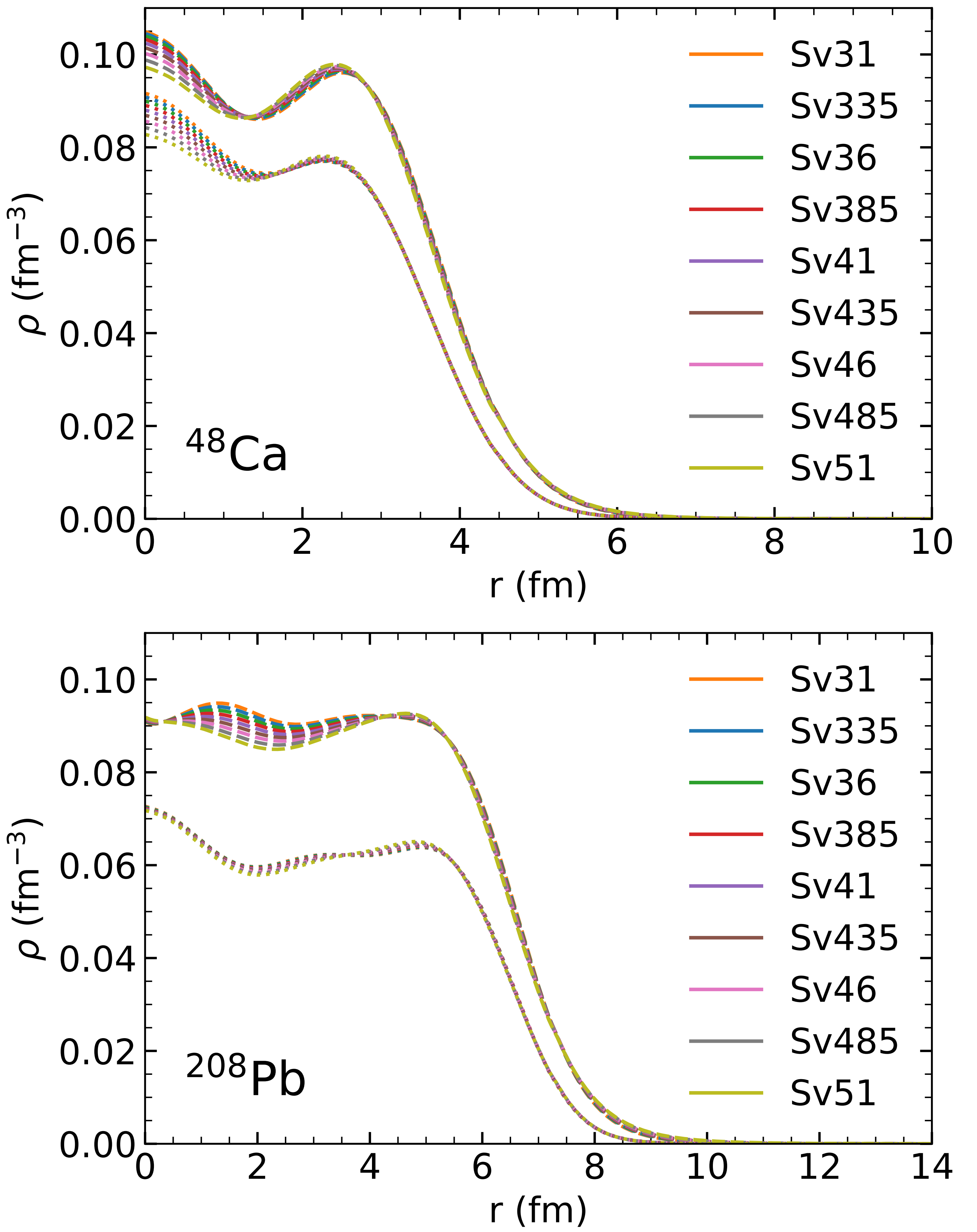
- L30-L130 are traditional
- Sv31-Sv51 are unusual



‘bulk’ density variation



‘localized’ density variation



Pearson Correlation

$$\mathrm{cov}[\mathrm{X}, \mathrm{p}(\mathrm{n_B}), \mathrm{P}] = \sum_{\mathrm{i}} \mathrm{P_i} \frac{(\mathrm{X_i} - \bar{\mathrm{X}})(\mathrm{p_i} - \bar{\mathrm{p}})}{\sigma_{\mathrm{X}}\sigma_{\mathrm{p}}}$$

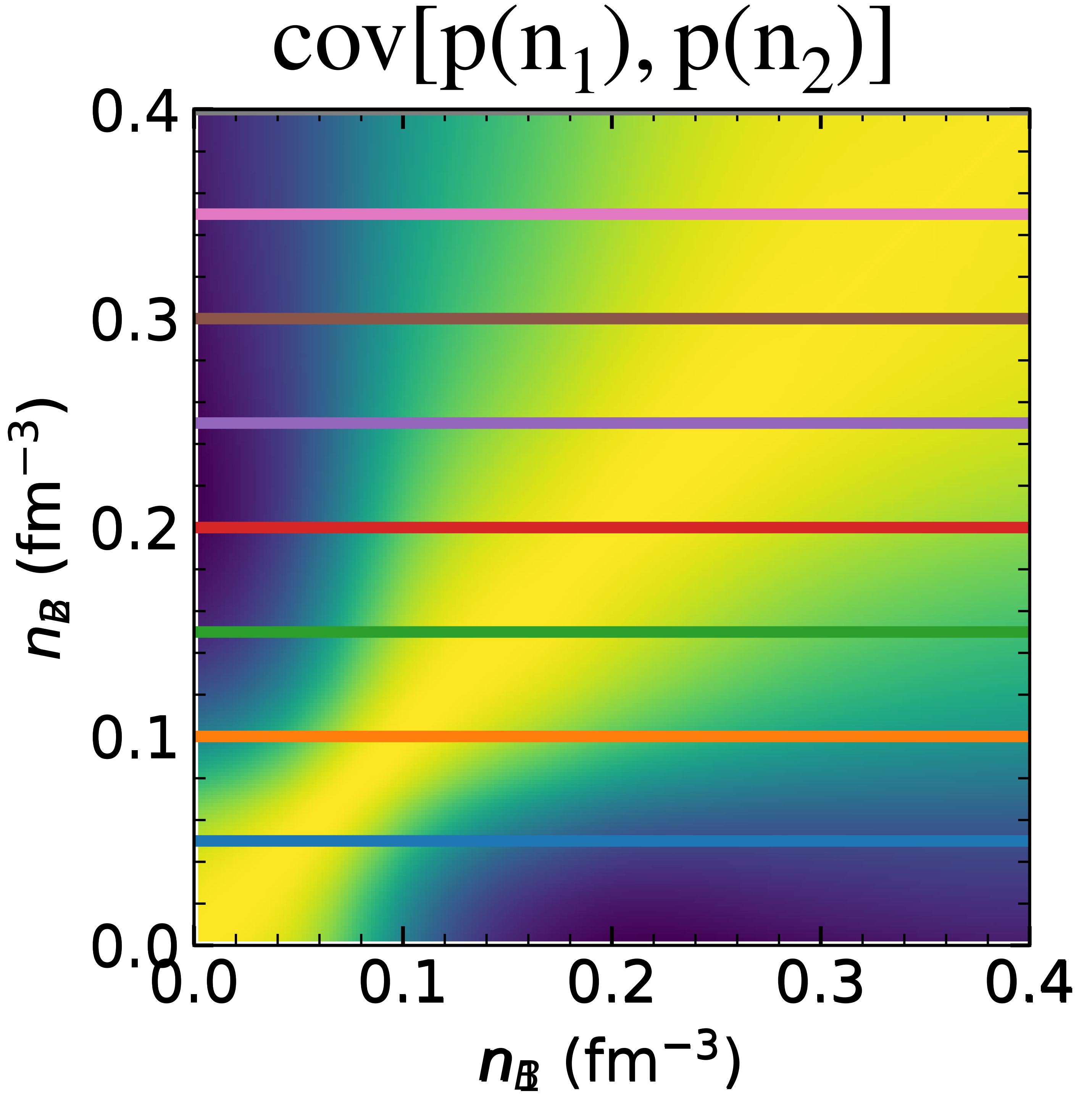
Observable

Pressure at given density

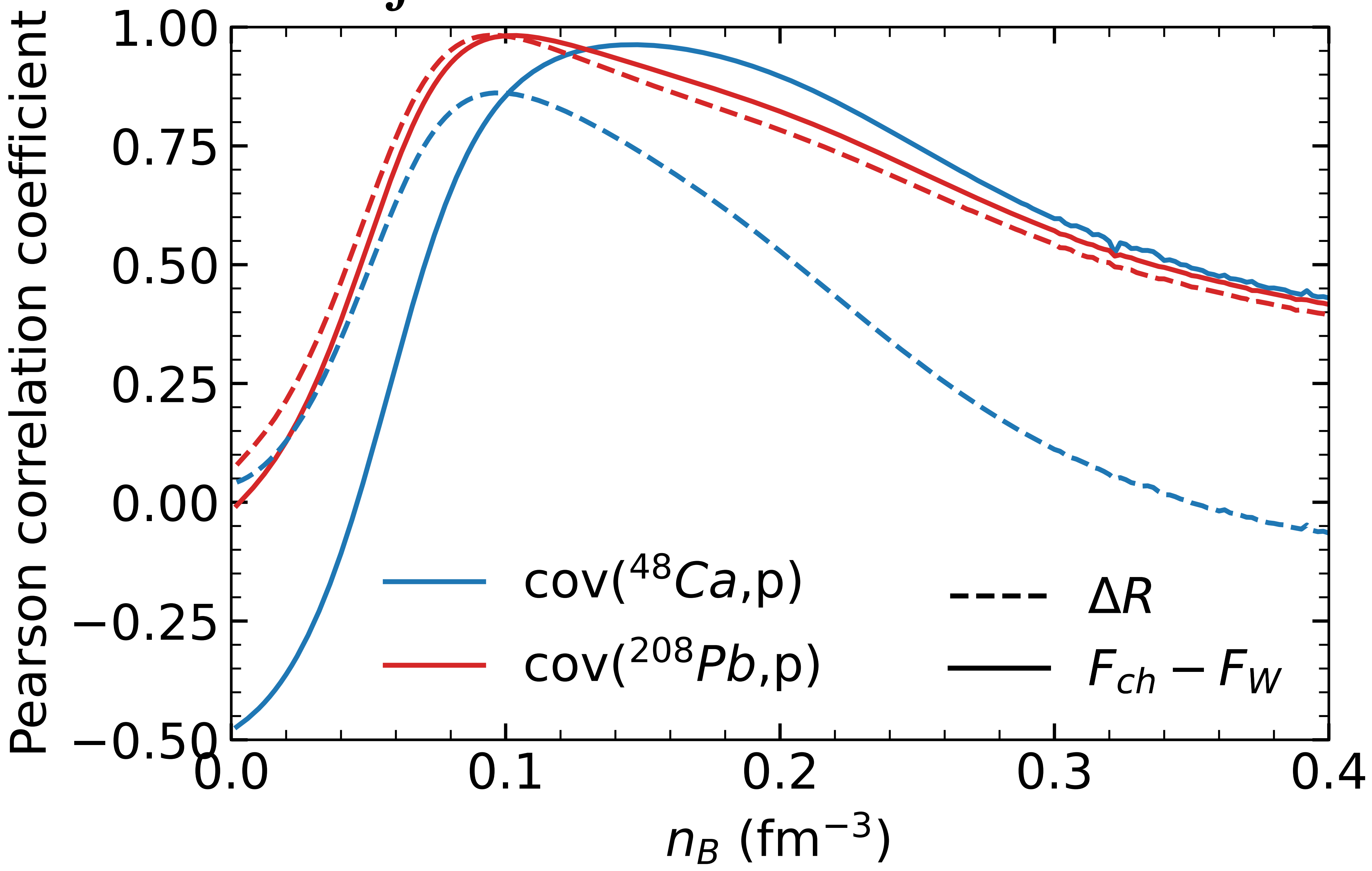
Likelihood

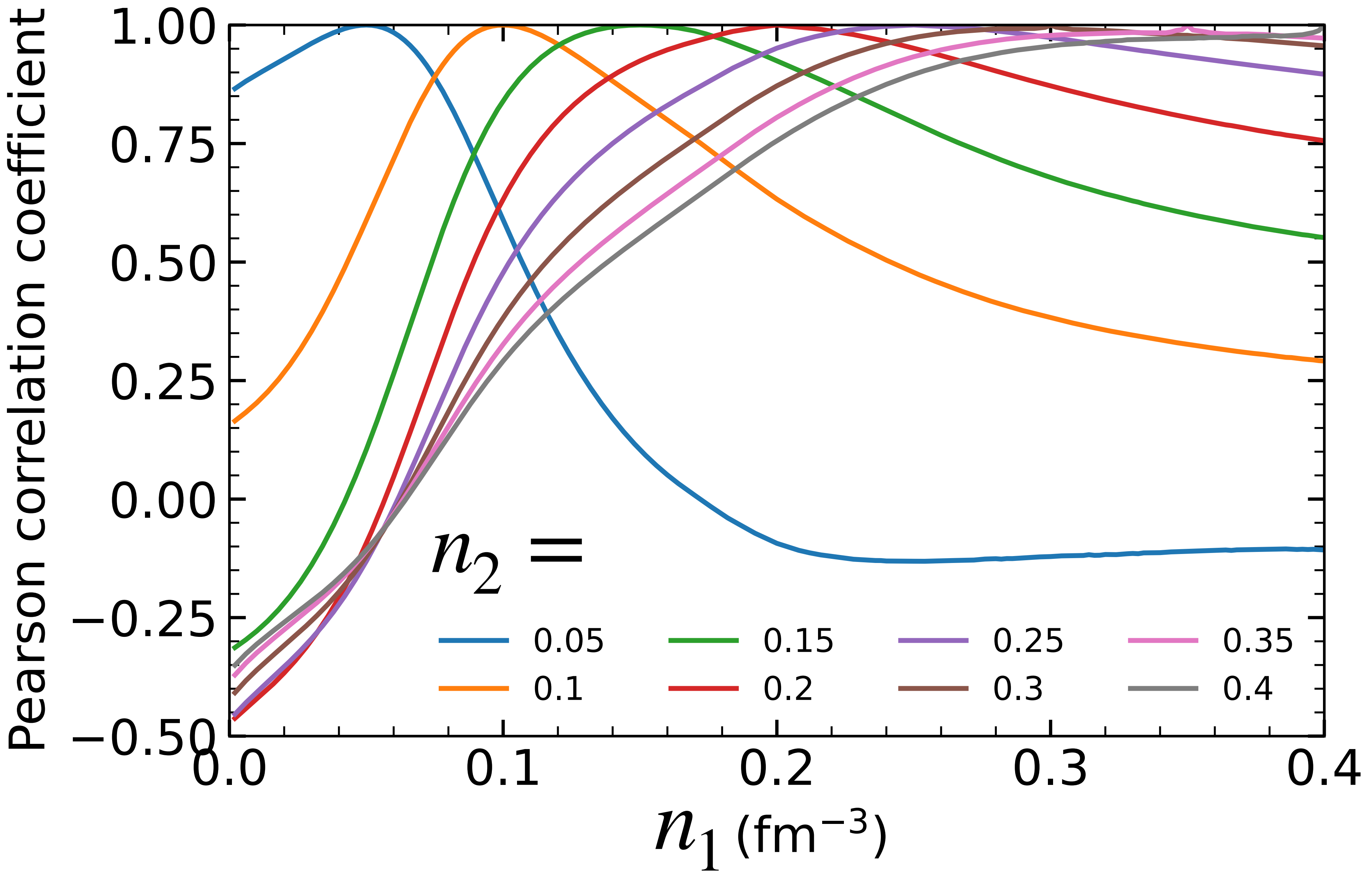
$$S(X, n_1) = \frac{r_X}{\sqrt{2\pi}\sigma_X} \exp[-\frac{(n_1 - \mu_X)^2}{2\sigma_X^2}]$$

[fm]	Fch-Fw	Skin	Fch-Fw	Skin
μ_X	0.147	0.0932	0.121	0.113
σ_X	6E-04	7E-04	0.0522	0.0616
r_X	0.00214	0.00197	0.00250	0.00268



$$\mathrm{cov}[\mathrm{X}, \mathrm{p}(\mathrm{n_2})] = \int \mathrm{cov}[\mathrm{p}(\mathrm{n_1}), \mathrm{p}(\mathrm{n_2})] S(\mathrm{X}, \mathrm{n_1}) d\mathrm{n_1}$$

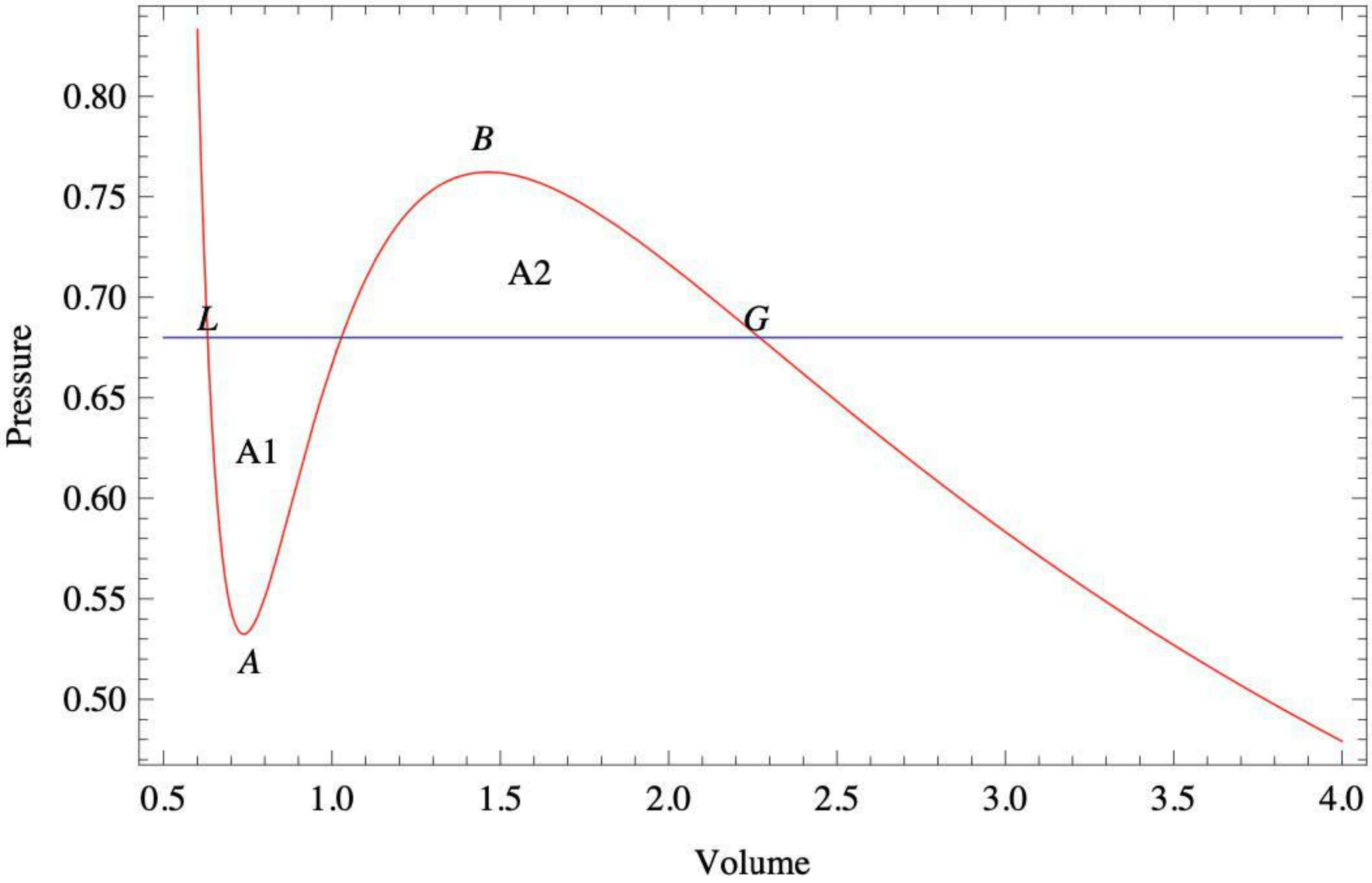




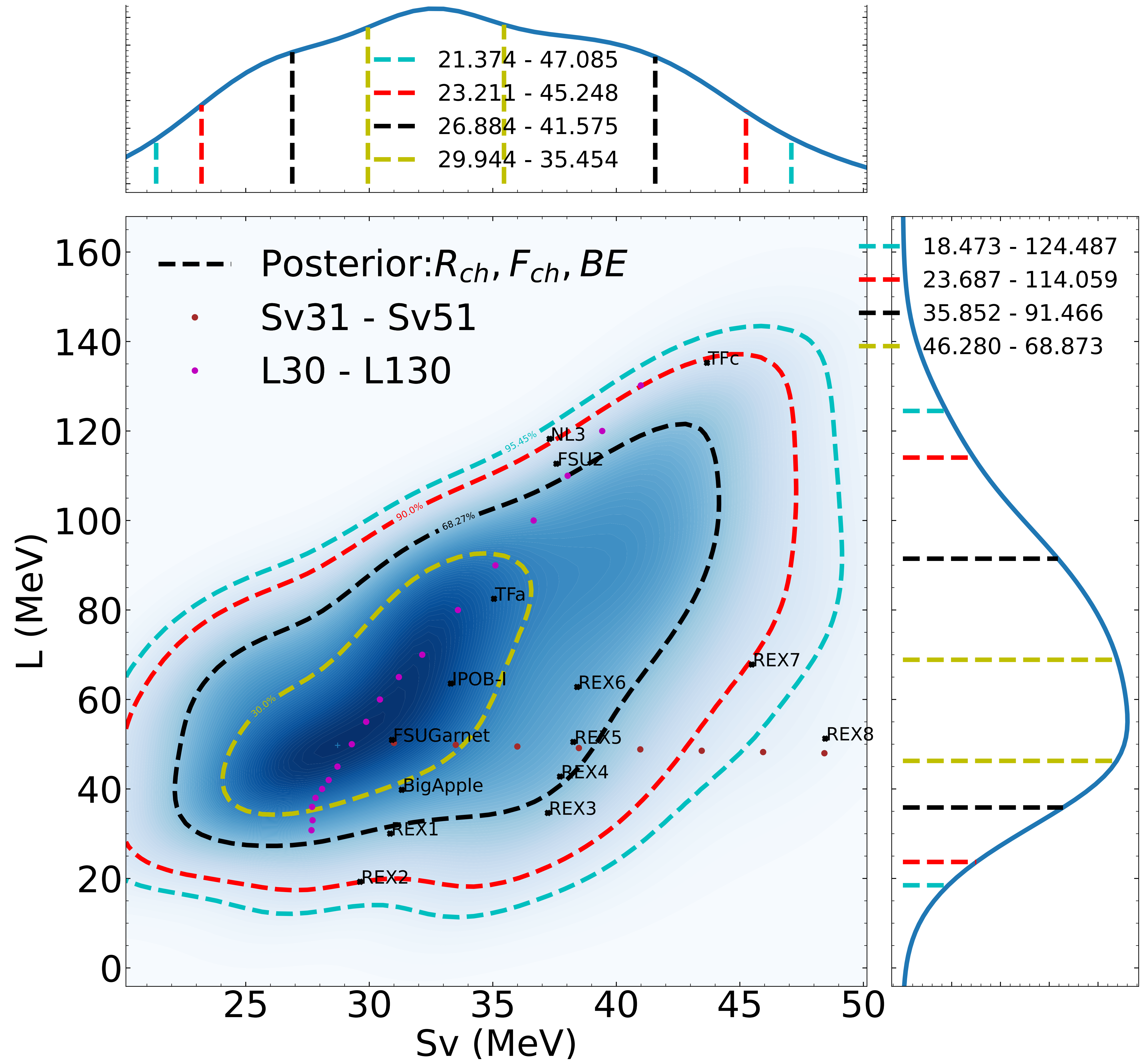
Conclusion

- There's 1 to 2 sigma tension between PREX + CREX and the model constrained by nuclei.
- The mild tension helps constrain density dependence of symmetry energy
- CREX indeed provides more information than PREX.
- PREX + CREX is consistent with ab-initio calculation, dipole polarizability, and other experiments regarding symmetry energy.
- Astronomical observation has very limited constraints on EOS below saturation.
- Absent of a direct Urca process is supported by PREX + CREX.
- Looking forward to MREX confirming PREX.

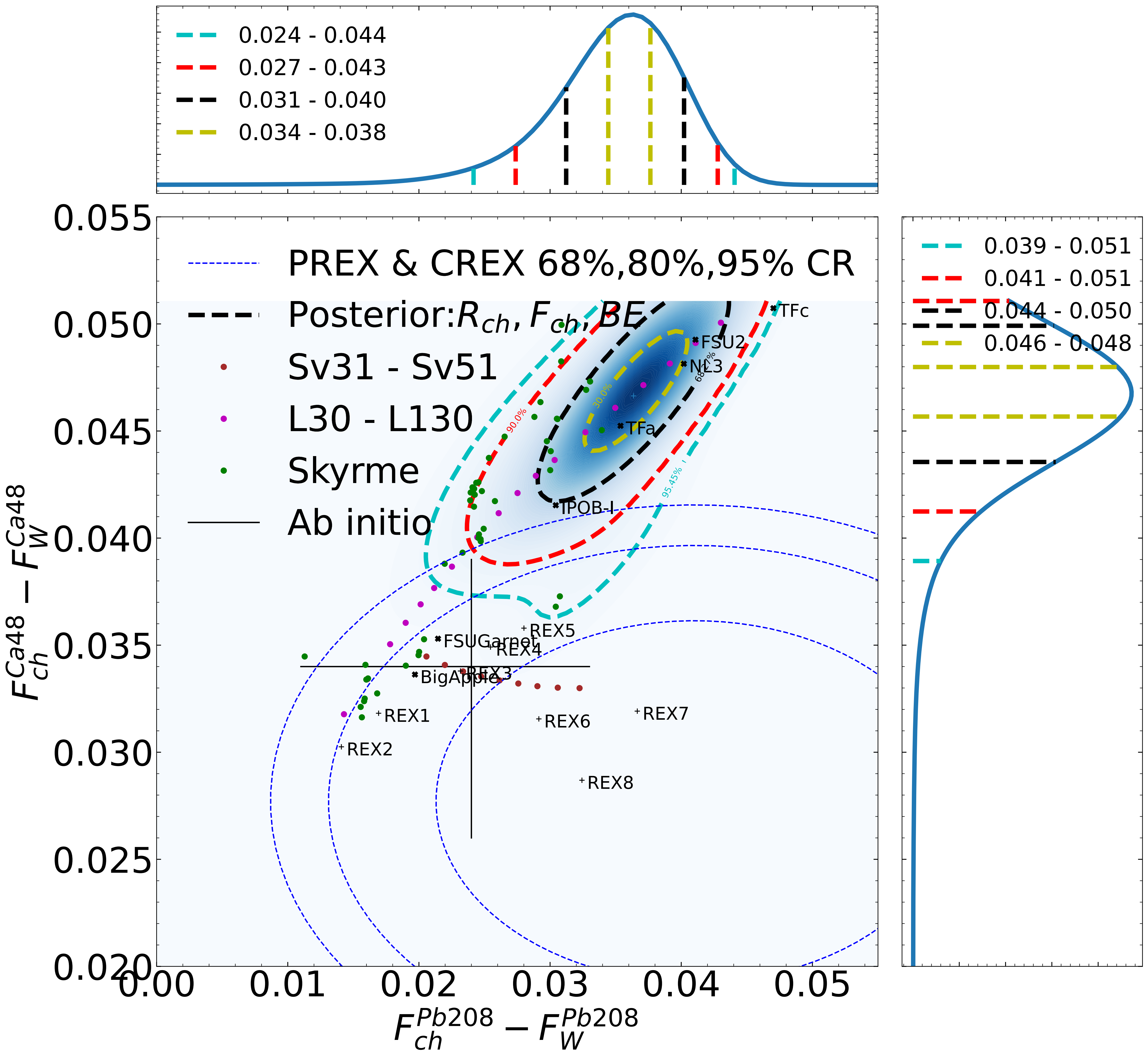
Maxwell and Gibbs in Van der Waals Gas

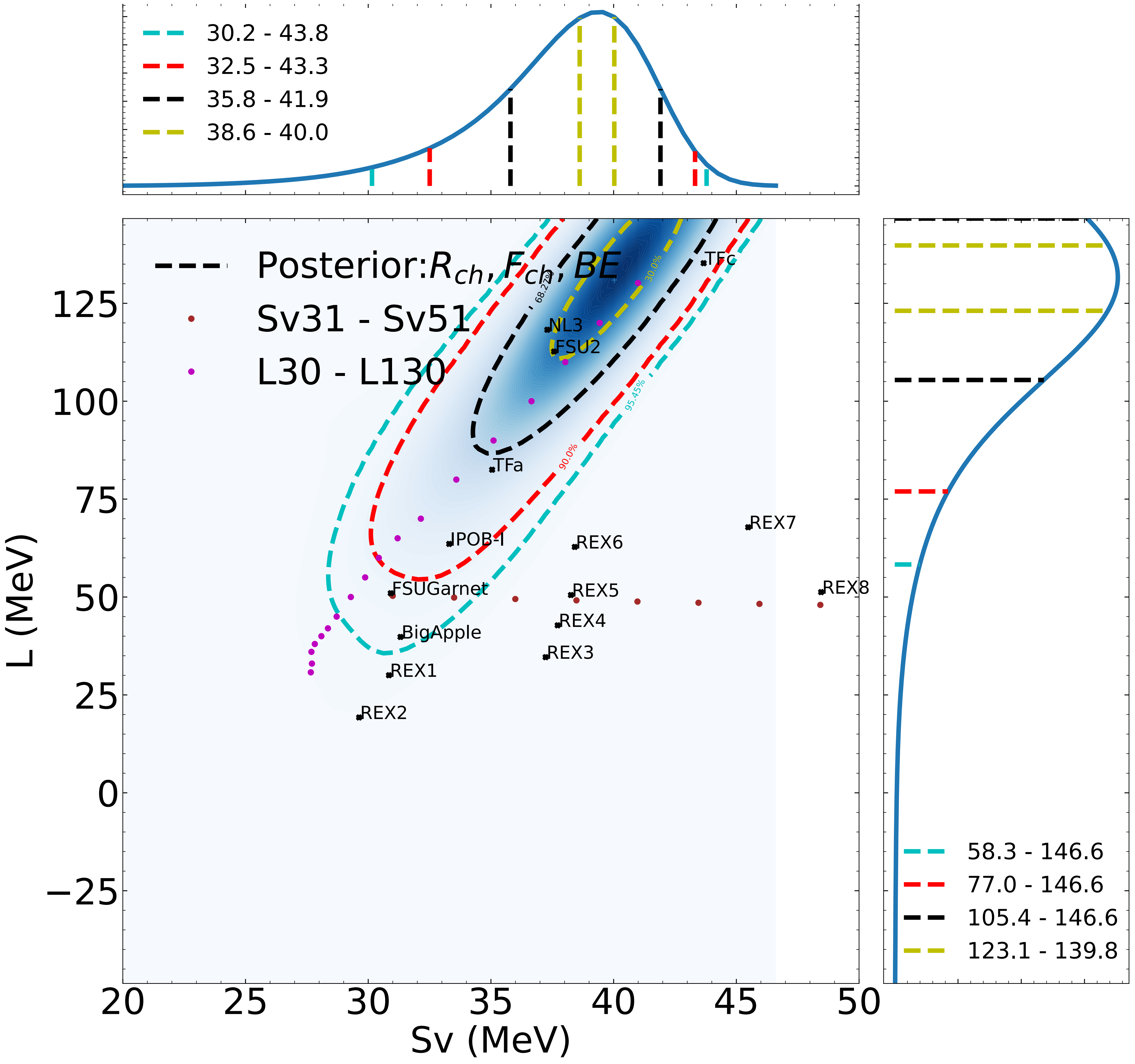


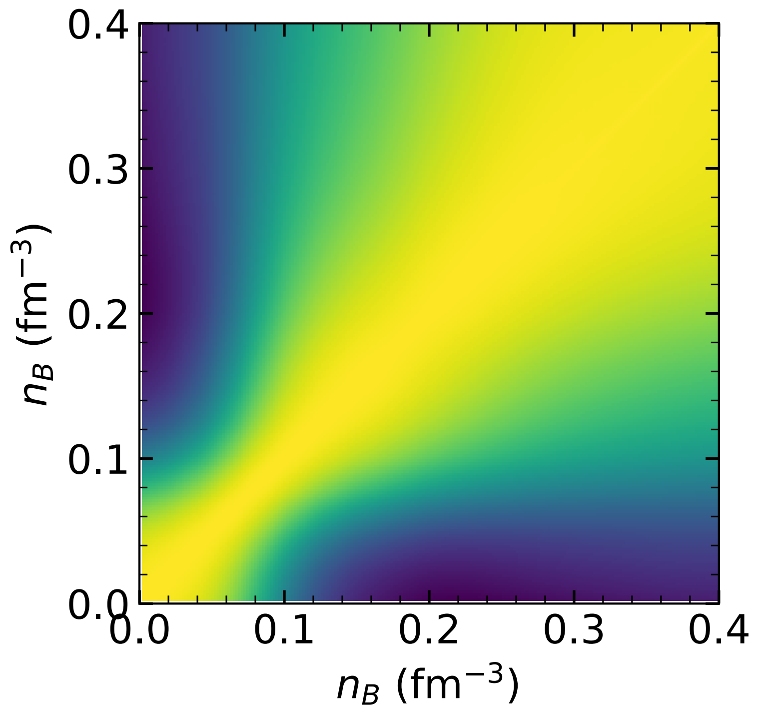
Sv-L Posterior with RMF

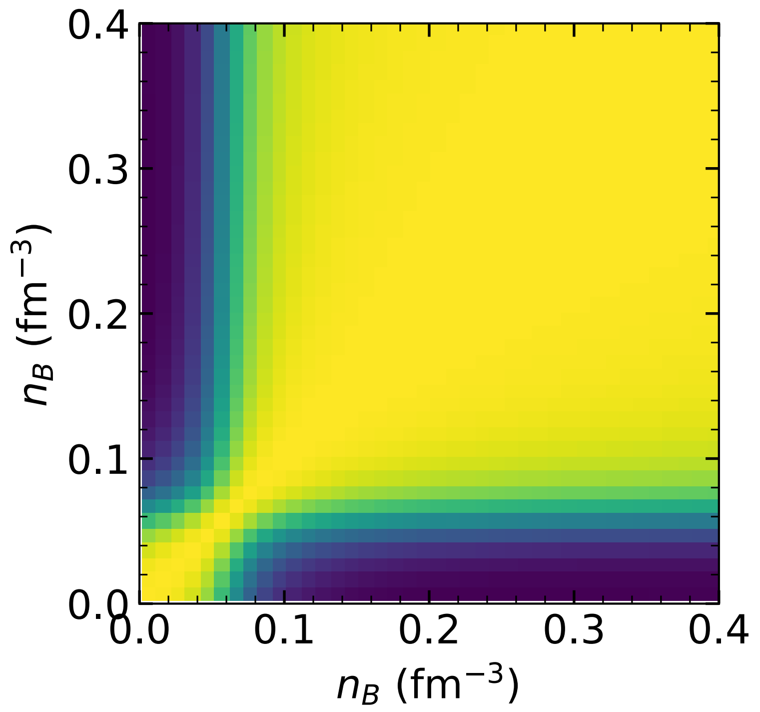


Skyrme Result

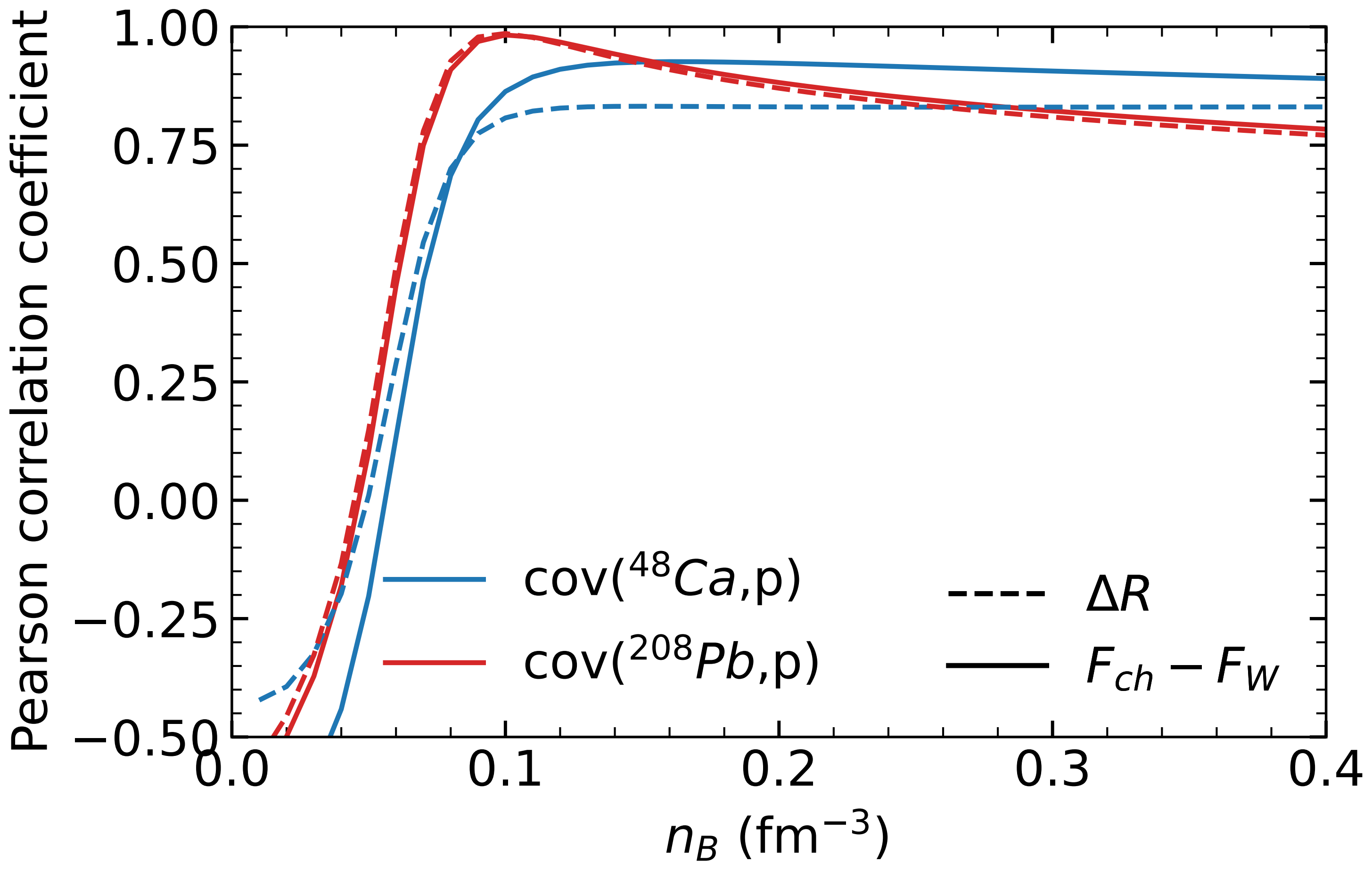


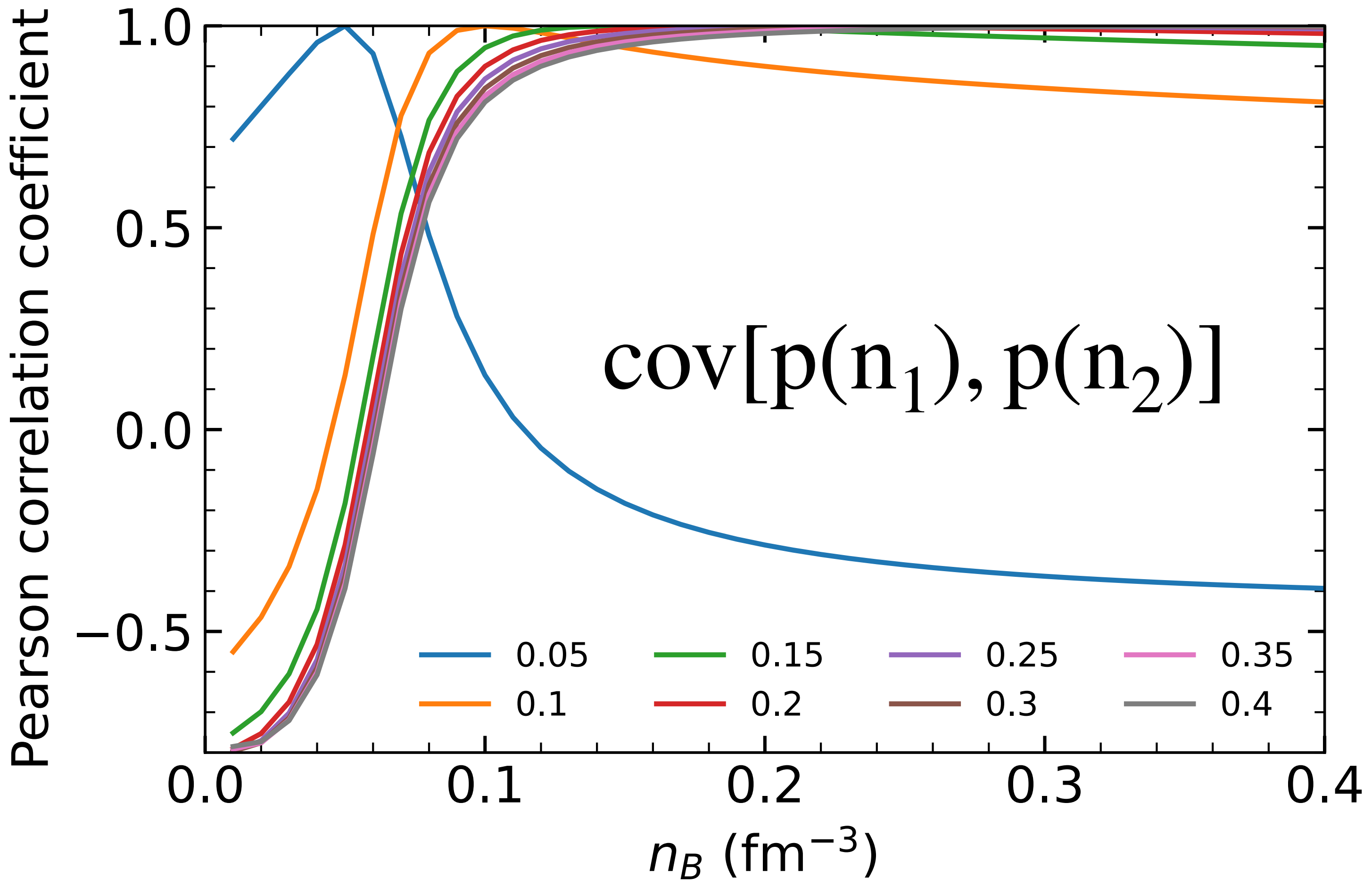




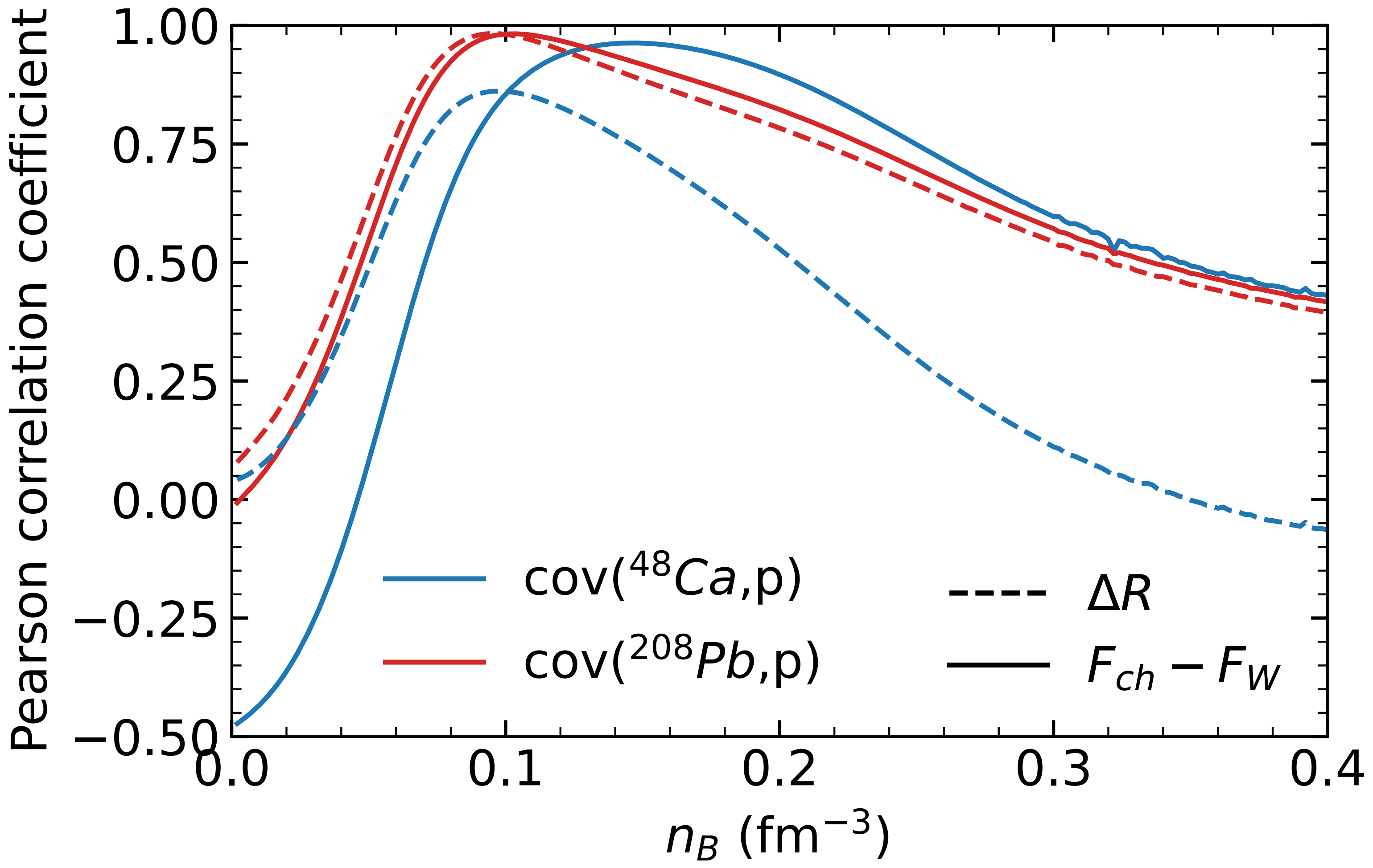


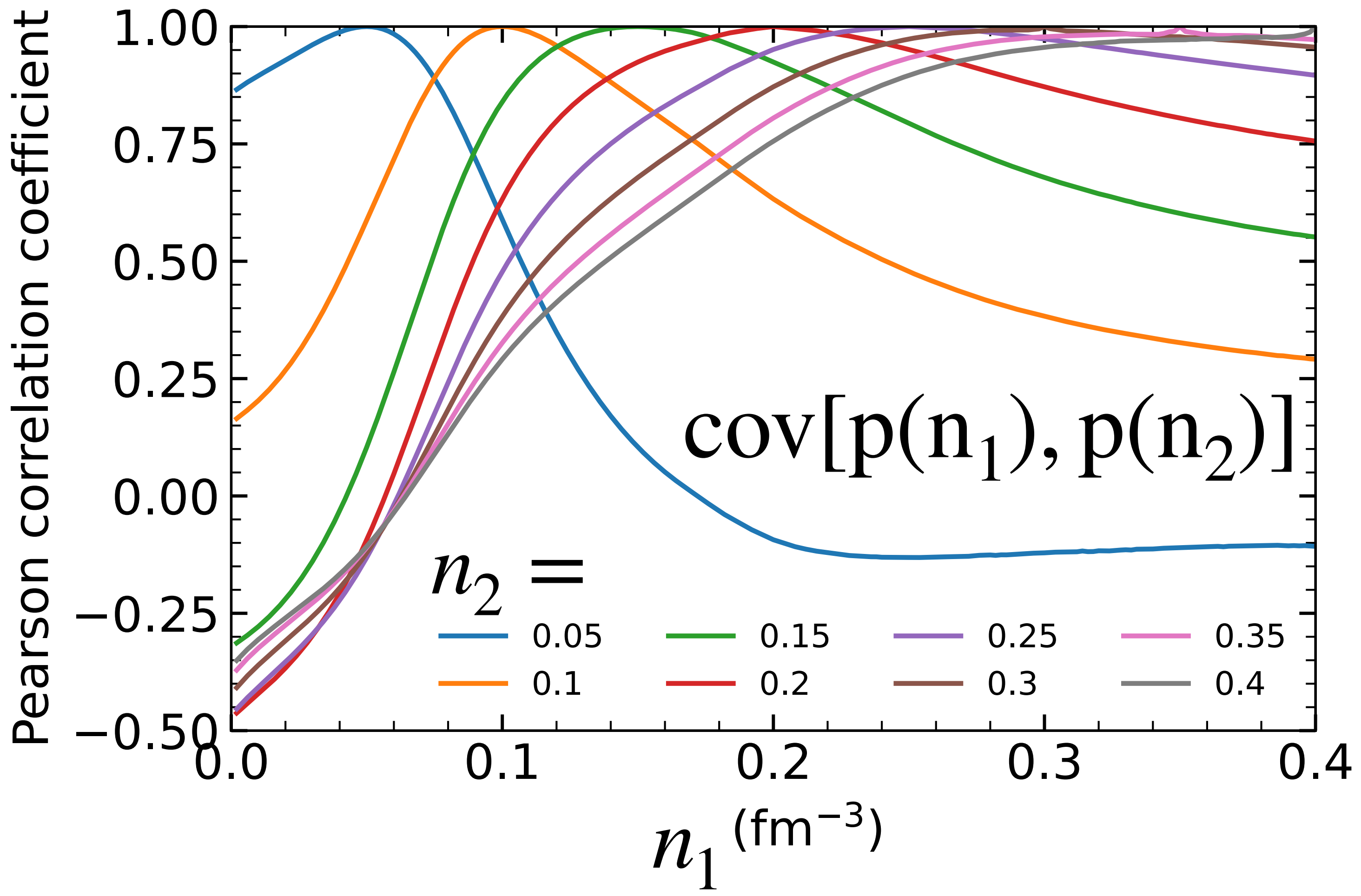
Skyrme Result





RMF Result





Finite nuclei with MFT

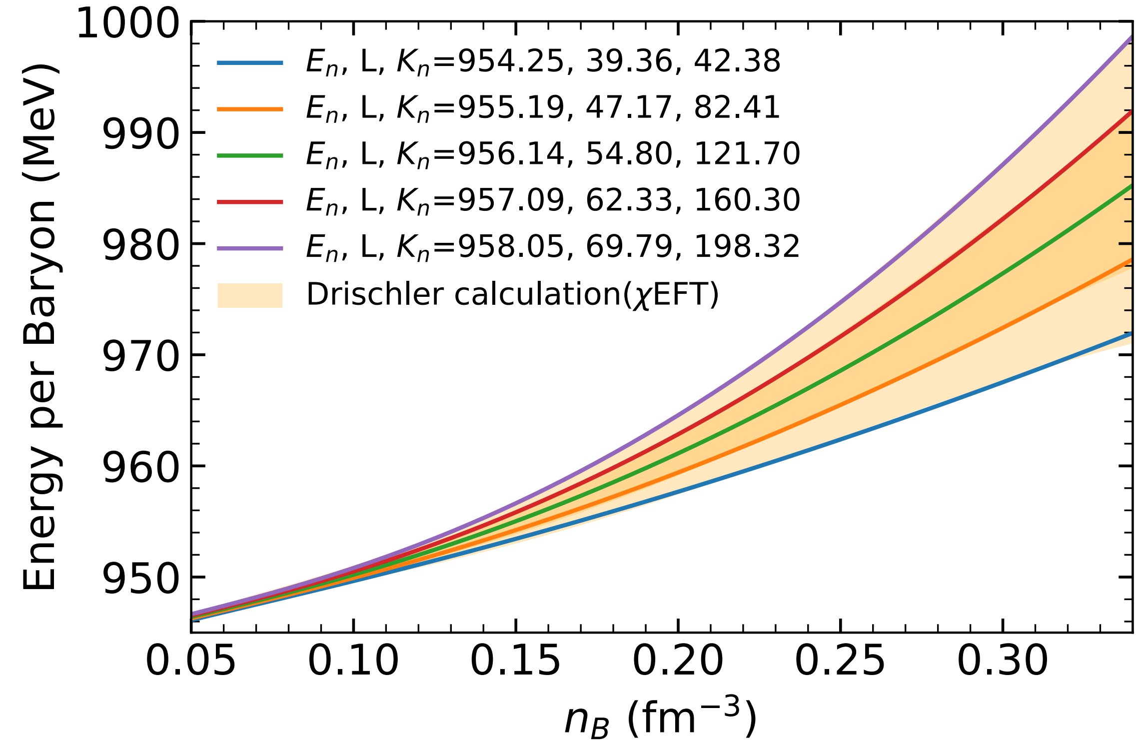
- Nucleon densities of ${}^{A}_{Z}X$:
  (1) Guess the initial 4 density profiles.
  (2) Solve Klein-Gorden Eq for 4 mesons fields.
  (3) Construct local 4 single-particle potentials.
  (4) Solve Dirac Eq for lowest Z (N) proton (neutron) levels.
  (5) Compute ground state 4 density profiles.
  (6) Repeat (2) To (5) until converge.

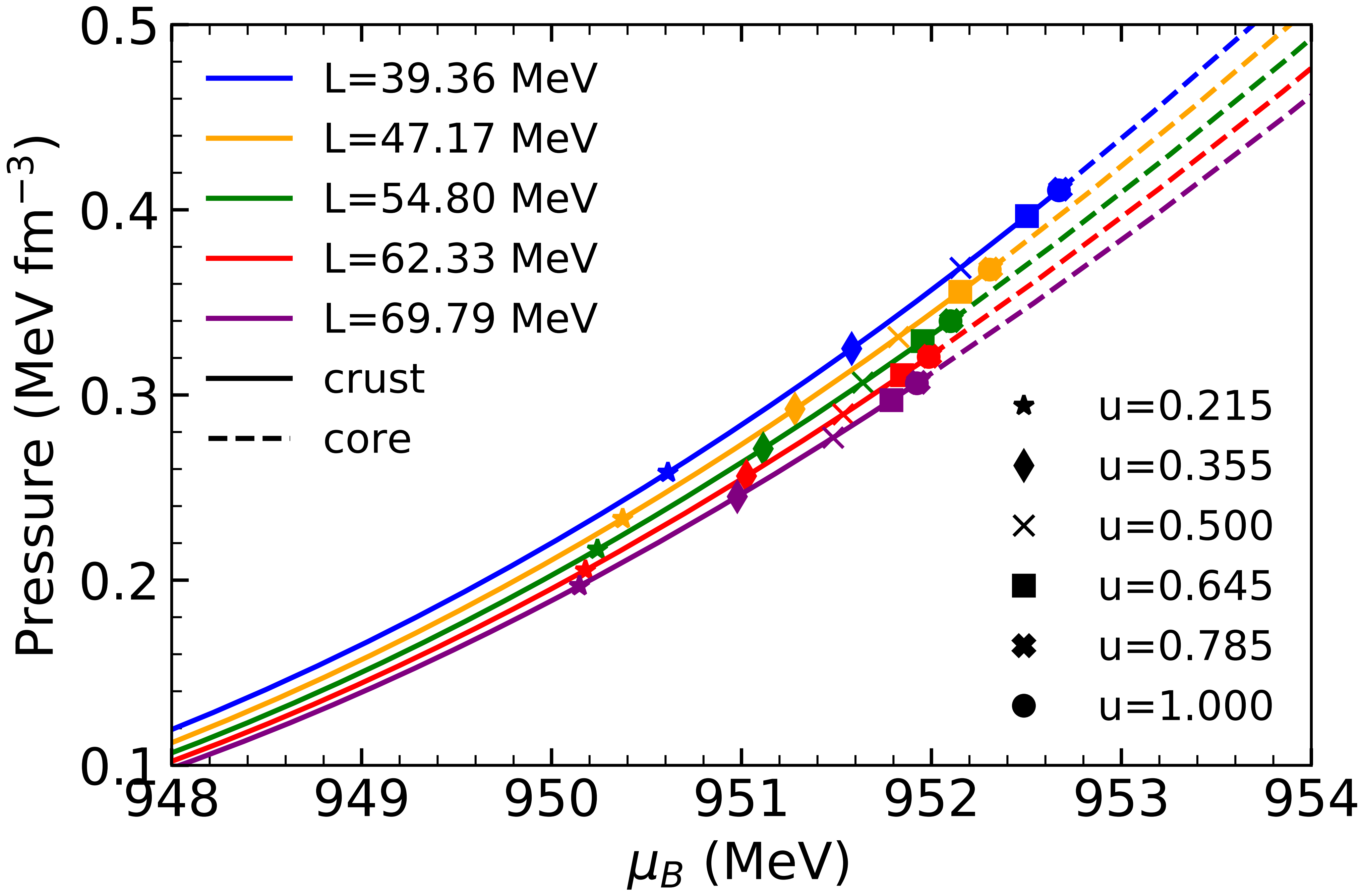
	value	σ_i
$R_{ch}^{^{48}Ca}$	3.48	0.0696
$R_{ch}^{^{90}Zr}$	4.27	0.0854
$R_{ch}^{^{208}Pb}$	5.5	0.11
$\mathrm{BE}^{^{48}Ca}$	8.67	0.1734
$\mathrm{BE}^{^{90}Zr}$	8.71	0.1742
$\mathrm{BE}^{^{208}Pb}$	7.87	0.1574
$F_{ch}^{^{48}Ca}$	0.1581	0.001
$F_{ch}^{^{208}Pb}$	0.409	0.001

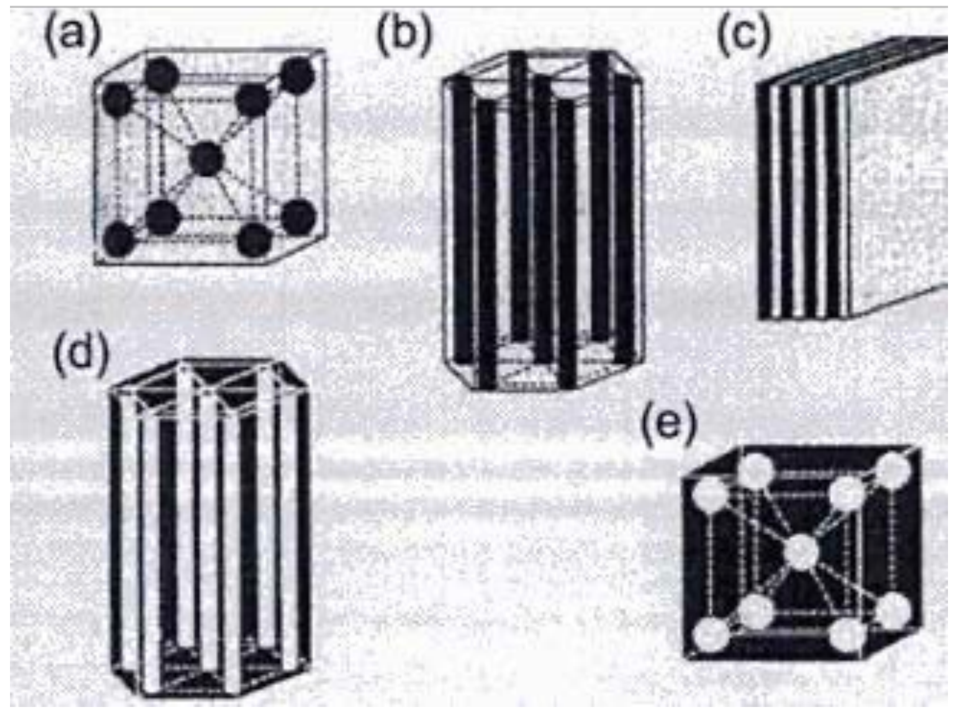
Impacts of Crust EOS and Core EOS on NS Properties

Impact on crust-core transition density

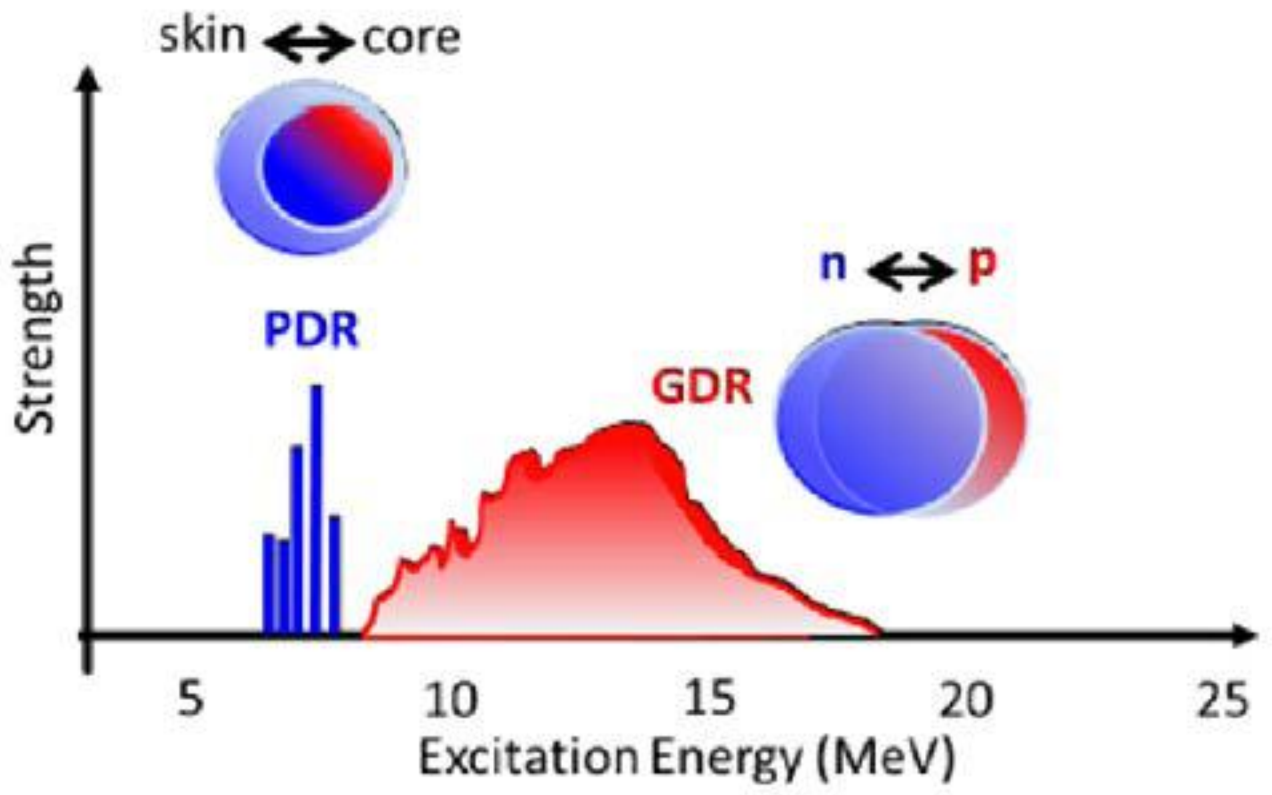
- Transition density decrease with symmetry energy slope *L*.
- With current χEFT interaction, core-crust transition density can be fixed to $n_B^{3\text{DB-UNI}} 0.074^{+0.004}_{-0.003}(\sigma)^{+0.010}_{-0.006}(2\sigma)\ \text{fm}^{-3}$.







EOS at crust-core transition



Dipole Polarizability

E.M. interaction probe: photo-absorption

- Strength function of Dipole operator D:

$$S(\omega) = \sum_{k>0} \left| < k | D | 0 > \right|^2 \delta(\omega - \omega_k) \propto \frac{\sigma_{abs}(\omega)}{\omega}$$

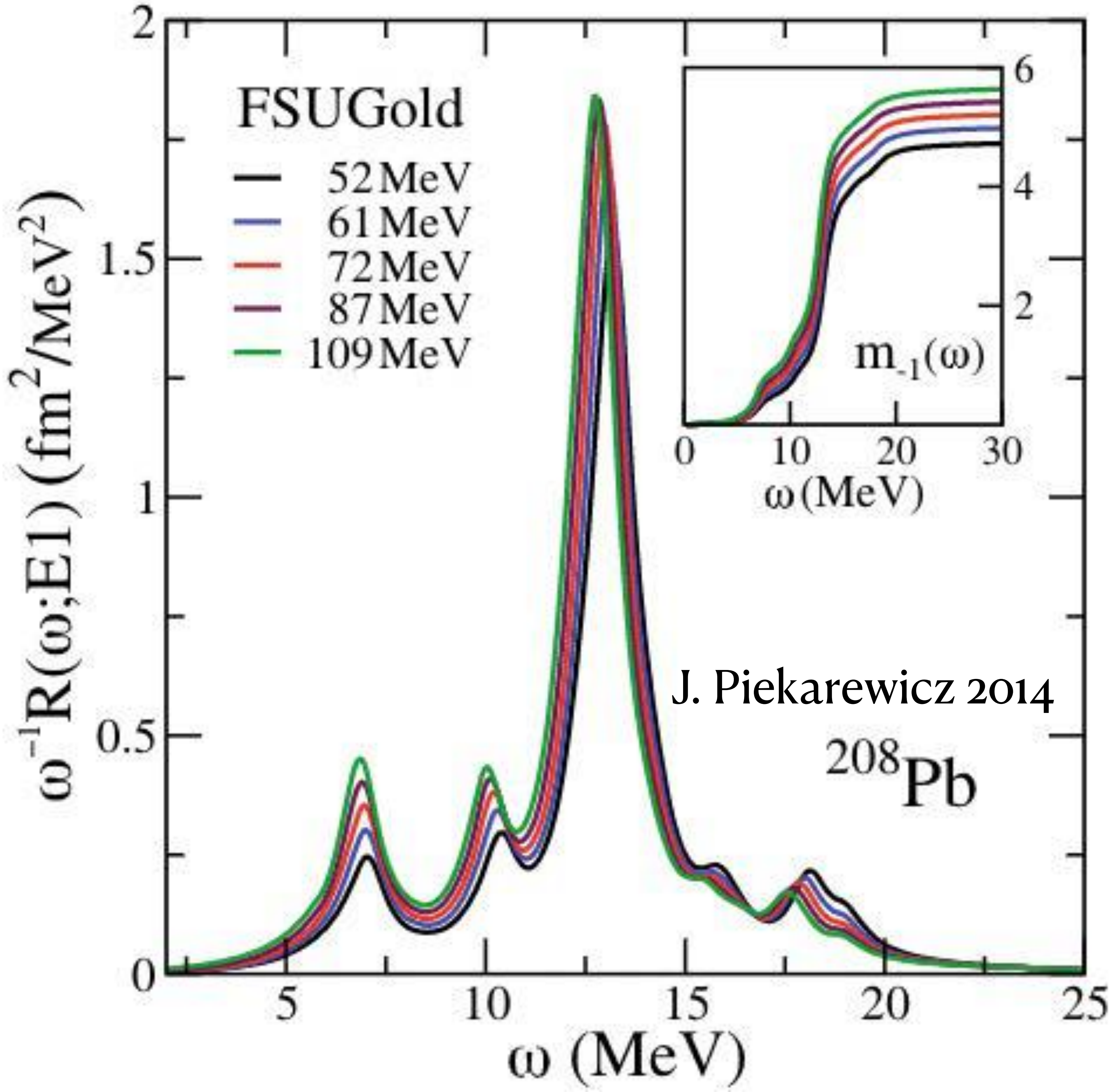
- Moments of strength function:

$$m_n = \int_0^\infty S(\omega)\omega^n d\omega \qquad m_1 \approx 15\frac{NZ}{A}\text{fm}^2\text{MeV}$$

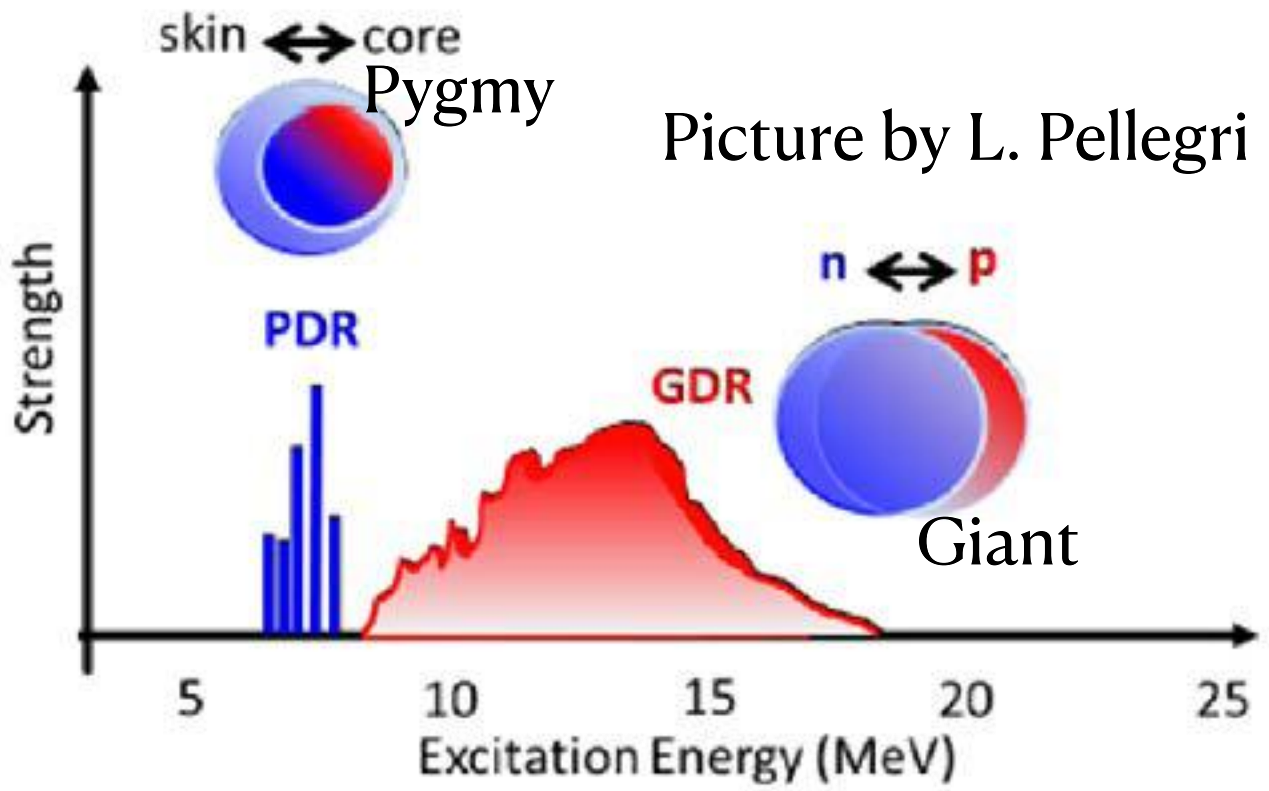
- Common Observable:

Centroid energy $E_0 = \frac{m_1}{m_0}, E_{-1} = \sqrt{\frac{m_1}{m_{-1}}}$

Polarizability $\alpha_D = \frac{8\pi e^2}{9} m_{-1}$, in terms of photo-absorption, $\alpha_D = \frac{\hbar c}{2\pi^2 e^2}\int \frac{\sigma_{abs}}{\omega^2} d\omega$

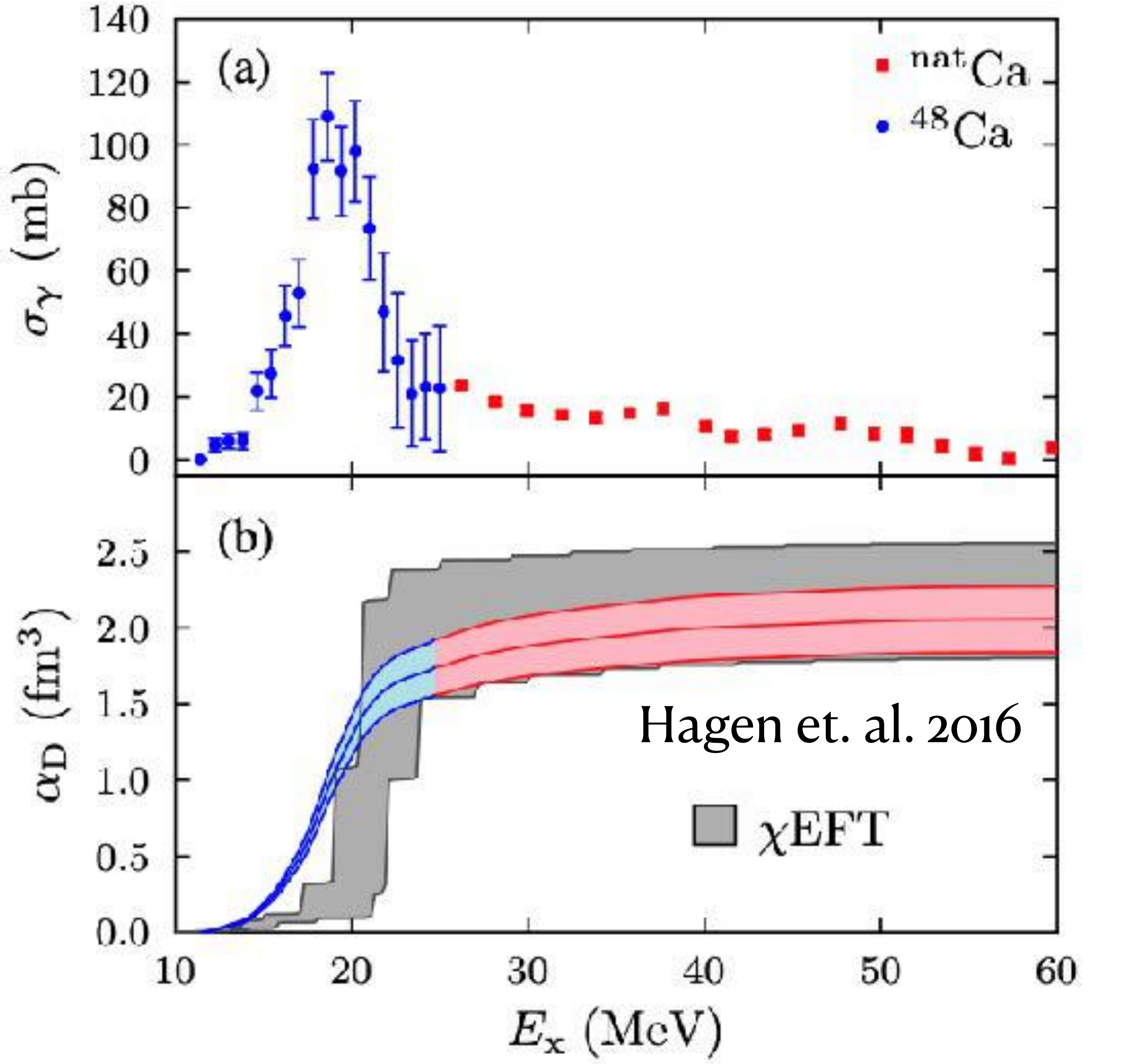


J. Piekarewicz 2014



Picture by L. Pellegri

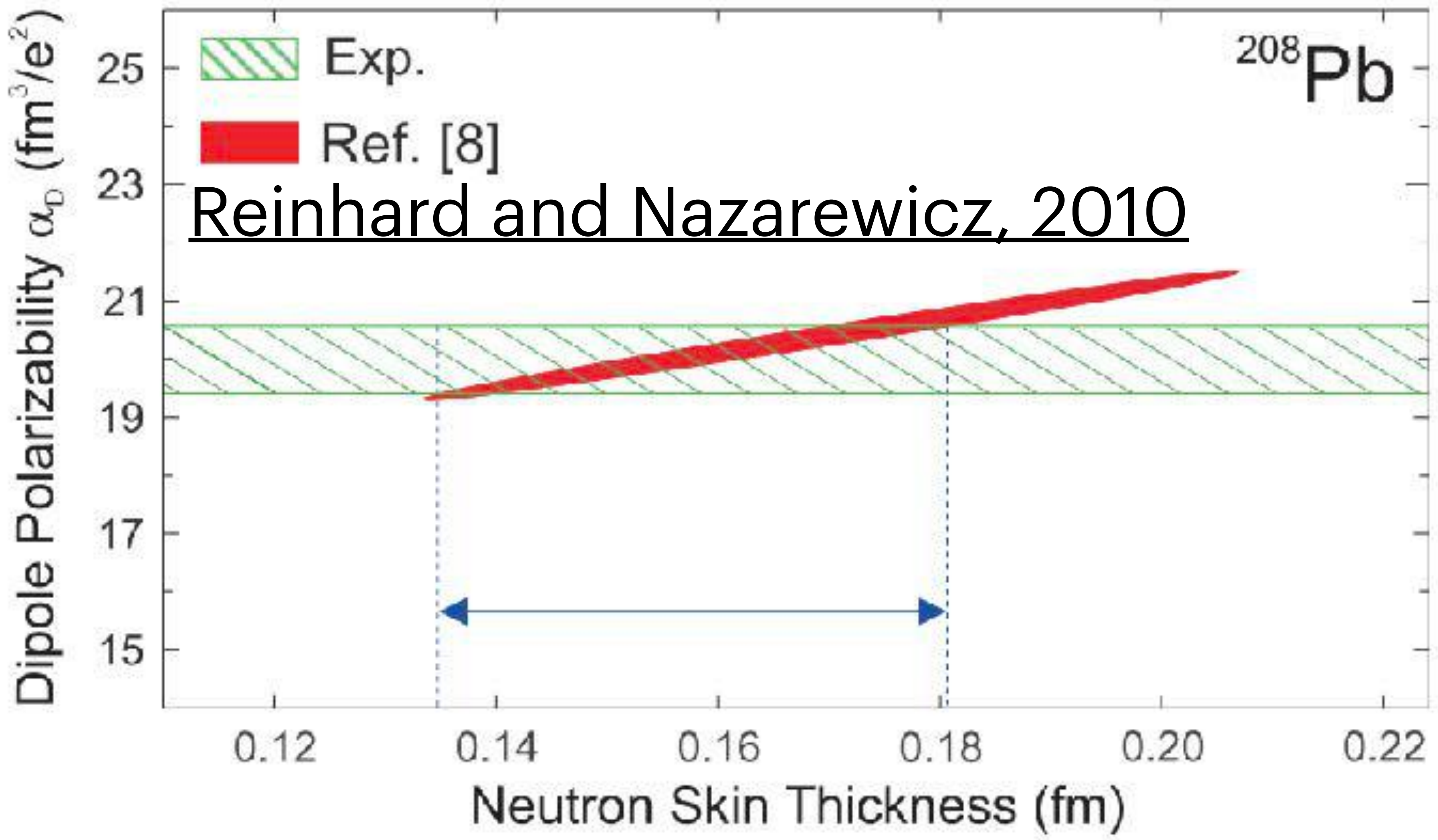
Dipole Polarizability



Hagen et. al. 2016

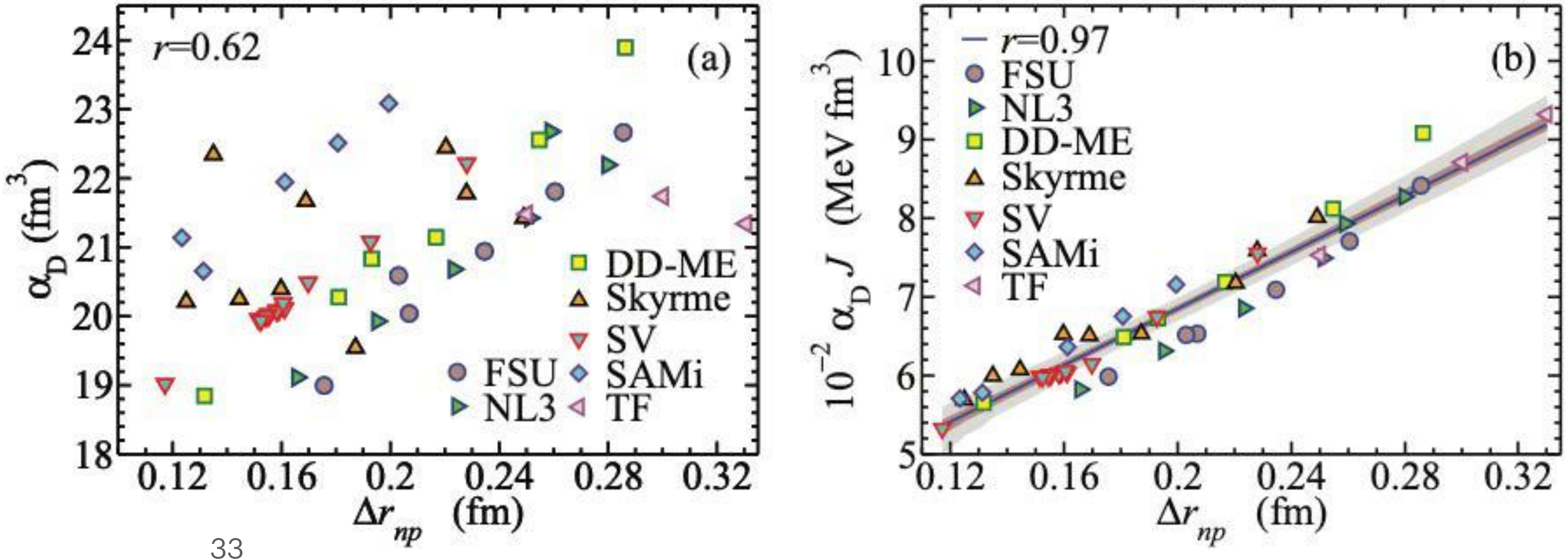
Electric Dipole Polarizability of 48Ca

- Photon excitation works below neutron ionization.
- Proton Coulomb excitation at RCNP, Osaka
- $\alpha_D S_V$ is a better observable than α_D



Reinhard and Nazarewicz, 2010

Complete Electric Dipole Response and the Neutron Skin in 208Pb



Moca-Maza et. al. 2013

Born approximation

- Axial weak potential, $A(r) = \frac{G_F}{2^{3/2}}\rho_W(r)$
- Scattering amplitude: $\int < \psi_{in}|A(r)|\psi_{out} > d^3r = \frac{G_F}{2^{3/2}}\int e^{i\mathbf{q}\cdot\mathbf{r}}\rho_W(\mathbf{r})d^3r = \frac{G_F Q_W}{2^{3/2}q^2}F_W(q)$
- $A_{PV} = \frac{\sigma_R - \sigma_L}{\sigma_R + \sigma_L} \approx \frac{G_F q^2 |Q_W|}{4\sqrt{2}\pi\alpha Z}\frac{F_W(q)}{F_E(q)} \propto \frac{(F_E + F_W)^2 - (F_E - F_W)^2}{(F_E + F_W)^2 + (F_E - F_W)^2}$

  where $F(q) = \frac{\int j_0(qr)\rho(r)d^3r}{\int \rho(r)d^3r}$, and $j_0(qr) = \frac{\sin(qr)}{qr}$ is spherical Bessel function

Back

Weak Charge of Nuclei

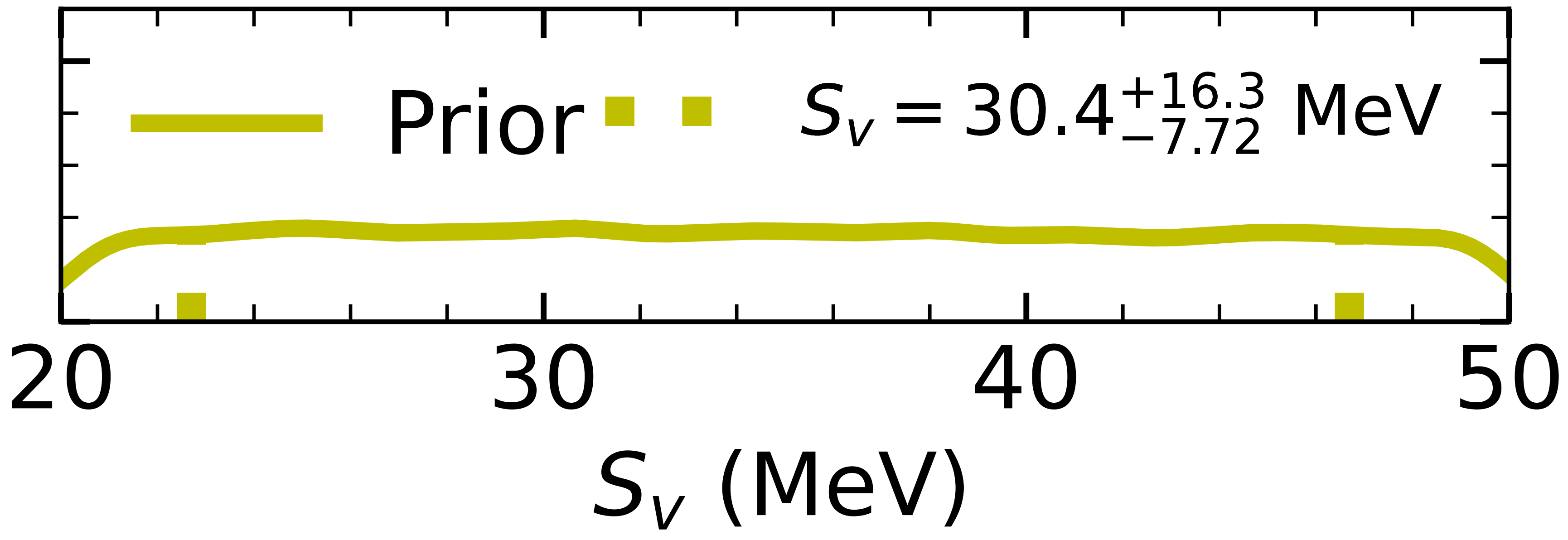
- Weak charge: $Q_W = 2T_3 - 4Q_E \sin^2(\Theta_W)$
  where weak isospin $T_3 = -\dfrac{1}{2}$ for $e^-, u^{+\frac{2}{3}}, n,$ and $T_3 = \dfrac{1}{2}$ for $\nu, d^{-\frac{1}{3}}, p^+$
  Neutron weak charge: $Q_n = -1$ (–0.9878 with radiative correction)
  Proton weak charge $Q_p = 1 - 4\sin^2(\Theta_W)$ (0.0721 with radiative correction)
- Neutron form factor: $G_n^W = Q_n G_p^E + Q_p G_n^E + Q_n G_s^E$
  Proton form factor: $G_p^W = Q_p G_p^E + Q_n G_n^E + Q_n G_s^E$
- Weak charge distribution: $\rho_W(r) = \int d^3r' \left[G_n^W(r-r')\rho_n(r) + G_p^W(r-r')\rho_p(r) \right]$
- Electric charge distribution: $\rho_E(r) = \int d^3r' \left[G_n^E(r-r')\rho_n(r) + G_p^E(r-r')\rho_p(r) \right]$
  Additional complicity: many-body correction, center-of-mass correction, the magnetic contribution from spin-orbital current(SHF) or tensor density (RMF)

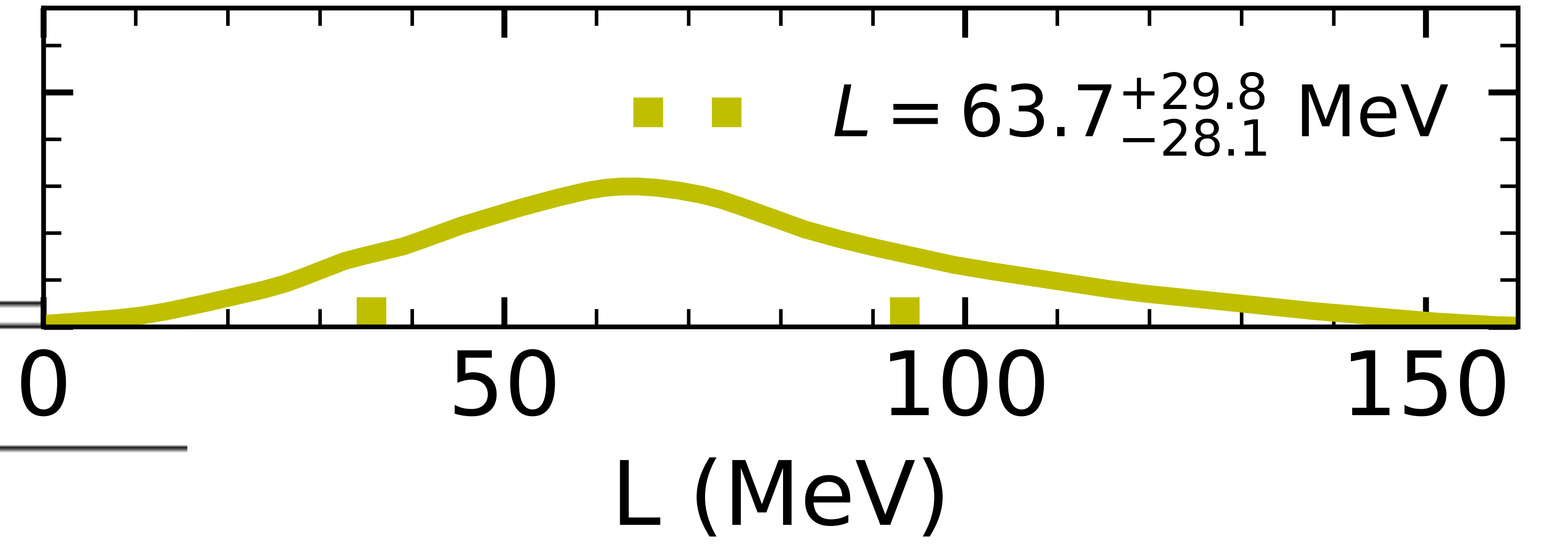
FSU-type RMF model

	Scalar	Vector
Isoscalar	σ	$\gamma^\mu\omega_\mu$
Isovector	$\vec\tau\,\vec\delta$	$\gamma^\mu\vec\tau\,\vec\rho_\mu$

$$\mathscr{L} = \mathscr{L}_0 + \bar\psi\left(g_\sigma\sigma - g_\omega\gamma^\mu\omega_\mu - \frac{g_\rho}{2}\gamma^\mu\boldsymbol{\tau\rho}_\mu\right)\psi - \frac{\kappa}{3!}(g_\sigma\sigma)^3 - \frac{\lambda}{4!}(g_\sigma\sigma)^4$$

$$+\frac{\zeta}{4!}(g_\omega^2\omega^\mu\omega_\mu)^2 + \Lambda_{\omega\rho}(g_\rho^2\boldsymbol{\rho}^\mu\boldsymbol{\rho}_\mu)(g_\omega^2\omega^\mu\omega_\mu)$$

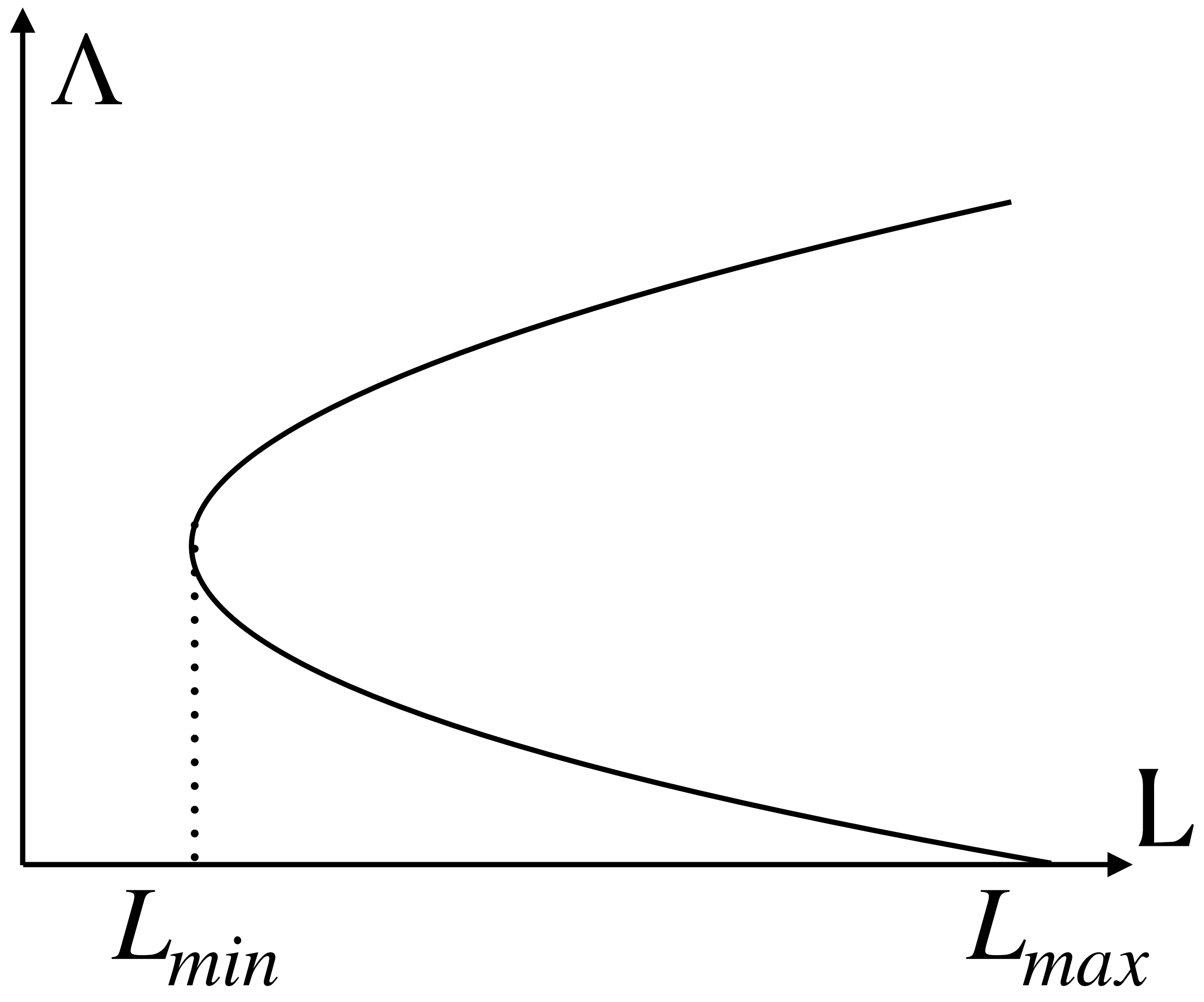


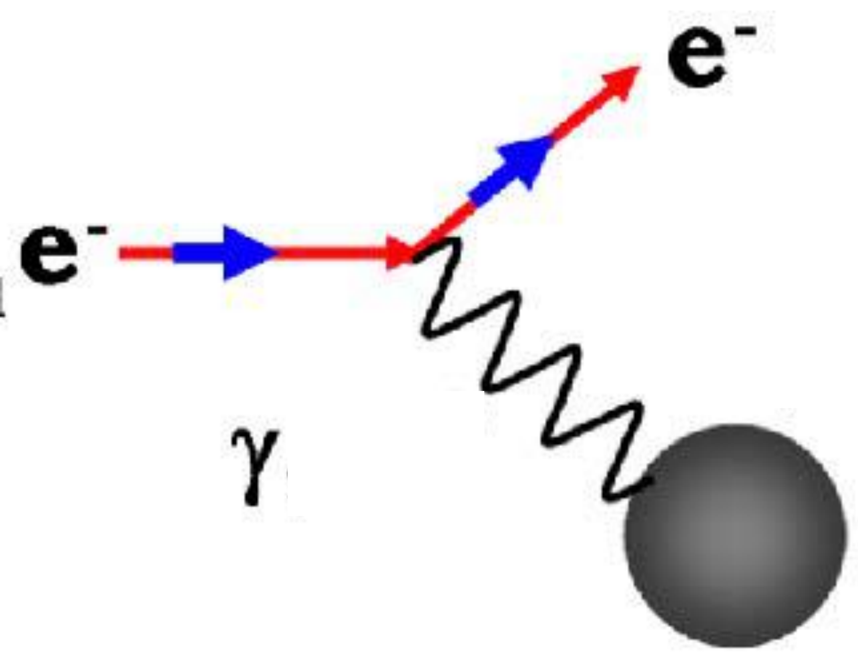


	NL3	FSU	FSU2
m_σ	508.194	491.5	497.479
m_ω	782.501	782.5	782.5
m_ρ	763	763	763
g_σ^2	104.3871	112.1996	108.0943
g_ω^2	165.5854	204.5469	183.7893
g_ρ^2	79.6	138.4701	80.4656
κ	3.8599	1.4203	3.0029
λ	-0.015905	0.023762	-0.000533
ζ	0	0.06	0.0256
Λ	0	0.03	0.000823

⟷

	prior
m_σ (MeV)	[450,550]
m_ω (MeV)	782.5
m_ρ (MeV)	763
n_s (MeV)	[0.14,0.165]
BE (MeV)	[-15.5,-16.5]
M^* (MeV)	[0.5,0.8]× 939
K (MeV)	[210,250]
S_V (MeV)	[20,50]
L (MeV)	$[L_{min},L(\Lambda=0)]$
ζ_ω	[0,0.03]



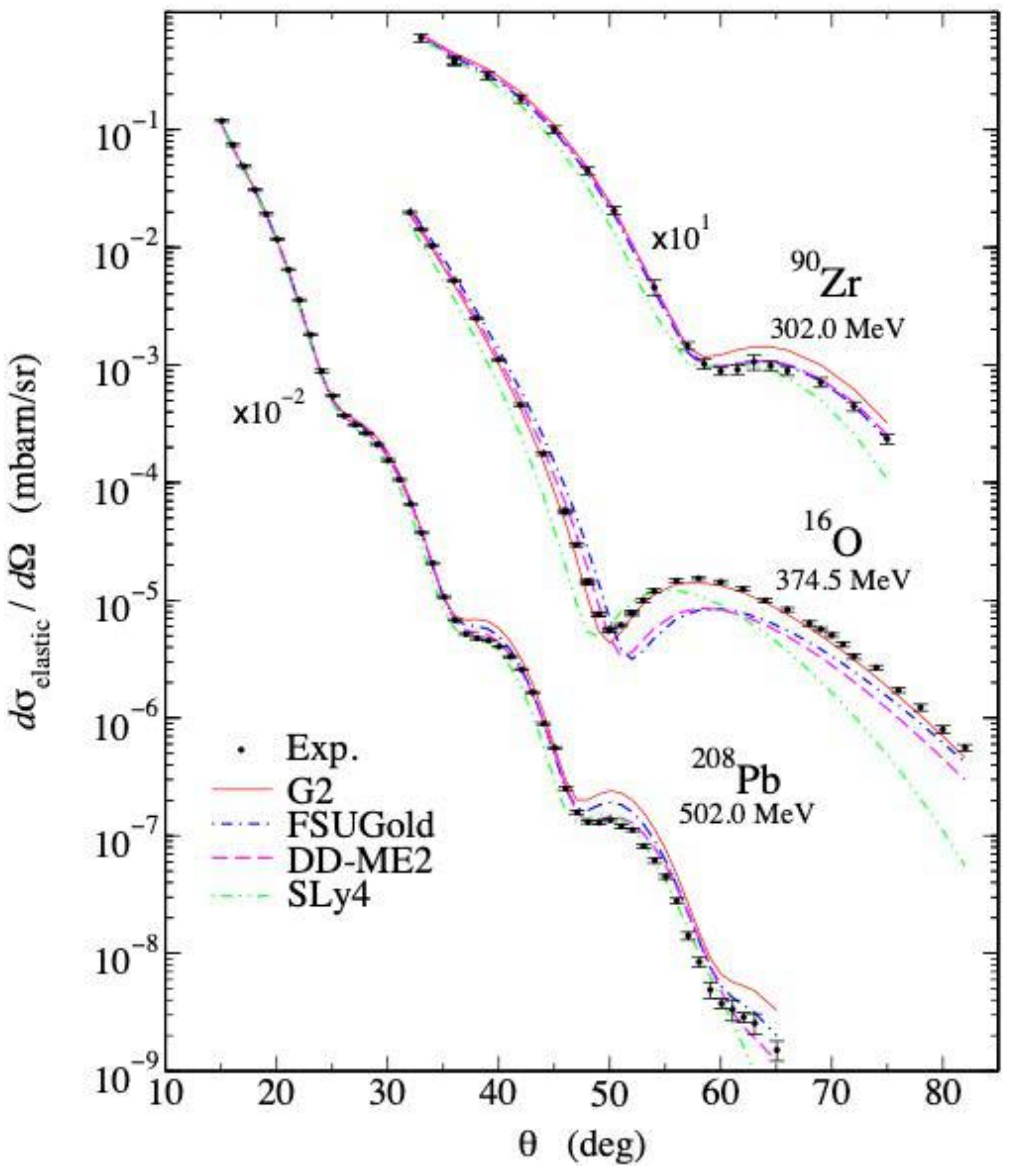


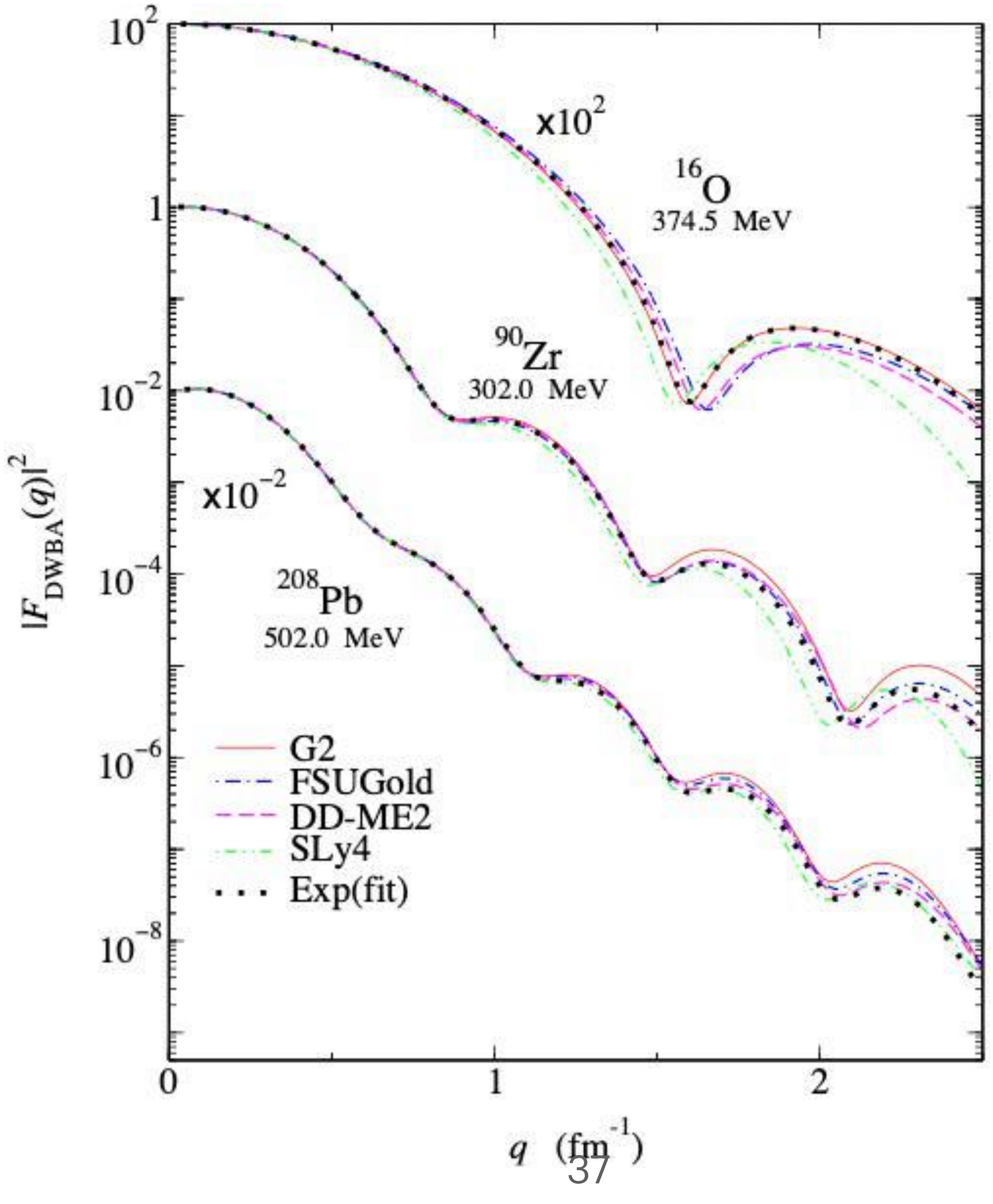
Electric Charge Distribution

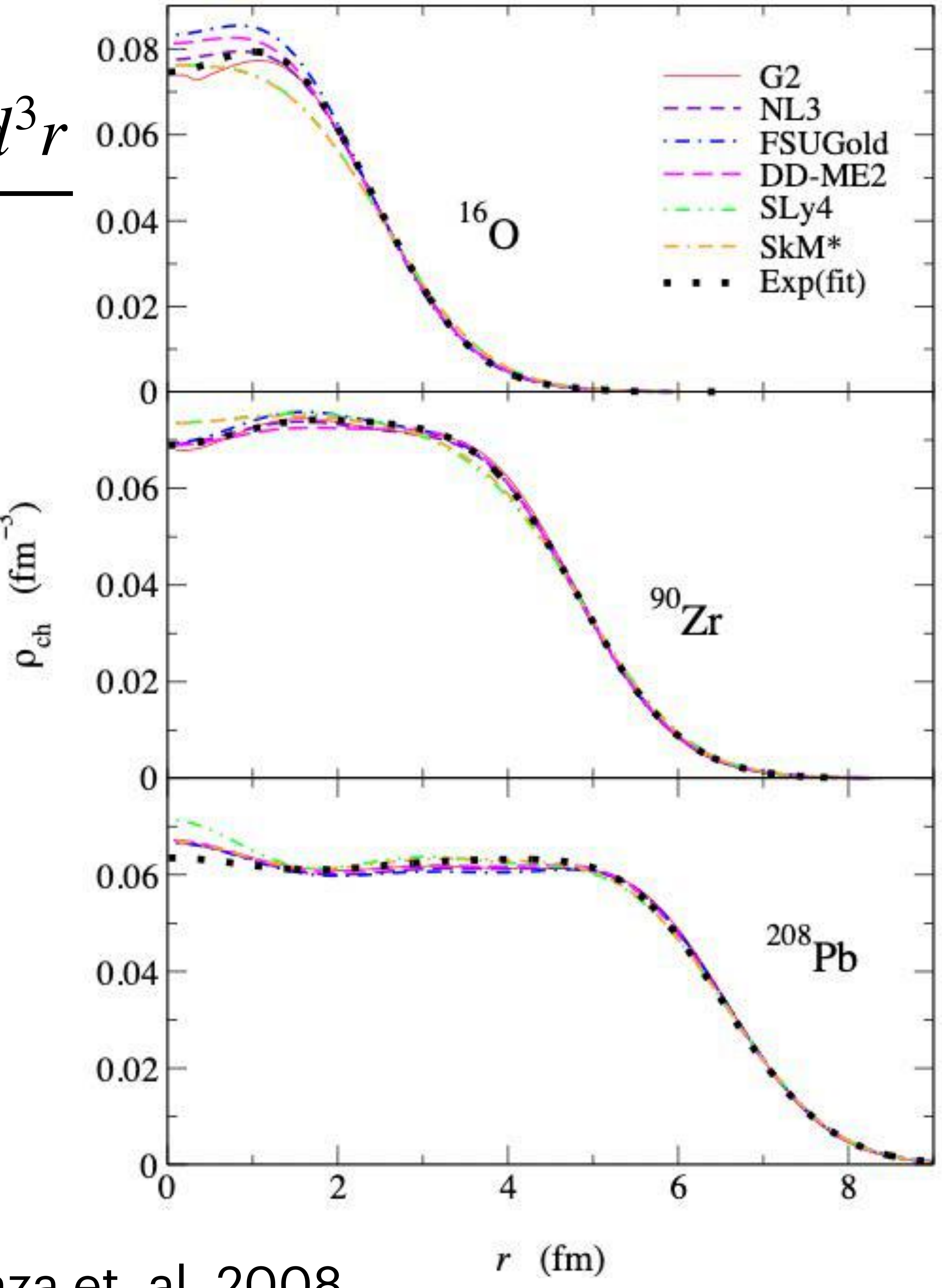
Well measured!!!

$$\frac{d\sigma}{d\Omega} = \left[\frac{d\sigma}{d\Omega}\right]_{Mott} |F(\mathbf{q})|^2$$

$$F(q) = \frac{\int j_0(qr)\rho(r)d^3r}{\int \rho(r)d^3r}$$







Moca-Maza et. al. 2008

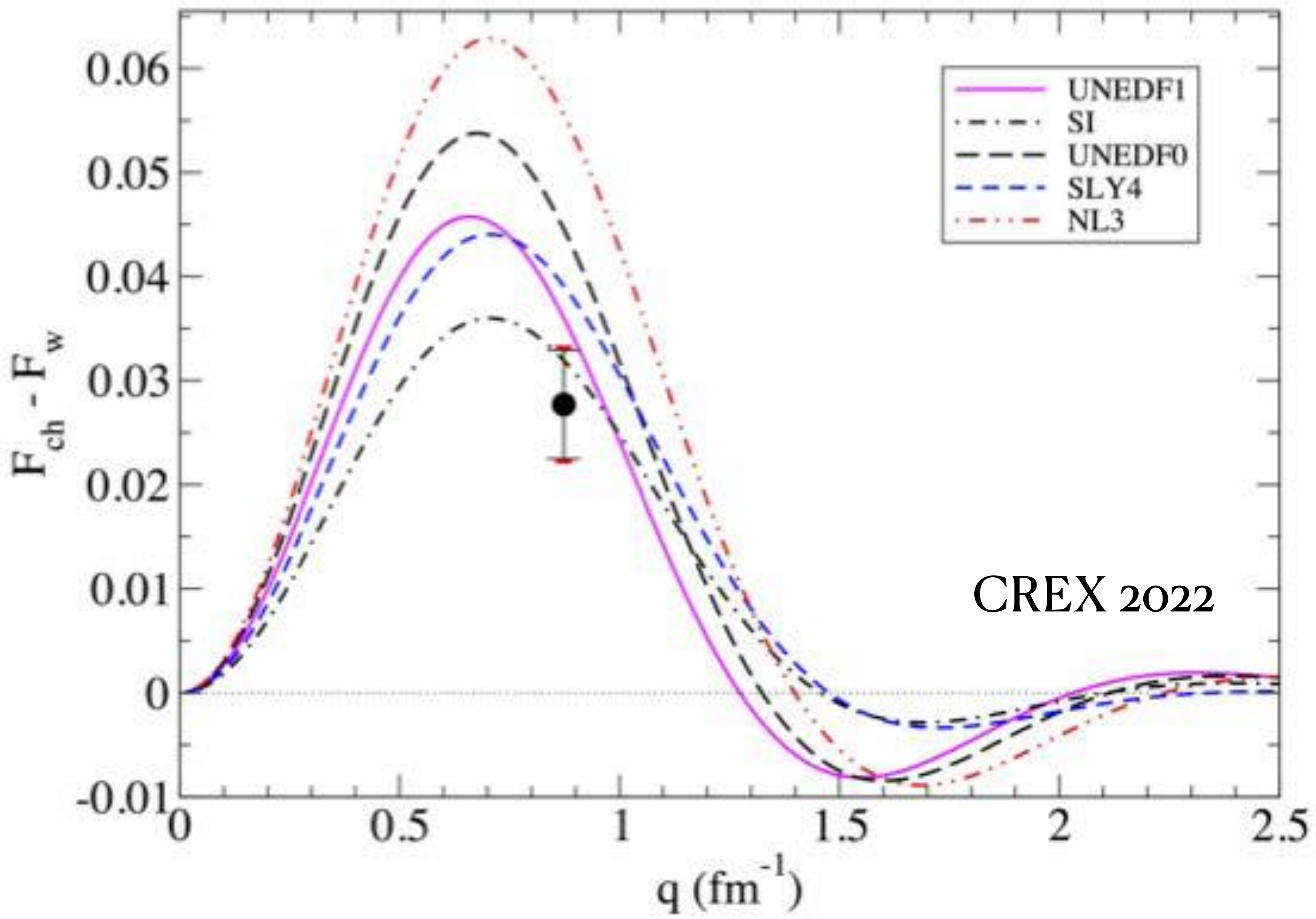
Weak Charge Distribution

[Jefferson Lab](#)
[Krishna Kumar 2018](#)
[Tao Ye 2021](#)
[Robert Radloff 2022](#)

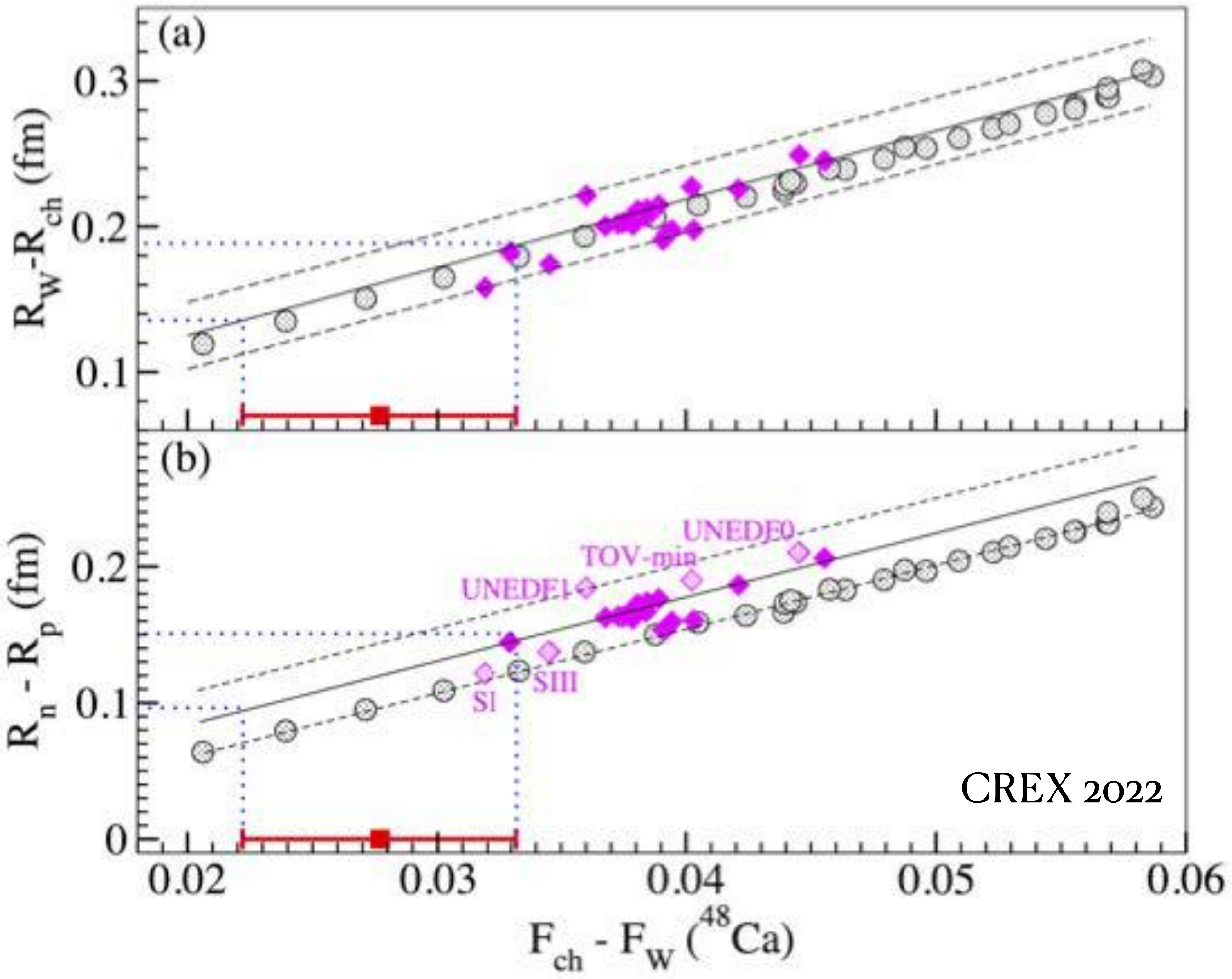
Extremely hard!!!

- $$\begin{aligned} F(\mathbf{q}) &= \frac{1}{Q}\int e^{i\mathbf{q}\cdot\mathbf{r}}\rho(\mathbf{r})d^3r \\ &= \frac{1}{Q}\int\left(1 + i\mathbf{q}\cdot\mathbf{r} - \frac{1}{2}(\mathbf{q}\cdot\mathbf{r})^2 + \dots\right)\rho(\mathbf{r})d^3r \\ &= 1 - \frac{1}{6}q^2 < r^2 > + \dots \quad \text{(spherical symmetry)} \end{aligned}$$

$$\lim_{q<<\sqrt{<r^2>}} < r^2 > = \frac{6[1 - F(q)]}{q^2}$$

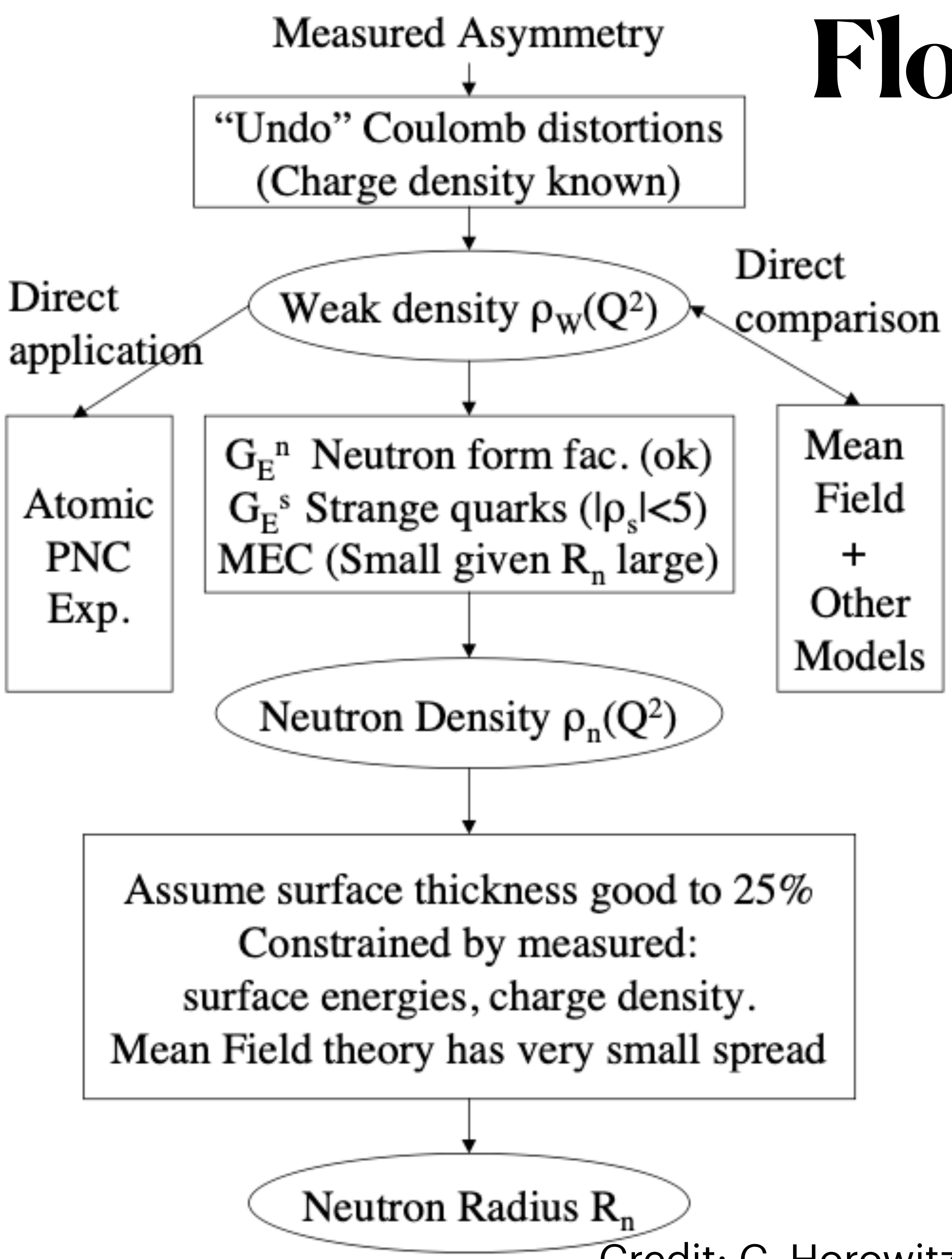


CREX 2022



CREX 2022

Flowchart of PREX and CREX



Credit: C. Horowitz

Fixed

Measured

Coulomb Distortion

MFT model uncertainty

	CREX	PREX
(N,Z)	(28,20)	(126,82)
q (fm-1)	0.8733	0.3977
Fch, Rch(fm)	0.1581, 3.481	0.409, 5.503
Apv	2668±106(stat) ±40(syst)	550±16(stat) ±8(syst)
Fw	0.1304±0.0052(stat) ±0.002(syst)	0.368±0.013(exp) ±0.001(theo)
Fch-Fw	0.0277±0.0052(stat) ±0.002(syst)	0.041±0.013(exp) ±0.001(theo)
Rw	3.64±0.026(exp) ±0.023(theo)	5.8±0.075(tot)
Rw-Rch	0.159±0.026(exp) ±0.023(theo)	0.297±0.075(tot)
Rn-Rp	0.121±0.026(exp) ±0.024(theo)	0.283±0.071(tot)

CREX 2022 PREX I 2012 PREX II 2021

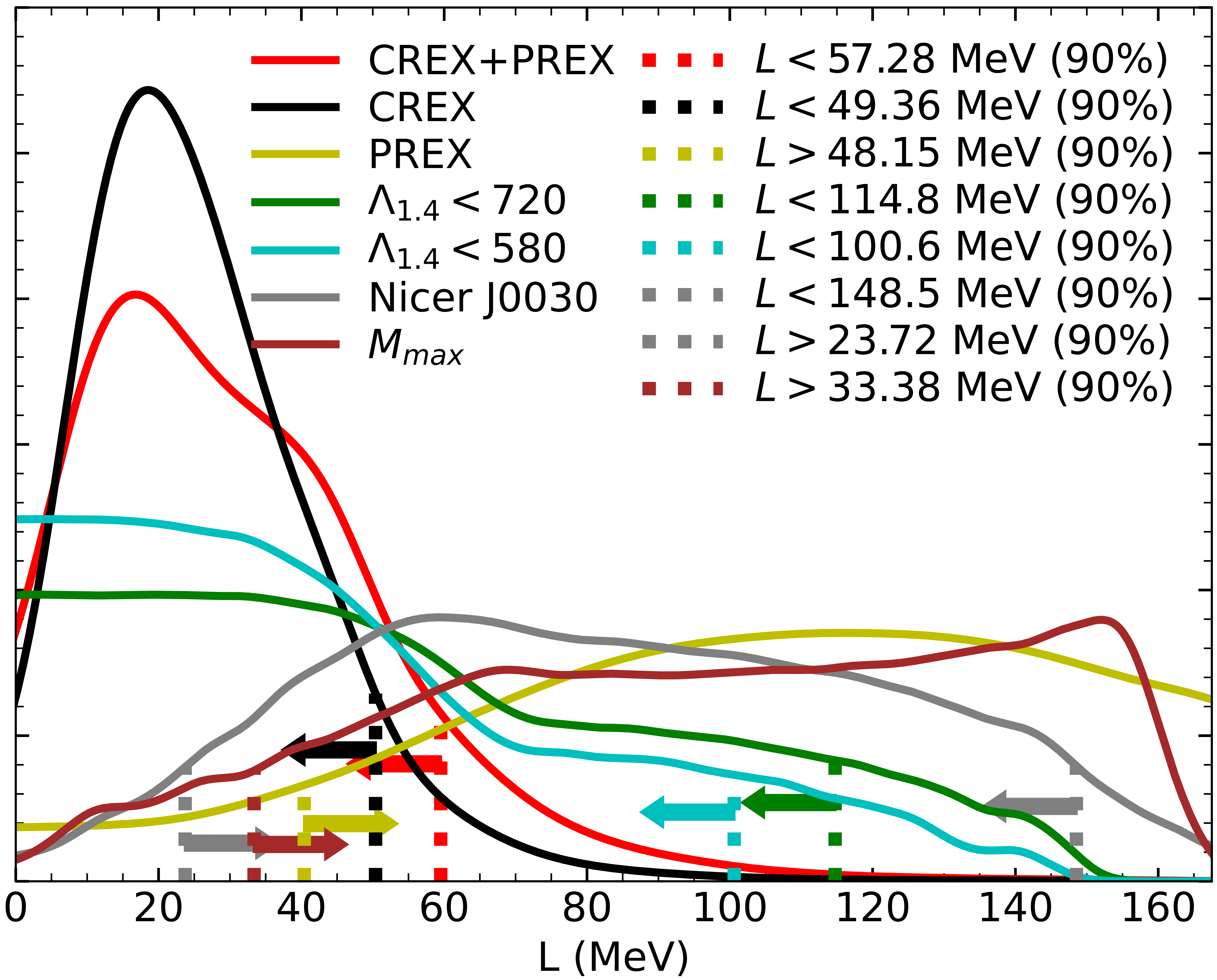
Compared with Astro

- Likelihood in uniform L

$$P(\mathscr{M}\,|\,\mathcal{O}_{M_{max}}) = \exp\left[-\frac{(M_{max}-M^{\mathcal{O}}_{max})^2}{2\sigma_{M_{max}}}\right]$$

$$P(\mathscr{M}\,|\,\mathcal{O}) = \int dM \exp\left[-\frac{(\beta_{\mathcal{O}}-\beta(M\,|\,\mathscr{M}))^2}{2\sigma^2_{\beta_{\mathcal{O}}}} - \frac{(M_{\mathcal{O}}-M)^2}{2\sigma^2_{M_{\mathcal{O}}}}\right]$$

$$P(\mathscr{M}\,|\,\mathcal{O}_{GW170817}) = \Theta(\Lambda^{\mathcal{O}}_{1.4}-\Lambda_{1.4})$$



$$p + e \rightarrow n + \nu$$

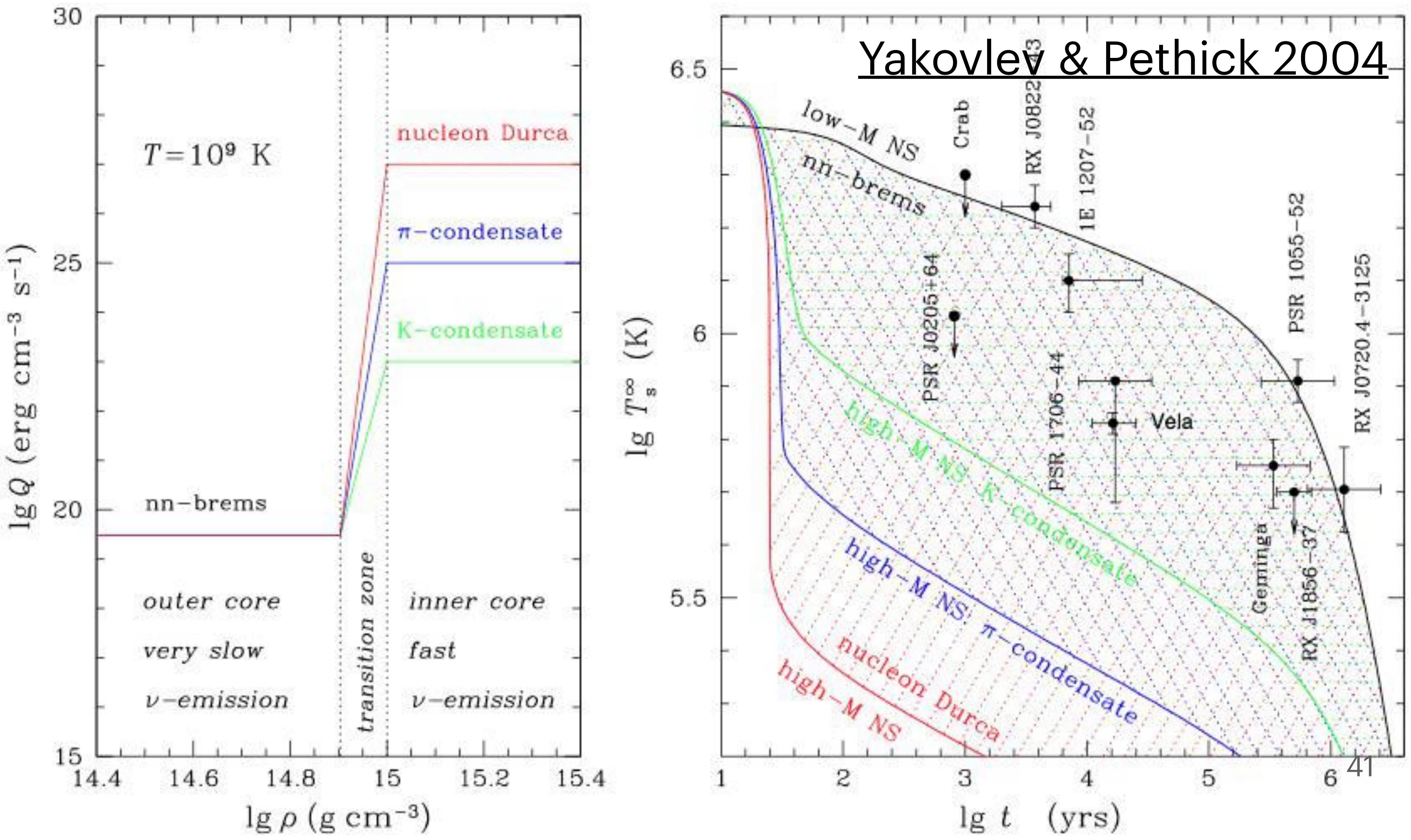
$$\bar{\nu} + p + e \leftarrow n$$

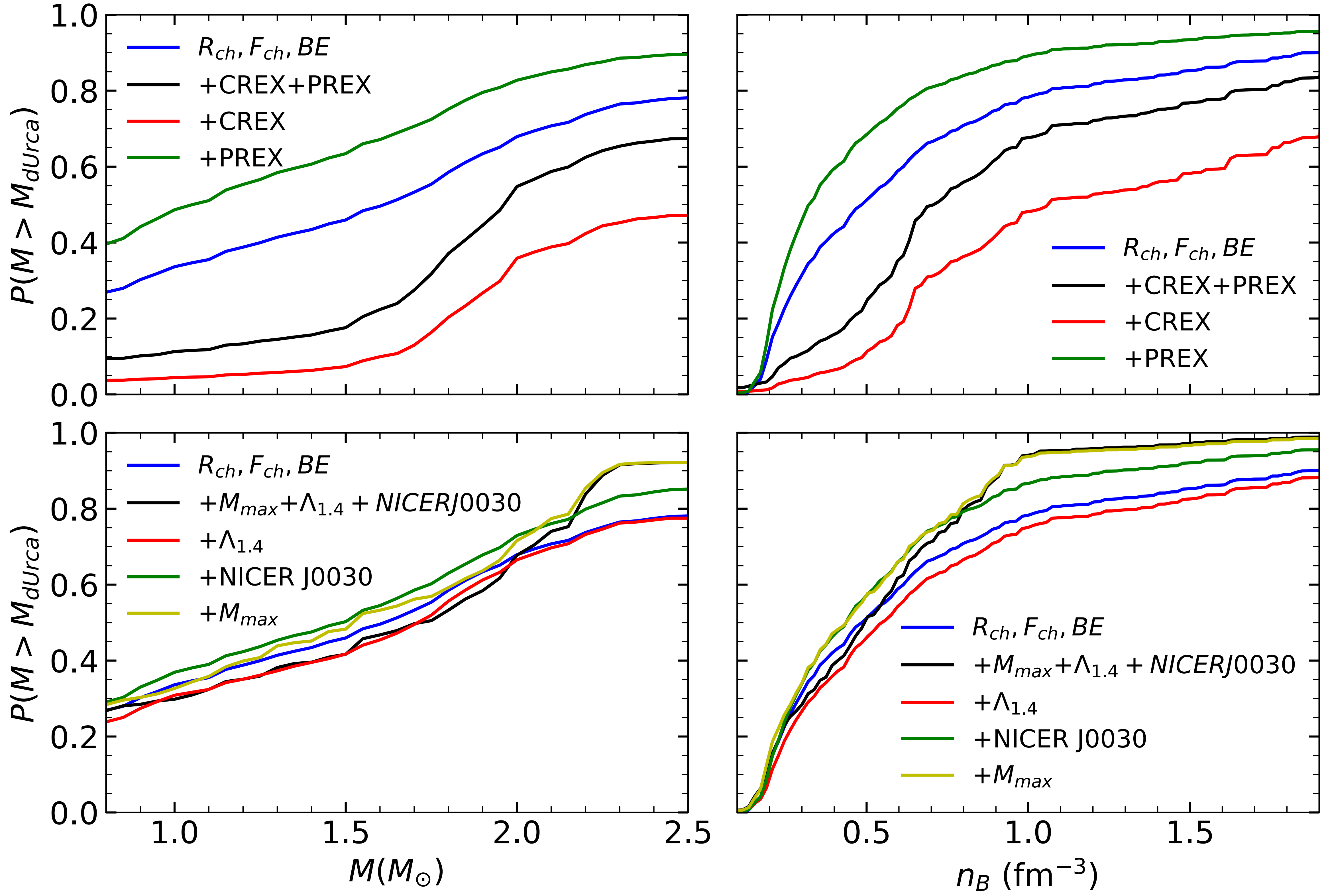
Direct Urca Process

$$p_F^e + p_F^p > p_F^n$$

$$n_p/n_n \gtrsim 0.14$$

- Modified Urca process with superfluidity explains most cooling data.
- PREX+CREX is consistent with the absence of evidence for the direct Urca process.





Conclusion

- There's 1 to 2 sigma tension between PREX + CREX and the model constrained by nuclei.
- The mild tension helps constrain density dependence of symmetry energy
- CREX indeed provides more information than PREX.
- PREX + CREX is consistent with ab-initio calculation, dipole polarizability, and other experiments regarding symmetry energy.
- Astronomical observation has very limited constraints on EOS below saturation.
- Absent of a direct Urca process is supported by PREX + CREX.
- Looking forward to MREX confirming PREX.

QED and Weak interaction

- Lagrange involving electron:

$$\mathscr{L} = \mathbf{i}\bar{\psi}\gamma^{\mu}\partial_{\mu}\psi - m\bar{\psi}\psi + eJ^{\mu}A_{\mu} + \frac{g_W}{\cos(\Theta_W)}J_Z^{\mu}Z_{\mu} - \frac{M_Z^2}{2}Z^{\mu}Z_{\mu} + \dots$$

$J^{\mu} = (\rho_E, \mathbf{j}) = \bar{\psi}\gamma^{\mu}\psi$ is electron 4-current,

and $J_Z^{\mu} = -\frac{1}{2}\bar{\psi}_L\gamma^{\mu}\psi_L - \sin^2(\Theta)\bar{\psi}\gamma^{\mu}\psi = -\frac{1}{4}\bar{\psi}\left[1 - 4\sin^2(\Theta_W) - \gamma^5\right]\psi$

Weak mixing angle: $\cos(\Theta_W) = \frac{M_W}{M_Z} = 0.882$

$M_W = 80.4\ \text{GeV},\ \ M_Z = 91.2\ \text{GeV},\ \ \sin^2(\Theta_W) = 0.223$

- Z boson propagator: $\frac{g_{\mu\nu}}{M_Z^2 - q^2}$
- 4-Fermi effective interaction at zero momentum : $G_F = \frac{g_W^2}{4\sqrt{2}M_W^2}$

Back

Maxwell Equations of E.M. and Weak fields

- Lagrange involving photon and Z boson:

$$\mathscr{L} = \left[-\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + eJ^{\mu}A_{\mu}\right] + \left[-\frac{1}{4}Z^{\mu\nu}Z_{\mu\nu} + \frac{g_W}{\cos(\Theta_W)}J_Z^{\mu}Z_{\mu} - \frac{1}{2}M_Z^2 Z^{\mu}Z_{\mu}\right]$$

where $F_{\mu\nu} = \partial^{\mu}A_{\nu} - \partial^{\nu}A_{\mu}$, $Z_{\mu\nu} = \partial^{\mu}Z_{\nu} - \partial^{\nu}Z_{\mu}$

$A_{\mu} = (\Phi, \mathbf{A})$, $Z_{\mu} = (\Phi_Z, \mathbf{Z})$ are gauge boson fields,

and $J^{\mu} = (\rho_E, \mathbf{j}) = \bar{\psi}\gamma^{\mu}\psi$ is E.M. 4-current of an electron.

- E.M. field follows Maxwell Equations: $\nabla^2\Phi - \dfrac{\partial^2\Phi}{\partial_t^2} = \rho_E$ + ($M^2\Phi$ for massive Z boson)

Static electric potential: $\Phi(r) = \displaystyle\int \frac{\rho_E(r')}{4\pi|r-r'|}dr'^3$

Static Z-boson potential: $\Phi_Z(r) = \displaystyle\int \frac{\rho_Z(r')e^{-M_Z|r-r'|}}{4\pi|r-r'|}dr'^3 \approx \rho_Z(r')\int \frac{e^{-M_Z|r-r'|}}{4\pi|r-r'|}dr'^3 = \frac{\rho_Z(r')}{M_Z^2}$

Weak interaction is approximately zero-range, since $M_Z \approx 500\ fm^{-1}$

Back

Dirac equation in E.M. and weak field

V-A theory

- Lagrange involving electron:

$$\mathscr{L} = \mathbf{i}\bar{\psi}\gamma^{\mu}\partial_{\mu}\psi - m\bar{\psi}\psi + eJ^{\mu}A_{\mu} + \frac{g_W}{\cos(\Theta_W)}J^{\mu}_{Z}Z_{\mu} - \frac{M_Z^2}{2}Z^{\mu}Z_{\mu} + \dots$$

- Electron weak 4-current:

$$J^{\mu}_{Z} = -\frac{1}{2}\bar{\psi}_L\gamma^{\mu}\psi_L + \sin^2(\Theta)\bar{\psi}\gamma^{\mu}\psi = -\frac{1}{4}\bar{\psi}\gamma^{\mu}\left[1 - 4\sin^2(\Theta_W) - \gamma^5\right]\psi \approx \frac{1}{4}\bar{\psi}\gamma^{\mu}\gamma^5\psi$$

- Dirac equation: $\left[\alpha\mathbf{p} + \beta m + \hat{V}(r)\right]\Psi = E\psi$

where $\hat{V}(r) = V(r) + \gamma_5 A(r),\ \ V(r) = \int d^3\mathbf{r}'\frac{\rho_p(\mathbf{r})}{|\mathbf{r}' - \mathbf{r}|},\ \ A(r) = \frac{G_F}{2^{3/2}}\rho_W(r)$

- In the massless limit(Weyl basis): $\left[\alpha\mathbf{p} + V_{L,R}(r)\right]\Psi_{L,R} = E\psi_{L,R}$, where $V_{L,R}(r) = V(r) \pm A(r)$

Back

Parity violating asymmetry A_{PV}

The observable in PREX and CREX

- Parity violating asymmetry: $A_{PV} = \dfrac{\sigma_R - \sigma_L}{\sigma_R + \sigma_L}$

  where σ_R, σ_L are cross-section of the scattering problem:
  $\left[\alpha \mathbf{p} + V_{L,R}(r)\right] \Psi_{L,R} = E\psi_{L,R}$, where $V_{L,R}(r) = V(r) \pm A(r)$,
  $V(r) = \int d^3\mathbf{r}' \dfrac{\rho_p(\mathbf{r})}{|\mathbf{r}' - \mathbf{r}|}, \;\; A(r) = \dfrac{G_F}{2^{3/2}} \rho_W(r)$
  which is called "Coulomb distortion" in this context:
  Coulomb distortion stands for repeated electromagnetic interactions with the nucleus remaining in its ground state. This is of order Zα/π, 20 % for 208Pb.

Back

Form Factor

- Point charge Coulomb (Mott) scattering: $\left[\dfrac{d\sigma}{d\Omega}\right]_{Mott} = \dfrac{Z^2e^4(1-\beta^2\sin^2\frac{\theta}{2})}{64\pi^2\varepsilon_0^2p^2\beta^2\sin^2\frac{\theta}{2}}$
- Extended Coulomb scattering: $-\dfrac{Ze^2}{r} \longrightarrow e^2\displaystyle\int\frac{\rho(r')d^3r'}{|r-r'|}$

$$\frac{d\sigma}{d\Omega} = \left[\frac{d\sigma}{d\Omega}\right]_{Mott} |F(\mathbf{q})|^2$$

$$F(\mathbf{q}) = \frac{1}{Q}\int e^{i\mathbf{q}\cdot\mathbf{r}}\rho(\mathbf{r})d^3r = \frac{1}{Q}\int\left(1 + i\mathbf{q}\cdot\mathbf{r} - \frac{1}{2}(\mathbf{q}\cdot\mathbf{r})^2 + \ldots\right)\rho(\mathbf{r})d^3r$$

$$= 1 - \frac{1}{6}q^2 < r^2 > + \ldots$$ assuming spherical symmetry

- At small q, $\displaystyle\lim_{q<<\sqrt{<r^2>}} < r^2 > = \frac{6[1-F(q)]}{q^2}$

Back

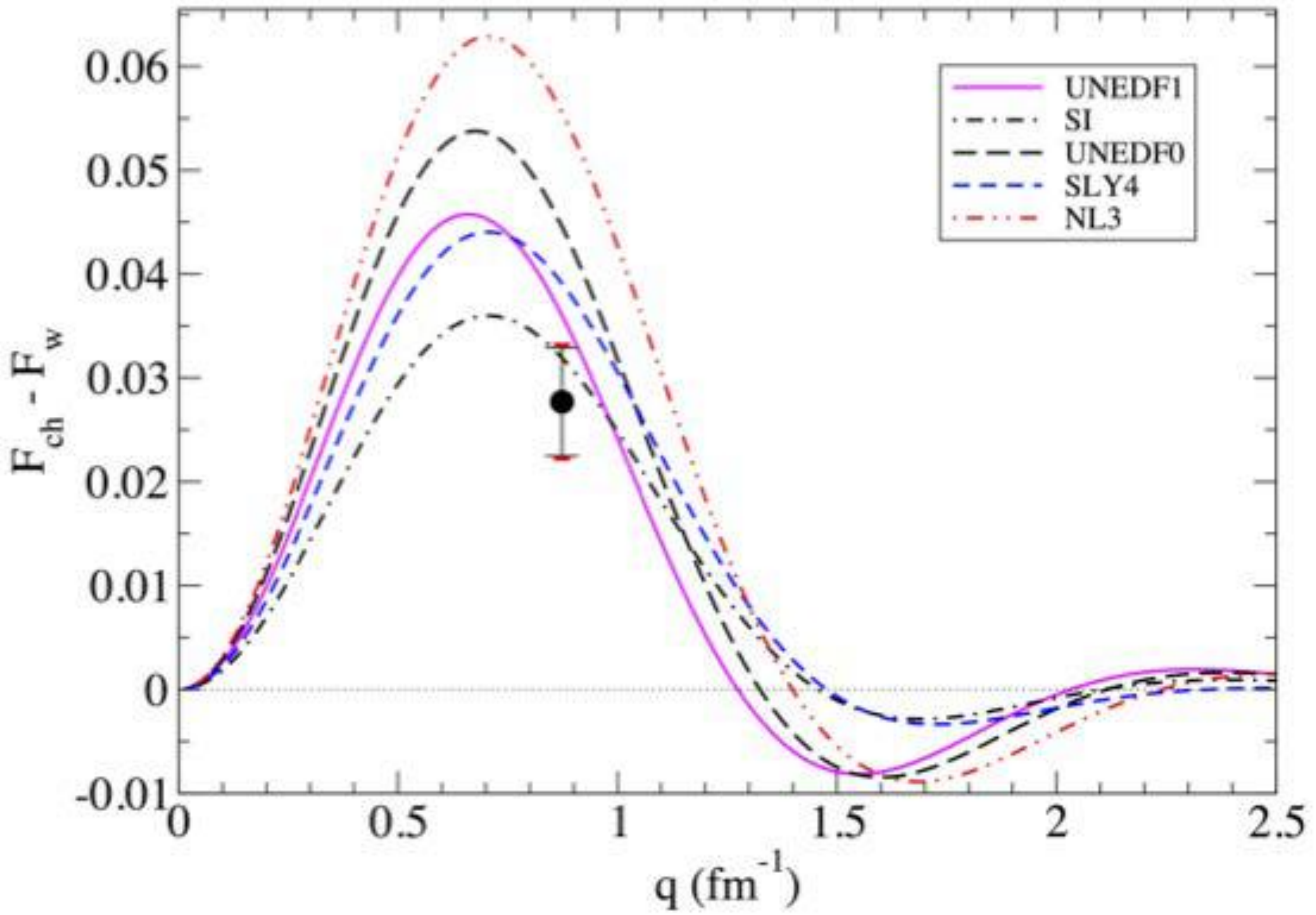
Born approximation

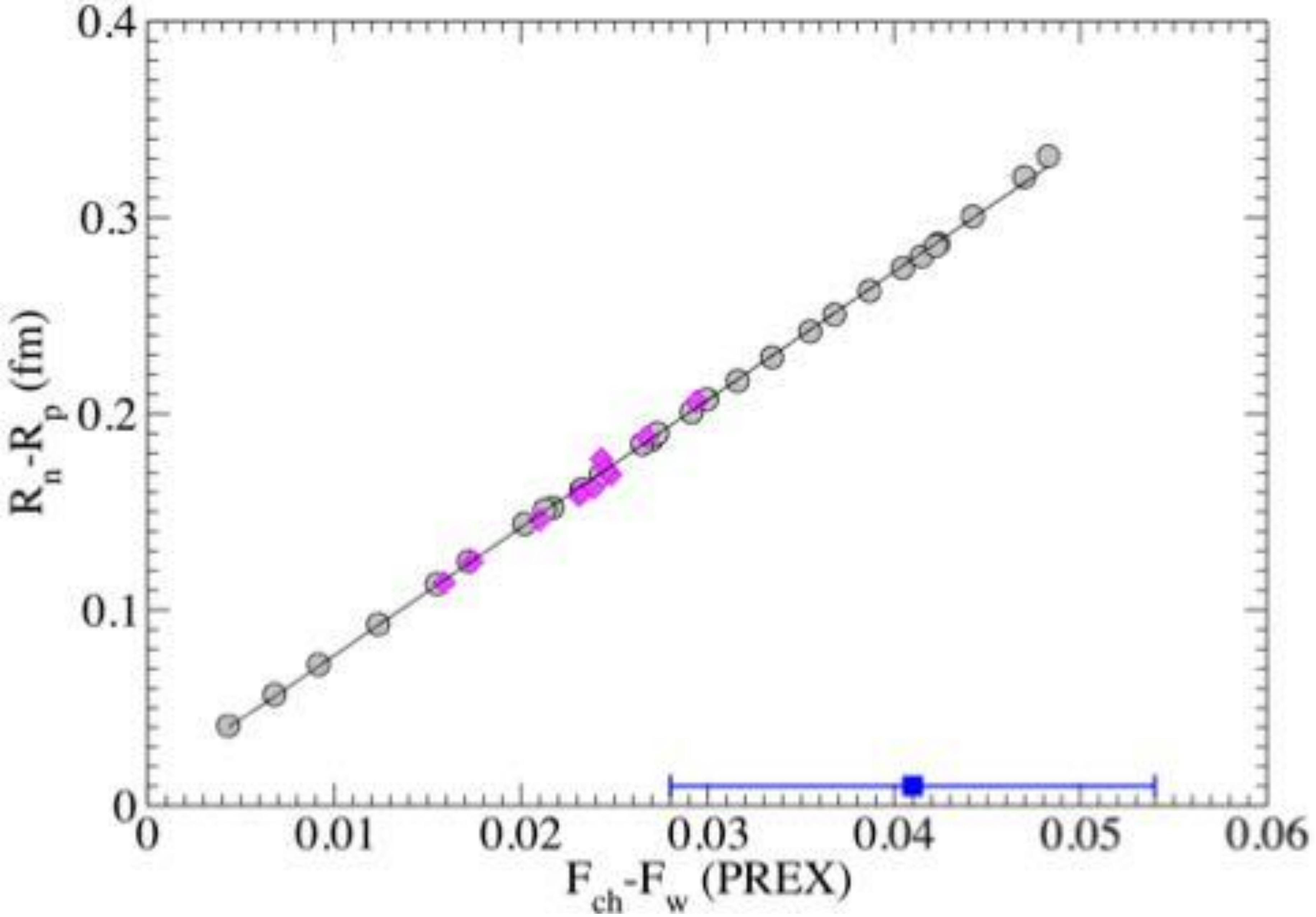
- Axial weak potential, $A(r) = \frac{G_F}{2^{3/2}}\rho_W(r)$
- Scattering amplitude: $\int < \psi_{in}|A(r)|\psi_{out} > d^3r = \frac{G_F}{2^{3/2}}\int e^{i\mathbf{q}\cdot\mathbf{r}}\rho_W(\mathbf{r})d^3r = \frac{G_F Q_W}{2^{3/2}q^2}F_W(q)$
- $A_{PV} = \frac{\sigma_R - \sigma_L}{\sigma_R + \sigma_L} \approx \frac{G_F q^2 |Q_W|}{4\sqrt{2}\pi\alpha Z}\frac{F_W(q)}{F_E(q)} \propto \frac{(F_E + F_W)^2 - (F_E - F_W)^2}{(F_E + F_W)^2 + (F_E - F_W)^2}$

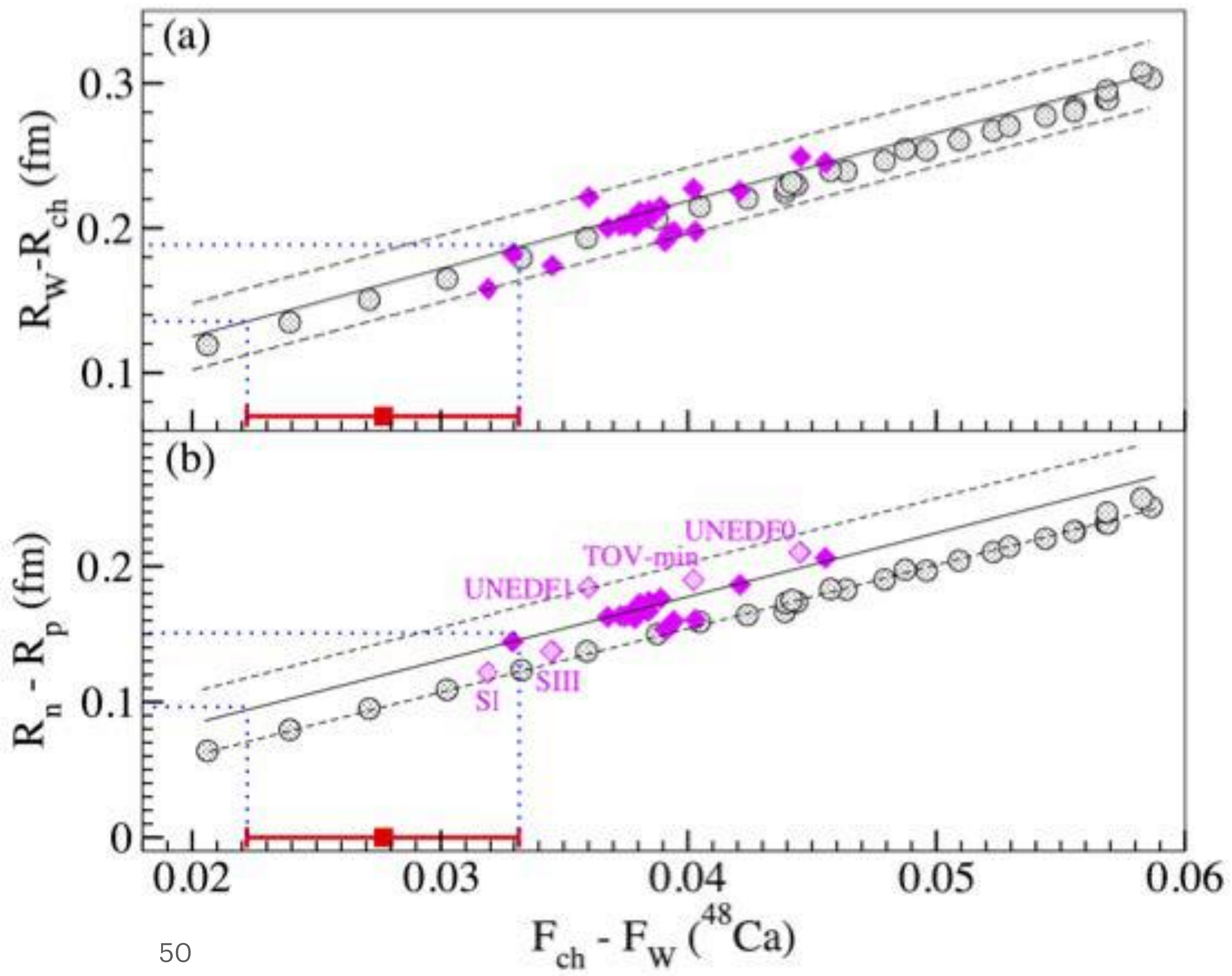
  where $F(q) = \frac{\int j_0(qr)\rho(r)d^3r}{\int \rho(r)d^3r}$, and $j_0(qr) = \frac{\sin(qr)}{qr}$ is spherical Bessel function

Weak Charge of Nuclei

- Weak charge: $Q_W = 2T_3 - 4Q_E \sin^2(\Theta_W)$
  where weak isospin $T_3 = -\frac{1}{2}$ for electron, up quark and neutron, $\frac{1}{2}$ for neutrino, down quark and proton
- Neutron weak charge: $Q_n = -1$ (–0.9878 with radiative correction)
  Proton weak charge $Q_p = 1 - 4\sin^2(\Theta_W)$ (0.0721 with radiative correction)
- Neutron form factor: $G_n^W = Q_n G_p^E + Q_p G_n^E + Q_n G_s^E$
  Proton form factor: $G_p^W = Q_p G_p^E + Q_n G_n^E + Q_n G_s^E$
- Weak charge distribution: $\rho_W(r) = \int d^3r' \left[G_n^W(r-r')\rho_n(r) + G_p^W(r-r')\rho_p(r)\right]$
- Electric charge distribution: $\rho_E(r) = \int d^3r' \left[G_n^E(r-r')\rho_n(r) + G_p^E(r-r')\rho_p(r)\right]$
- Additional complicity: many-body correction, center-of-mass correction, the magnetic contribution from spin-orbital current(SHF) or tensor density (RMF)


UNEDF1
SI
UNEDF0
SLY4
NL3
$F_{ch} - F_w$
q (fm^{-1})
0.06
0.05
0.04
0.03
0.02
0.01
0
-0.01
0
0.5
1
1.5
2
2.5


R_n-R_p (fm)
F_{ch}-F_w (PREX)
0.4
0.3
0.2
0.1
0
0
0.01
0.02
0.03
0.04
0.05
0.06


(a)
R_W-R_{ch} (fm)
0.3
0.2
0.1
(b)
R_n - R_p (fm)
UNEDF0
TOV-min
UNEDF1
SI
SIII
0.2
0.1
0
0.02
0.03
0.04
0.05
0.06
F_{ch} - F_W (^{48}Ca)

Back

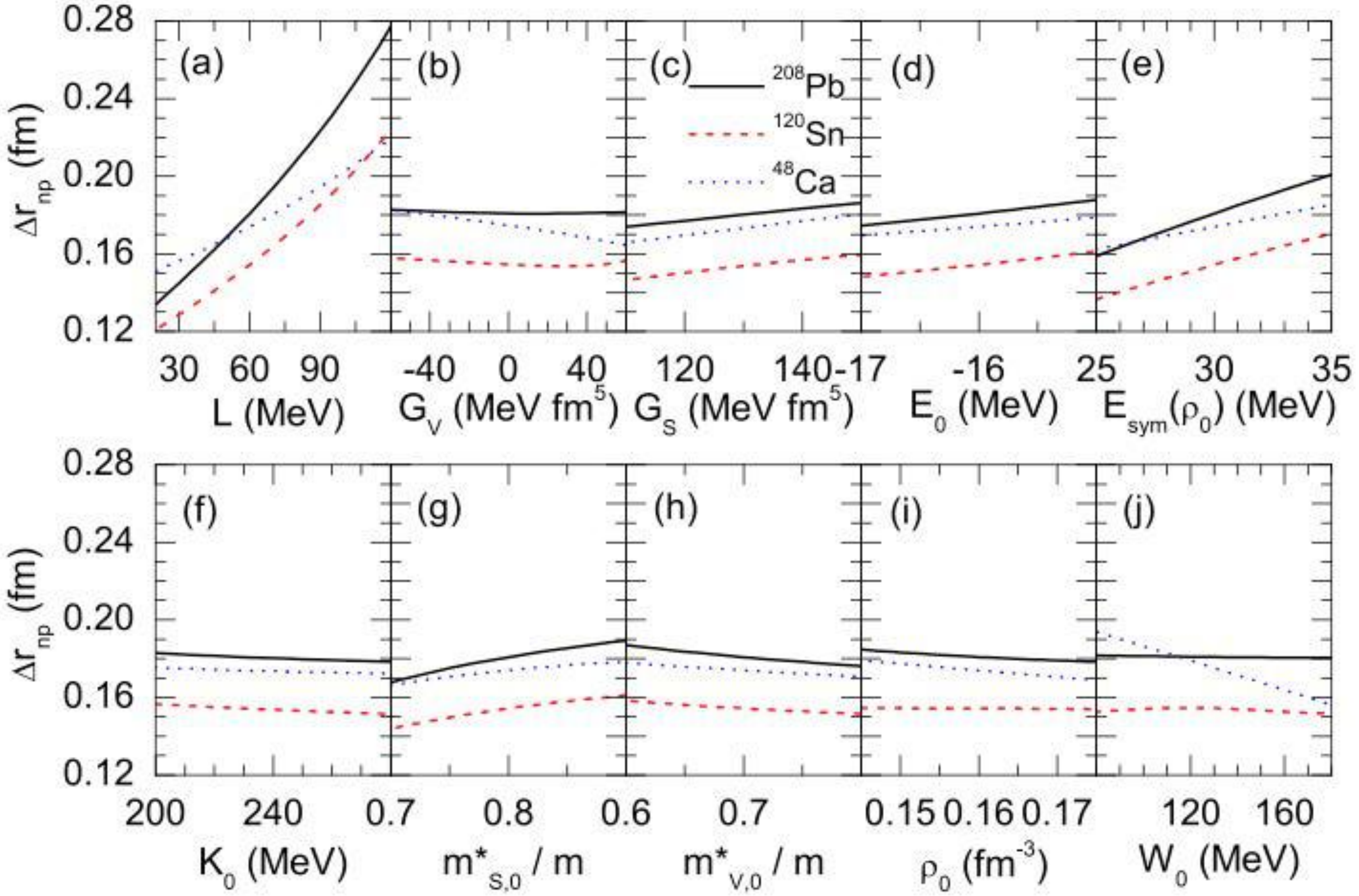
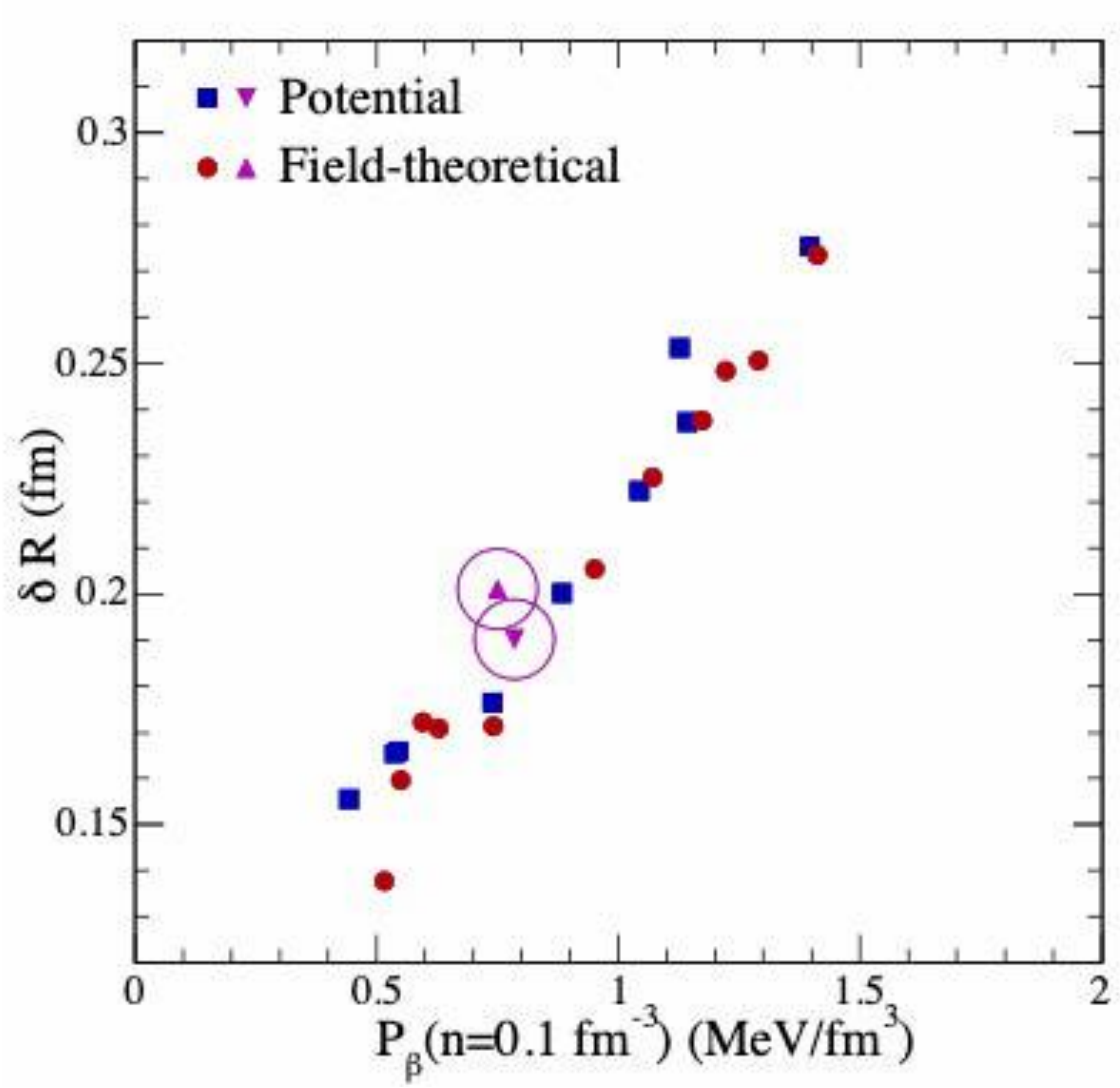
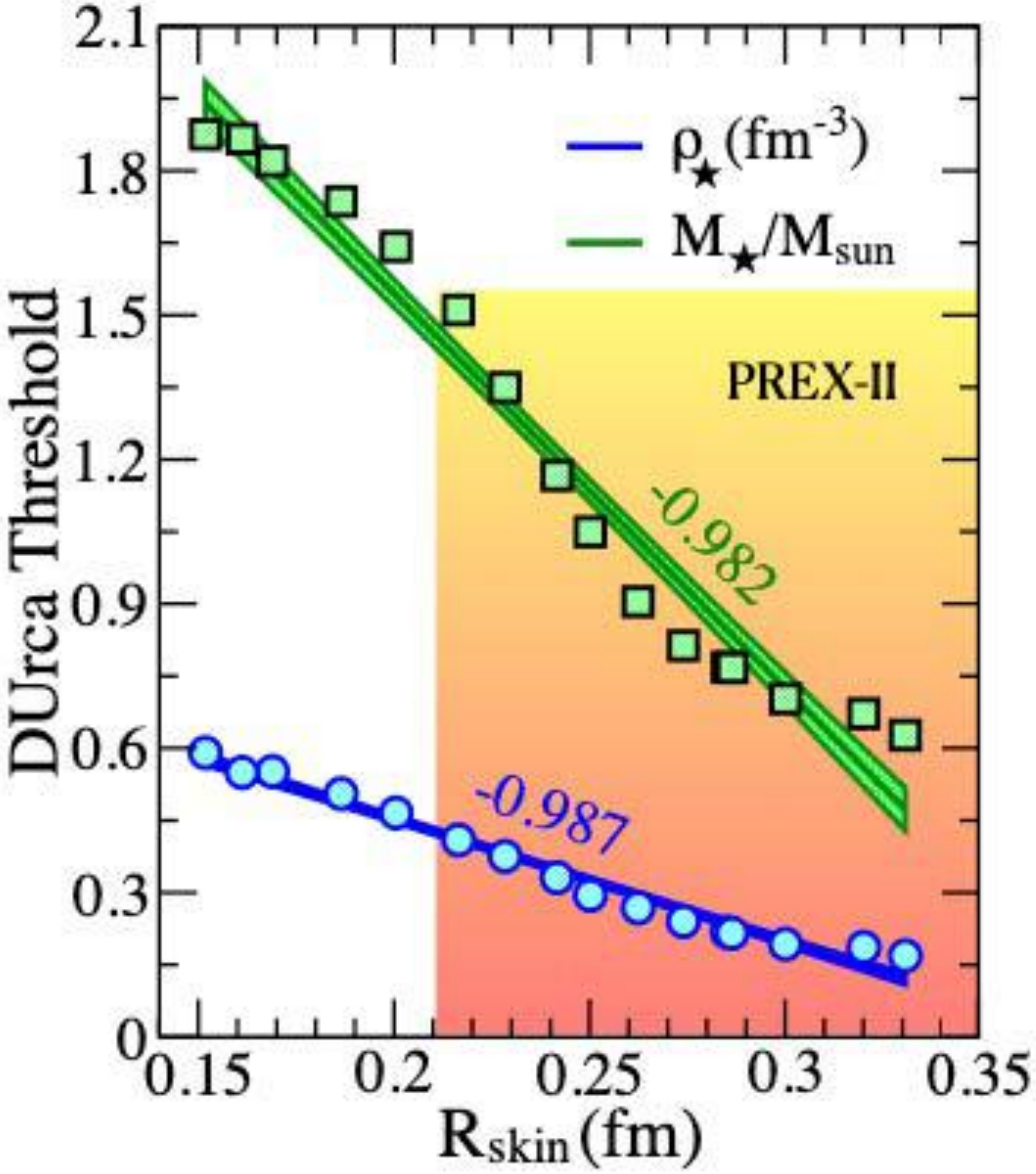
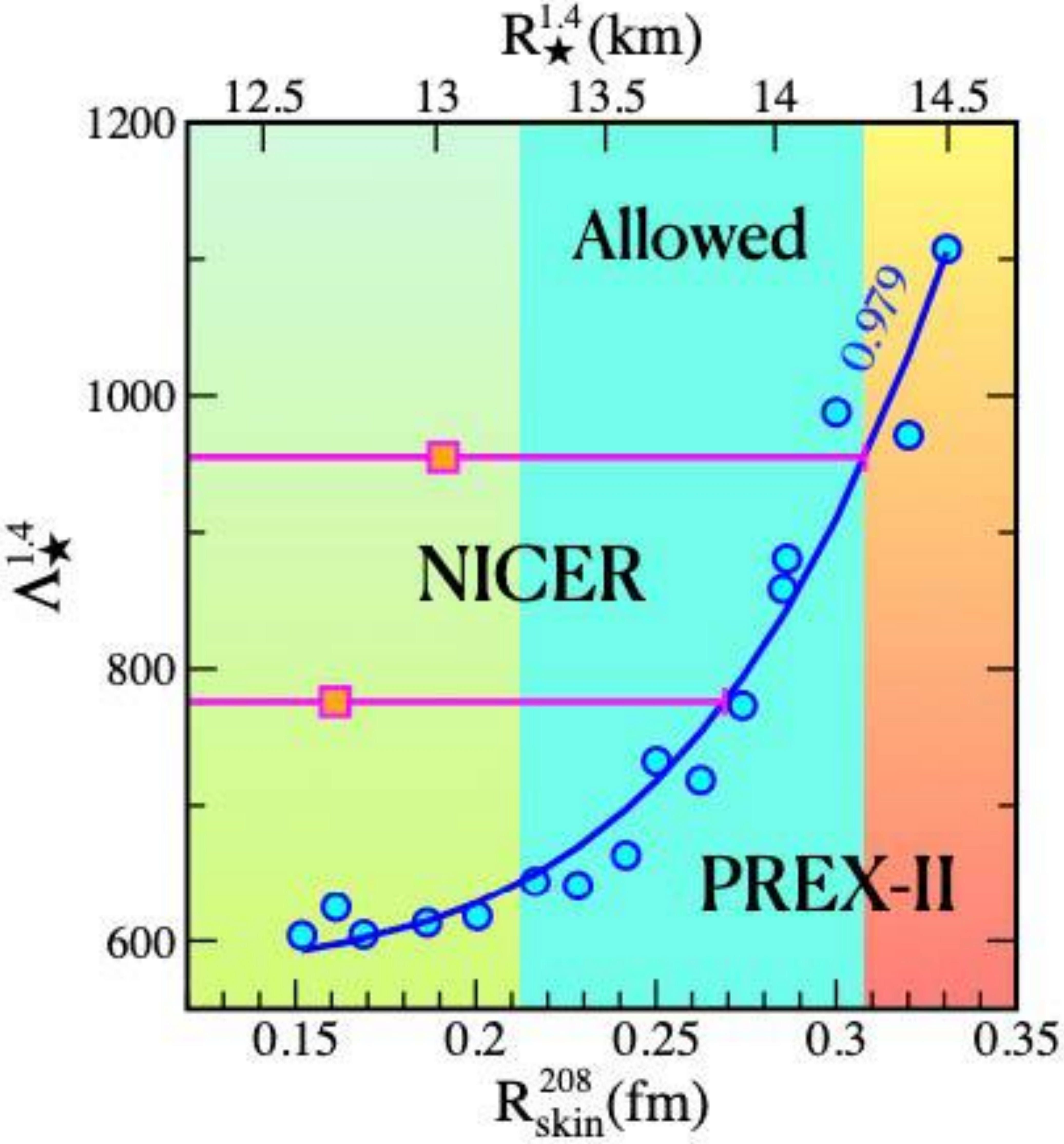


FIG. 3: (Color online) The neutron skin thickness Δr_{np} of ^{208}Pb, ^{120}Sn and ^{48}Ca from SHF with MSL0 by varying individually L (a), G_V (b), G_S (c), $E_0(\rho_0)$ (d), $E_{\rm sym}(\rho_0)$ (e), K_0 (f), $m^*_{s,0}$ (g), $m^*_{v,0}$ (h), ρ_0 (i), and W_0 (j).

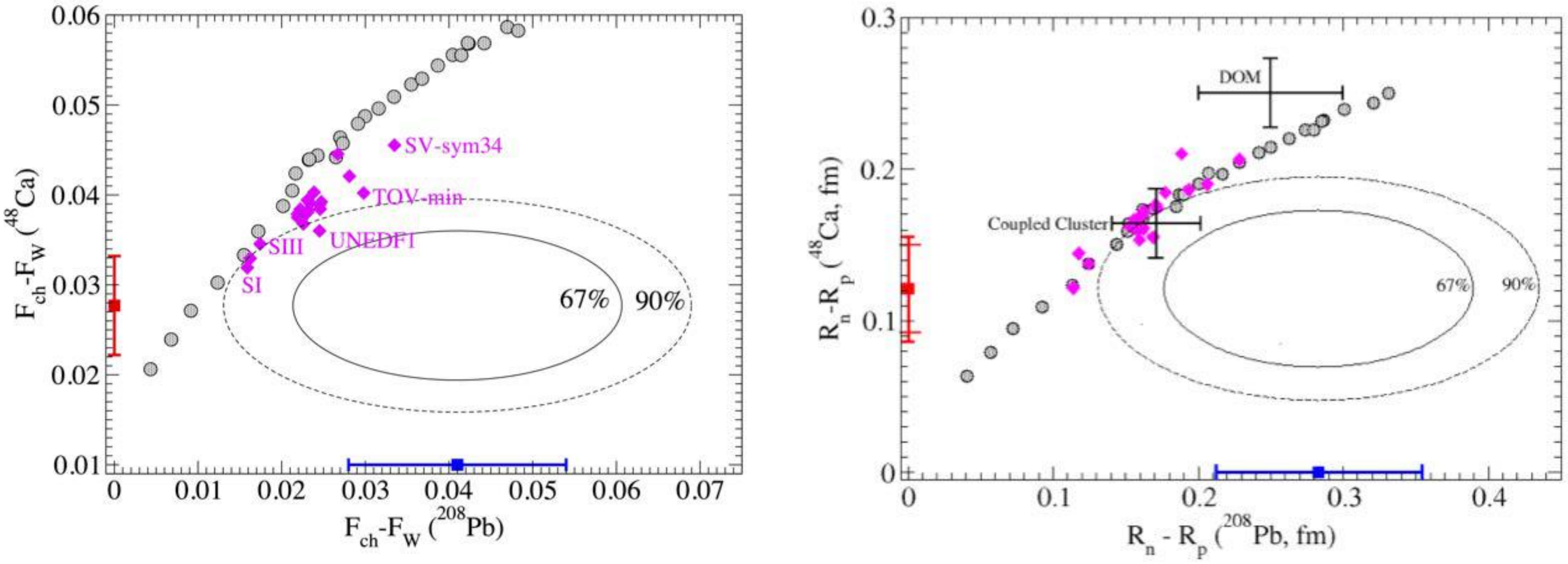


Back


DUrca Threshold
2.1
1.8
1.5
1.2
0.9
0.6
0.3
0
0.15
0.2
0.25
0.3
0.35
R_{skin} (fm)
$\rho_\star$ (fm^{-3})
$M_\star/M_{sun}$
PREX-II
-0.982
-0.987


$R_\star^{1.4}$(km)
12.5
13
13.5
14
14.5
1200
1000
800
600
$\Lambda_\star^{1.4}$
Allowed
NICER
PREX-II
0.979
0.15
0.2
0.25
0.3
0.35
R_{skin}^{208}(fm)

Back


0.06
0.05
0.04
0.03
0.02
0.01
F_{ch}-F_W (^{48}Ca)
0
0.01
0.02
0.03
0.04
0.05
0.06
0.07
F_{ch}-F_W (^{208}Pb)
SV-sym34
TOV-min
SIII
UNEDF1
SI
67%
90%
0.3
0.2
0.1
0
R_n-R_p (^{48}Ca, fm)
0
0.1
0.2
0.3
0.4
R_n - R_p (^{208}Pb, fm)
DOM
Coupled Cluster
67%
90%

Finite nuclei with MFT

(1) guess a set of initial density/current profiles $\{\rho(\mathbf{x})\}$ (this notation denotes the baryon and isospin densities, plus any other spin or scalar densities); most can be set to zero or initialized with a Fermi shape with appropriate parameters;

(2) evaluate a functional of the $\{\rho(\mathbf{x})\}$, yielding a *local* single-particle potential $V_{\mathrm{s}}(\mathbf{x})$;

(3) solve the Dirac or Schrödinger equation for the lowest A eigenvalues and eigenfunctions $\{\epsilon_\alpha, \psi_\alpha\}$:

$$\left[-i\boldsymbol{\alpha}\cdot\boldsymbol{\nabla} + \beta M + \beta V_{\mathrm{s}}(\mathbf{x})\right]\psi_\alpha(\mathbf{x}) = \epsilon_\alpha \psi_\alpha(\mathbf{x}),$$

or

$$\left[-\frac{\nabla^2}{2M} + V_{\mathrm{s}}(\mathbf{x})\right]\psi_\alpha(\mathbf{x}) = \epsilon_\alpha \psi_\alpha(\mathbf{x});$$

(4) compute new densities, e.g.,

$$\rho(\mathbf{x}) = \sum_{\alpha=1}^{A} \left|\psi_\alpha(\mathbf{x})\right|^2,$$

and other observables (which are functionals of $\{\epsilon_\alpha, \psi_\alpha\}$);

(5) repeat steps (2)–(4) until the changes from iteration to iteration are acceptably small, i.e., until the solution is self-consistent.

$$\sigma_0'' = m_\sigma^2 \sigma_0 - g_\sigma n_s + \frac{\kappa}{2} g_\sigma^3 \sigma_0^2 + \frac{\lambda}{6} g_\sigma^4 \sigma_0^3 - g_\rho^2 \rho_0^2 \frac{\partial f}{\partial \sigma_0} \ ,$$

$$\omega_0'' = m_\omega^2 \omega_0 - g_\omega n + \frac{\zeta}{6} g_\omega^4 \omega_0^3 + g_\rho^2 \rho_0^2 \frac{\partial f}{\partial \omega_0} \ ,$$

$$\rho_0'' = m_\rho^2 \rho_0 - \frac{1}{2} g_\rho \alpha + 2 g_\rho^2 \rho_0 f + \frac{\xi}{6} g_\rho^4 \rho_0^3 \ ,$$

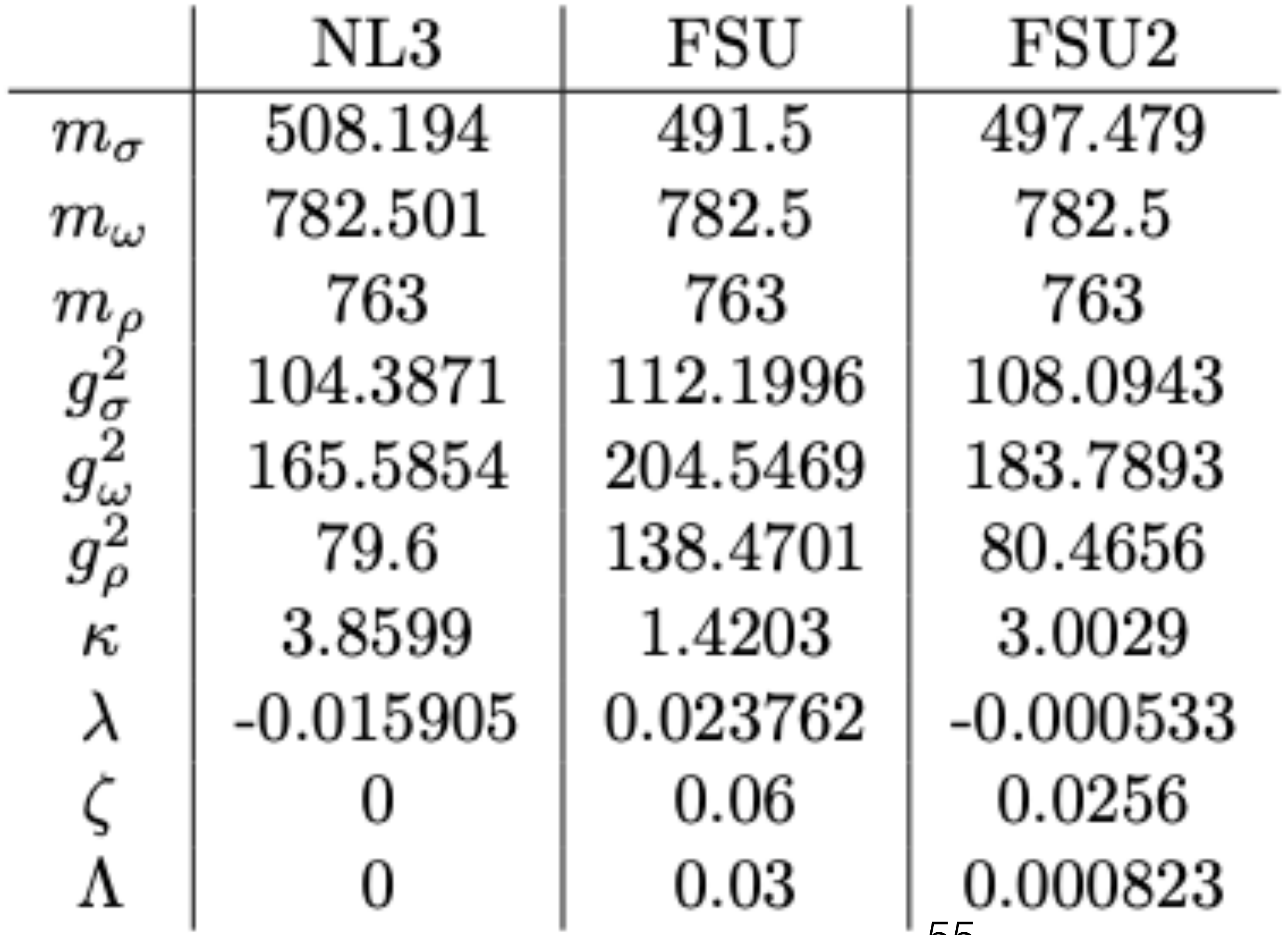
$$0 = (i\not{\partial} - g_\omega \omega_0 \gamma_0 + \tfrac{1}{2} g_\rho \rho_0 \gamma_0 - M + g_\sigma \sigma_0)\psi_n \ ,$$

$$0 = (i\not{\partial} - g_\omega \omega_0 \gamma_0 - \tfrac{1}{2} g_\rho \rho_0 \gamma_0 - M + g_\sigma \sigma_0)\psi_p \ ,$$

FSU-type RMF model

$$\mathscr{L}_0 = \bar{\psi}(i\gamma^\mu\partial_\mu - m)\psi + \frac{1}{2}\left(\partial^\mu\sigma\partial_\mu\sigma - m_\sigma^2\sigma^2\right) + \frac{1}{2}m_\omega^2\omega^\mu\omega_\mu - \frac{1}{4}\Omega^{\mu\nu}\Omega_{\mu\nu} + \frac{1}{2}m_\rho^2\boldsymbol{\rho}^\mu\boldsymbol{\rho}_\mu - \frac{1}{4}\boldsymbol{\rho}^{\mu\nu}\boldsymbol{\rho}\mu\nu$$

$$\mathscr{L} = \mathscr{L}_0 + \bar{\psi}\left(g_\sigma\sigma - g_\omega\gamma^\mu\omega_\mu - \frac{g_\rho}{2}\gamma^\mu\boldsymbol{\tau}\boldsymbol{\rho}_\mu\right)\psi - \frac{\kappa}{3!}(g_\sigma\sigma)^3 - \frac{\lambda}{4!}(g_\sigma\sigma)^4 + \frac{\zeta}{4!}(g_\omega^2\omega^\mu\omega_\mu)^2 + \Lambda_{\omega\rho}(g_\rho^2\boldsymbol{\rho}^\mu\boldsymbol{\rho}_\mu)(g_\omega^2\omega^\mu\omega_\mu)$$



	NL3	FSU	FSU2
m_σ	508.194	491.5	497.479
m_ω	782.501	782.5	782.5
m_ρ	763	763	763
g_σ^2	104.3871	112.1996	108.0943
g_ω^2	165.5854	204.5469	183.7893
g_ρ^2	79.6	138.4701	80.4656
κ	3.8599	1.4203	3.0029
λ	-0.015905	0.023762	-0.000533
ζ	0	0.06	0.0256
Λ	0	0.03	0.000823

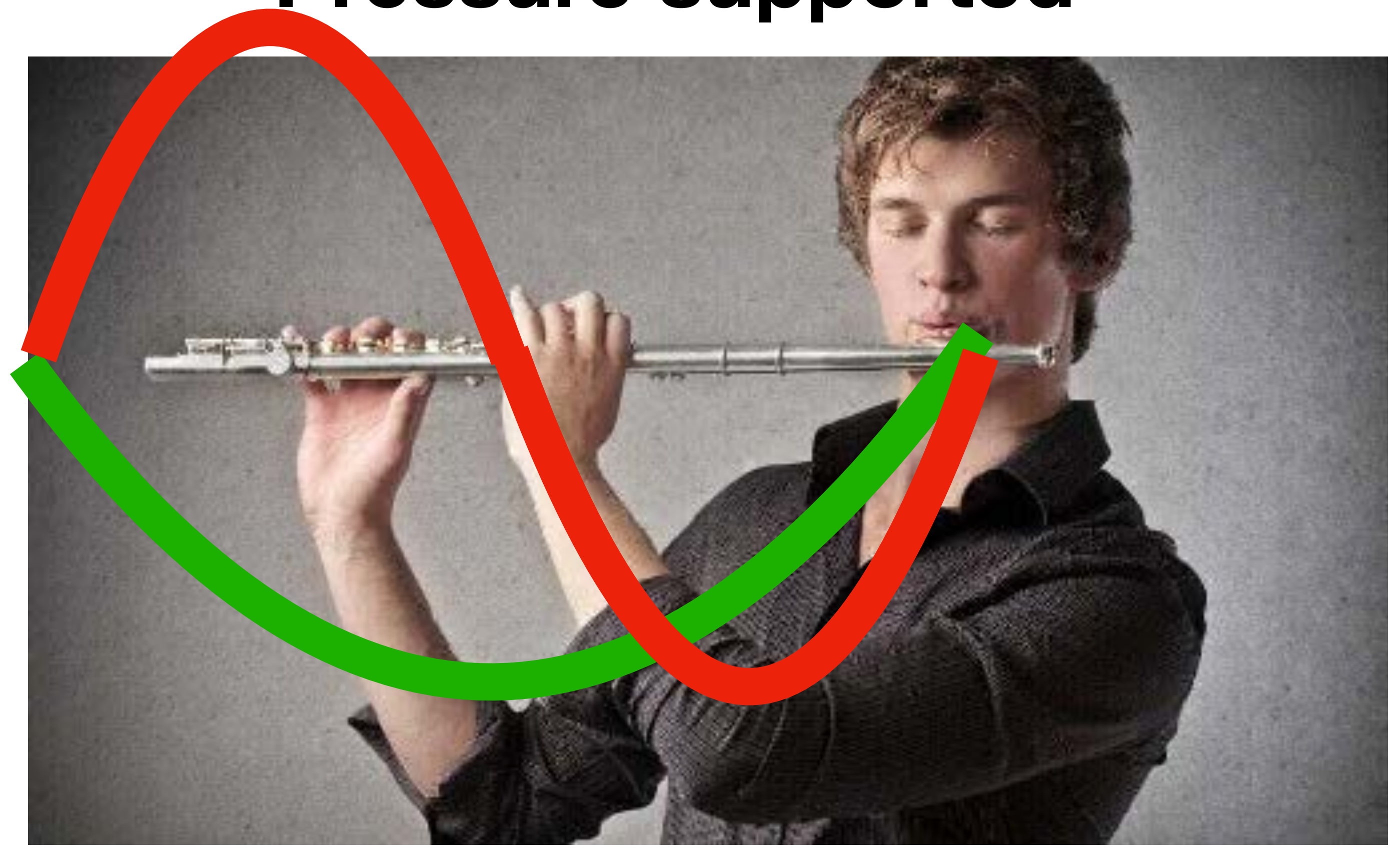
Scalar isoscalar
Vector isoscalar
Vector isovector

Back up slides
NS Oscillations

Oscillation modes

$$A(r)e^{i\omega t} \qquad \omega = 2\pi\nu + \frac{\mathbf{i}}{\tau}$$

Pressure supported



Standing sound wave of order n:

$$\omega^2 \approx \frac{dp}{d\varepsilon}k^2 \qquad k = \frac{\sqrt{l(l+1)}n}{2\pi R}$$

n=0 f-mode (fundamental)

n=1 p-mode (pressure)

Gravity & interface

Stratified fluid in uniform gravity g:

$$\omega^2 = \frac{(\varepsilon_+ - \varepsilon_-)gk}{\varepsilon_+/\tanh(kd_+) + \varepsilon_-/\tanh(kd_-)}$$

Discontinuity g-mode (interface gravity mode)

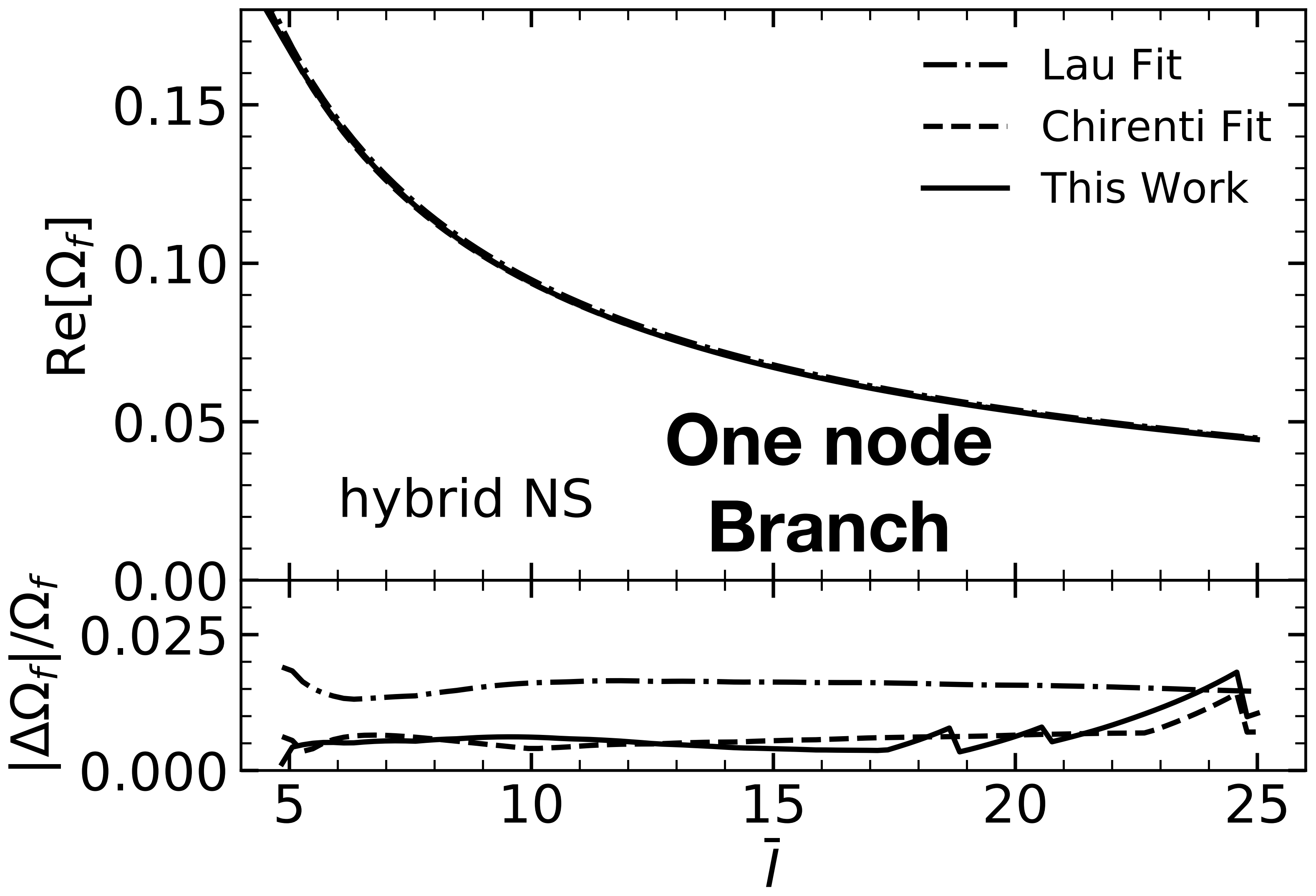
Gravity & x gradient

Buoyancy oscillation in uniform gravity g:

$$\omega^2 \approx \mathcal{N}^2 = -\frac{g}{\varepsilon}\left(\frac{\partial\varepsilon}{\partial x}\right)_p \frac{dx}{dr} = g^2\left(\frac{1}{c_{eq}^2} - \frac{1}{c_{ad}^2}\right) \qquad x = \{\frac{n_p}{n_B}, \frac{n_p}{n_B}, T, \ldots\}$$

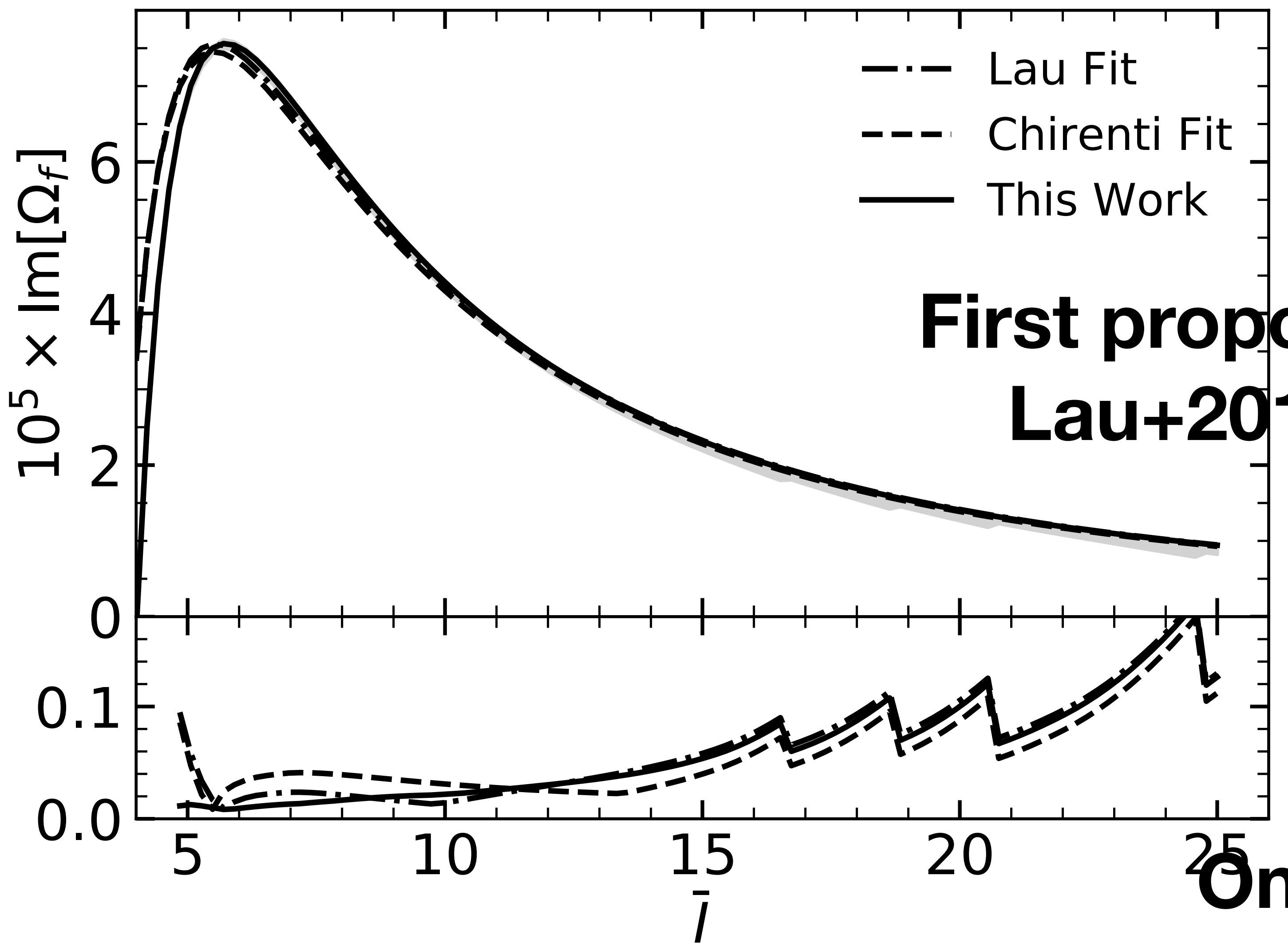
Chemical g-mode (gravity with composition gradient)

f-mode universal relations



One node Branch

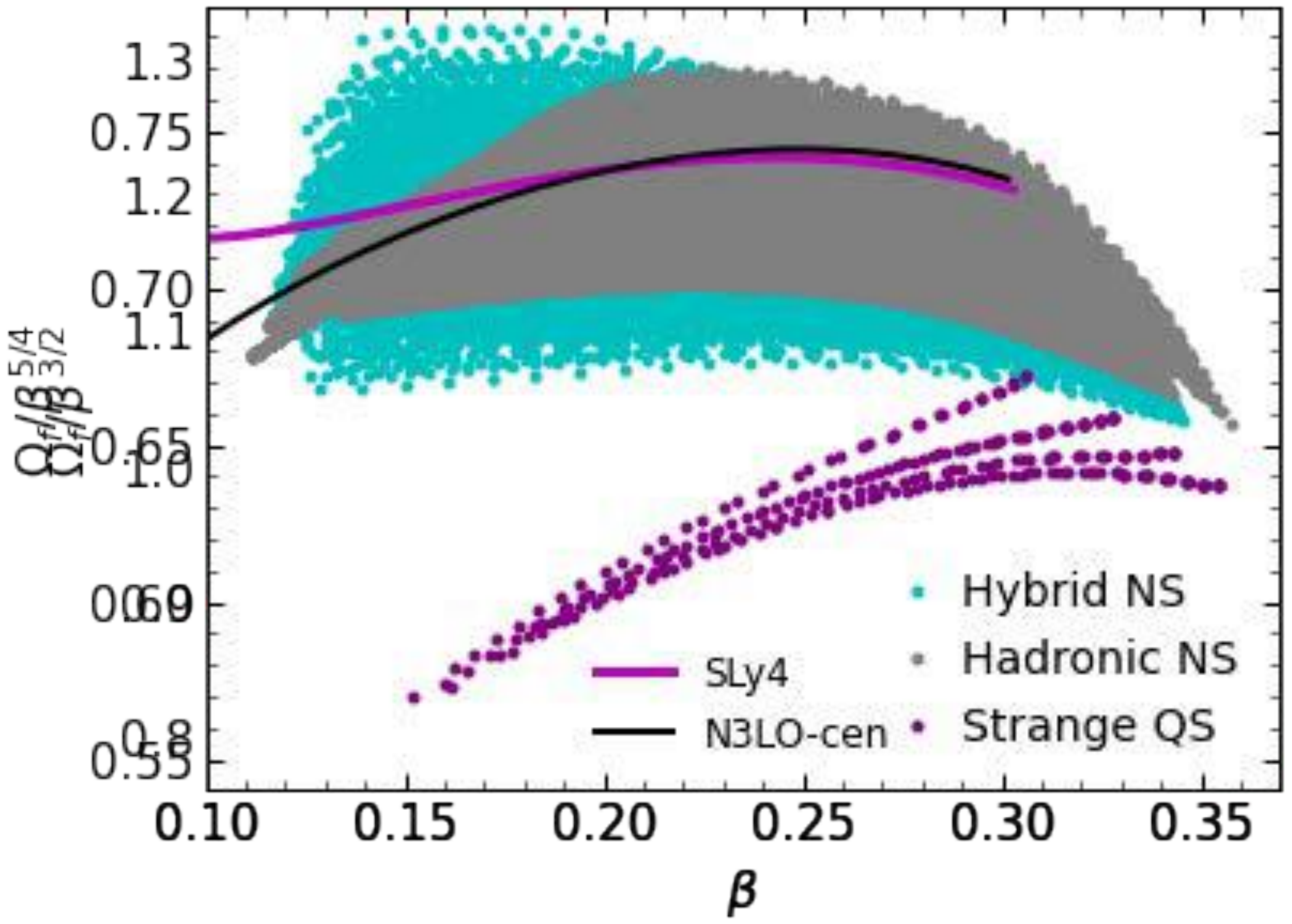
0.1% deviation except for one-node branches



First proposed Lau+2010

One-node Branch

1% deviation except for one-node branches



$$\Omega_f = GM\omega_f/c^3 \qquad (\propto \beta^{3/2} \text{ in Newtonian})$$

1. $\Omega_f - \Lambda$ **is slightly weaker than** $\Omega_f - \bar{I}$
2. Ω_f **is close related to compactness** β

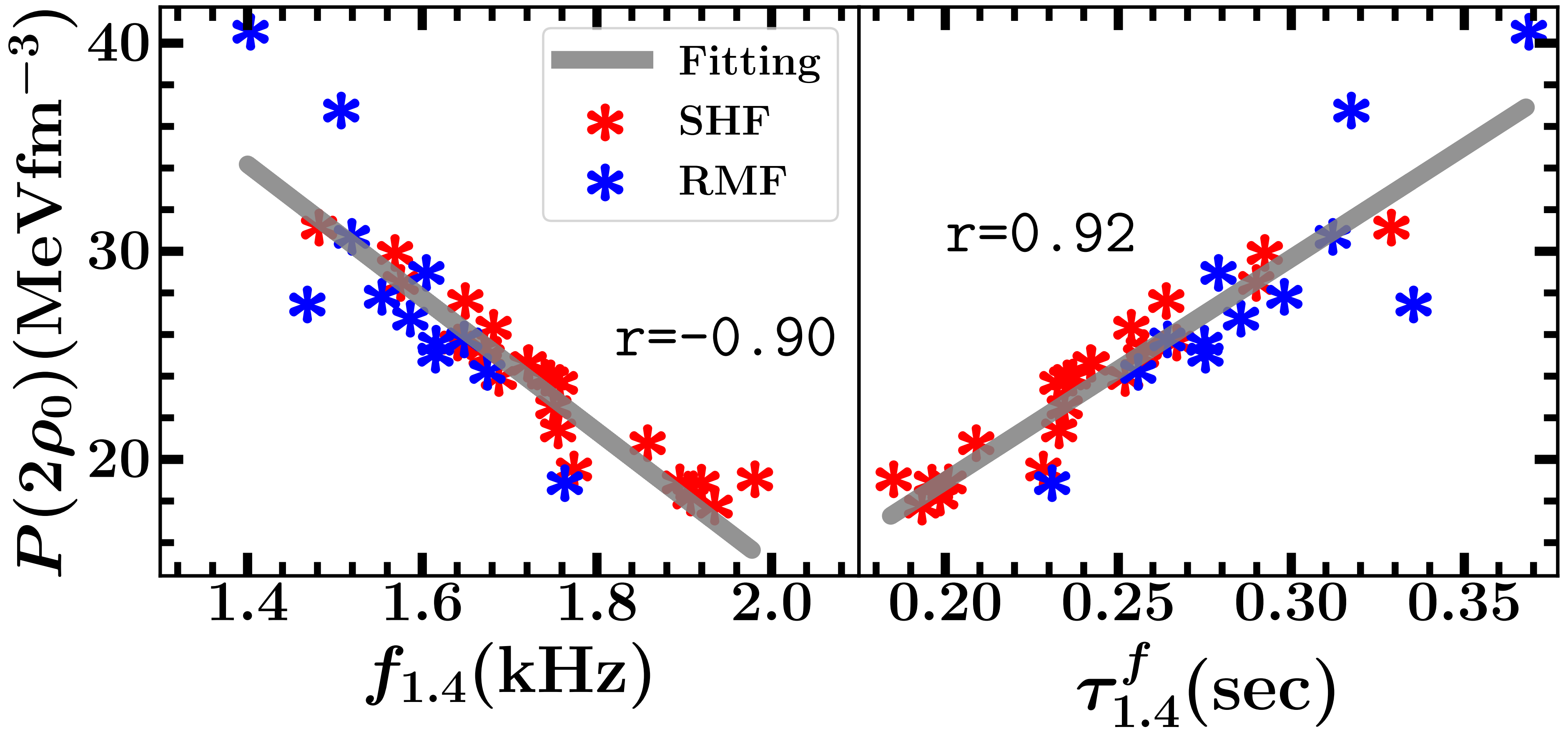
$$\Omega_f = (0.887 \pm 0.061)\ \beta^{3/2}$$ **Quark star**

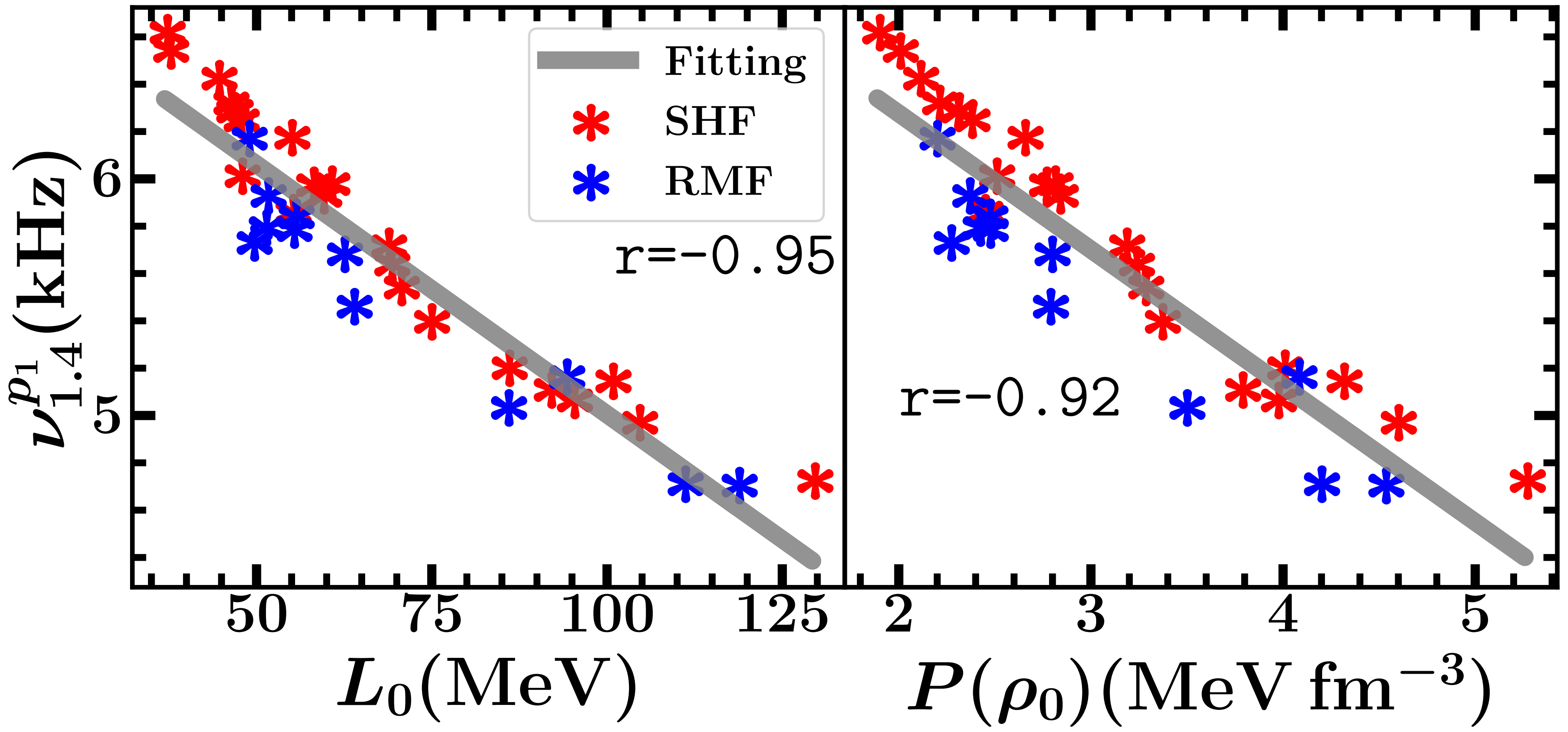
$$\Omega_f = (0.714 \pm 0.056)\ \beta^{5/4}$$ **Hadronic & hybrid NS**

Zhao & Lattimer 2022

BACK

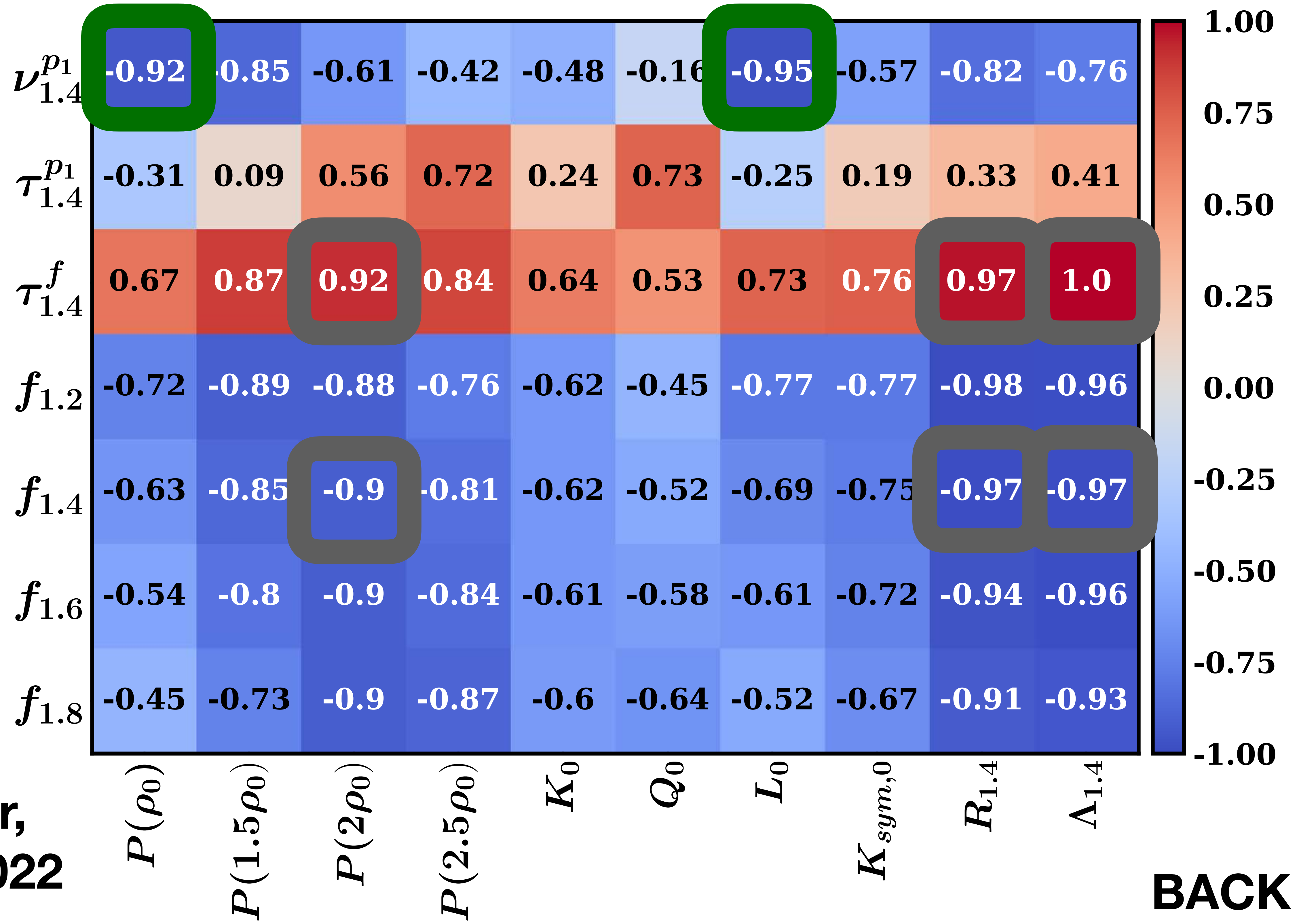
p-modes with SHF and RMF EOSs





- $\nu^{p_1}_{1.4}$ is sensitive to EOS around saturation
- $f_{1.4}$ and $\tau^{f}_{1.4}$ are sensitive to EOS around twice saturation density

Higher order p-mode is sensitive to EOS at lower density



	$P(\rho_0)$	$P(1.5\rho_0)$	$P(2\rho_0)$	$P(2.5\rho_0)$	K_0	Q_0	L_0	$K_{sym,0}$	$R_{1.4}$	$\Lambda_{1.4}$
$\nu^{p_1}_{1.4}$	-0.92	-0.85	-0.61	-0.42	-0.48	-0.16	-0.95	-0.57	-0.82	-0.76
$\tau^{p_1}_{1.4}$	-0.31	0.09	0.56	0.72	0.24	0.73	-0.25	0.19	0.33	0.41
$\tau^{f}_{1.4}$	0.67	0.87	0.92	0.84	0.64	0.53	0.73	0.76	0.97	1.0
$f_{1.2}$	-0.72	-0.89	-0.88	-0.76	-0.62	-0.45	-0.77	-0.77	-0.98	-0.96
$f_{1.4}$	-0.63	-0.85	-0.9	-0.81	-0.62	-0.52	-0.69	-0.75	-0.97	-0.97
$f_{1.6}$	-0.54	-0.8	-0.9	-0.84	-0.61	-0.58	-0.61	-0.72	-0.94	-0.96
$f_{1.8}$	-0.45	-0.73	-0.9	-0.87	-0.6	-0.64	-0.52	-0.67	-0.91	-0.93

Athul, Zhao, Kumar, Agrawal, Prakash 2022

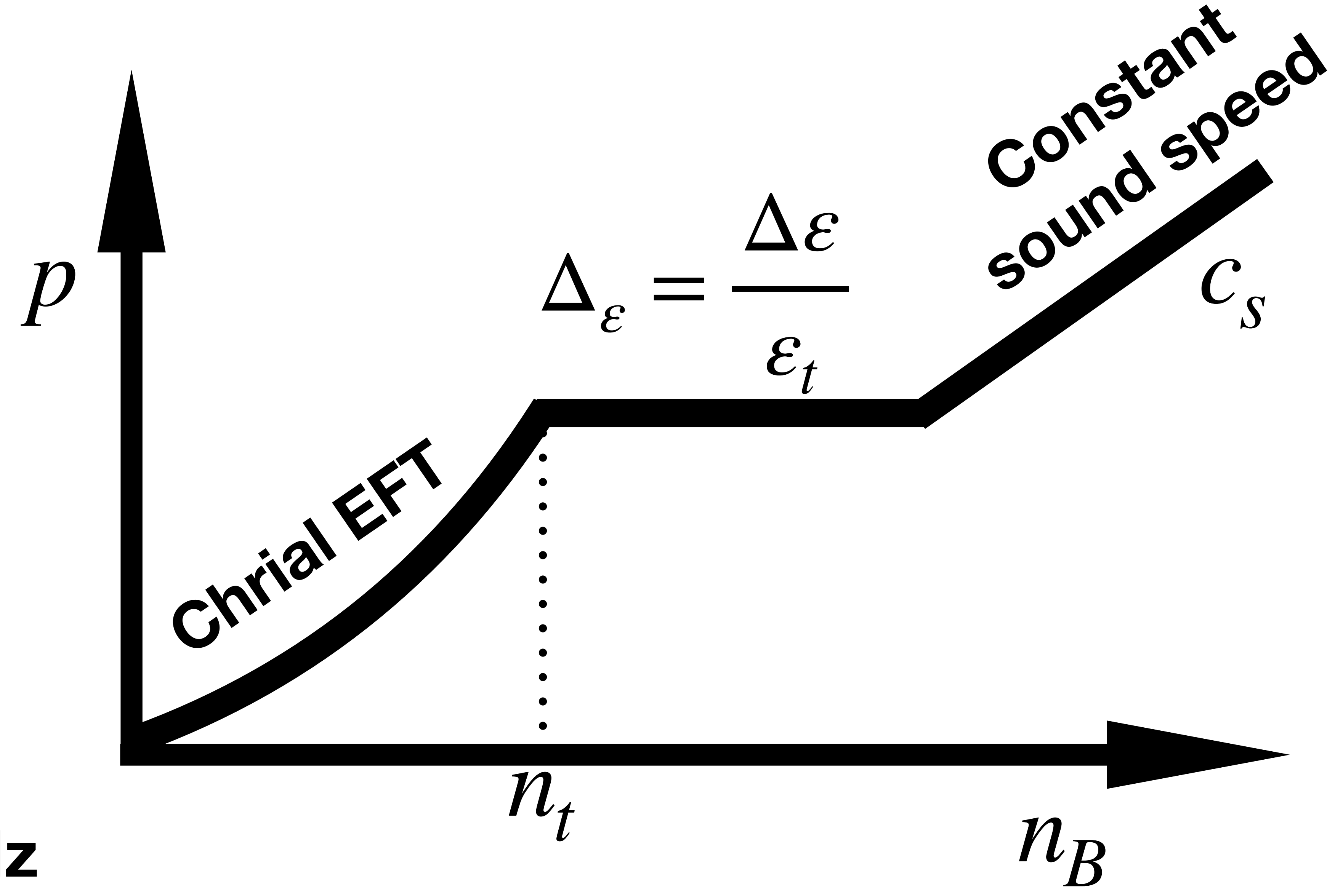
BACK

Discontinuity g-mode

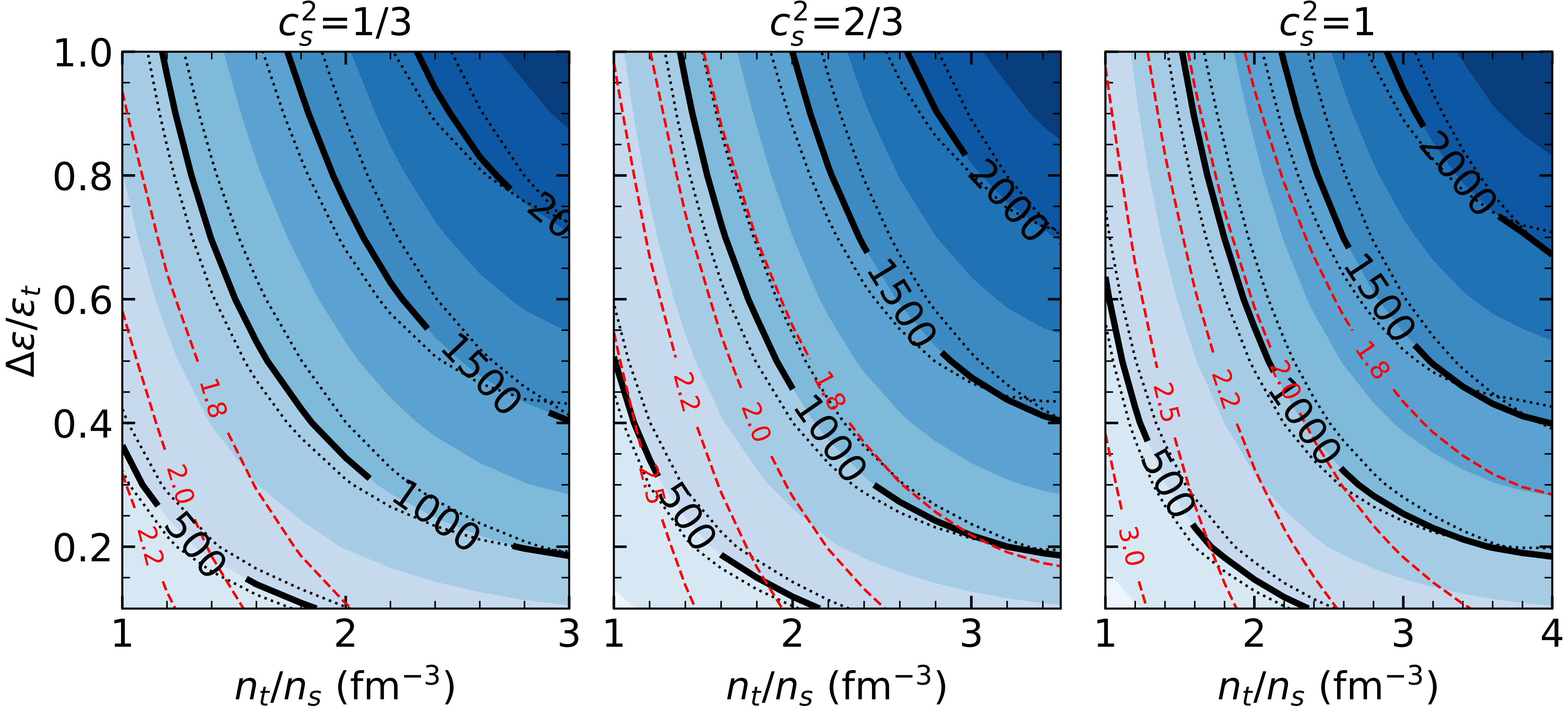
First order transition

$$\Omega_g^2 \approx \frac{\beta^3 (M_t/M)(R/R_t)^3 (\Delta\varepsilon/\varepsilon_t) D \tanh[D]}{1 + \Delta\varepsilon/\varepsilon_t + \tanh[D]/\tanh[D(R/R_t - 1)]}$$

Ω_g **is sensitive to structure factors,** $\frac{R_t}{R}, \frac{M_t}{M}, \frac{\Delta\varepsilon}{\varepsilon}$



Contour of $\nu_g(n_t, \Delta_\varepsilon, c_s^2)|_{M=M_{max}}$ **Hz**



Dotted: Chrial EFT Uncertanty 5%

Dashed: maximum mass

Frequency: $\nu_g < 0.8\ (1.5)$ kHz for $c_s^2 = c^2/3\ (c^2)$

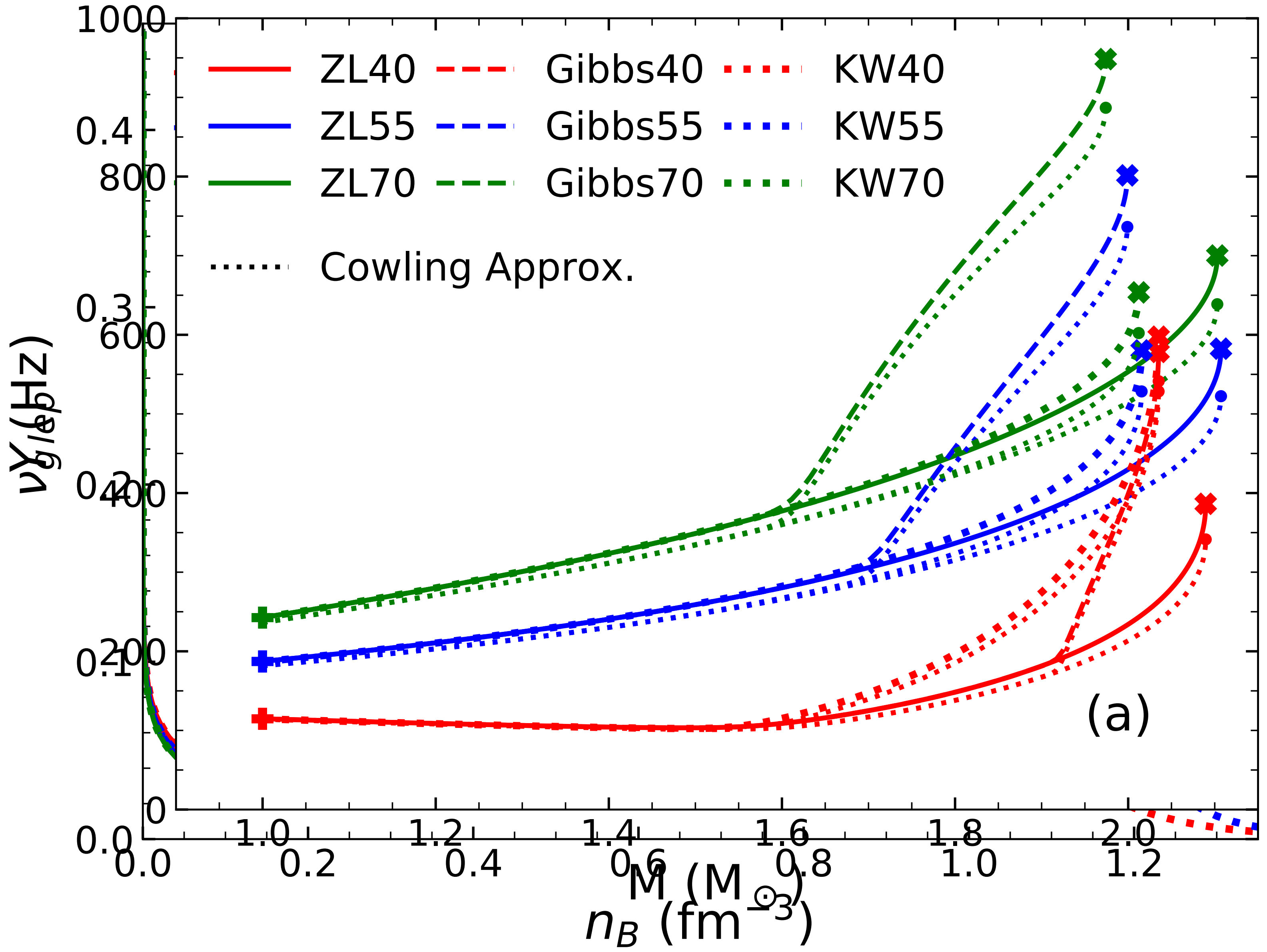
Damping time: $\tau > 100\ (10000)$ s

Zhao & Lattimer 2022

BACK

Compositional g-mode universal relation

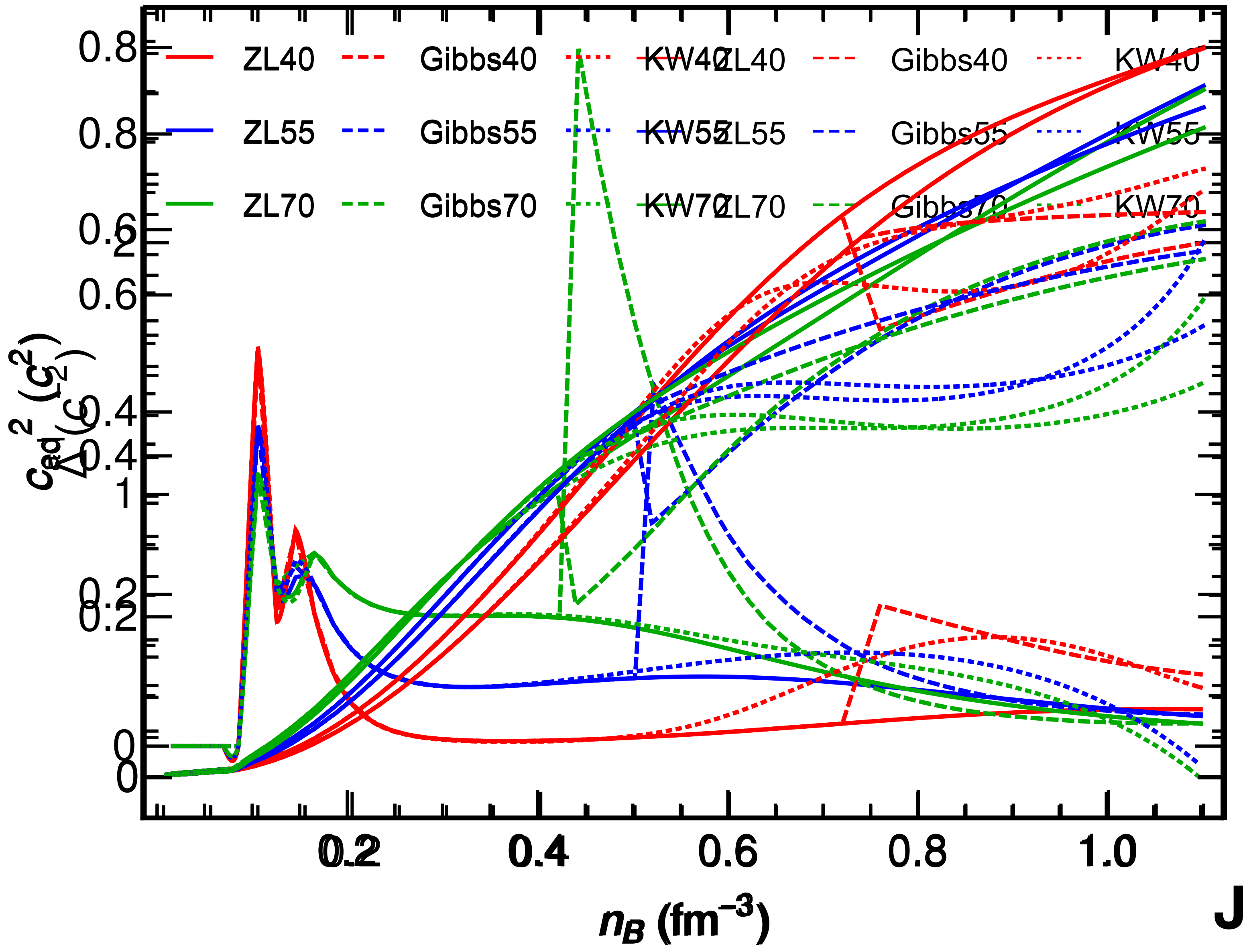
Hadronic: ZL First-order: Gibbs Crossover: KW

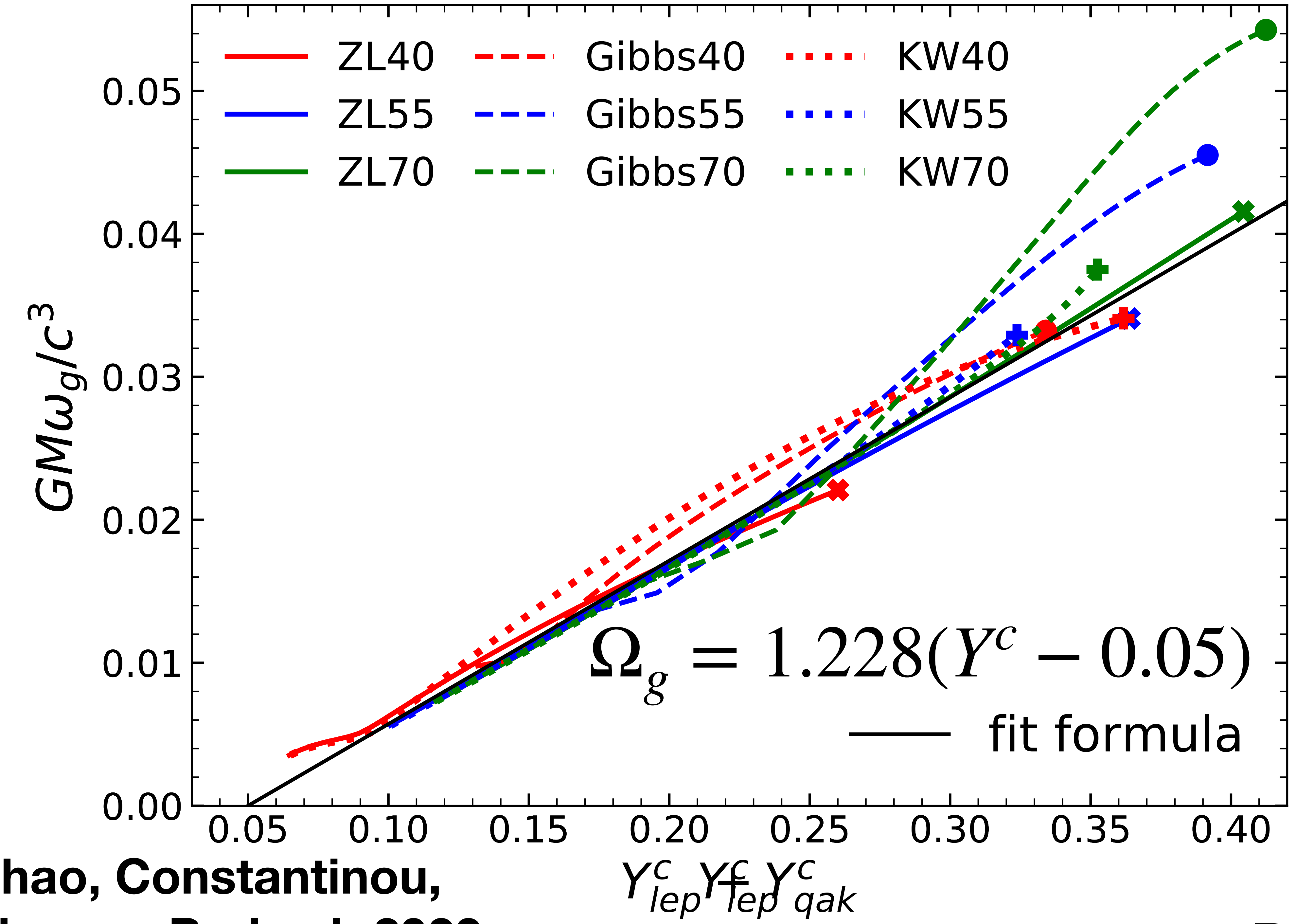


$$\nu_g^2 = g^2 e^{\nu-\lambda}\Delta(c^{-2}) \qquad \Delta(c^{-2}) = g^2(\frac{1}{c_{ad}^2} - \frac{1}{c_{eq}^2})$$

is sensitive to:

1. **Symmetry energy S(n)**
2. **Number of particle species**
3. **Proto-NS neutrino emission**
4. **Bulk viscosity**





Zhao, Constantinou, Jaikumar, Prakash 2022

BACK

Gravitational radiation of NS oscillation

- The amplitude of observed oscillations is

$$h(t) = h_0 e^{-t/\tau} \cos \omega t \qquad\qquad A(r)e^{i\omega t} \quad \omega = 2\pi\nu + \frac{\mathbf{i}}{\tau}$$

- The observed GW energy flux is

$$F(t) = \frac{c^3\omega^2h_0^2}{16\pi G}e^{-2t/\tau} = 3.17e^{-2t/\tau}\left(\frac{\nu}{\textbf{kHz}}\right)^2\left(\frac{h_0}{10^{-22}}\right)^2 \ \textbf{ergs cm}^{-2}\textbf{s}^{-1}$$

- The total GW energy is

$$E = \frac{c^3\omega^2h_0^2\tau D^2}{8G} = 4.27\times10^{49}\left(\frac{\nu}{\textbf{kHz}}\right)^2\left(\frac{h_0}{10^{-23}}\right)^2\left(\frac{\tau}{0.1\ \textbf{s}}\right)\left(\frac{D}{\textbf{15 Mpc}}\right)^2 \textbf{ergs}$$

- supernovae remnant: 10^44-10^47 ergs

  merger remnant: 10^51-10^52 ergs

$D < 20$ **kpc**	$D < 200$ **kpc**
	A few per century
$D \lesssim 20-45$ **Mpc**	$D \lesssim 200-450$ **Mpc**
	0.06-4 per year
aLIGO	**3G**

Observation of Oscillations of NS

- Direct observation:

matter motion
1. BNS merger remnant
2. Core-collapse SNe
3. Star quake (glitches)
4. NS close encounter

↓

spacetime variation

↓

gravitational wave radiation

- Indirect observation:

Binary NS inspiring

↓

Orbital angular momentum transfer

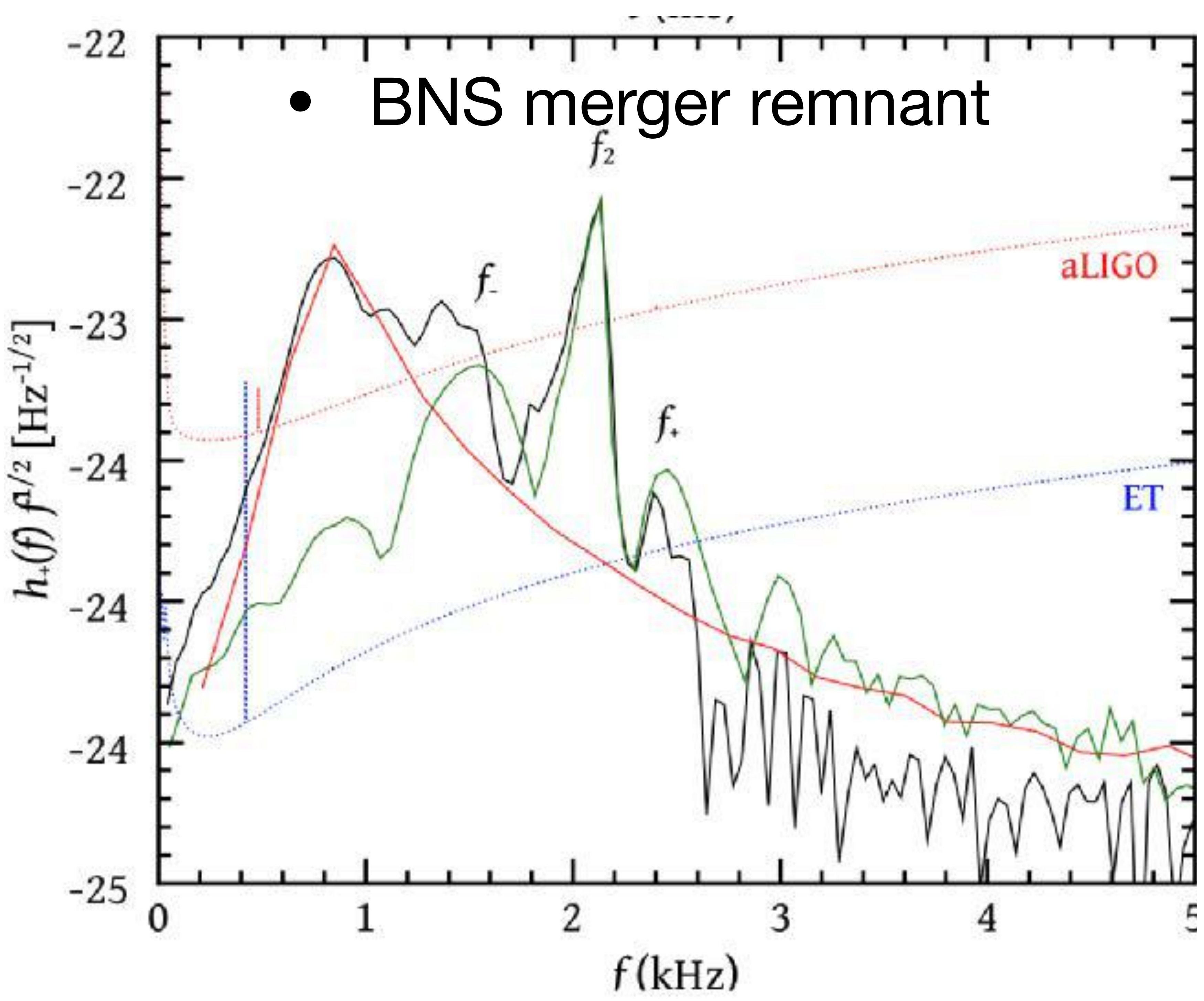
↓

gravitational wave form information

- Instrument:
  Comic explore (US)
  Einstein Telescope (Europe)

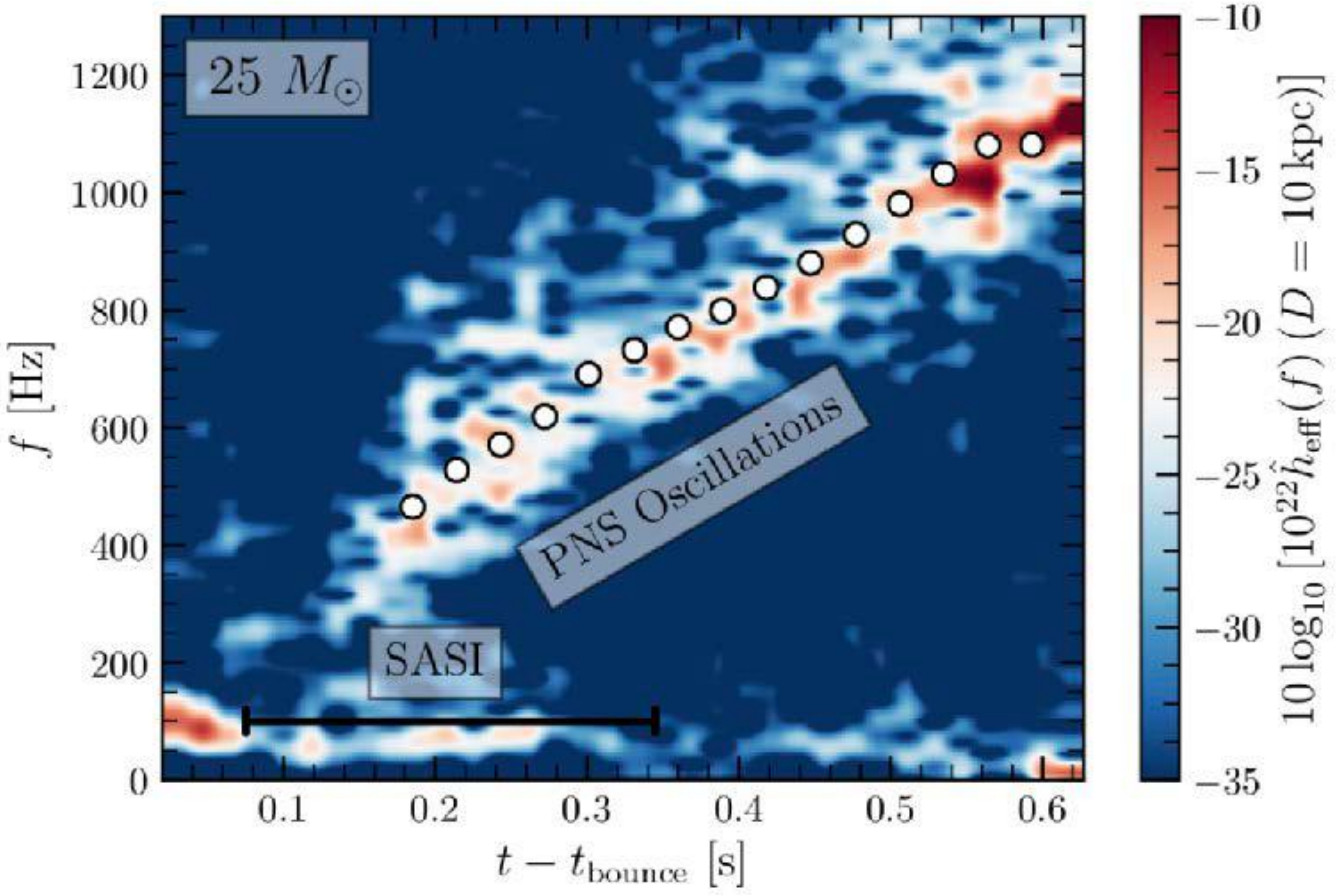
Oscillations of NS in simulation

- BNS merger remnant



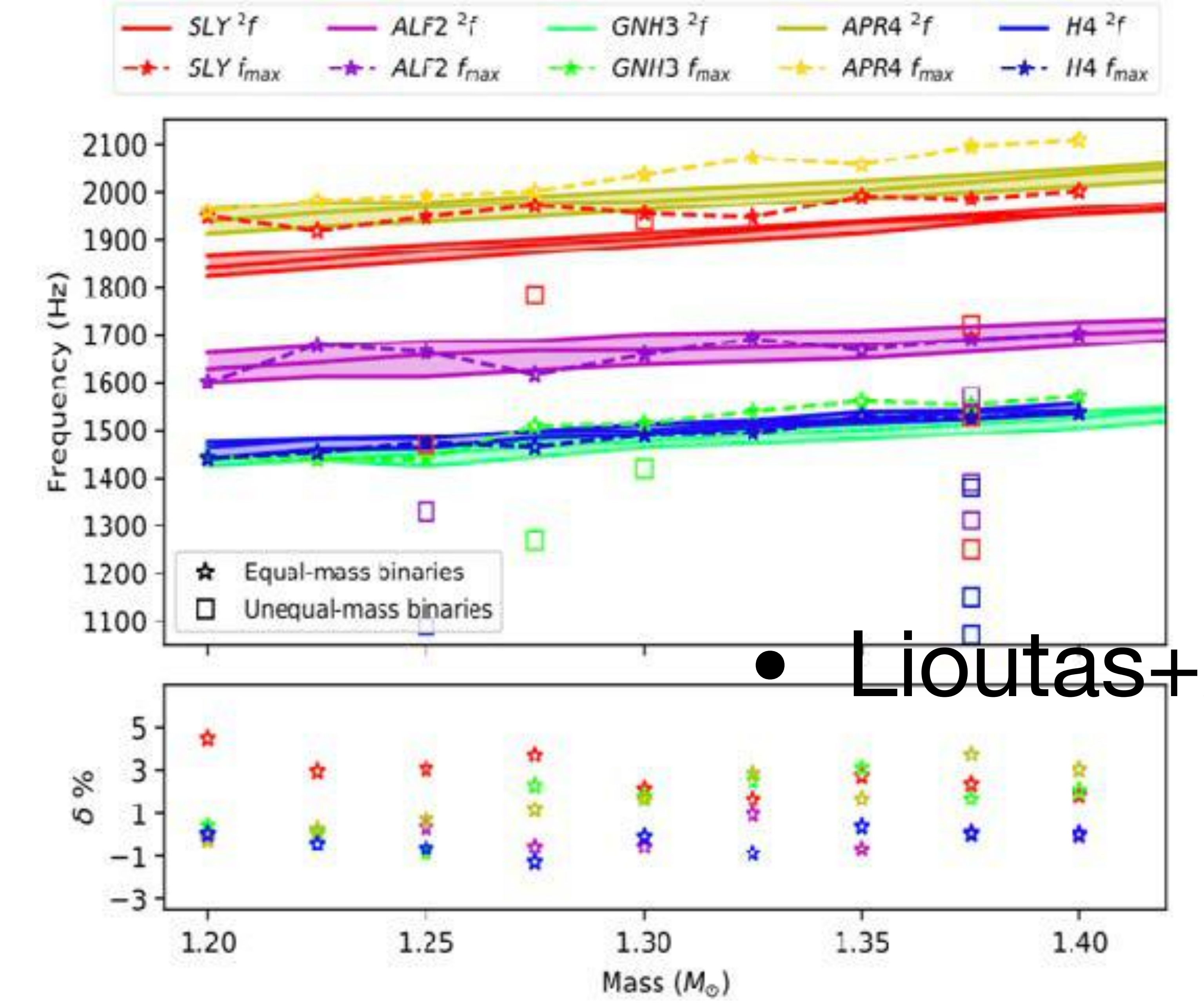
- Stergioulas+2011

- Core-collapse SNe

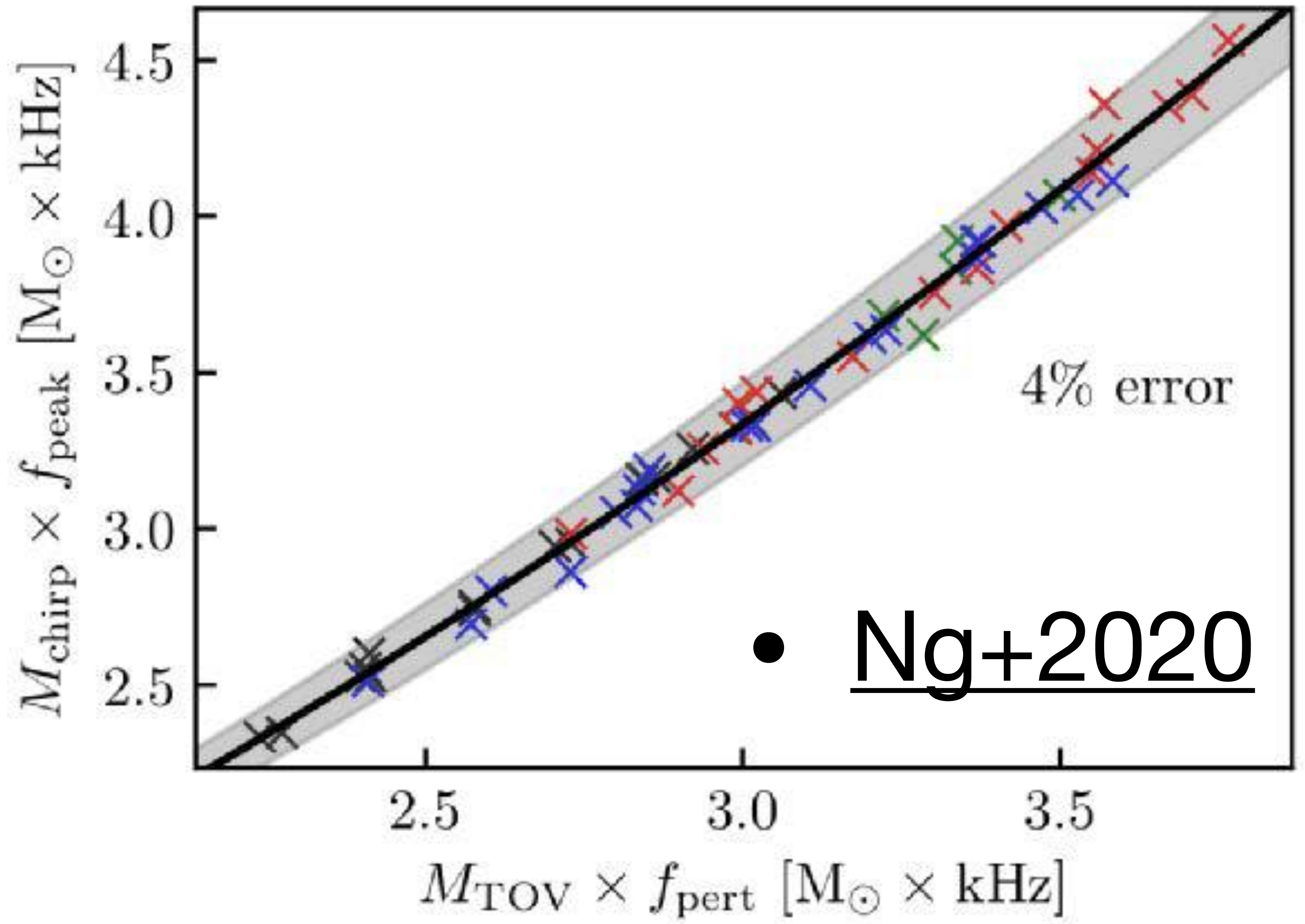


- Radice+2019

Isolated oscillation VS merger remnant



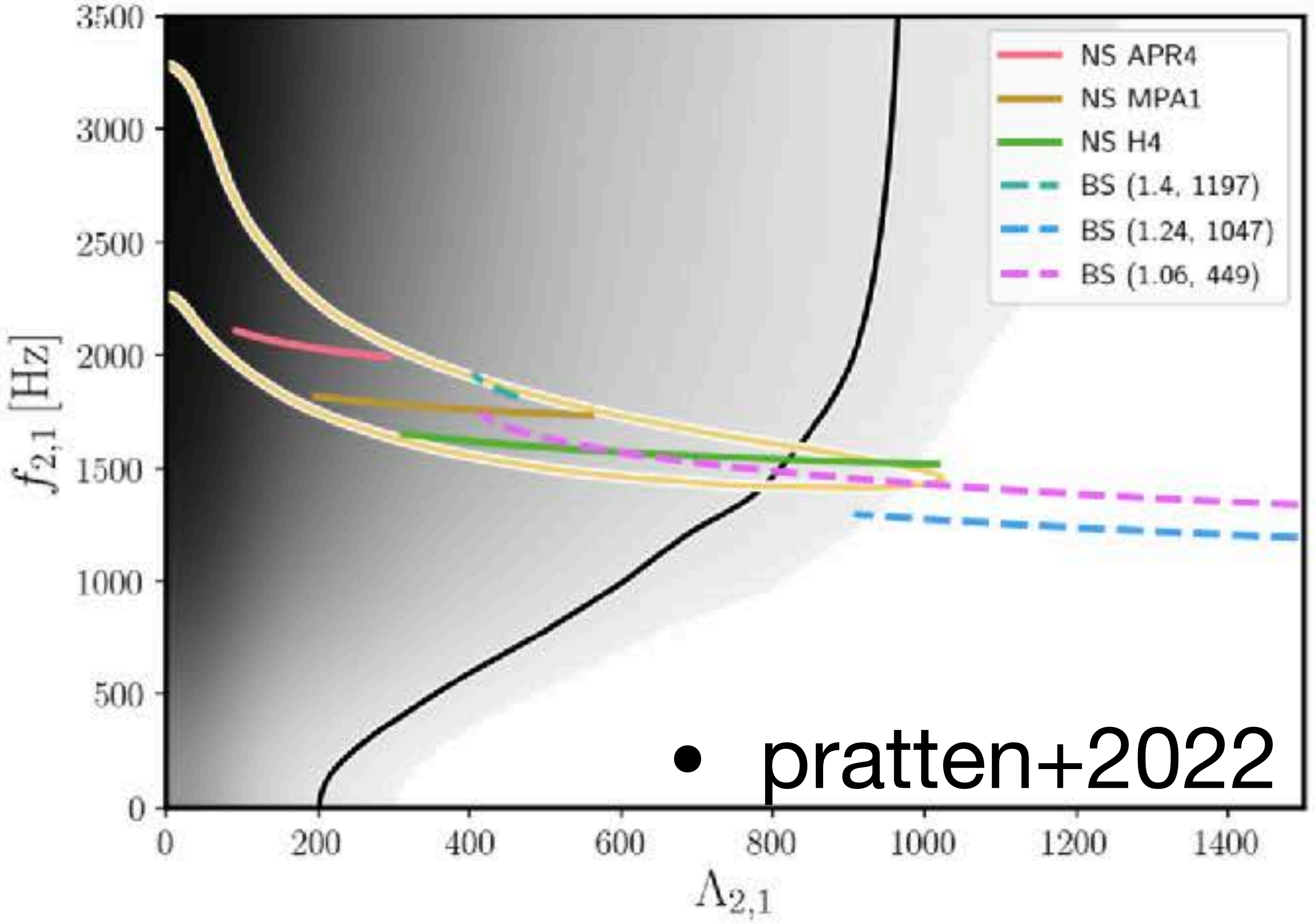
- Lioutas+2021



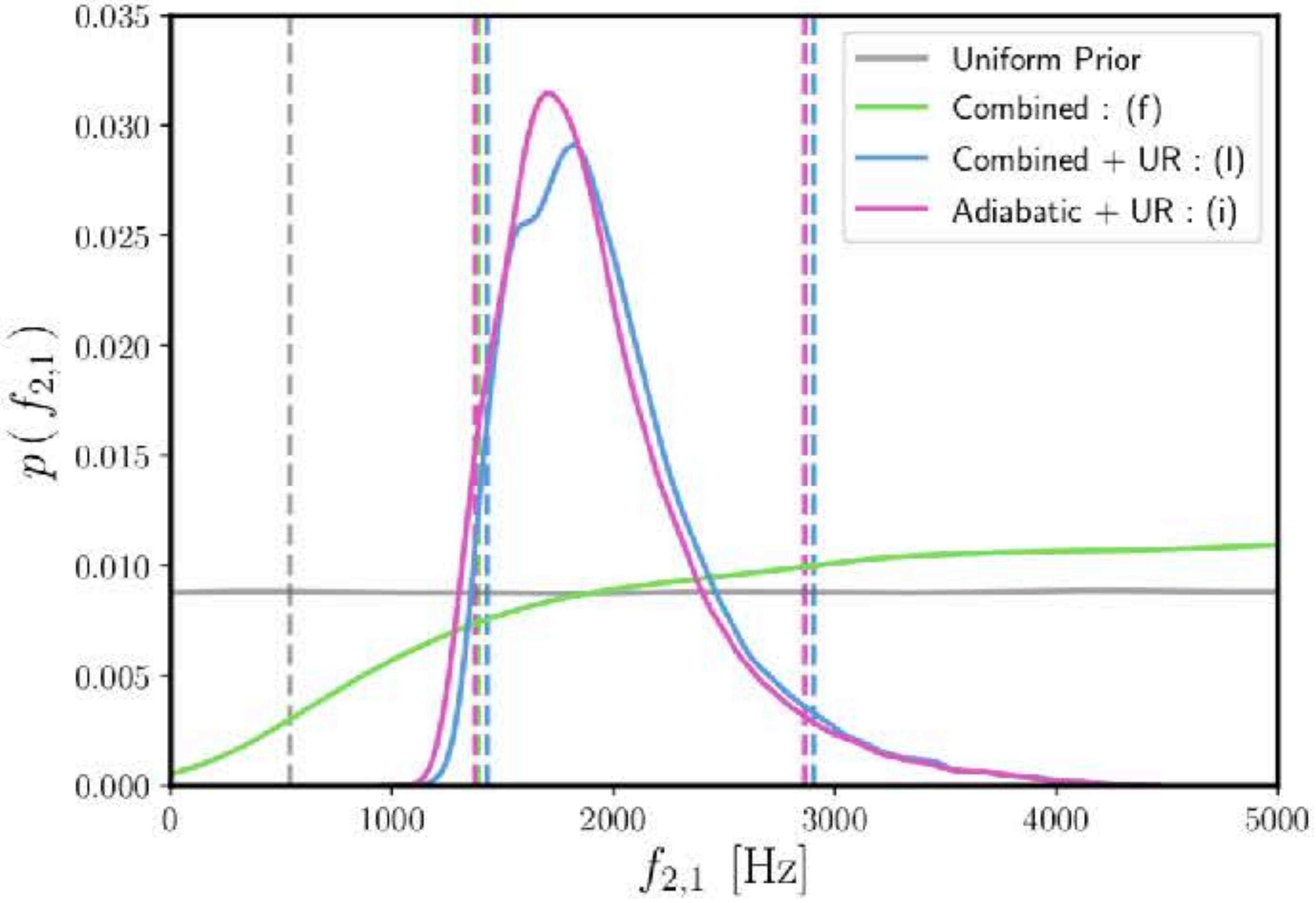
- Ng+2020

- Strong correlation with the isolated NS f-mode frequency and the peak frequency in post merger.
- case of equal-mass mergers, the peak frequency in supramassive NSs is almost equal to that of the non-rotating f-mode frequency of isolated NSs with the same mass as each of the merging components

Dynamical tidal effect of GW170817



- pratten+2022



- 90% credible interval of f-mode frequency for GW170817:
  1.43 kHz ~ 2.90 kHz for the more massive star
  1.48 kHz ~ 3.18 kHz for the less massive star

Oscillations of NS

Fluid perturbations

- Radial oscillation (l=0): $\varepsilon^{\boxed{r}} = R_n^r(r)e^{\mathbf{i}\omega t}$
  don’t couple to gravitational waves

- Non-radial oscillation (l>=2):
  $$\varepsilon^{\boxed{r,\theta,}\boxed{\phi}} = \partial_{r,\,\theta,\,\phi}\left(R_n^{r,\,\theta,\,\phi}(r)Y_m^l(\theta,\phi)e^{\mathbf{i}\omega t}\right)$$
  f-mode (fundamental n=0) (even),
  p-modes (pressure n=1,2,…) (even)
  g-modes (gravity n=1,2,…) (even)
  r-modes (rotation m=+-1,+-2,…) (odd)

$$\omega = 2\pi\nu + \frac{\mathbf{i}}{\tau}$$

	ν (kHz)	τ (s)
f-mode	1.3-2.8	0.1-1
g-mode	<0.8	>100
p-mode	>2.7	1-1000
r-mode	~ spin	<0
w-mode	~10	~1E-5

- **Spacetime perturbations**:
  Family I w-modes (even)
  Family II w-modes (odd)
  important for BBH ring-down

even-parity
(polar mode)

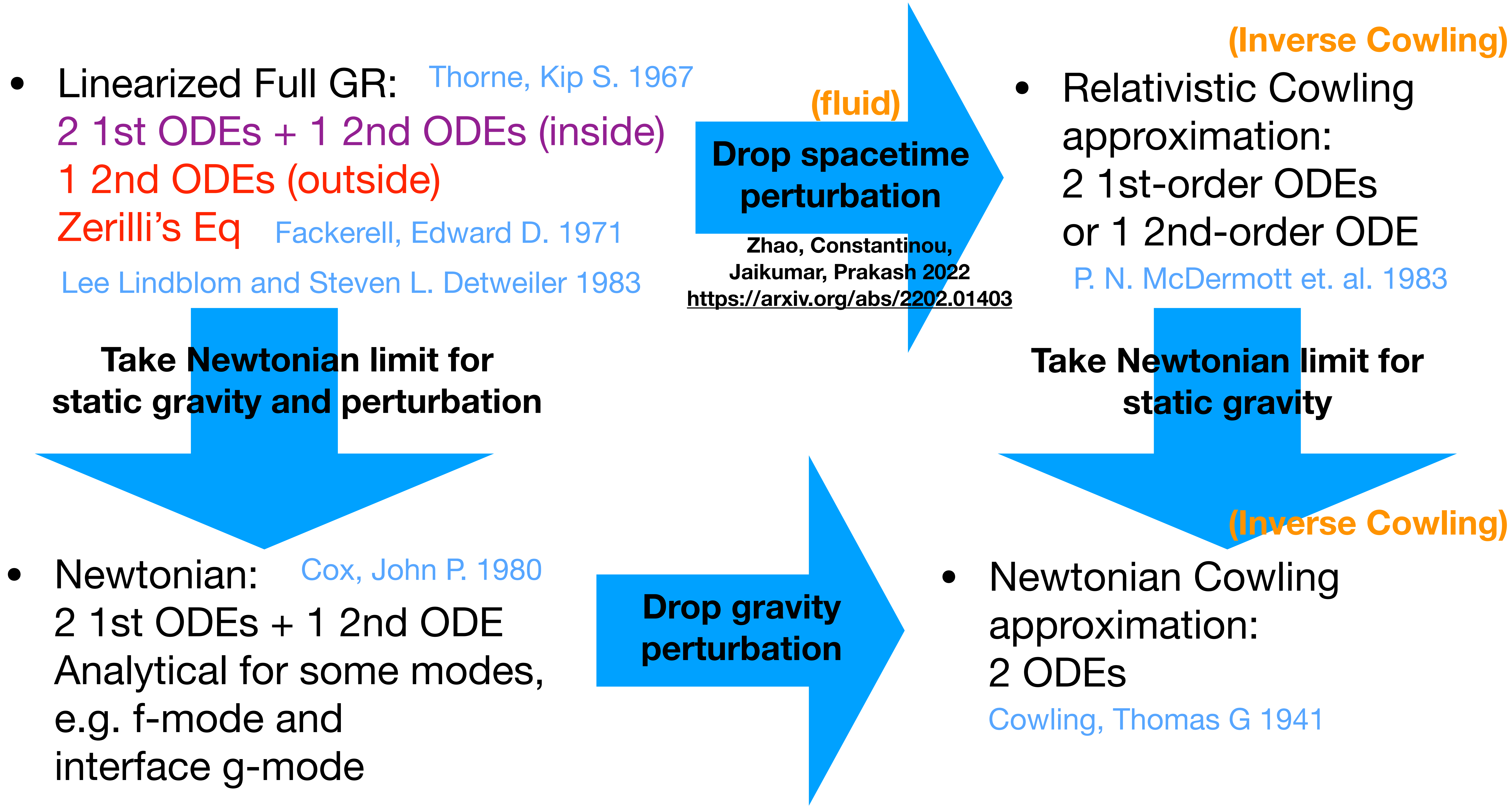
$$h_{\mu\nu}^{even} = \begin{pmatrix} H_0e^{\nu} & H_1 & 0 & 0 \\ H_1 & H_2 & 0 & 0 \\ 0 & 0 & r^2K & 0 \\ 0 & 0 & 0 & r^2\sin^2\theta K \end{pmatrix} P_l(\cos\theta)$$

odd-party
(axial modes)

$$h_{\mu\nu}^{odd} = \begin{pmatrix} 0 & 0 & 0 & H_0' \\ 0 & 0 & 0 & H_1' \\ 0 & 0 & 0 & 0 \\ H_0' & H_1' & 0 & 0 \end{pmatrix} \sin\theta\partial_\theta P_l(\cos\theta)$$

ODEs of Non-radial Adiabatic Oscillation

Eigen value problem of even quasi-normal modes



- Linearized Full GR: Thorne, Kip S. 1967
  2 1st ODEs + 1 2nd ODEs (inside)
  1 2nd ODEs (outside)
  Zerilli's Eq Fackerell, Edward D. 1971
  Lee Lindblom and Steven L. Detweiler 1983

(fluid)

Drop spacetime perturbation

Zhao, Constantinou, Jaikumar, Prakash 2022
https://arxiv.org/abs/2202.01403

(Inverse Cowling)

- Relativistic Cowling approximation:
  2 1st-order ODEs
  or 1 2nd-order ODE
  P. N. McDermott et. al. 1983

Take Newtonian limit for static gravity and perturbation

Take Newtonian limit for static gravity

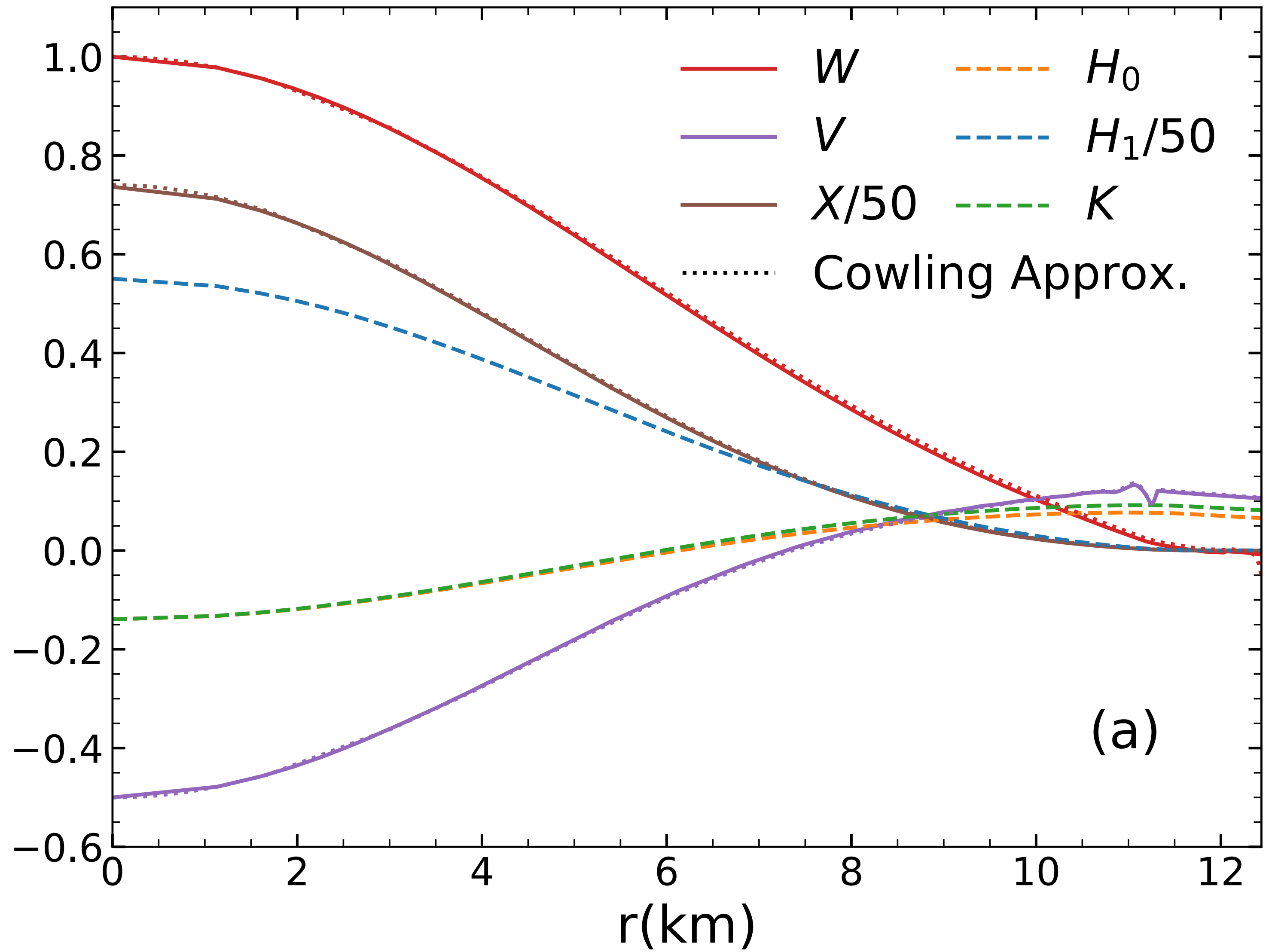
- Newtonian: Cox, John P. 1980
  2 1st ODEs + 1 2nd ODE
  Analytical for some modes,
  e.g. f-mode and
  interface g-mode

Drop gravity perturbation

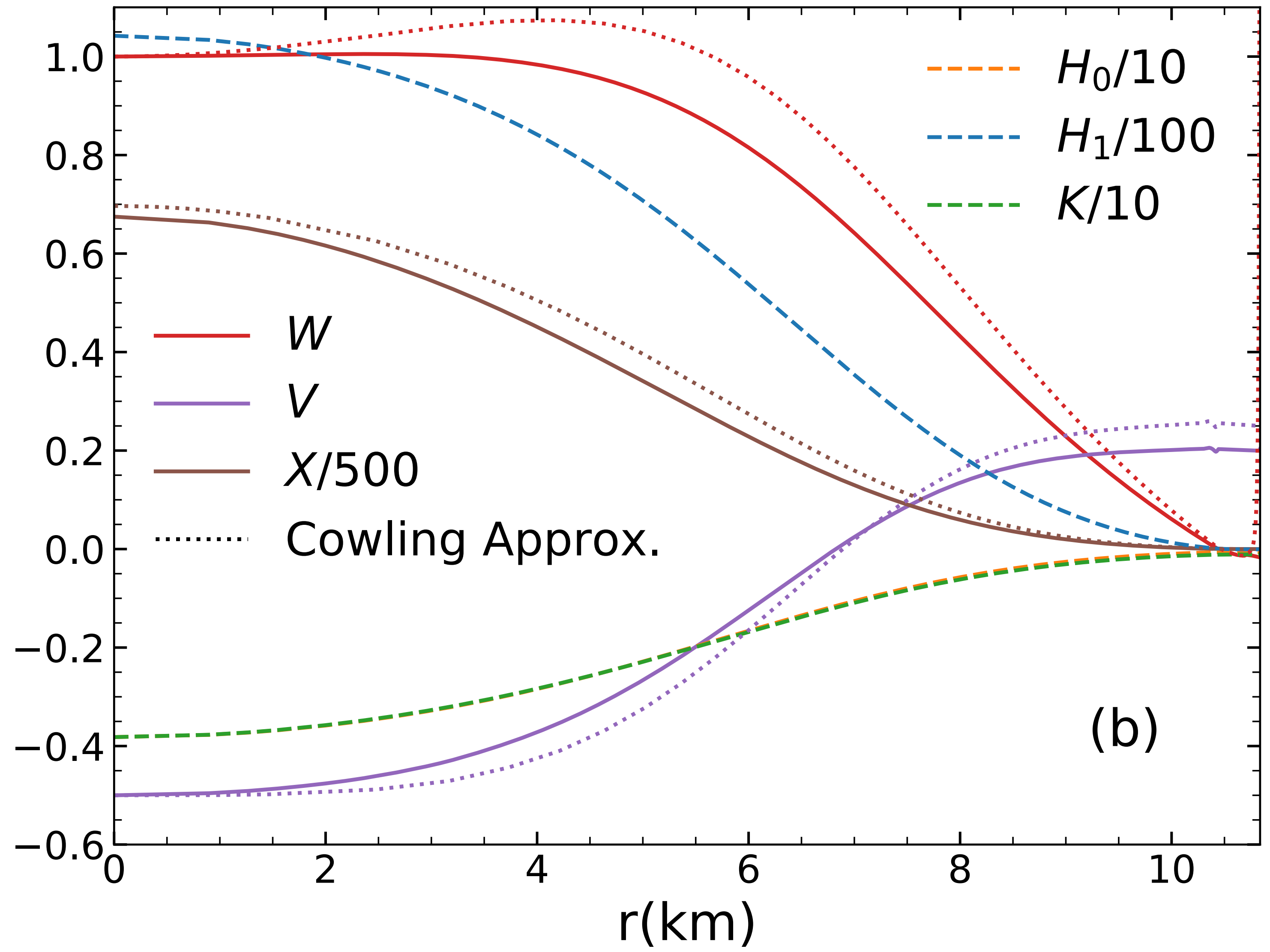
(Inverse Cowling)

- Newtonian Cowling approximation:
  2 ODEs
  Cowling, Thomas G 1941

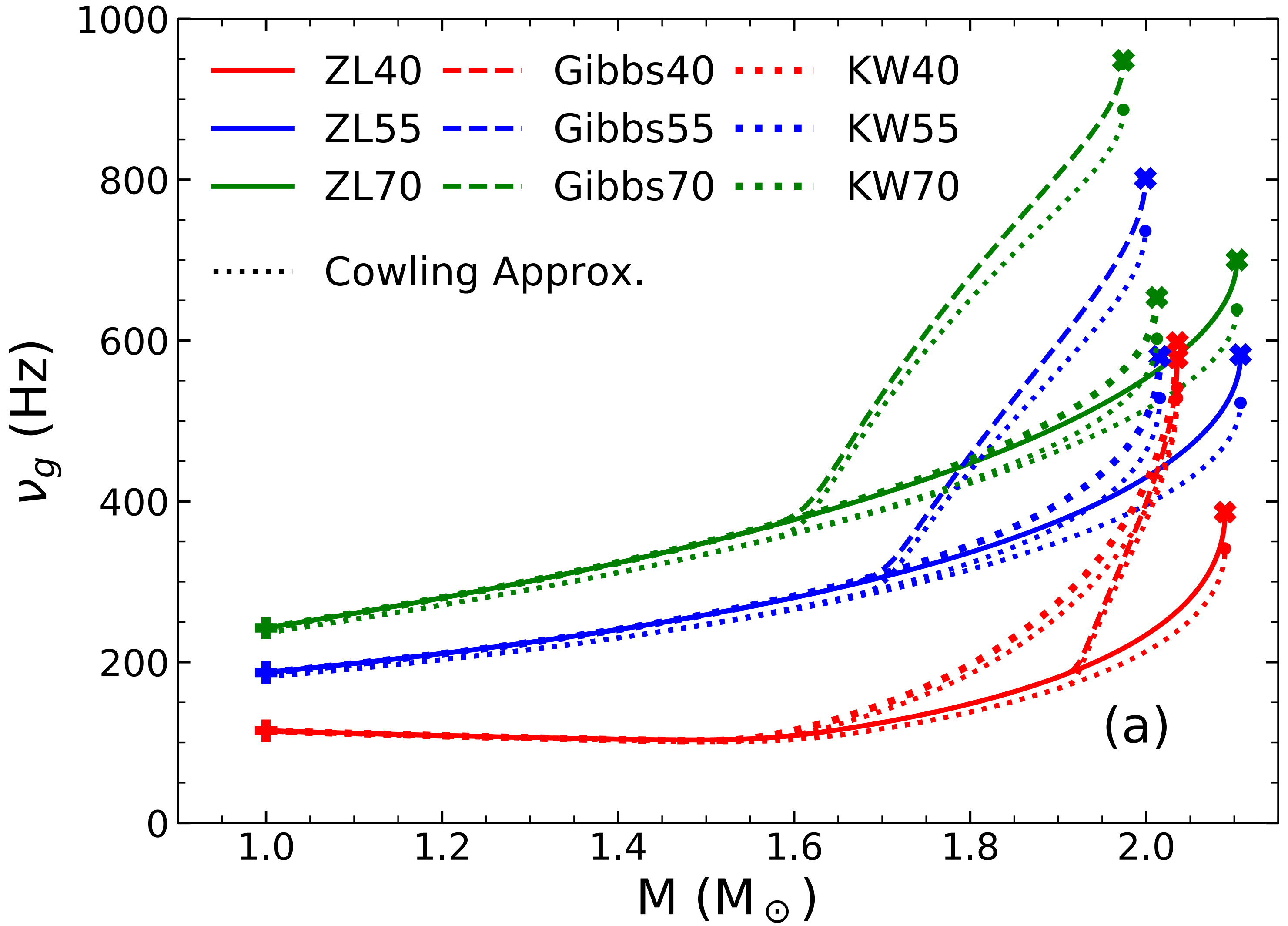
Cowling Approximation in Compositional g-modes



Low mass compositional g-mode



High mass compositional g-mode



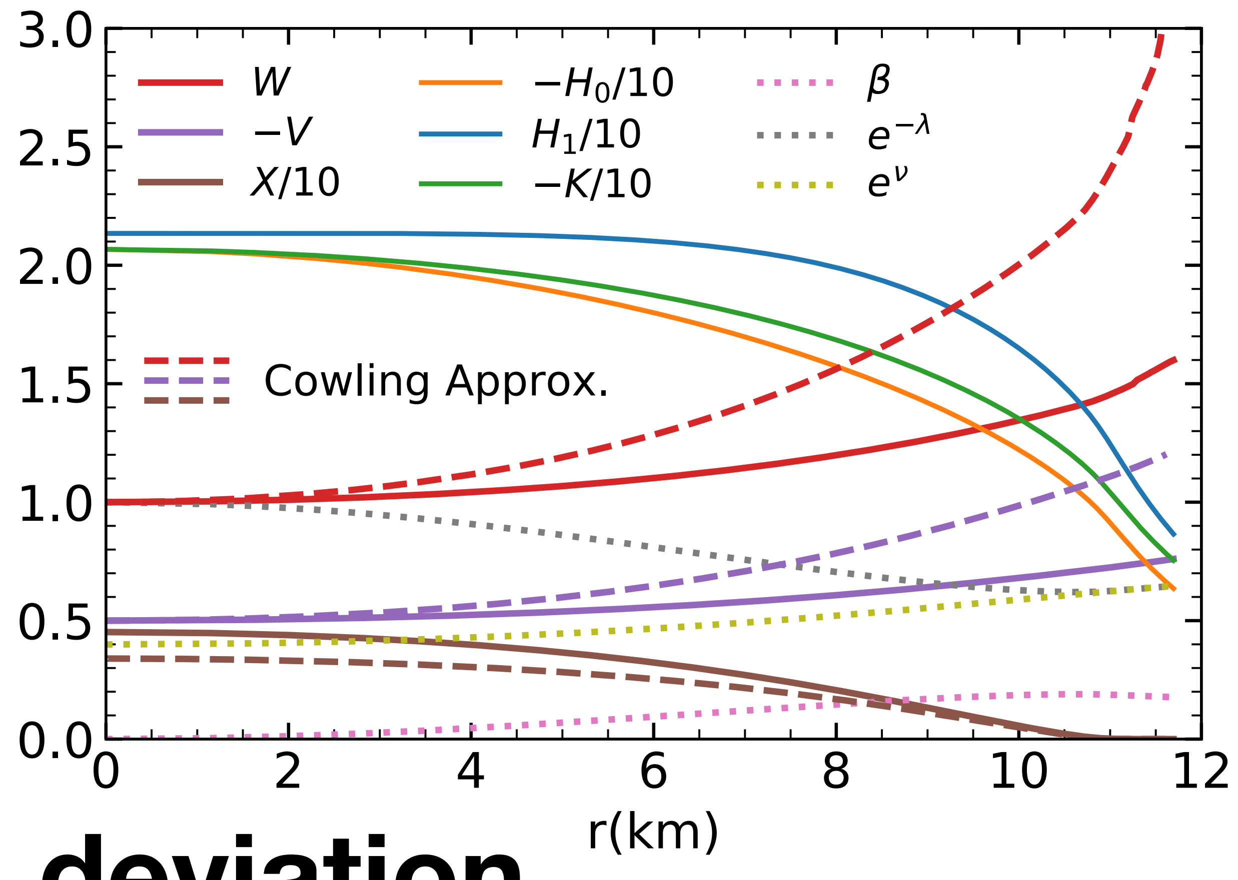
Cowling approximation: up to 10% deviation from the linearized full GR

Zhao, Constantinou, Jaikumar and Prakash 2022
https://arxiv.org/abs/2202.01403

f-mode with Analytical TOV Solutions

- Dimensionless frequency:

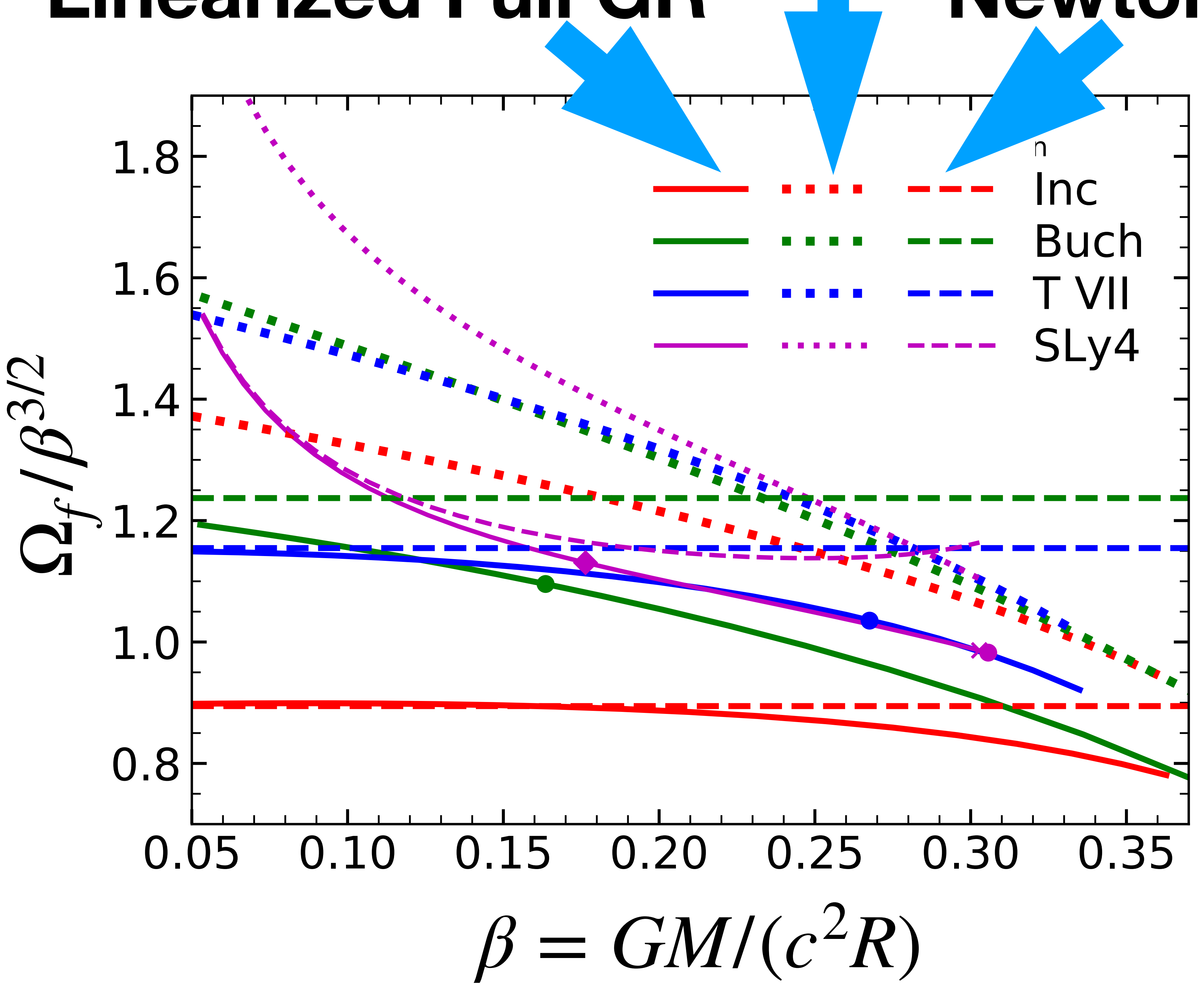
$$\Omega_f = GM\omega_f/c^3 \qquad (\propto \beta^{3/2} \text{ in Newtonian})$$



Cowling approximation: up to 30% deviation

Linearized Full GR

Newtonian: up to 15% deviation

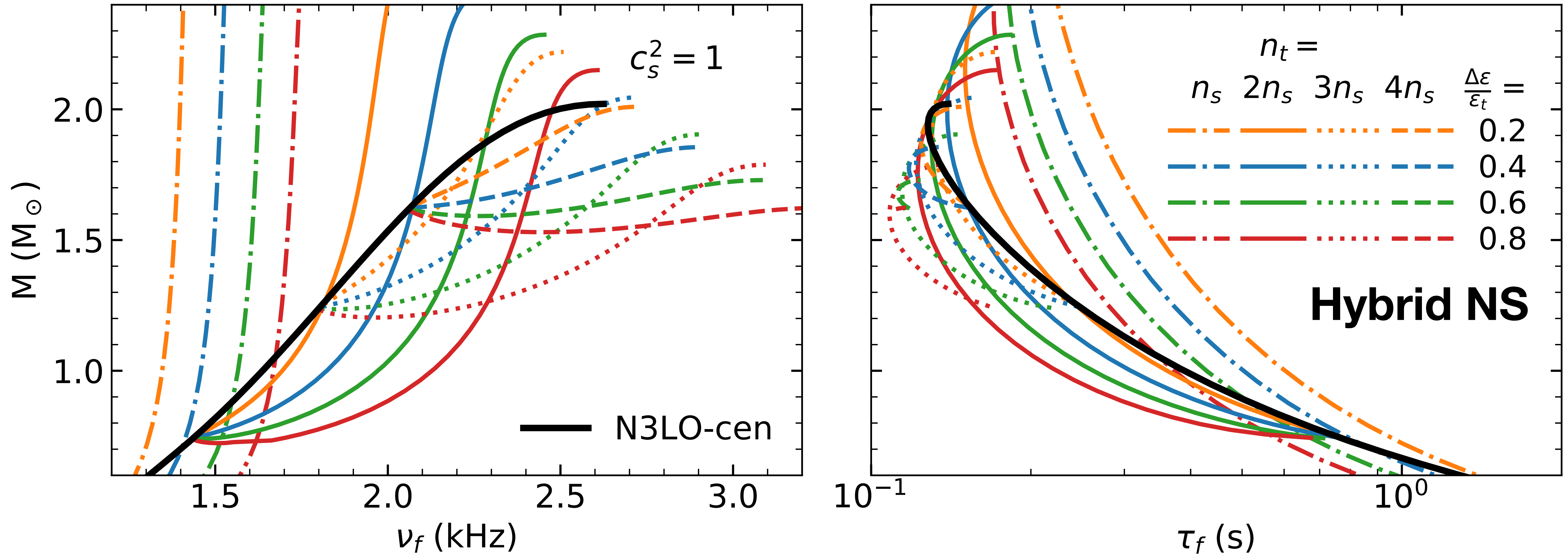


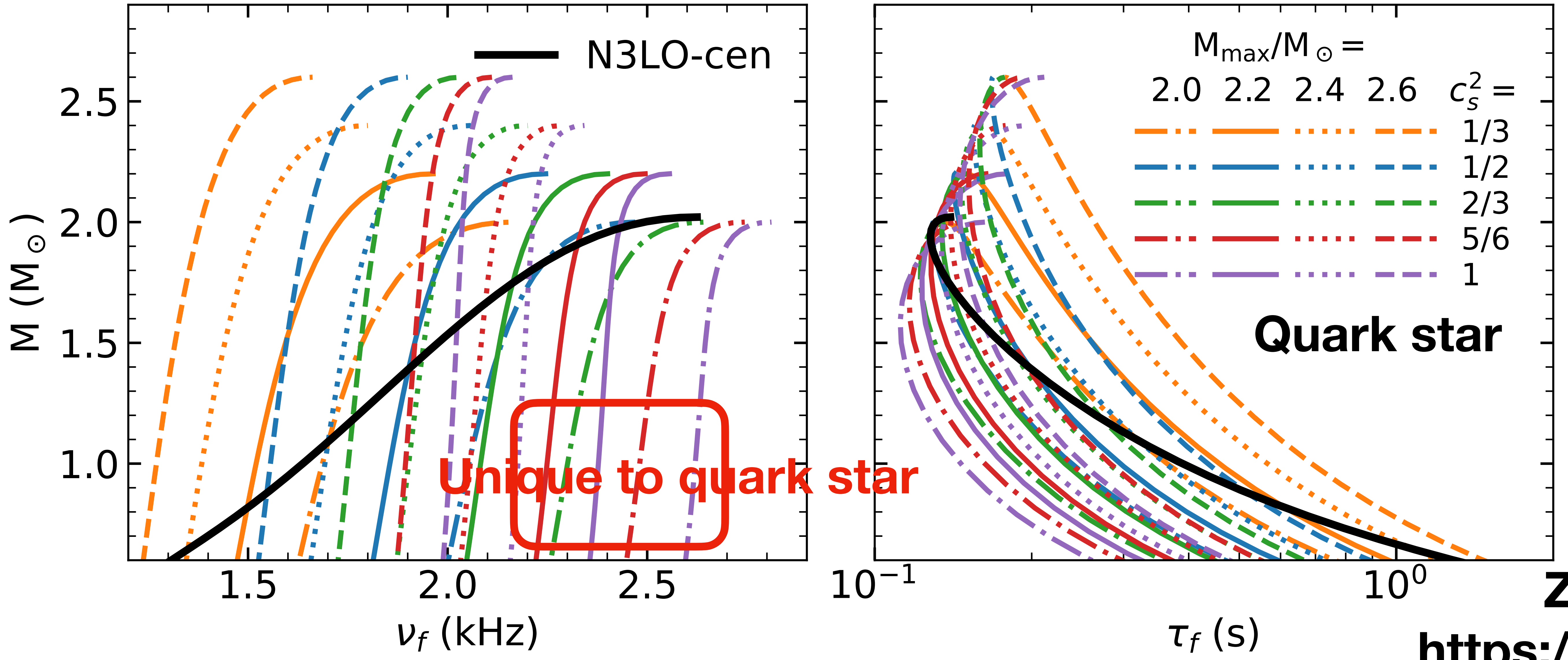
EOS	$\ell = 2$	$\ell = 3$	$\ell = 4$
Inc	$4/5$	$12/7$	$8/3$
T VII	$4/3$	$204/77$	$152/39$
Buch	$3\pi^2(5\pi^2 - 30)^{-1}$	2.94766	4.24121

Table: Newtonian $\Omega_f/\beta^{3/2}$

Zhao & Lattimer 2022
https://arxiv.org/abs/2204.03037

f-mode with Hybrid and Quark EOS





Unique to quark star

Frequency:

$\nu_f \in (1.3 - 2.8)$ kHz

Damping time:

$\tau \in (0.1,1)$ s

Zhao & Lattimer 2022

https://arxiv.org/abs/2204.03037

One node branch

- Lowest order pressure mode have zero node which is named as f-mode (fundamental).

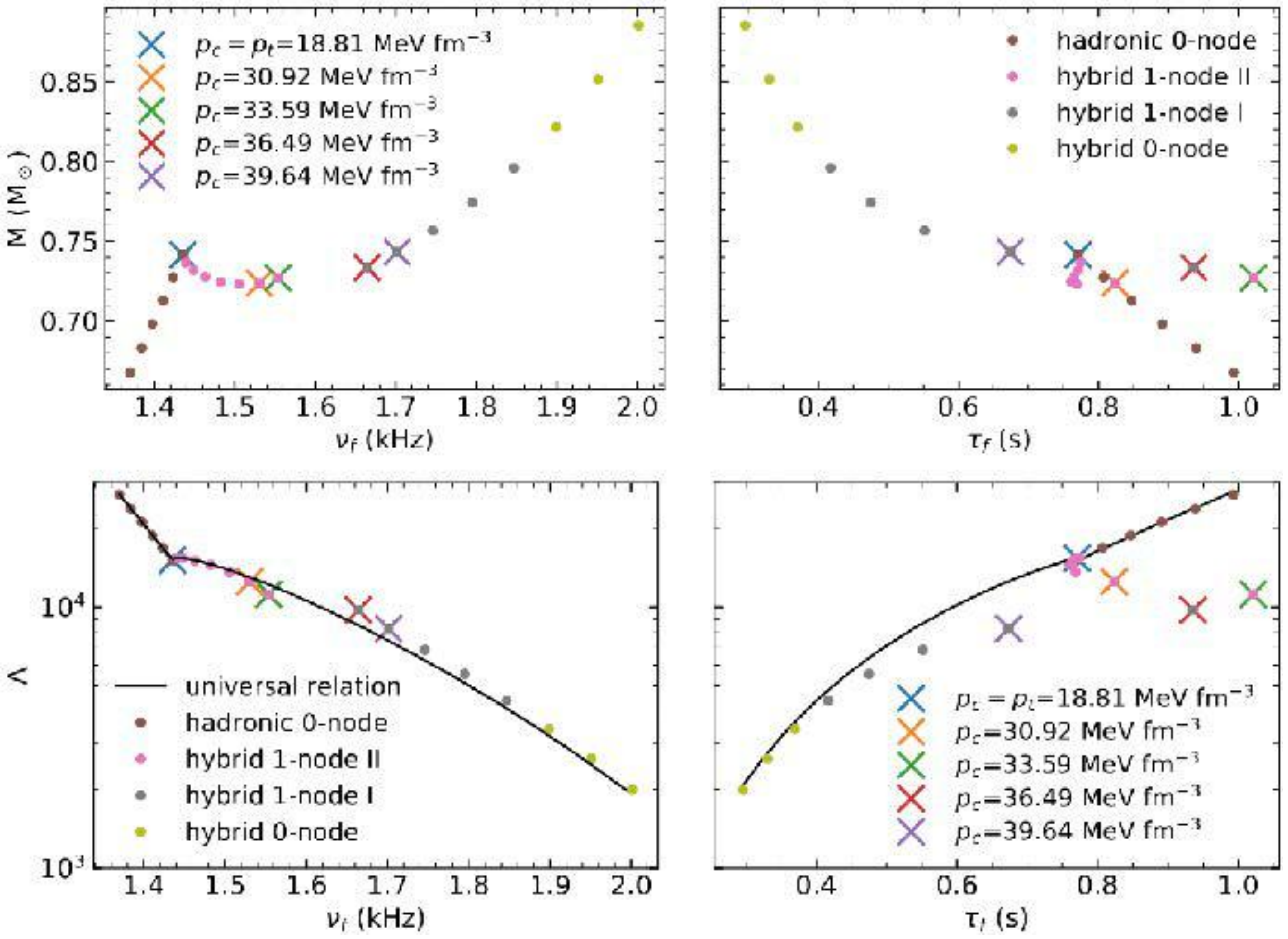
- However, in case of hybrid NS lowest order pressure mode sometimes have one node due to strong density discontinuity.

- Stars with radial nodes in V only we refer to as 1-node I.

- hybrid stars have a radial node (zero) in the fluid and metric perturbation amplitudes X, W, H_0, H_1, K (but not V, which, however discontinuously changes sign) at a radius slightly larger than the phase transition radius R_t. We will call this type of behavior 1-node II

One node branch

hybrid star with 1-node deviates away from f-I-love-Q relation



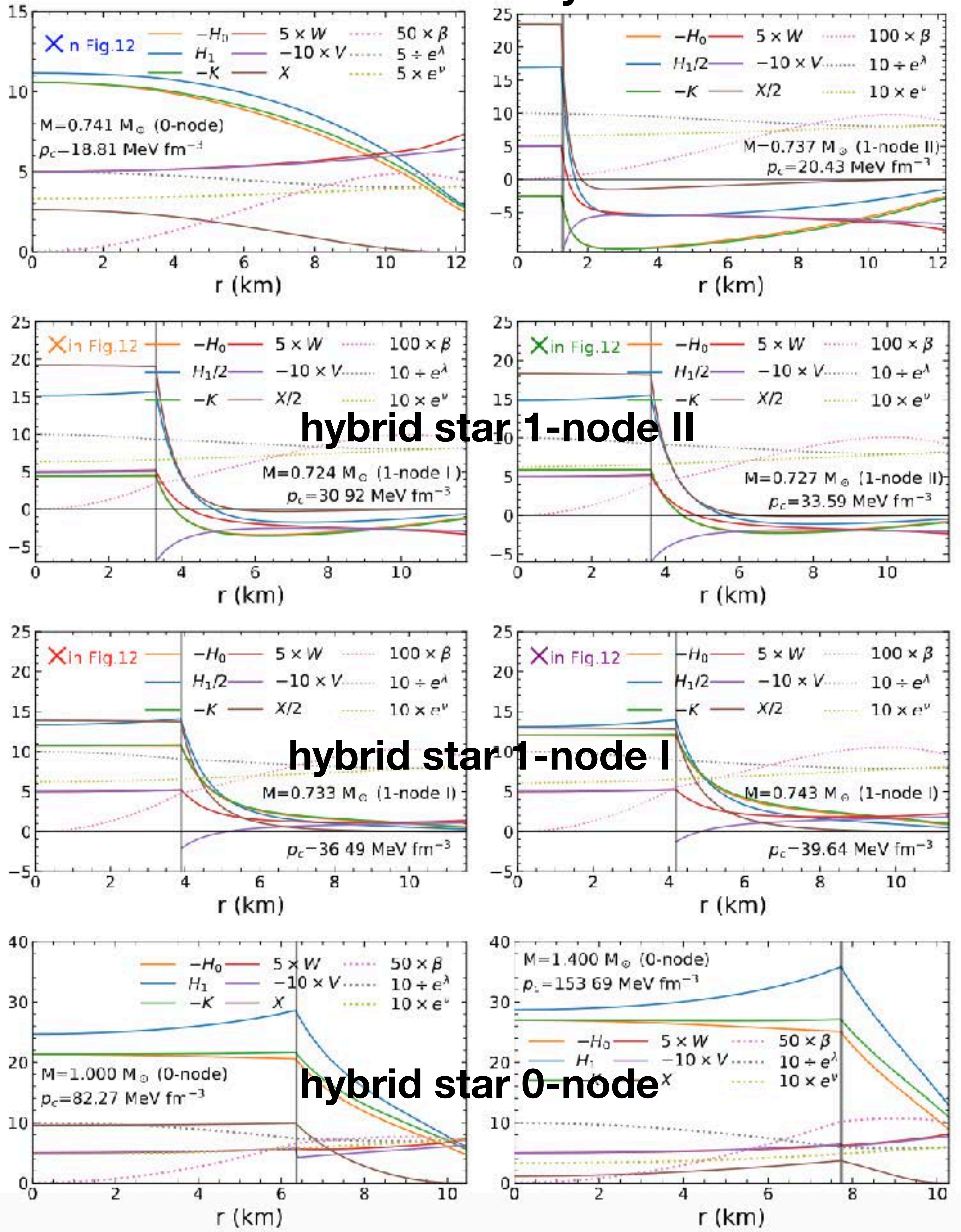
Zhao & Lattimer 2022

hadronic star 0-node **hybrid star 1-node II**

hybrid star 1-node II

hybrid star 1-node I

hybrid star 0-node



Typical f-mode oscillation

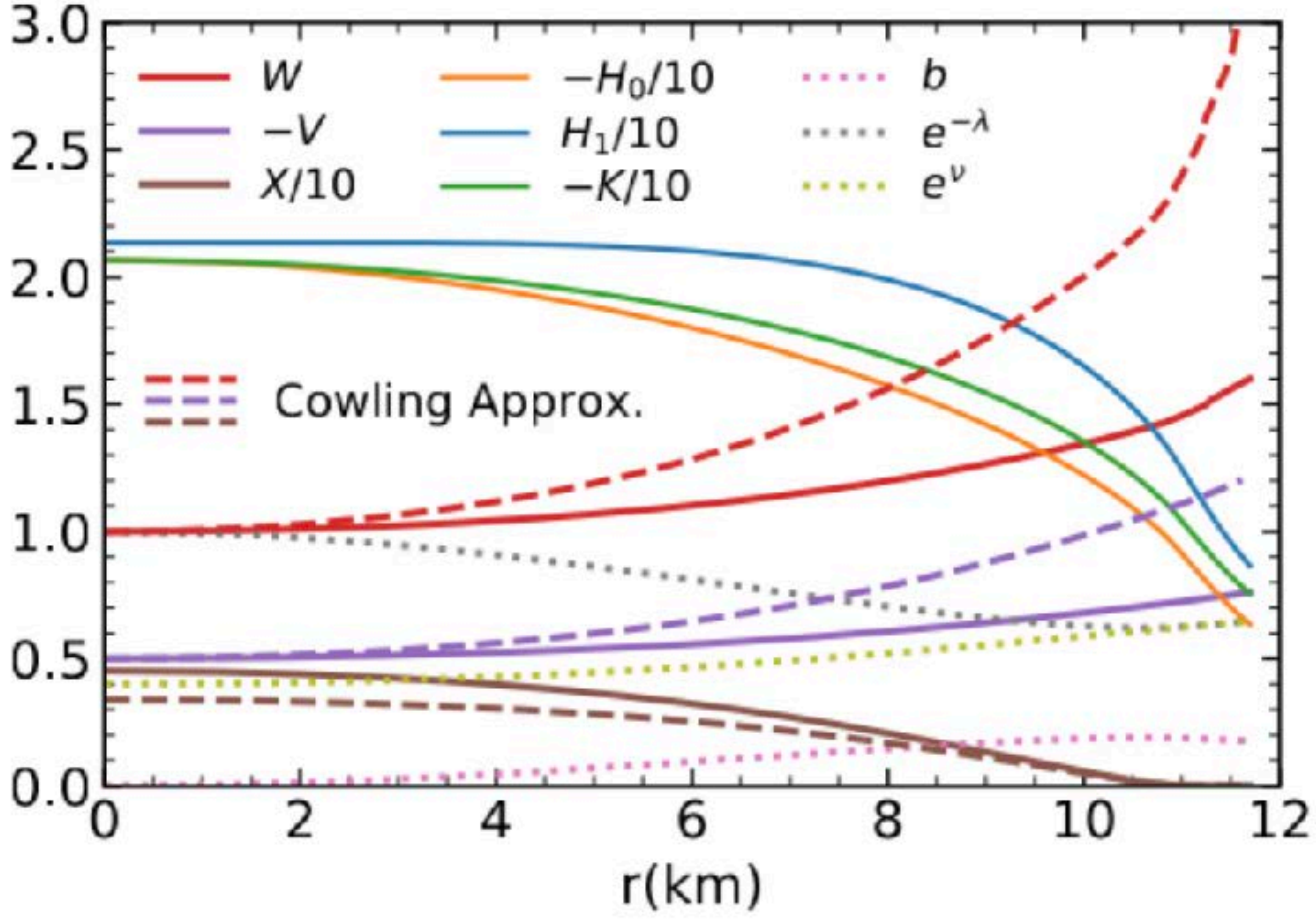
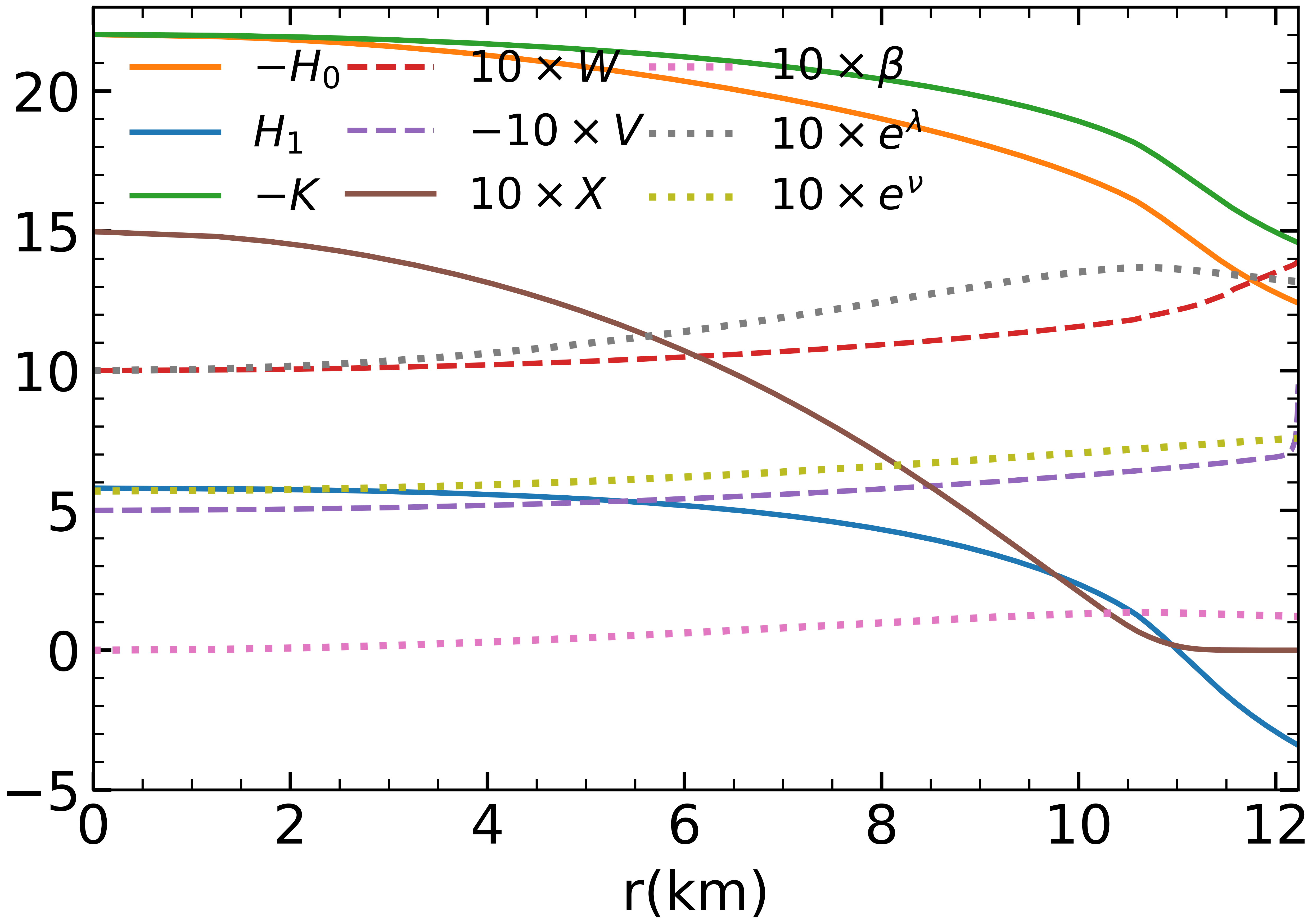
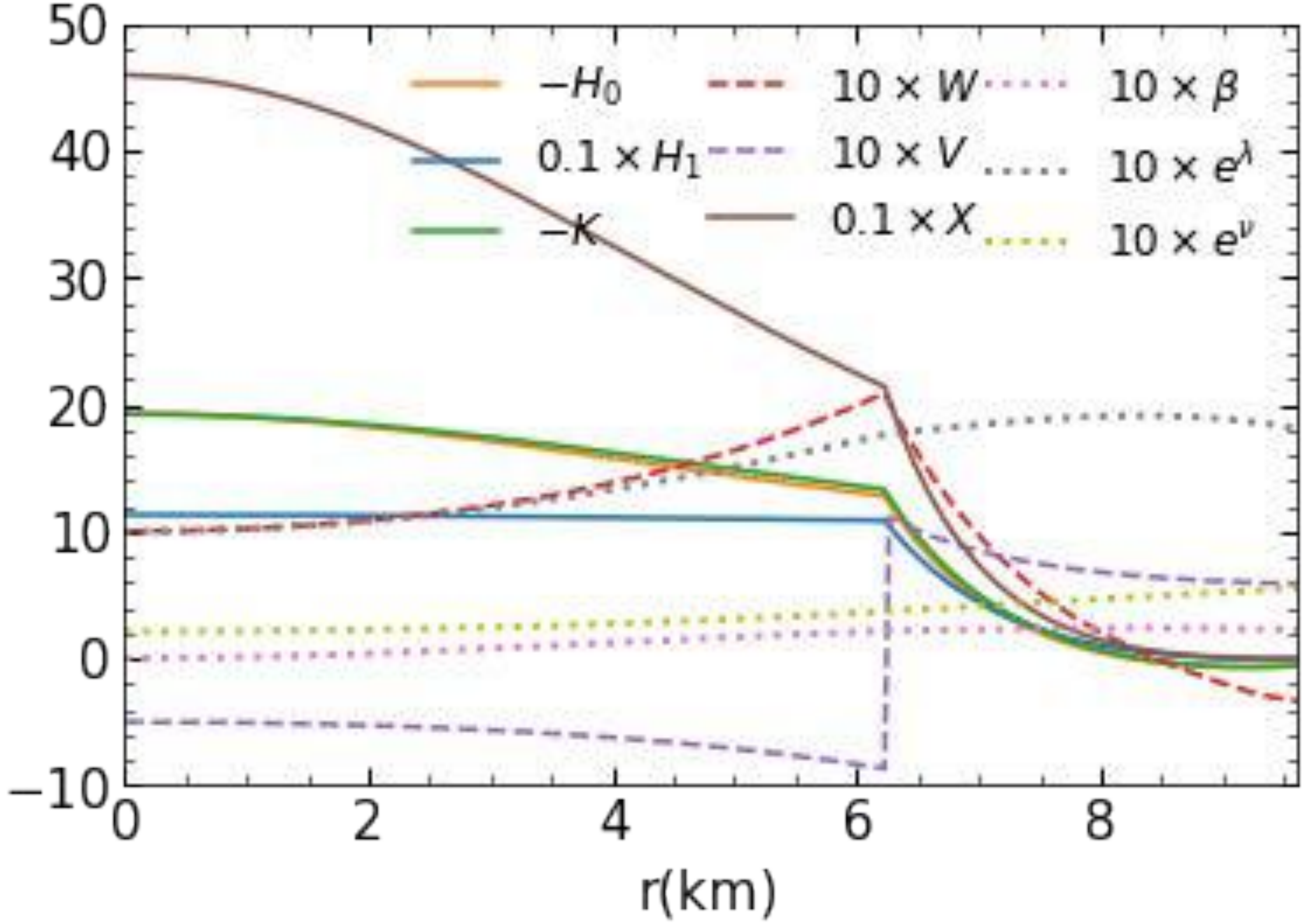


FIG. 14. Metric perturbation amplitudes, fluid perturbation amplitudes for non-radial oscillations with $\ell = 2$ with (dashed curves) and without (solid curves) the Cowling approximation, and static metric functions (dotted curves) inside a $1.4M_{\odot}$ NS computed with the Sly4 EOS [85]. H_0, H_1 and K are in units of $\varepsilon_s = 152.26$ MeV fm^{-3}, X is in units of ε_s^2, and W, V, ν and λ are dimensionless. Only real parts of the perturbation amplitudes are plotted.

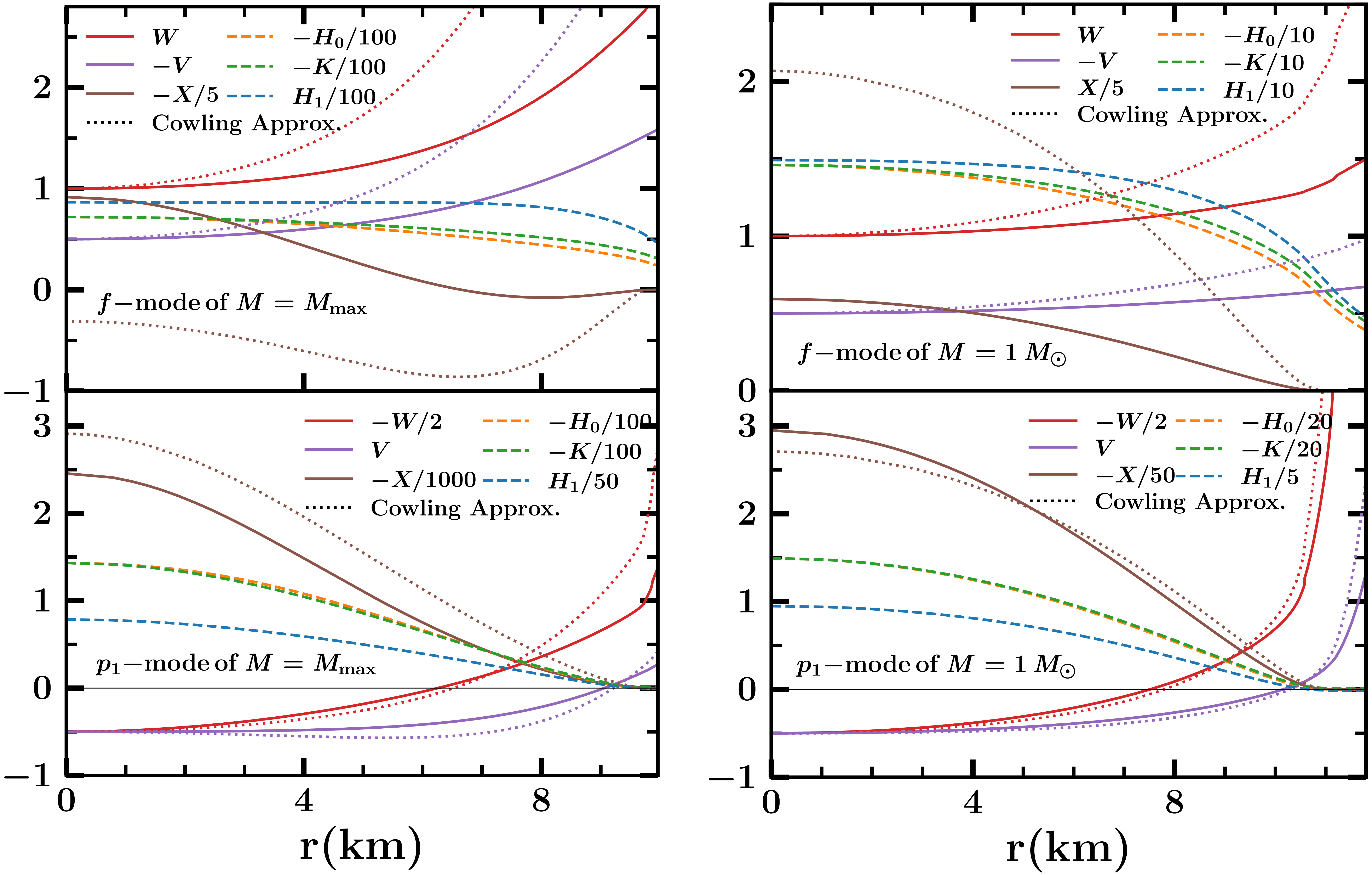
Compositional g-mode of hadronic NS



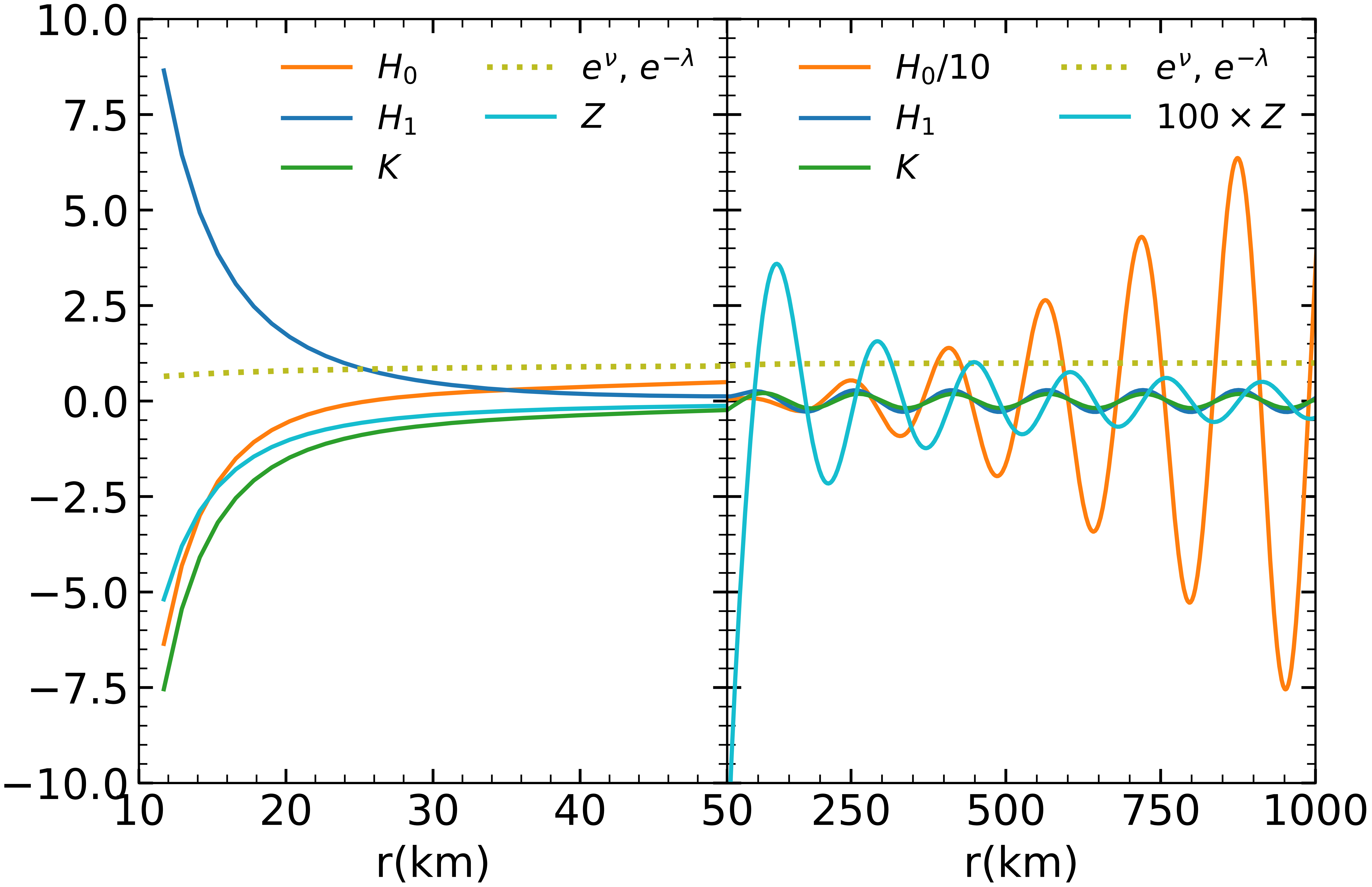
Discontinuity g-mode of hadronic NS



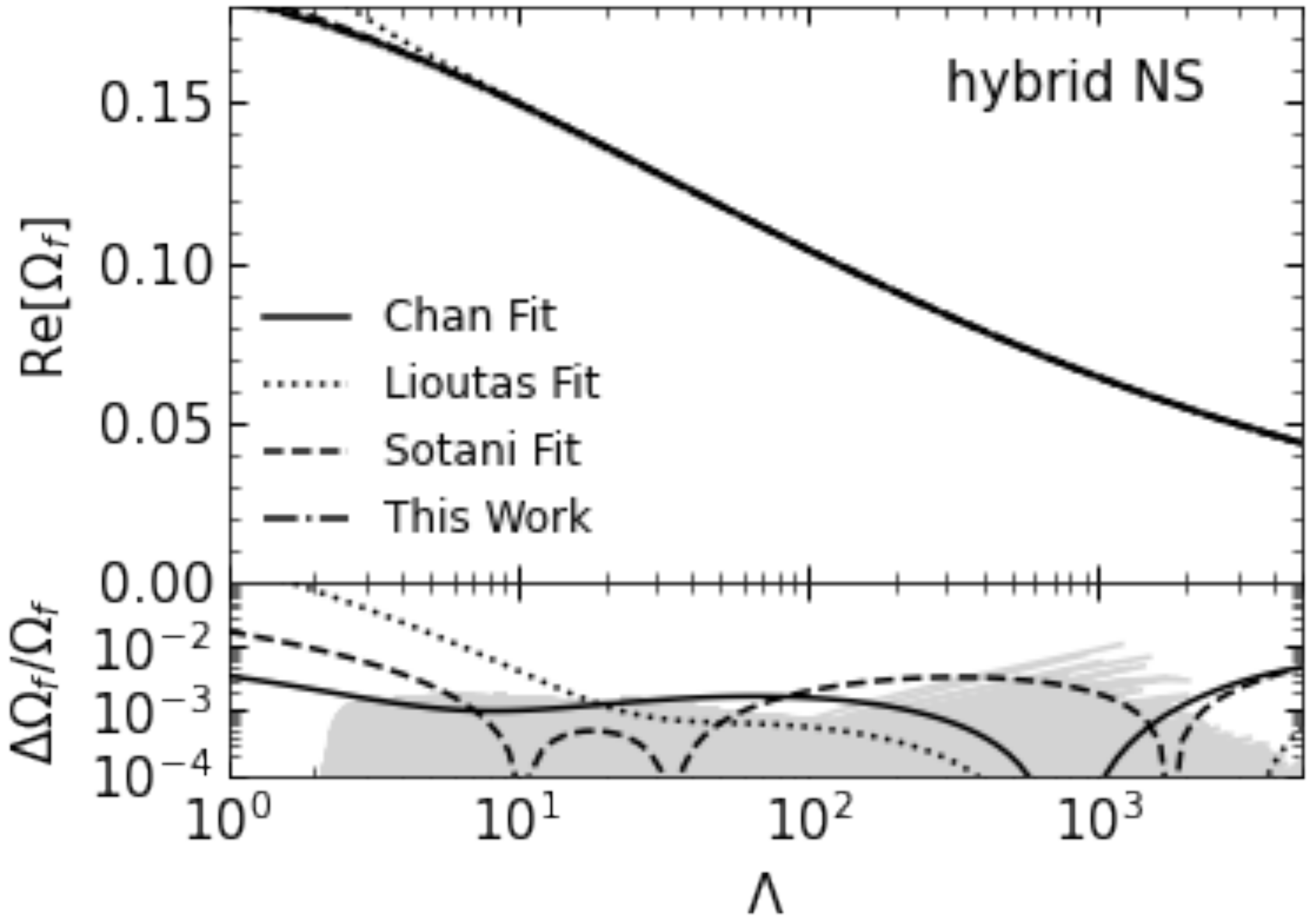
f-mode vs p-mode oscillations



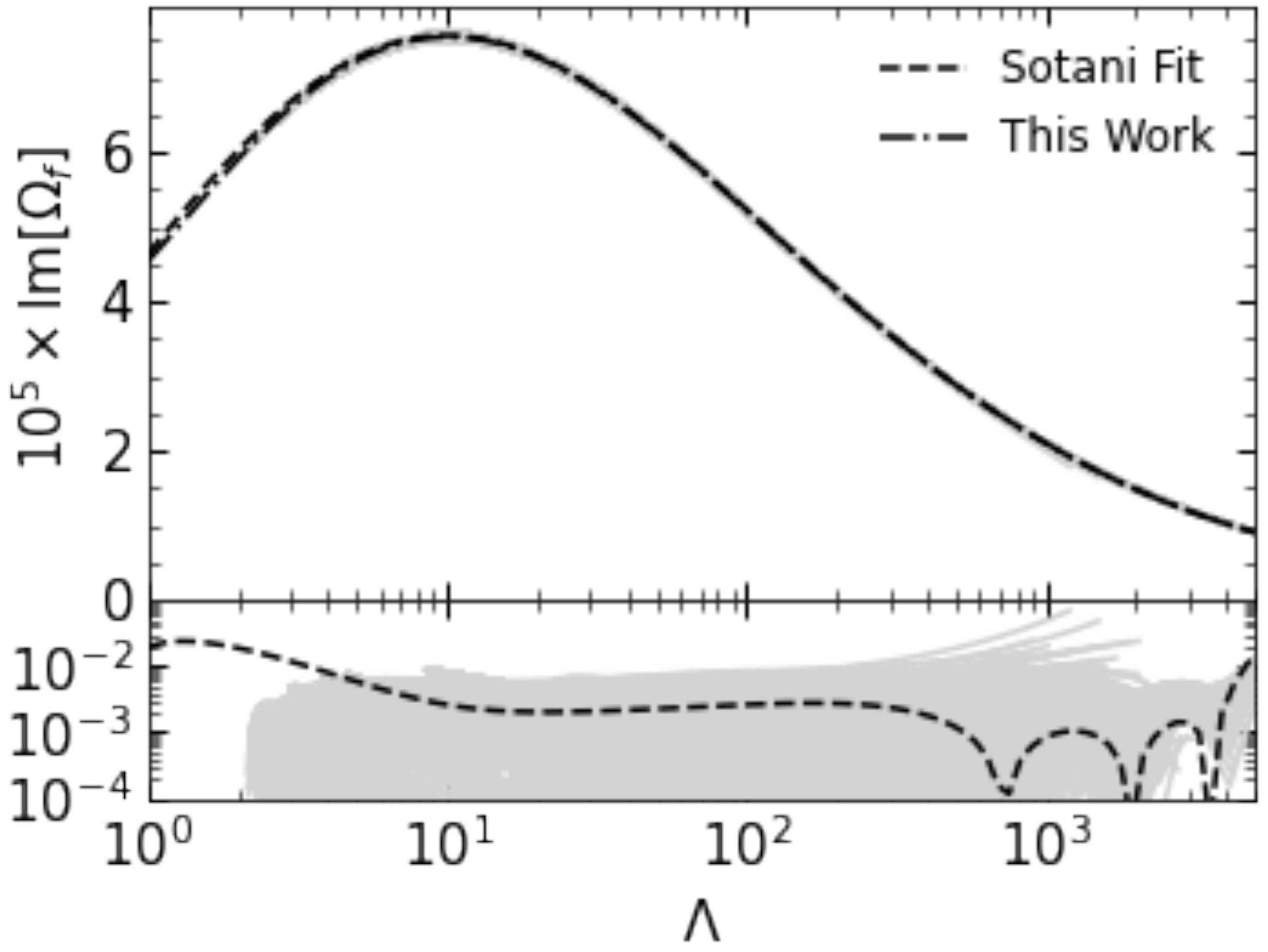
Outside metric perturbation (f-mode as example)



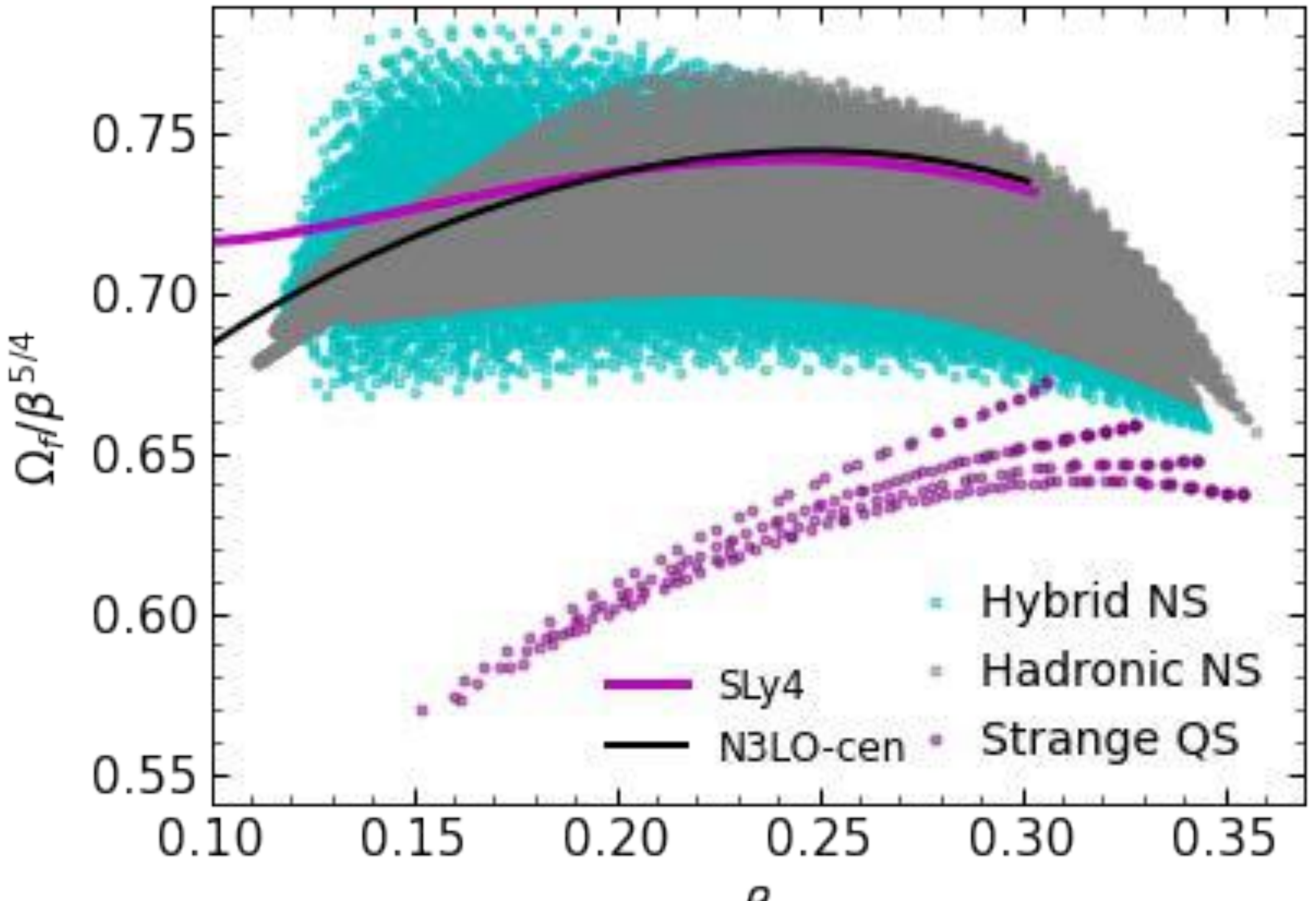
f-mode universal relations



0.1% deviation except for one-node branches



1% deviation except for one-node branches

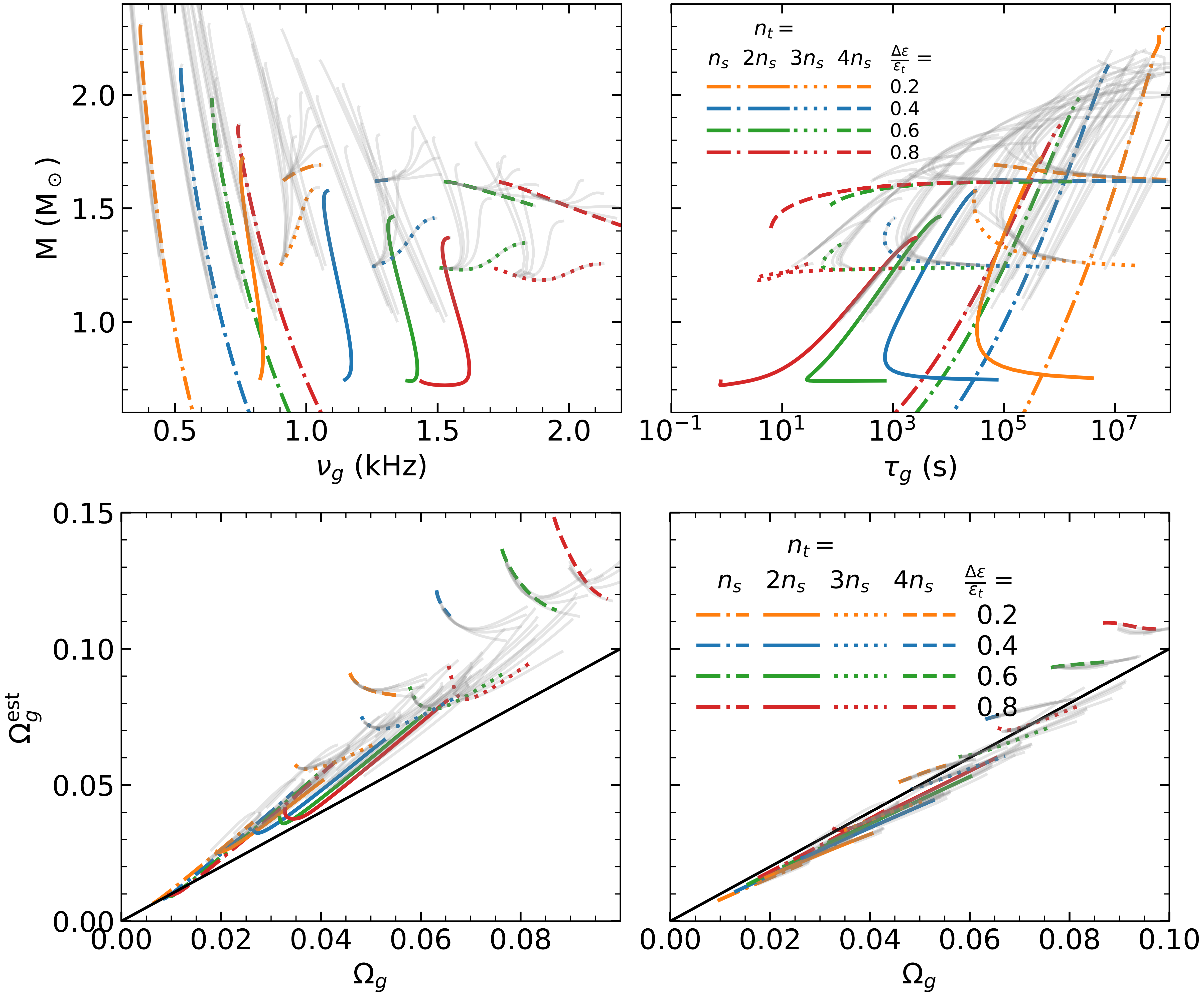


Slightly weaker for $\Omega_f - \bar{I}$ **, see in the paper**

$$\Omega_f = (0.714 \pm 0.056)\ \beta^{5/4}$$

Zhao & Lattimer 2022

Discontinuity g-mode for hybrid NS



Newtonian 1D gravity wave:

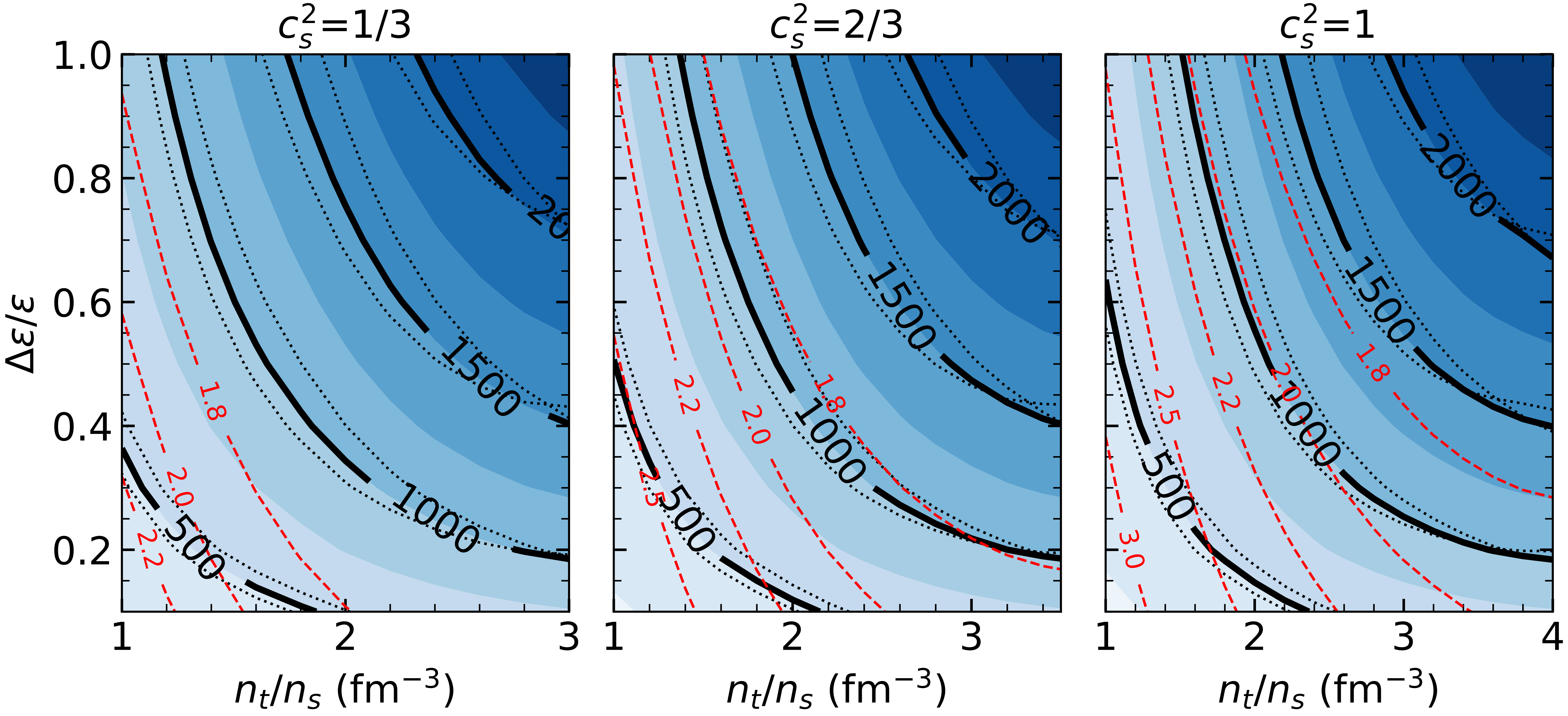
$$\omega_g^2 = \frac{(\varepsilon_+ - \varepsilon_-)gk}{\varepsilon_+/\tanh(kd_+) + \varepsilon_-/\tanh(kd_-)}$$

Landau and Lifshitz 1960

Zhao & Lattimer 2022

Discontinuity g-mode semi-universal relation

Contour of $\nu_g(n_t,\ \Delta_\varepsilon,\ c_s^2)\,|_{M=M_{max}}$



Dotted: Chrial EFT Uncertanty 5%

Dashed: maximum mass

$$\nu_g = (326.4 \pm 36.1\,\mathbf{Hz})\left(\frac{p_t}{\mathbf{MeV\ \ fm}^{-3}}\right)^{0.268+0.146c_s/c}\sqrt{\frac{\Delta\varepsilon}{\varepsilon_t}\left(\frac{c}{c_s}\right)}$$

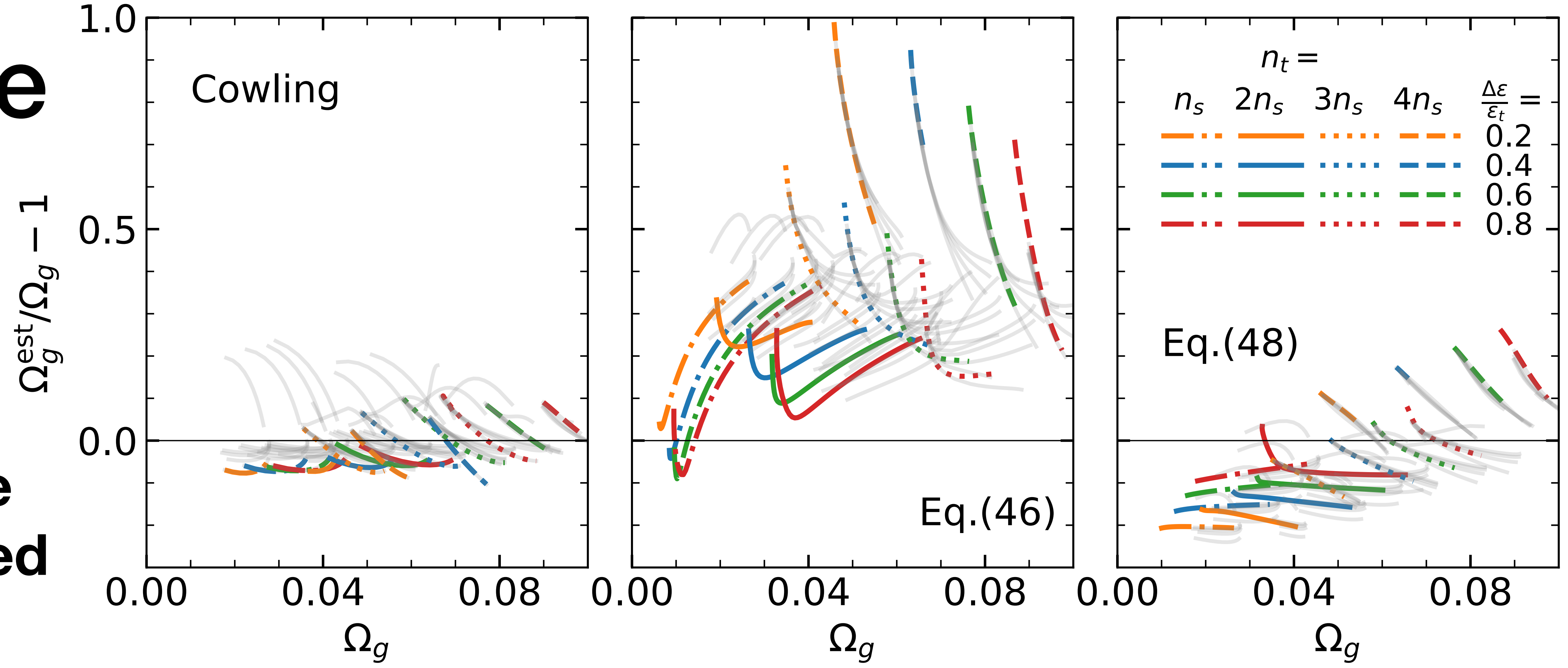
Frequency: $\nu_g < 0.8\ (1.5)$ kHz for $c_s^2 = c^2/3\ (c^2)$

Damping time: $\tau > 100\ (10000)$ s

Zhao & Lattimer 2022

Discontinuity g-mode

First order transition

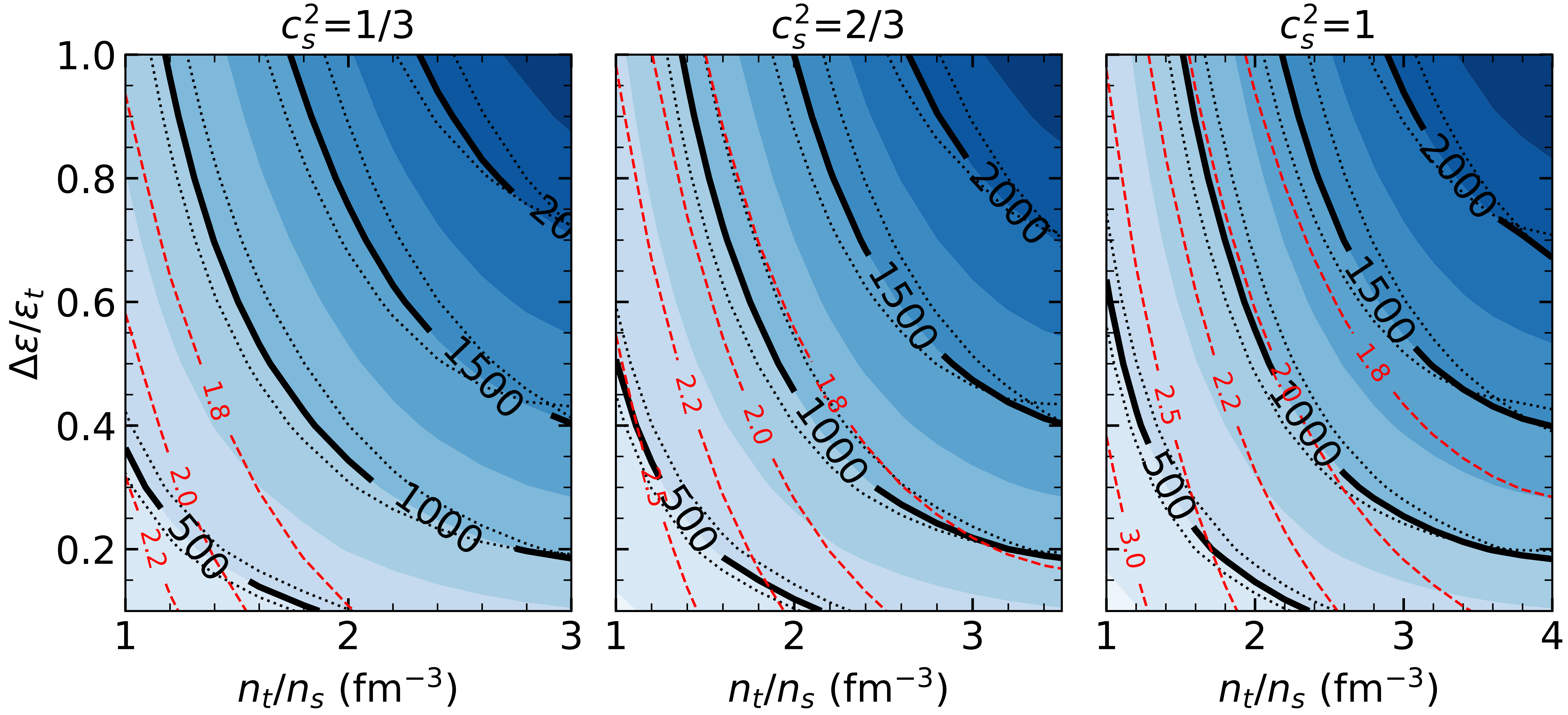


density discontinuity

transition density

Inner core sound speed

Contour of $\nu_g(n_t,\ \Delta_\varepsilon,\ c_s)|_{M=M_{max}}$ **Hz**



Dotted: Chrial EFT Uncertanty 5%

Dashed: maximum mass

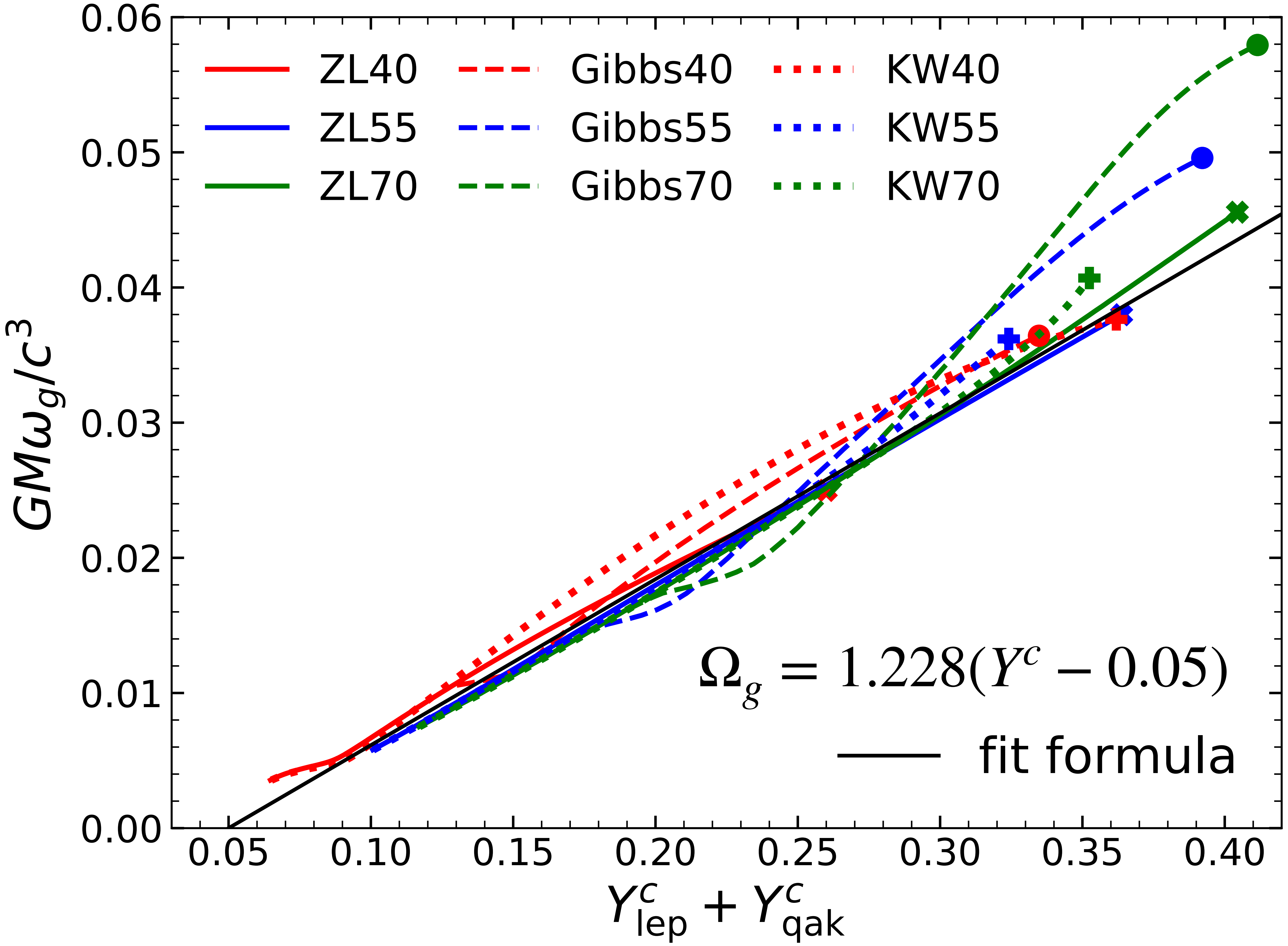
Frequency: $\nu_g < 0.8\ (1.5)\ \mathrm{kHz}$ for $c_s^2 = c^2/3\ (c^2)$

Damping time: $\tau > 100\ (10000)\ \mathrm{s}$

Zhao & Lattimer 2022

BACK

Compositional g-mode universal relation



Y_{lep} **impact Proto-NS neutrino emission**

symmetry energy S(n)

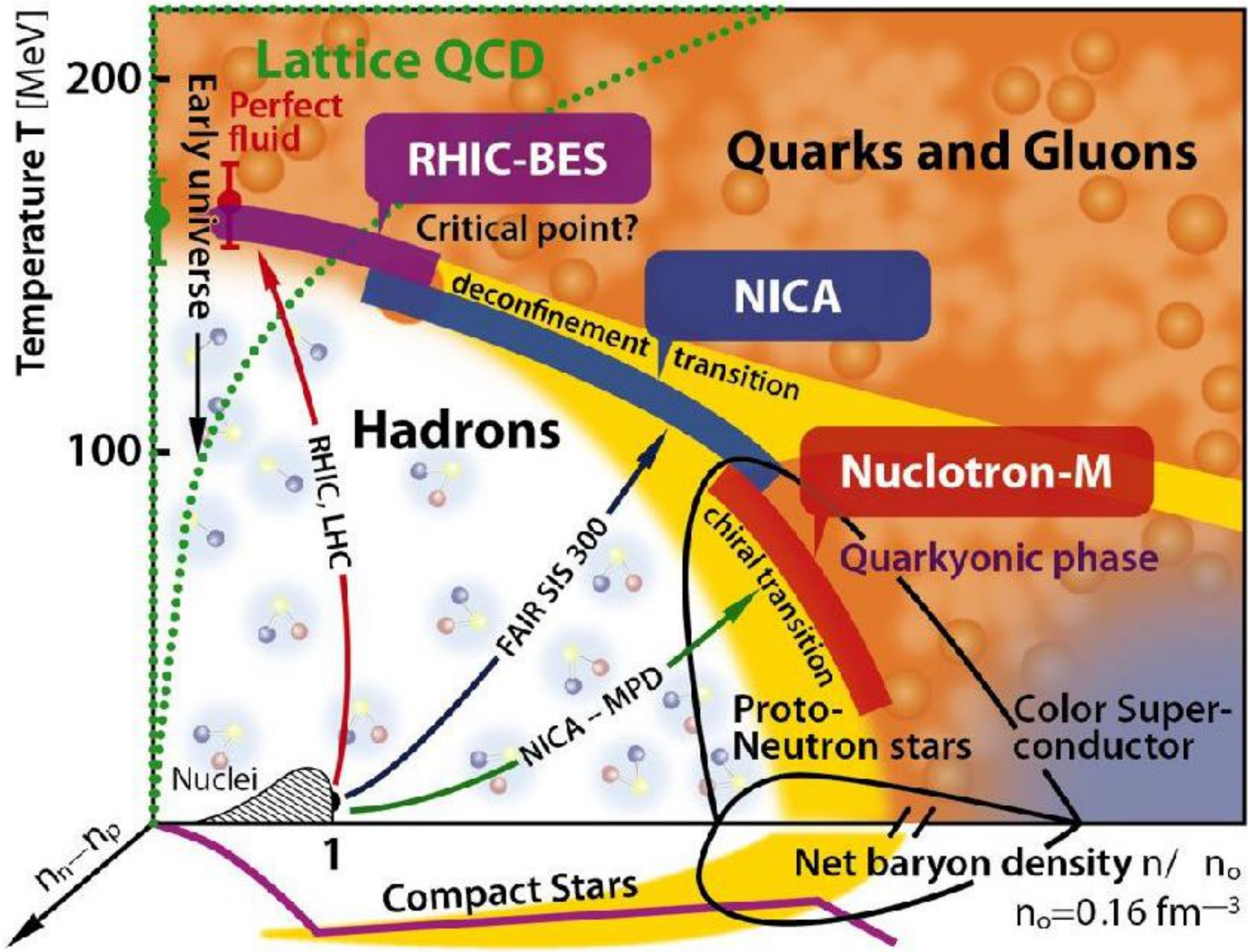
G-mode frequency linearly correlated with lepton fraction and quark fraction at center of NS

Zhao, Constantinou, Jaikumar and Prakash 2022
https://arxiv.org/abs/2202.01403

Back up slides
Quarkyonic matter EOS in beta-equilibrium

Quarkyonic Phase

- Hypothetical phase between hadronic matter and deconfined quark matter(McLerran 2008).



https://nica.jinr.ru/physics.php

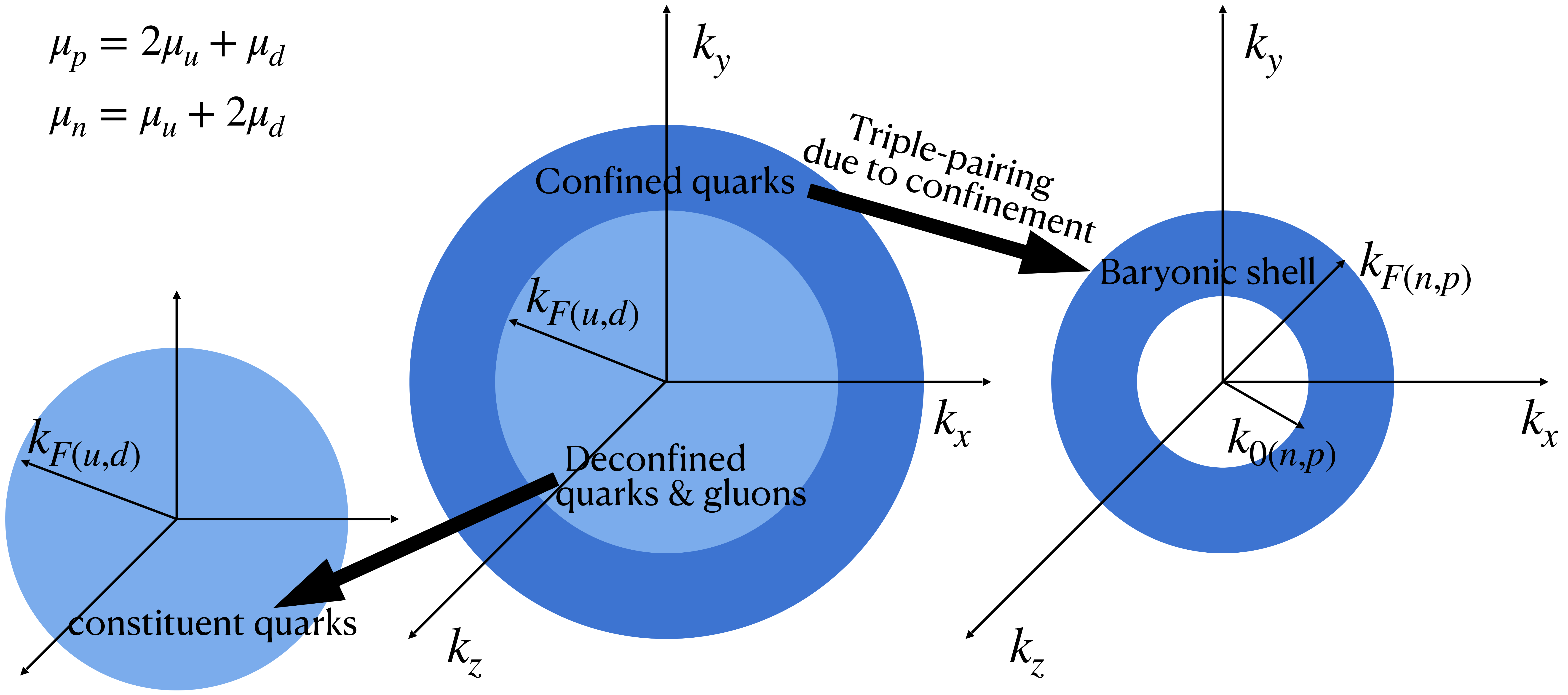
Quarkyonic Momentum Space

- Perturbative quarks = quarks deep inside Fermi sphere
- Baryons = triple-pair of quarks near Fermi surface
- Quarks ⟷ baryons

chemical equilibrium

$$\mu_p = 2\mu_u + \mu_d$$

$$\mu_n = \mu_u + 2\mu_d$$



Quarkyonic Transition

- Outer core: Neutron rich uniform $npe\mu$, $\varepsilon_B = \varepsilon_{kin,n} + \varepsilon_{kin,p} + n_B V(n_p, n_n)$
- Critical point: $n_B = n_t$
  (1) baryonic shells start to 'saturate', $k_{0(n,p)} \geq 0$,
  (2) quarks drip out of nucleons,

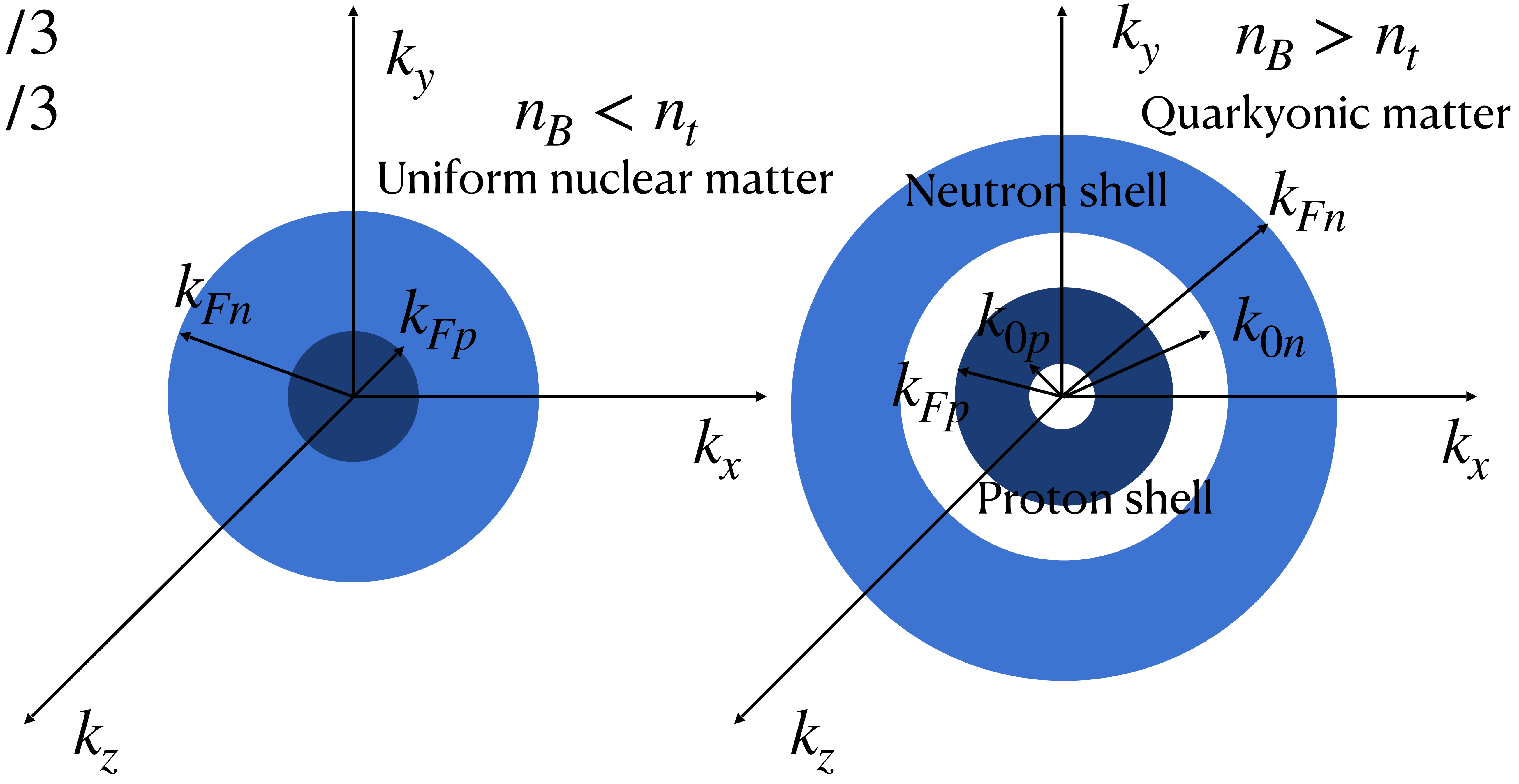
$$k_{0(n,p)} = k_{F(n,p)} \left[1 - \left(\frac{\Lambda}{k_{F(n,p)}} \right)^2 - \frac{\kappa_{n,p}\Lambda}{9k_{F(n,p)}} \right]$$

e.g.
$$m_u = (2\mu_{tp} - \mu_{tn})/3$$
$$m_d = (2\mu_{tn} - \mu_{tp})/3$$

$$L = 45\text{MeV}$$
$$n_t = 0.3\text{fm}^{-3}$$
$$\Lambda = 1400\text{MeV}$$

⬇

$$m_u \approx 250\text{MeV}$$
$$m_d \approx 390\text{MeV}$$
$$\kappa_p \approx -70$$
$$\kappa_p \approx -30$$



Beta-equilibrium & Global Charge Neutrality

- Total baryon number density: $n_B = n_p + n_n + \frac{n_u + n_d}{3}$
- Total lepton number fraction: $Y_L = \frac{n_{e^-} + n_{\mu^-}}{n_B}$
- Weak interaction time scale is small. Neutrino is free.

$p + e^- \leftrightarrow n + (\nu_e), u + e^- \leftrightarrow d + (\nu_e)\,.$

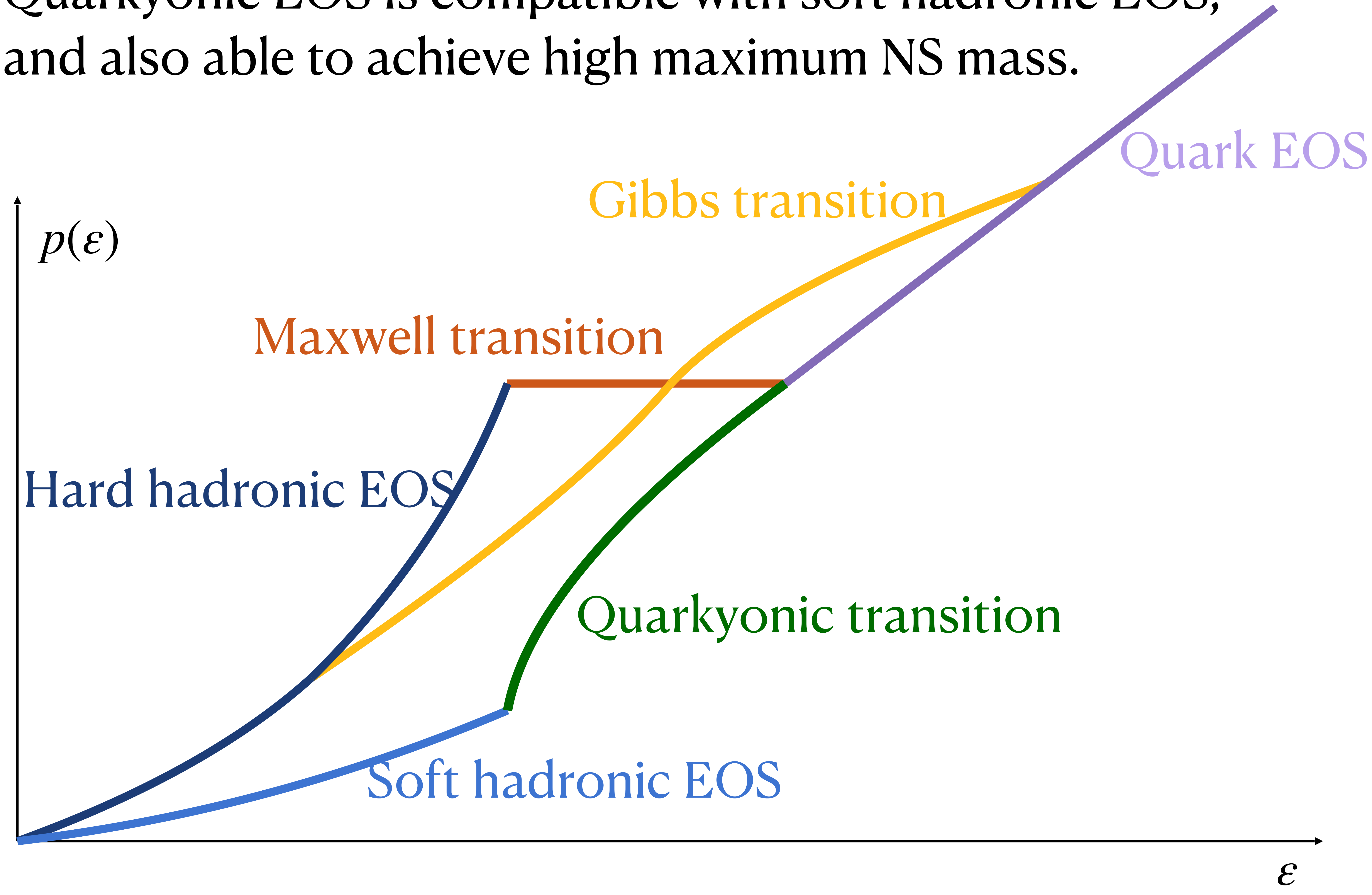
$$\frac{\partial \varepsilon(n_B, Y_L)}{\partial Y_L} = 0 \Longrightarrow \mu_{e^-} = \mu_{\mu^-} = \mu_n - \mu_p = \mu_d - \mu_u$$

- npude$_\mu$ mixture is charge neutral,

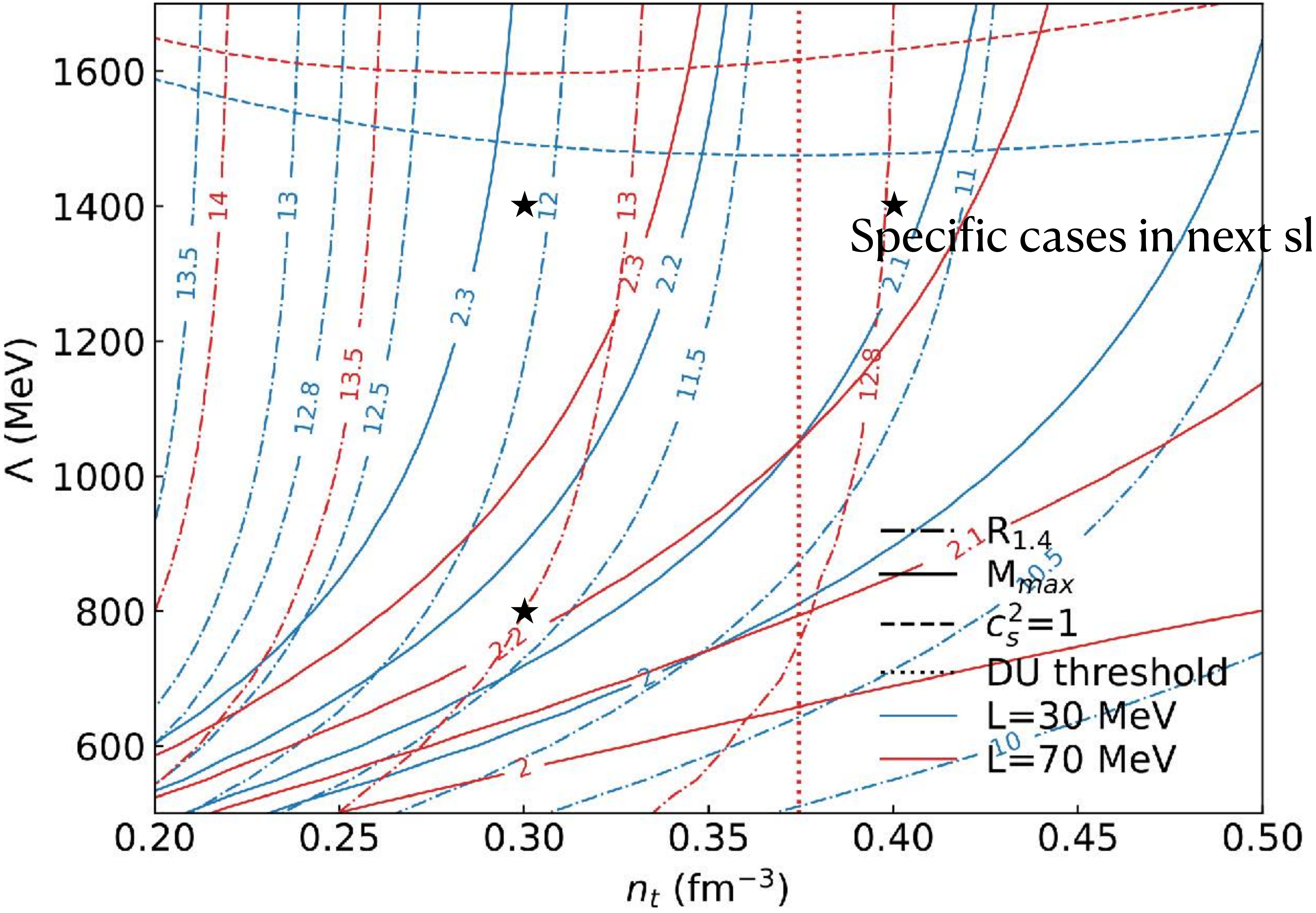
$n_B Y_L = n_p + \frac{2n_u - n_d}{3}$

Uniqueness of Quarkyonic EOS

- Quarkyonic EOS is compatible with soft hadronic EOS, and also able to achieve high maximum NS mass.

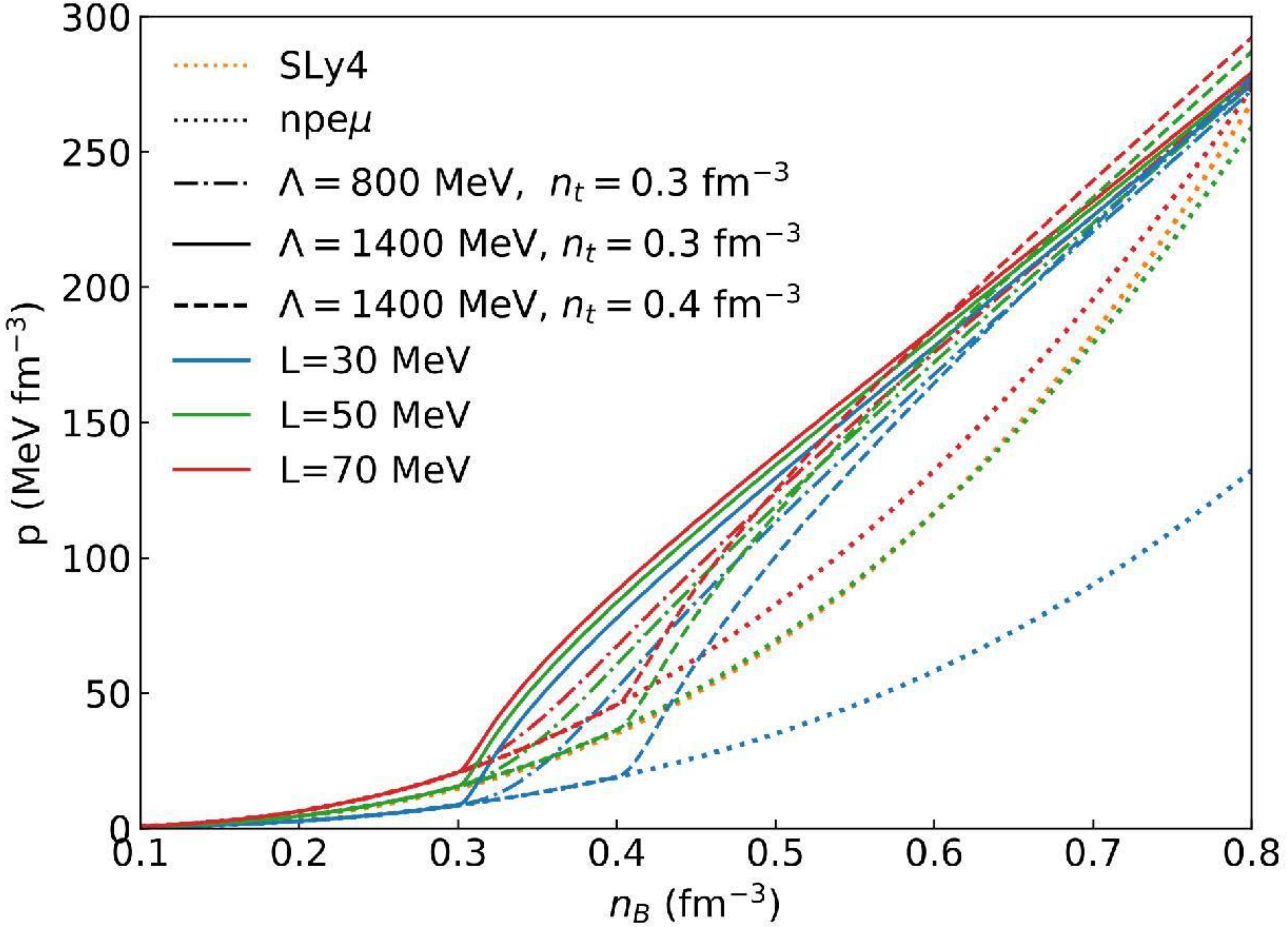


Parameter Space

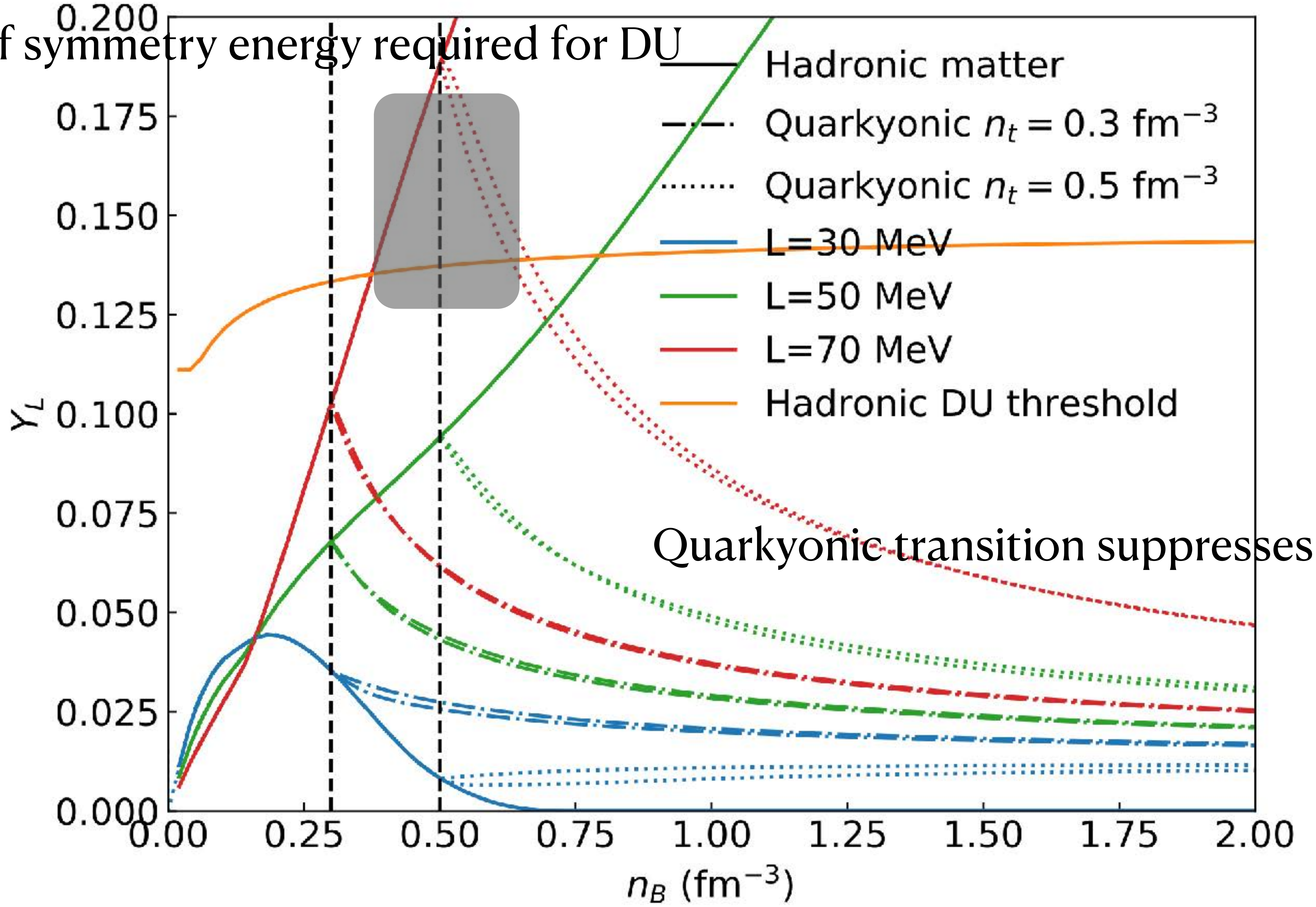


★ Specific cases in next slides

EOS and M-R of 9 Specific Cases ★



Direct Urca Process



Stiff symmetry energy required for DU

Quarkyonic transition suppresses DU