New developments on the physics of neutrino fast flavor conversion

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Based on: I. Padilla-Gay, I. Tamborra, G. G. Raffelt
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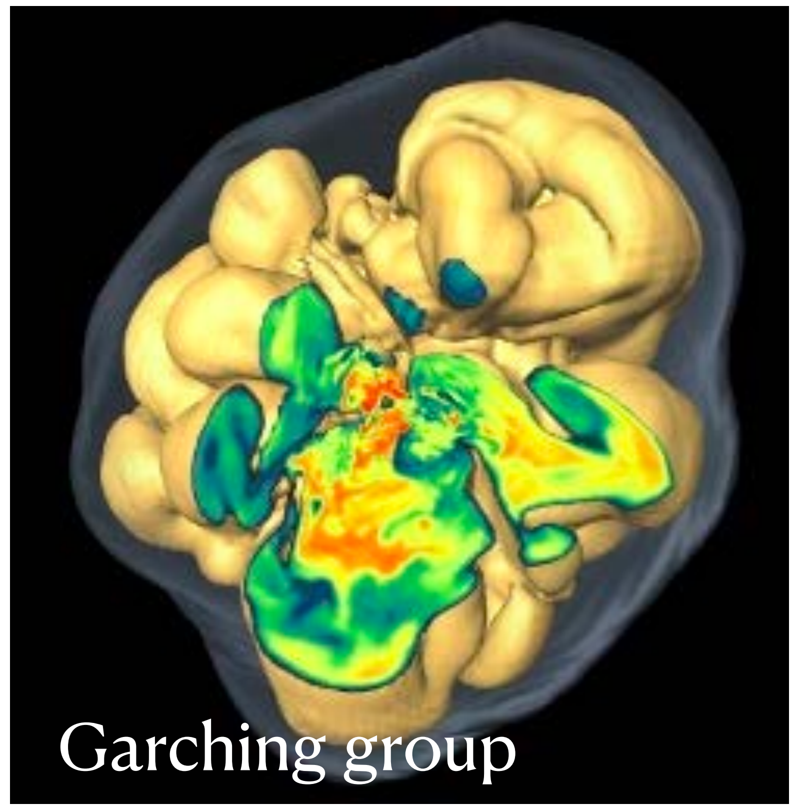
Neutrinos in dense astrophysical environments

Copiously produced ~10^{58} (99% energy)

Flavor conversion rate proportional to neutrino number density

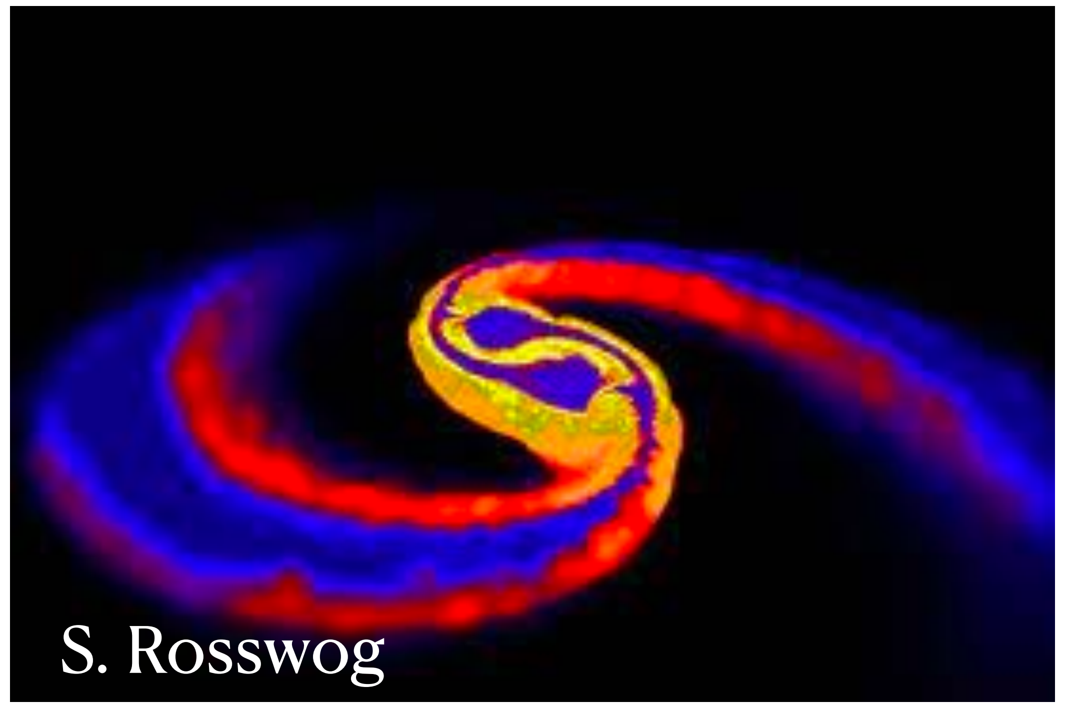
In core-collapse supernovae ν:

- can heat matter outflows in the gain region
- are responsible for the delayed neutrino-driven explosion mechanism



In compact binary mergers ν:

- can affect proton-to-neutron ratio
- modify the nucleosynthesis of elements heavier than iron

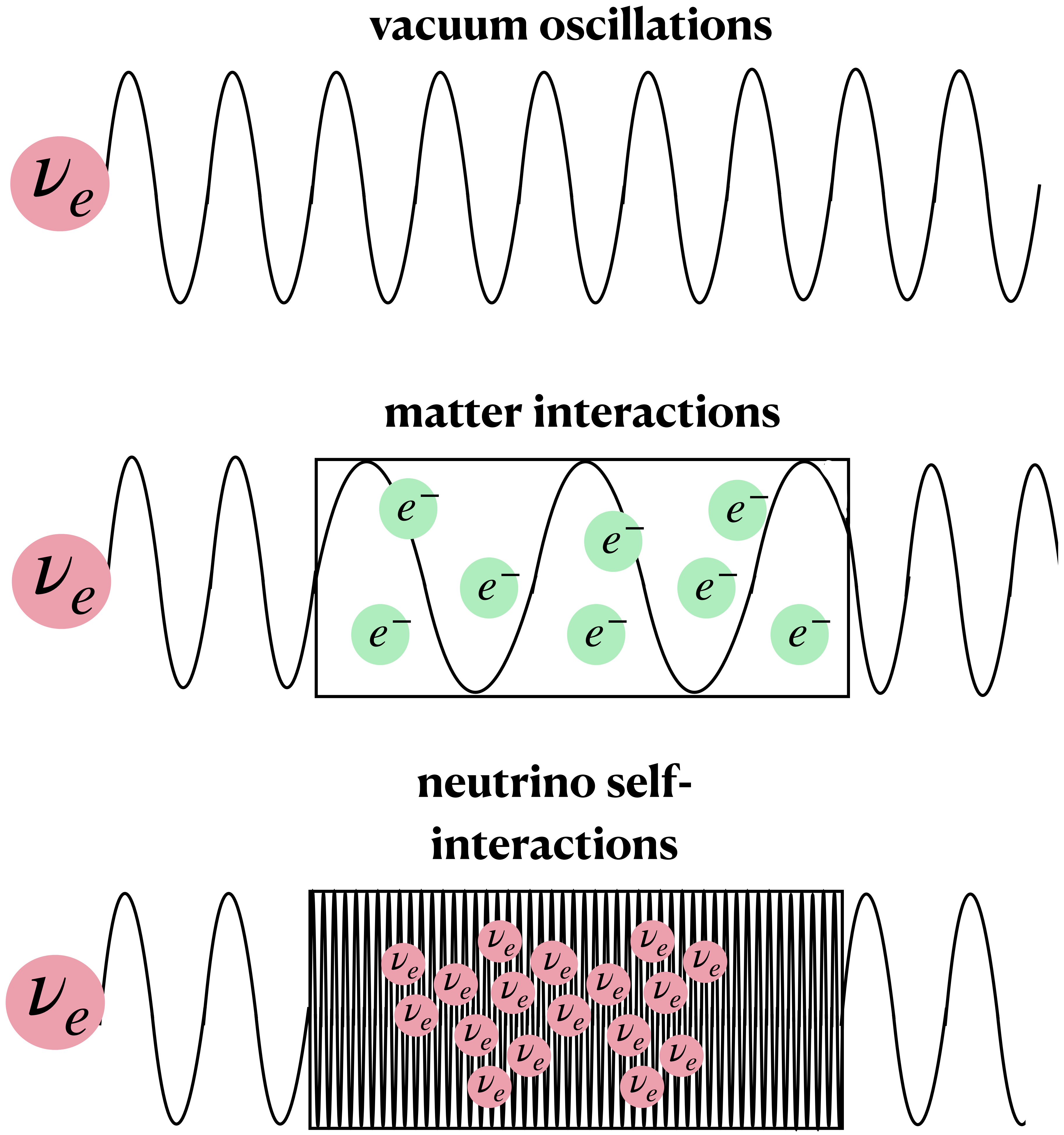


Neutrino oscillations

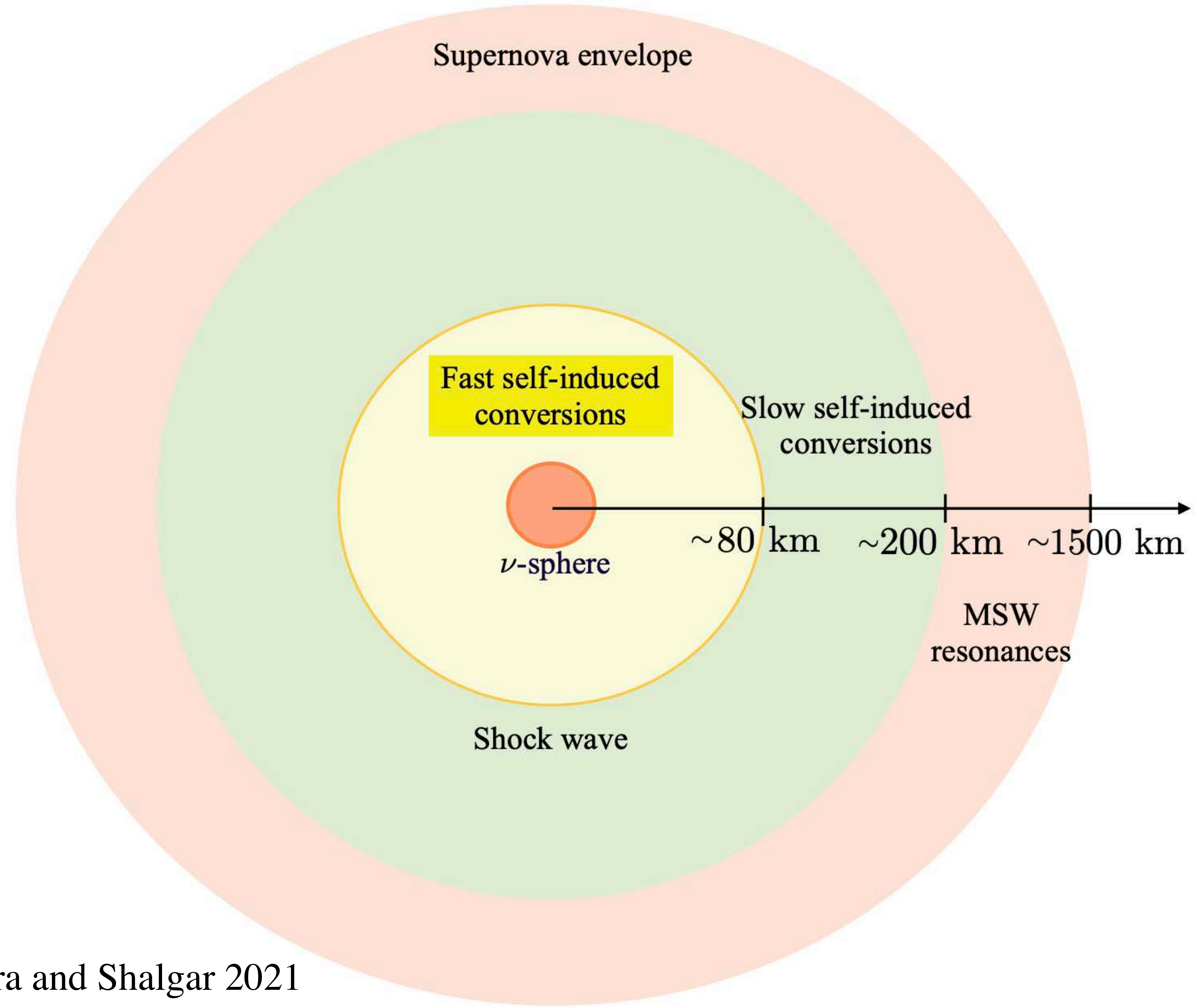
- **Vacuum oscillations -** driven by Δm^2

- **MSW effect -** coherent forward scattering with electrons

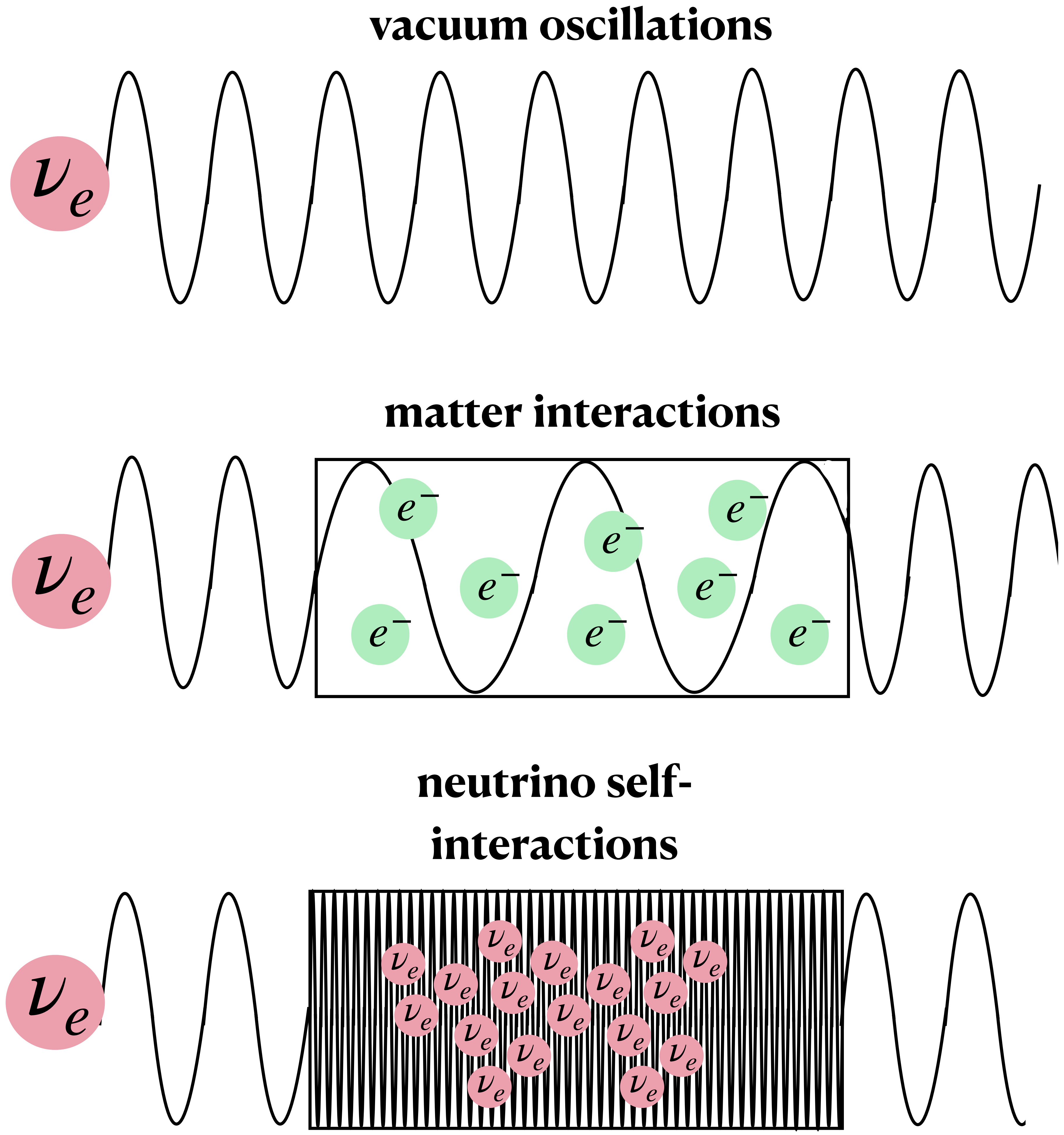
- **$\nu - \nu$ coherent forward scattering -** Neutrinos also constitute a background to other neutrinos → Special case: *Fast pairwise neutrino flavor conversion (FFC)*



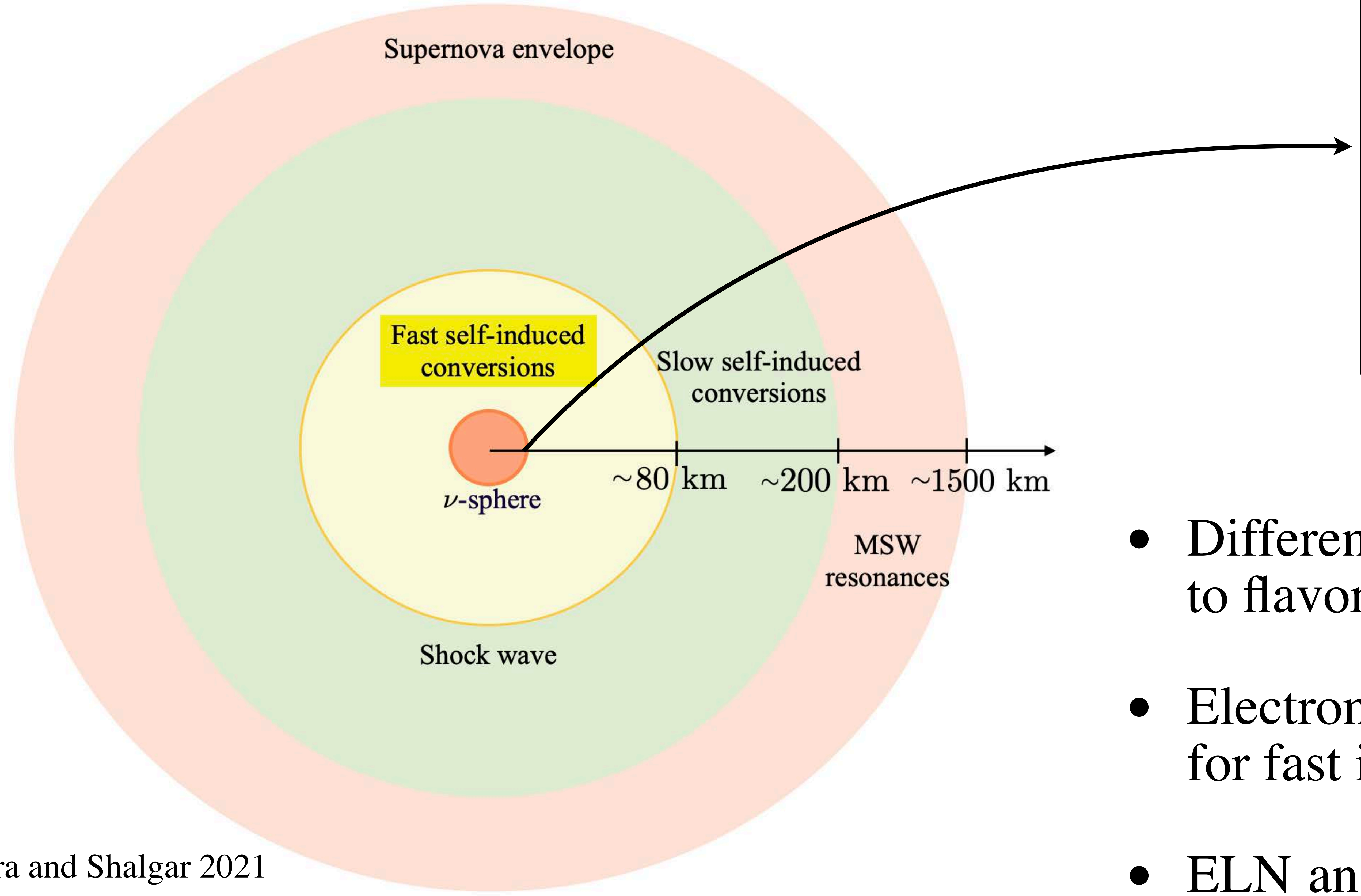
Different regimes for neutrino oscillations



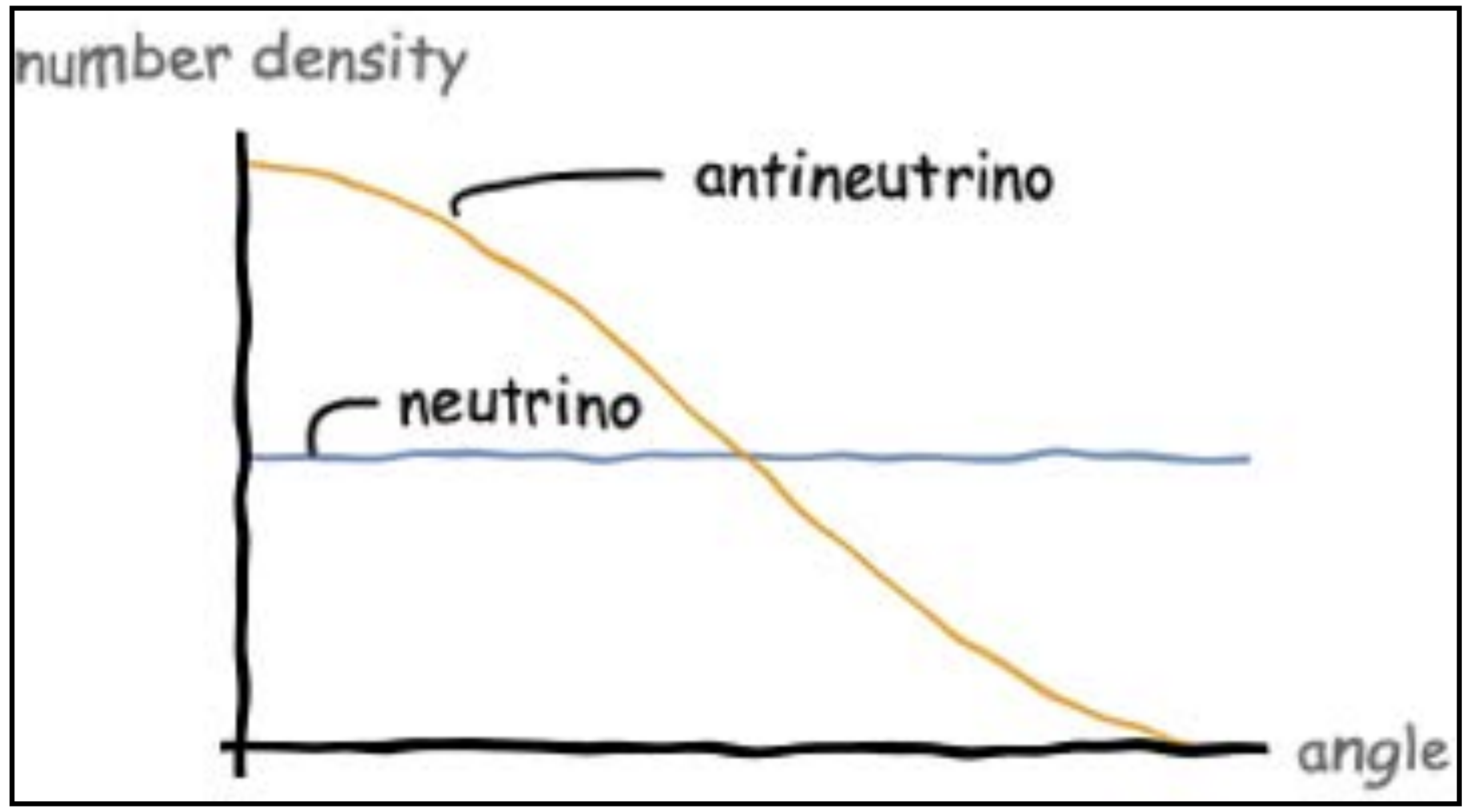
Tamborra and Shalgar 2021



Flavor-dependent angular distributions



Tamborra and Shalgar 2021



- Different collision rates for neutrinos and antineutrinos lead to flavor-dependent angular distributions
- Electron-lepton-number (ELN) crossing is a key ingredient for fast instabilities
- ELN angular distribution is input for the flavor evolution of neutrinos

How does the neutrino flavor evolve?

Neutrino flavor field described by $\varrho(\vec{p}, \vec{r}, t)$ and $\bar{\varrho}(\vec{p}, \vec{r}, t)$

Quantum Kinetic Equations (QKE's)

$$i\,(\partial_t + \vec{v}\cdot\vec{\nabla})\varrho_{\vec{p}} = +\,[\Omega_E, \varrho] + \sqrt{2}\,G_F[\mathrm{H}_{\vec{v}}, \varrho] + i\,\mathrm{C}(\varrho_{\vec{\mathrm{p}}}, \bar{\varrho}_{\vec{\mathrm{p}}})$$

$$i\,(\partial_t + \vec{v}\cdot\vec{\nabla})\bar{\varrho}_{\vec{p}} = -\,[\Omega_E, \bar{\varrho}] + \sqrt{2}\,G_F[\mathrm{H}_{\vec{v}}, \bar{\varrho}] + i\,\bar{\mathrm{C}}(\varrho_{\vec{\mathrm{p}}}, \bar{\varrho}_{\vec{\mathrm{p}}})$$

Neutrino advective term neglected	Vacuum oscillations $\Omega_E = \mathrm{M}^2/2E$ neglected	Neutrino self-interaction	Collisions neglected
✗	✗	✓	✗

How does the neutrino flavor evolve?

Particle number and lepton number density matrices

$$\mathsf{S}_{\vec{p}} = \varrho_{\vec{p}} + \bar{\varrho}_{\vec{p}}$$

$$\mathsf{D}_{\vec{p}} = \varrho_{\vec{p}} - \bar{\varrho}_{\vec{p}}$$

$$\longrightarrow \quad \mathsf{H}_{\vec{v}} = \int \frac{d^3\vec{q}}{(2\pi)^3} \mathsf{D}_{\vec{q}} (1 - \vec{v}_{\vec{q}} \cdot \vec{v})$$

$$i\,\partial_t \mathsf{S}_{\vec{p}} = \sqrt{2}\, G_F [\mathsf{H}_{\vec{v}}, \mathsf{S}_{\vec{p}}]$$

$$i\,\partial_t \mathsf{D}_{\vec{p}} = \sqrt{2}\, G_F [\mathsf{H}_{\vec{v}}, \mathsf{D}_{\vec{p}}]$$

The equations for $\mathsf{D}_{\vec{p}}$ form a closed system

$\mathsf{D}_{\vec{p}}$ determines the instability condition and the FFC dynamics

How does the neutrino flavor evolve?

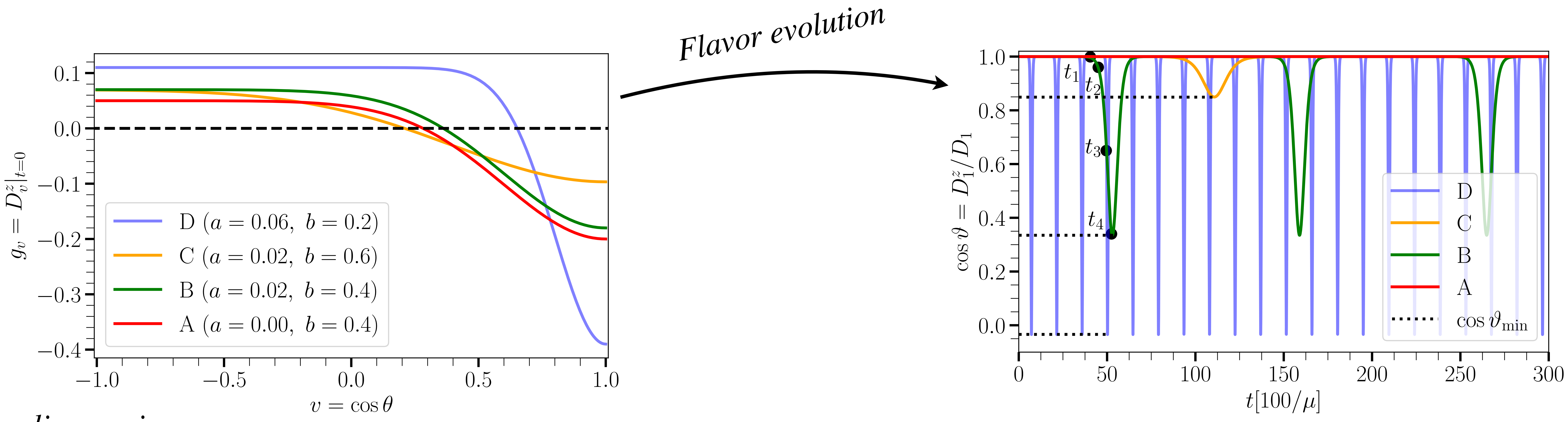
- Two-flavor system with axial symmetry
- For fixed $\mu = \sqrt{2} G_F n_{\nu\bar{\nu}}$, FFC is driven by the velocity distribution of $\mathbf{D}_v$ (ELN spectrum)
- Oscillations are *bipolar*

$$\dot{\mathbf{S}}_v = \mu \mathbf{D}_0 \times \mathbf{S}_v - \mu v \mathbf{D}_1 \times \mathbf{S}_v$$

$$\dot{\mathbf{D}}_v = \mu \mathbf{D}_0 \times \mathbf{D}_v - \mu v \mathbf{D}_1 \times \mathbf{D}_v$$

$\mathbf{D}_0$ conserved: no net flavor conversions

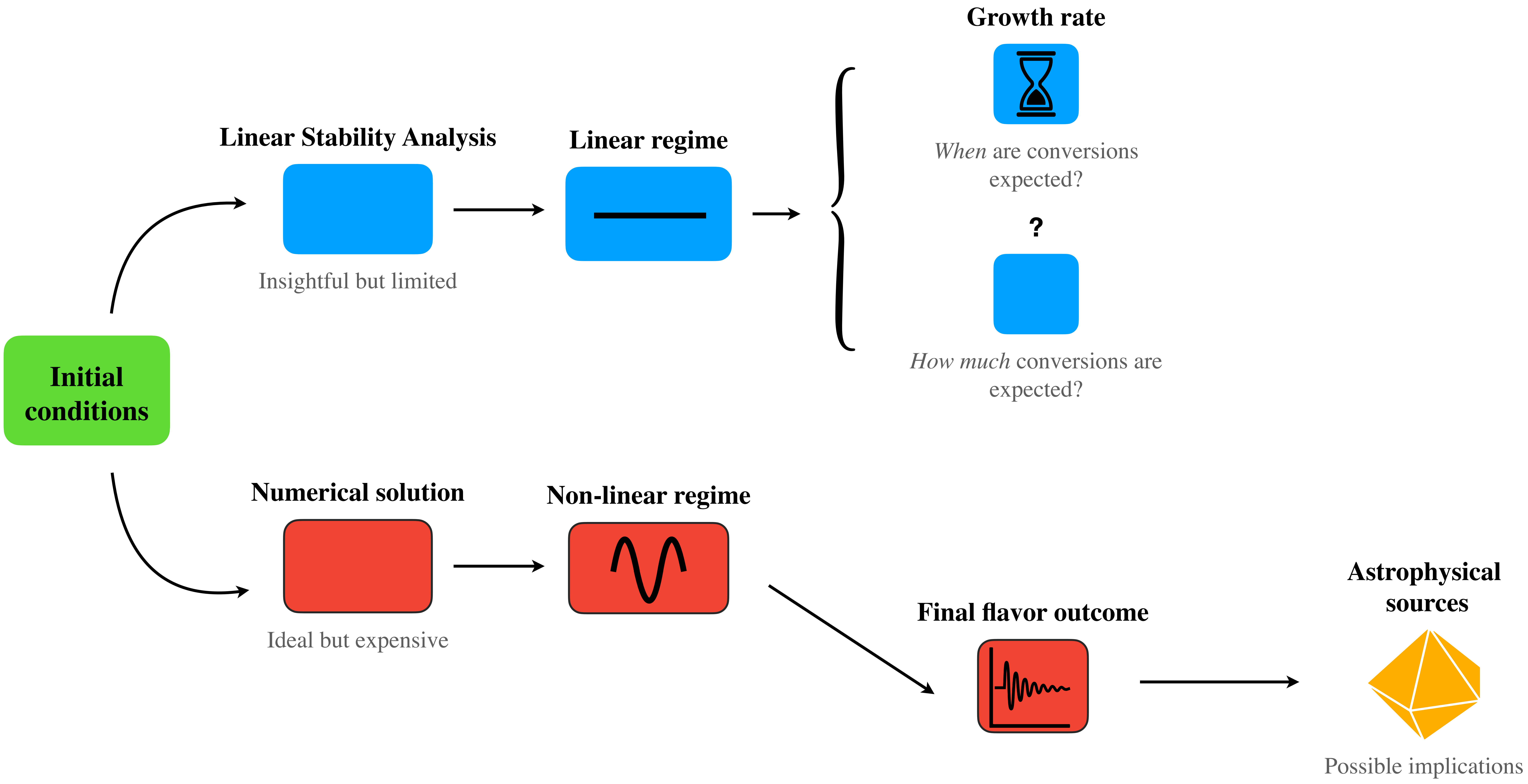
$\mathbf{D}_1$ dynamic, conserves its length $|\mathbf{D}_1|$



Find a method to systematically and analytically predict FFC based on initial ELN spectra

Can we gauge the amount of conversions without evolving the QKEs??

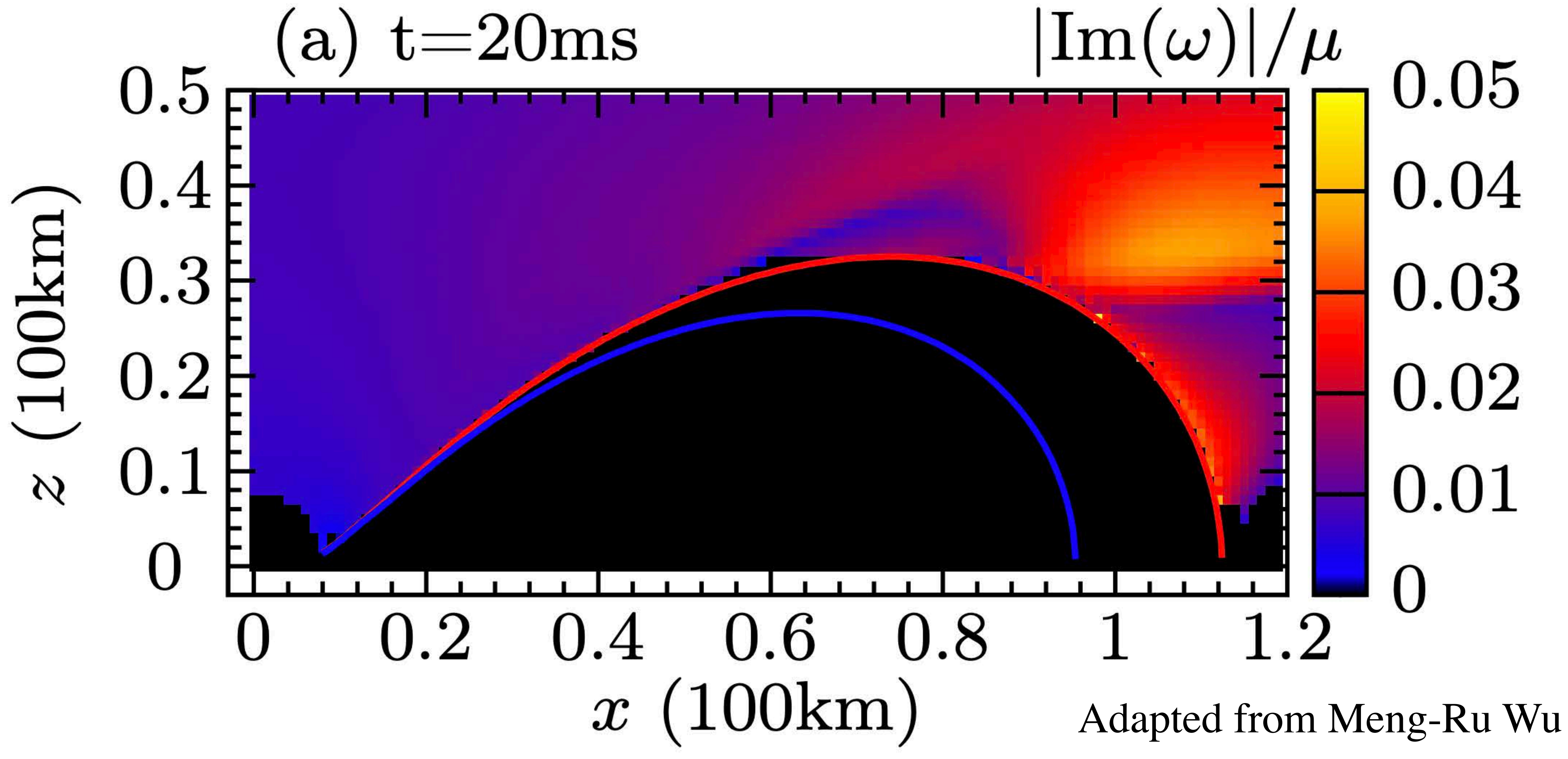
How are FFC approached?



How are FFC approached? Example 1

Growth rate

- Neutrino fast conversions are predicted to be ubiquitous in neutron star merger remnants (M-R Wu & I. Tamborra 2017, M-R Wu et al. 2017)
- Semi-analytical estimates suggest mergers as perfect sites for FFC

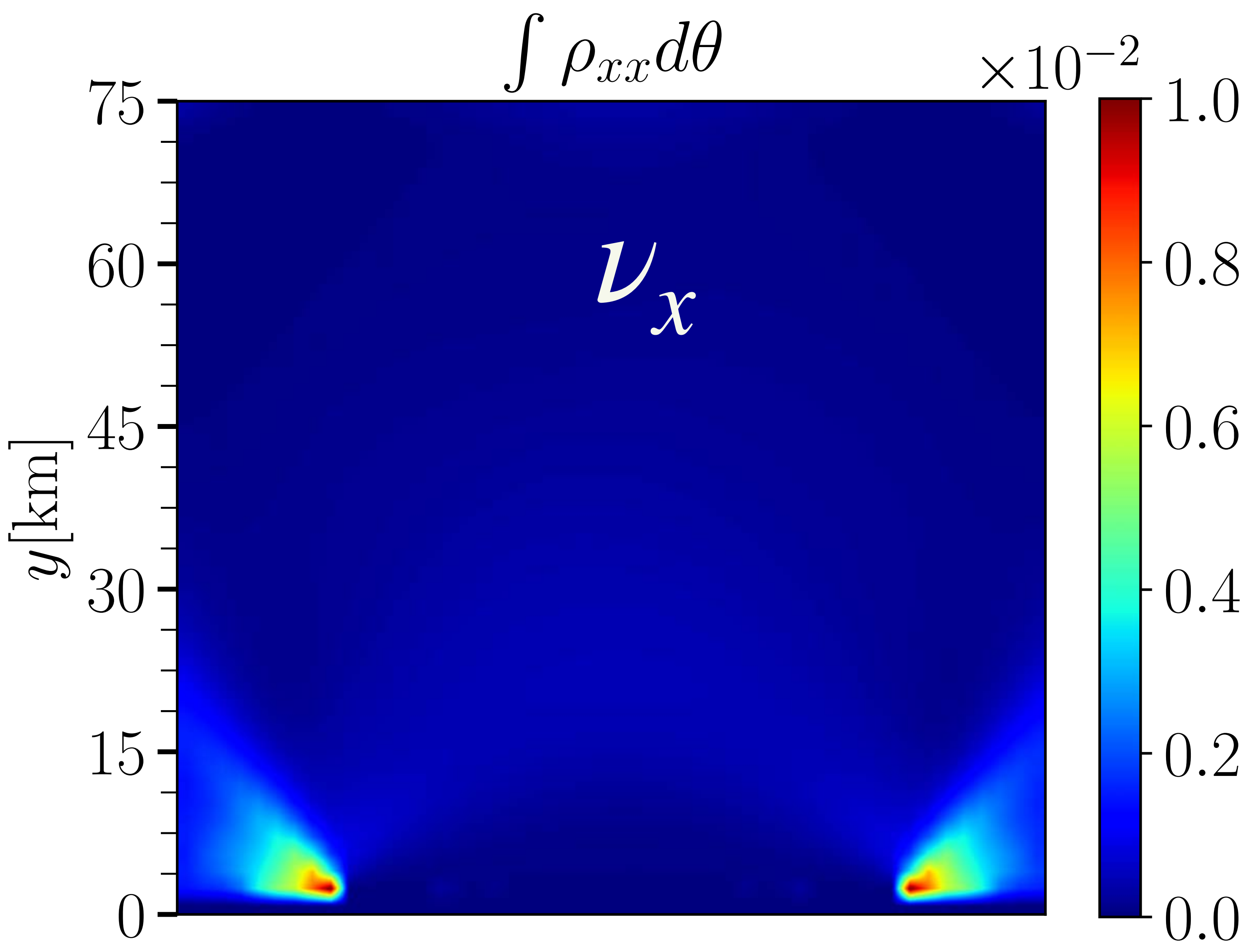


Adapted from Meng-Ru Wu et at. 2017

Final flavor outcome

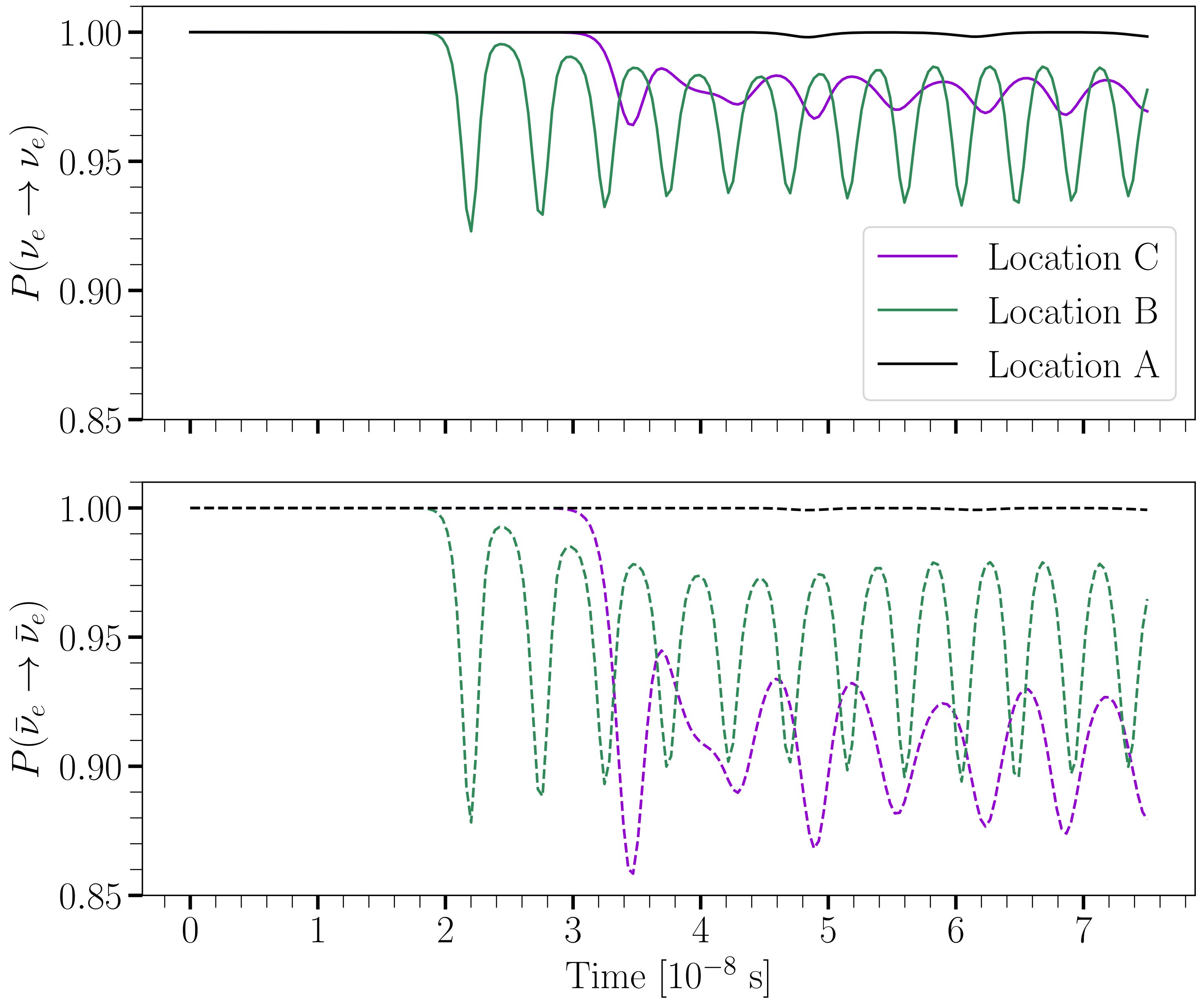
How are FFC approached? Example 2

- Multi-dimensional solution gives similar growth rates, but very little overall flavor conversion (I. Padilla-Gay et al 2021)
- Large growth rates do not imply large amount of flavor conversion… so, what does?



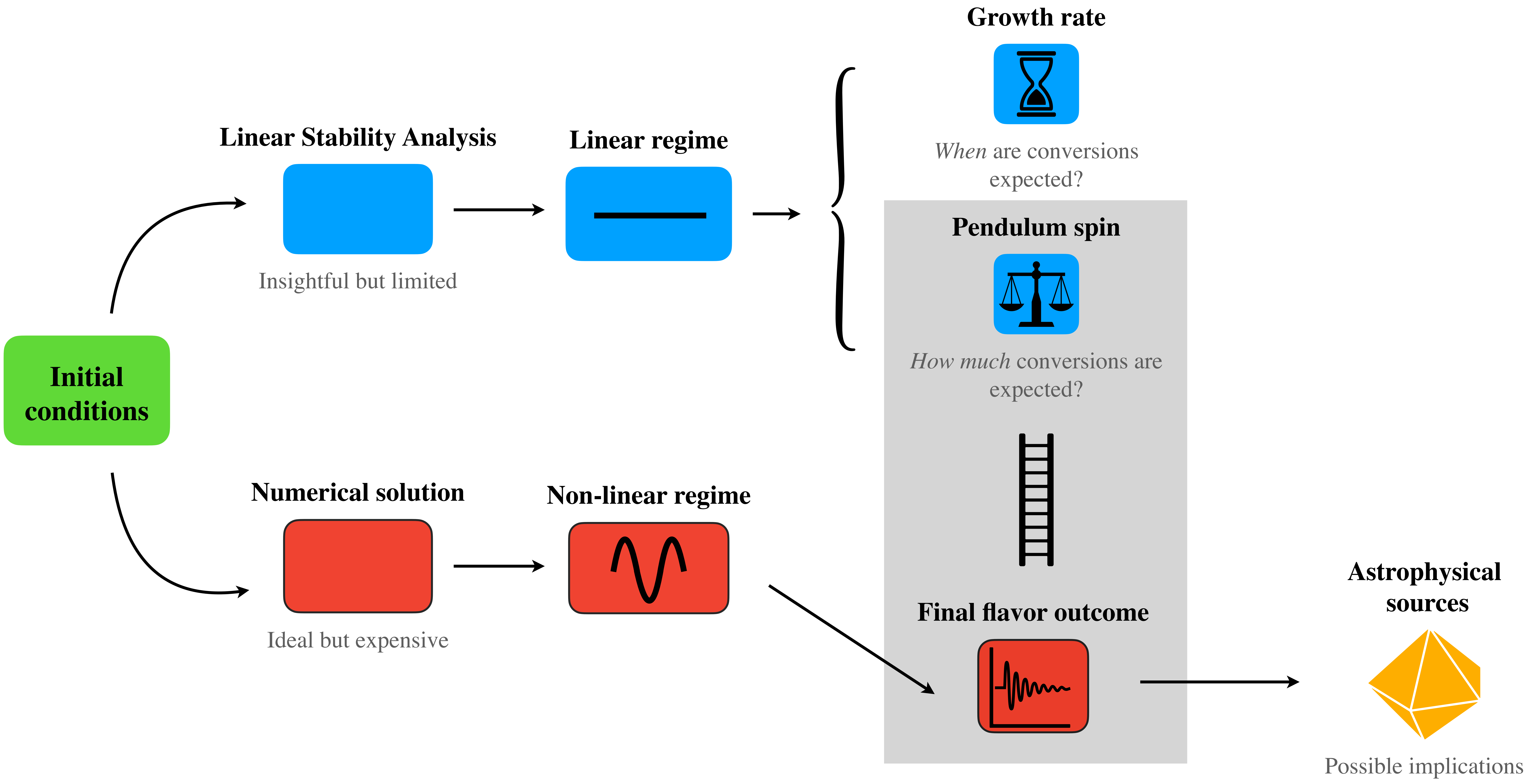
Oscillated $\nu_e \rightarrow v_x$ distribution

I. Padilla-Gay et al 2021



Minimal neutrino conversion <1%

Where have we innovated?



Main findings in a nutshell

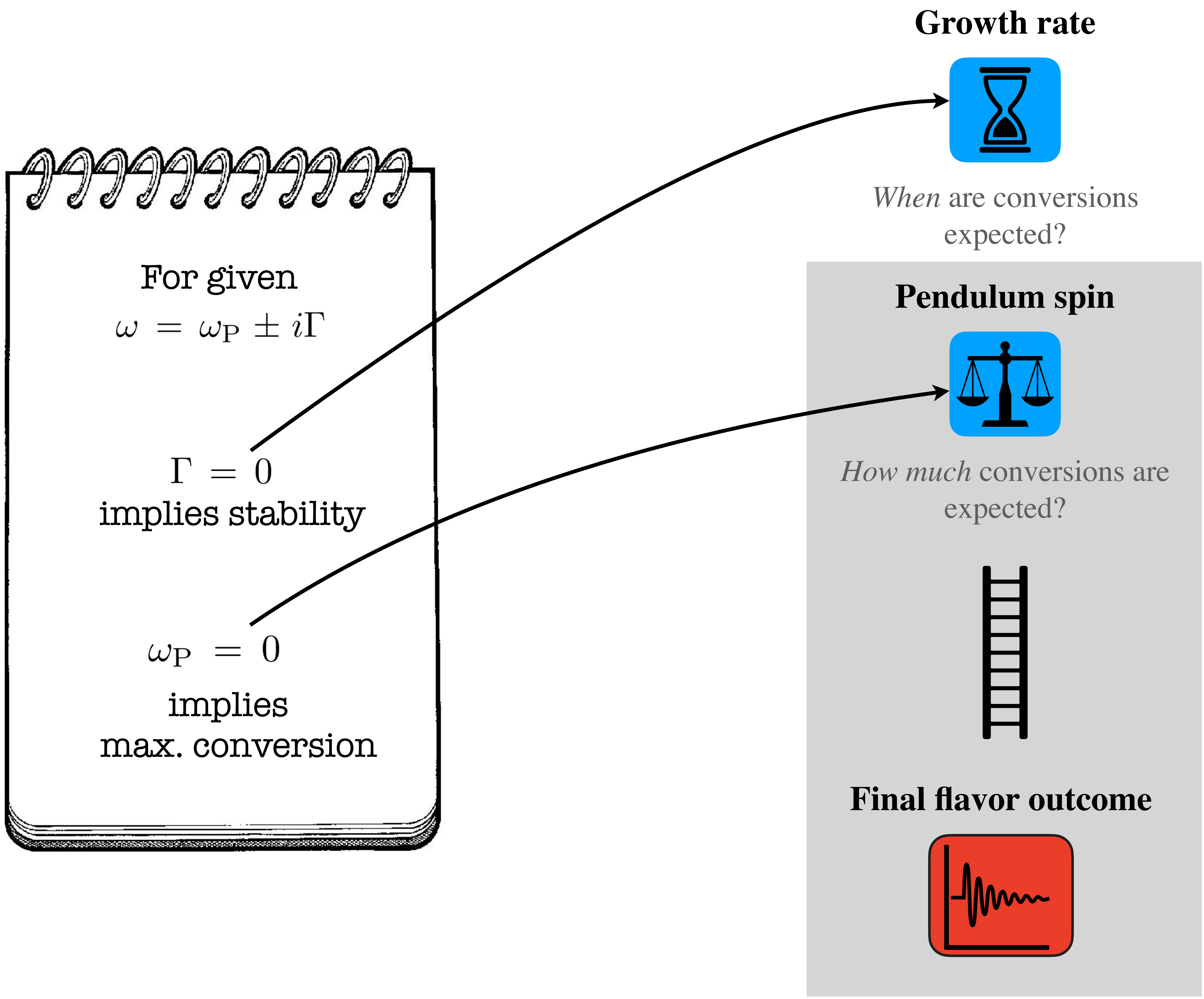
The linear stability analysis provides much more information than we thought:

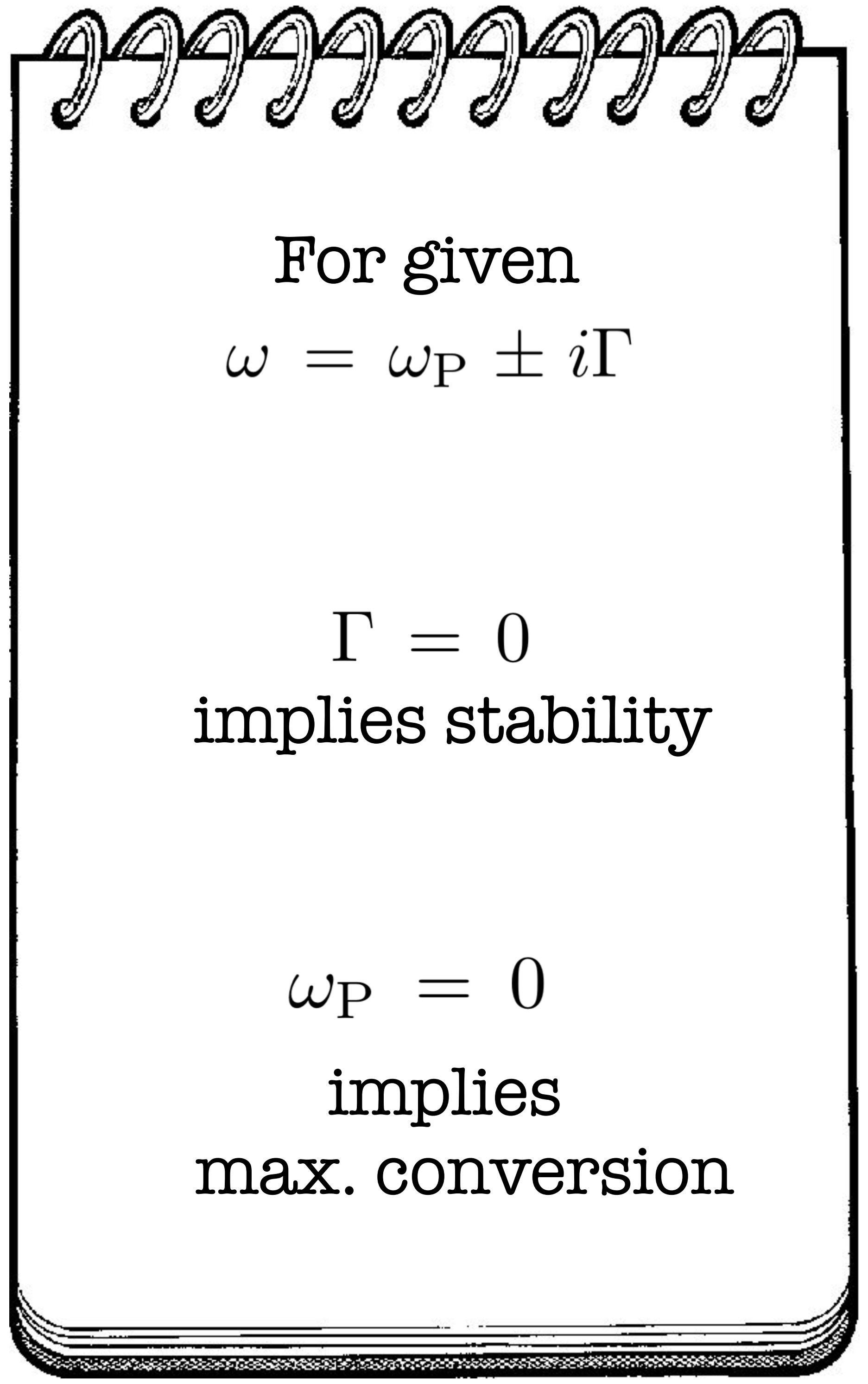
Collective normal mode:

$$D_v^{xy} = g_v Q_v e^{-i\omega t}$$

Complex eigenfrequency:

$$\omega = \omega_{\rm P} \pm i\Gamma$$





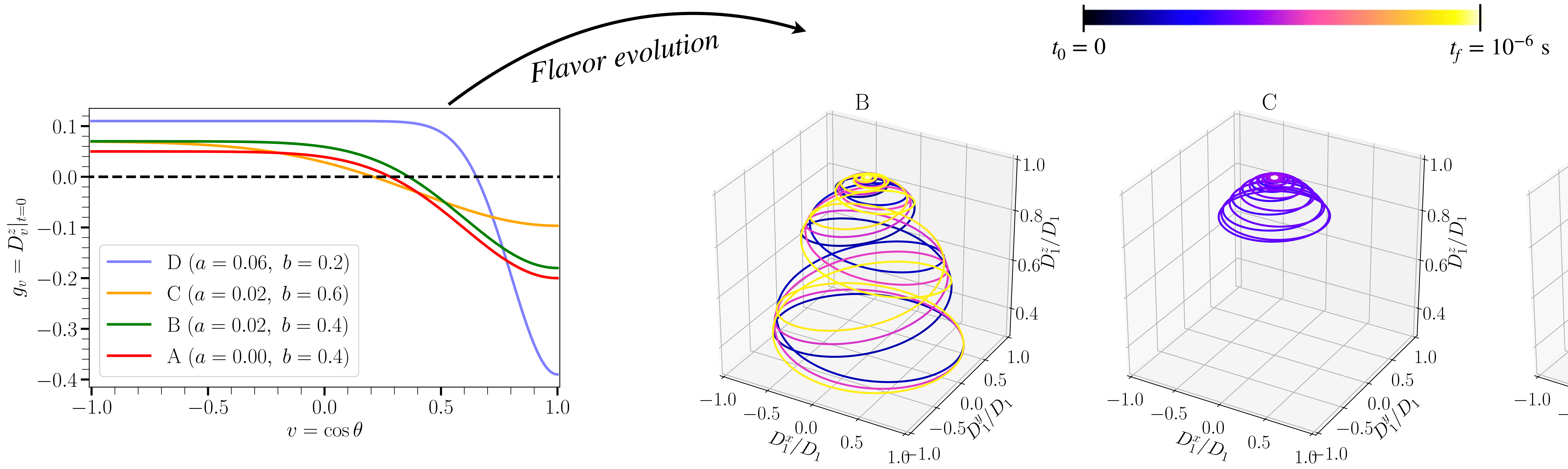
Find a method to systematically and analytically predict FFC based on initial ELN spectra

How do we arrive to this criterion?

Pendulum-like behaviour

- Solution behaves like a pendulum. Is it strictly a pendulum? (S. Hannestad 2006, G. Folgi 2007, L. Johns et al 2018, L. Johns et al 2020)

- Yes! Flavor pendulum with *spin*, subject to *gravity:*



<u>Challenge</u>: identify pendulum parameters from initial conditions

Connection to previous works (L. Johns et al 2020)

- Expansion of the QKEs in angular multipoles: truncating the tower of equations → pendulum-like equation

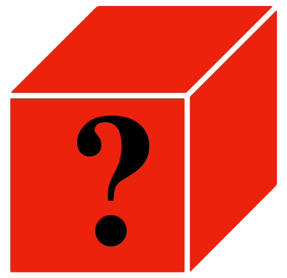
$$\dot{\mathbf{D}}'_1 = \mu \mathbf{L}' \times \mathbf{D}'_1,$$
$$\dot{\mathbf{D}}'_2 = \frac{3}{2}\mu \mathbf{G}' \times \mathbf{D}'_1$$

- Approximate system: representative of the full solution?

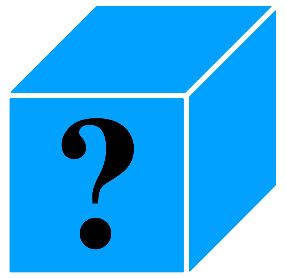
$$\mathbf{L}' = (\mathbf{D}'_0 + 2\mathbf{D}'_2)/3 \qquad \mathbf{G}' = 2\mathbf{D}'_3/5$$

- We take a different approach

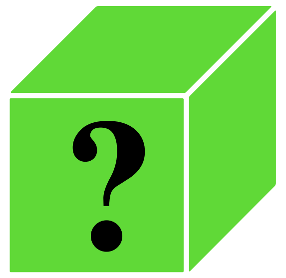
Pendulum-like behaviour



1. **Find** independent d.o.f. (Gram-matrix test)



2. **Represent** the gyroscopic pendulum (flavor space)



3. **Match** parameters with full system

Challenge: identify pendulum parameters from initial conditions

1. Gram-matrix test

- The coherence of all velocity modes suggest a small number of d.o.f.

$$G_{ij} = \int_{t_1}^{t_2} dt\, \boldsymbol{D}_{v_i}(t) \cdot \boldsymbol{D}_{v_j}(t)$$

- Rank of G (i.e. N+1) is the number of linearly independent functions.
- One *time-independent* solution in the form of $\boldsymbol{D}_0 = \sum_{i=1}^{n} \boldsymbol{D}_{v_i}(t)$
- We find N = 2: Solutions are equivalent to 2 *dynamical* d.o.f., equivalent to 3 discrete angle modes

2. Gyroscopic pendulum in flavor space

The Gram-matrix test suggests the following linearly independent functions:

Mechanical analogy

"Gravity" = Lepton-number density vector

$$\boldsymbol{G} = \boldsymbol{D}_0 = \int dv\, \boldsymbol{D}_v(t)$$

Pendulum = Lepton-number flux vector

$$\boldsymbol{R}(t) = \boldsymbol{D}_1(t) = \int dv\, v \boldsymbol{D}_v(t)$$

Total angular momentum = to be determined

$$\boldsymbol{J}(t) = \int dv\, w_v \boldsymbol{D}_v(t)$$

EOMs of a gyroscopic pendulum

$$\dot{\boldsymbol{G}} = 0\ , \quad \dot{\boldsymbol{R}} = \mu \boldsymbol{J} \times \boldsymbol{R} \quad \text{and} \quad \dot{\boldsymbol{J}} = \gamma \boldsymbol{G} \times \boldsymbol{R}\ .$$

2. Gyroscopic pendulum in flavor space

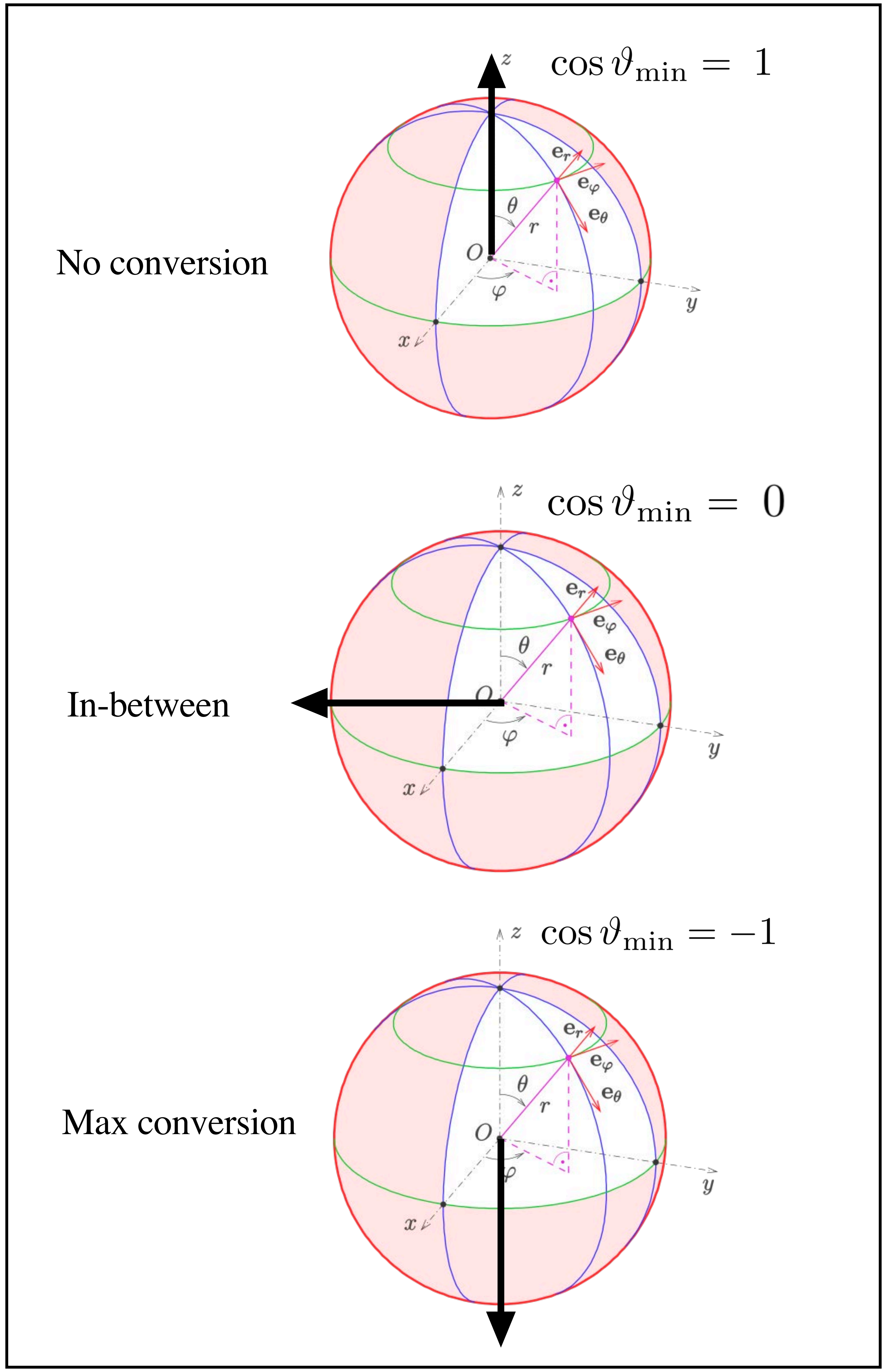
- By solving the EOMs for $\vartheta(t)$ and $\varphi(t)$, one finds:

$$\dot{\varphi} = \mu \frac{2\lambda\sigma}{1+\cos\vartheta} \, ,$$

$$\dot{c}_\vartheta^2 = \mu^2\lambda^2 \, 2\left(1-c_\vartheta\right)^2\left(1+c_\vartheta-2\sigma^2\right)$$

- For unstable solutions, the pendulum oscillates between its initial position and a minimal value given by

$$\boxed{\cos\vartheta_{\min} = -1 + 2\sigma^2}$$



3. Match parameters with full system

In the linear regime $(\vartheta \ll 1)$, we match the *real* and *imaginary* parts of ω with the pendulum parameters:

Uniform precession

ω_{P}

$$\dot{\varphi} = \mu\lambda\sigma$$

$$\dot{\vartheta} = \pm\mu\lambda\sqrt{1-\sigma^2}\,\vartheta$$

Γ

Polar angle grows/
shrinks exponentially

$$\omega_{\mathrm{P}} = \mu\lambda\sigma$$

$$\Gamma = \mu\lambda\sqrt{1-\sigma^2}$$

3. Match parameters with full system

- Solving for the spin parameter and the natural frequency:

$$\sigma = \sqrt{\frac{\omega_{\mathrm{P}}^2}{\omega_{\mathrm{P}}^2 + \Gamma^2}} \quad \text{and} \quad \lambda = \frac{1}{\mu}\sqrt{\omega_{\mathrm{P}}^2 + \Gamma^2}$$

Remarks

$\omega_{\mathrm{P}} = 0$ implies $\sigma = 0$
(max. conversion)

$\Gamma = 0$ implies $\sigma = 1$
(stable)

- The minimum point reached by the pendulum is then:

$$\cos\vartheta_{\min} = -1 + 2\sigma^2$$

$$\cos\vartheta_{\min} = -1 + 2\,\frac{\omega_{\mathrm{P}}^2}{\omega_{\mathrm{P}}^2 + \Gamma^2}$$

1. **Find** independent d.o.f. (Gram-matrix test)

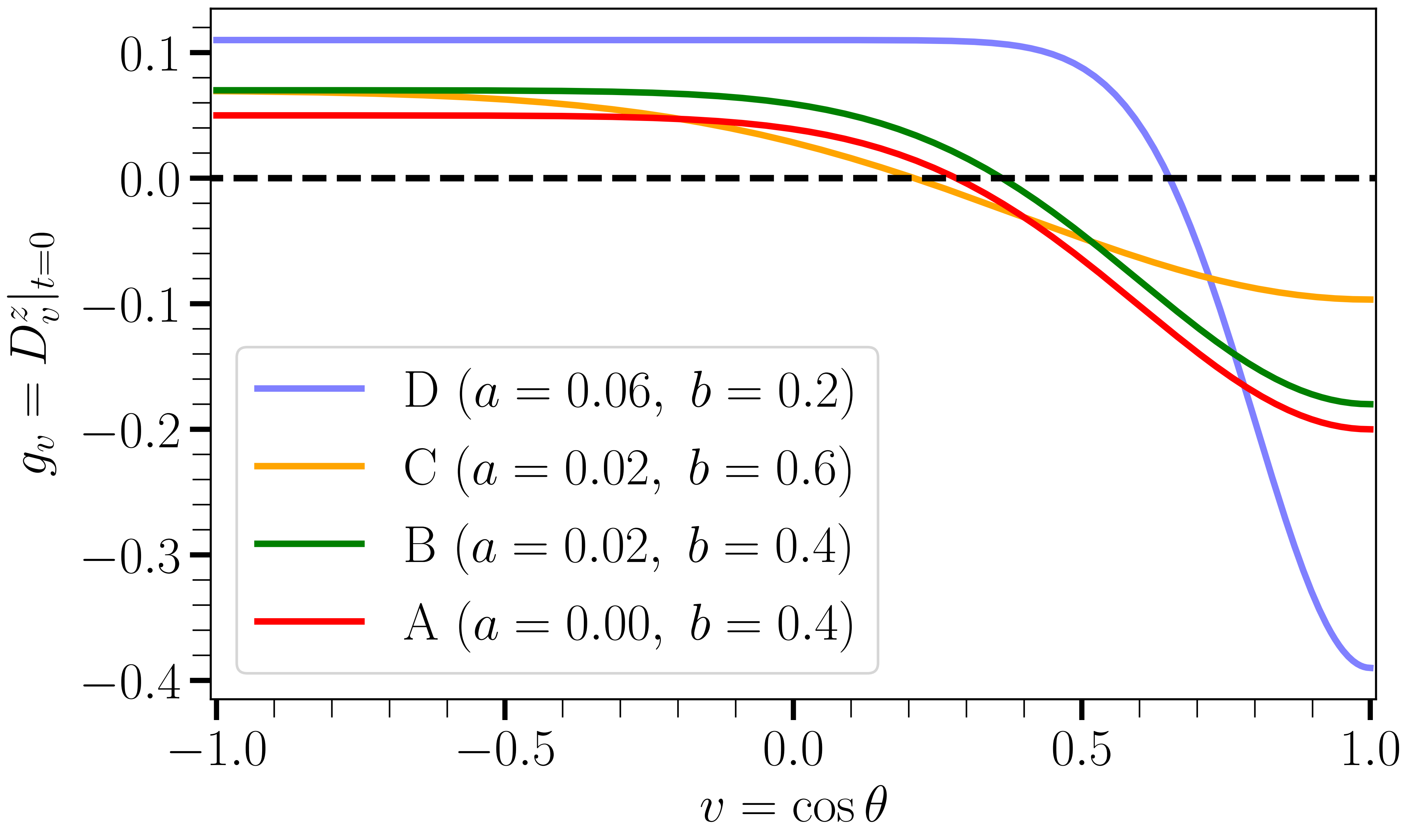
2. **Represent** the gyroscopic pendulum (flavor space)

3. **Match** parameters with full system

Pendulum parameters from initial conditions

Systematic method: our criterion in practice

- Given an initial ELN spectra, find $\omega = \omega_\mathrm{P} \pm i\Gamma$
- Then, compute λ and σ to obtain the maximum amount of conversions



$$\sigma = \sqrt{\frac{\omega_\mathrm{P}^2}{\omega_\mathrm{P}^2 + \Gamma^2}} \quad \text{and} \quad \lambda = \frac{1}{\mu}\sqrt{\omega_\mathrm{P}^2 + \Gamma^2}$$

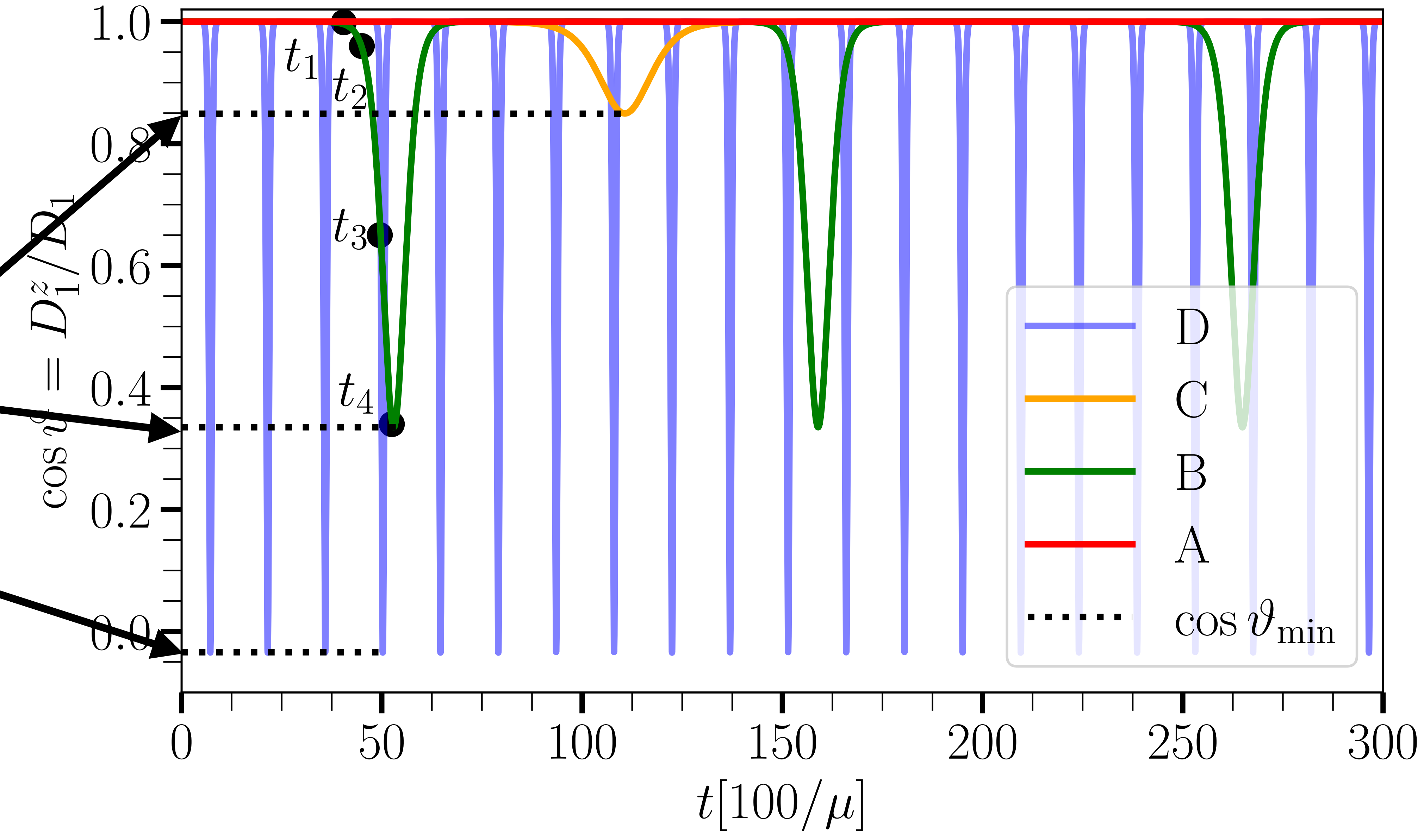
$$\cos\vartheta_\mathrm{min} = -1 + 2\,\frac{\omega_\mathrm{P}^2}{\omega_\mathrm{P}^2 + \Gamma^2}$$

Systematic method: our criterion in practice

Excellent agreement: our prediction and full FFC solution

TABLE I. Solutions for the complex eigenfrequencies of our benchmark ELN configurations A–D (see main text).

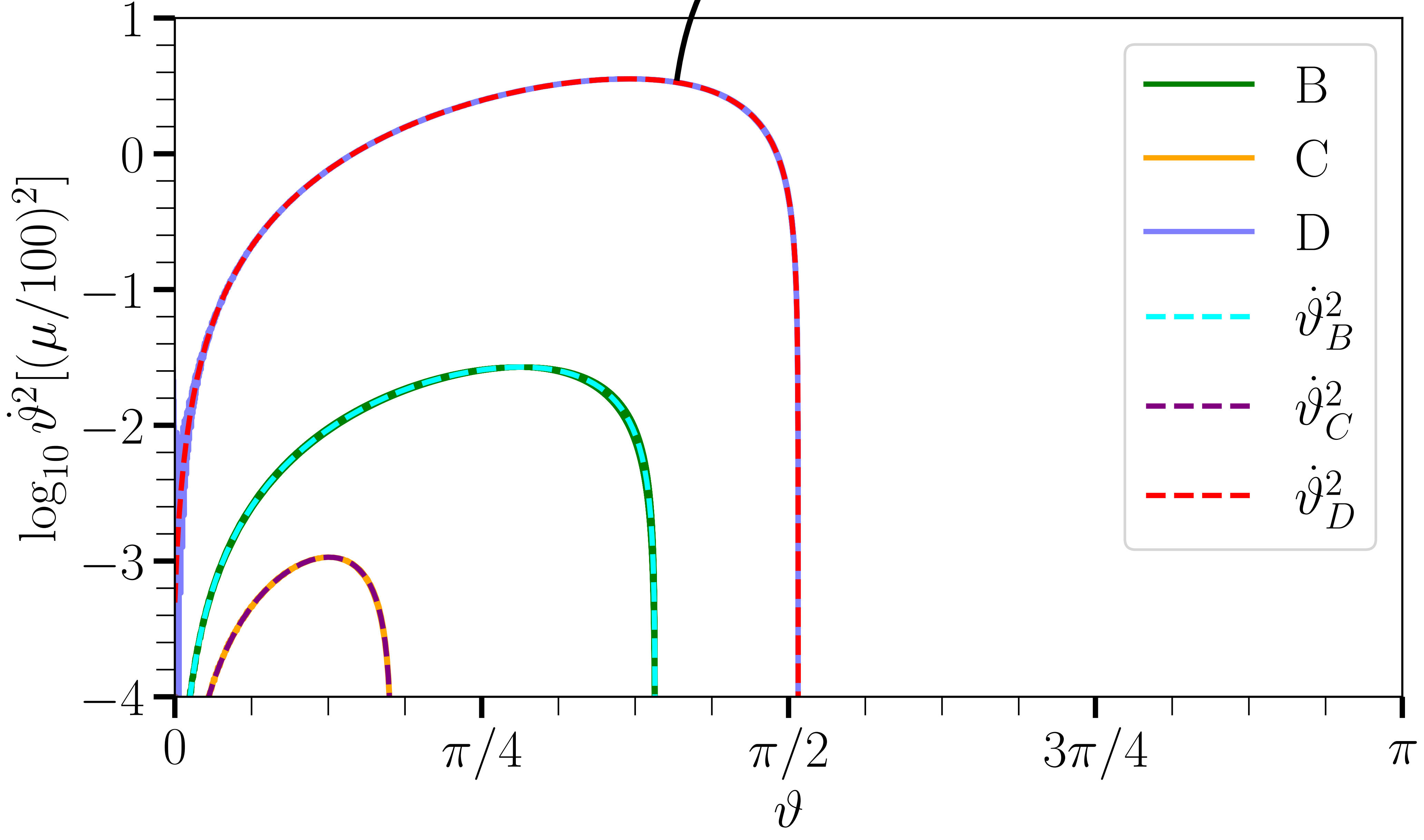
Case	Λ_0 $[\mu/100]$	Λ_1 $[\mu/100]$	$\omega_\pm = \omega_\mathrm{P} \pm i\Gamma$ $[\mu/100]$	σ	$\cos\vartheta_\mathrm{min}$
A	-1.2666	-4.2666	stable	—	—
B	$+0.7334$	-4.2666	$0.1828 \pm 0.1291\,i$	0.817	$+0.335$
C	$+0.7388$	-3.2728	$0.2047 \pm 0.0584\,i$	0.962	$+0.849$
D	$+4.7334$	-5.2665	$1.0743 \pm 1.1121\,i$	0.694	-0.034

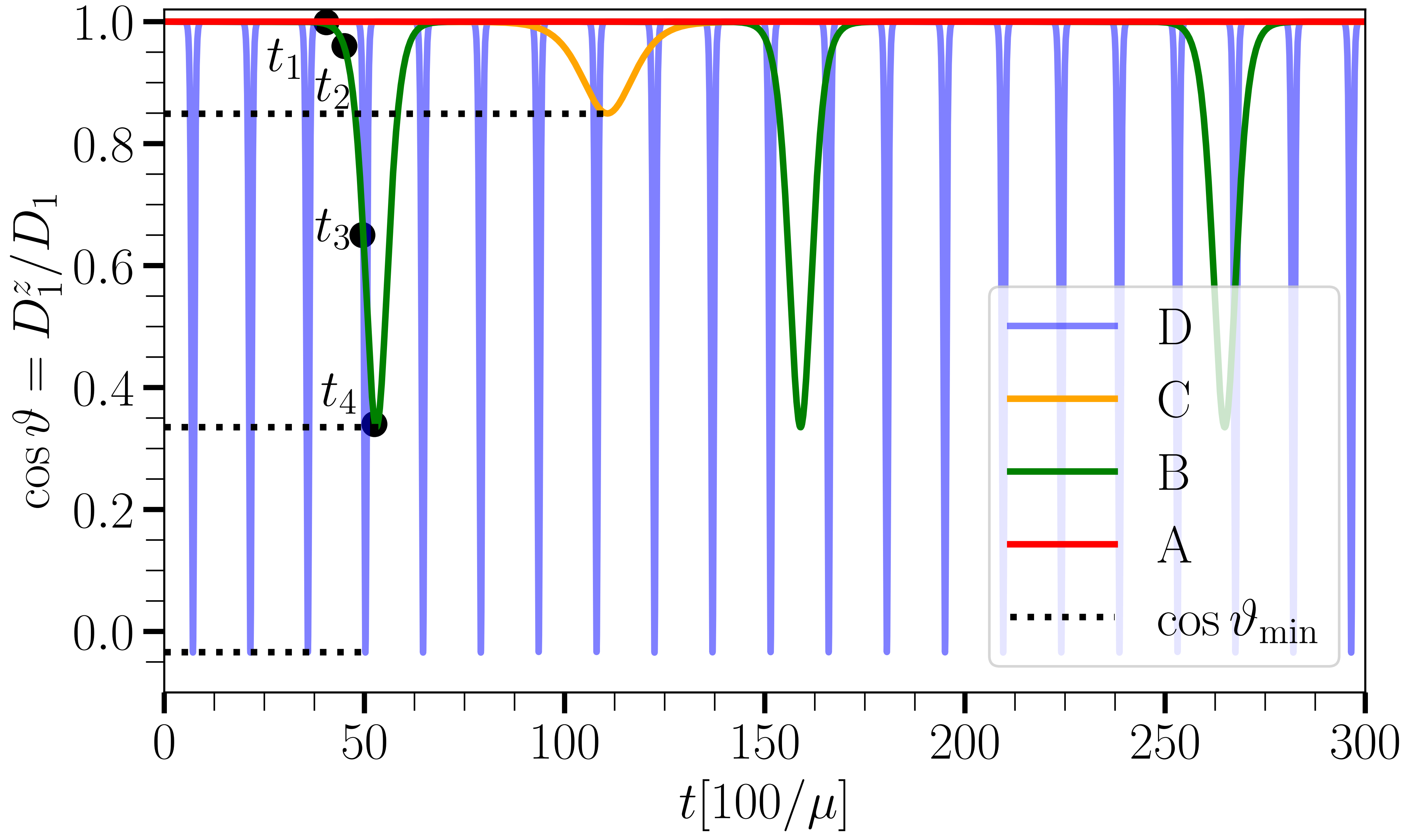


Systematic method: our criterion in practice

Excellent agreement: Phase-space comparison (analytical and full solution)

dashed = analytical
solid = numerical





Conclusions

- FFC strongly depends on initial ELN spectrum
- Amount of flavor conversion does not directly correlate with the growth rate
- Evolution of ELN flux vector is equivalent to a gyroscopic pendulum
- One can identify the pendulum parameters:
  1. Growth rate of instability ~ imaginary part of eigenfrequency
  2. Maximum amount of conversions ~ real part of eigenfrequency
- Based on initial ELN spectrum, we can predict *when* and *how much* FCC occur

Thank you for the attention!

Backup slides

The neutrino QKE's

- Mean field approximation (one-body problem)

$$i\,(\partial_t + \vec{v}\cdot\vec{\nabla})\varrho_{\vec{p}} = \sqrt{2}\,G_{\rm F}\,[\mathsf{H}_{\vec{v}},\,\varrho_{\vec{p}}]$$

- Neutrino self-interactions Hamiltonian

$$\mathsf{H}_{\vec{v}} = \int \frac{d^3\vec{q}}{(2\pi)^3}(\varrho_{\vec{q}} - \bar{\varrho}_{\vec{q}})\,(1-\vec{v}_{\vec{q}}\cdot\vec{v})$$

- QKE's do not depend on the neutrino energy

$$\mathsf{D}_{\vec{p}} = \varrho_{\vec{p}} - \bar{\varrho}_{\vec{p}} \qquad \mathsf{D}_{\vec{v}} \equiv n_{\nu_e}^{-1}\int_0^\infty E^2 dE/(2\pi^2)\,\mathsf{D}_{E,\vec{v}}$$

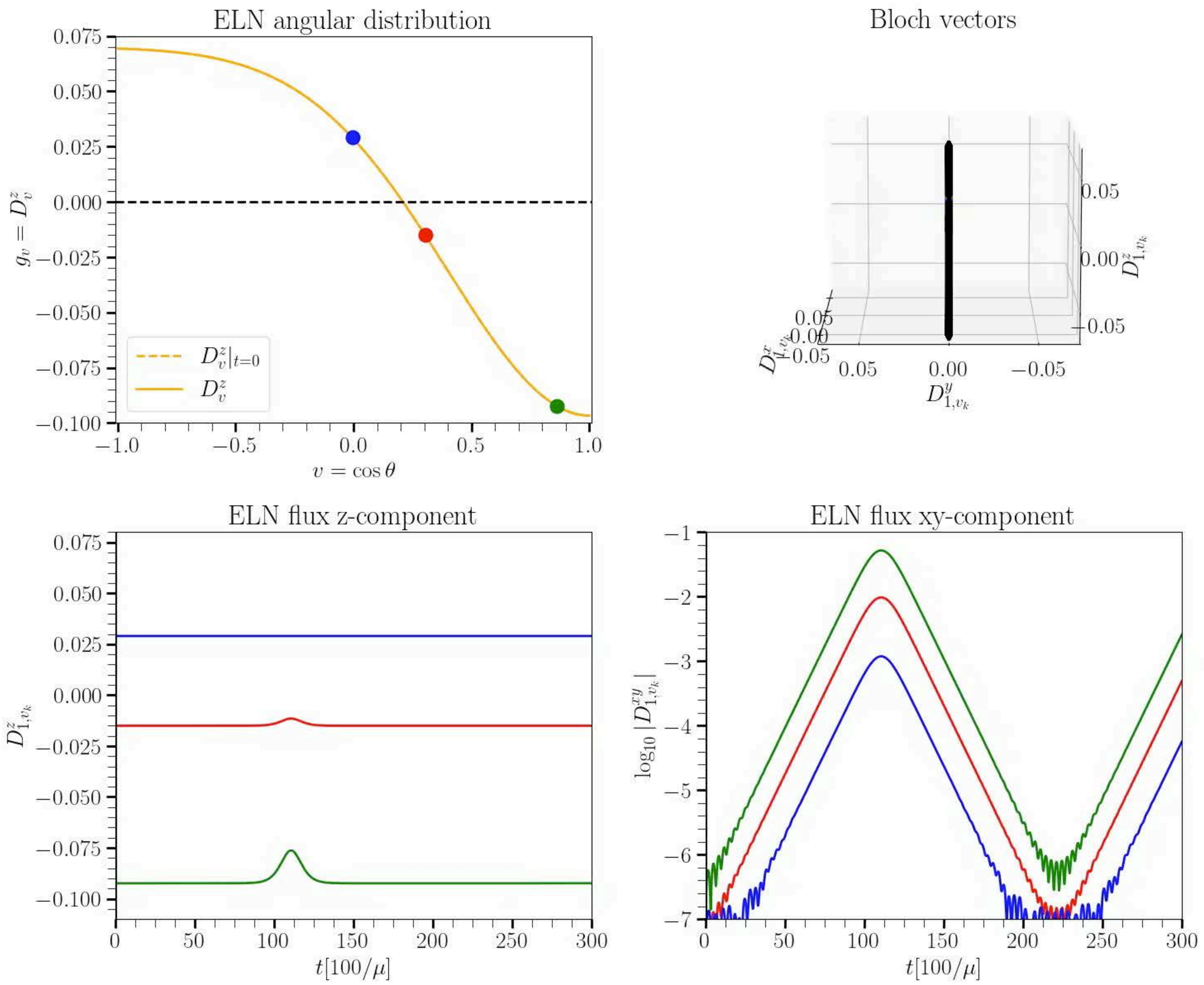
- Closed set of equations

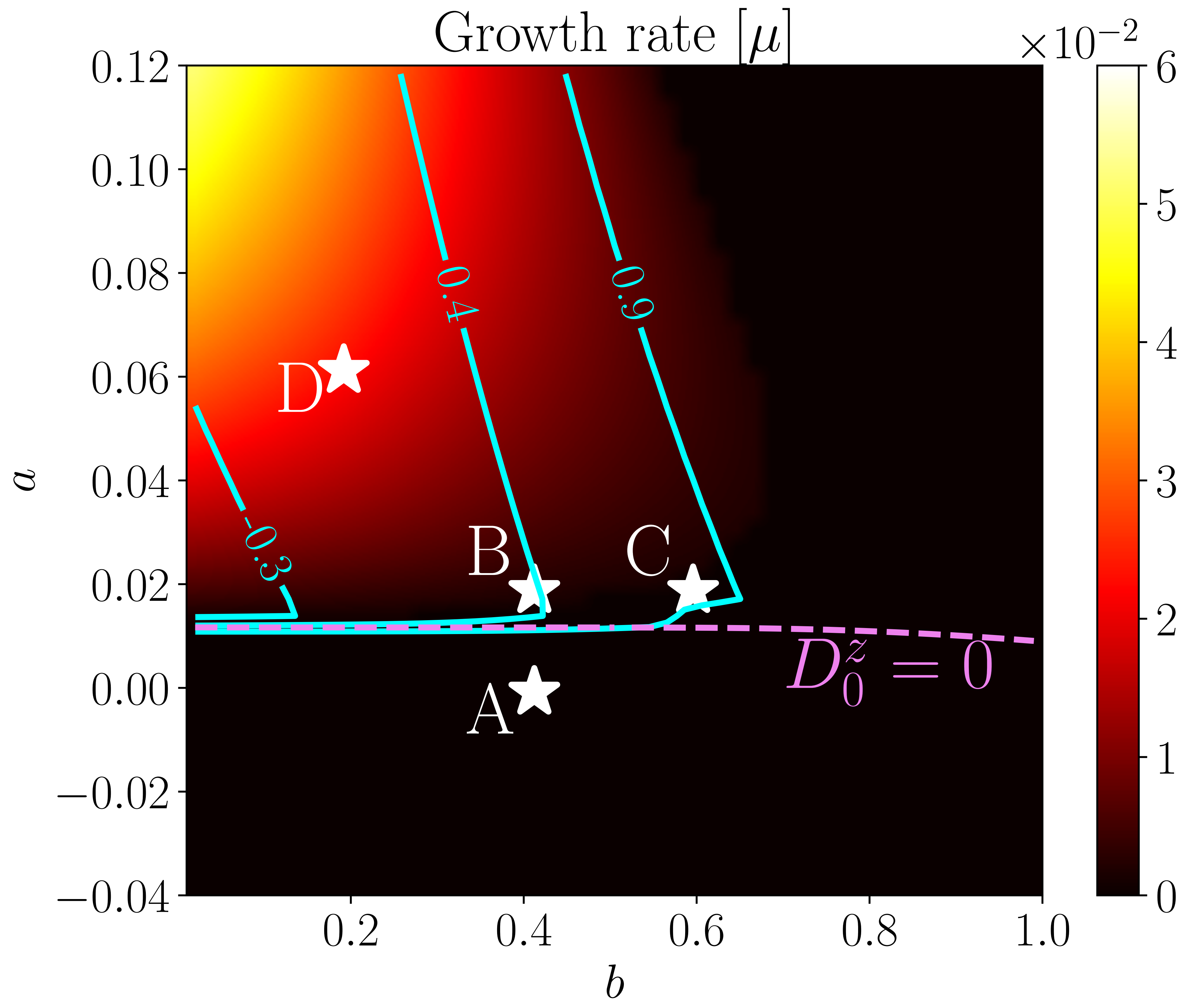
$$i\,(\partial_t + \vec{v}\cdot\vec{\nabla})\mathsf{D}_{\vec{v}} = \mu\,[\mathsf{H}_{\vec{v}},\mathsf{D}_{\vec{v}}]$$

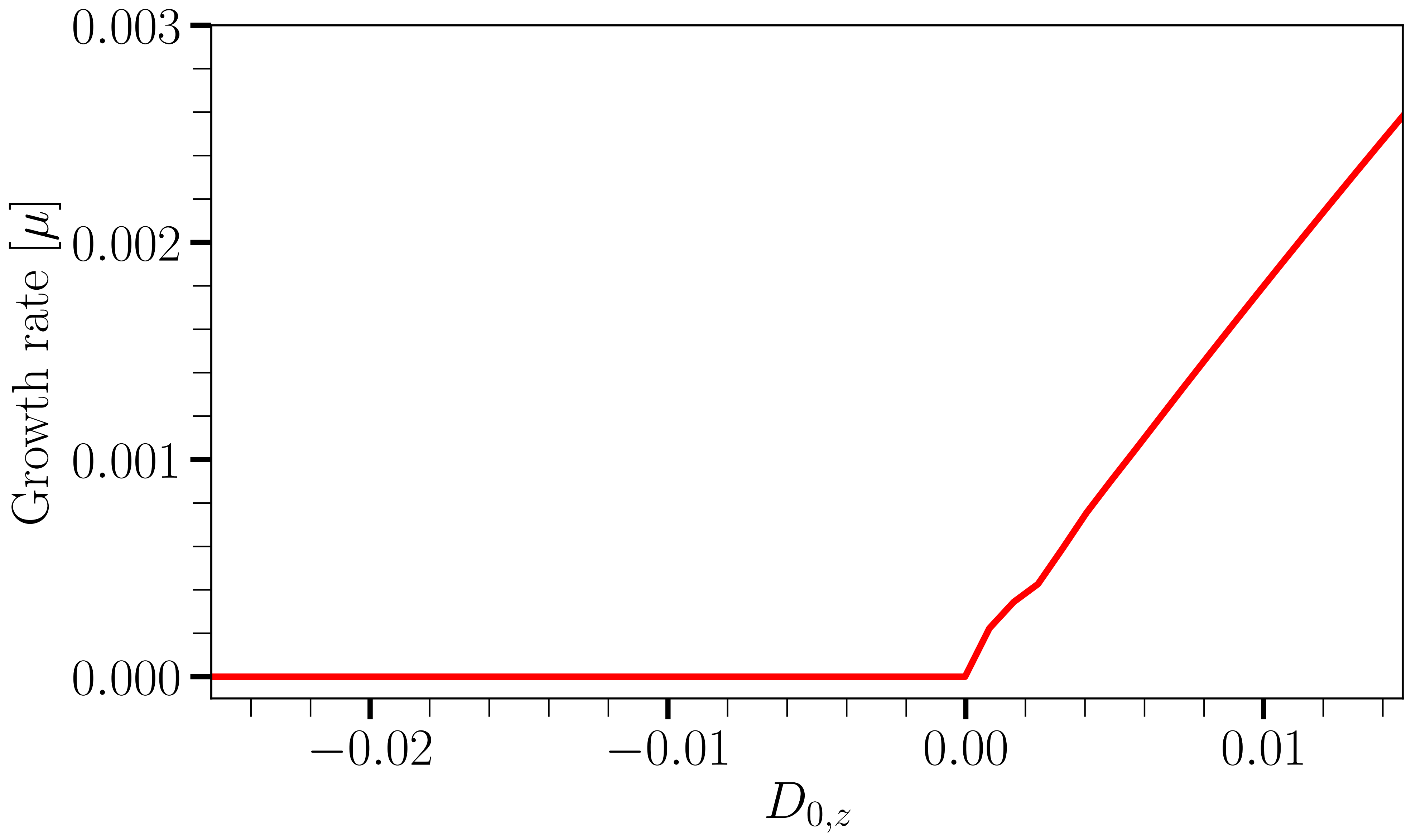
- For fixed $\mu \equiv \sqrt{2}G_{\rm F}n_{\nu_e}$, FFC dynamics completely driven by $\mathsf{D}_{\vec{v}}$

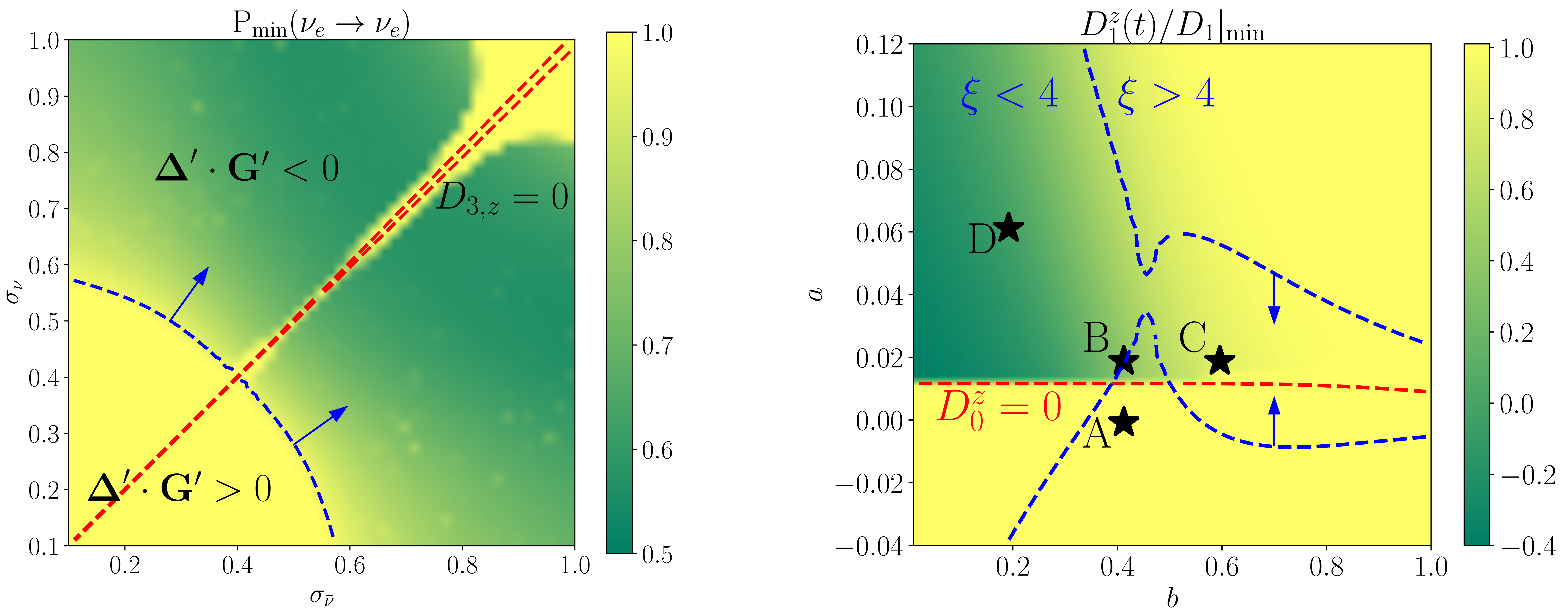
$$\mathsf{H}_{\vec{v}} = \int (d^2\vec{u}/4\pi)\,D_{\vec{u}}(1-\vec{u}\cdot\vec{v})$$

$t = 0.0 \times 10^{-6}$ s

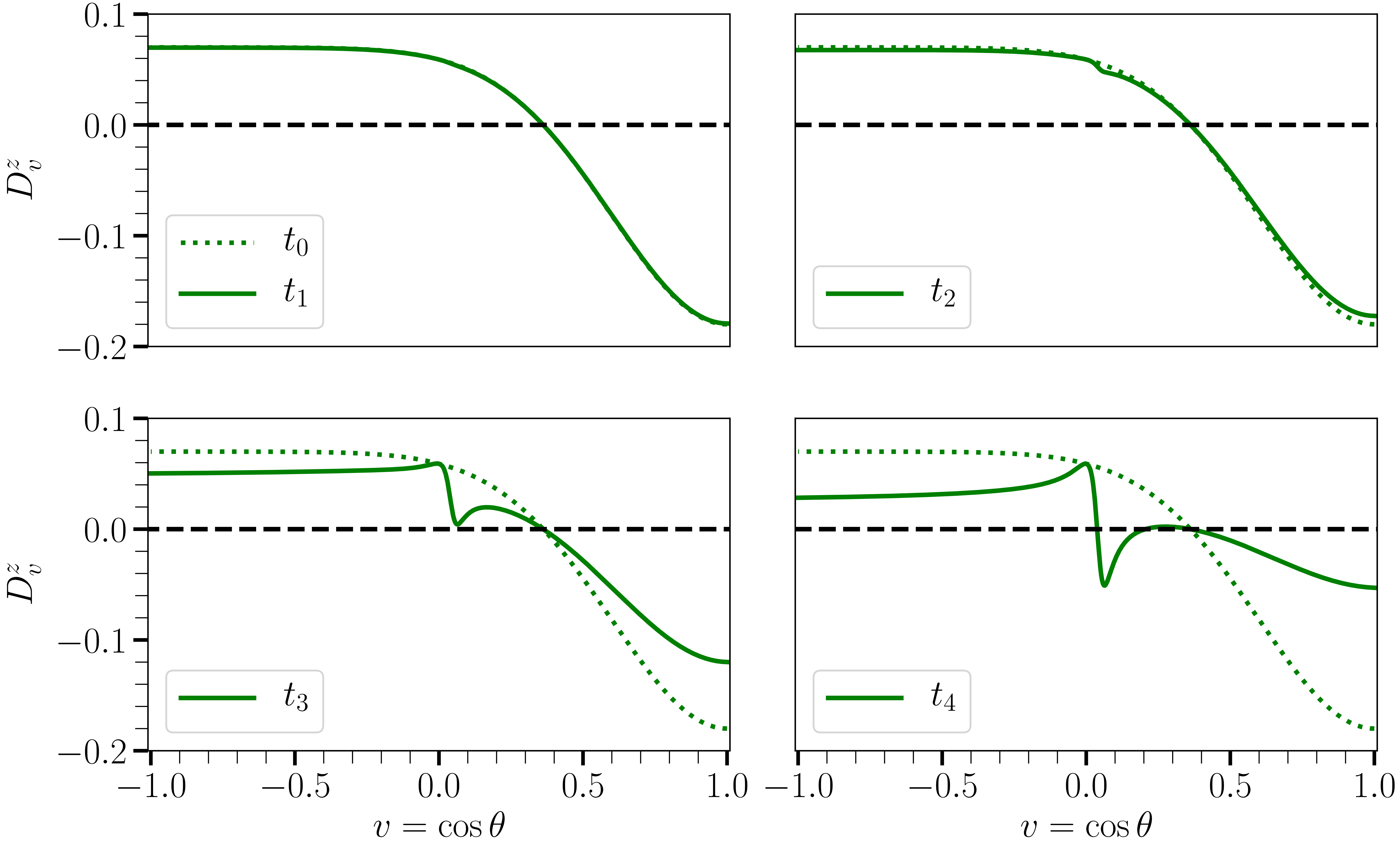


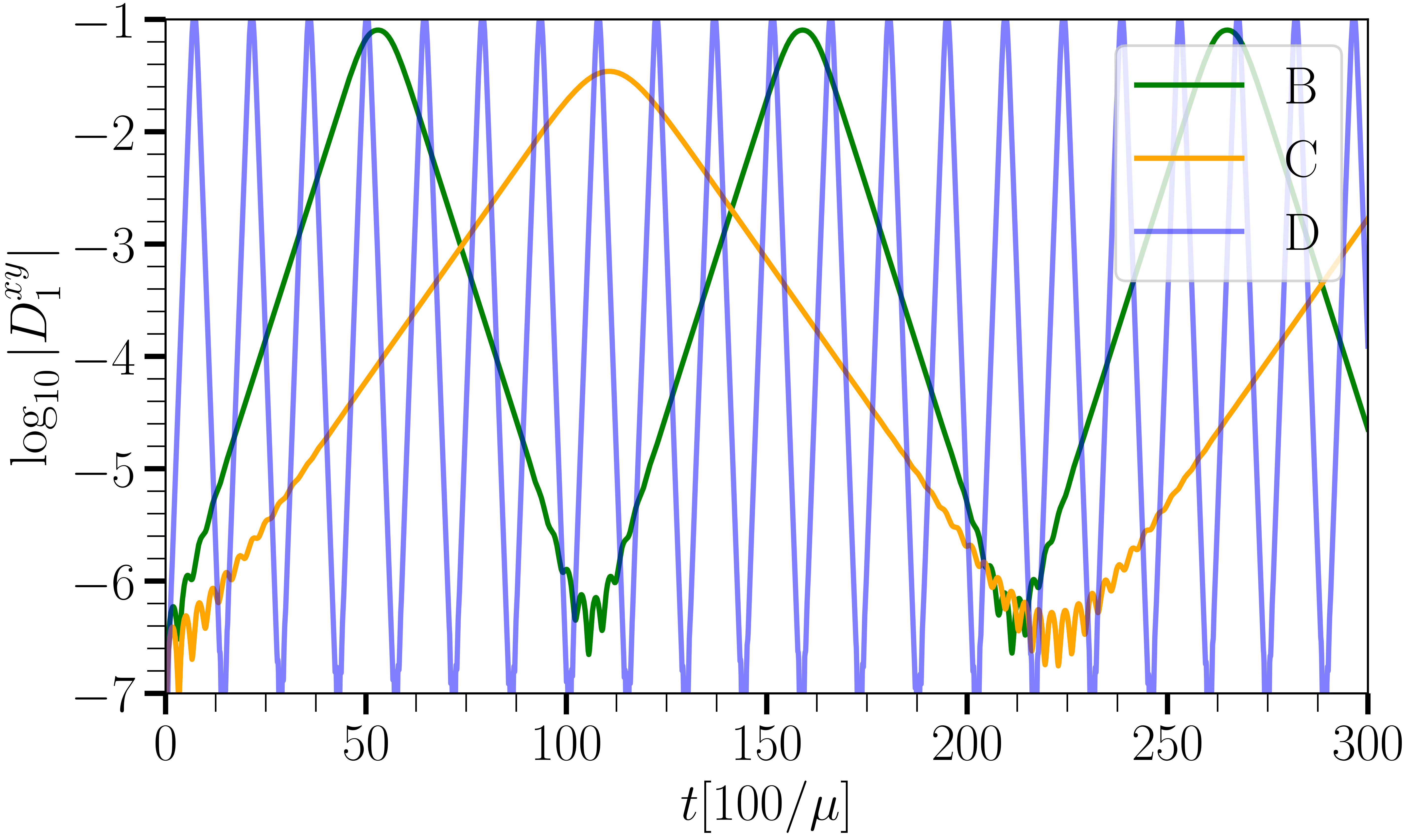

Growth rate [μ]
$\times 10^{-2}$
0.12
0.10
0.08
0.06
0.04
0.02
0.00
−0.02
−0.04
a
0.2
0.4
0.6
0.8
1.0
b
6
5
4
3
2
1
0
-0.3
0.4
0.9
A
B
C
D
$D_0^z = 0$


0.003
0.002
0.001
0.000
Growth rate [μ]
−0.02
−0.01
0.00
0.01
$D_{0,z}$



$\mathrm{P}_{\min}(\nu_e \to \nu_e)$
$\mathbf{\Delta}' \cdot \mathbf{G}' < 0$
$D_{3,z} = 0$
$\mathbf{\Delta}' \cdot \mathbf{G}' > 0$
σ_ν
$\sigma_{\bar{\nu}}$
$D_1^z(t)/D_1|_{\min}$
$\xi < 4$
$\xi > 4$
D
B
C
A
$D_0^z = 0$
a
b


0.1
0.0
−0.1
−0.2
D_v^z
t_0
t_1
t_2
t_3
t_4
−1.0
−0.5
0.0
0.5
1.0
$v = \cos\theta$


−1
−2
−3
−4
−5
−6
−7
$\log_{10}|D_1^{xy}|$
0
50
100
150
200
250
300
$t[100/\mu]$
B
C
D

The flavor dynamics

With $\mu = \sqrt{2} G_F n_{\nu\bar{\nu}}$ the EOM is given by

$$i\,\partial_t \mathsf{D}_{\vec{v}} = \mu[\mathsf{H}_{\vec{v}}, \mathsf{D}_{\vec{v}}]$$

where we have defined dimensionless, energy-integrated density matrices

$$\mathsf{D}_{\vec{v}} = \frac{1}{n_{\nu\bar{\nu}}} \int_0^\infty \frac{E^2 dE}{2\pi^2} \mathsf{D}_{E,\vec{v}}$$

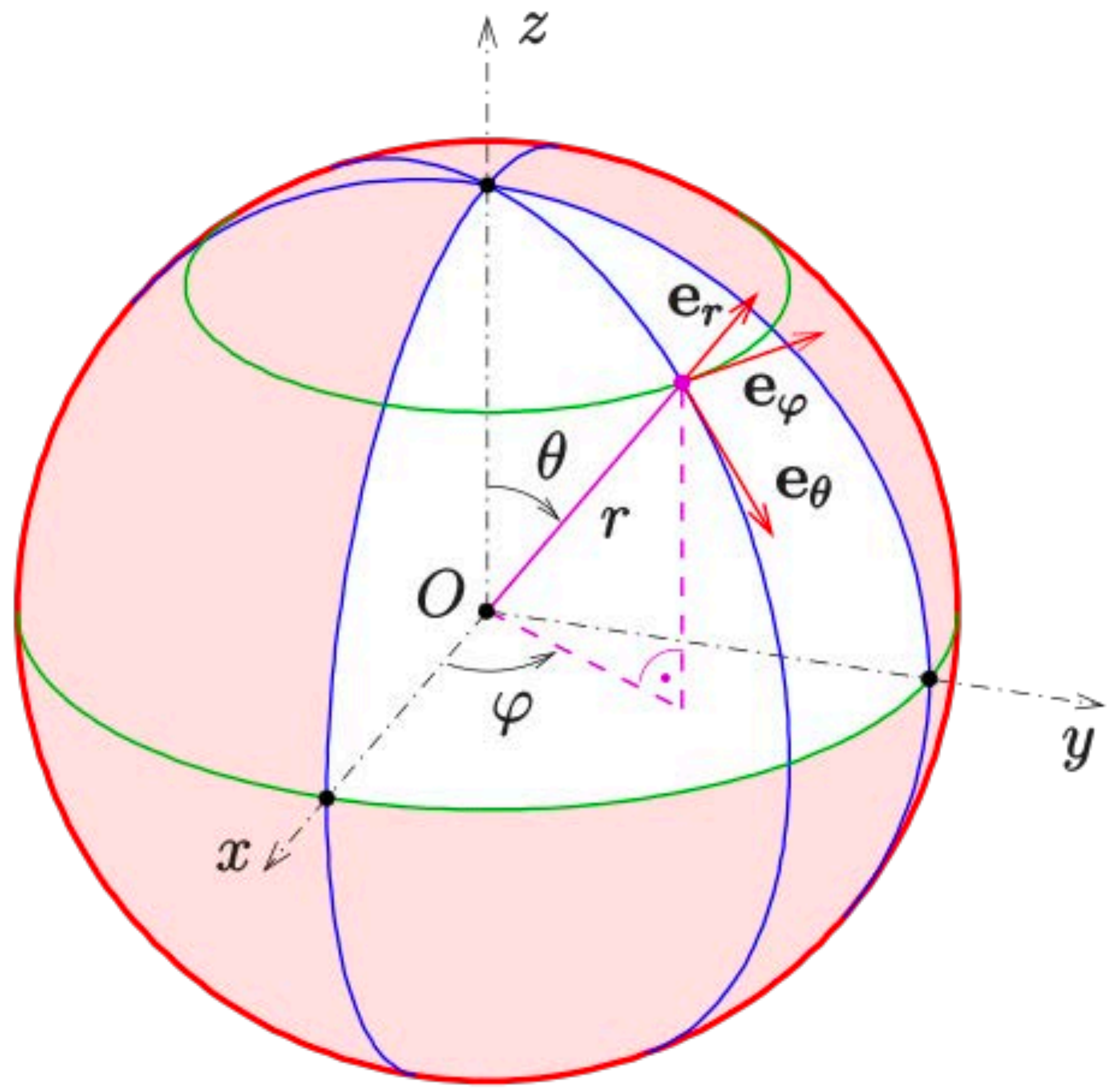
The flavor dynamics driven by the velocity distribution of $\mathsf{D}_{\vec{v}}$

2. Gyroscopic pendulum in flavor space

- By solving the EOMs for $\vartheta(t)$ and $\varphi(t)$, one finds:

$$\dot{\varphi} = \mu \, \frac{2\lambda\sigma}{1+\cos\vartheta} \, ,$$

$$\dot{c}_\vartheta^2 = \mu^2\lambda^2\, 2\left(1-c_\vartheta\right)^2\left(1+c_\vartheta-2\sigma^2\right)$$



- For fixed μ, the pendulum is fully described by natural frequency λ and the spin parameter σ

$$\lambda = \sqrt{\gamma G R/\mu} \qquad\qquad S = 2\lambda\sigma$$

2. Gyroscopic pendulum in flavor space

- Take $c_\vartheta = \cos\vartheta$ as independent variable:

$$\dot{c}_\vartheta^2 = \mu^2\lambda^2\, 2\big(1 - c_\vartheta\big)^2\big(1 + c_\vartheta - 2\sigma^2\big)$$

- In the region where $c_\vartheta = 1$, the condition for instability reads $\sigma < 1$, and for larger values solution is stable.

- For unstable solutions, the pendulum oscillates between its initial position and a minimal value given by

$$\boxed{\cos\vartheta_{\rm min} = -1 + 2\sigma^2}$$