The effect of correlations in models of the nuclear equation of state on neutron star inference

Isaac Legred (Caltech)

N3AS Seminar

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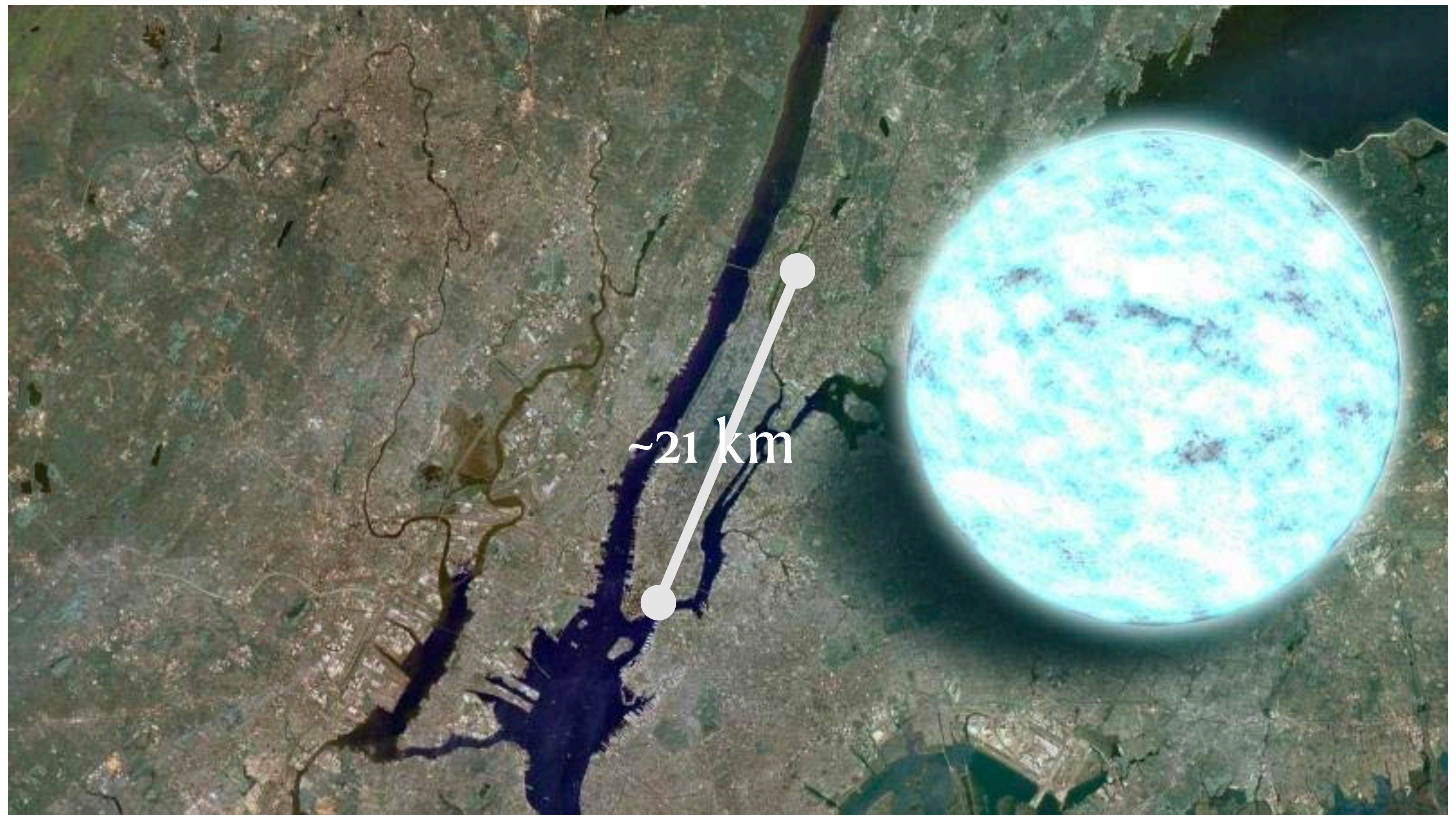
Work with: Katerina Chatziioannou, Reed Essick, and Philippe Landry

10.1103/PhysRevD.105.043016

https://arxiv.org/abs/2201.06791

Why Study Neutron Stars?



GR matters when GM/Rc^2 is not small

NS: GM/Rc^2 ~ 1/3

Behavior of nuclear matter is uncertain when n/n_{nuc} is not small*

NS: $n_{\text{max}}/n_{\text{nuc}}$ $4 - 7$?

Neutron Stars give us laboratories to Study nuclear physics along with general relativity

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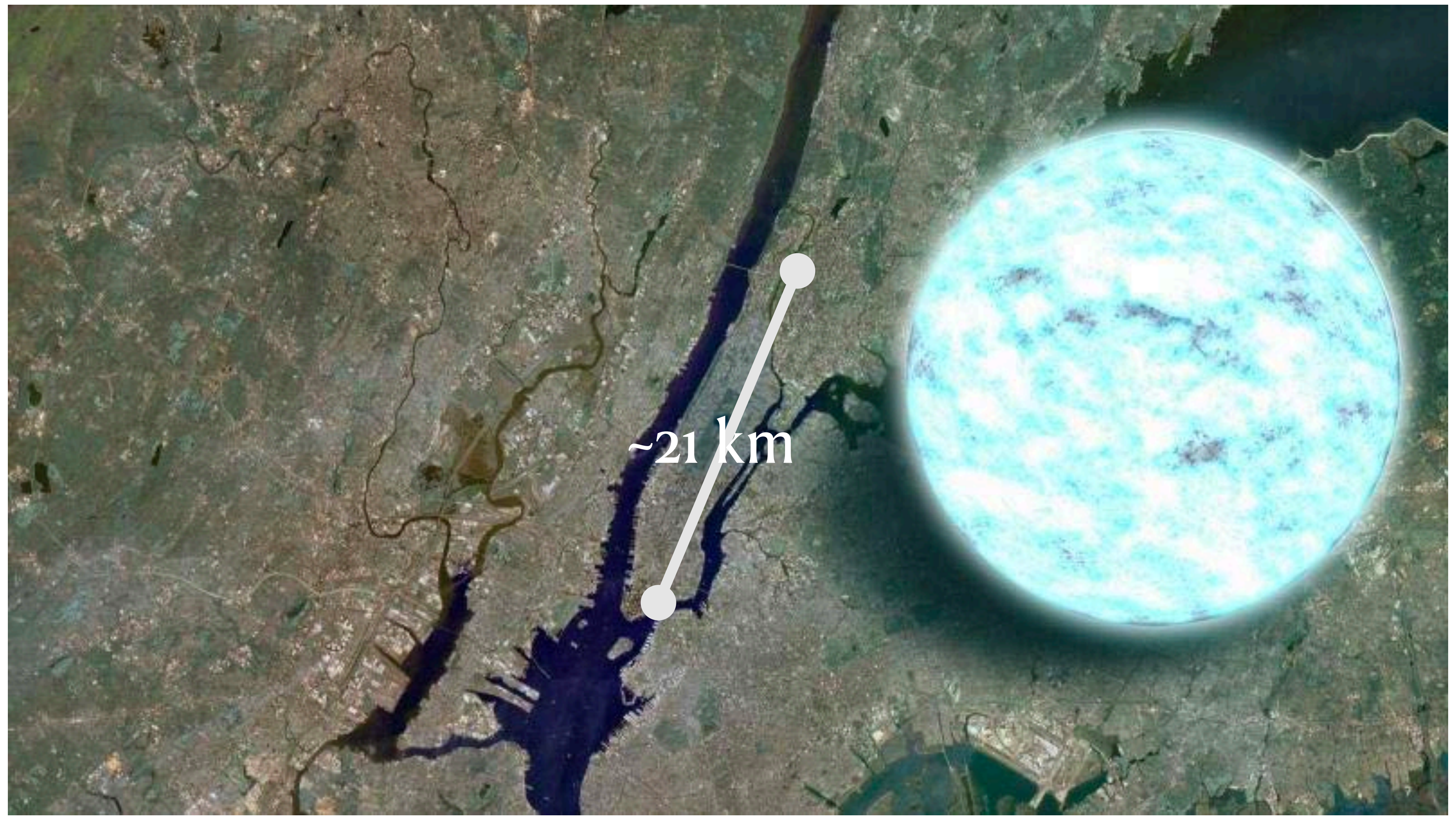

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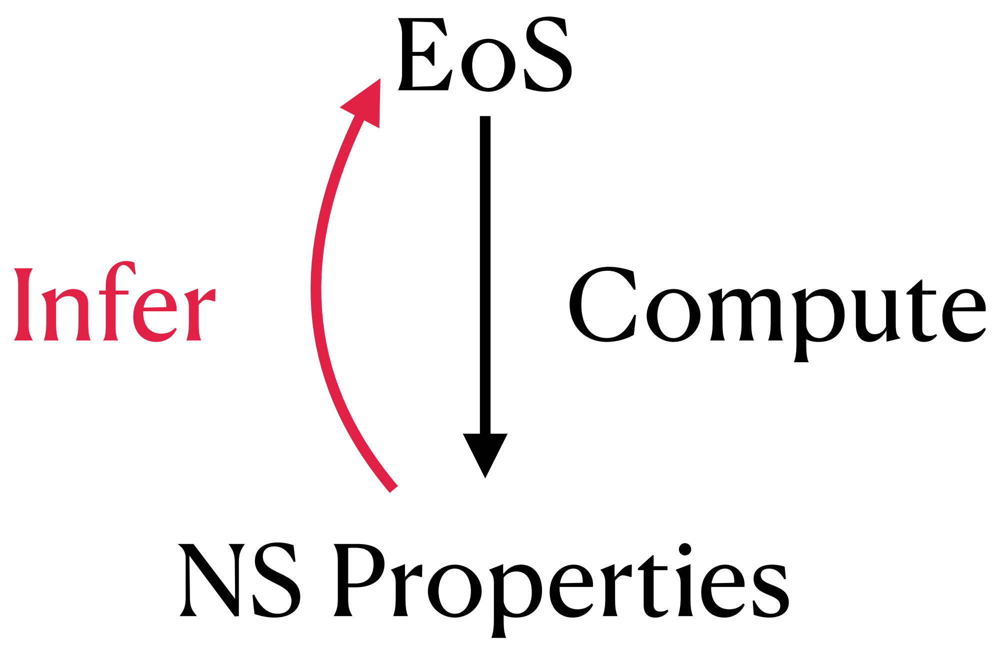
Neutron Stars give us laboratories to Study nuclear physics along with general relativity

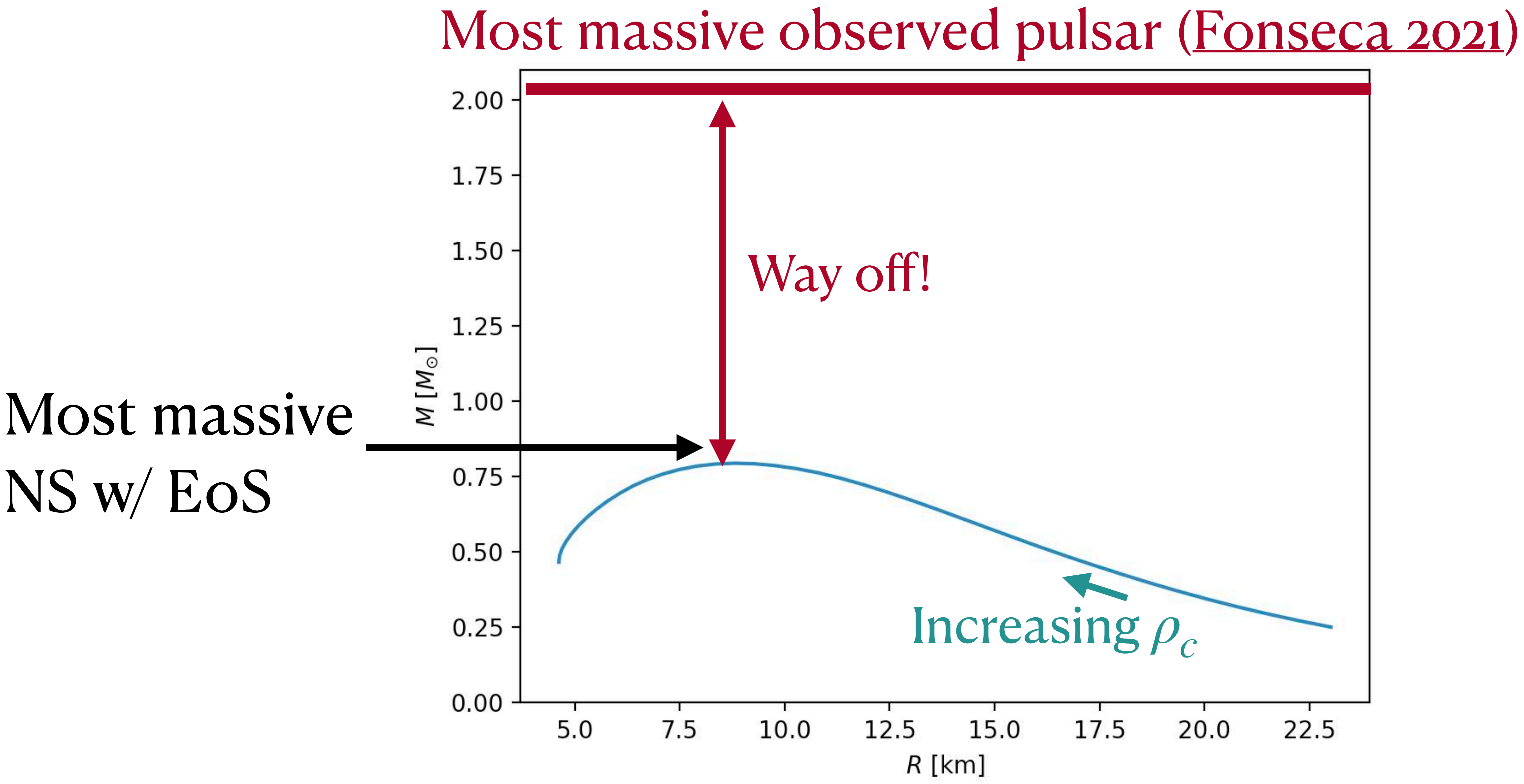
We Really do need Both!

Inferring the EoS — Motivation

"Microphsyics <=> Macrophysics"



5/3 Polytrope "degenerate neutrons (Non-Rel)"



Inferring the EoS — In practice

- Want to establish a probability distribution on candidate equations of state given observed astrophysical data

Equation of state candidate

$$P(\varepsilon_i \,|\, d) = P(\varepsilon \,|\, d_1, d_2, \ldots) \propto P(d_1, d_2, \ldots \,|\, \varepsilon_i) \times P(\varepsilon_i)$$

Prior

Astrophysical data

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$$P(\varepsilon_i \mid d) = P(\varepsilon \mid d_1, d_2, \ldots) \propto P(d_1, d_2, \ldots \mid \varepsilon_i) \times P(\varepsilon_i)$$

Astrophysical data

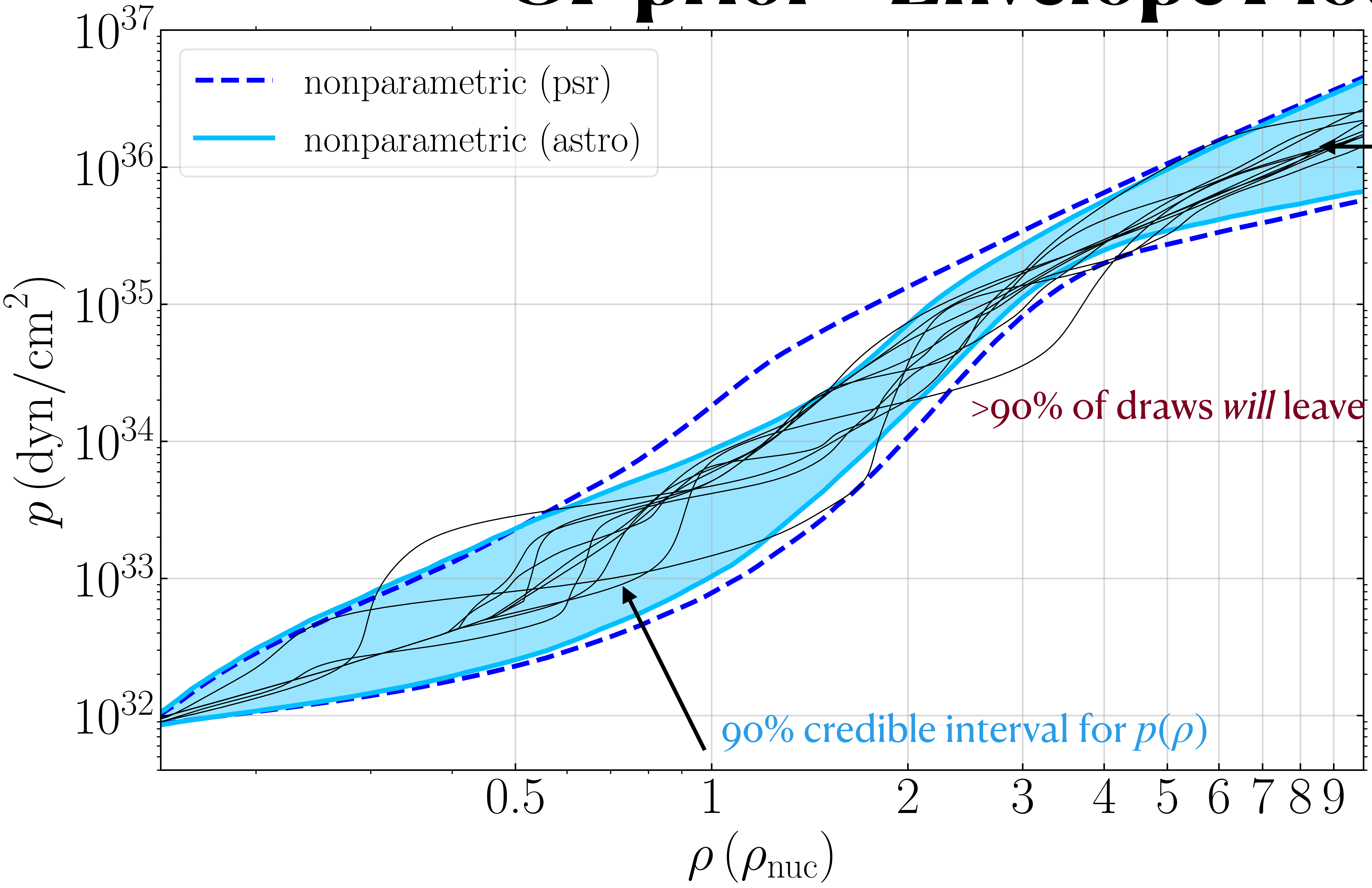
Phenomenological

Parametrize a *functional form* (i.e. Spectral, Piecewise-polytrope)

Nonparametric methods, i.e. Gaussian process (GP)

Tabulated models from nuclear theory

GP prior + Envelope Plots



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Candidates produced by model (plotted satisfy some posterior weight criteria)

psr -> just heavy pulsar mass measurements

astro -> Heaviest pulsar, 2 NICER x-ray, 2 GWs

Nonparametric: Gaussian Process

Gaussian Process Regression (Landry and Essick 2018)

Tabulate a draw $\phi(p_i) = \ln(1/c_s^2(p_i) - 1)$ @ Pressures p_i from a multivariate Gaussian distribution

Parameters for the covariance kernel are chosen to Control "shape" of EoS distribution

Model-Agnostic Prior (broadest range of models)

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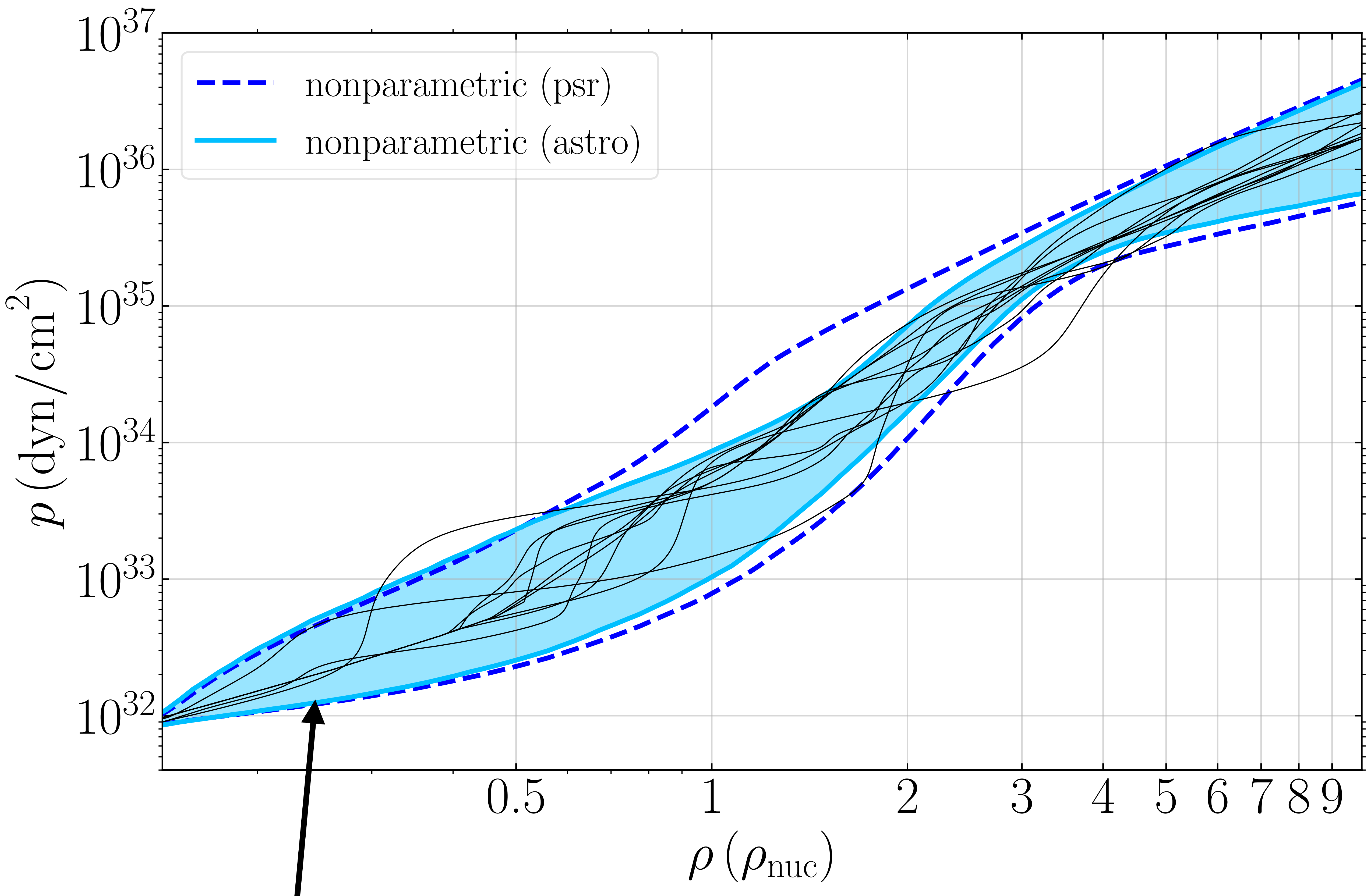


90% credible interval for $p(\rho)$

psr -> just heavy pulsar mass measurements

astro -> Heaviest pulsar, 2 NICER x-ray, 2 GWs

Parametric

Spectral (Lindblom 2010)

Parametrize the adiabatic index

$$p(\rho) = \rho^{\Gamma(x)} \qquad \Gamma(x) = \sum_{i=0}^{n} \gamma_i \left(\log(x)\right)^i$$

Piecewise-polytrope (Read 2008)

A polytrope with multiple segments

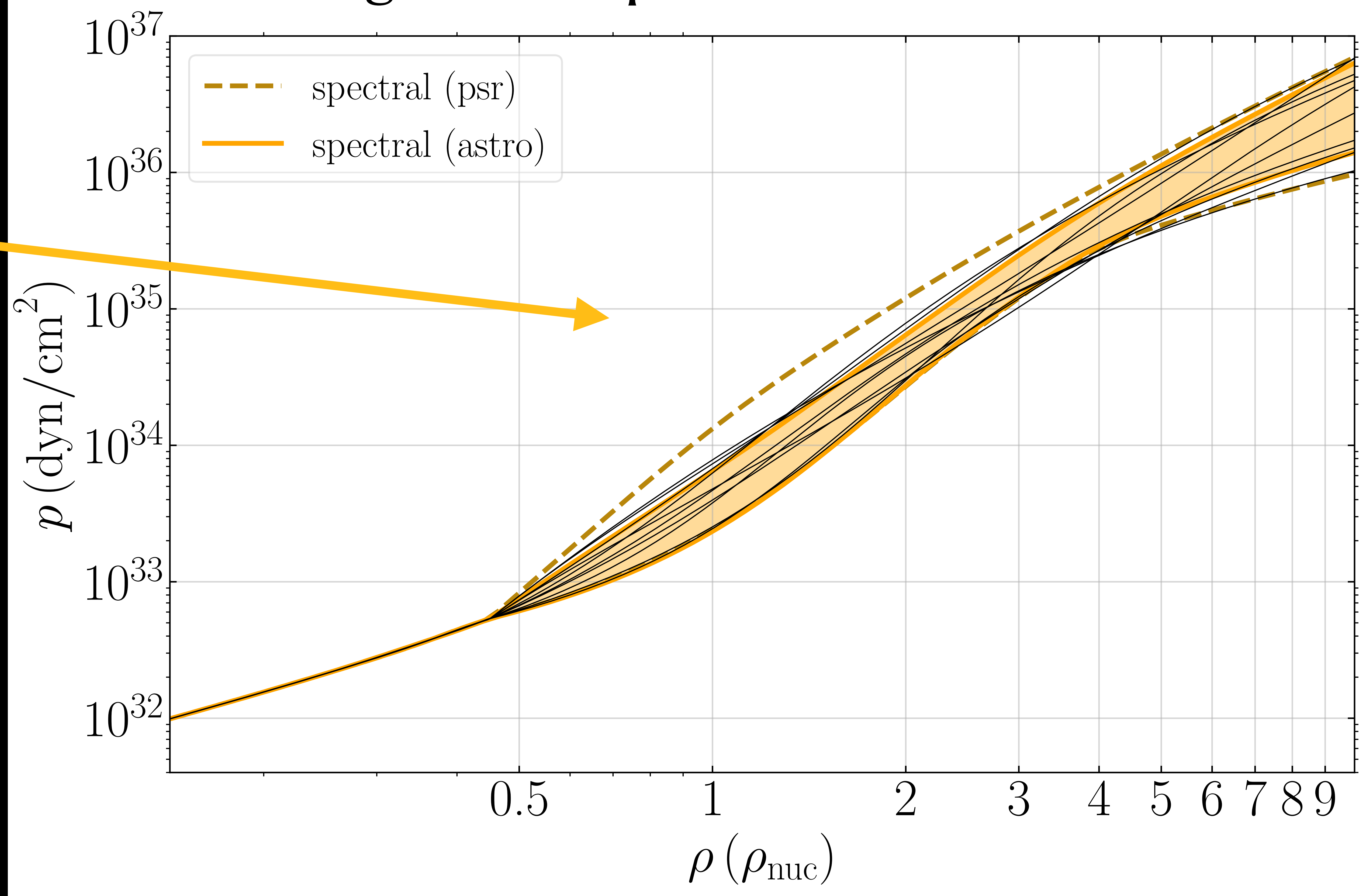
$$p(\rho) = \begin{cases} K_1 \rho^{\Gamma_1} : \rho < \rho_1 \\ K_2 \rho^{\Gamma_2} : \rho_1 < \rho < \rho_2 \\ K_3 \rho^{\Gamma_3} : \rho_2 < \rho \end{cases}$$

Direct **speed-of-sound** (Greif 2018)

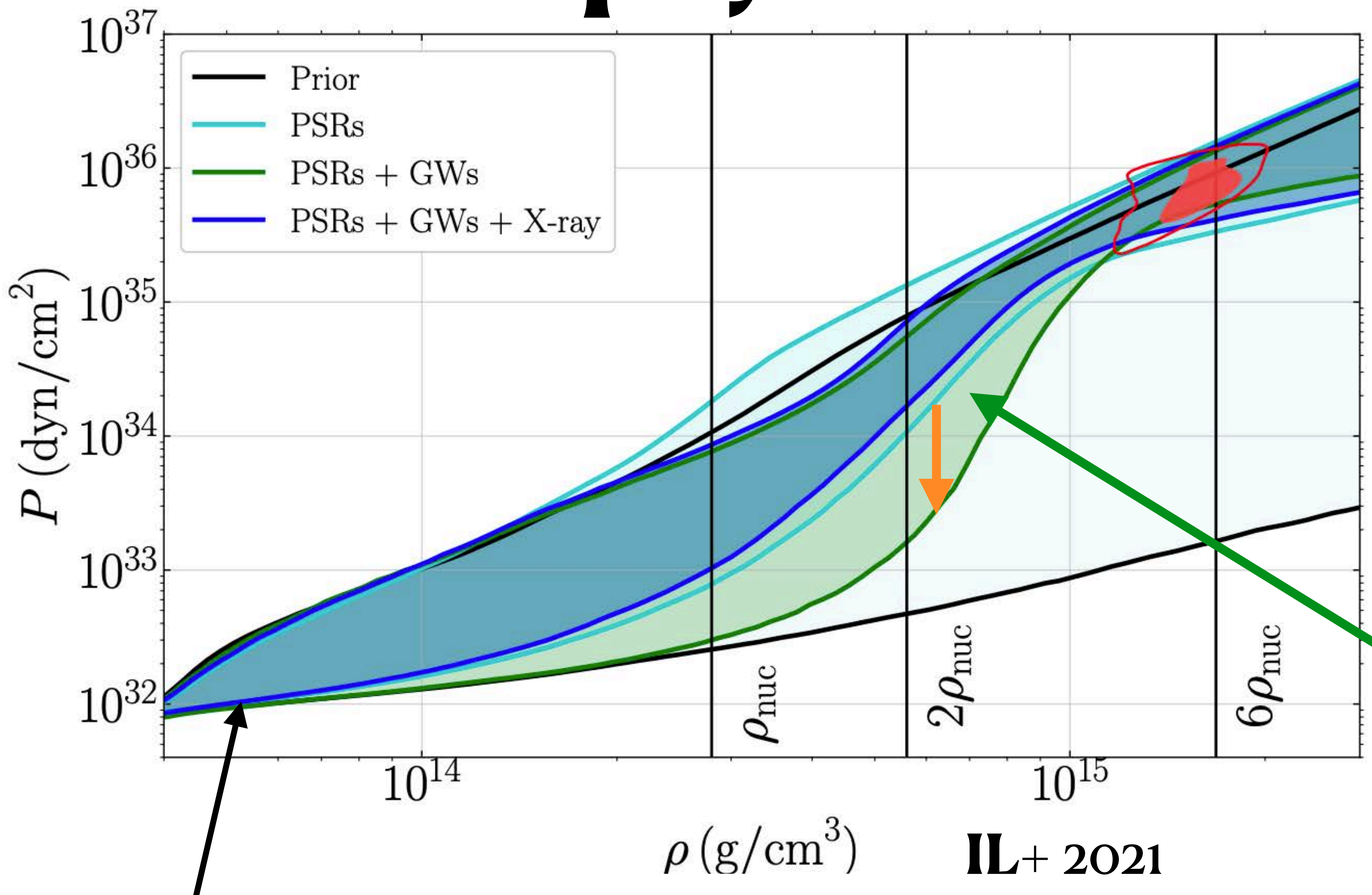
A bump in the speed of sound before asymptotic behavior

$$\frac{c_s^2(z)}{c^2} = a_1 e^{-\frac{1}{2}(z-a_2)^2/a_3^2} + a_6 + \frac{\frac{1}{3} - a_6}{1 + e^{-a_5(z-a_4)}}$$

E.g. for the Spectral Parametrization



Astrophysical Data and Density Scales



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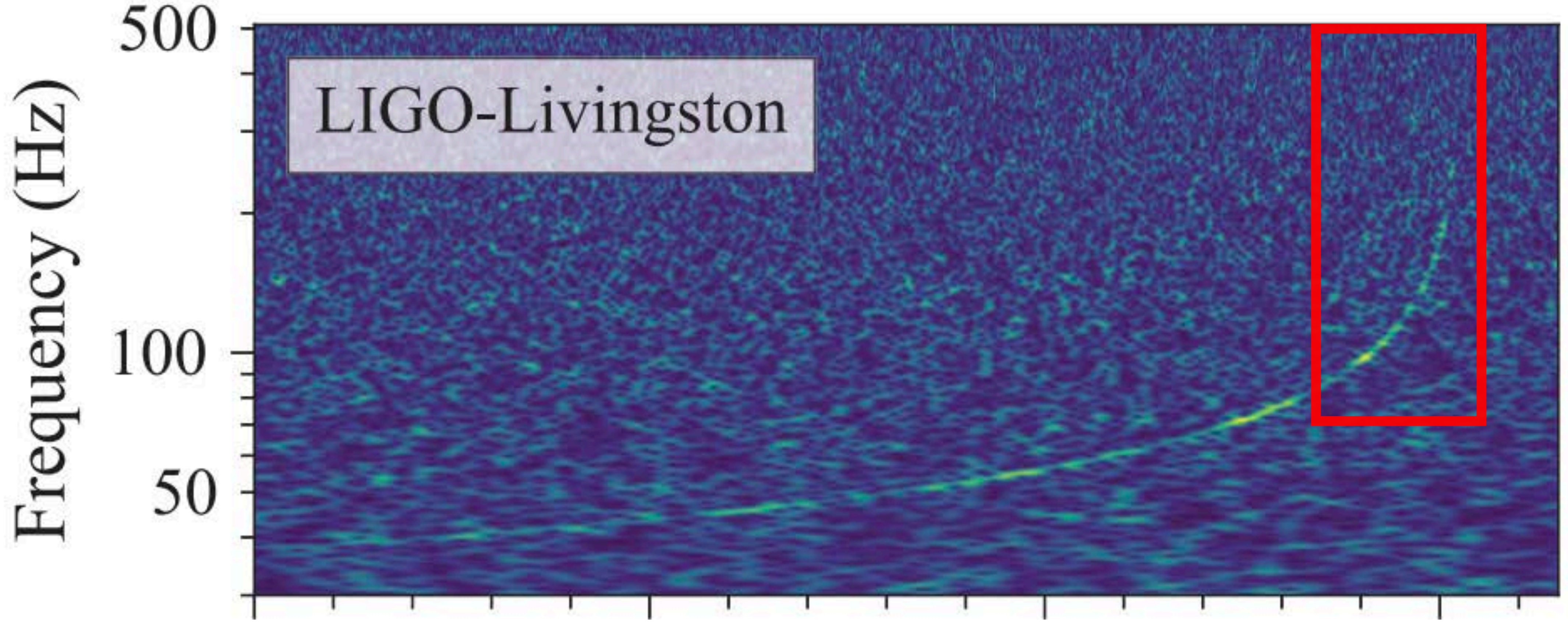
We *infer* the EoS at different density scales
Using different types of astrophysical **data**

e.g.

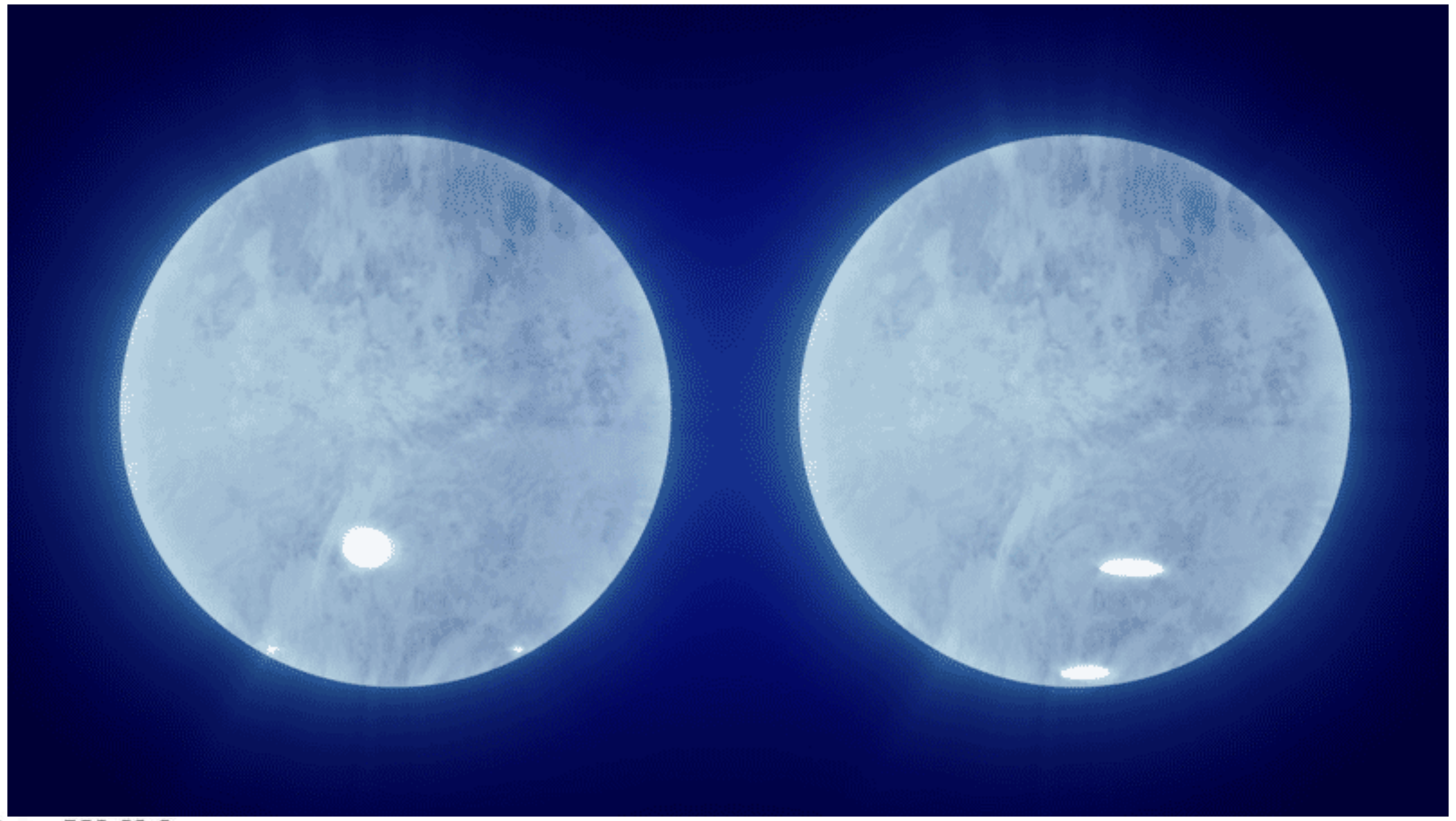
Abbott+ (LVC) 2017



90% credible region for $p(\rho)$

$$P(\varepsilon_i\,|\,d) = P(\varepsilon\,|\,d_1, d_2, \ldots) \propto P(d_1, d_2, \ldots\,|\,\varepsilon_i) \times P(\varepsilon_i)$$

Astrophysical Data and Density Scales



What about measuring the NS radius?

Neutron Star Interior Composition Explorer (NICER)
(Currently operating on ISS!)

Measured Mass and radius of two pulsars

J0030 (2019)

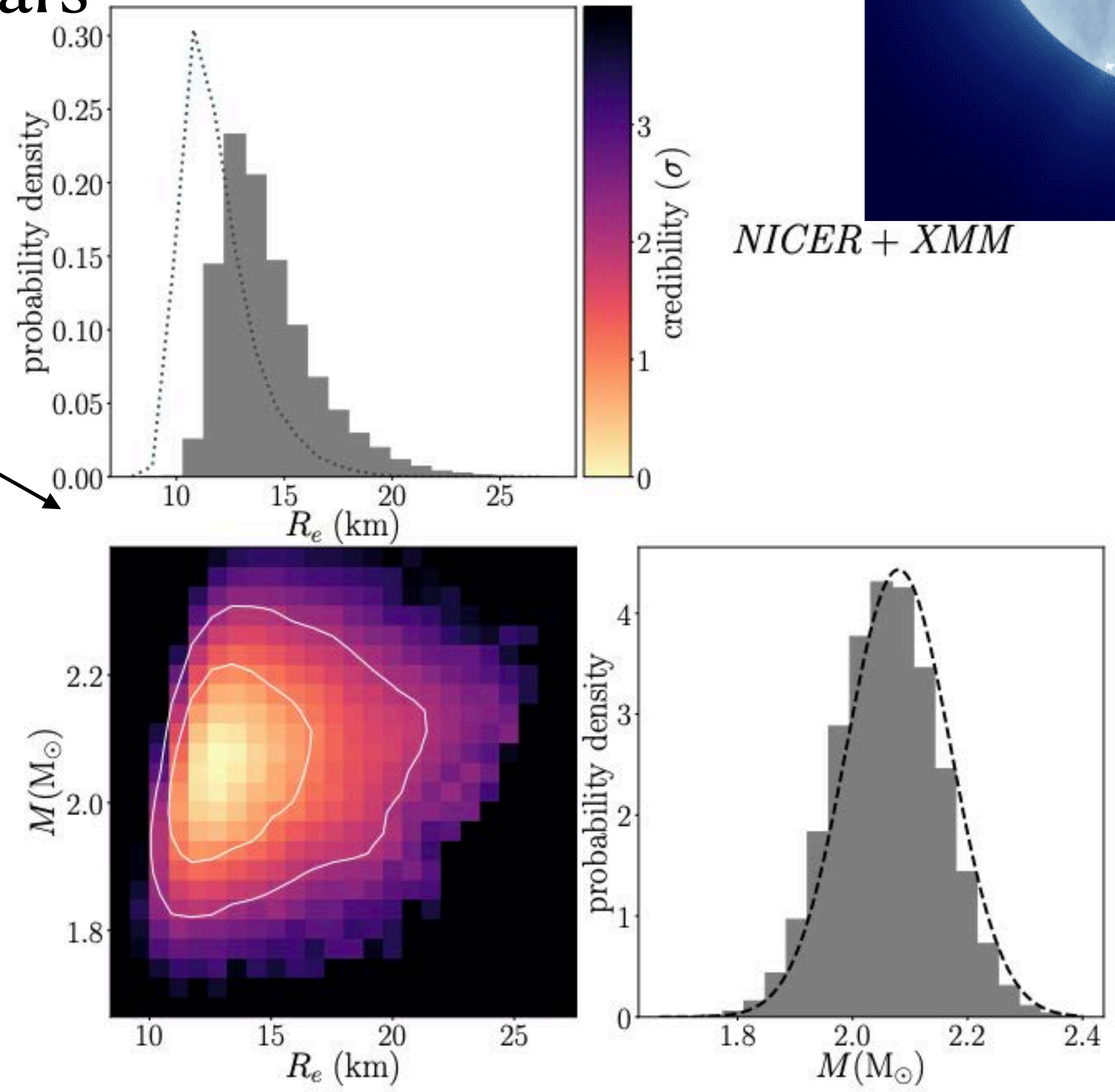
Miller et al.

Raaijmakers et al.

J0740 (2021)

Miller et al.

Riley et al.



Use gravitational lensing of x-rays to infer the compactness of the star
Also used XMM-Newton to calibrate pulse rate

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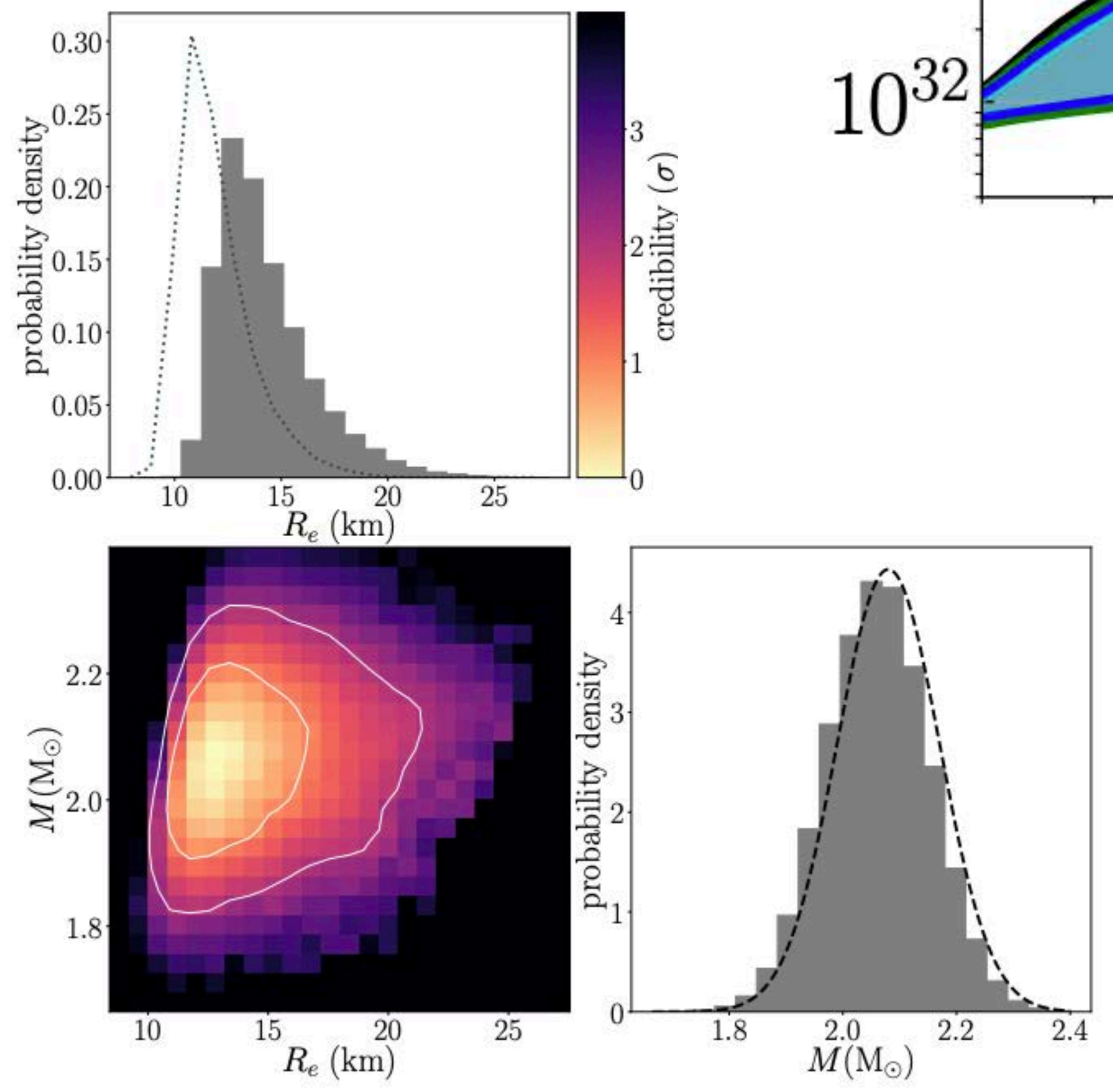

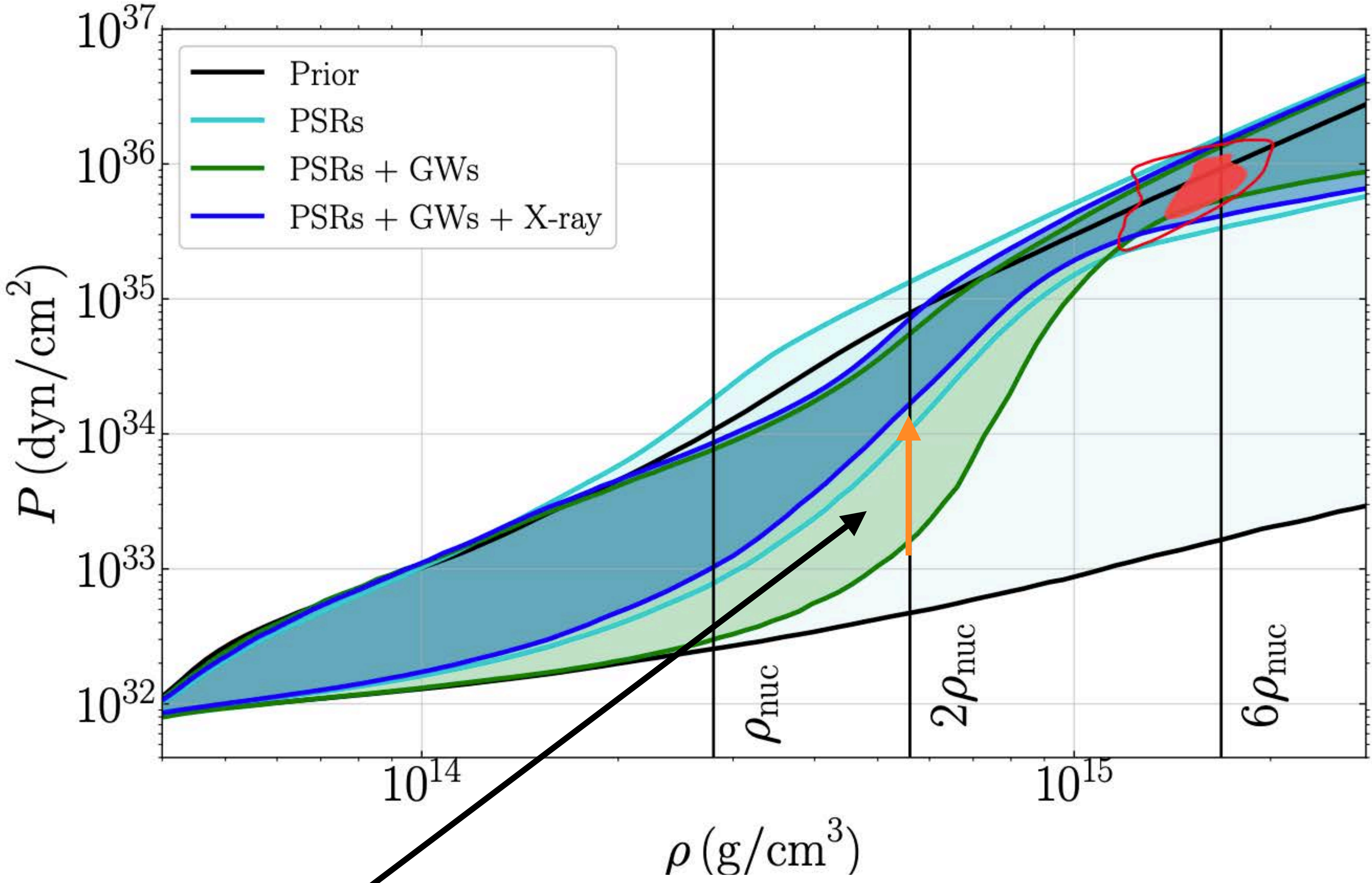

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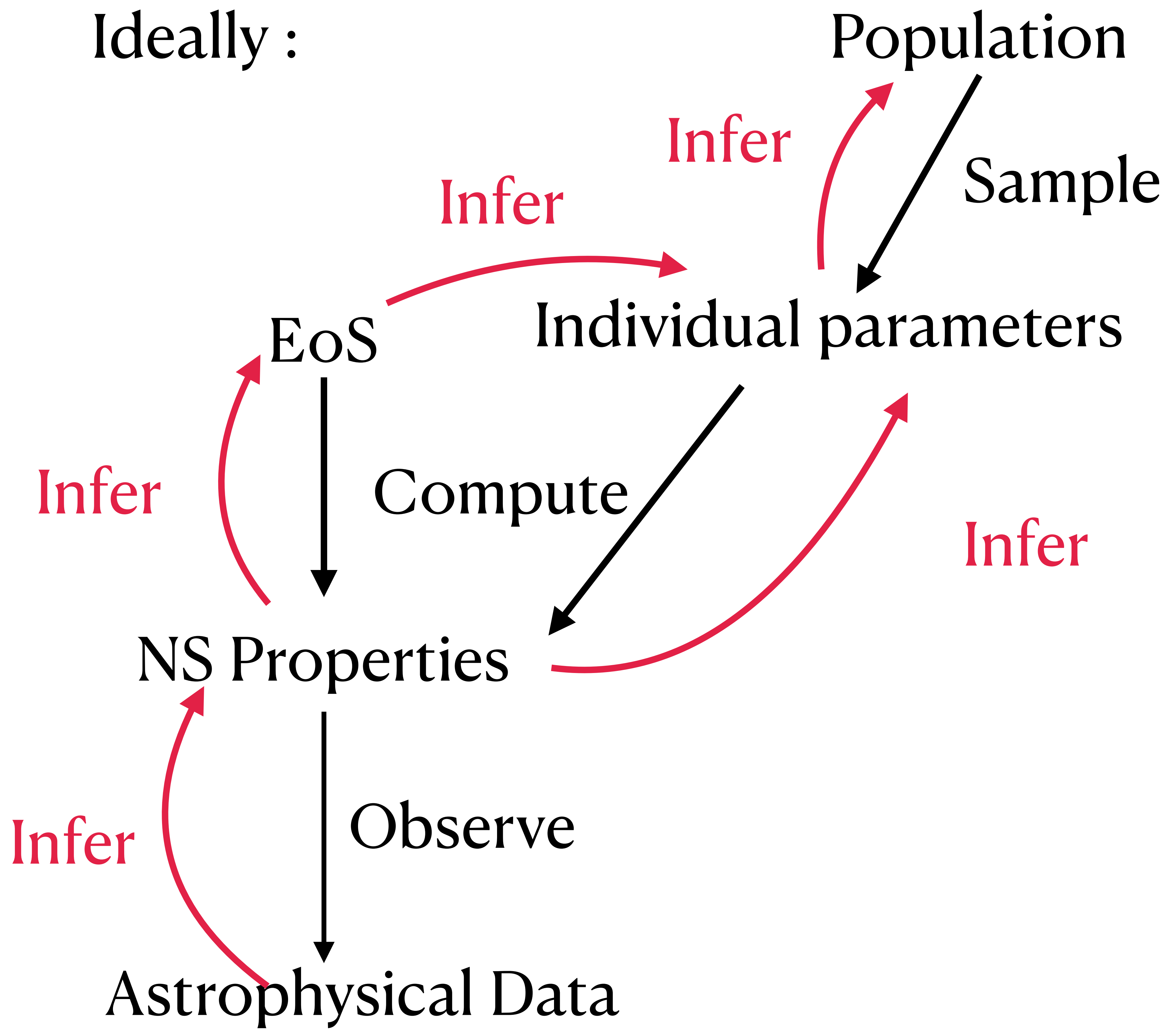
Astrophysical Data (Brief Aside)

Ideally :



Lots of details

(1) Selection Effects
(2) Interpreting data (GW waveforms, x-ray pulse profiles)
(3) Poorly characterized population of NSs

$$P(\varepsilon_i \,|\, d) = P(\varepsilon \,|\, d_1, d_2, \ldots) \propto \boxed{P(d_1, d_2, \ldots \,|\, \varepsilon_i)} \times P(\varepsilon_i)$$

$$P(d_1, d_2, \ldots \,|\, \varepsilon_i) = P(d_1 \,|\, \varepsilon_i) \times P(d_2 \,|\, \varepsilon_i))\ldots$$

$$P(d_1 \,|\, \varepsilon_i) = \sum_{\text{NS sources}} P(d_1 \,|\, \text{NS source}) \times P(\text{NS Source} \,|\, \varepsilon_i, (\text{Population Model}))$$

Astrophysical Data (Brief Aside)

In practice:

Population

Sample

Individual parameters

EoS

Infer

Compute

NS Properties

Infer

Observe

Astrophysical Data

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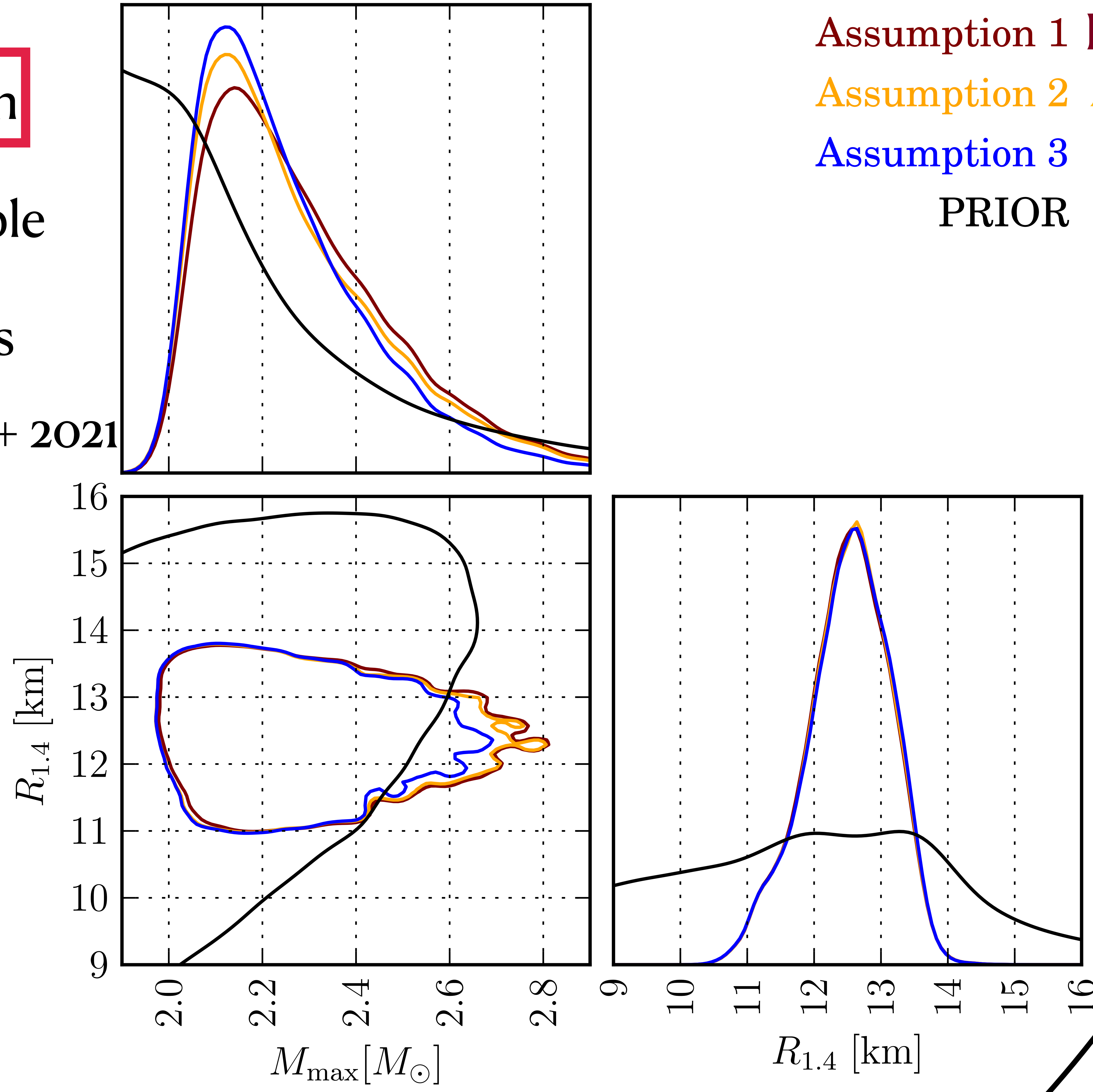

Assumption 1 BHs are just small NSs

Assumption 2 Astrophysically lmtd.

Assumption 3 TOV mass lmtd.

PRIOR

$$P(d_1 \mid \varepsilon_i) = \sum_{\text{NS sources}} P(d_1 \mid \text{NS source}) \times P(\text{NS Source} \mid \varepsilon_i, (\text{Population Model}))$$

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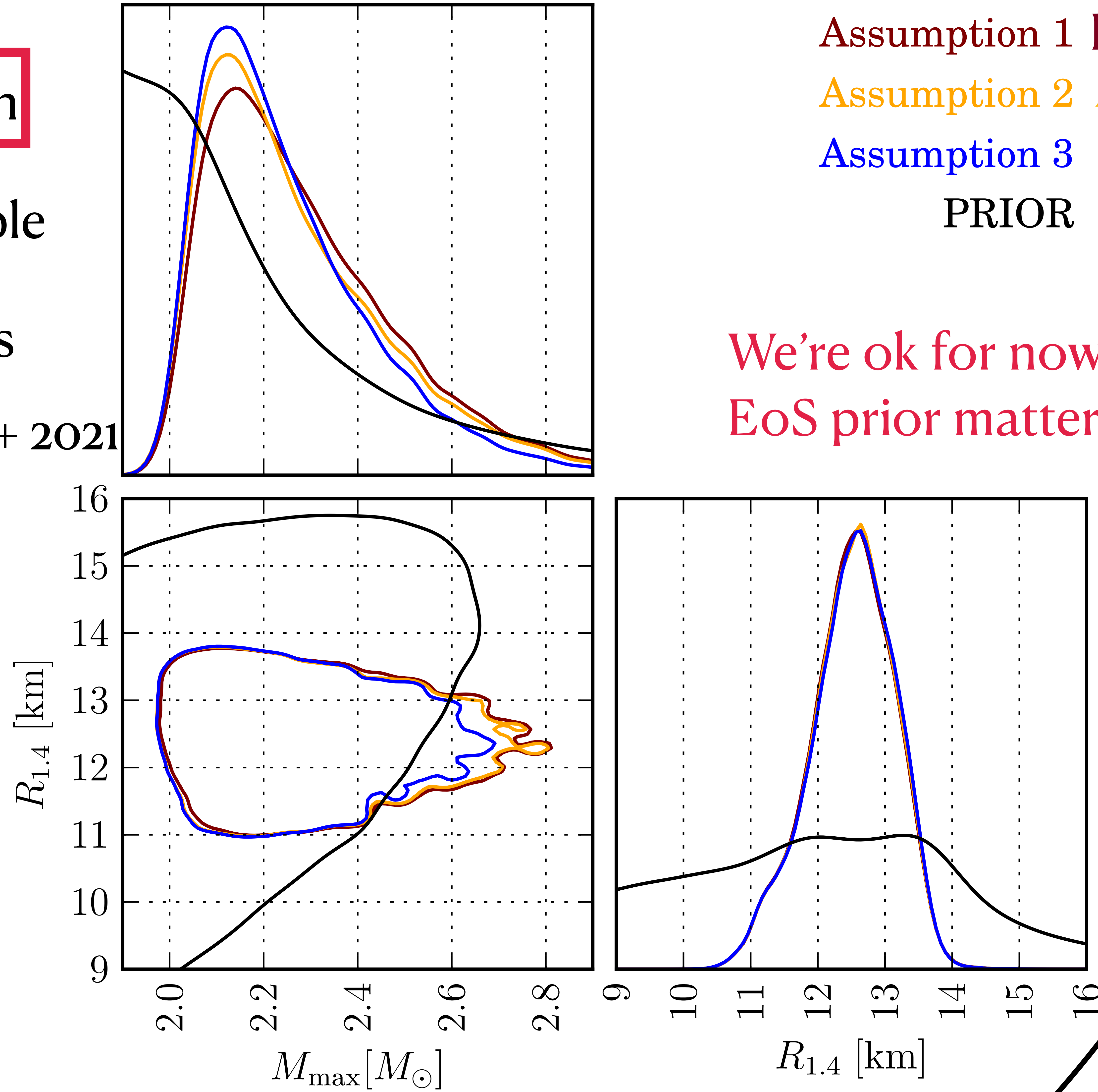

Assumption 1 BHs are just small NSs

Assumption 2 Astrophysically lmtd.

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PRIOR

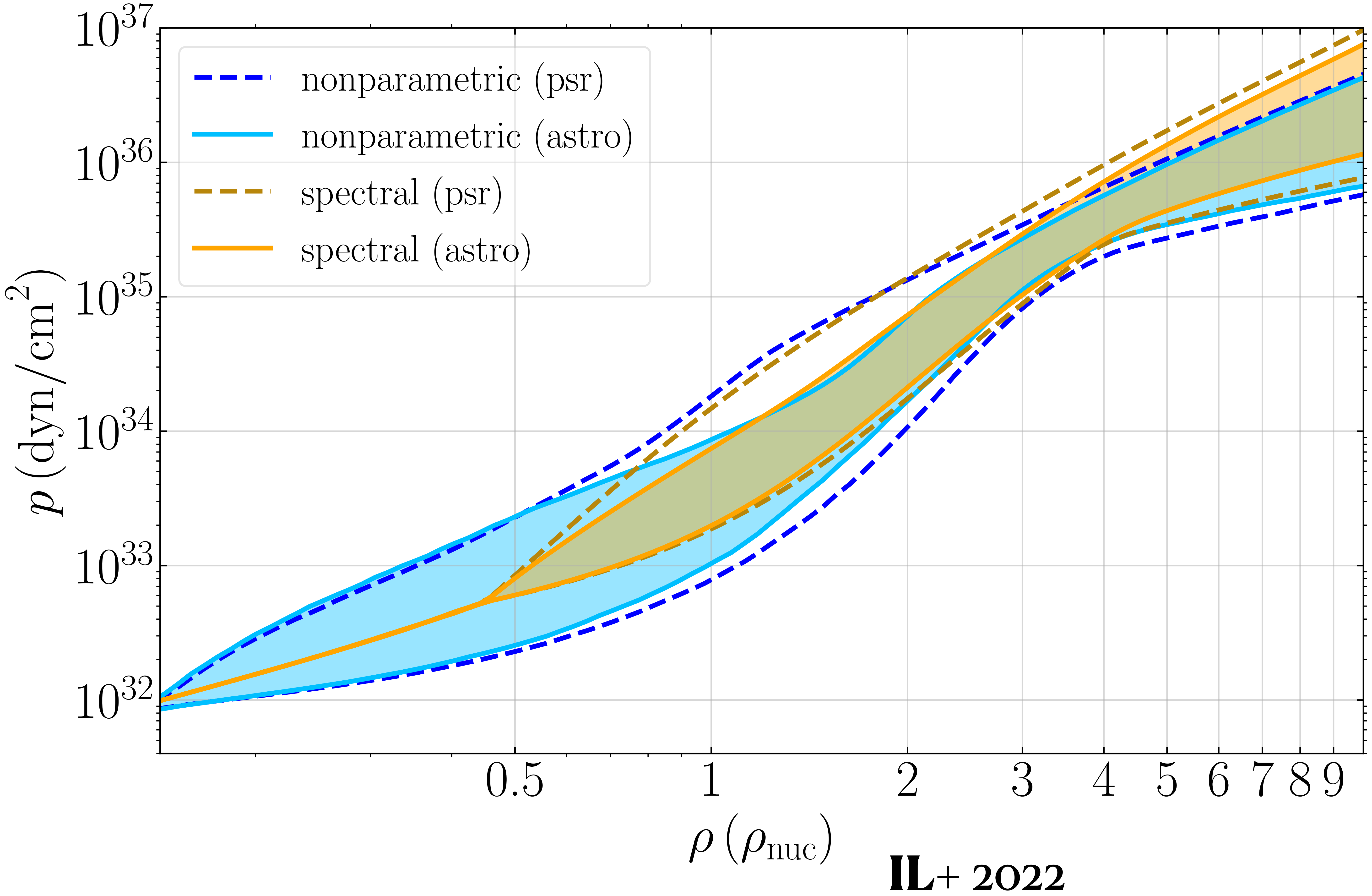
We're ok for now =>
EoS prior matters more

$$P(d_1 \mid \varepsilon_i) = \sum_{\text{NS sources}} P(d_1 \mid \text{NS source}) \times P(\text{NS Source} \mid \varepsilon_i, (\text{Population Model}))$$

Different Priors, Different Results

We know that the choice of prior does impact our recovered distribution on the EoS

Hold all other things constant (astrophysical data, pop.)



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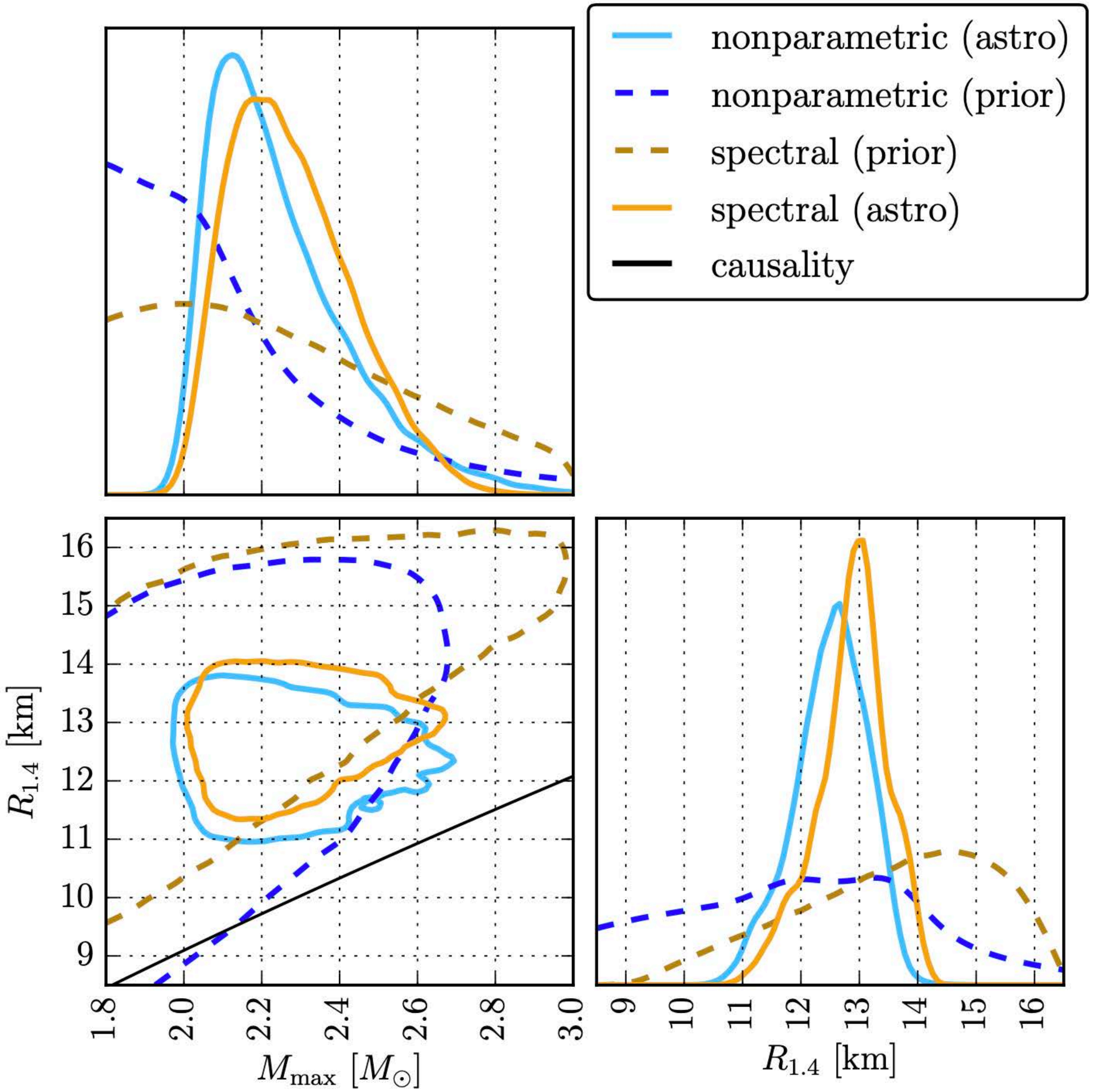
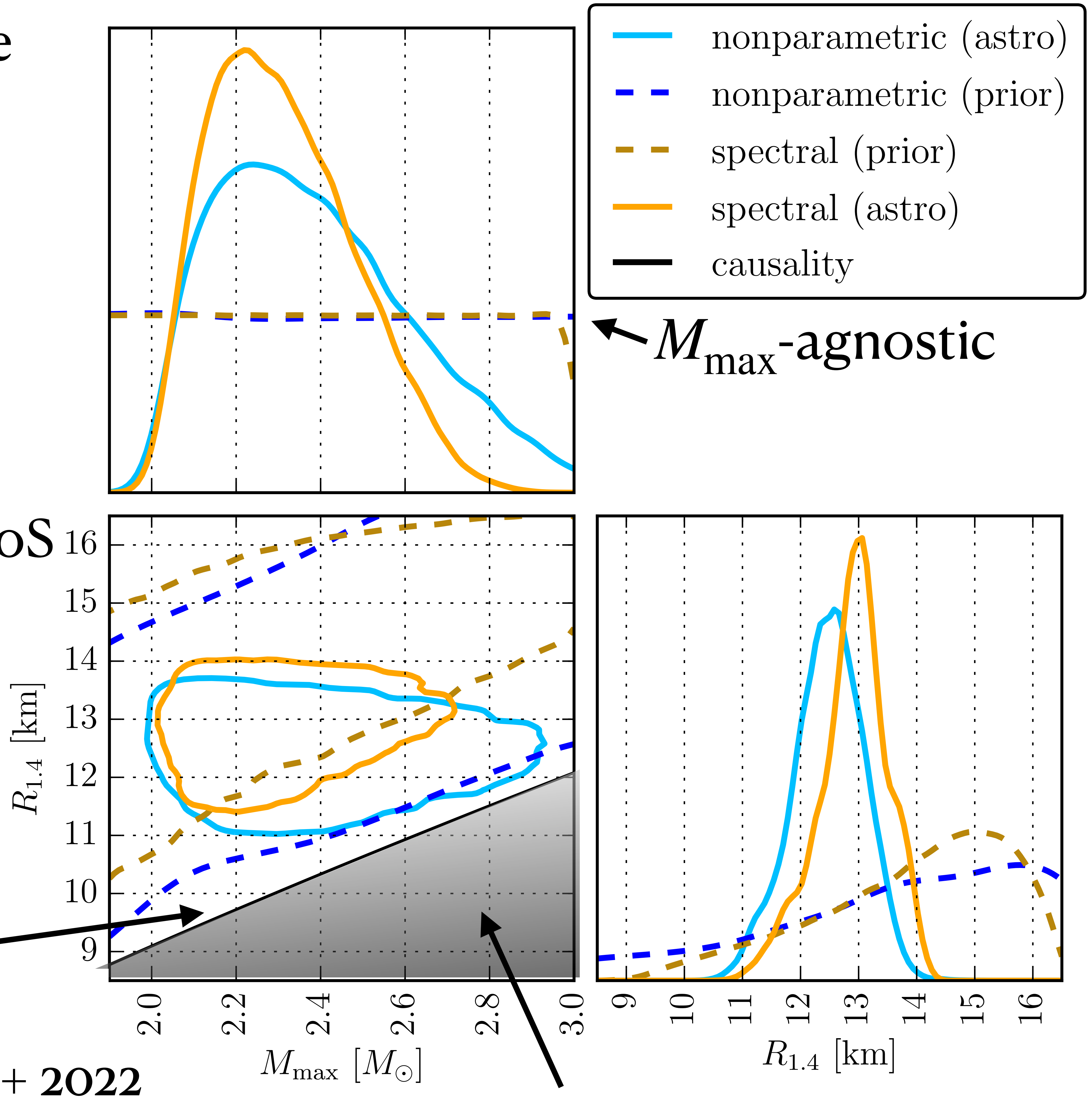


Different Priors, Different Results

TOV maximum mass and radius of a 1.4 solar mass NS are correlated among equation of state candidates due to causality

This rules out certain configurations in "$M_{\max} - R_{1.4}$" space

The boundary is "fuzzy" — depends on the low density EoS



Parametrized by Stitching density to $c_s^2 = c^2$ EoS

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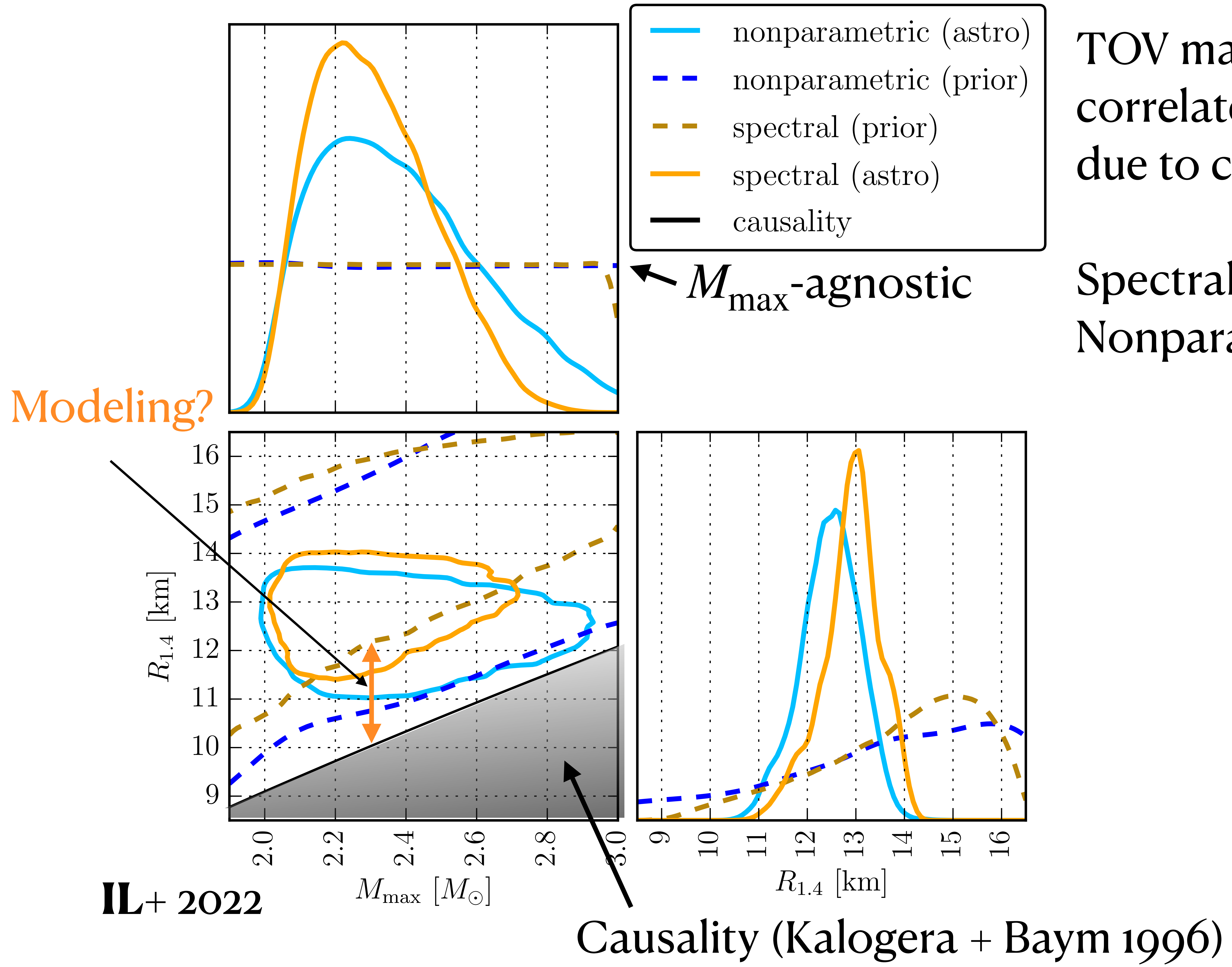
Causality (Kalogera + Baym 1996)

Different Priors, Different Results



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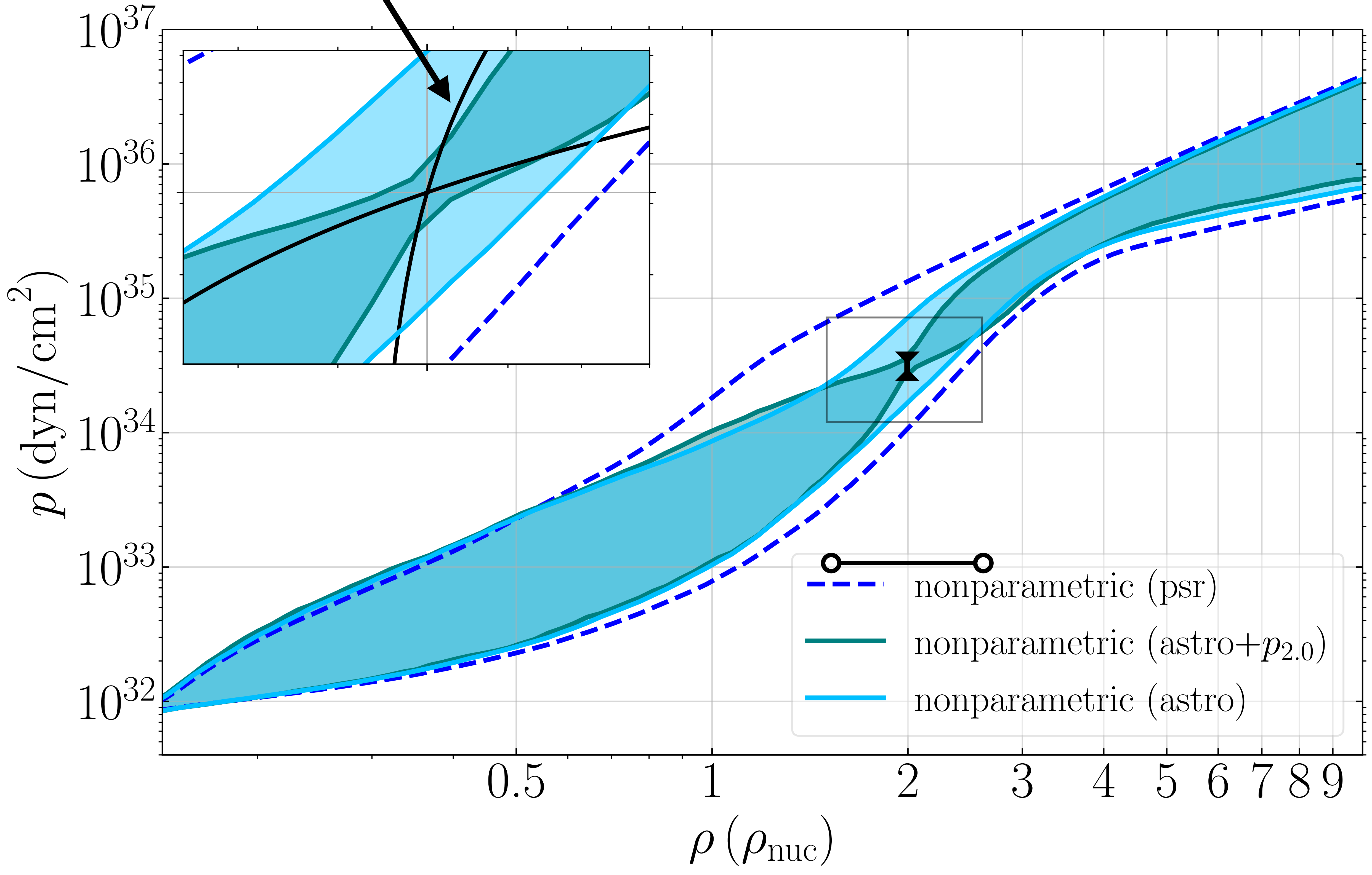
TOV maximum mass and radius of a 1.4 solar mass NS are correlated among equation of state candidates due to causality

Spectral model sees a "tighter correlation" than the Nonparametric model — not likely due to causality!

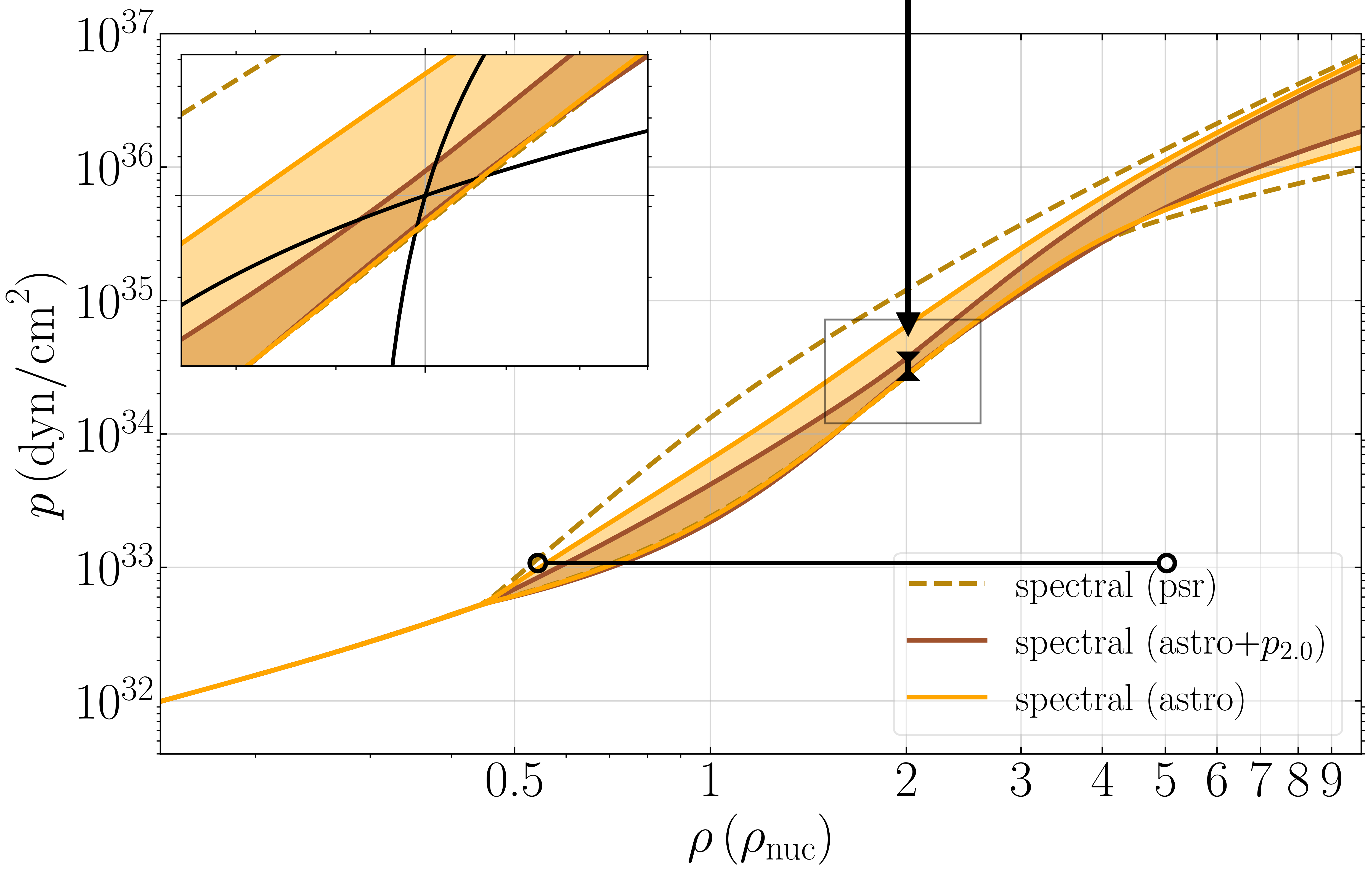
Correlations

Correlations between astro observables <=> Correlations between density scales

Approximate causality and stability limits



Tight constraint a $2\rho_{\mathrm{nuc}}$



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Implicit Correlations

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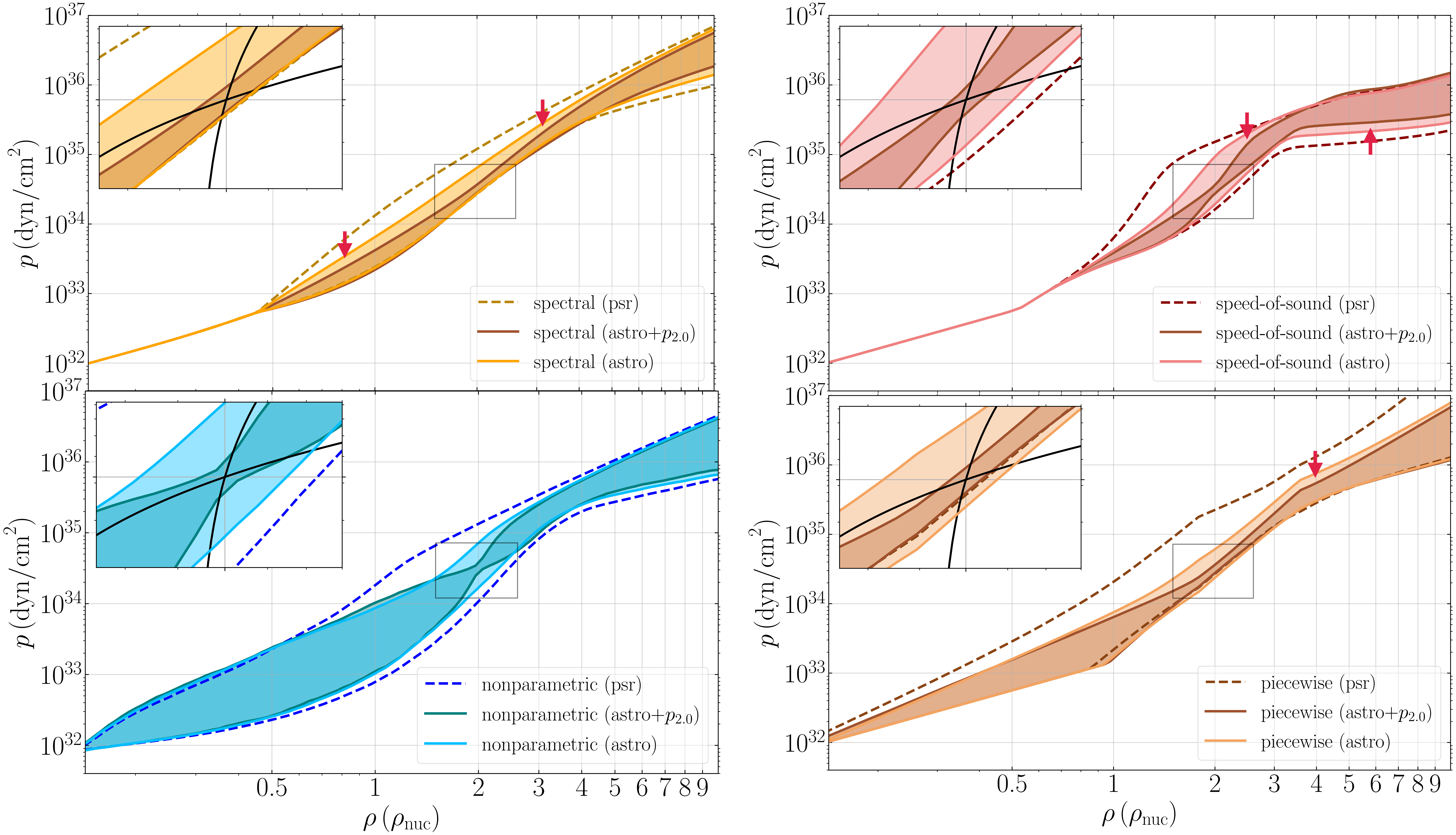

Implicit Correlations

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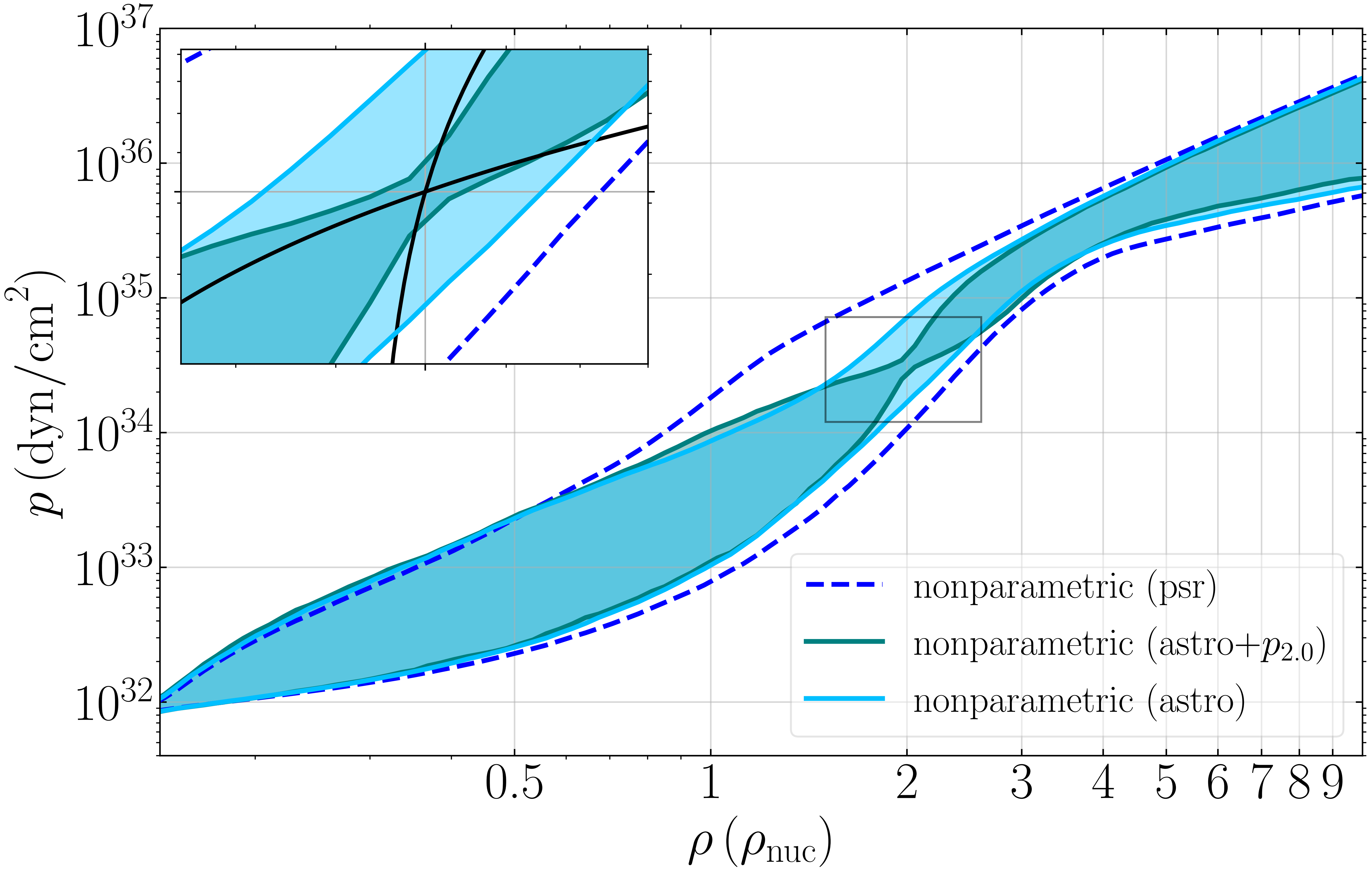


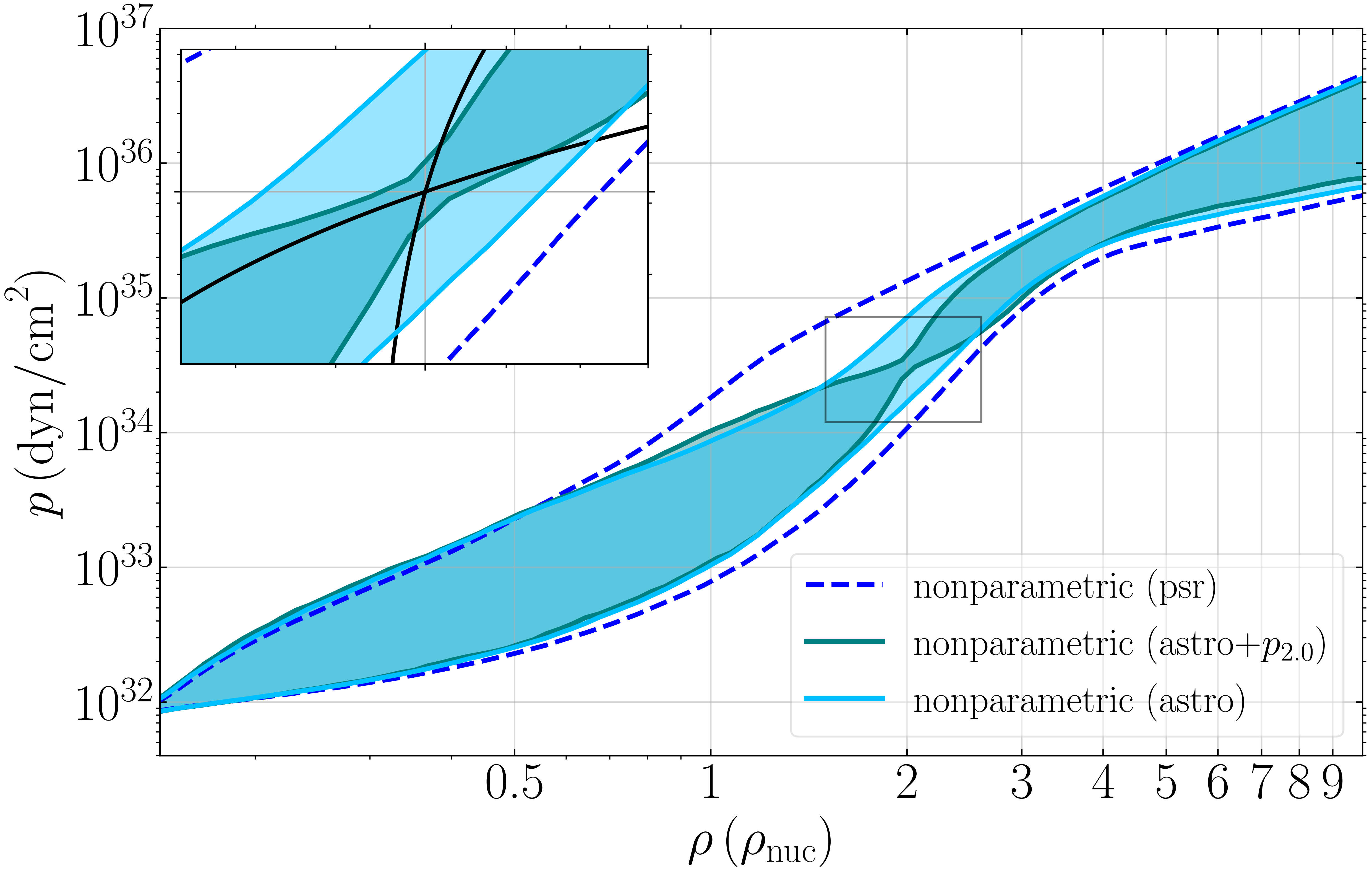
Quantifying correlations — Mutual Information

How much information is gained about other density Scales by knowing the EoS at some fixed density

$$I(p_a, p_b) \equiv \int dp_a dp_b P(p_a, p_b) \ln \left(\frac{P(p_a, p_b)}{P(p_a)P(p_b)} \right)$$

Implicit Correlations

IL+ 2022



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Also a K-L divergence!

$$I(p_a, p_b) = \int dp_a P(p_a) \int dp_b P(p_b | p_a) \ln\left(\frac{P(p_b | p_a)}{P(p_b)}\right)$$

Difference in knowledge about p_b after learning p_a

Changing this analogous to adding a tight Pressure “mock-measurement”

Implicit Correlations

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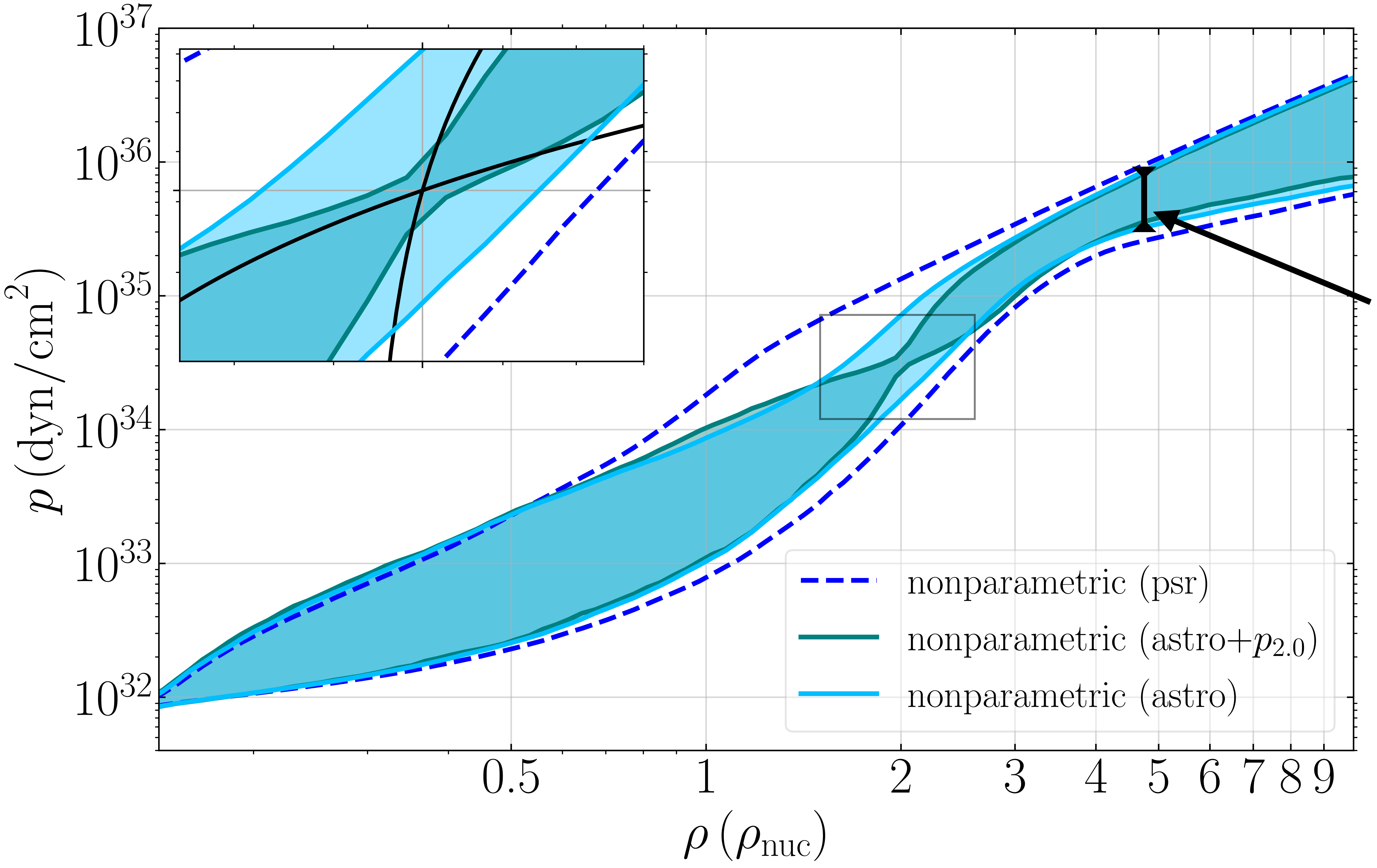

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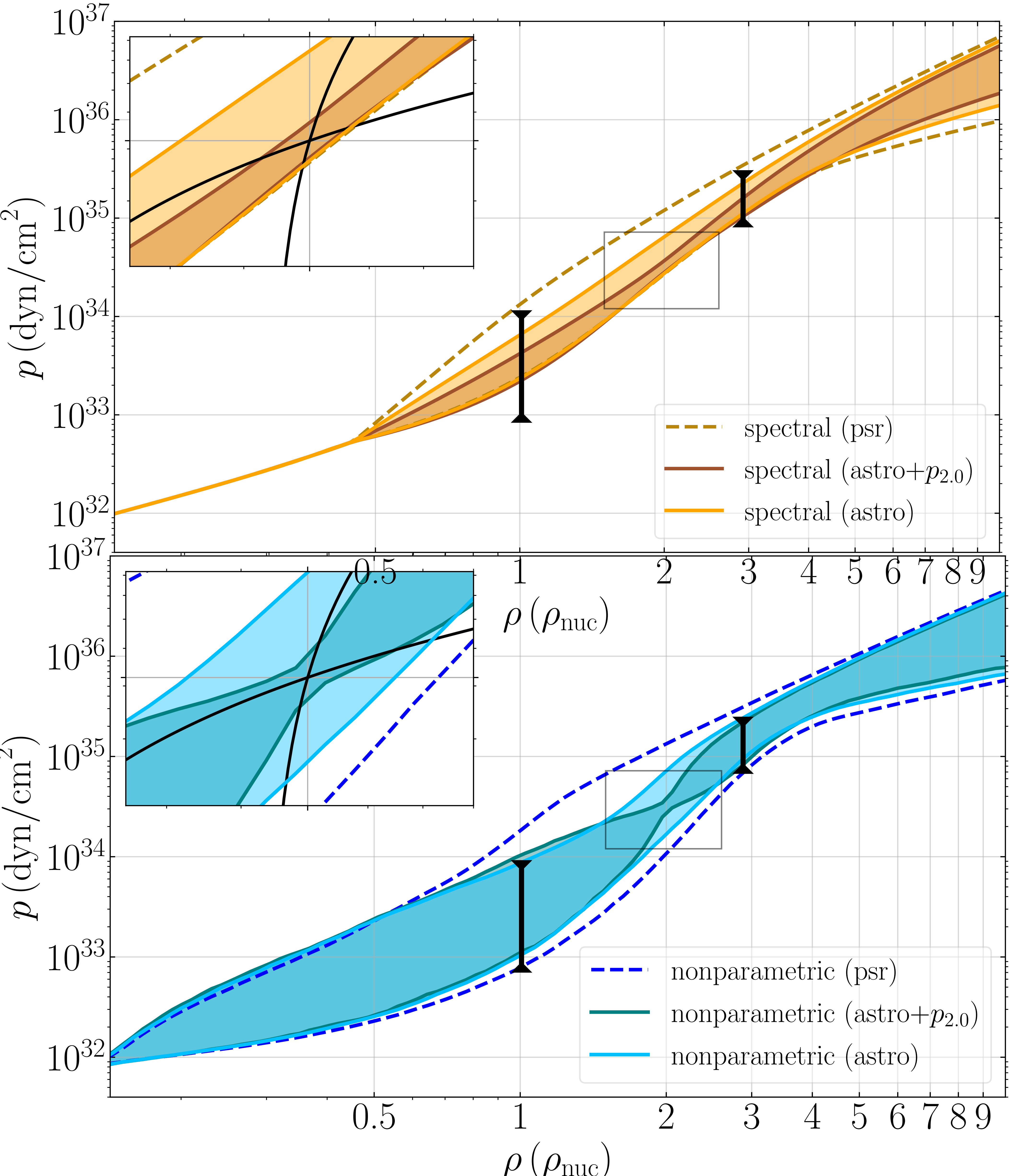
Caveats!

Scales with overall uncertainty of marginal distributions

Want to keep I small even with large entropy in Marginal distributions $P(p_a)$, …

Implicit Correlations

IL+ 2022



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Caveats!

Scales with overall uncertainty of marginal distributions

Want to keep I small even with large entropy in Marginal distributions $P(p_a)$, …

$$I\left(\ln(p_{1.0}), \ln(p_{1.5}), \ln(p_{2.0}), \ln(p_{3.0}), \ln(p_{4.0})\right)$$

	PSR	Astro	Astro+$p_{2.0}$
Nonparametric	3.7	3.1	2.9
Spectral	6.6	5.5	4.7
Polytrope	5.7	4.6	3.8
Speed of sound	5.0	4.7	4.3

Simulated Astrophysical Data

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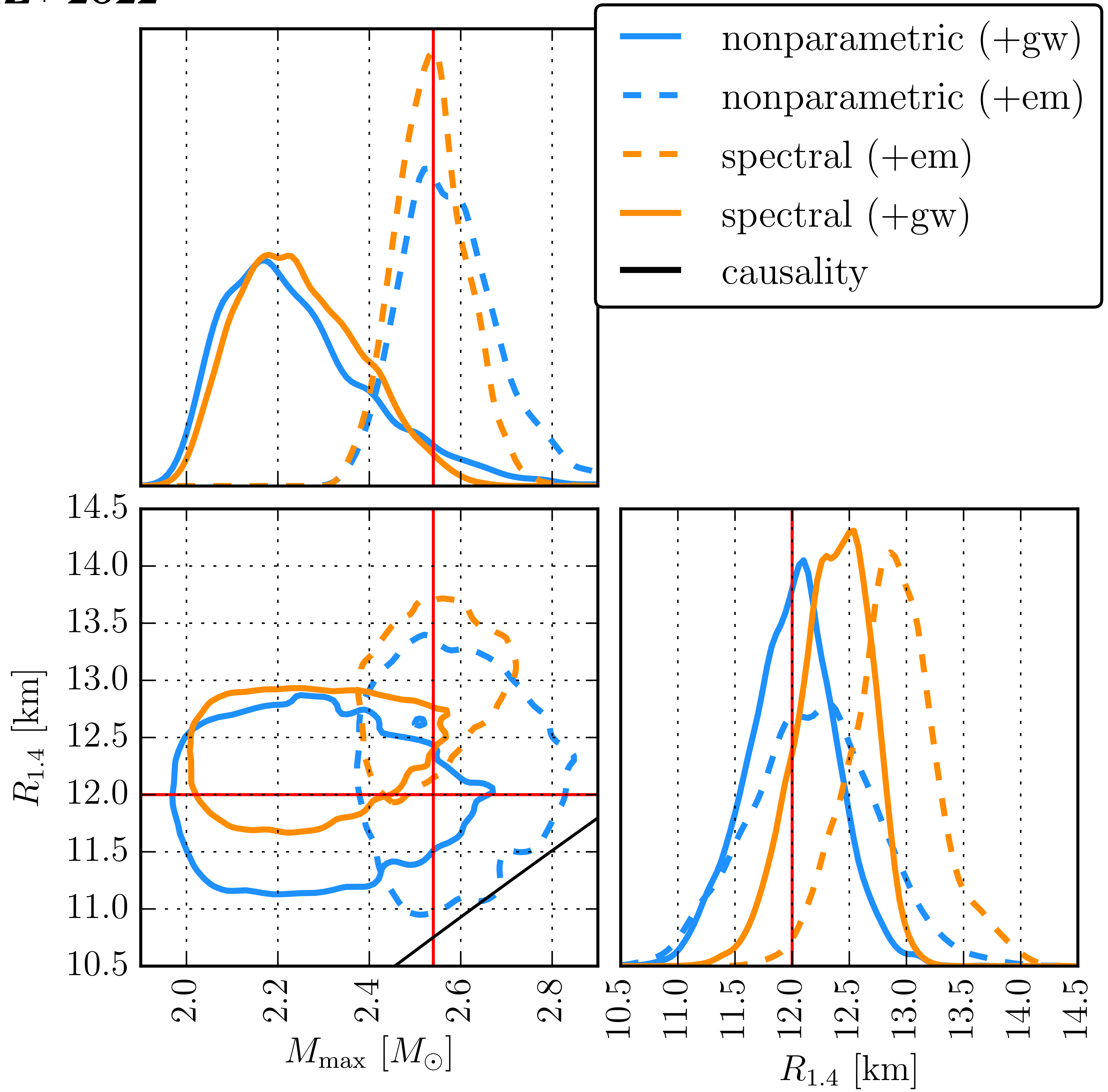
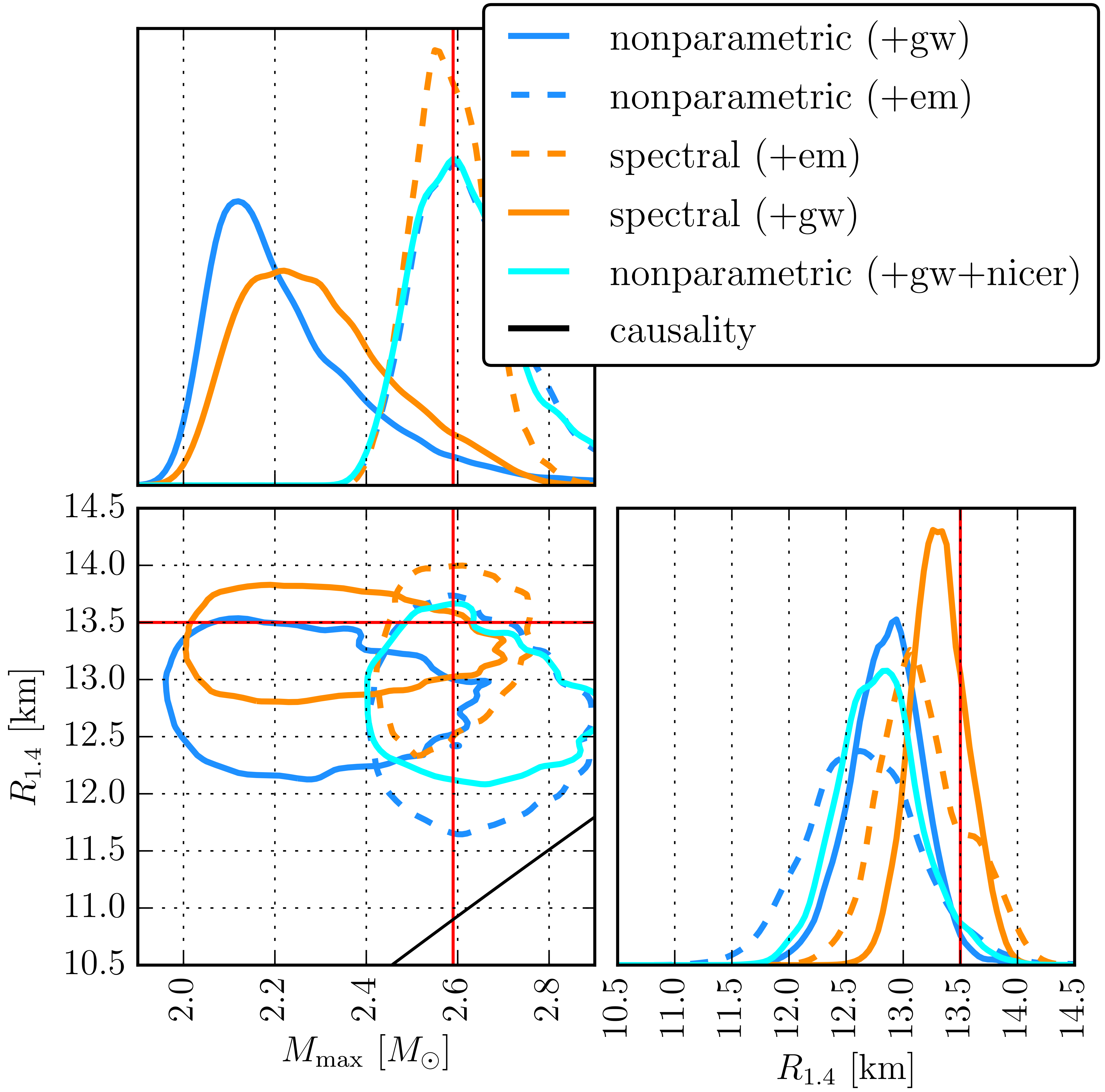


We inject gravitational-wave (gw) and x-ray-radio (em) observations on top of existing constraints

We intentionally choose an EoS that we expect the Spectral model to fail to recover

Gives a sense of tension that may arise from combining constraints using models with unphysical correlations

Simulated Astrophysical Data



We inject gravitational-wave (gw) and x-ray-radio (em) observations on top of existing constraints

Inverse Problem : Spectral Eos -> NP analysis

Slow convergence, but no bias

Modified parametric priors

Why not just modify the parametric models to get more flexibility?

E.g.

$$p(\rho) = \rho^{\Gamma}; \quad \Gamma(p) = \sum_{i=0}^{3} \gamma_i \log(p/p_0)^i + \text{more terms}$$

Modified parametric priors

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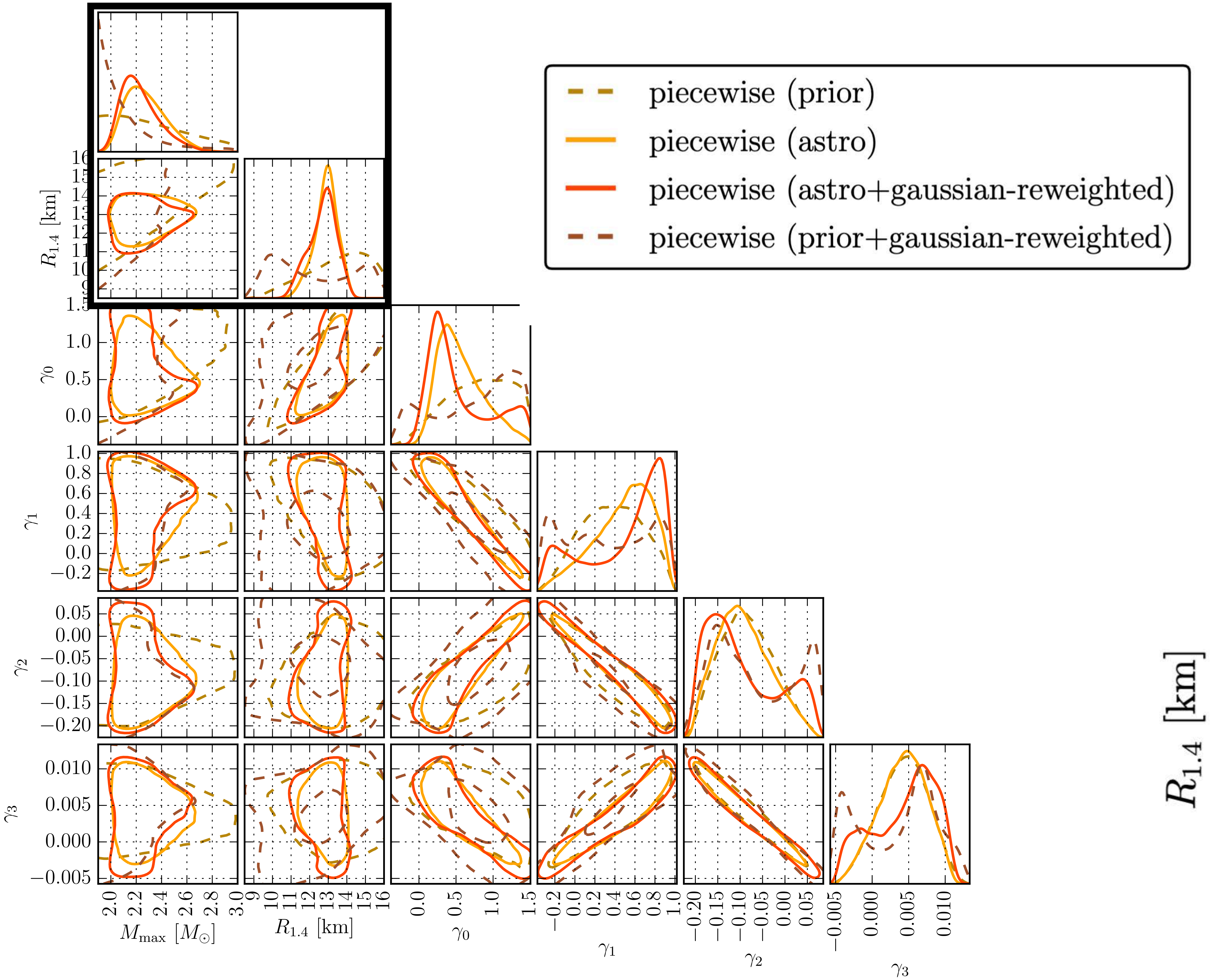
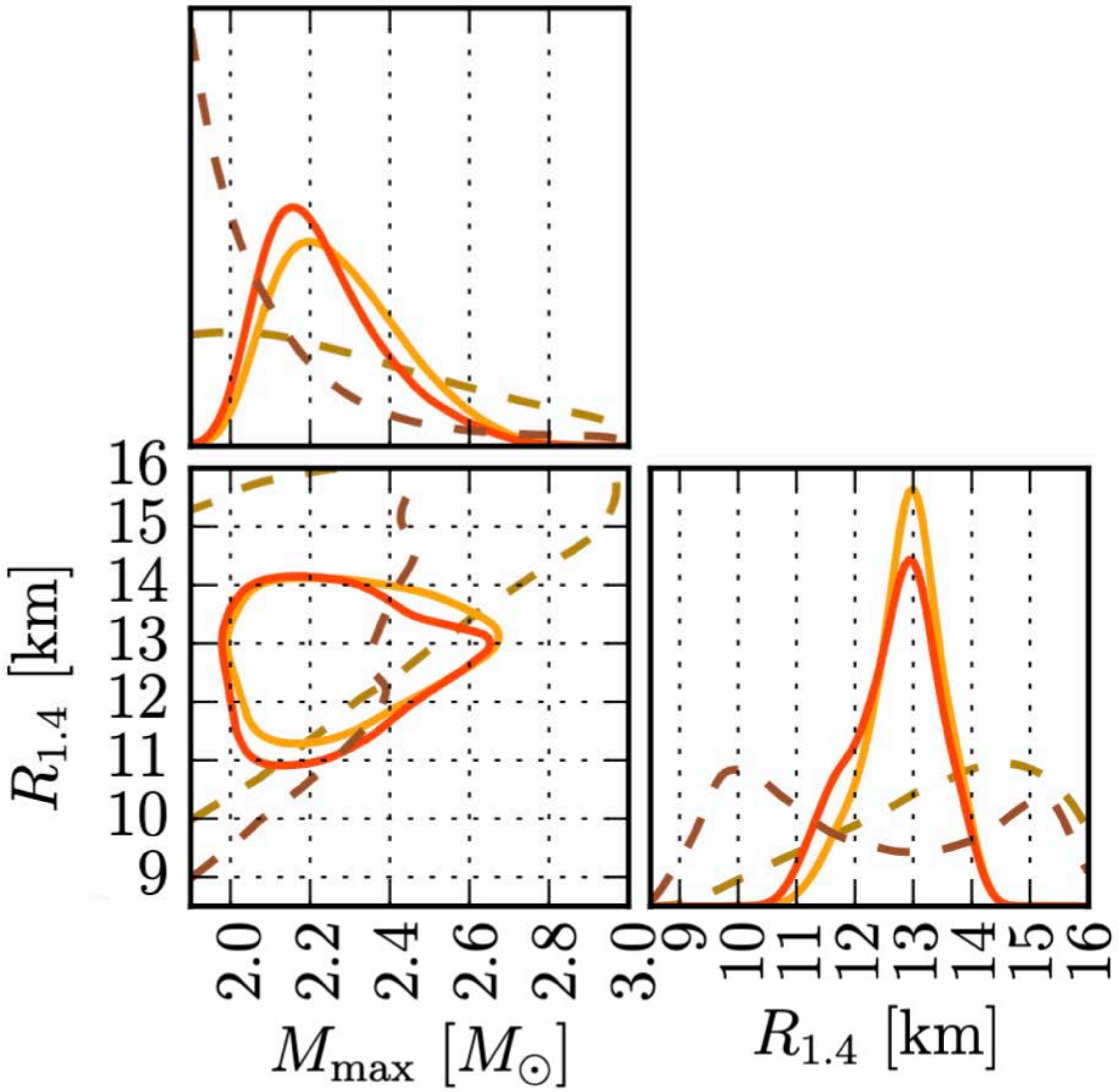


We find changing the
Prior on parameters doesn't
Remove the correlations



Modified parametric priors

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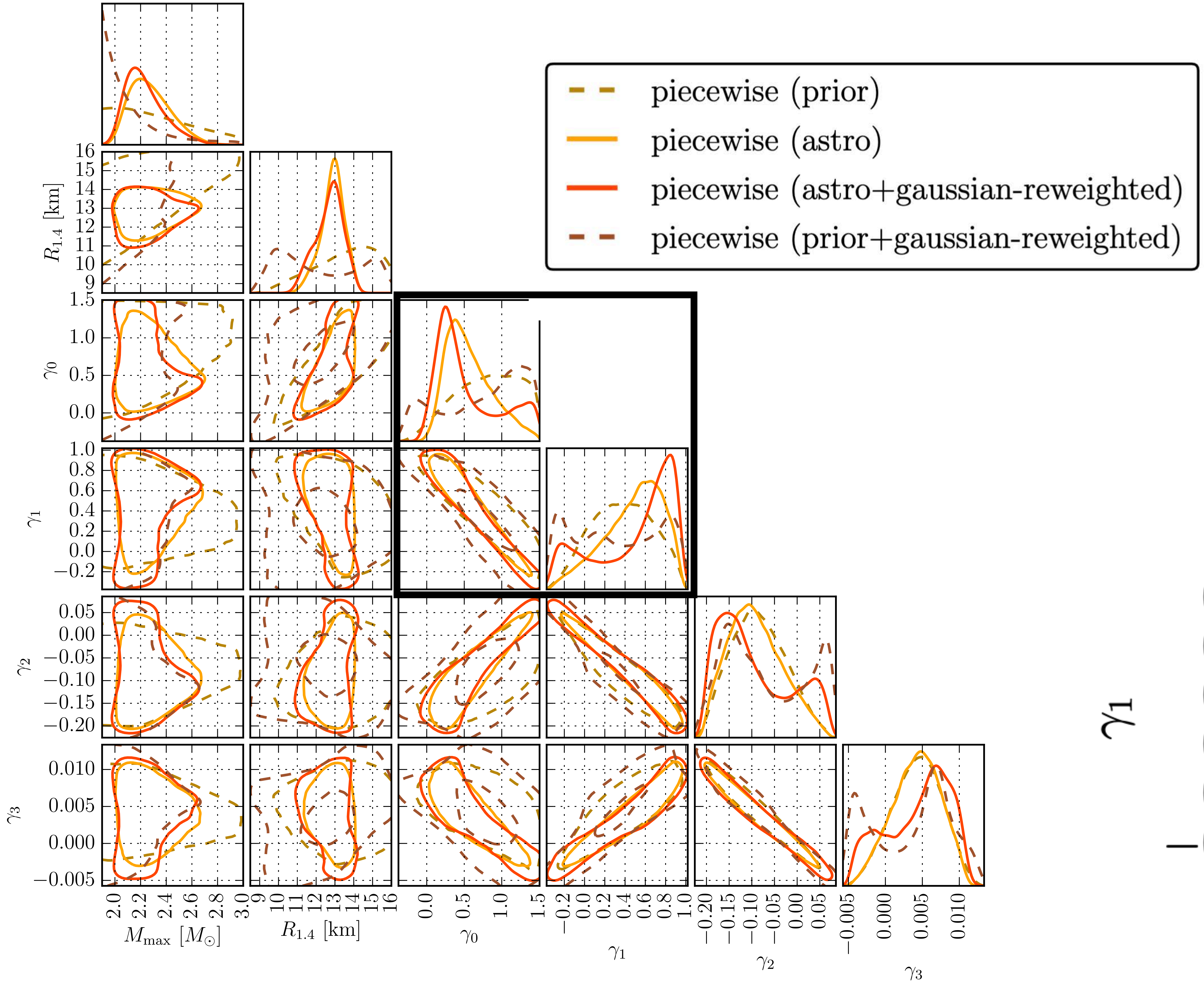
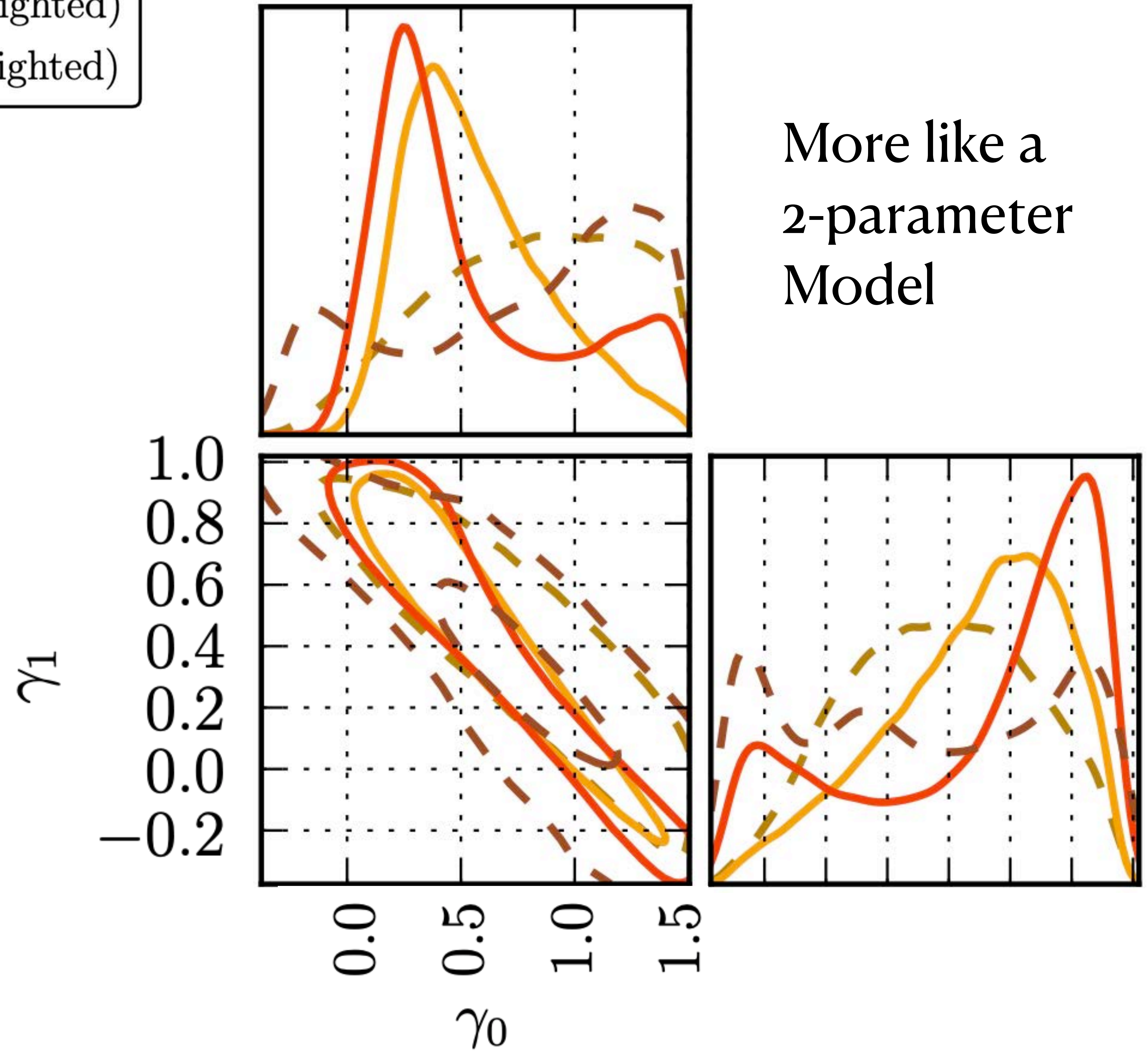


With the current data we only
Really only infer γ_0 & γ_1



More like a
2-parameter
Model

Modified parametric priors

Why not just modify the parametric models to get more flexibility?

Models are either

(1) fine-tuned => extending them without breaking is difficult (spectral + speed of sound)
(2) Need overhaul-type improvements (piecewise-polytrope + speed of sound)

This is already being done!
i.e. Steiner+ 2016 -> better piecewise-polytrope models

But... Extensions are nontrivial.
Best to understand limitations of each model while using it

Not all Correlations are Bad!

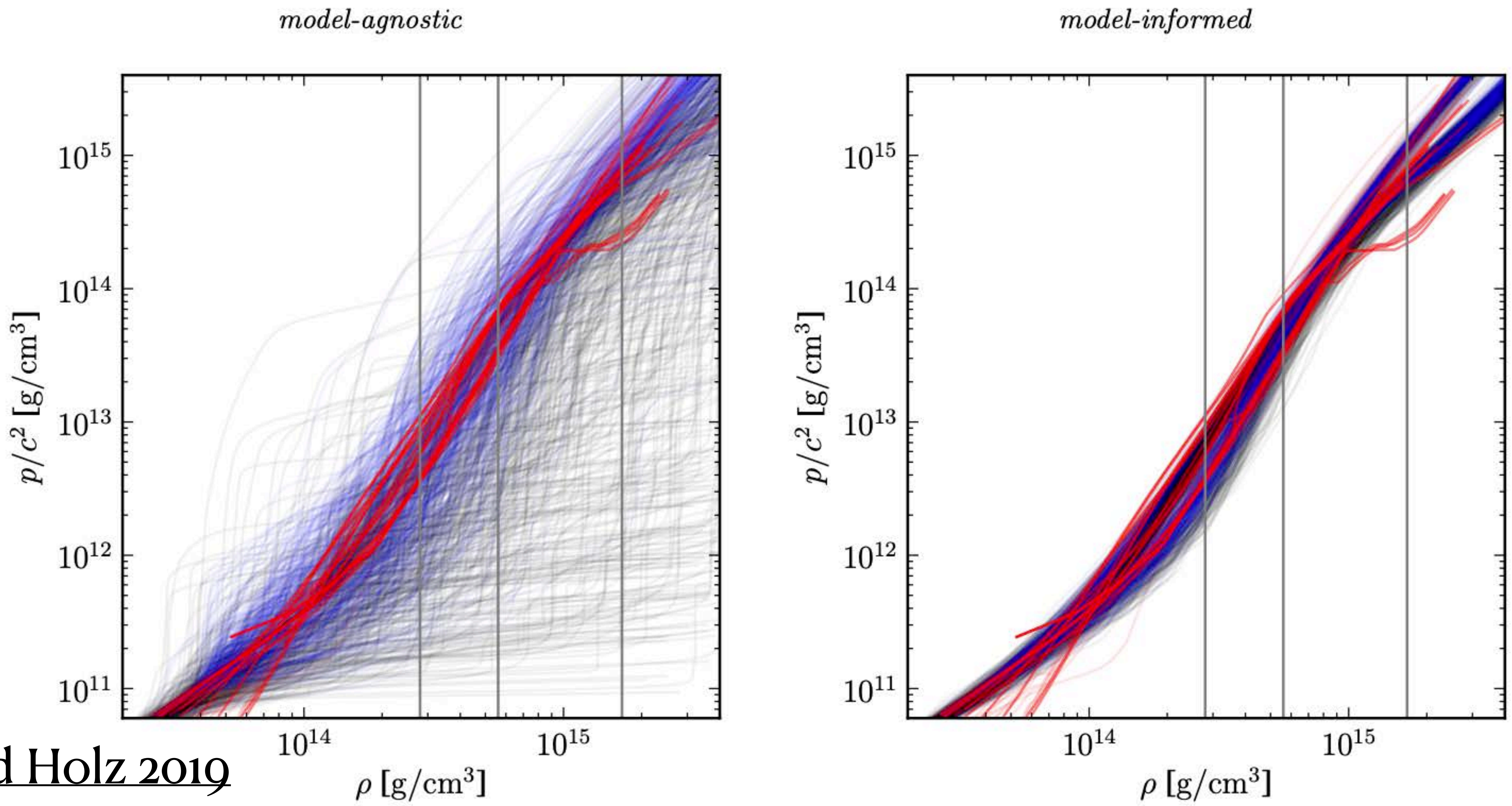
Physical theories have correlations between quantities “F=ma”

Correlated

Goal is to give flexibility in the choice of correlations

See e.g. Miller+2021 : GP with “tight” correlations

Eventually one should *infer* the correlations



Essick, Landry, and Holz 2019

Conclusions

- Phenomenological models of the nuclear equation of state can build in (often hidden) correlations due to the functional form of the EoS

- Nonparametric models (such as the Gaussian Process model), can provide more flexibility in inference of the EoS (but do not guarantee it)

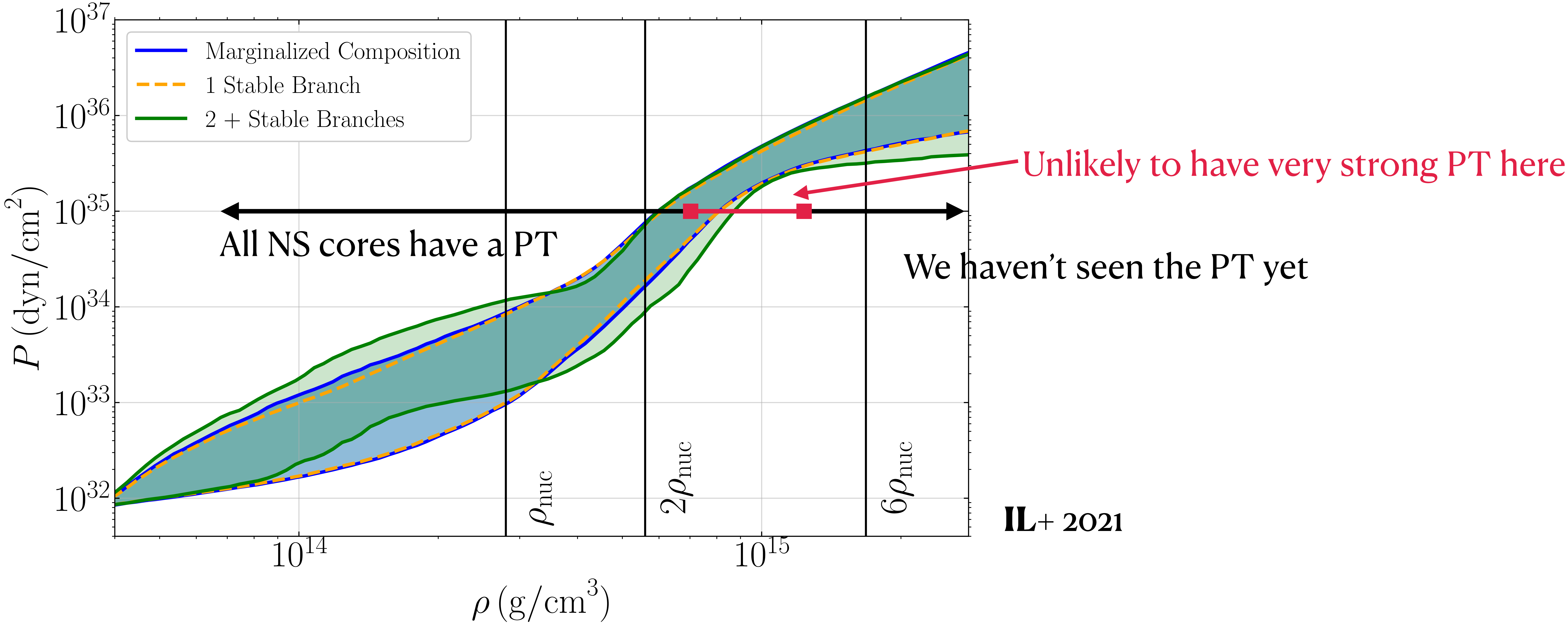
- These correlations affect both astrophysical inference and inference of properties of dense matter

(Backup) : Strong Phase Transitions

Parametric models struggle to model phase transitions

Piecewise-polytope models with variable stitching densities may be able to -> need fine tuning

The GP model can produce EoSs mimicking phase transitions generically



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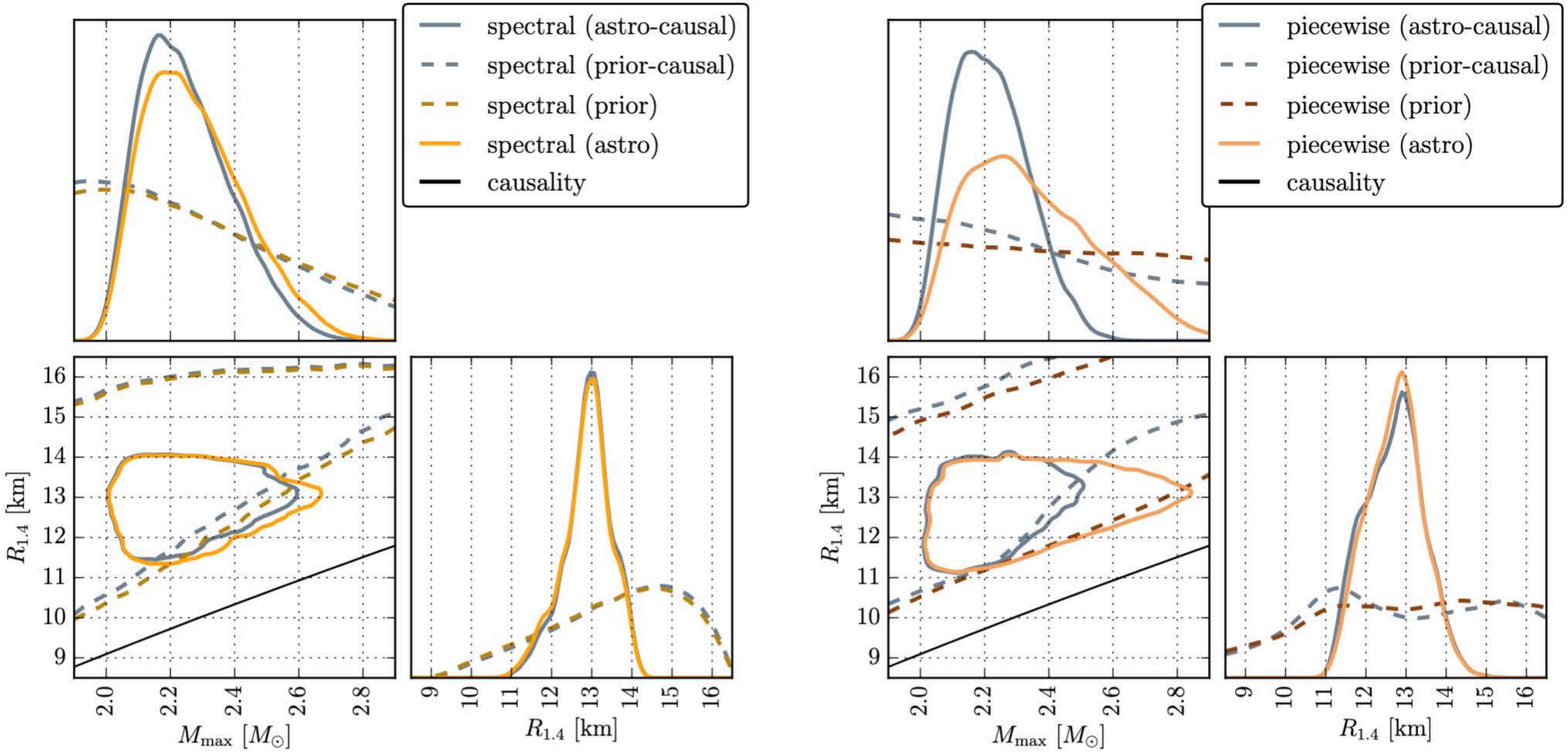
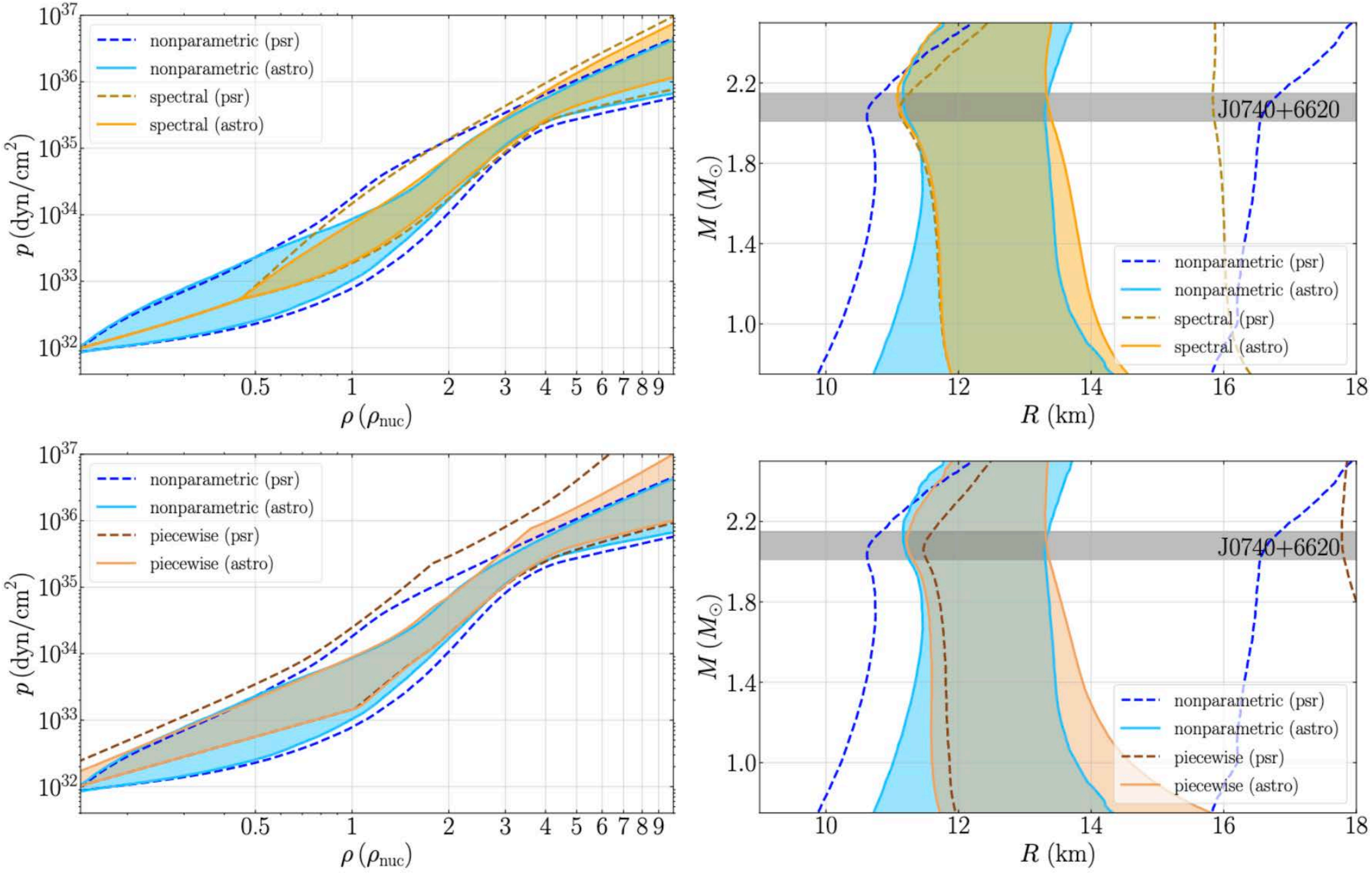
(Backup): Causality in Parametric Models



(Backup): More Parametric Results



(Backup): Corner Plot