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[A. Watts]

Christian Drischler

October 21, 2021 | N3AS: Biweekly Neutron Star Merger Meetings

These *biweekly meetings* aim to bring together researchers working on neutron star mergers.

Topics will include:

- computational astrophysics,
- dense matter physics,
- gravitational waves and data analysis,
- neutrino and particle astrophysics.

Meetings will include a 30-40 min talk on a related topic, and 20-30 minutes of discussion...

Today:

- + Chiral EFT + MBPT
- + Bayesian UQ
- + infinite nuclear matter
- + symmetry energy
- + nuclear saturation
- + N³LO NN + 3N forces



see also Jeremy Holt's talk (November 4) Equation of State of Dense Matter at Finite Temperature

+ ...

Recent review article

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Equation of State C. Drischler,^{1,2,3} J. W. Holt,⁴ and C. Wellenhofer,^{5,6}

¹Department of Physics, University of California, Berkeley, California 94720, USA

CD, Holt, and Wellenhofer, ARNPS 71, 403

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Chiral Effective Field

High-Density Nuclear

Theory and the

invited contribution to Annu. Rev. Nucl. Part. Sci. 71, 403 see also: Lattimer, Annu. Rev. Nucl. Part. Sci. 71, 433

Keywords

chiral effective field theory, nuclear matter, neutron stars, many-body perturbation theory, bayesian uncertainty quantification

Abstract

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Annu. Rev. Nucl. Part. Sci. in press.

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Ab initio calculations | outline



nuclear equation of state neutron matter | symmetric matter

many-body perturbation theory

computationally efficient many-body uncertainty estimates

chiral effective field theory

systematic expansion of nuclear forces truncation error estimates

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Microscopic nuclear forces

e.g., Machleidt, Entem, Phys. Rep. 503, 1

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Chiral EFT: modern approach to deriving *microscopic* nuclear forces consistent with the symmetries of low-energy QCD

 use relevant instead of the fundamental degrees of freedom: *e.g.*, **nucleons** and **pions**

Microscopic nuclear forces

e.g., Machleidt, Entem, Phys. Rep. 503, 1

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Expansion

Weinberg, van Kolck, Kaplan, Savage, Wise, Epelbaum, Kaiser, Krebs, Machleidt, Meißner, ...

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Chiral EFT: modern approach to deriving *microscopic* nuclear forces consistent with the symmetries of low-energy QCD

- use relevant instead of the fundamental degrees of freedom: *e.g.*, **nucleons** and **pions**
- pion exchanges and short-range contact interactions (∝ LECs)
- systematic expansion enables improvable uncertainty estimates

$$Q = \max\left(\frac{p}{\Lambda_b}, \frac{m_{\pi}}{\Lambda_b}\right) \ge \frac{1}{3}$$

Hierarchy of nuclear forces in chiral EFT

e.g., Machleidt, Entem, Phys. Rep. 503, 1

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Many-body forces

Expansion

Weinberg, van Kolck, Kaplan, Savage, Wise, Epelbaum, Kaiser, Krebs, Machleidt, Meißner, ...

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Many chiral potentials available!

Hoppe, CD, Furnstahl et al., PRC 96, 054002



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Outline



CD & Bogner, Few Body Syst. 62, 109

many-body perturbation theory

computationally efficient many-body uncertainty estimates

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Efficient Monte Carlo framework

CD, Hebeler, Schwenk, PRL 122, 042501



efficient evaluation of MBPT diagrams with NN, 3N, and 4N forces (single-particle basis)

- implementing diagrams has become straightforward (incl. particle-hole terms)
- multi-dimensional momentum integrals: (improved) VEGAS algorithm
- acceleration: openMP, MPI, and CUDA
- controlled computation of arbitrary interaction and many-body diagrams

improved sampling: Brady, Wen, and Holt, PRL **127**, 062701



High-order MBPT

Stevenson, Int. J. Mod. Phys. C 14, 1135

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The number of diagrams increases rapidly!



Integer sequence A064732:

Number of labeled Hugenholtz diagrams with *n* nodes.



ADG: Automated generation and evaluation of many-body diagrams I. Bogoliubov many-body perturbation theory

Pierre Arthuis, Thomas Duguet, Alexander Tichai, Raphaël-David Lasseri, Jean-Paul Ebran Comput. Phys. 240, 202

fully automated approach to MBPT

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Outline



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nuclear equation of state neutron matter | symmetric matter

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Nuclear matter calculations

e.g., Hebeler, Holt et al., ARNP 65, 457



great progress in predicting the **EOS** of infinite matter and the structure of **neutron stars** at densities $\leq n_0$



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Nuclear matter calculations

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great progress in predicting the **EOS** of infinite matter and the structure of **neutron stars** at densities $\leq n_0$

Hebeler, Lattimer *et al.*, APJ **773**, 11 Carbone, Rios *et al.*, PRC **88**, 044302 Hagen, Papenbrock *et al.* PRC **89**, 014319

needed: statistically robust comparisons between nuclear theory and recent observational constraints

> Lonardoni, Tews *et al.*, PRR **2**, 022033(R) Piarulli, Bombaci *et al.*, PRC **101**, 045801

But: existing predictions only provided rough estimates for the with-densitygrowing EFT truncation error, and did *not* account for correlations

New framework for UQ of EFT calculations

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buqeye.github.io



CD, Furnstahl, Melendez, and Phillips

How well do we know the neutron-matter equation of state at the densities inside neutron stars? A Bayesian approach with correlated uncertainties, PRL **125**, 202702

CD, Melendez, Furnstahl, and Phillips

Effective Field Theory Convergence Pattern of Infinite Nuclear Matter, PRC **102**, 054315

> See also: Melendez *et al.*, PRC **100**, 044001 Wesolowski *et al.*, JPG **43**, 074001



Bayesian Uncertainty Quantification: Errors for Your EFT

UQ framework available at <u>https://buqeye.github.io</u>

New framework for UQ of EFT calculations

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buqeye.github.io



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Weinberg, van Kolck, Kaplan, Savage, Wise, Epelbaum, Kaiser, Krebs, Machleidt, Meißner, ...

predict observable y_k order by order in EFT

$$y_k = y_{ ext{ref}} \sum_{n=0}^k c_n Q^n$$

c_n are not the EFT's LEC

treat all *c_n* as independent draws from a Gaussian Process

learn GP's hyperparameters & infer EFT truncation error

$$\delta y_k = y_{ ext{ref}} \sum_{n=k+1}^\infty c_n Q^n$$

geometric sum



For example: $y_k = E/A$ in SNM at chiral order k



Propagating type-*x* uncertainties

CD, Melendez *et al.*, PRC **102**, 054315



Bayesian inference

CD, Melendez et al., PRC 102, 054315

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Bayesian inference

CD, Melendez et al., PRC 102, 054315

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Parameters of the low-density EOS

CD, Holt, and Wellenhofer, ARNPS. 71, 403



FFG: free Fermi gas; $\delta = (n_n - n_p)/n$: isospin asymmetry

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Parameters of the low-density EOS

CD, Holt, and Wellenhofer, ARNPS. 71, 403





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Confronting chiral EFT with empirical constraints CD, Furnstahl et al., PRL 125, 202702



$$S_2(n) \approx \frac{E}{N}(n) - \frac{E}{A}(n)$$

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Confronting chiral EFT with empirical constraints CD, Furnstahl et al., PRL 125, 202702



$$S_2(n) \equiv S_v + rac{L}{3} \left(rac{n-n_0}{n_0}
ight) + \dots$$

Excellent agreement with experiment Lattimer and Lim, APJ 771, 51 $pr(S_v, L \mid D) = \int dn_0 pr(S_2, L \mid n_0, D) pr(n_0 \mid D)$ $pr(n_0 \mid D) \approx 0.17 \pm 0.01 \text{ fm}^{-3}$

 2σ ellipse (light yellow) is completely within the *conjectured* unitary gas limit

predicted range in S_v agrees with other theoretical constraints; but ~15 MeV stronger density-dependence of $S_2(n_0)$

GP–B (500): two-dimensional Gaussian

$$\begin{bmatrix} \mu_{S_v} \\ \mu_L \end{bmatrix} = \begin{bmatrix} 31.7 \\ 59.8 \end{bmatrix} \qquad \Sigma = \begin{bmatrix} 1.11^2 & 3.27 \\ 3.27 & 4.12^2 \end{bmatrix}$$



Compilation of recent terrestrial and astrophysical **constraints on S_v and** *L*



 $\Sigma =$

 1.11^{2}

3.27

3.27

31.7

 μ_{S_v}

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PREX–II vs theory and observation

see also Yue et al., arXiv:2102.05267



Take away from PREX-II-informed results:

- uncertainties are still large
- allows for stiffer EOS at ~n₀, but within the large uncertainties consistent with chiral EFT
- tension between A_{PV} and α_D

Parity violating elastic e scattering $R_{\rm skin} \left({}^{208} {\rm Pb} \right) = 0.283 \pm 0.071 \, {\rm fm}$ PREX collaboration, PRL **126**, 172502

Exploiting strong correlations (EDFs)

 $S_v = 38.1 \pm 4.7 \,\mathrm{MeV}$ $L = 105.9 \pm 36.9 \,\mathrm{MeV}$

Reed et al., PRL 126, 172503

Astron. data + chiral EFT only (incl. GP-B)

 $R(^{208}\text{Pb}) = 0.18^{+0.04}_{-0.04} \text{ fm}$ $S_v = 34^{+3}_{-2} \text{ MeV} \quad L = 52^{+20}_{-18} \text{ MeV}$

Essick et al., arXiv:2102.10074

Different EDFs (closest to RCNP & PREX)

 $R(^{208}\text{Pb}) = 0.19 \pm 0.02 \,\text{fm}$

 $S_v = 32 \pm 1 \,\mathrm{MeV}$ $L = 54 \pm 8 \,\mathrm{MeV}$

Reinhard, Roca-Maza et al., arXiv:2105.15050

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PREX–II vs theory and observation

see also Yue et al., arXiv:2102.05267



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Nonquadratic contributions to the nuclear symmetry energy

Kaiser, PRC **91**, 065201 Wellenhofer, Holt, and Kaiser, PRC **93**, 055802 Somasundaram, CD, Tews *et al.*, PRC **103**, 045803

$$\frac{E}{A}(n,\delta) = \frac{E}{A}(n,\delta=0) + S_2(n)\delta^2 + \sum_{i>1} (A_{2i}(n)) \delta^{2i}$$



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MBPT beyond Hartree-Fock gives rise to (nonanalytic) logarithmic contributions

Precision MBPT calculations can **extract** high-order symmetry energy **coefficients**

Overall small contribution from nonquadratic terms (but can impact β-equilibrium)





Direct correspondence: *M*–*R* relation and EOS





Limiting neutron star radii

CD, Han, Lattimer et al., PRC 103, 045808





Limiting neutron star radii

CD, Han, Lattimer et al., PRC 103, 045808



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New developments: symmetric nuclear matter



Functional Renormalization Group: *complementary* constraints at > $3n_0$ (beyond the range of chiral EFT) from the QCD action New insights into the high-density EOS: remarkable consistency between the constraints, which suggests that they can be combined via simple extrapolations

+ LIGO + Virgo + GEO600 + KAGRA



What is the secondary object in GW190425 and GW190814



Summit @ Oak Ridge 122.3 peta flops

+ STROBE-X

eXTP

#1 (U.S

CER

275

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Conclusion





Microscopic EOS constraints | *statistically* robust uncertainties

- excellent agreement of predicted S_v -L correlation with experiment
- PNM and SNM show a regular EFT convergence pattern with increasing order
- extracted Λ_b is consistent with NN scattering N²LO coefficient may be an outlier



full Bayesian UQ: sample over LECs & hyperparameters

- in future: include consistently uncertainties in the LECs of chiral interactions
- promising: new potentials up to N²LO by Wesolowski et al., arXiv:2104.04441



thanks to my collaborators:

R. Furnstahl J. Melendez K. McElvain D. Phillips S. Han J. Lattimer M. Prakash S. Reddy T. Zhao



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Equation of State of Dense (Neutron-rich) Matter at Zero Temperature

Key questions for *ab initio* many-body theory

CD & Holt, PAX-VII Workshop

How can neutron star observations help improve nuclear effective field theories



