

Christian Drischler

October 21, 2021 | N3AS: Biweekly Neutron Star Merger Meetings

These biweekly meetings aim to bring together researchers working on neutron star mergers.

Topics will include:

- computational astrophysics,
- dense matter physics,
- gravitational waves and data analysis,
- neutrino and particle astrophysics.

Meetings will include a 30-40 min talk on a related topic, and 20-30 minutes of discussion...

Today:

- + Chiral EFT + MBPT
- + Bayesian UQ
- + infinite nuclear matter
- + symmetry energy
- + nuclear saturation
- + N³LO NN + 3N forces

+ ...



see also Jeremy Holt's talk (November 4)

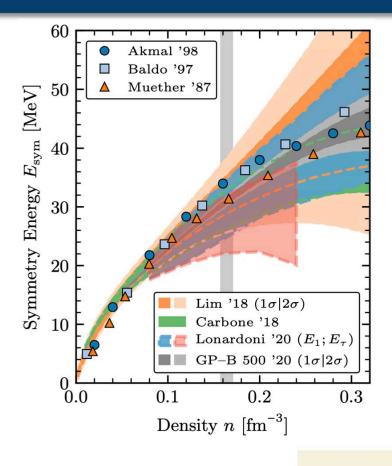
Equation of State of Dense Matter at Finite Temperature

[A. Watts]

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Recent review article

CD, Holt, and Wellenhofer, ARNPS 71, 403





Chiral Effective Field
Theory and the
High-Density Nuclear
Equation of State



C. Drischler, 1,2,3 J. W. Holt, 4 and

C. Wellenhofer,^{5,6}

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⁴Cyclotron Institute and Department of Physics and Astronomy, Texas A&M University, College Station, Texas 77843, USA; email: holt@physics.tamu.edu

⁵Institut für Kernphysik, Technische Universität Darmstadt, 64289 Darmstadt, Germany; email: wellenhofer@theorie.ikp.physik.tu-darmstadt.de

⁶ExtreMe Matter Institute EMMI, GSI Helmholtzzentrum für Schwerionenforschung GmbH, 64291 Darmstadt, Germany

invited contribution to Annu. Rev. Nucl. Part. Sci. 71, 403 see also: Lattimer, Annu. Rev. Nucl. Part. Sci. 71, 433

see also, *e.g.*: Tews, Front. in Phys. **8**, 153

Annu. Rev. Nucl. Part. Sci. 2021. 71:1-30

This article's doi: 10.1146/annurev-nucl-102419-041903

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Annu. Rev. Nucl. Part. Sci. in press.

Keywords

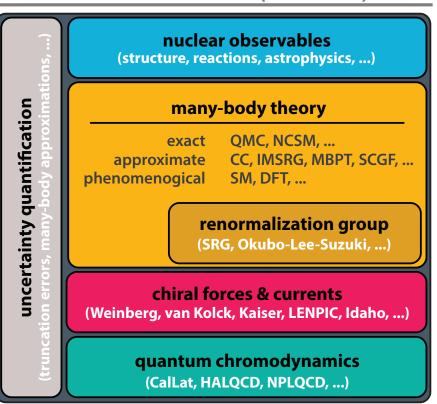
chiral effective field theory, nuclear matter, neutron stars, many-body perturbation theory, bayesian uncertainty quantification

Abstract



Ab initio calculations | outline





CD & Bogner, Few Body Syst. 62, 109

nuclear equation of state neutron matter | symmetric matter

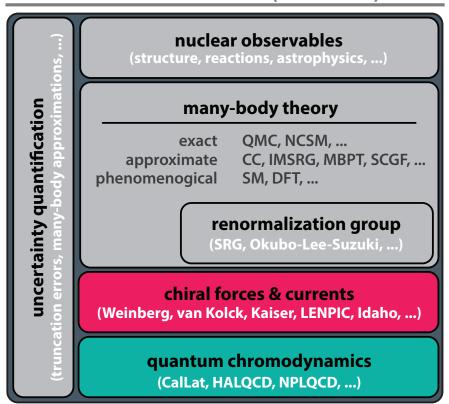
many-body perturbation theory computationally efficient many-body uncertainty estimates

chiral effective field theory
systematic expansion of nuclear forces
truncation error estimates



Ab initio calculations | outline

Ab initio workflow (idealized)



CD & Bogner, Few Body Syst. 62, 109

nuclear equation of state neutron matter | symmetric matter

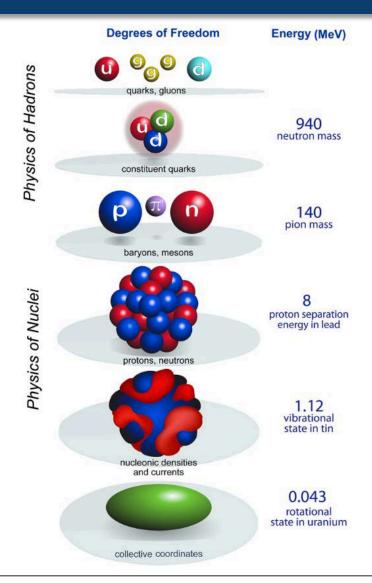
many-body perturbation theory computationally efficient many-body uncertainty estimates

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Microscopic nuclear forces

e.g., Machleidt, Entem, Phys. Rep. 503, 1



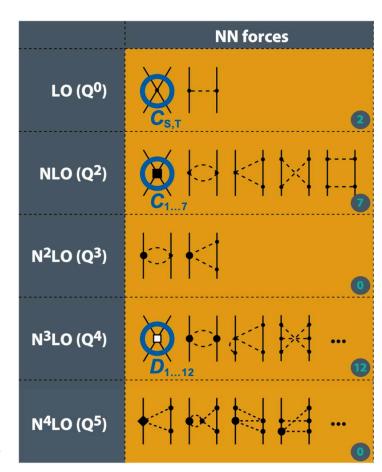
Chiral EFT: modern approach to deriving *microscopic* nuclear forces consistent with the symmetries of low-energy QCD

use relevant instead of the fundamental degrees of freedom:
 e.g., nucleons and pions



Microscopic nuclear forces

e.g., Machleidt, Entem, Phys. Rep. 503, 1



Chiral EFT: modern approach to deriving *microscopic* nuclear forces consistent with the symmetries of low-energy QCD

- use relevant instead of the fundamental degrees of freedom:
 e.g., nucleons and pions
- pion exchanges and short-range contact interactions (∝ LECs)
- systematic expansion enables improvable uncertainty estimates

$$Q = \max\left(\frac{p}{\Lambda_b}, \frac{m_\pi}{\Lambda_b}\right) \ge \frac{1}{3}$$

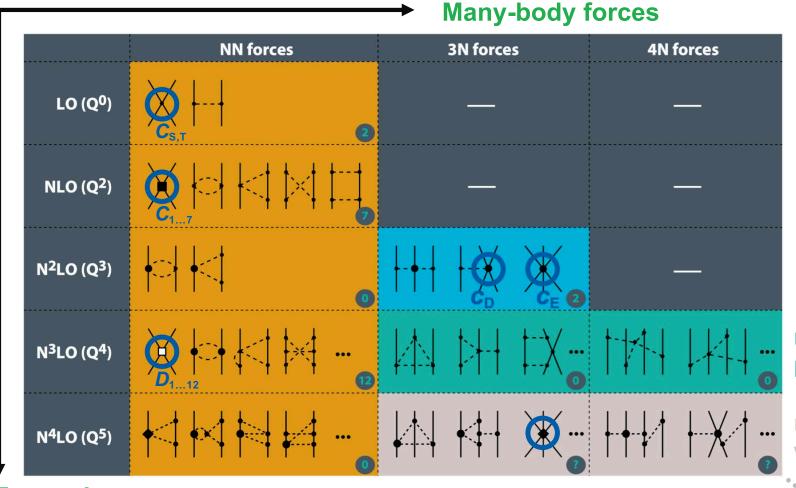
Expansion

Weinberg, van Kolck, Kaplan, Savage, Wise, Epelbaum, Kaiser, Krebs, Machleidt, Meißner, ...



Hierarchy of nuclear forces in chiral EFT

e.g., Machleidt, Entem, Phys. Rep. 503, 1





S. Weinberg

no unknown parameters

not (completely) worked out

Expansion

Weinberg, van Kolck, Kaplan, Savage, Wise, Epelbaum, Kaiser, Krebs, Machleidt, Meißner, ...



Many chiral potentials available!

Hoppe, CD, Furnstahl et al., PRC 96, 054002

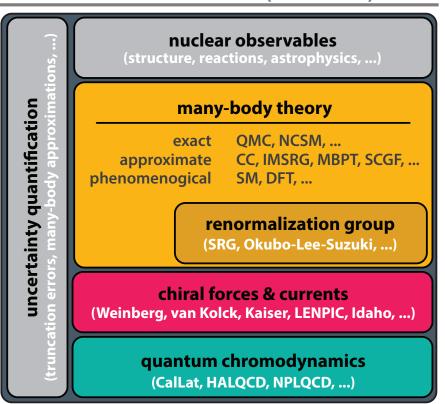
Semilocal momentum-space regularized chiral two-nucleon potentials up to fifth High-quality two-nucleon potentials up to fifth order of the chiral expansion P. Reinert,¹,* H. Krebs,¹,† D. R. Entem,^{1,*} R. Machleidt,^{2,†} and Y. Nosyk² 008 Salamanca, Spain Uncertainty Analysis and Order-by-Order Optimization of Chiral Nuclear Interactions 83844, USA B. D. Carlsson,^{1,*} A. Ekström,^{2,3,†} C. Forssén,^{1,2,3,‡} D. Fahlin Strömberg,¹ G. R. Jansen,^{3,4} O. Lilia,¹ M. Lindby,¹ B. A. Mattsson,¹ and K. A. Wendt^{2,3} Minimally nonlocal nucleon-nucleon potentials with chiral two-pion exchange Depart ²Departmen including A resonances M. Piarulli, ¹ L. Girlanda, ^{2,3} R. Schiavilla, ^{1,4} R. Navarro Pérez, ⁵ J. E. Amaro, ⁵ and E. Ruiz Arriola ⁵ ginia 23529, USA Δ isobars and nuclear saturation I-73100 Lecce, Italy A. Ekström, G. Hagen, T. D. Morris, T. Papenbrock, and P. D. Schwartz^{2,3} 23606. USA d de Granada. ¹Department of Physics, C Three-nucleon force in chiral EFT with explicit $\Delta(1232)$ degrees of freedom: ²Physics Division, Oak I Longest-range contributions at fourth order ³Department of Physics and A H. Krebs,¹,* A. M. Gasparyan,^{1,2},† and E. Epelbaum¹,‡ Local chiral effective field theory interactions and quantum Monte Carlo applications A. Gezerlis, ^{1,*} I. Tews, ^{2,3,†} E. Epelbaum, ^{4,‡} M. Freunek, ⁴ S. Gandolfi, ⁵ K. Hebeler, ^{2,3} A. Nogga, ⁶ and A. Schwenk, ^{2,3,§} ¹Department of Physics, University of Guelph, Guelph, Ontario, Canada NIG 2W1 ²Institut für Kernphysik, Technische Universität Darmstadt, 64289 Darmstadt, Germany ³ExtreMe Matter Institute EMMI, GSI Helmholtzzentrum für Schwerionenforschung GmbH, 64291 Darmstadt, Germany

e.g., Carlsson, Ekström, Entem, Epelbaum, Forssén, Gezerlis, Krebs, Machleidt, Piarulli, Reinert, Tews



Outline

Ab initio workflow (idealized)



CD & Bogner, Few Body Syst. 62, 109

many-body perturbation theory

computationally efficient many-body uncertainty estimates

chiral effective field theory

systematic expansion of nuclear forces truncation error estimates

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Efficient Monte Carlo framework

CD, Hebeler, Schwenk, PRL **122**, 042501



efficient evaluation of MBPT diagrams with NN, 3N, and 4N forces (single-particle basis)

- implementing diagrams has become straightforward (incl. particle-hole terms)
- multi-dimensional momentum integrals: (improved) VEGAS algorithm
- acceleration: openMP, MPI, and CUDA
- controlled computation of arbitrary interaction and many-body diagrams

improved sampling: Brady, Wen, and Holt, PRL 127, 062701

EOS up to high orders



analytic form of diagrams/forces



High-order MBPT

Stevenson, Int. J. Mod. Phys. C 14, 1135

The number of diagrams increases rapidly!

1, 3, 39, 840, 27 300, 1232 280, ...

$$n = 2$$
 3 4 5 6 7

Integer sequence A064732:

Number of labeled Hugenholtz diagrams with *n* nodes.



ADG: Automated generation and evaluation of many-body diagrams I. Bogoliubov many-body perturbation theory

Pierre Arthuis, Thomas Duguet, Alexander Tichai, Raphaël-David Lasseri, Jean-Paul Ebran

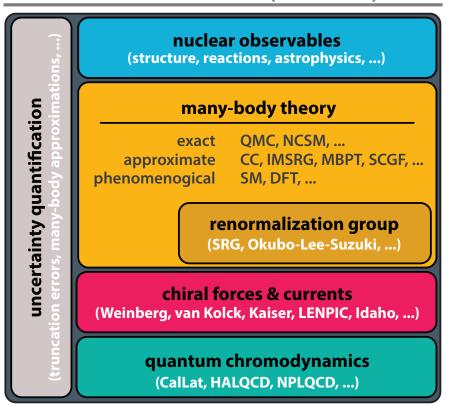
Comput. Phys. **240**, 202

fully automated approach to MBPT



Outline





CD & Bogner, Few Body Syst. 62, 109

nuclear equation of state neutron matter | symmetric matter

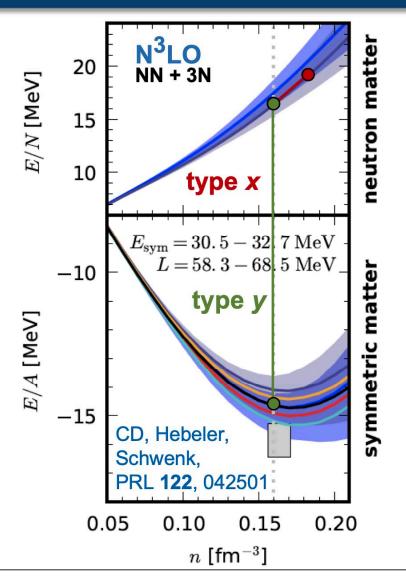
many-body perturbation theory computationally efficient many-body uncertainty estimates

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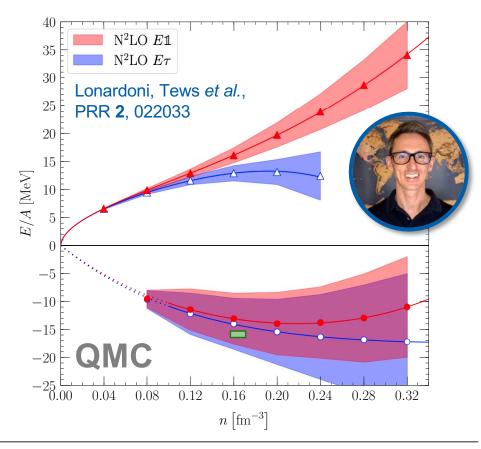


Nuclear matter calculations

e.g., Hebeler, Holt et al., ARNP 65, 457



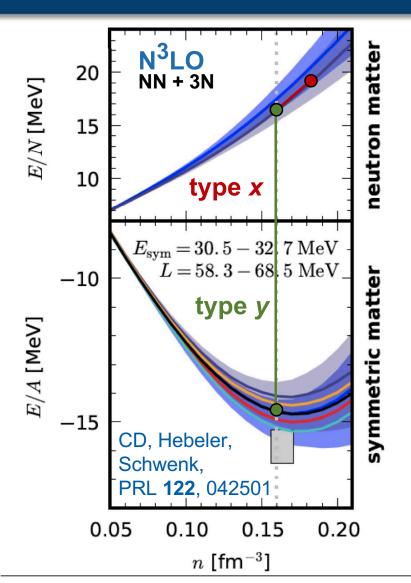
great progress in predicting the **EOS** of infinite matter and the structure of **neutron stars** at densities $\lesssim n_0$





Nuclear matter calculations

e.g., Hebeler, Holt et al., ARNP 65, 457



great progress in predicting the **EOS** of infinite matter and the structure of **neutron stars** at densities $\lesssim n_0$

Hebeler, Lattimer *et al.*, APJ **773**, 11 Carbone, Rios *et al.*, PRC **88**, 044302 Hagen, Papenbrock *et al.* PRC **89**, 014319

needed: statistically robust comparisons
between nuclear theory and recent
observational constraints

Lonardoni, Tews et al., PRR 2, 022033(R) Piarulli, Bombaci et al., PRC 101, 045801

But: existing predictions **only** provided **rough estimates** for the with-density-

growing EFT truncation error, and did

not account for correlations

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buqeye.github.io

New framework for UQ of EFT calculations



















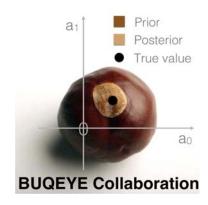
CD, Furnstahl, Melendez, and Phillips

How well do we know the neutron-matter equation of state at the densities inside neutron stars? A Bayesian approach with correlated uncertainties, PRL **125**, 202702

CD, Melendez, Furnstahl, and Phillips

Effective Field Theory Convergence Pattern of Infinite Nuclear Matter, PRC 102, 054315

See also: Melendez et al., PRC **100**, 044001 Wesolowski et al., JPG **43**, 074001



Bayesian
Uncertainty
Quantification:
Errors for
Your
EFT

UQ framework available at https://buqeye.github.io

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New framework for UQ of EFT calculations

bugeye.github.io



















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ation:

CD, Furnstahl, Melendez, and Phillips

How equa stars

unce

Correlated EFT truncation errors are important!

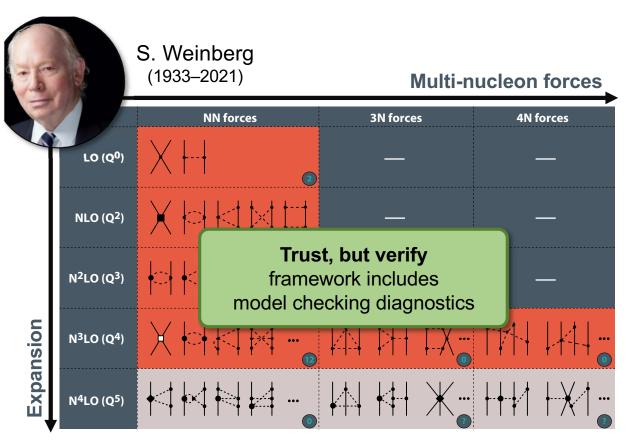
statistically robust uncertainty estimates for key quantities of neutron stars

CD.

Effective Field Theory Convergence Pattern of Infinite Nuclear Matter, PRC 102, 054315

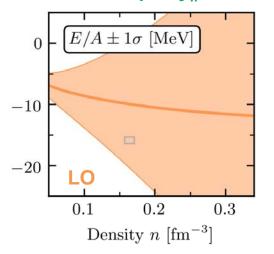
> See also: Melendez et al., PRC 100, 044001 Wesolowski et al., JPG 43, 074001

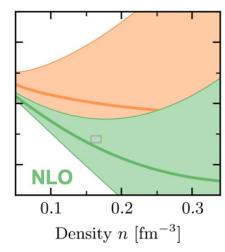
UQ framework available at https://buqeye.github.io

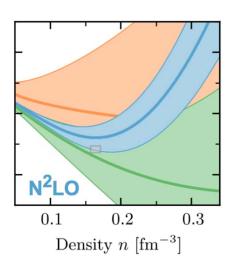


Weinberg, van Kolck, Kaplan, Savage, Wise, Epelbaum, Kaiser, Krebs, Machleidt, Meißner, ...

For example: $y_k = E/A$ in SNM at chiral order k







predict observable y_k order by order in EFT

$$y_k = y_{ ext{ref}} \sum_{n=0}^k c_n Q^n$$

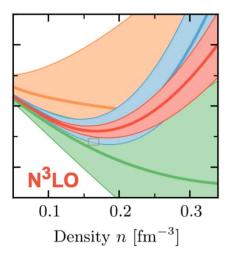
c_n are not the EFT's LEC

treat all c_n as independent draws from a Gaussian Process

learn GP's hyperparameters & infer EFT truncation error

$$\delta y_k = y_{ ext{ref}} \sum_{n=k+1}^{\infty} c_n Q^n$$

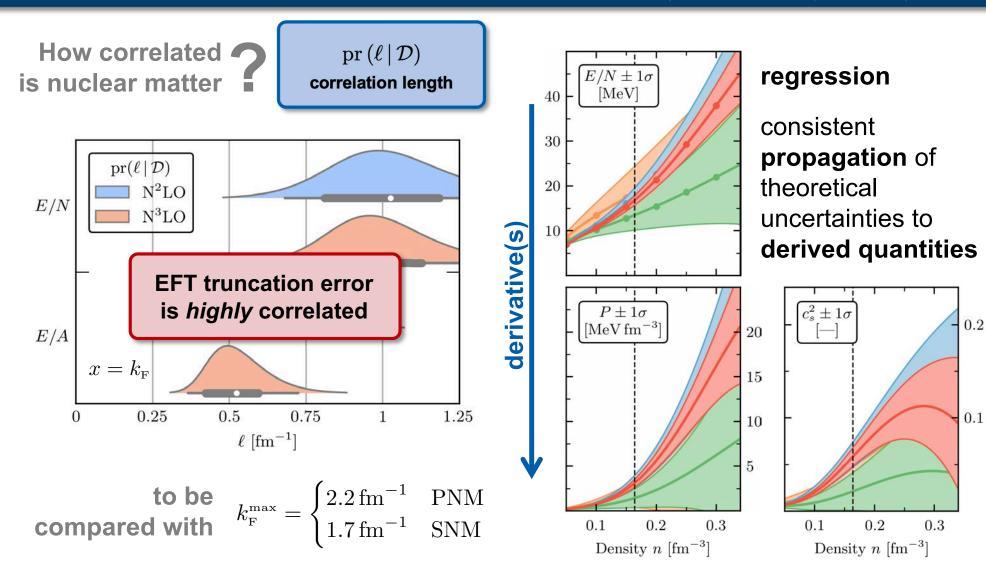
geometric sum



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Propagating type-*x* uncertainties

CD, Melendez et al., PRC 102, 054315





Bayesian inference

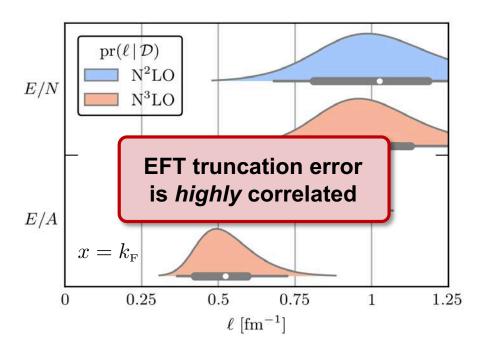
CD, Melendez et al., PRC 102, 054315

How correlated ?

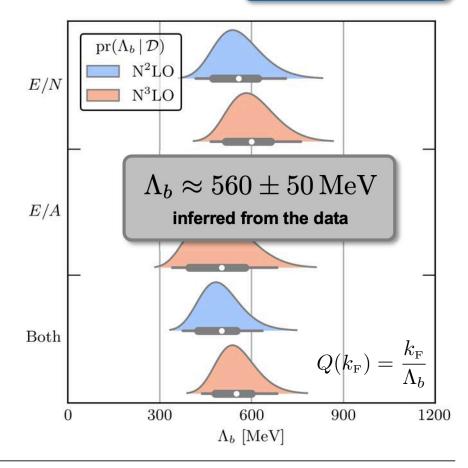
 $\operatorname{pr}\left(\ell\,|\,\mathcal{D}
ight)$ correlation length

Where does the EFT break down

 $\operatorname{pr}\left(\Lambda_{b} \mid \mathcal{D}
ight)$ breakdown scale



$$egin{aligned} extbf{to be} \ extbf{compared with} \end{aligned} k_{ ext{F}}^{ ext{max}} = egin{cases} 2.2 \, ext{fm}^{-1} & ext{PNM} \ 1.7 \, ext{fm}^{-1} & ext{SNM} \end{cases}$$





Bayesian inference

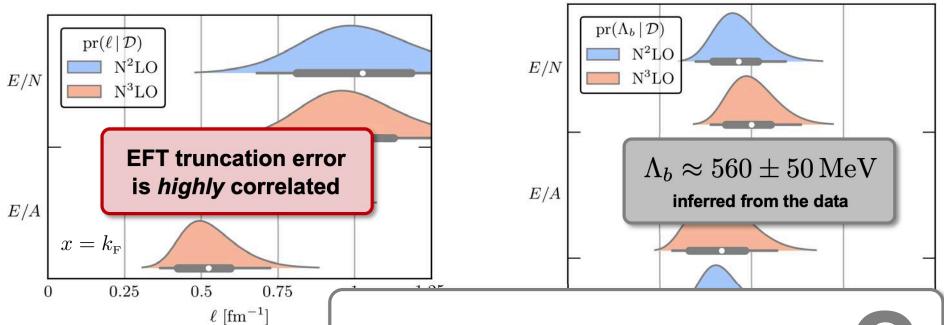
CD, Melendez et al., PRC 102, 054315

How correlated 7 is nuclear matter

 $\operatorname{pr}\left(\ell\,|\,\mathcal{D}
ight)$ correlation length

Where does the EFT break down

 $\operatorname{pr}\left(\Lambda_{b} \,|\, \mathcal{D}\right)$ breakdown scale



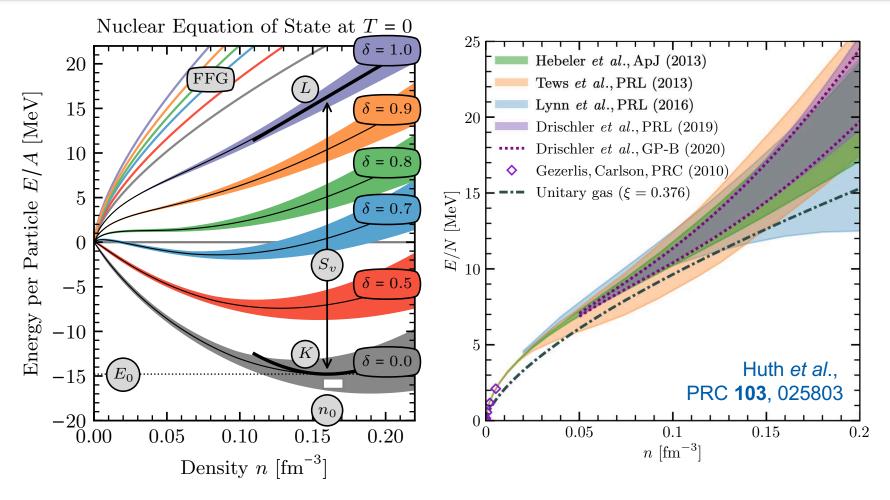
 ${f compared\ with} \quad k_{
m F}^{
m max} =$

At which density scale does nuclear effective field theory break down

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Parameters of the low-density EOS

CD, Holt, and Wellenhofer, ARNPS. 71, 403

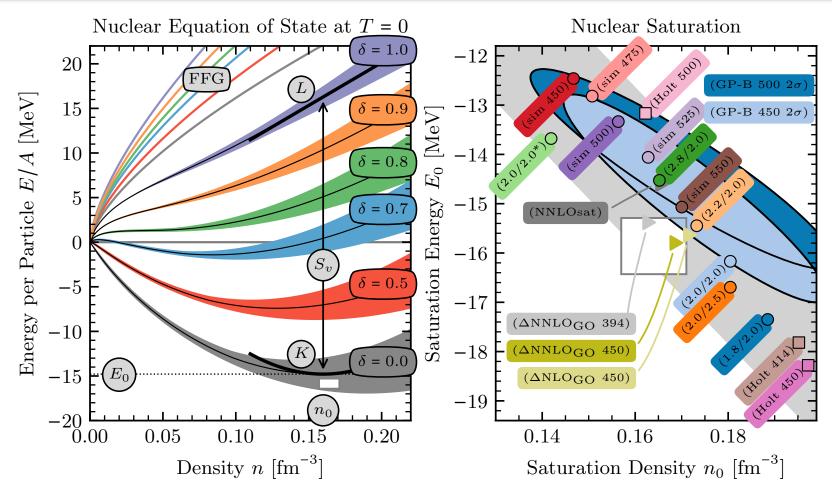


FFG: free Fermi gas; $\delta = (n_n - n_p)/n$: isospin asymmetry



Parameters of the low-density EOS

CD, Holt, and Wellenhofer, ARNPS. 71, 403



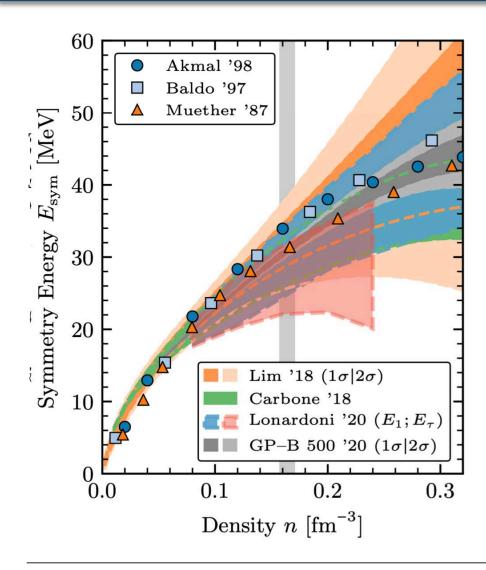
FFG: free Fermi gas; $\delta = (n_n - n_p)/n$: isospin asymmetry

Annotations: (λ / Λ_{3N}) in fm⁻¹ or (Λ) in MeV

for nuclear saturation, see also Atkinson et al., PRC 102, 044333; Dewulf et al, PRL 90, 152501



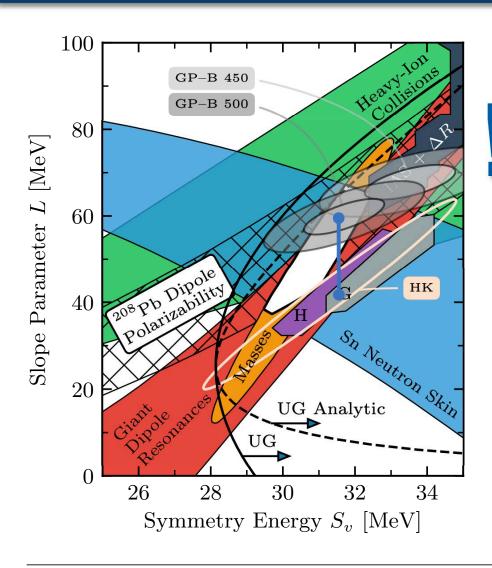
Confronting chiral EFT with empirical constraints CD, Furnstahl et al., PRL 125, 202702



$$S_2(n) \approx \frac{E}{N}(n) - \frac{E}{A}(n)$$



Confronting chiral EFT with empirical constraints CD, Furnstahl et al., PRL 125, 202702



$$S_2(n) \equiv S_v + rac{L}{3} \left(rac{n - n_0}{n_0}
ight) + \dots$$

Excellent agreement with experiment Lattimer and Lim, APJ 771, 51

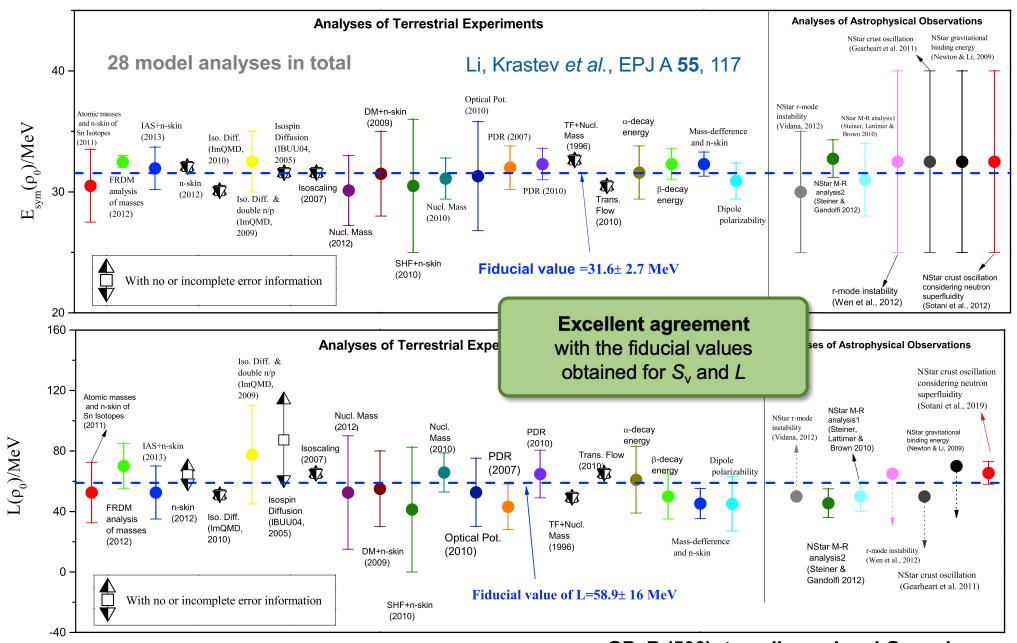
$$\operatorname{pr}(S_v, L \mid \mathcal{D}) = \int dn_0 \operatorname{pr}(S_2, L \mid n_0, \mathcal{D}) \operatorname{pr}(n_0 \mid \mathcal{D})$$
$$\operatorname{pr}(n_0 \mid \mathcal{D}) \approx 0.17 \pm 0.01 \operatorname{fm}^{-3}$$

 2σ ellipse (light yellow) is completely within the *conjectured* unitary gas limit

predicted range in S_v agrees with other theoretical constraints; but ~15 MeV stronger density-dependence of $S_2(n_0)$

GP-B (500): two-dimensional Gaussian

$$\begin{bmatrix} \mu_{S_v} \\ \mu_L \end{bmatrix} = \begin{bmatrix} 31.7 \\ 59.8 \end{bmatrix} \qquad \Sigma = \begin{bmatrix} 1.11^2 & 3.27 \\ 3.27 & 4.12^2 \end{bmatrix}$$



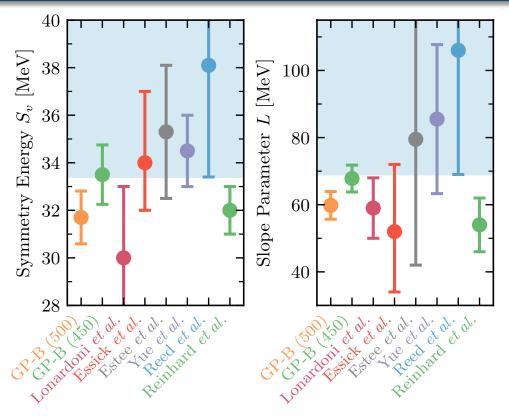
Compilation of recent terrestrial and astrophysical constraints on S_v and L

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PREX-II vs theory and observation

see also Yue et al., arXiv:2102.05267



Take away from PREX-II-informed results:

- uncertainties are still large
- allows for stiffer EOS at ~n₀, but within the large uncertainties consistent with chiral EFT
- tension between A_{PV} and α_{D}

Parity violating elastic e scattering

 $R_{\rm skin} \left(^{208} {\rm Pb}\right) = 0.283 \pm 0.071 \, {\rm fm}$ PREX collaboration, PRL **126**, 172502

Exploiting strong correlations (EDFs)

$$S_v = 38.1 \pm 4.7 \, {
m MeV}$$
 $L = 105.9 \pm 36.9 \, {
m MeV}$ Reed *et al.*, PRL **126**, 172503

Astron. data + chiral EFT only (incl. GP-B)

$$R(^{208}\text{Pb}) = 0.18^{+0.04}_{-0.04} \,\text{fm}$$

 $S_v = 34^{+3}_{-2} \,\text{MeV} \quad L = 52^{+20}_{-18} \,\text{MeV}$
Essick et al., arXiv:2102.10074

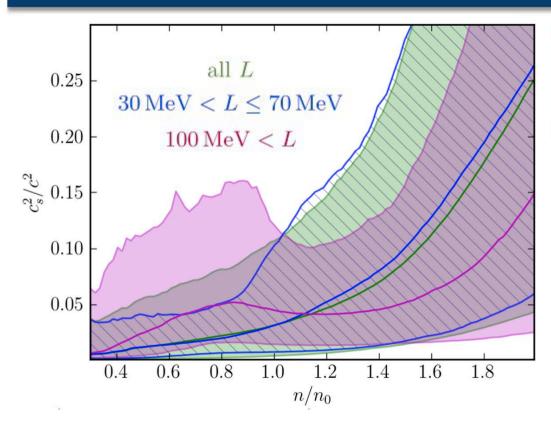
Different EDFs (closest to RCNP & PREX)

$$R\left(^{208}\mathrm{Pb}\right)=0.19\pm0.02\,\mathrm{fm}$$
 $S_v=32\pm1\,\mathrm{MeV}$ $L=54\pm8\,\mathrm{MeV}$ Reinhard, Roca-Maza *et al.*, arXiv:2105.15050

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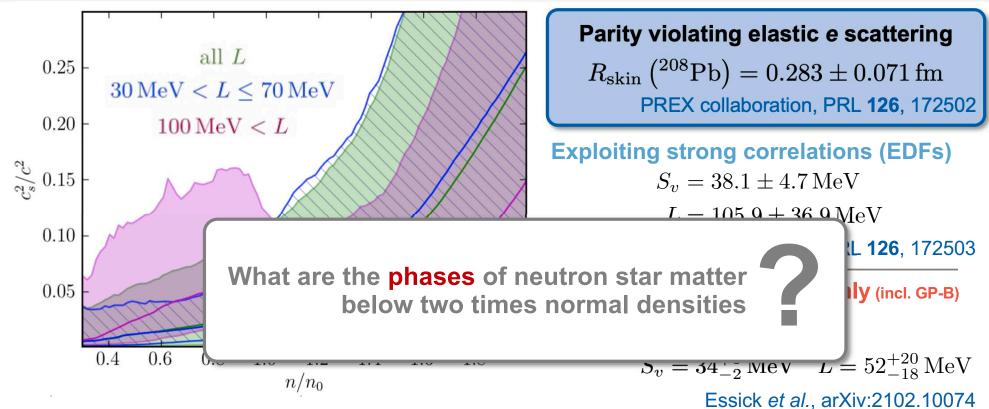
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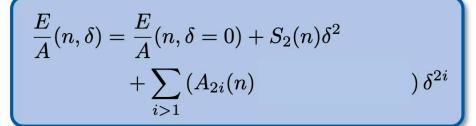
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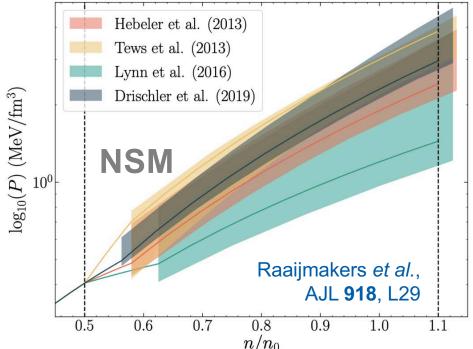
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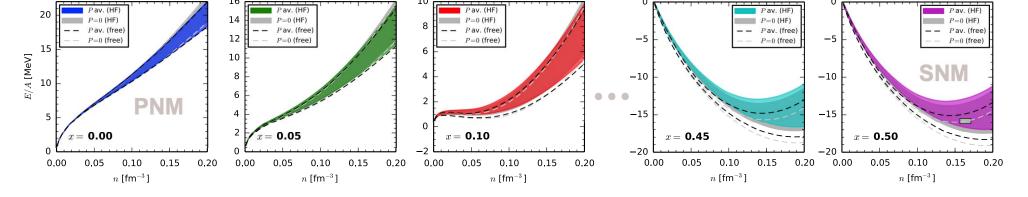
Nonquadratic contributions to the nuclear symmetry energy

Kaiser, PRC **91**, 065201 Wellenhofer, Holt, and Kaiser, PRC **93**, 055802 Somasundaram, CD, Tews *et al.*, PRC **103**, 045803



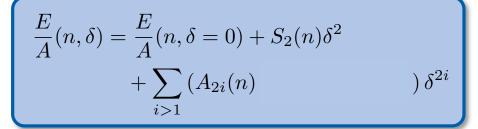


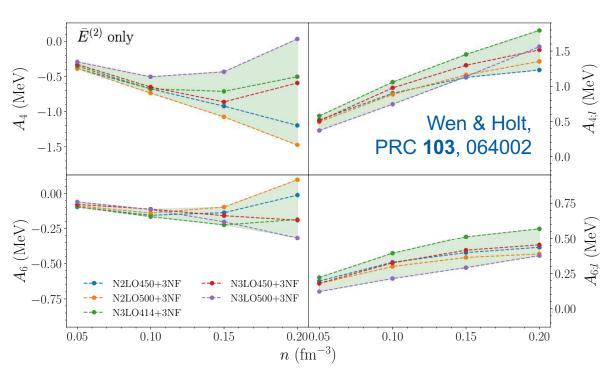
CD, Hebeler, and Schwenk, PRC 93, 054314 (explicit) asymmetric matter calculations



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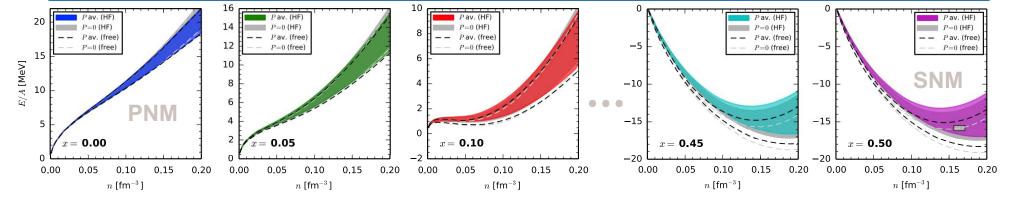


MBPT beyond Hartree-Fock gives rise to (nonanalytic) logarithmic contributions

Precision MBPT calculations can **extract** high-order symmetry energy **coefficients**

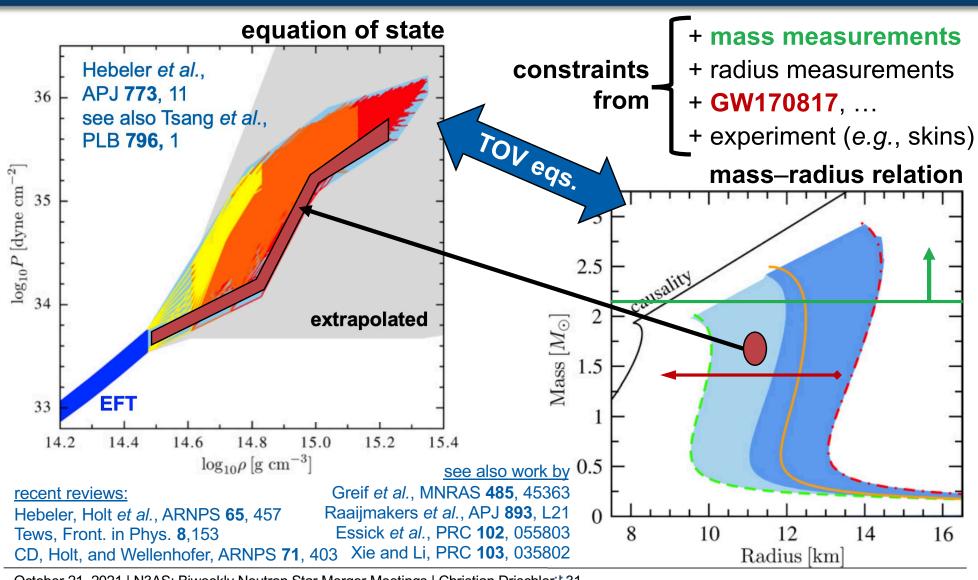
Overall small contribution from nonquadratic terms (but can impact β-equilibrium)

CD, Hebeler, and Schwenk, PRC 93, 054314 (explicit) asymmetric matter calculations





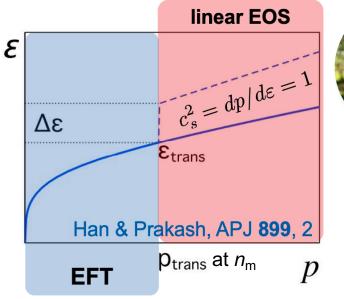
Direct correspondence: M-R relation and EOS



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Limiting neutron star radii

CD, Han, Lattimer et al., PRC 103, 045808



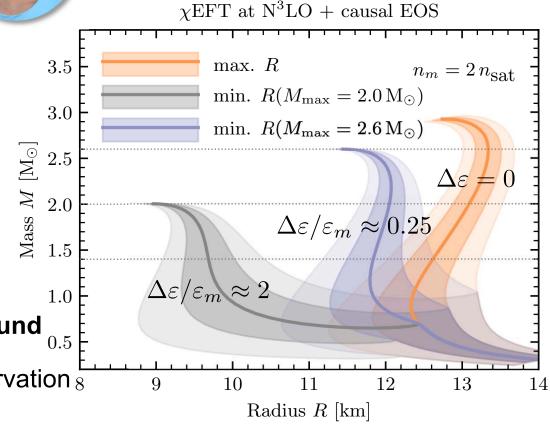
extend EFT EOS at density $n_{\rm m}$ to a linear EoS with finite discontinuity

see also: Alford et al., JPG: NPP 46, 114001

 $\Delta \varepsilon$ anticorrelates with $M_{\rm max}$ and R

continuous match sets upper bound

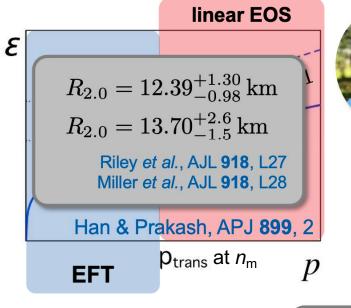
use **lower limit on M_{\text{max}}** from observation $\bar{8}$ to adjust $\Delta \varepsilon$ and constrain R_{min}



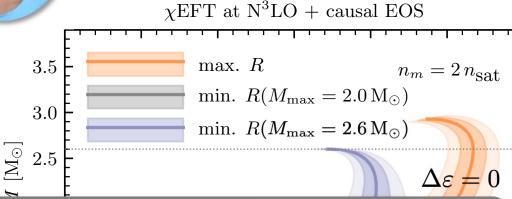


Limiting neutron star radii

CD, Han, Lattimer et al., PRC **103**, 045808



extend EFT EOS at density $n_{\rm m}$ to a linear EoS with finite discontinuity



see also: Alford et al., JPG: NPP 46

 $\Delta \varepsilon$ anticorrelates with $M_{\rm max}$

continuous match sets u

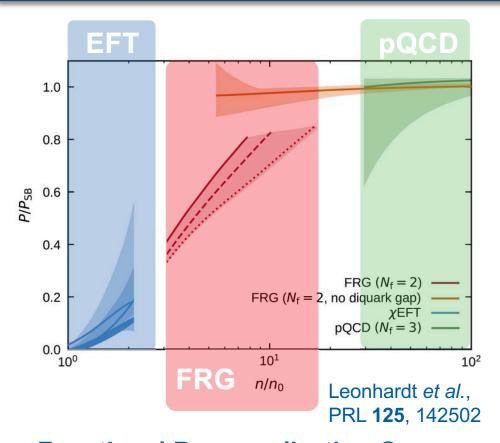
to adjust $\Delta \varepsilon$ and constrain R_{\min}

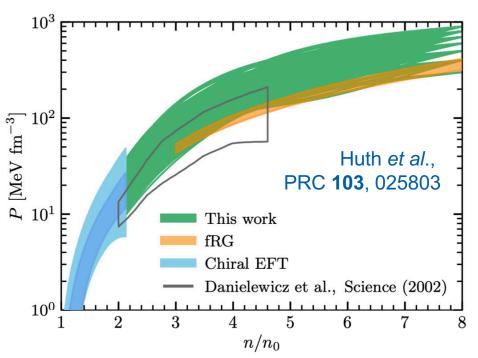
How can neutron star observations help improve nuclear effective field theories

use lower limit on M_{max} from observation 8 10 11 12 13 14 Radius R [km]

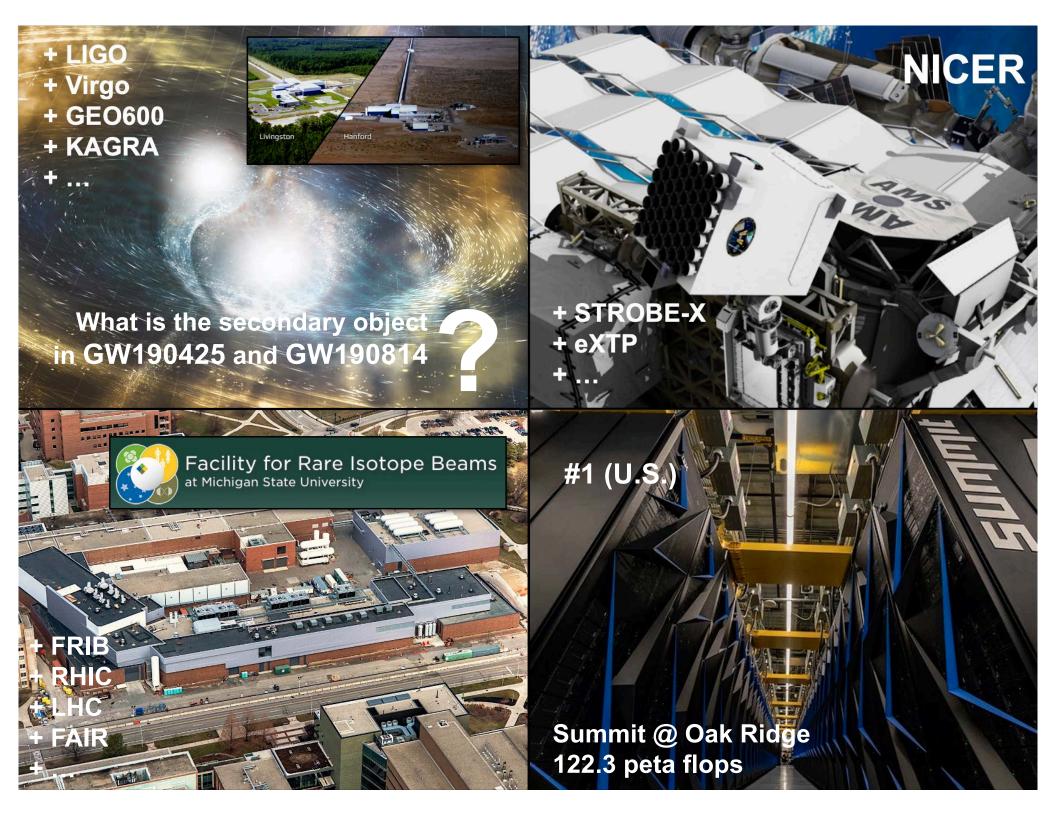


New developments: symmetric nuclear matter



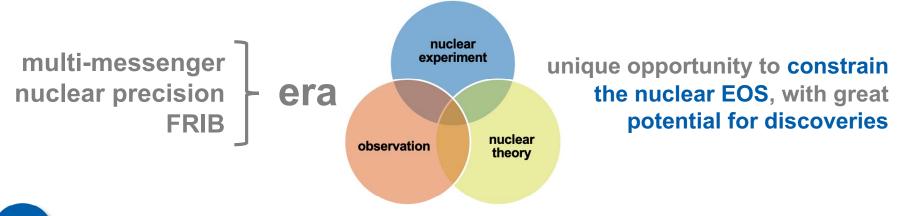


Functional Renormalization Group: complementary constraints at > $3n_0$ (beyond the range of chiral EFT) from the QCD action New insights into the high-density EOS: remarkable consistency between the constraints, which suggests that they can be combined via simple extrapolations





Conclusion

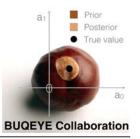


- Microscopic EOS constraints | statistically robust uncertainties
 - excellent agreement of predicted S_v –L correlation with experiment
 - PNM and SNM show a regular EFT convergence pattern with increasing order
 - extracted Λ_b is consistent with NN scattering N²LO coefficient may be an outlier
- full Bayesian UQ: sample over LECs & hyperparameters
 - in future: include consistently uncertainties in the LECs of chiral interactions
 - promising: new potentials up to N²LO by Wesolowski et al., arXiv:2104.04441



thanks to my collaborators:

R. Furnstahl J. Melendez K. McElvain D. Phillips S. Han J. Lattimer M. Prakash S. Reddy T. Zhao





Key questions for ab initio many-body theory

CD & Holt, PAX-VII Workshop

How can neutron star observations help improve nuclear effective field theories



At which density scale does nuclear effective field theory break down



What are the phases of neutron star matter below two times normal densities

