Bayesian Analysis of multi-dimensional neutrino emission simulations with detection events from Supernova 1987A

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Supernova 1987A (SN87A)

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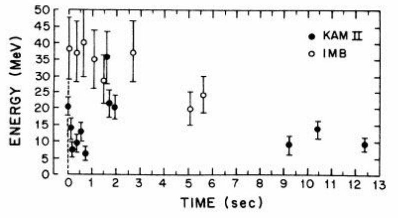
NASA, ESA, and R. Kirshner (Harvard-Smithsonian Center for Astrophysics and Gordon and Betty Moore Foundation) and P. Challis (Harvard-Smithsonian Center for Astrophysics)

$\bar{\nu}_e$ Detection

Hirata et al, Phys. Rev. Lett. 58, 1490 (1987)
Hirata et al, Phys. Rev. D 38, 448 (1988)
Bionta et al, Phys. Rev. Lett. 58, 1494 (1987)

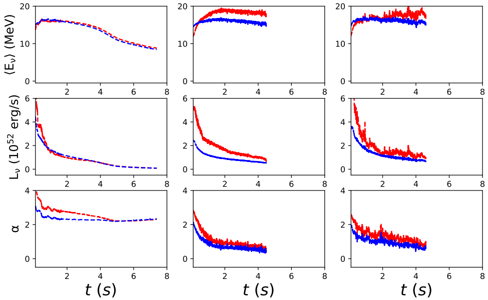


KII		IMB	
$t_{\rm det}$ (s)	E (MeV)	$t_{\rm det}$ (s)	E (MeV)
0	20.0	0	38
0.107	13.5	0.412	37
0.303	7.5	0.650	28
0.324	9.2	1.141	39
0.507	12.8	1.562	36
1.541	35.4	2.684	36
1.728	21.0		
1.915	19.8		
		5.010	19
		5.582	22
9.219	8.6		
10.433	13.0		
12.439	8.9		

Inverse Beta Decay (IBD): $\bar{\nu}_e + p \rightarrow n + e^+$

Multi-dimensional Simmulations

Bollig et al, APJ 915, 28 (2021)
Nagakura et al, Monthly Notices of the Royal Astronomical Society 506, 1462 (2021)



First column is the 3D model from the Garching group, and the next columns are 2D models with $M = 12, 27 M_\odot$ from the Princeton group. Red — $\bar{\nu}_e$, blue — $\bar{\nu}_x$.

- $\alpha_{\nu_\beta} = \frac{2\langle E_{\nu_\beta}\rangle^2 - \langle E^2_{\nu_\beta}\rangle}{\langle E^2_{\nu_\beta}\rangle - \langle E_{\nu_\beta}\rangle^2}$, $T_{\nu_\beta} = \frac{\langle E_{\nu_\beta}\rangle}{1+\alpha_{\nu_\beta}}$
- $f_{\nu_\beta}(E_\nu, t) = \frac{T^{-1}_{\nu_\beta}(t)}{\Gamma(1+\alpha_{\nu_\beta}(t))} \left(\frac{E_\nu}{T_{\nu_\beta}(t)}\right)^{\alpha_{\nu_\beta}(t)} e^{-E_\nu / T_{\nu_\beta}(t)}$
- $F_{\nu_\beta}(E_\nu, t) = \frac{L_{\nu_\beta}}{4\pi d^2 \langle E_{\nu_\beta}\rangle} f_{\nu_\beta}(E_\nu, t)$

Bayesian Approach

- $F_{\rm det}(E_\nu, t) = p_{\bar{\nu}_e} F_{\bar{\nu}_e}(E_\nu, t) + (1 - p_{\bar{\nu}_e}) F_{\bar{\nu}_x}(E_\nu, t)$
- $\frac{d^2N}{dtdE}(E, t) = B(E) + N_p \int F_{\rm det}(E_\nu, t)\sigma_{\rm IBD}(E_\nu)\frac{\epsilon(E_e)}{\sigma_E\sqrt{2\pi}} \exp\left[-\frac{(E-E_e)^2}{2\sigma_E^2}\right] dE_\nu$
- $\langle N_\alpha \rangle = \int_0^{t_{\alpha,\rm cut-off}} dt \int_{E_{\alpha,\rm cut-off}}^{\infty} dE \frac{d^2N_\alpha}{dtdE}$
- $p(E, t|\boldsymbol{\theta}, M_i) = \frac{1}{\langle N_\alpha \rangle}\frac{d^2N_\alpha}{dtdE}$
- $p(D_\alpha|\boldsymbol{\theta}, M_i) = \frac{\langle N_\alpha \rangle^{N_\alpha} e^{-\langle N_\alpha \rangle}}{N_\alpha!} \prod_{\alpha_j=1}^{N} p(E, t|\boldsymbol{\theta}, M_i)$
- $B_{ij} = \frac{p(M_i|D_\alpha)}{p(M_j|D_\alpha)} = \frac{\frac{p(D_\alpha|M_i)p(M_i)}{p(D_\alpha)}}{\frac{p(D_\alpha|M_j)p(M_j)}{p(D_\alpha)}} = \frac{p(D_\alpha|M_i)}{p(D_\alpha|M_j)}$

Free Parameters

d / K , t_{KII}, t_{IMB}, $p_{\bar{\nu}_e}$ (optional)

Results: No flavor oscillations

Model	d (pc)	$t_{\rm KII}$ (s)	$t_{\rm IMB}$ (s)	$\langle N_{\rm KII}\rangle$	$\langle N_{\rm IMB}\rangle$	$p(M_i\|D)$
2D - 12 $M_\odot$	52.17	0.28	0.25	19.05	10.34	3×10^{-5}
2D - 13 $M_\odot$	52.01	0.34	0.43	19.17	10.41	0.0033
2D - 14 $M_\odot$	52.29	0.35	0.42	18.97	10.3	0.039
2D - 15 $M_\odot$	52.51	0.34	0.51	18.81	10.21	5.1×10^{-5}
2D - 16 $M_\odot$	52.11	0.34	0.31	19.1	10.4	0.17
2D - 17 $M_\odot$	52.71	0.36	0.52	18.67	10.14	2.3×10^{-5}
2D - 18 $M_\odot$	52.03	0.29	0.27	19.15	10.4	0.068
2D - 19 $M_\odot$	52.0	0.32	0.43	19.18	10.41	0.0042
2D - 20 $M_\odot$	52.88	0.43	0.52	18.55	10.07	1.096×10^{-6}
2D - 21 $M_\odot$	53.3	0.44	0.62	18.26	9.91	2.91×10^{-9}
2D - 22 $M_\odot$	52.72	0.41	0.48	18.66	10.13	5.8×10^{-5}
2D - 23 $M_\odot$	52.31	0.38	0.42	18.95	10.29	4.3×10^{-5}
2D - 25 $M_\odot$	51.98	0.4	0.77	19.19	10.42	0.0012
2D - 26 $M_\odot$	52.75	0.37	0.45	18.64	10.12	7.5×10^{-7}
2D - 27 $M_\odot$	52.39	0.38	0.52	18.9	10.26	5.2×10^{-6}
3D - 19 $M_\odot$	52.1	0.04	0.11	19.1	10.37	0.72

Model	K	$t_{\rm KII}$ (s)	$t_{\rm IMB}$ (s)	$\langle N_{\rm KII}\rangle$	$\langle N_{\rm IMB}\rangle$	$p(M_i\|D)$
2D - 12 $M_\odot$	0.49	0.27	0.41	9.84	5.28	0.003
2D - 13 $M_\odot$	0.48	0.35	0.43	9.7	5.2	0.009
2D - 14 $M_\odot$	0.56	0.35	0.42	11.29	6.07	0.05
2D - 15 $M_\odot$	0.43	0.32	0.51	8.59	4.6	0.01
2D - 16 $M_\odot$	0.64	0.34	0.42	12.92	6.95	0.048
2D - 17 $M_\odot$	0.37	0.36	0.52	7.41	3.96	0.004
2D - 18 $M_\odot$	0.62	0.31	0.27	12.45	6.69	0.032
2D - 19 $M_\odot$	0.51	0.31	0.32	10.24	5.5	0.013
2D - 20 $M_\odot$	0.35	0.38	0.52	7.04	3.76	0.003
2D - 21 $M_\odot$	0.33	0.44	0.52	6.71	3.59	0.005
2D - 22 $M_\odot$	0.41	0.39	0.49	8.19	4.38	0.008
2D - 23 $M_\odot$	0.39	0.38	0.51	7.84	4.2	0.007
2D - 25 $M_\odot$	0.42	0.4	0.77	8.55	4.58	0.074
2D - 26 $M_\odot$	0.34	0.37	0.44	6.91	3.69	0.02
2D - 27 $M_\odot$	0.39	0.37	0.52	7.82	4.19	0.006
3D - 19 $M_\odot$	0.6	0.06	0.11	12.02	6.46	0.73

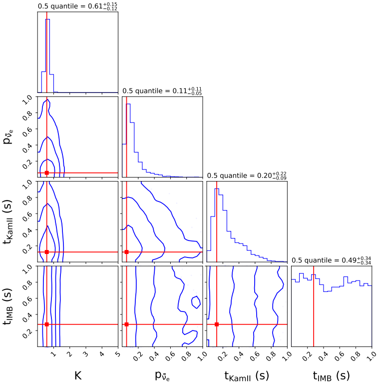
Results: flavor oscillations

Model	d (kpc)	$p_{\overline{\nu}_e}$	$t_{\rm KII}$ (s)	$t_{\rm IMB}$ (s)	$\langle N_{\rm KII}\rangle$	$\langle N_{\rm IMB}\rangle$	$p(M_i\|D)$
2D - 12 $M_\odot$	51.18	0.0068	0.27	0.44	18.64	11.93	1.17×10^{-5}
2D - 13 $M_\odot$	52.36	0.023	0.34	0.43	18.52	11.82	3.27×10^{-4}
2D - 14 $M_\odot$	51.82	0.0083	0.35	0.34	18.9	12.09	0.013
2D - 15 $M_\odot$	52.68	0.0054	0.34	0.44	18.29	11.71	2.94×10^{-6}
2D - 16 $M_\odot$	51.61	0.033	0.33	0.46	19.06	12.15	0.045
2D - 17 $M_\odot$	52.95	0.012	0.36	0.51	18.1	11.57	1.86×-6
2D - 18 $M_\odot$	51.76	0.0014	0.29	0.38	18.93	12.13	0.0079
2D - 19 $M_\odot$	52.39	0.0044	0.31	0.33	18.49	11.84	3.22×10^{-4}
2D - 20 $M_\odot$	53.07	0.0056	0.40	0.54	18.02	11.53	9.70×10^{-8}
2D - 21 $M_\odot$	53.76	0.013	0.44	0.52	17.56	11.23	4.02×10^{-10}
2D - 22 $M_\odot$	52.69	0.011	0.39	0.53	18.28	11.69	8.90×10^{-6}
2D - 23 $M_\odot$	52.47	0.0079	0.37	0.53	18.43	11.79	3.46×10^{-6}
2D - 25 $M_\odot$	52.22	0.036	0.40	0.77	18.62	11.86	1.02×10^{-4}
2D - 26 $M_\odot$	53.02	0.014	039	0.48	18.05	11.54	4.71×10^{-8}
2D - 27 $M_\odot$	52.97	0.014	0.38	0.49	18.09	11.57	5.52×10^{-7}
3D - 19 $M_\odot$	51.69	0.98	0.083	0.14	19.40	10.58	0.933

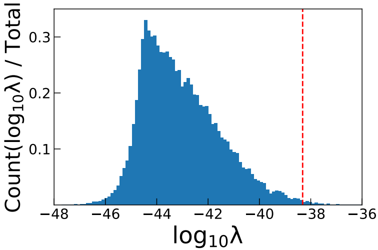
Model	K	$p_{\overline{\nu}_e}$	$t_{\rm KII}$ (s)	$t_{\rm IMB}$ (s)	$\langle N_{\rm KII}\rangle$	$\langle N_{\rm IMB}\rangle$	$p(M_i\|D)$
2D - 12 $M_\odot$	0.26	0.309	0.63	0.67	5.00	3.02	0.0012
2D - 13 $M_\odot$	0.32	0.36	0.43	0.19	6.21	3.72	0.012
2D - 14 $M_\odot$	0.43	0.35	0.55	0.46	8.39	5.05	0.12
2D - 15 $M_\odot$	0.258	0.34	0.54	0.01	5.05	3.03	0.0077
2D - 16 $M_\odot$	0.52	0.37	0.48	0.96	10.19	6.13	0.049
2D - 17 $M_\odot$	0.28	0.39	0.51	0.79	5.48	3.26	0.0056
2D - 18 $M_\odot$	0.39	0.33	0.58	0.71	7.65	4.62	0.029
2D - 19 $M_\odot$	0.32	0.34	0.51	0.08	6.24	3.75	0.0066
2D - 20 $M_\odot$	0.28	0.40	0.53	0.58	5.52	3.28	0.007
2D - 21 $M_\odot$	0.24	0.45	0.62	0.33	4.71	2.77	0.024
2D - 22 $M_\odot$	0.32	0.43	0.61	0.49	6.26	3.71	0.023
2D - 23 $M_\odot$	0.28	0.42	0.64	0.39	5.52	3.27	0.0034
2D - 25 $M_\odot$	0.30	0.42	0.86	0.85	5.96	3.53	0.12
2D - 26 $M_\odot$	0.25	0.41	0.45	0.76	4.93	2.92	0.0021
2D - 27 $M_\odot$	0.28	0.42	0.60	0.25	5.43	3.22	0.012
3D - 19 $M_\odot$	0.61	0.055	0.13	0.33	11.79	7.47	0.57

Results: 3D Simmulation

Corner Plot



P-Value Test



$$\lambda = \prod_{\alpha_j=1}^{N} p(E_{\alpha_j}, t_{\alpha_j} | \boldsymbol{\theta}_{\rm bf}, M_i)$$

Summary

- ≈ 20 events from SN87A (KII + IMB)
- 2D / 3D simmulations (short time)
- Bayesian analysis of $d\ /K,\ t_{KII},\ t_{IMB},\ p_{\bar{\nu}_e}$
- 3D best at explaining the detection
- flavor mixing highly likely

Thank you

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Results: 3D Simmulation (t = 4.5 s)