Introduction of the GRRMHD code Gmunu and its applications

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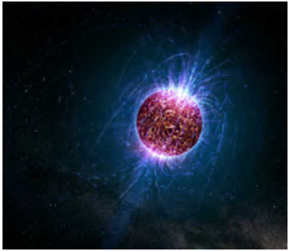
Agenda

Introduction

Neutron stars (NSs)

- the most compact with an internal structure
- power many astrophysical sources of high-energy emission
- could contain strong $\vec{B}$ field ($\sim 10^{8-16}$G)
- sometime related to extreme astrophysical events
- extreme physics play important roles



Artist concept of a neutron star. Credit: NASA

NS modelling

- Magnetars
- Neutron star mergers
- Core-collapse supernovae

Equations needed to be solved

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = T_{\mu\nu}, \quad \text{(Einstein equation)}$$

$$\nabla_\mu \left(\rho u^\mu \right) = 0, \quad \text{(cons. rest mass)}$$

$$\nabla_\mu T^{\mu\nu} = 0, \quad \text{(cons. energy/momentum)}$$

$$p = p\left(\rho, \epsilon, Y_\mathrm{e} \cdots \right), \quad \text{(equation of state)}$$

$$\nabla_\mu F^{\mu\nu} = \mathcal{J}^\nu, \quad \nabla_\mu {}^*F^{\mu\nu} = 0, \quad \text{(Maxwell equations)}$$

$$\left(p^\mu \frac{\partial}{\partial x^\mu} - \Gamma^\mu_{\alpha\beta} p^\alpha p^\beta \frac{\partial}{\partial p^\mu} \right) f = \left(\frac{\partial f}{\partial \tau} \right)_\mathrm{coll} \quad \text{(Boltzmann equation)}$$

$$\vdots$$

$$T^\mathrm{total}_{\mu\nu} = T^\mathrm{fluid}_{\mu\nu} + T^\mathrm{EM}_{\mu\nu} + T^\mathrm{rad}_{\mu\nu} + \cdots$$

Gmunu: A new code for generic astrophysical simulations

Gmunu (**G**eneral-relativistic **mu**ltigrid **nu**merical solver) ([1, 2, 3, 4])

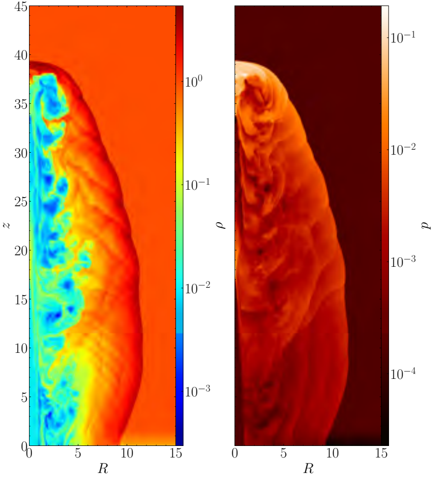
Physics modules

- Consternated-evolution scheme for Einstein equation
  - Conformally flat condition (CFC)
- GRMHD
  - ideal/(resistive + dynamo)
  - hyperbolic cleaning
  - constrained transport
  - elliptic cleaning
- Radiative transfer
  - Two-moment scheme
  - grey/multi-group

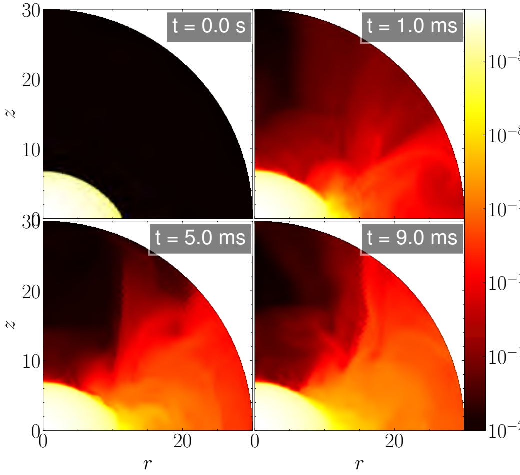
Numerical features

- Block-based Adaptive Mesh Refinement (AMR) (provided by `MPI-AMRVAC`)
- Parallelised with MPI (provided by `MPI-AMRVAC`)
- Multi-dimensional (1-3D)
- Curvilinear geometries

Examples

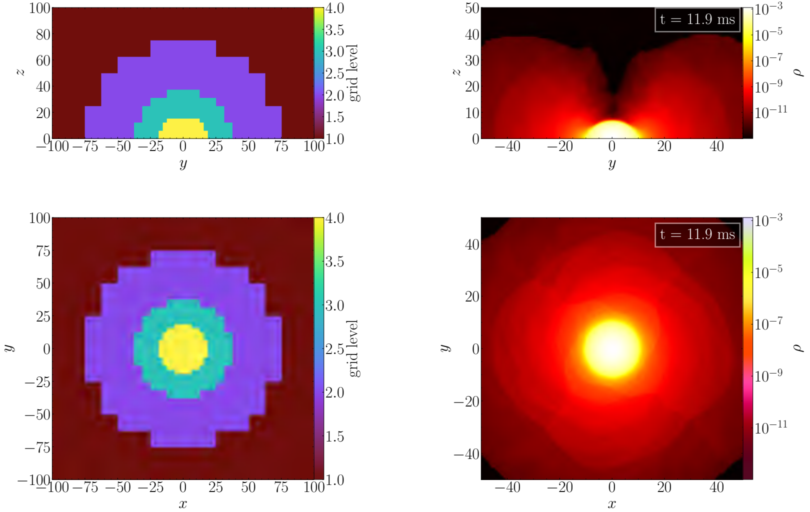


Cylindrical



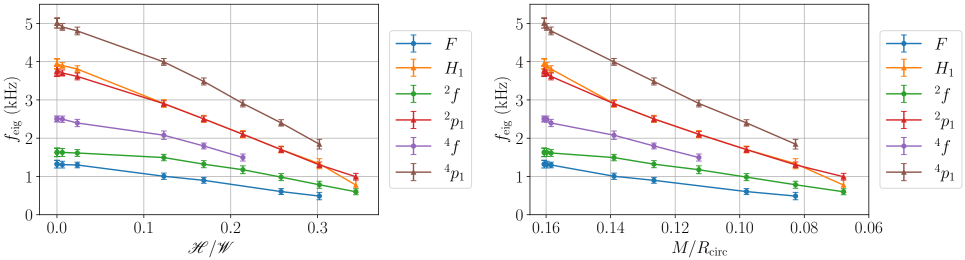
Spherical

Examples



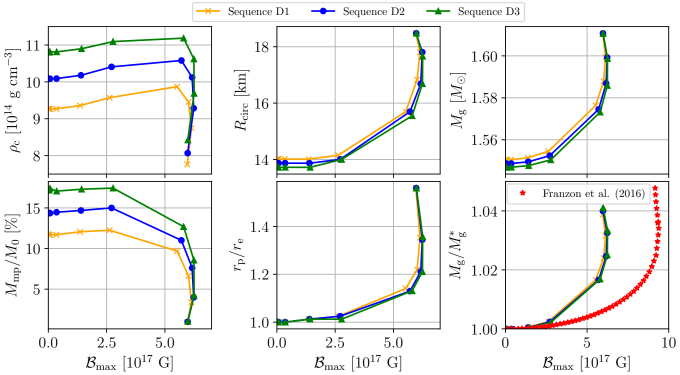
Applications

Oscillation modes of strongly magnetised neutron stars



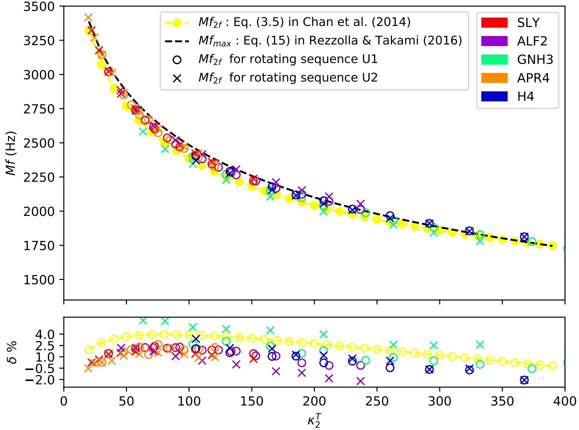
- Leung+ 2022 (arXiv:2303.05684)
- 2D, purely toroidal B fields, GRMHD simulations
- the stellar oscillations are insensitive to magnetic fields until the magnetic to binding energy ratio goes beyond 10%
- $\vec{B}_{\rm tor}$ suppress the compactness of NSs, and thus frequency of the oscillation modes

Formation of Hybrid star



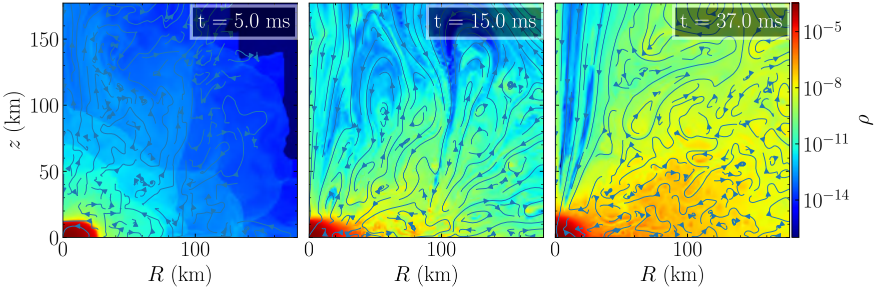
- Yip+ in prep.
- polytrope EOS + MIT bag model
- Macroscopic quantities of the hybrid stars are not sensitive to the magnetic field until $\mathcal{B}^*_{max} \gtrsim 5 \times 10^{17}$ G, where all quantities change significantly.

Gravitational-wave Asteroseismology



- Ng+ 2021
- $f_{\max} \approx f_{2f}$

Magnetic winding

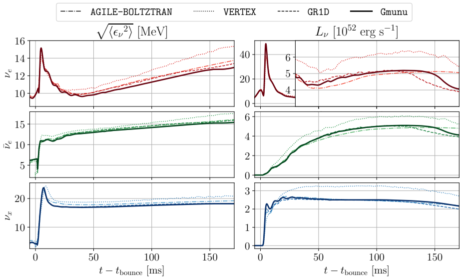


- Carlin Will (N3AS mentorship program)
- 2D GRMHD simulations of differential rotating neutron stars
- Study how the rotation and MHD effects in NS affects the surrounding disk materials

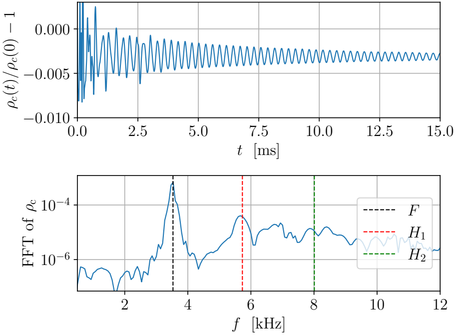
Future work

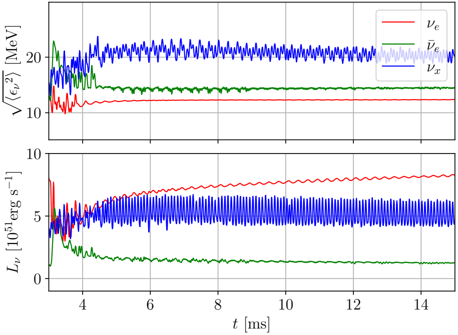
Radiation transport

- Cheong+ 2023 (arXiv: 2303.03261)
- General relativistic
- two-moment based
- multi-energy
- multi-species, multi-energy radiation-matter couplings
- nuclear/neutrino microphysics inputs are essential



Radiation transport





Summary

Summary

- Briefly presented `Gmunu`
  - Dynamical spacetime (CFC)
  - GRMHD
  - radiation transport
- Nuclear and neutrino microphysics are essential inputs
- Post-merger & CCSN simulations

Thank you for your attention.

Q & A

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References I

[1] P. C.-K. Cheong, L.-M. Lin, and T. G. F. Li, "Gmunu: toward multigrid based Einstein field equations solver for general-relativistic hydrodynamics simulations," *Classical and Quantum Gravity*, vol. 37, p. 145015, July 2020.

[2] P. C.-K. Cheong, A. T.-L. Lam, H. H.-Y. Ng, and T. G. F. Li, "Gmunu: paralleled, grid-adaptive, general-relativistic magnetohydrodynamics in curvilinear geometries in dynamical space-times," *MNRAS*, vol. 508, pp. 2279–2301, Dec. 2021.

[3] P. C.-K. Cheong, D. Y. T. Pong, A. K. L. Yip, and T. G. F. Li, "An Extension of Gmunu: General-relativistic Resistive Magnetohydrodynamics Based on Staggered-meshed Constrained Transport with Elliptic Cleaning," *ApJS*, vol. 261, p. 22, Aug. 2022.

[4] P. C.-K. Cheong, H. H.-Y. Ng, A. T.-L. Lam, and T. G. F. Li *arXiv e-prints*, p. arXiv:2303.03261, Mar. 2023.

[5] E. B. Abdikamalov, H. Dimmelmeier, L. Rezzolla, and J. C. Miller, "Relativistic simulations of the phase-transition-induced collapse of neutron stars," *MNRAS*, vol. 392, pp. 52–76, Jan. 2009.

[6] B. Franzon, V. Dexheimer, and S. Schramm, "A self-consistent study of magnetic field effects on hybrid stars," *MNRAS*, vol. 456, pp. 2937–2945, Mar. 2016.

Backup slides

Formation of hybrid stars

Presenting a recent work by Yip et al. (in prep.)

- The formation of a hybrid star (phase transition from hadronic to deconfined quark matter) $\rightarrow$ EM/GW signals
- These signals provide probes for the properties of NS
- dynamical study of magnetized hybrid star formation has yet to be realized
- We consider MIT bag model + normal hadronic matter in such systems

Hybrid star model

The MIT bag model equation of state for massless and non-interacting quarks at zero temperature is given by

$$P_{\mathrm{q}} = \frac{1}{3}(e - 4B), \tag{1}$$

where e is the total energy density and B is the bag constant. For the normal hadronic matter, we adopt an ideal gas type of equation of state for the evolution

$$P_{\mathrm{h}} = (\gamma - 1)\rho\epsilon \tag{2}$$

Hybrid star model

EOS for evolution:

$$P = \begin{cases} P_{\mathrm{h}} & \text{for } \rho < \rho_{\mathrm{hm}}, \\ \alpha_{\mathrm{q}} P_{\mathrm{q}} + (1-\alpha_{\mathrm{q}}) P_{\mathrm{h}} & \text{for } \rho_{\mathrm{hm}} \le \rho \le \rho_{\mathrm{qm}}, \\ P_{\mathrm{q}} & \text{for } \rho_{\mathrm{qm}} < \rho, \end{cases} \quad (3)$$

where

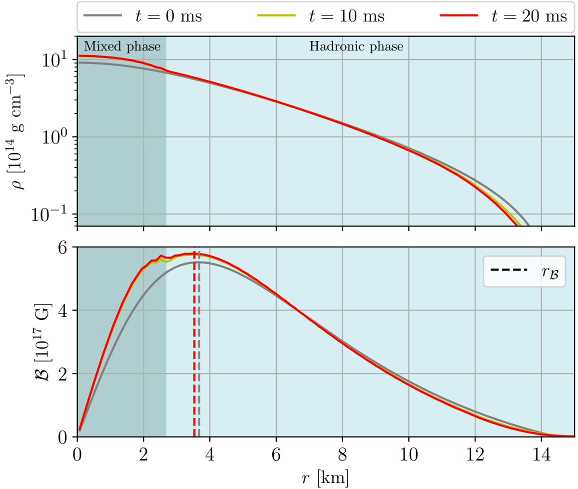
$$\alpha_{\mathrm{q}} = 1 - \left(\frac{\rho_{\mathrm{qm}} - \rho}{\rho_{\mathrm{qm}} - \rho_{\mathrm{hm}}} \right)^{\delta} \quad (4)$$

We choose $\rho_{\mathrm{hm}} = 6.97 \times 10^{14}$ g cm^{-3}, $\rho_{\mathrm{qm}} = 24.3 \times 10^{14}$ g cm^{-3} and $B^{1/4} = 170$ MeV [5].

Equilibrium model

	M ($M_\odot$)	ρ_c (10^{14} g cm^{-3})	r_e (km)	$\mathscr{H}/\mathscr{W}$	B_{max} (10^{17} G)
REF	1.55	8.56	11.85	0.00	0.00
T1K1	1.55	8.56	11.85	3.97×10^{-6}	3.45×10^{-2}
T1K2	1.55	8.56	11.85	1.58×10^{-5}	6.89×10^{-2}
T1K3	1.55	8.57	11.85	3.95×10^{-4}	3.44×10^{-1}
T1K4	1.55	8.63	11.92	6.21×10^{-3}	1.36
T1K5	1.56	8.81	12.15	2.35×10^{-2}	2.63
T1K6	1.58	9.10	14.43	0.12	5.52
T1K7	1.59	8.81	16.21	0.17	6.01
T1K8	1.60	8.27	18.64	0.21	6.14
T1K9	1.61	7.53	21.97	0.26	5.96
T1K10	1.62	6.64	26.62	0.30	5.53
T1K11	1.63	5.69	33.19	0.34	4.93

Formation of Hybrid star



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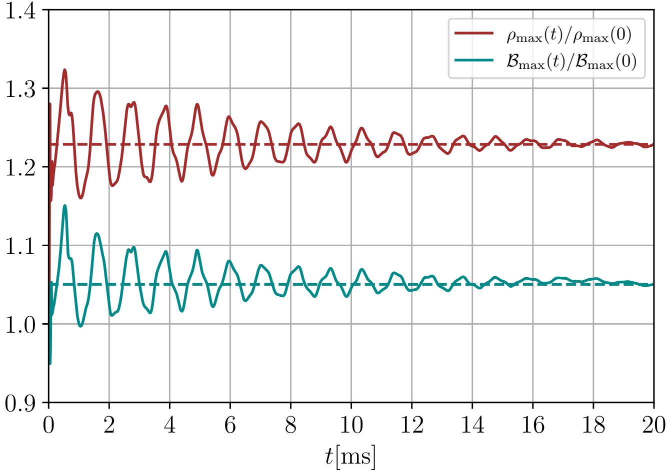

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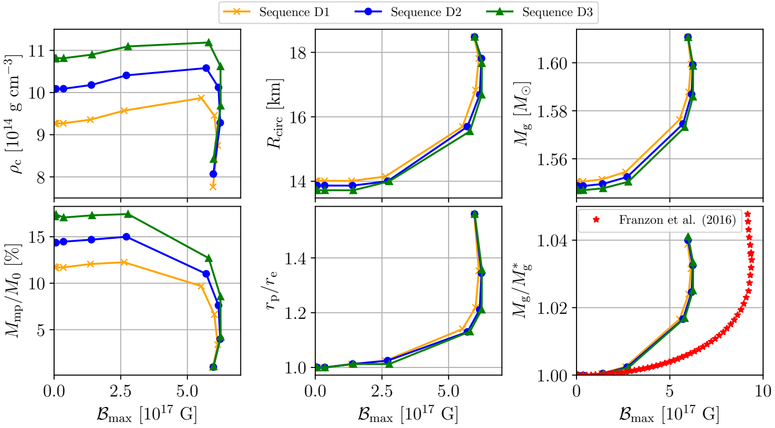

Formation of Hybrid star

Short summary

- Macroscopic quantities of the hybrid stars are not sensitive to the magnetic field until $\mathcal{B}^*_{\mathrm{max}} \gtrsim 5 \times 10^{17}$ G, where all quantities change significantly.
- the magnetic deformation decreases the rest-mass density dramatically, leading to a substantial reduction in the matter fraction in the mixed phase
- our results agree with [6], in which they considered realistic EoS with B fields

Future work

- the analysis of the corresponding GW is ongoing.
- different rotation/magnetic fields geometries
- 3D simulations
- realistic EOS