Moment-based Neutrino Flavor Transformation in BNS Mergers

Evan Grohs (he/him/his)
North Carolina State University

N3AS Annual Meeting, Berkeley, CA

18 Mar 2023

In Collaboration with:
Julien Froustey, Gail McLaughlin, Jim Kneller (NCSU);
Sherwood Richers (UTK); Francois Foucart (UNH);
Sean Couch (MSU)

NC STATE UNIVERSITY

THE UNIVERSITY OF CALIFORNIA BERKELEY · LET THERE BE LIGHT · 1868

Outline

1. Fast Flavor Instability (FFI) Motivation
2. Evolution variables and Quantum Kinetic Equations (QKEs)
3. Previous Oscillation Tests in 0D and 1D
4. FFI Tests in 1D and 3D
5. FFI simulation in a BNS merger
6. Summary and plan moving forward

Fast Flavor Instability -- Qualitative

1. Neutrinos and anti-neutrinos carry weak charges

2. As neutrino moves through compact object environment:
   a. Neutrinos propagate through field of other neutrinos
   b. Self-energy diagrams have (relatively) large amplitudes $\sim G_F$
   c. Momentum-preserving neutrino forward scattering (i.e., self-interactions)

3. Non-linear self-interactions sensitive to asymmetry of neutrinos vs. anti-neutrinos $\Longrightarrow$ lepton number

4. Lepton number crossings (in angle) can give rise to rapid flavor transformation: Fast Flavor Conversion

Neutrino Density Matrices & QKEs

1. Use “generalized neutrino density matrices” to characterize the neutrino ensemble

$$\varrho(t, \vec{r}, \vec{p}) = \begin{pmatrix} \varrho_{ee} & \varrho_{ex} \\ \varrho^*_{ex} & \varrho_{xx} \end{pmatrix} \qquad \overline{\varrho}(t, \vec{r}, \vec{p}) = \begin{pmatrix} \overline{\varrho}_{ee} & \overline{\varrho}_{ex} \\ \overline{\varrho}^*_{ex} & \overline{\varrho}_{xx} \end{pmatrix}$$

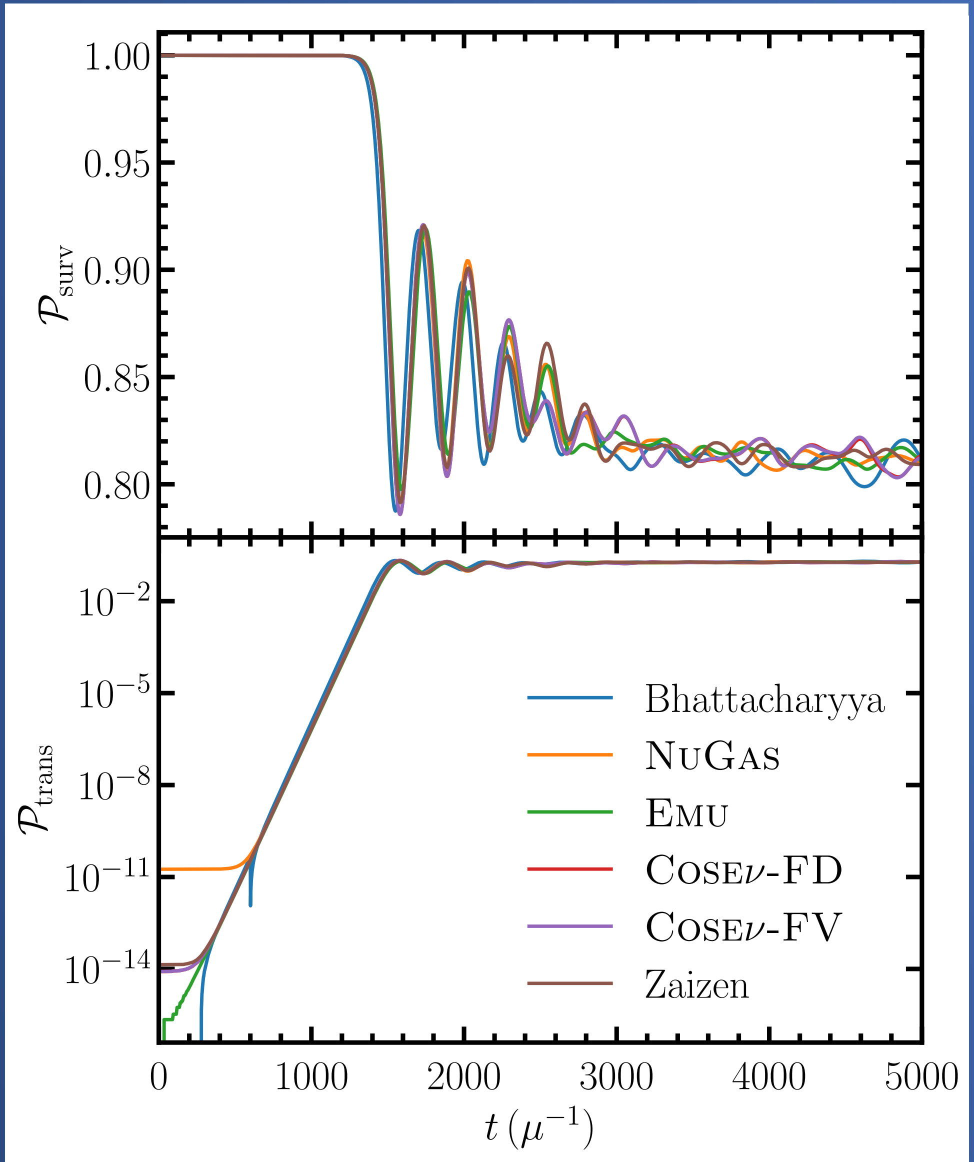
2. Only keep single-particle correlation functions – mean field

3. QKEs from Blaschke & Cirigliano (2016) – among others:

Drift Term ~~Force Term~~ Coherent Term ~~Collision Term~~

$$\frac{\partial \varrho}{\partial t} + \dot{\vec{x}} \cdot \frac{d\varrho}{d\vec{x}} + \dot{\vec{p}} \cdot \frac{d\varrho}{d\vec{p}} = -i[H, \varrho] + C$$

Code Comparison – Richers et al (2022)



Fast Flavor Instability test problem (Put in ELN)

PIC and other multi-angle codes

Excellent agreement

Different Approach: Neutrino Moments

FLASH uses angular moments

Take moments of generalized density matrices to eliminate angle dimensions ($j, k, l \in \{x, y, z\}; a, b \in \{e, x\}$):

$$E_{ab}(t, \vec{x}, p) = \frac{1}{4\pi} \frac{p^3}{(2\pi)^3} \int d\Omega_p \, \varrho_{ab}(t, \vec{x}, \vec{p}) \qquad (0^{th})$$

$$F^j_{ab}(t, \vec{x}, p) = \frac{1}{4\pi} \frac{p^3}{(2\pi)^3} \int d\Omega_p \frac{p^j}{p} \varrho_{ab}(t, \vec{x}, \vec{p}) \qquad (1^{st})$$

$$P^{jk}_{ab}(t, \vec{x}, p) = \frac{1}{4\pi} \frac{p^3}{(2\pi)^3} \int d\Omega_p \frac{p^j p^k}{p^2} \varrho_{ab}(t, \vec{x}, \vec{p}) \qquad (2^{nd})$$

$$T^{jkl}_{ab}(t, \vec{x}, p) = \ldots$$

Neutrino QKEs with moments for FFI

Zhang & Burrows (2013): Equations of motion for first 2 moments

$$\frac{\partial E}{\partial t} + \frac{\partial F^j}{\partial x^j} = -i[H_V + H_M + H_E, E] + i[H_{F_j}, F^j]$$

$$\frac{\partial F^j}{\partial t} + \frac{\partial P^{jk}}{\partial x^k} = -i[H_V + H_M + H_E, F^j] + i[H_{F_k}, P^{jk}]$$

$$H_V = -\frac{1}{2p} U M^2 U^\dagger \qquad H_E = 4\pi\sqrt{2} G_F \int \frac{dq}{q}(E - \overline{E})$$

$$H_M = \sqrt{2} G_F n_e I_e \qquad H_{F_j} = 4\pi\sqrt{2} G_F \int \frac{dq}{q}(F_j - \overline{F}_j)$$

Truncate tower of equations after F^j with closure relationship

Closure Relationship – Cartesian Geometry

1. For Cartesian box calculations: closure independent of radius
2. Need flux factors for diagonal and off-diagonal (in each cell)

$$f_{ab}(q) = f(q) = \frac{|\mathrm{Tr}[\vec{F}]|}{\mathrm{Tr}[E]}$$

Same for each density matrix component!

3. Eddington factor from flux factor via Maximum Entropy Closure

$$\chi = \frac{1}{3} + \frac{2}{15} f^2 (3 - f + 3f^2)$$

4. Calculate Eddington tensor (Pressure)

$$P_{ab}^{jk} = P_{ab}^{jk}(\chi, E_{ab}, \vec{F}_{ab})$$

No guarantee moments will capture FFI

FLASH – Multi-D Hydrodynamics Code

1. Code for modeling CCSN in multiple dimensions;
   See Fryxell et al (2000)
2. Advanced M1 *neutrino transport*, i.e., non-unitary processes;
   See O'Connor & Couch (2018) *[3 species of ν]*

3. Initiate 2-flavor oscillation physics and infrastructure –
   M. Warren & S. Richers *[8 species of ν]*

4. 1D/2D/3D calculations
5. Cartesian or Spherical geometries
6. Force and Collision terms neglected at this point

Bipolar Oscillation Test: Homogeneous

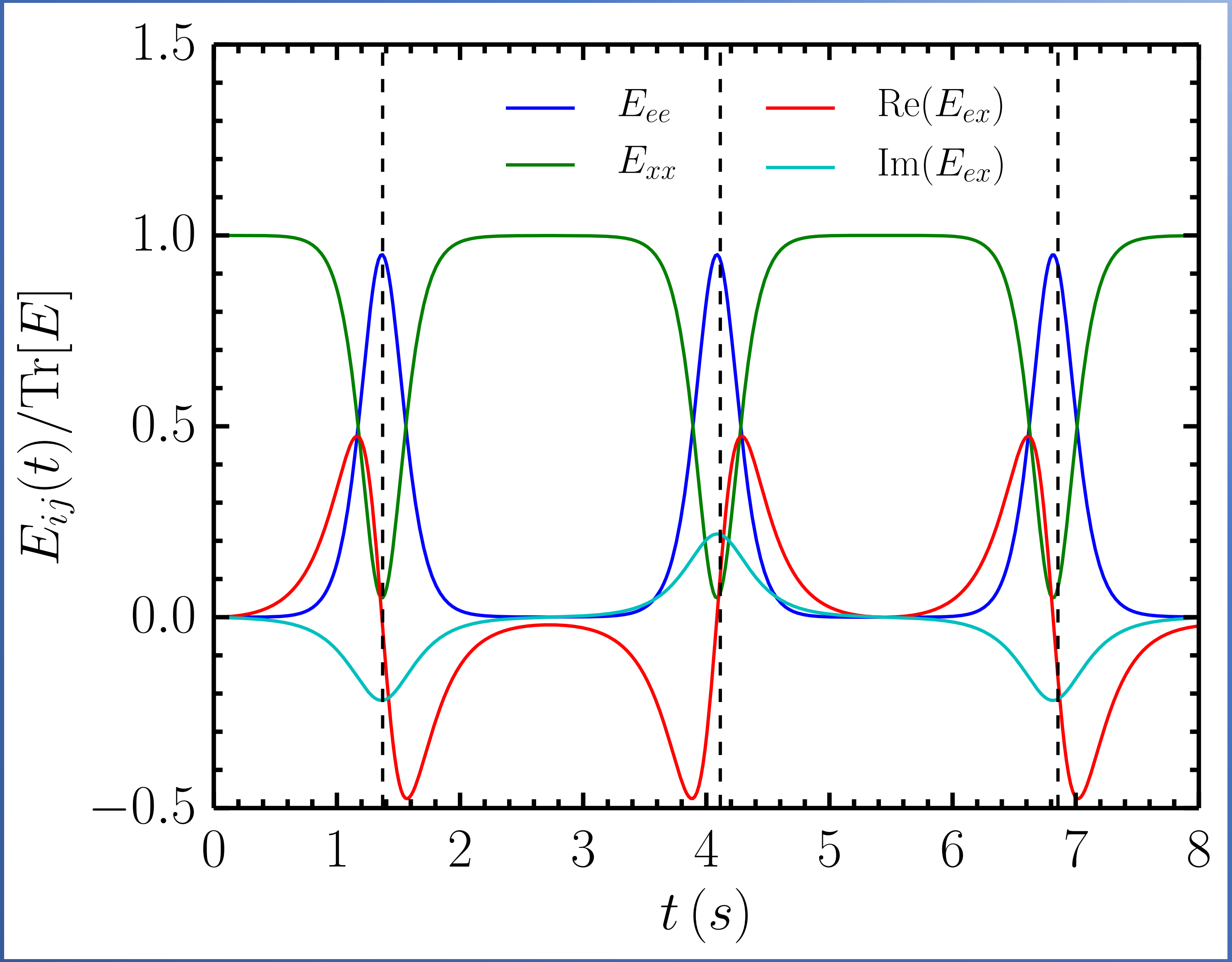
Flux Moment identically zero at all times

Use Vacuum and Self-Interacting terms in QKEs

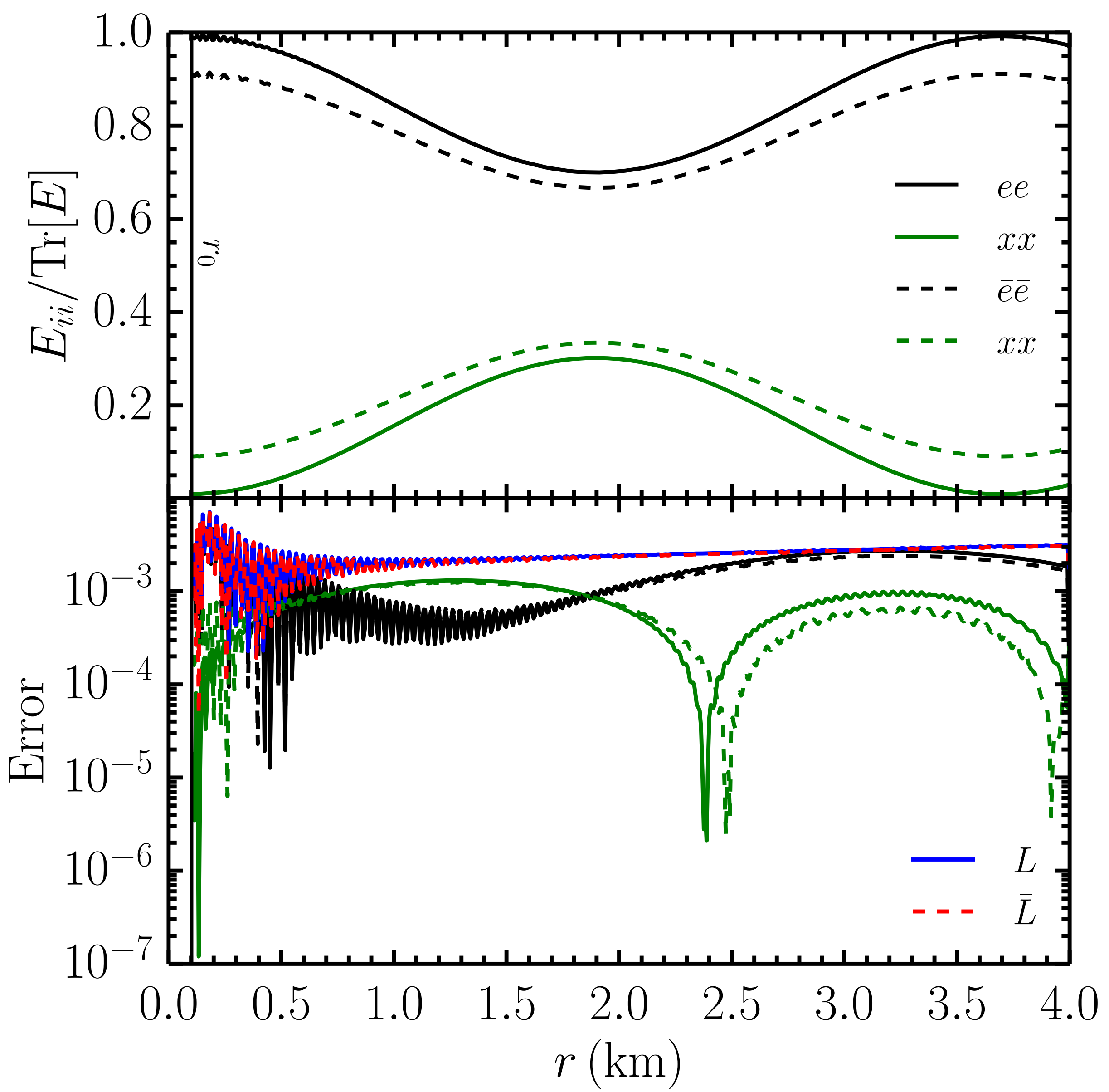
No analytical prediction for shape of the curves

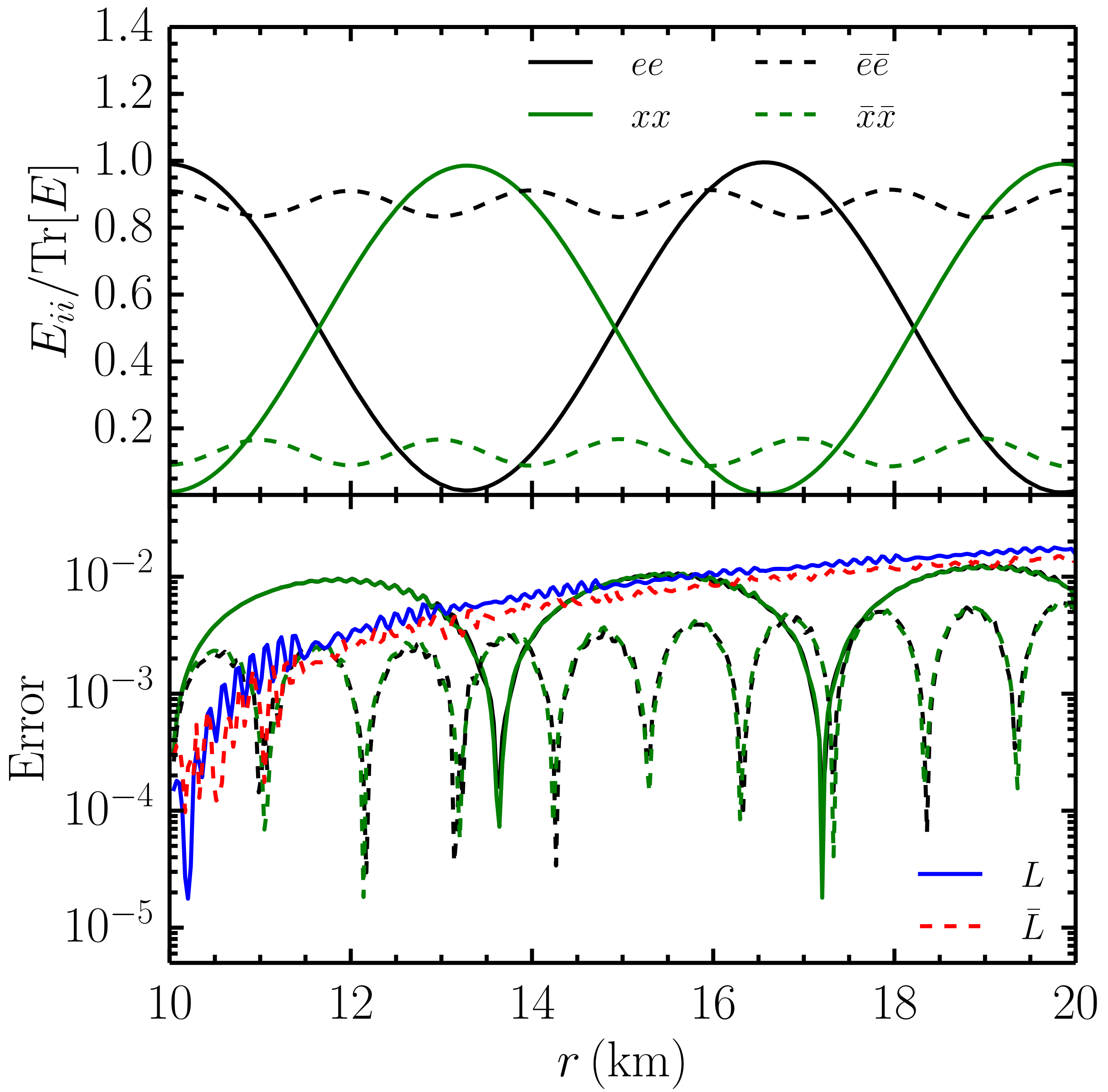
Plots are for first cell

Dashed black line analytic prediction for period



Vacuum & MSW Test: Spherical 1D





Three FFI Tests in 3D

1. Fiducial: beams/slabs of neutrinos vs. anti-neutrinos at 180 degrees

2. TwoThirds: isotropic neutrinos, beamed anti-neutrinos; anti-neutrino number density 2/3 of neutrino

3. 90Degree: beams of neutrinos vs. anti-neutrinos at right angles

Name	n_{ν_e} $(10^{32}\mathrm{cm}^{-3})$	$n_{\overline{\nu}_e}$ $(10^{32}\mathrm{cm}^{-3})$	Σn_{ν_x} $(10^{32}\mathrm{cm}^{-3})$	$\mathbf{f}_{\nu_e}$	$\mathbf{f}_{\overline{\nu}_e}$	$\mathbf{f}_{\nu_x}$	L (cm)	N_{gp}
Fiducial	4.89	4.89	0	$(0,0,1/3)$	$(0,0,-1/3)$	$(0,0,0)$	8	128^3
TwoThirds	4.89	3.26	0	$(0,0,0)$	$(0,0,-1/3)$	$(0,0,0)$	32	128^3
90Degree	4.89	4.89	0	$(0,0,1/3)$	$(0,1/3,0)$	$(0,0,0)$	8	128^3

$$N_{ab}(\vec{x},p) = \frac{4\pi}{p} E_{ab}(\vec{x},p)$$

$$\mathbf{f}_{ab} = \frac{\vec{F}_{ab}}{E_{ab}}$$

Compare against EMU

Initial Conditions: Off-Diagonal components

Turn off Vacuum term

Seed off-diagonal components at $t = 0$ with perturbation $(a \neq b)$

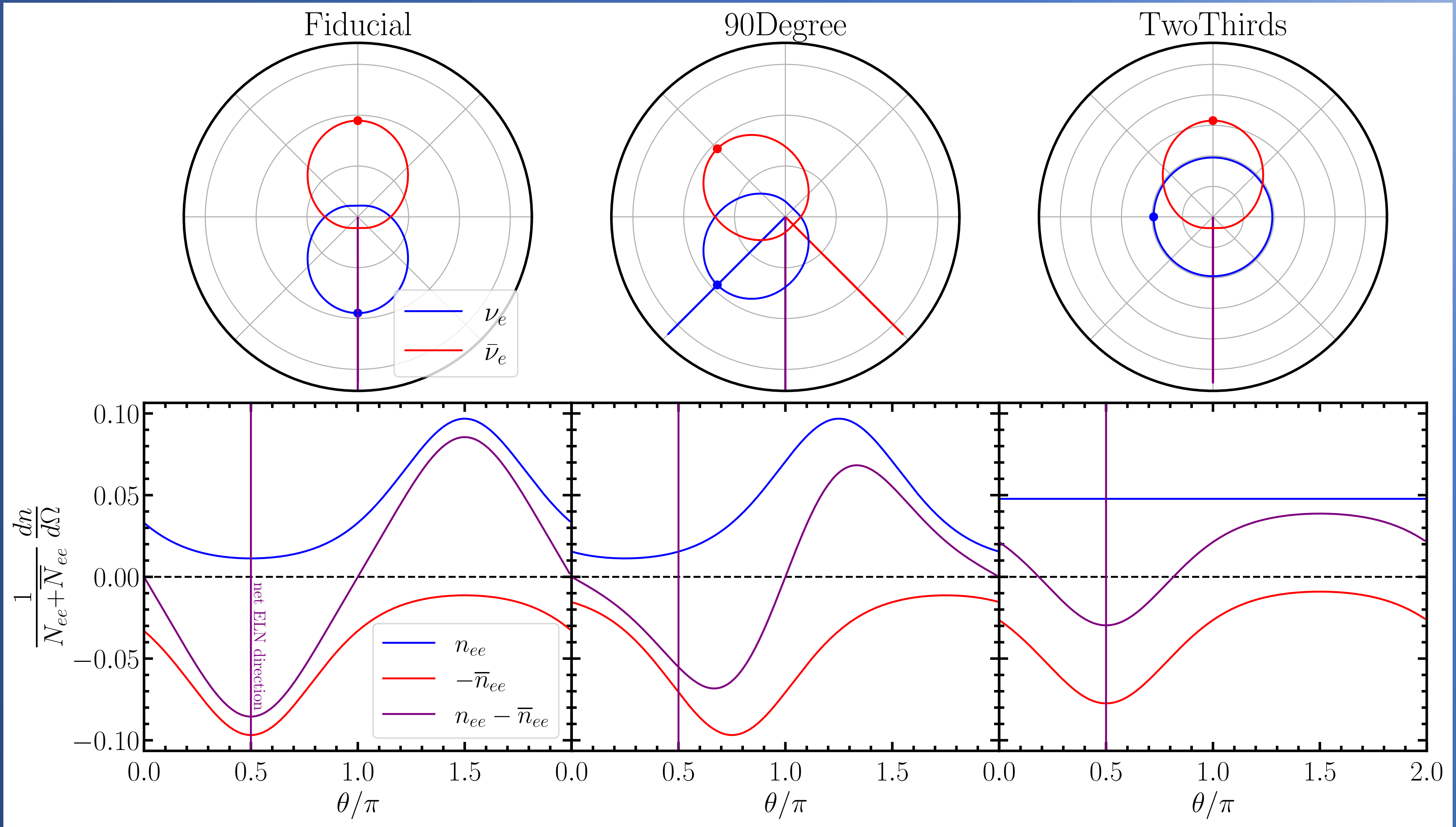
$$\delta E_{ab}(\vec{x}) = 10^{-6}\frac{p}{4\pi}\max\{N_{cc}\}[A_{ab}(\vec{x}) + iB_{ab}(\vec{x})]$$

Random numbers $-1 < A_{ab}, B_{ab} < 1$ in each cell

Weighted fluences correlated with energy density

$$\delta\vec{F}_{ab}(\vec{x}) = \delta E_{ab}(\vec{x})\frac{\Sigma_c N_{cc}\mathbf{f}_{cc}}{\Sigma_c N_{cc}}$$

Geometry of FFI Tests in 3D

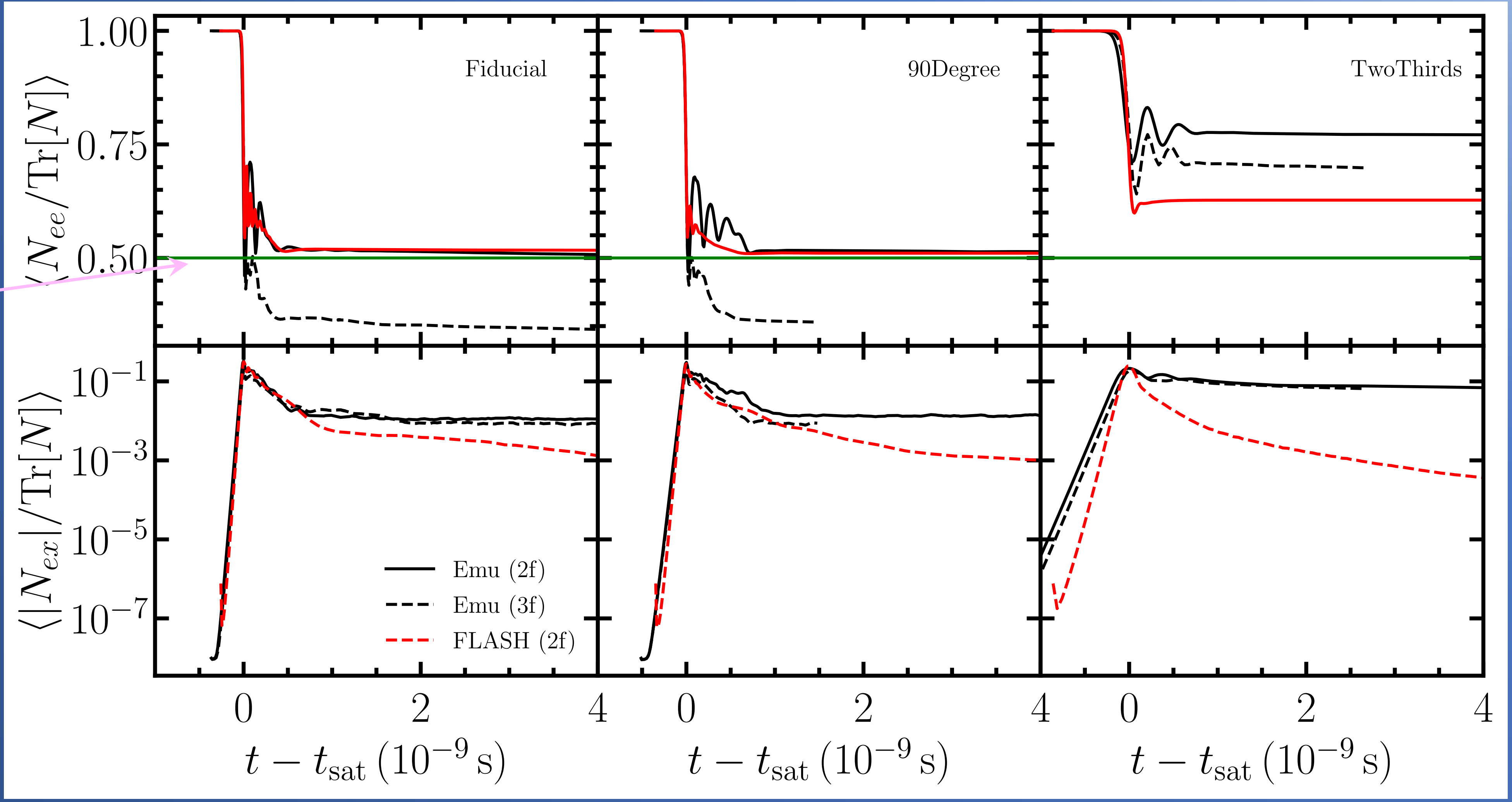


Averaged Results: Three Initial Tests

Diagonal component: Number density

Flavor Equilibration

Off-Diagonal component: Flavor Coherence



Grohs et al (In Prep.)

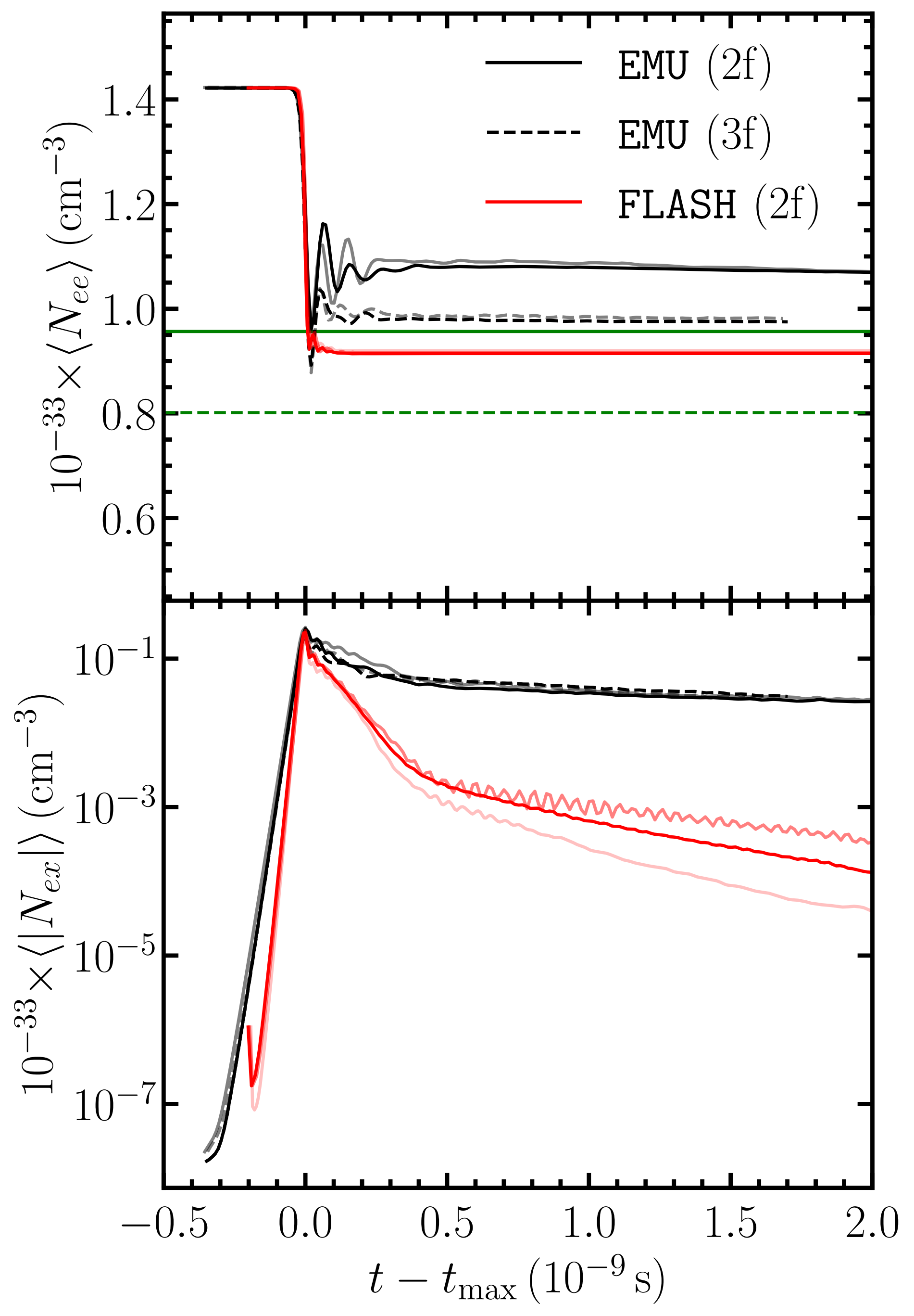
Fast Flavor Instability(FFI) in a Neutron Star Merger

Red Curves: Moment method (FLASH code)
Black curves: Particle-in-a-Cell method (EMU code)

Lepton number crossings (in angle) give rise to rapid flavor transformation: Fast Flavor Conversion

Foucart et al (2016): GR simulation with M1 scheme for NSM with Transport & Maximum Entropy Closure

Success: Moment method captures FFI



arXiv: 2207.02214

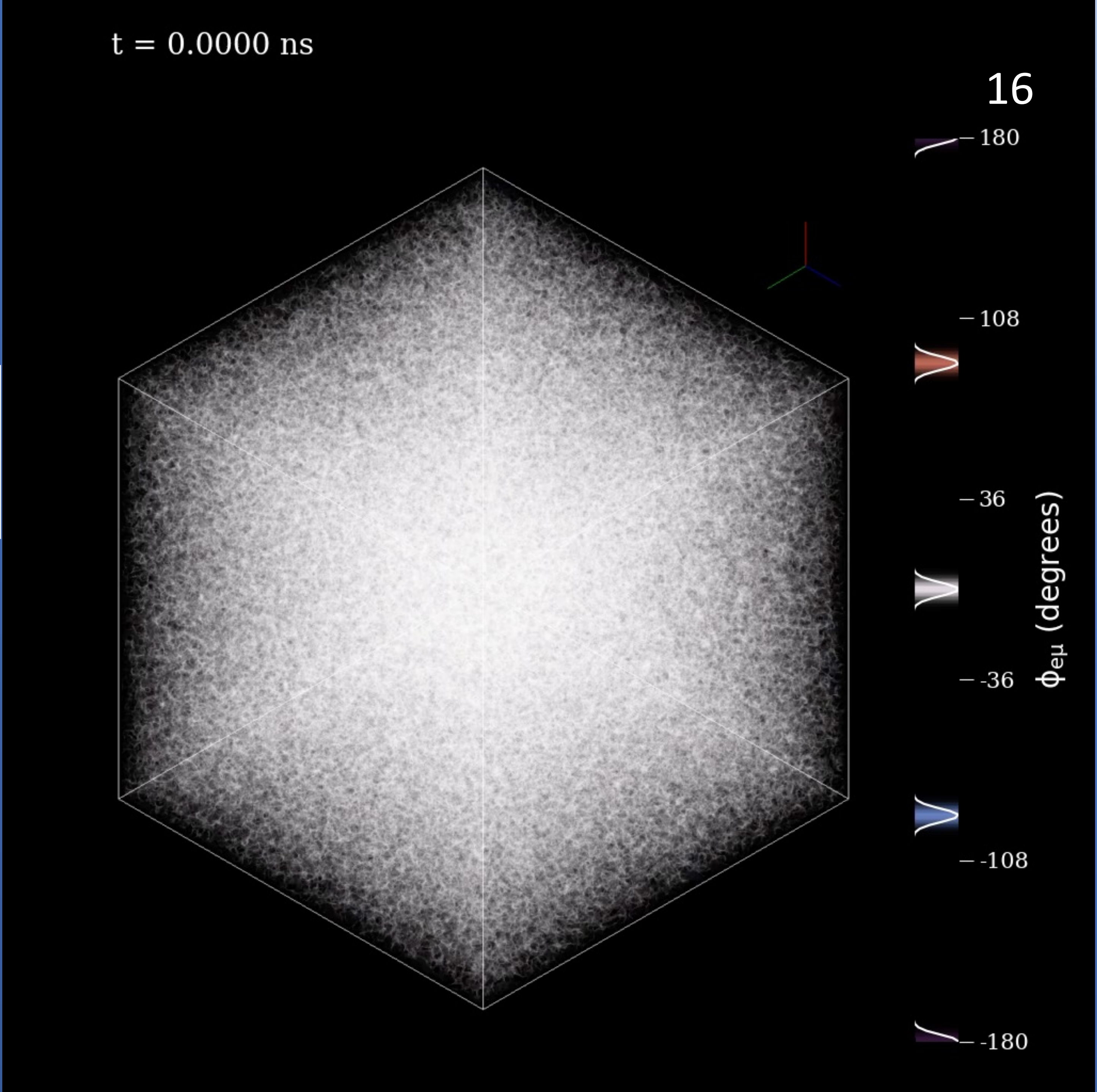
Phase of Off-Diagonal

$$\phi_{ex} = \text{arctan2}\left[\frac{\text{Im}(N_{ex})}{\text{Re}(N_{ex})}\right]$$

White: $\phi_{ex} = 0$

Red: $\phi_{ex} = \pi/2$

Blue: $\phi_{ex} = -\pi/2$

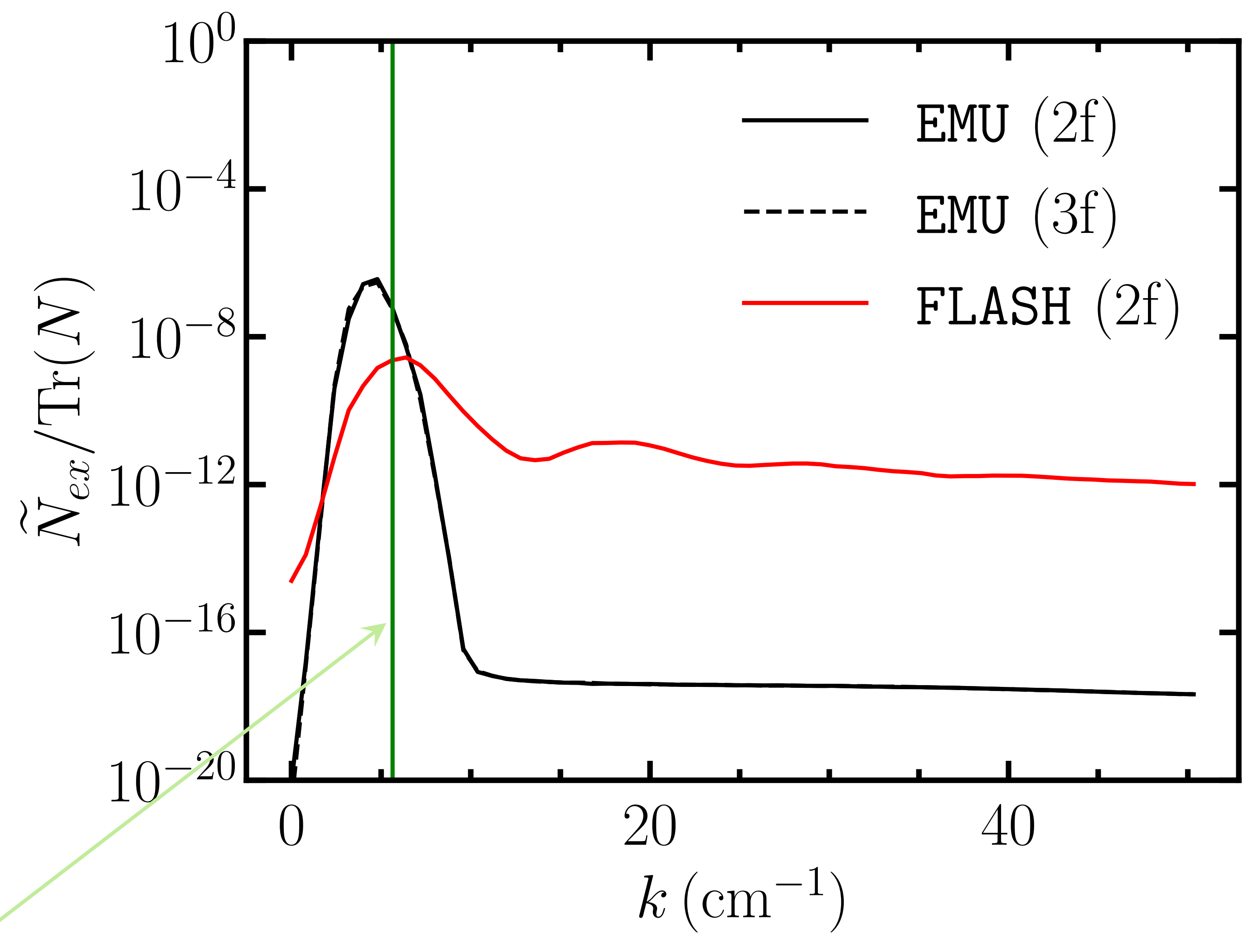


Fourier Transform

Discrete Fourier transform of Off-Diagonal over space

Shows which scales have the largest power

Time ~0.1 ns before saturation



Prediction from
Linear Stability Analysis

Comparisons

Name	$\mathrm{Im}(\Omega)$ $(10^{10}\mathrm{s}^{-1})$	$\lvert k\rvert_{\max}$ (cm^{-1})
LSA	7.62	5.64
EMU (2f)	5.58	4.79
EMU (3f)	5.47	4.79
FLASH (2f)	8.09	6.39

Prediction

Growth Rate

Fastest Growing Mode

CPU Time Comparison:

$$\frac{\mathrm{FLASH}}{\mathrm{EMU}} = \frac{1}{30}$$

Summary & Plan moving forward

1. Moments/FLASH capture the FFI

2. Closure relationship is and will continue to be crucial
   (Quantum Closures – Jim Kneller)

3. Need to look at more coherent physics cases in compact objects

4. Need to introduce the QKE collision term

5. Long term: calculate electron fraction(s)

6. Longer term: model CCSNe and BNS mergers with full flavor transformation in the Flash-X code and other GRMHD (N3AS)