# Collective flavor evolution, collisions, and advection in core-collapse supernovae: The full solution

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Based on arXiv:2206.00676 and upcoming work (with Irene Tamborra)

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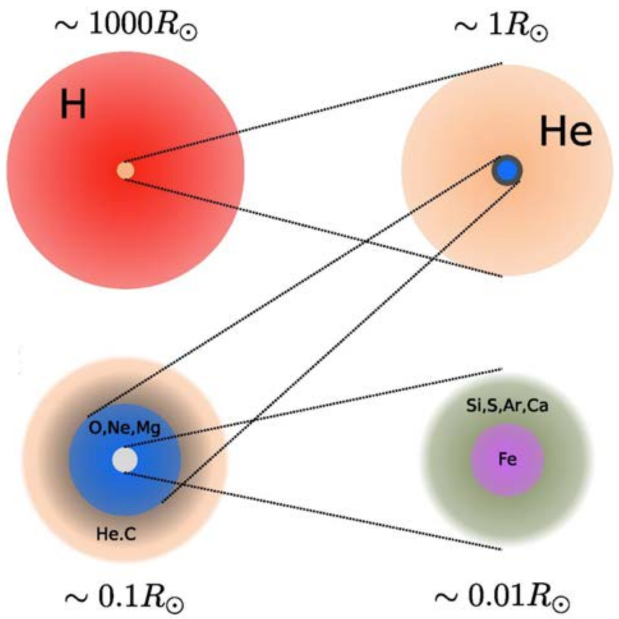
# Outline of the talk

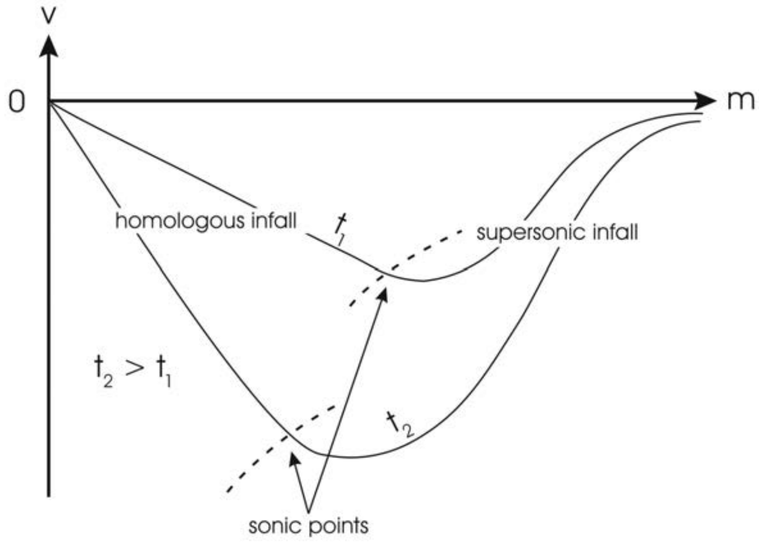
- Introduction
- Flavor evolution
- Collisions and advection
- Numerical results
- Conclusions

# What is a (core-collapse) supernova?



- Stars heavier than about 8 times the mass of the Sun for a onion like structure with heavier elements at the center.
- Iron is the most stable element (in terms of binding energy per nucleon)
- Energy cannot be released by fusion of Iron and heavier elements.

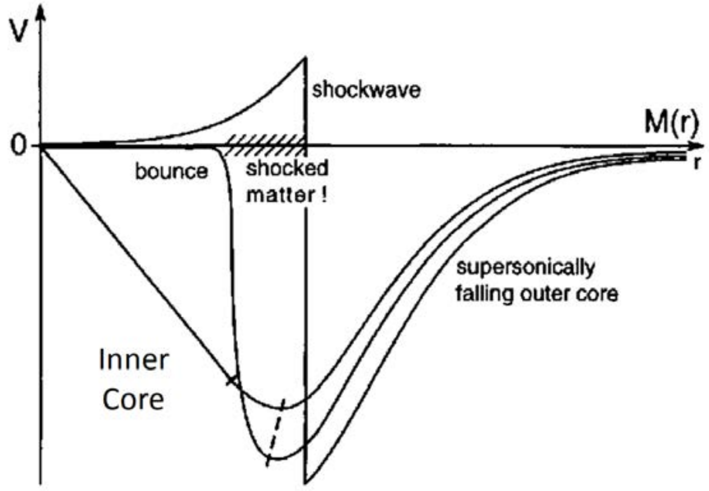
# The collapse of the core ...



Stellar Structure and Evolution: Kippenhahn et.al. (1990)

- Inner core undergoes a “homologous” collapse, until nuclear densities are reached.
- The outer part, which is in free-fall slams onto the inner core with supersonic speeds.

# ... and the bounce.



E Müller: Saas-Fee Lectures (1998)

- When the inner core reaches nuclear density the infalling outer core slams into it.
- The inner core ‘bounces back’.
- This results in a shockwave that propagates outwards.

# The stalling of the shockwave ...

- The shock-wave produced by the bounce is supposed to blow up the outer envelope.

- Numerical simulations showed that the shockwave looses energy while propagating in the outer core.

- The shockwave looses energy because it dissociates Iron group nuclei.

# Neutrinos to the rescue …

- 99 percent of energy released by the supernova is in the form of elementary particles called neutrinos, which only interact via weak interactions (and gravity).

- Neutrinos come in three flavors: electron, muon, tau – and their anti-particles.

# Neutrinos to the rescue ...

- Bethe and Wilson proposed that if a small fraction of this energy, which is in the form of neutrinos

- If it is deposited in the right place at the right time, the resulting hydrodynamical instability and revive the shock.

# How much energy is deposited by neutrinos?

- Number of neutrinos emitted.
- Energy of the neutrinos. Because cross section of neutrinos with matter increases with energy.
- And their flavor, because ...

$$\nu_e + n \rightarrow p + e^- \qquad \bar{\nu}_e + p \rightarrow n + e^+$$

# Neutrino flavor evolution (Two flavor approximation)

Neutrinos

$$\rho = \begin{pmatrix} \rho_{ee} & \rho_{ex} \\ \rho_{ex}^* & \rho_{xx} \end{pmatrix}$$

$$i\frac{d}{dt}\rho = [H, \rho]$$

Anti-neutrinos

$$\bar{\rho} = \begin{pmatrix} \bar{\rho}_{ee} & \bar{\rho}_{ex} \\ \bar{\rho}_{ex}^* & \bar{\rho}_{xx} \end{pmatrix}$$

$$i\frac{d}{dt}\bar{\rho} = [\bar{H}, \bar{\rho}]$$

$$\begin{aligned} H &= H_{\text{vac}} + H_{\text{mat}} + H_{\nu\nu} \\ \bar{H} &= -H_{\text{vac}} + H_{\text{mat}} + H_{\nu\nu} \end{aligned}$$

# Linear part of Hamiltonian

$$\omega = \frac{\Delta m^2}{2E}$$

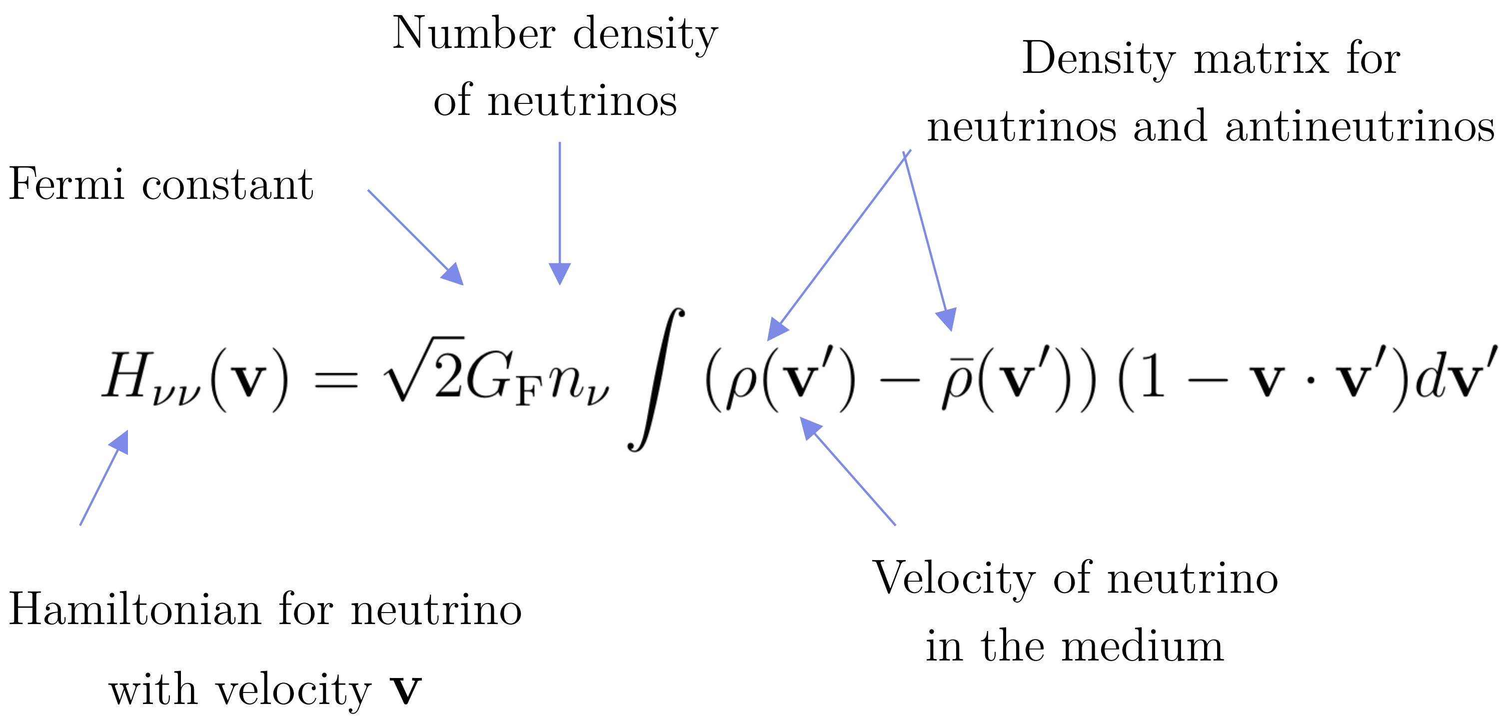
Vacuum frequency

$$H_{\text{vac}} = \frac{\omega}{2}\begin{pmatrix} -\cos 2\vartheta_{\text{V}} & \sin 2\vartheta_{\text{V}} \\ \sin 2\vartheta_{\text{V}} & \cos 2\vartheta_{\text{V}} \end{pmatrix}$$

$\vartheta_{\text{V}}$ Vacuum mixing angle

$H_{\text{mat}} = 0$ Throughout this talk

# Hamiltonian of neutrino self-interactions



Number density
of neutrinos

Density matrix for
neutrinos and antineutrinos

Fermi constant

$$H_{\nu\nu}(\mathbf{v}) = \sqrt{2} G_{\mathrm{F}} n_{\nu} \int \left(\rho(\mathbf{v}') - \bar{\rho}(\mathbf{v}')\right) (1 - \mathbf{v} \cdot \mathbf{v}') d\mathbf{v}'$$

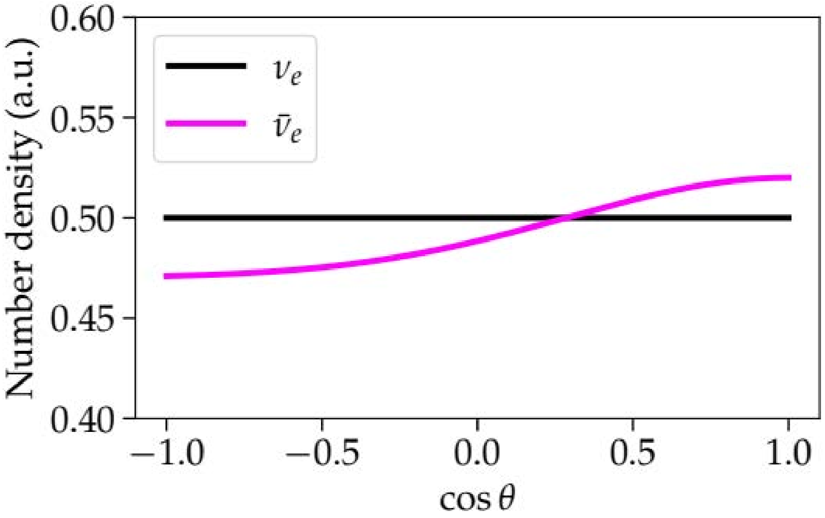
Hamiltonian for neutrino
with velocity $\mathbf{v}$

Velocity of neutrino
in the medium

# Slow Vs Fast collective flavor evolution

- Same equations of motion, but different initial angular distributions.
- Slow collective flavor evolution. Requires non-zero vacuum frequency for significant flavor evolution.
- Fast flavor evolution requires crossing in the angular distribution of electron neutrinos and electron anti-neutrinos (ELN-crossing).

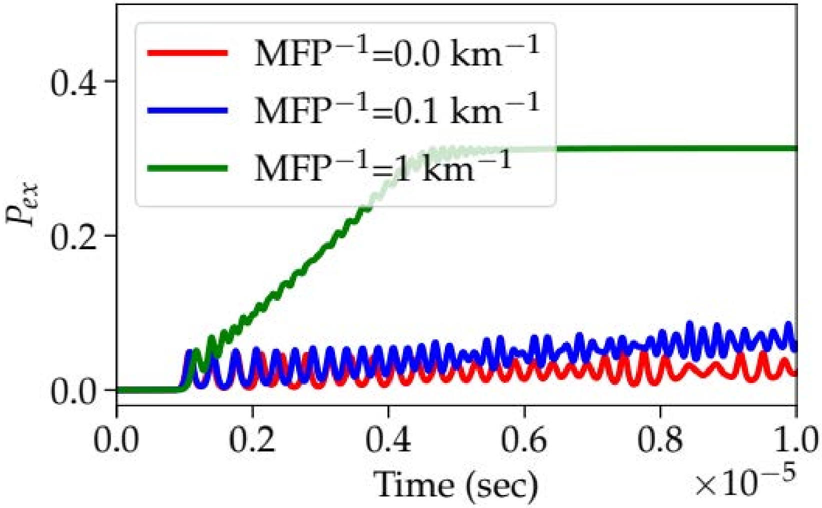
# ELN-crossing.



Tamborra and Shalgar, arXiv: 2011.01948
Shalgar and Tamborra, arXiv: 2106.15622

- In azimuthally symmetric case, flavor evolution more or less restricted to the region of ELN crossing.
- If the location of ELN-crossing changes dynamically, more flavor conversions possible.

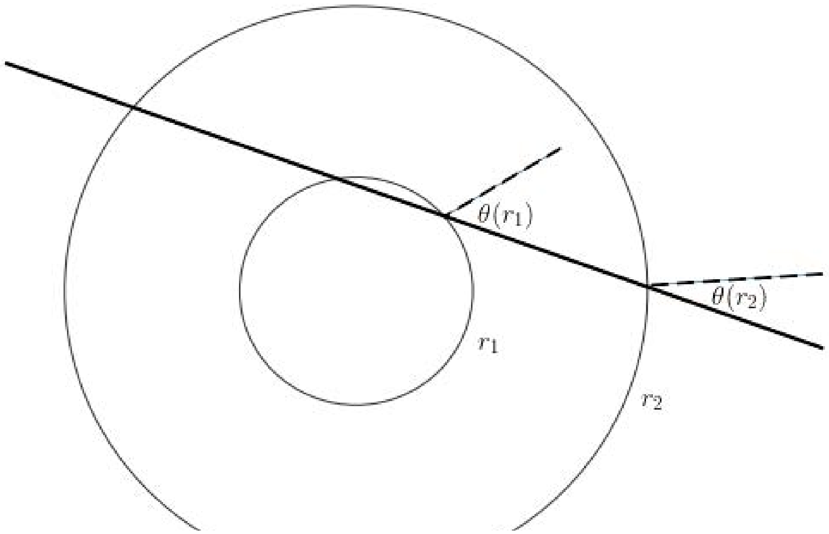
# Effect of collisions.



Shalgar and Tamborra, arXiv: 2011.00004

- Collisions lead to enhancement of fast flavor conversions in homogeneous neutrino gas.
- What if the neutrino gas is not homogeneous?
- Which spatial and angular distribution should we look at?

# Collisions and advection: The spherical geometry.



$$\vec{v}\cdot\vec{\nabla} = \cos\theta\frac{\partial}{\partial r} + \frac{\sin^2\theta}{r}\frac{\partial}{\partial\cos\theta}$$

- A given angle does not represent a neutrino trajectory.
- The advective term ensures that this is taken in to account.
- At each point neutrino can be absorbed, emitted or change momentum.

# Collision and advection: The equations of motion.

Advection

Does not depend on
neutrinos number
density

$$\left(\frac{\partial\rho(\cos\theta,r,t)}{\partial t}+\vec{v}\cdot\vec{\nabla}\rho(\cos\theta,r,t)\right)=\mathcal{C}_{\text{emission}}-\mathcal{C}_{\text{absorb}}\rho(\cos\theta,r,t)$$

$$+\int_{-1}^{1}\mathcal{C}_{\text{dir-ch}}\rho(\cos\theta',r,t)d\cos\theta'-\int_{-1}^{1}\mathcal{C}_{\text{dir-ch}}\rho(\cos\theta,r,t)d\cos\theta'$$

Together these two conserve number

# Neutrino decoupling

- Neutrinos undergo a random walk in the dense regions of the interior and escape when matter density is low.

- Different flavors of neutrinos escape at different radii.

- If flavor transformations happen in this high density regions there can be a interplay between the motion of neutrinos (advection), the collisions, and the flavor transformation.

# Collision terms.

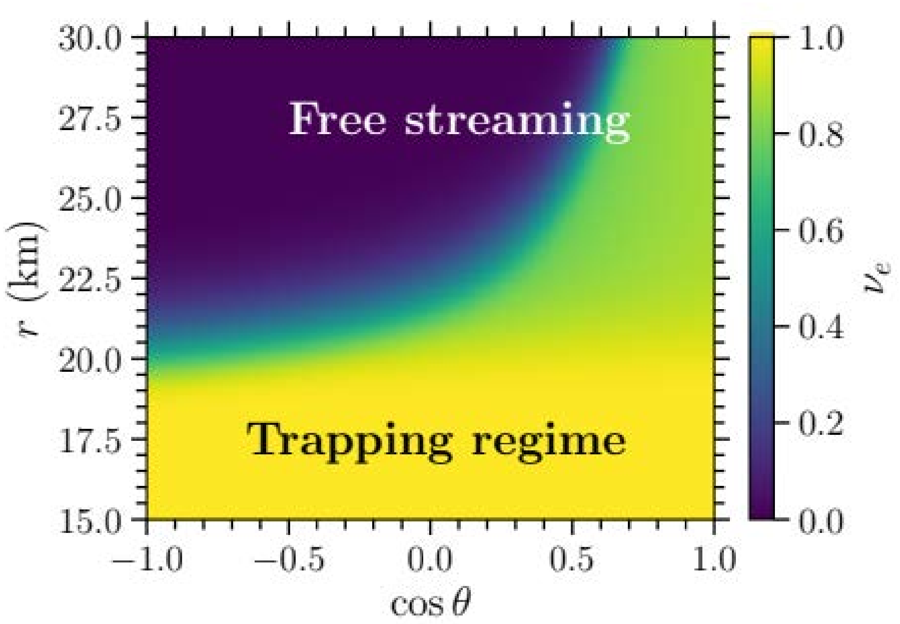
- Non-electron type neutrinos decouple at smallest radius, followed by electron anti-neutrinos and then electron neutrinos.

- The average energy for each species is determined by the temperature of the medium in the region of decoupling.

- These features are difficult to reproduce in the single energy approximation.

- We use collision terms with the right order of magnitude that reproduces this hierarchy of decoupling.

# Collision and advection: The classical steady state.



- Depends on the collision strength as a function of radius (due to variation in density, temperature, etc.)
- Depends on the flavor.
- Depends on energy.

# Challenges in numerical computation.

- Neutrino flavor has to be evolved at each location and each direction.
- Neutrinos can move from one location to another require finite-difference. Results in numerical instabilities.
- Neutrinos can be absorbed or emitted. So evolution is not unitary.
- Requires numerically solving several million coupled non-linear differential equations.

# Neutrino decoupling with flavor transformations

## <u>Simplifying assumptions:</u>

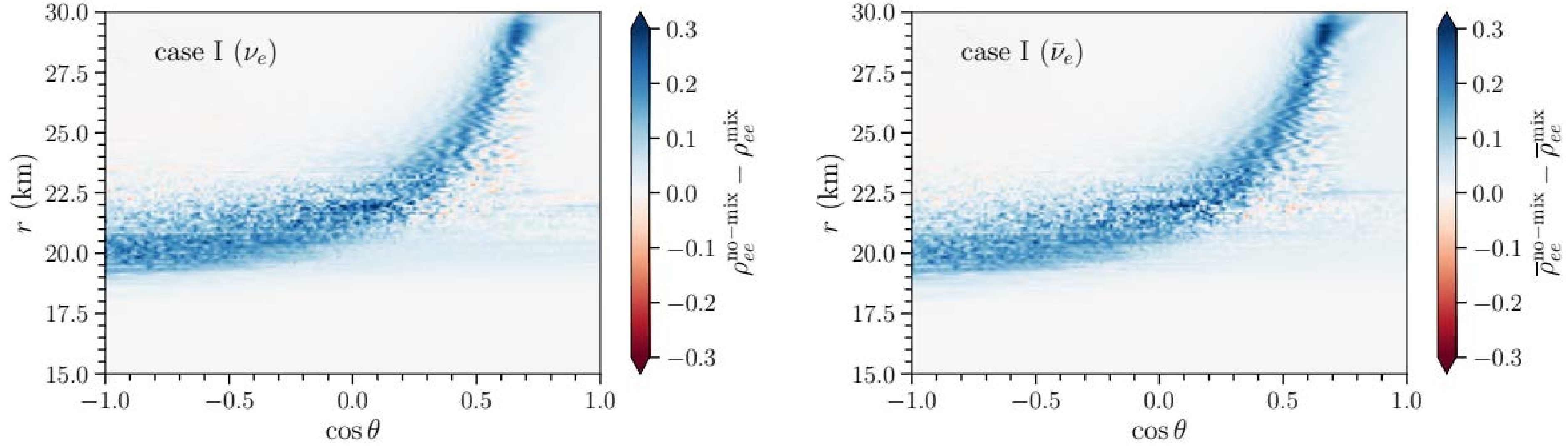
- Two flavor approximation
- Spherical symmetry
- Single energy approximation
- Simplified (exponentially decaying) radial profiles for collision terms (absorption, emission and momentum changing) – We tried out a few.

# Equations of motion.

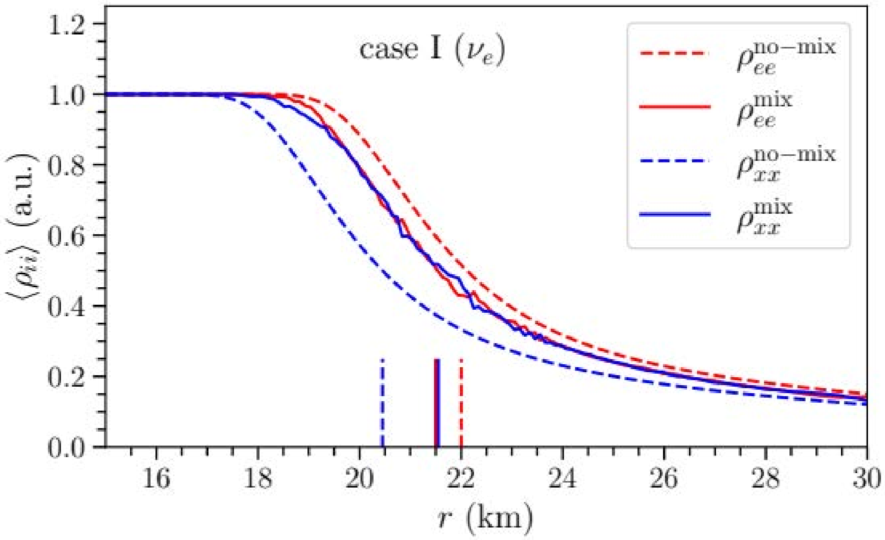
$$
\begin{aligned}
& \left(\frac{\partial\rho(\cos\theta,r,t)}{\partial t} + \vec{v}\cdot\vec{\nabla}\rho(\cos\theta,r,t)\right) = \mathcal{C}_{\text{emission}} - \mathcal{C}_{\text{absorb}}\rho(\cos\theta,r,t) \\
+ \quad & \int_{-1}^{1}\mathcal{C}_{\text{dir-ch}}\rho(\cos\theta',r,t)d\cos\theta' - \int_{-1}^{1}\mathcal{C}_{\text{dir-ch}}\rho(\cos\theta,r,t)d\cos\theta' \\
- \quad & i[H(\cos\theta,r,t),\rho(\cos\theta,r,t)]
\end{aligned}
$$

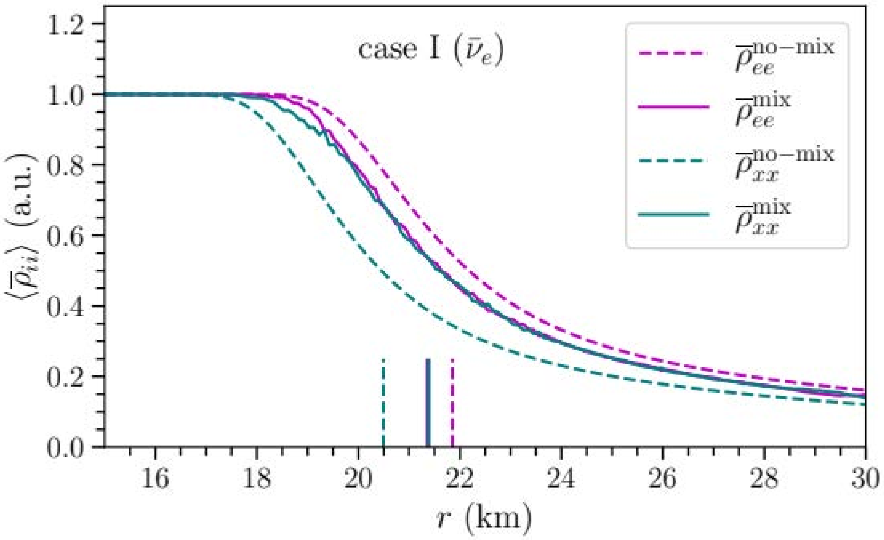
# Numerical results (case I)

Effect of flavor evolution (case I).

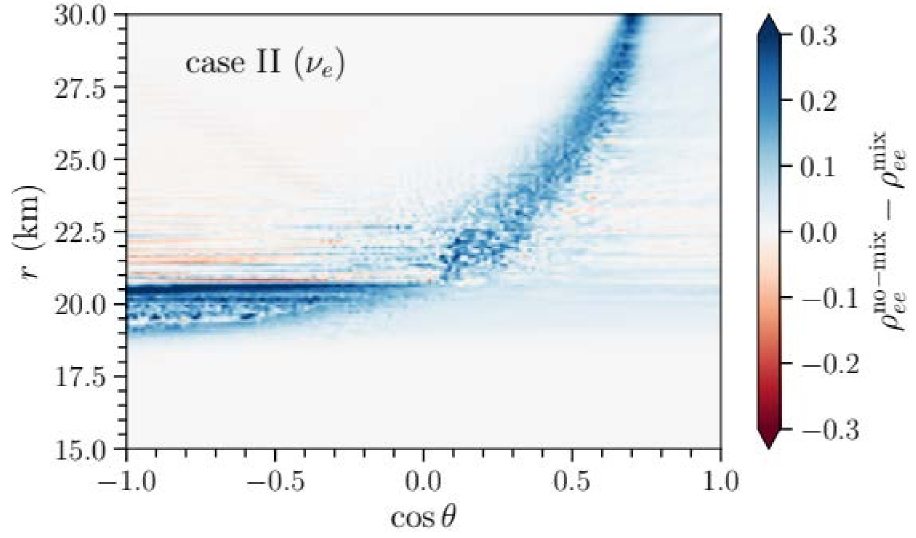


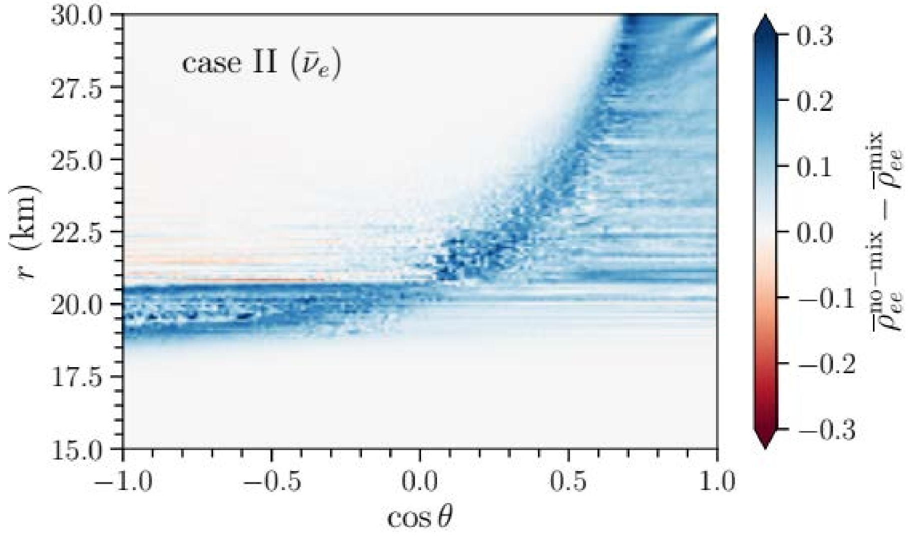
# Angle averaged flavor evolution (case I)



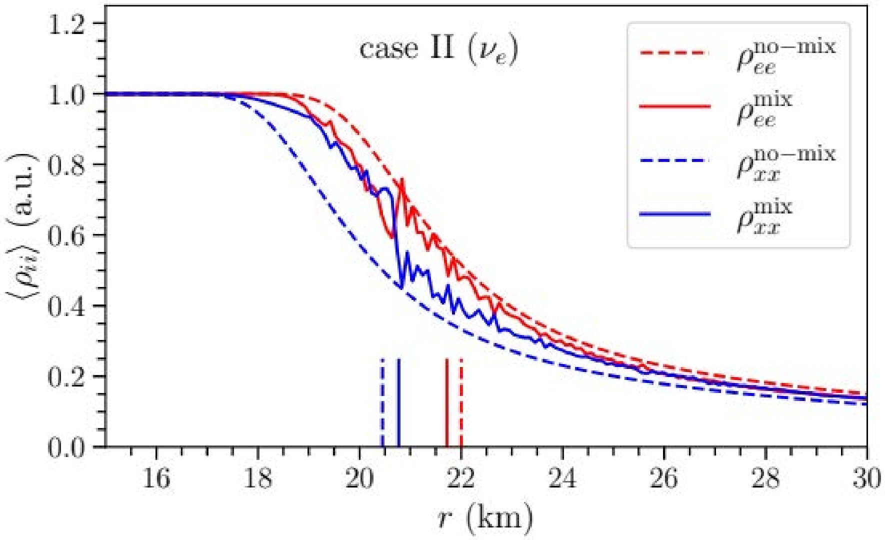


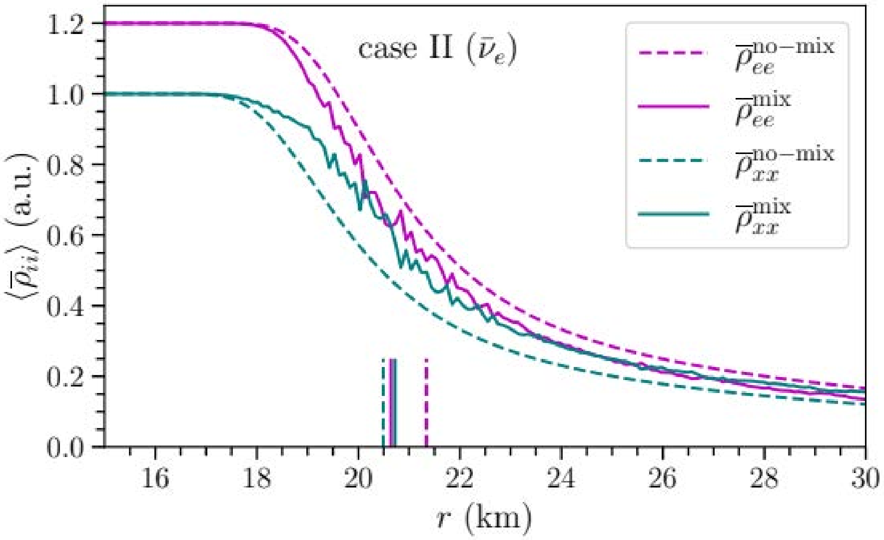
# Numerical results (case II)





# Angle averaged flavor evolution (case II)





# Salient features

- Flavor equiparition or depolarization, even partial, not a generic outcome.

- Fast flavor evolution is not very sensitive to energy, but collisions are. Multi-energy simulations needed in future.

# Implications ...

- The effective radius of neutrinos decoupling can change due to flavor evolution.
- The average energy of the neutrinos should be determined by the temperature at the radius of decoupling.
- Multi-energy calculations necessary for us to be sure about this.

# Implications ...

- The change in the radius of decoupling means that the emitting area changes. This can change the number flux.
- These effects are not included in hydrodynamical simulations of core-collapse supernovae.
- Enhancement of flavor conversions due collisions and advection. Difficult to separate the effects of two in inhomogeneous case.

# Conclusions

Validity of hydrodynamical simulations of core-collapse supernovae ?