Quarkyonic matter and neutron stars

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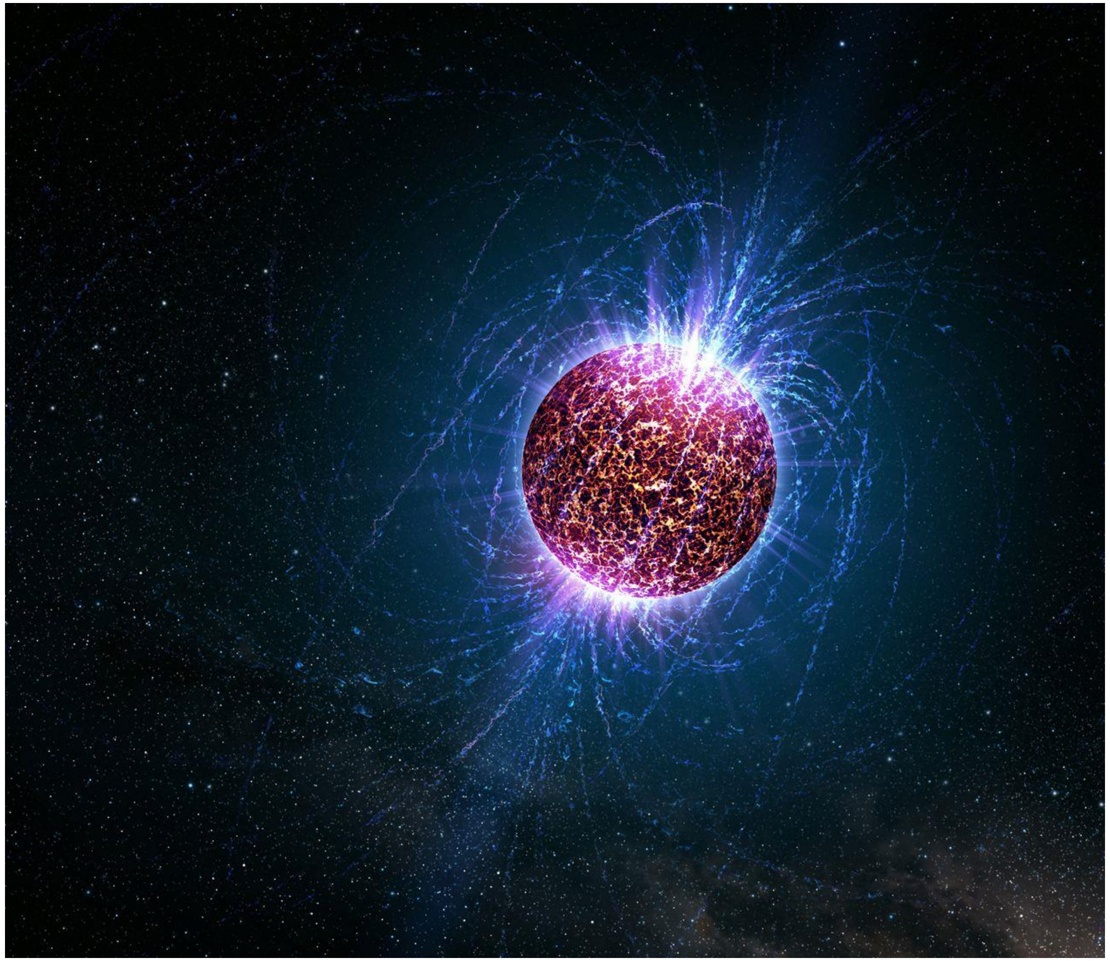
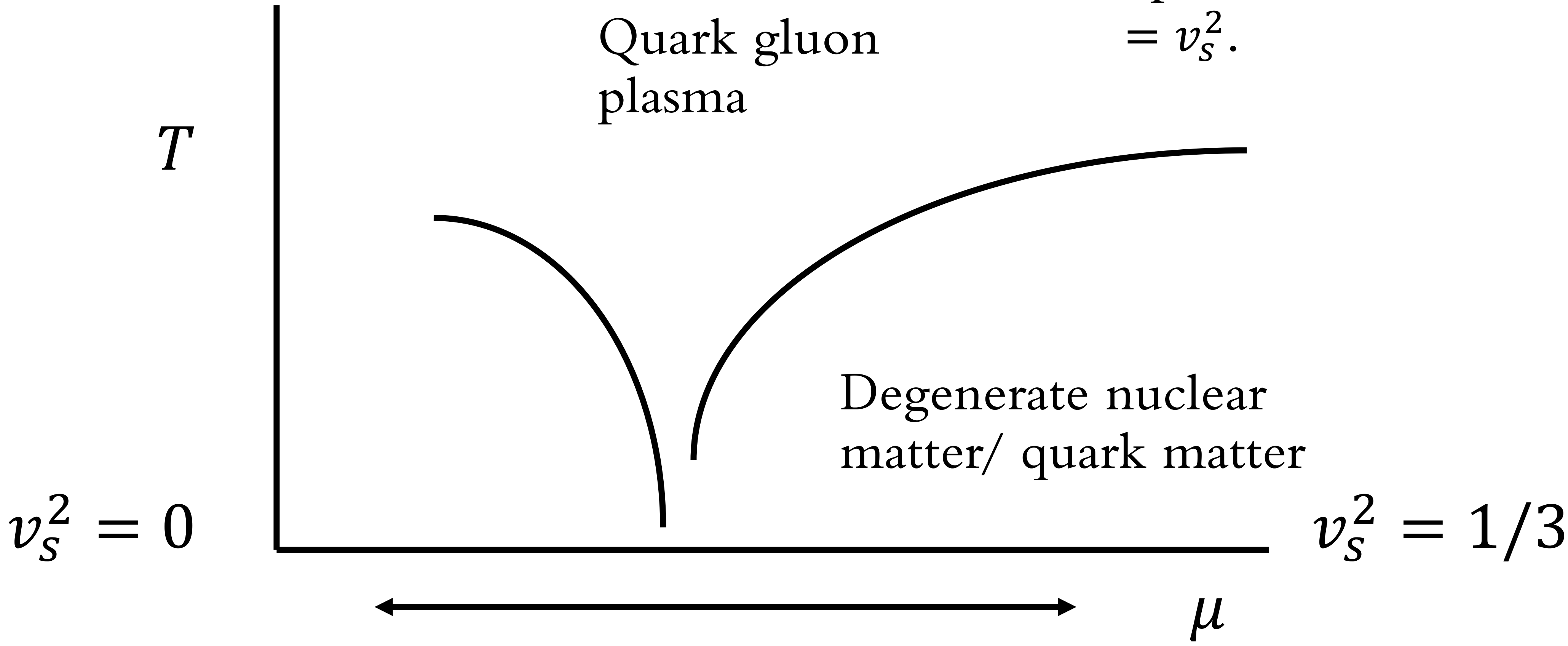
arXiv: 1908.04799, 2002.11133, 2011.04681



N3AS seminar, Nov 2021

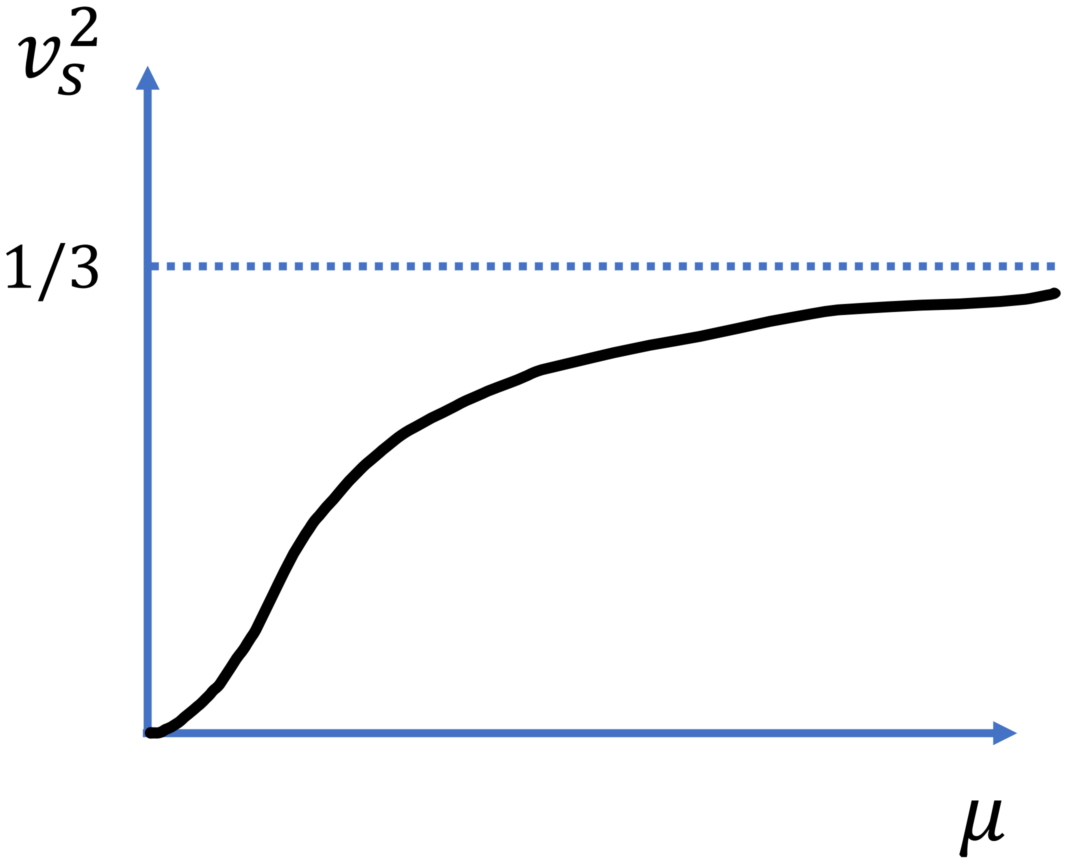
QCD phase diagram + Neutron stars

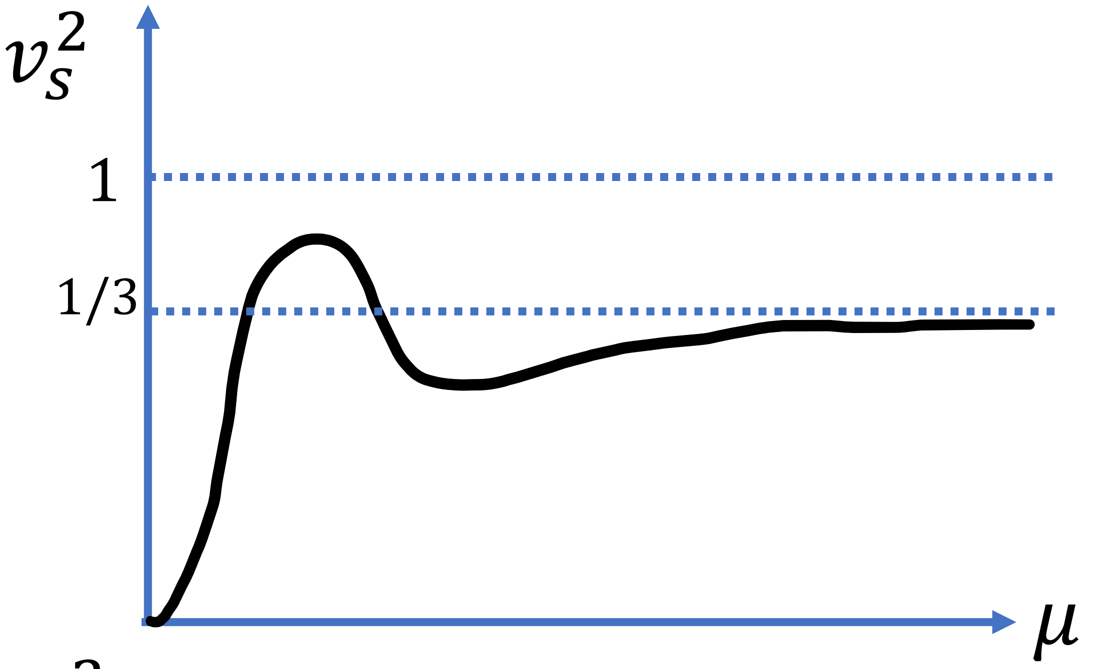
Speed of sound $= v_s^2$.

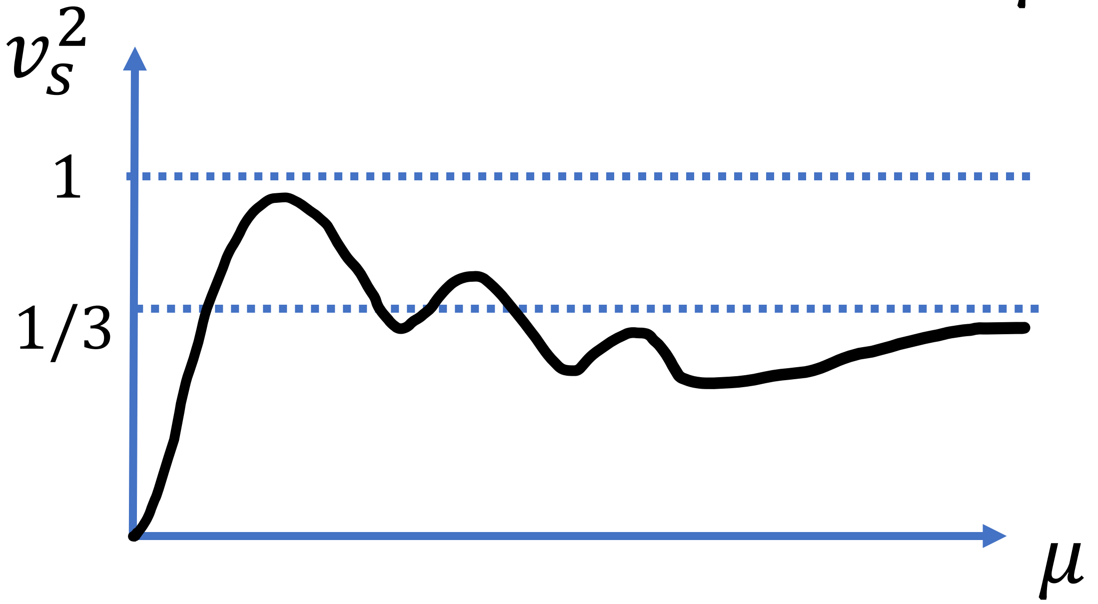


What happens here with v_s^2 here ?

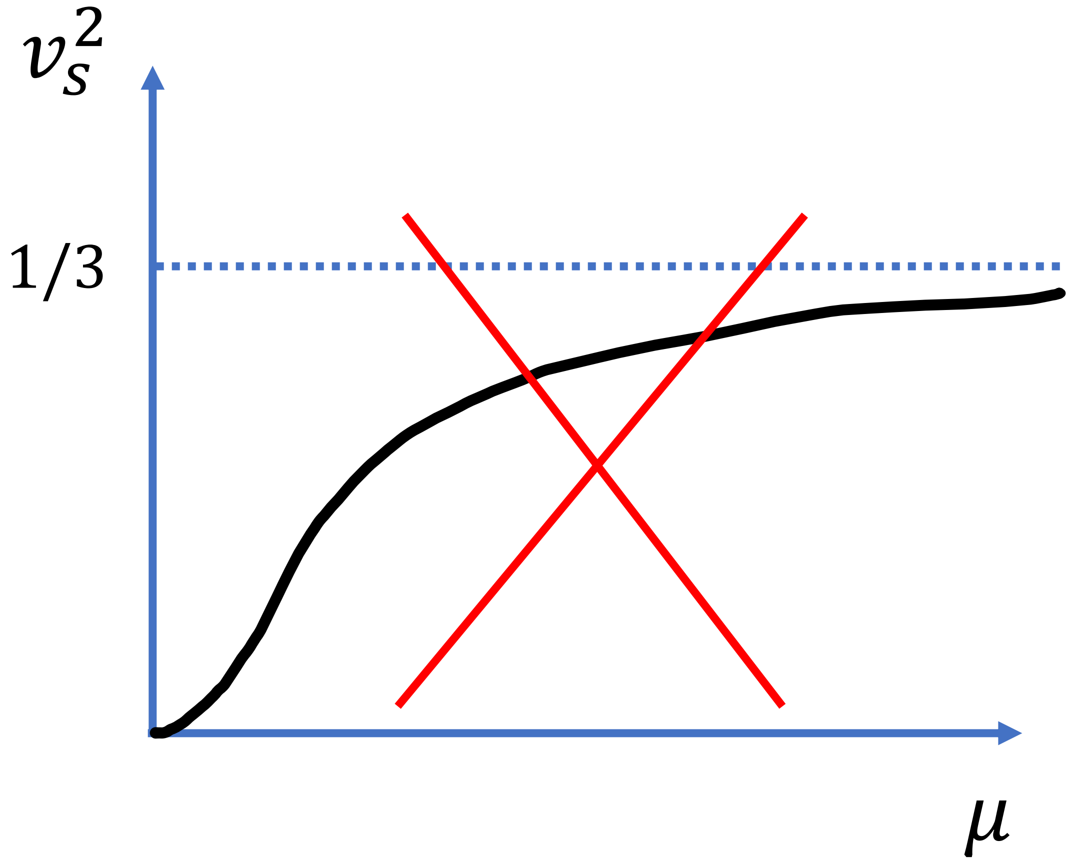
Speed of sound at finite density

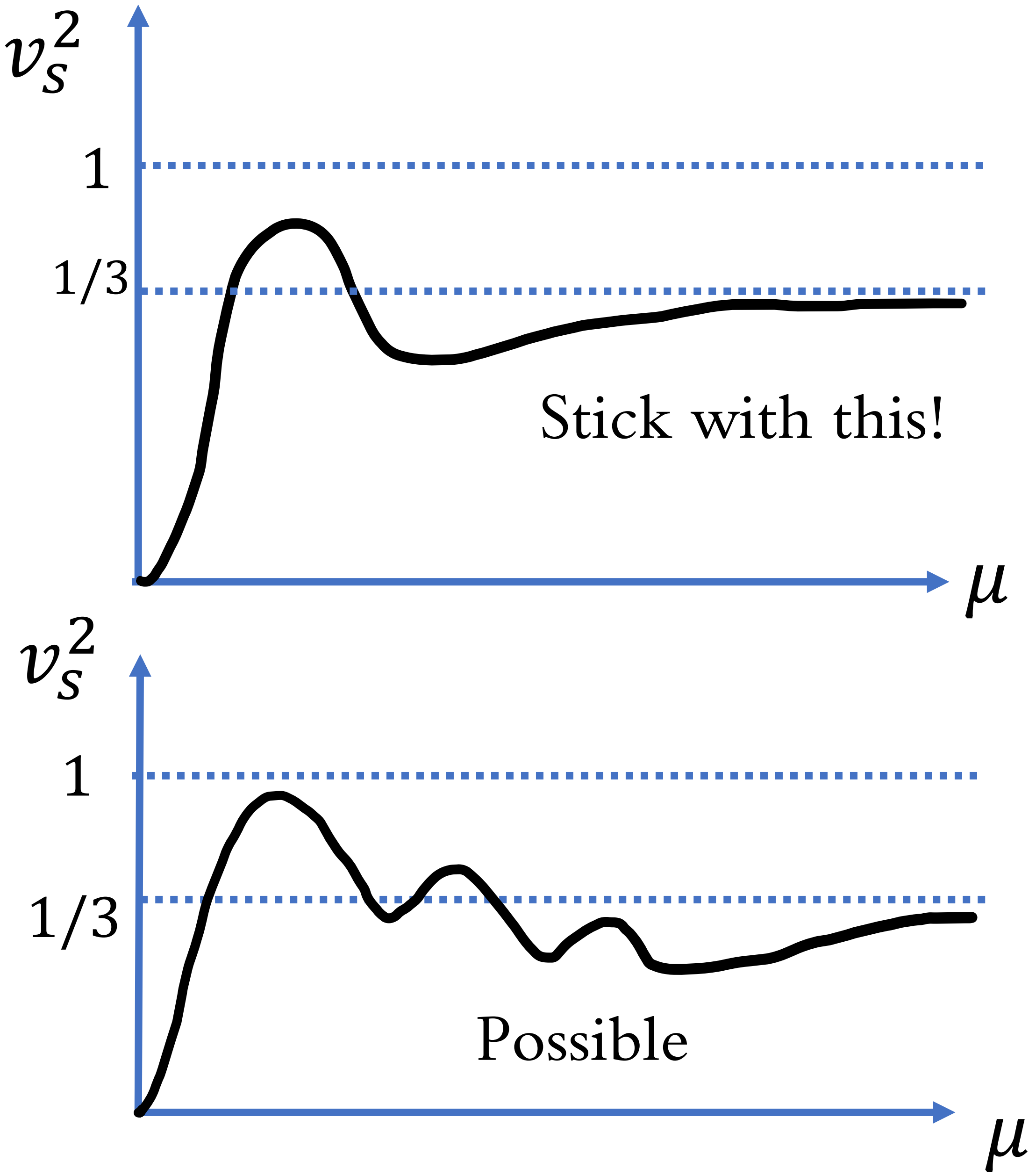






Probe with neutron stars





Bedaque, Steiner, 2014; Essick, Tews, Landry, Reddy, Holz, 2020

Speed of sound conundrum

Baryon density

$$v_s^2 = \frac{dP}{d\epsilon} = \frac{n_B}{\mu_B \frac{dn_B}{d\mu_B}} \sim 1$$

Baryon chemical potential

Nucleon mass = M_N

$$\frac{dn_B}{n_B} \sim 1 \qquad \frac{d\mu_B}{\mu_B} \sim 1 \quad \Rightarrow \quad \frac{d\mu_B}{M_N} \sim 1$$

Speed of sound conundrum

All-nucleon models achieve order 1 speed of sound using repulsive nucleon interactions.

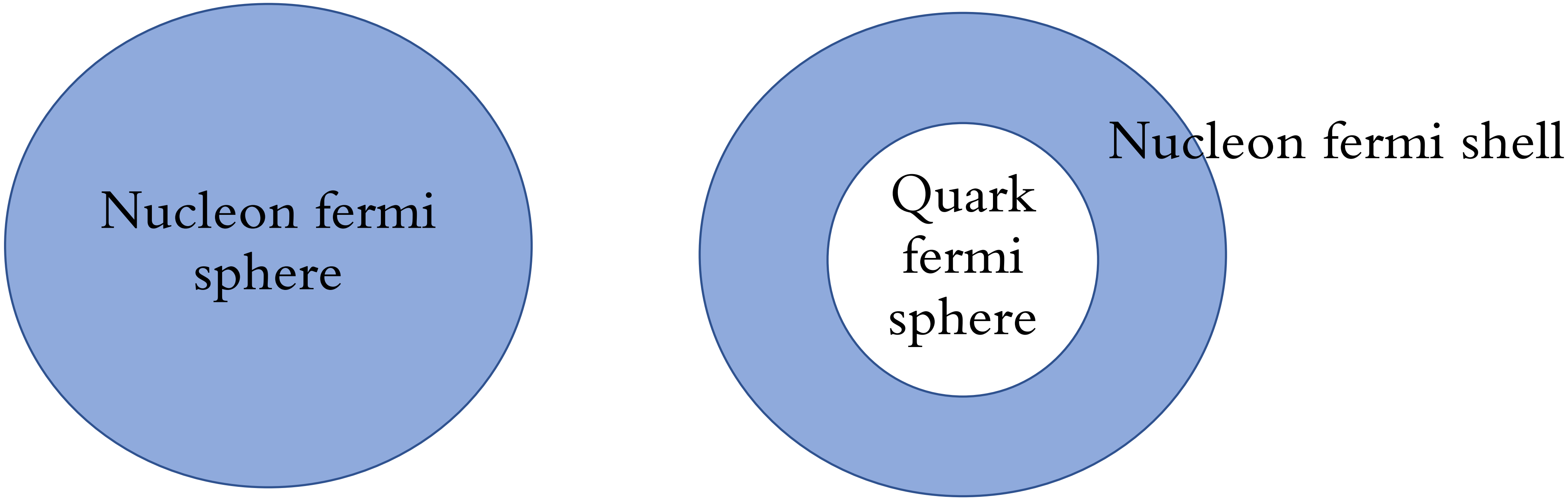
Nuclear Hamiltonians are problematic beyond twice nuclear saturation density $= 2n_0$. (Epelbaum, Hammer, Meissner, 2009)

Some exotic form matter, possibly quarks need to come in the picture at such densities.

Avoid a strong first order phase transition while introducing quarks.

Solution : quarkyonic matter ?

Possible to introduce quark Fermi liquid features in degenerate nuclear matter without a phase transition.



Increasing n_B, μ

McLerran, Pisarski, 2007

McLerran, Reddy, 2018

How does quarkyonic matter help ?

baryon density in nucleons

$$n_N = 4\int_{k_N} \frac{d^3k}{(2\pi)^3},$$

baryon density in quarks

$$n_Q = 4\frac{N_c}{N_c}\int_{k_Q} \frac{d^3k}{(2\pi)^3}$$

$n_N = n_Q = n_B =$ baryon density

k_N, k_Q as a function of n_B
have the same N_c dependence.

Constituent quark mass $= M_Q$, Nucleon mass $= M_N$, $M_N = N_c M_Q$

How does quarkyonic matter help ?

Nucleon energy density :

$$\epsilon_N = 4\int \left(M_N + \frac{k^2}{2M_N}\right)\frac{d^3k}{(2\pi)^3}$$

$$= M_N n_B + \frac{4\int k^2 \frac{d^3k}{(2\pi)^3}}{M_N}$$

Quark energy density :

$$\epsilon_Q$$

$$= 4N_c\int \left(M_Q + \frac{k^2}{2M_Q}\right)\frac{d^3k}{(2\pi)^3}$$

$$= M_N n_B + 4N_c^2 \frac{\int k^2 \frac{d^3k}{(2\pi)^3}}{M_N}$$

Pressure : $P = \mu_B n_B - \epsilon \sim M_N n_B - \epsilon$ Chemical potential $= \mu_B$

The increase in pressure in the case of quarks in enhanced by a factors of N_c^2.

A dynamical model.

We want quarkyonic matter to arise dynamically.

We would need the fermi distribution in question to arise via some type of energy minimization procedure.

Such that, there is only a nucleon fermi sphere at low density and then quarkyonic fermi sphere beyond a few times saturation density.

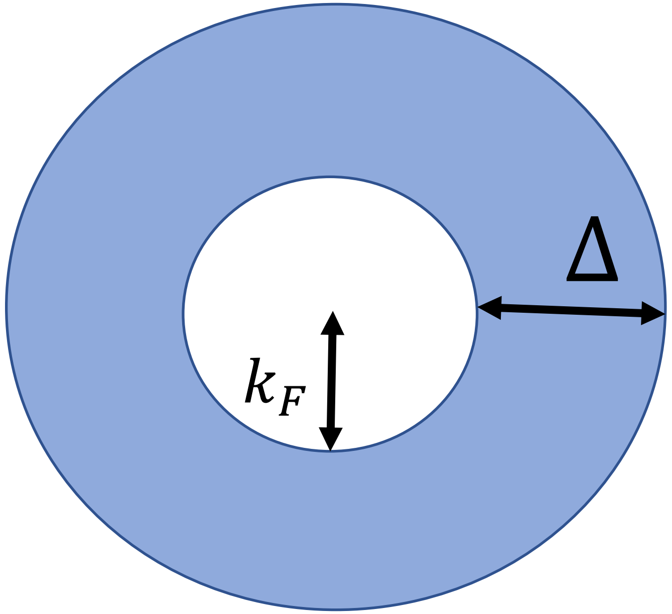
Not possible to see this from QCD directly.

Labeling the configurations :

Fix total baryon density : divide up in quarks n_Q and nucleons n_N.

$$n_B = n_N + n_Q$$

$$n_Q = \frac{2}{3\pi^2} k_F^3$$



Nucleon fermi momentum = $k_F + \Delta$

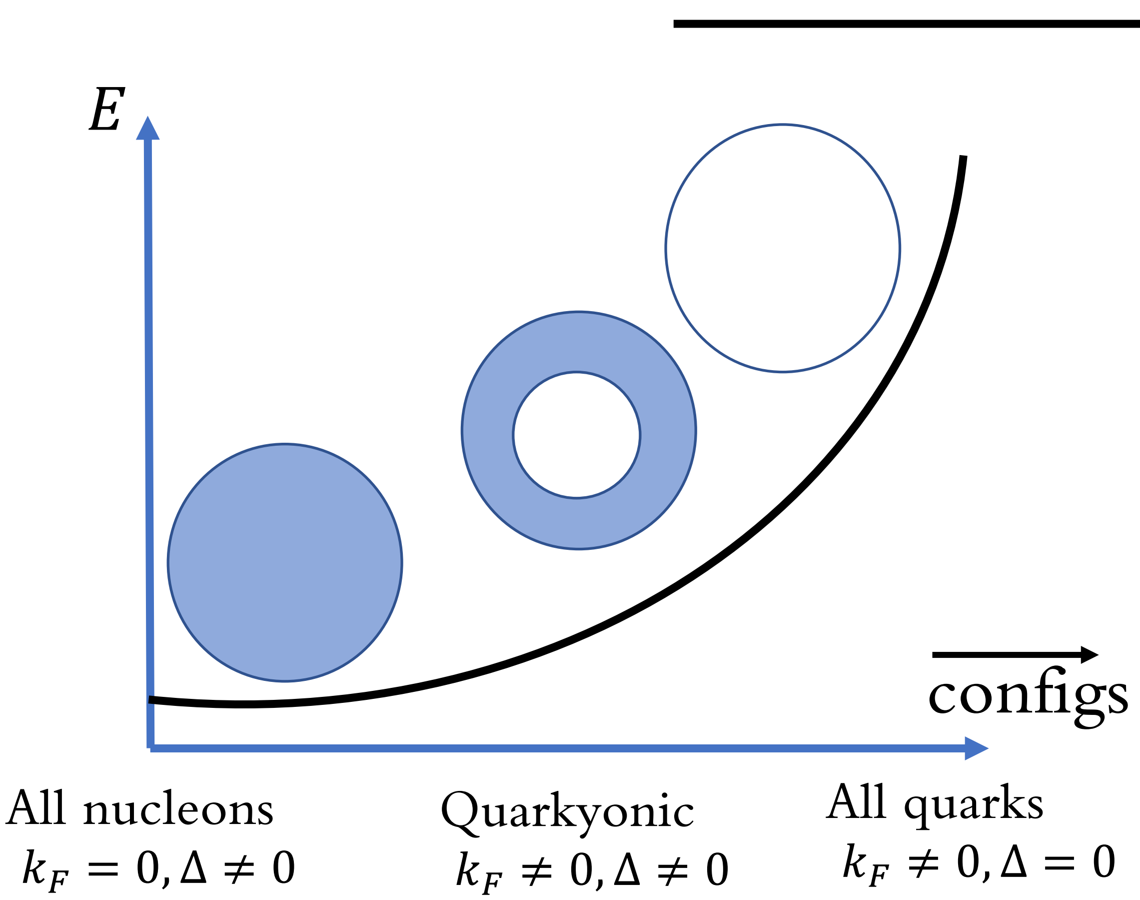
Quark fermi momentum = k_F

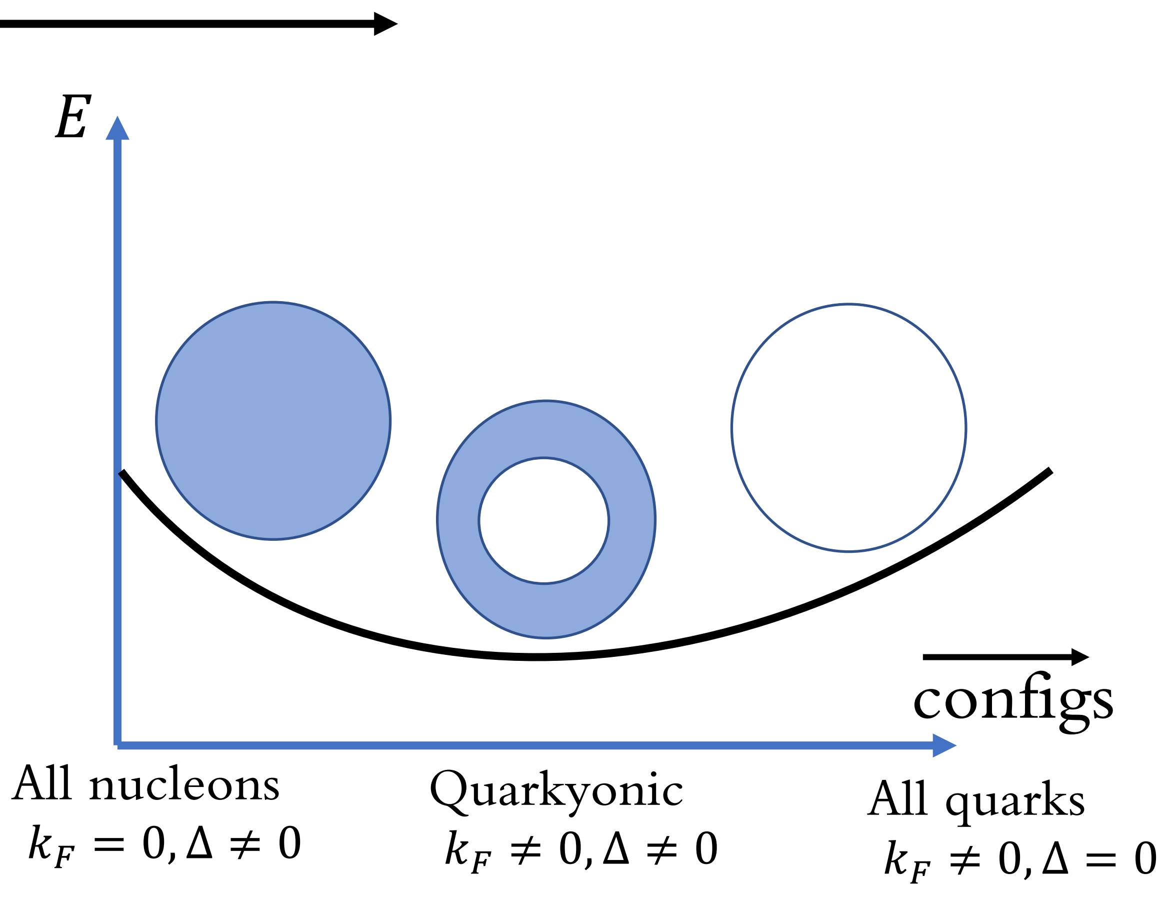
Minimizing free energy density over configs.

We will need an energy functional as a function of total baryon density.

Free energy density = E.

Increasing n_B, μ





Free energy density functional : free quarks and hard-core nucleons.

Nucleons as free
Particles, but with a finite size.

Cannot pack too many nucleons in a box of finite size.

Sets the maximum achievable nucleon density : also called Hardcore density.

Jeong, Sen, McLerran 2019

Energy functional : nucleons.

Available volume for N nucleons in a box of size V is = $V - N\,V_0$.

V_0 = size of a nucleon. Hardcore density $n_0 = \frac{1}{V_0}$ (is a parameter : set it to few times the saturation density.)

Density $n_N = N/V$.

Excluded density $n_{ex} \equiv \frac{N}{V - N\,V_0} = \frac{n_N}{1 - \frac{n_N}{n_0}}$

Effective fermi momentum for nucleons

$$\mathrm{n_{ex}} = \frac{n_N}{1 - \frac{n_N}{n_0}} = 4\int_{k_F}^{k_F+\Delta} \frac{d^3k}{(2\pi)^3} = \frac{2}{3\pi^2}\left((k_F + \Delta)^3 - k_F^3\right)$$

For finite n_0, Δ or the nucleon fermi momentum gets pushed up to higher values.

Express Δ as a function of n_{ex} and k_F . Nucleon energy density is then given by

$$\epsilon_N = 4\left(1 - \frac{n_N}{n_0}\right)\int_{k_F}^{k_F+\Delta} \frac{d^3k}{2\pi^3}\sqrt{k^2 + M_N^2}$$

Energy functional :

$$\text{Energy density} = 4\left(1-\frac{\mathrm{n_N}}{\mathrm{n_0}}\right)\int_{k_F}^{k_F+\Delta}\frac{d^3k}{(2\pi)^3}\sqrt{k^2+M_N^2}+4N_c\int_0^{k_F}\frac{d^3k}{(2\pi)^3}\sqrt{k^2+M_Q^2}$$

Nucleon mass = M_N

Quark mass = M_Q

$$M_N = N_c M_Q$$

Almost free nucleons

Free quarks
(constituent)

Energy density minimization :

Minimize energy functional as a function of k_F for a particular n_B.

For low density $k_F = 0$: no inner sphere of quarks.

For high density $k_F \neq 0$: quarkyonic matter.

All nucleon vs all quark config :

$$\frac{n_N}{1-\frac{n_N}{n_0}} \sim \Delta^3 \text{ with } k_F = 0.$$

The (kinetic) energy density of all nucleon config (nonrelativistic):

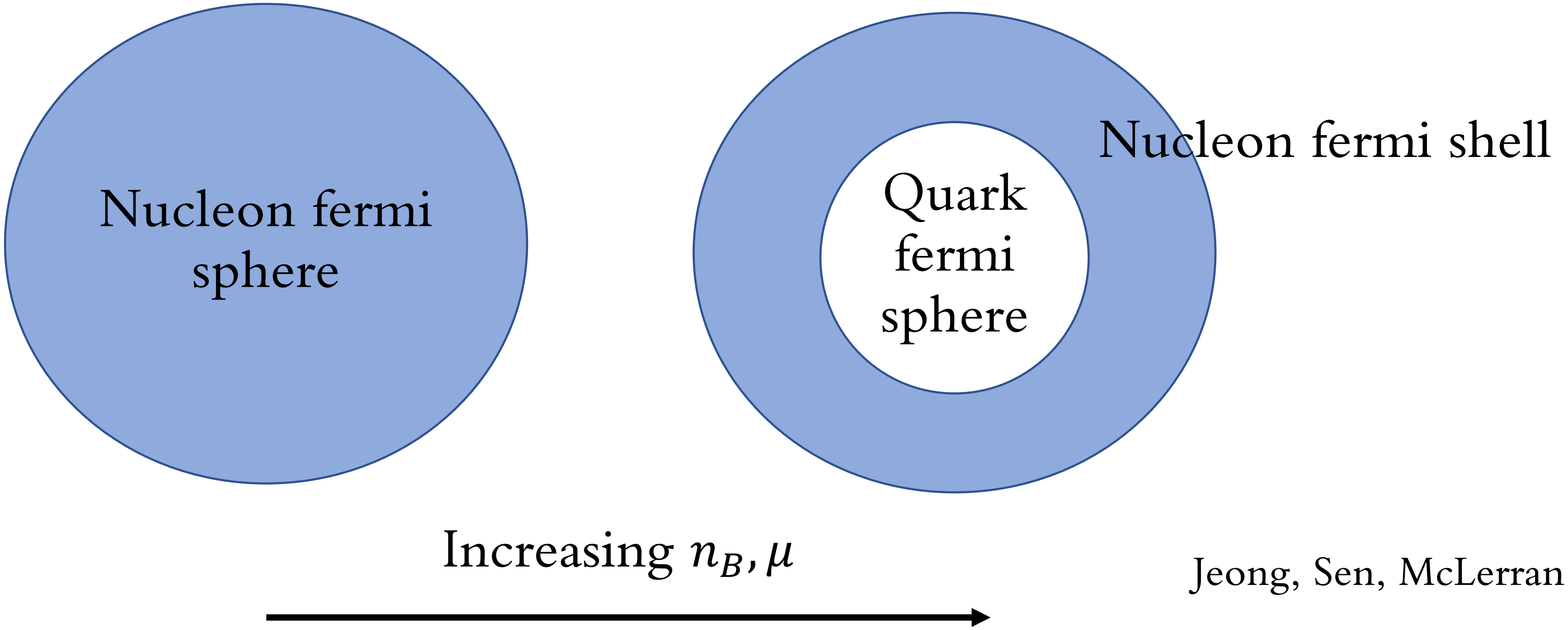
$$\sim \Delta^5 \left(1 - \frac{n_N}{n_0}\right) \sim \frac{n_N^{\frac{5}{3}}}{\left(1-\frac{n_N}{n_0}\right)^{\frac{2}{3}}} = \frac{n_B^{\frac{5}{3}}}{\left(1-\frac{n_B}{n_0}\right)^{\frac{2}{3}}}.$$

The energy density of all quark config : $\sim n_B^{\frac{5}{3}} N_c^2$.
(nonrelativistic)

Comparing : quarks lose at low density, but win for $\left(1 - \frac{n_B}{n_0}\right)^{\frac{2}{3}} \sim 1/N_c^2$

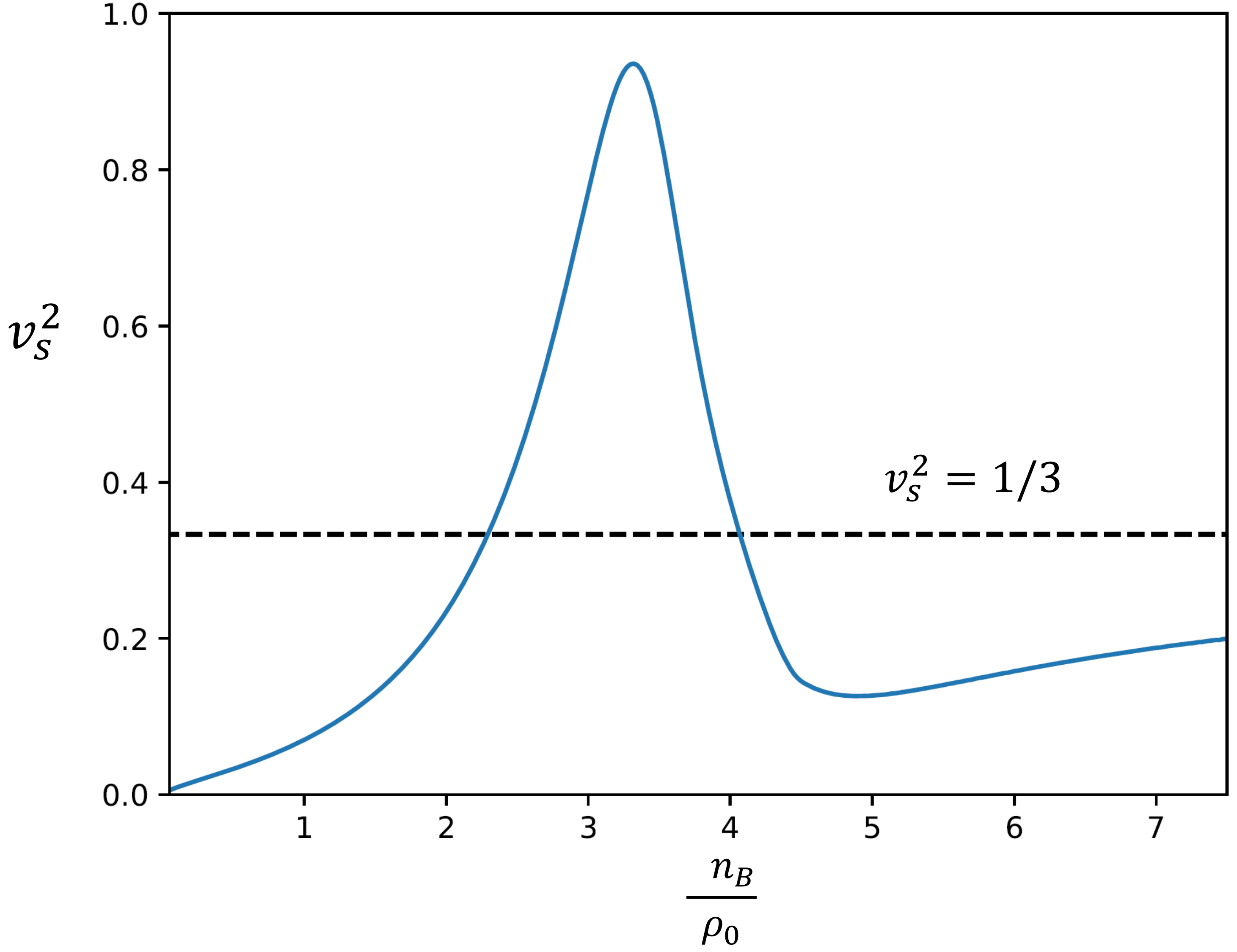
Quarkyonic matter ?

Hence, we have achieved the following:



Jeong, Sen, McLerran 2019

Speed of sound



ρ_0 = saturation density

Mass radius relations of quarkyonic NS

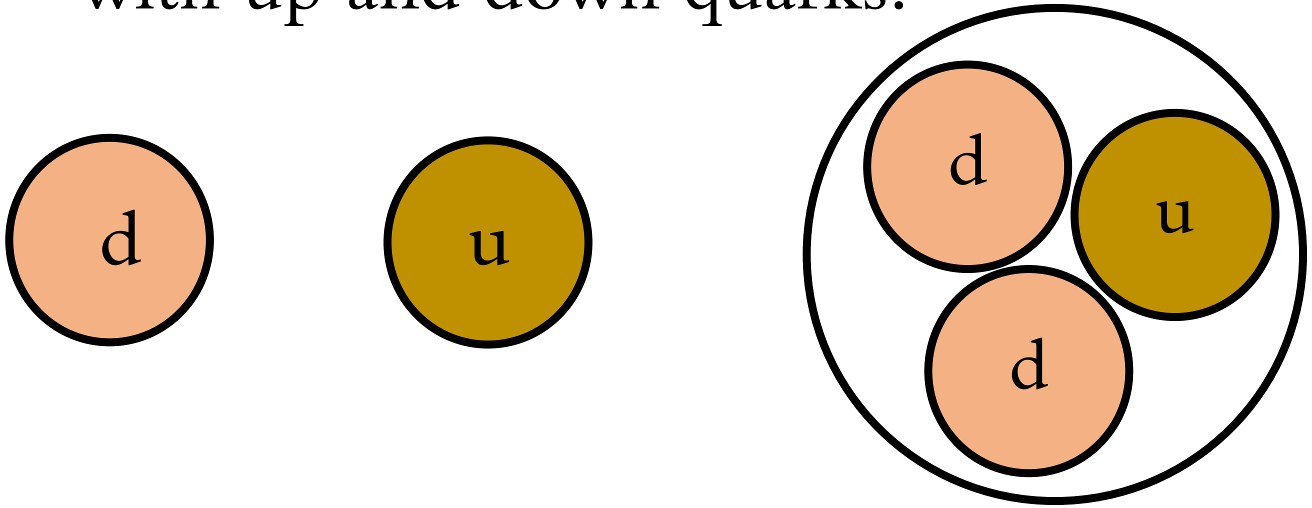
~~isospin symmetric, no nuclear interactions~~

→ pure neutron matter in hadronic sector

→ charge neutrality in quarks with up and down quarks.

↓

Add nuclear interactions



Sen, Sivertsen *Astrophys.J.* 915 (2021) 2, 109

Challenges

1. Avoid phase transition between nuclear matter and quark matter.

2. Eliminate the effect of hard-sphere interaction at low density

3. Eliminate abrupt onset of quarks.

Quarkyonic Neutron Matter

baryon density in quarks

Up and down quark density

$$n_B^Q = n_B^u + n_B^d$$

$$n_B = n_B^N + n_B^Q$$

Total baryon density

baryon density in nucleons and quarks

Up and down Fermi momenta $k_{u,d} = (3\pi^2 n_{u,d})^{\frac{2}{3}}$

Charge neutrality $n_B^d = 2n_B^u$

Neutron Fermi momentum $3k_d = k_F$

Energy functional

$$\mathrm{n}_{\mathrm{ex}} = \frac{n_B^N}{1 - \frac{n_B^N}{n_0}} = 2\int_{k_F}^{k_F+\Delta} \frac{d^3k}{(2\pi)^3} = \frac{2}{3\pi^2}\left((k_F+\Delta)^3 - k_F^3\right)$$

Energy density:

$$2\left(1 - \frac{\mathrm{n}_B^N}{\mathrm{n}_0}\right)\int_{k_F}^{k_F+\Delta} \frac{d^3k}{(2\pi)^3}\sqrt{k^2 + M_N^2} + \sum_{u,d} N_c \int_0^{k_F} \frac{d^3k}{(2\pi)^3}\sqrt{k^2 + M_Q^2}$$

Subtract contributions from here for low density + Add nuclear interactions

Energy functional

What we want at low density $2\int_{k_F}^{k_F+\Delta} \frac{d^3k}{(2\pi)^3} \sqrt{k^2 + M_N^2} + V_n$

$$V_{n_B^N} = a\, n_B^N \frac{n_B^N}{\rho_0} + b\, n_B^N \left(\frac{n_B^N}{\rho_0}\right)^2$$

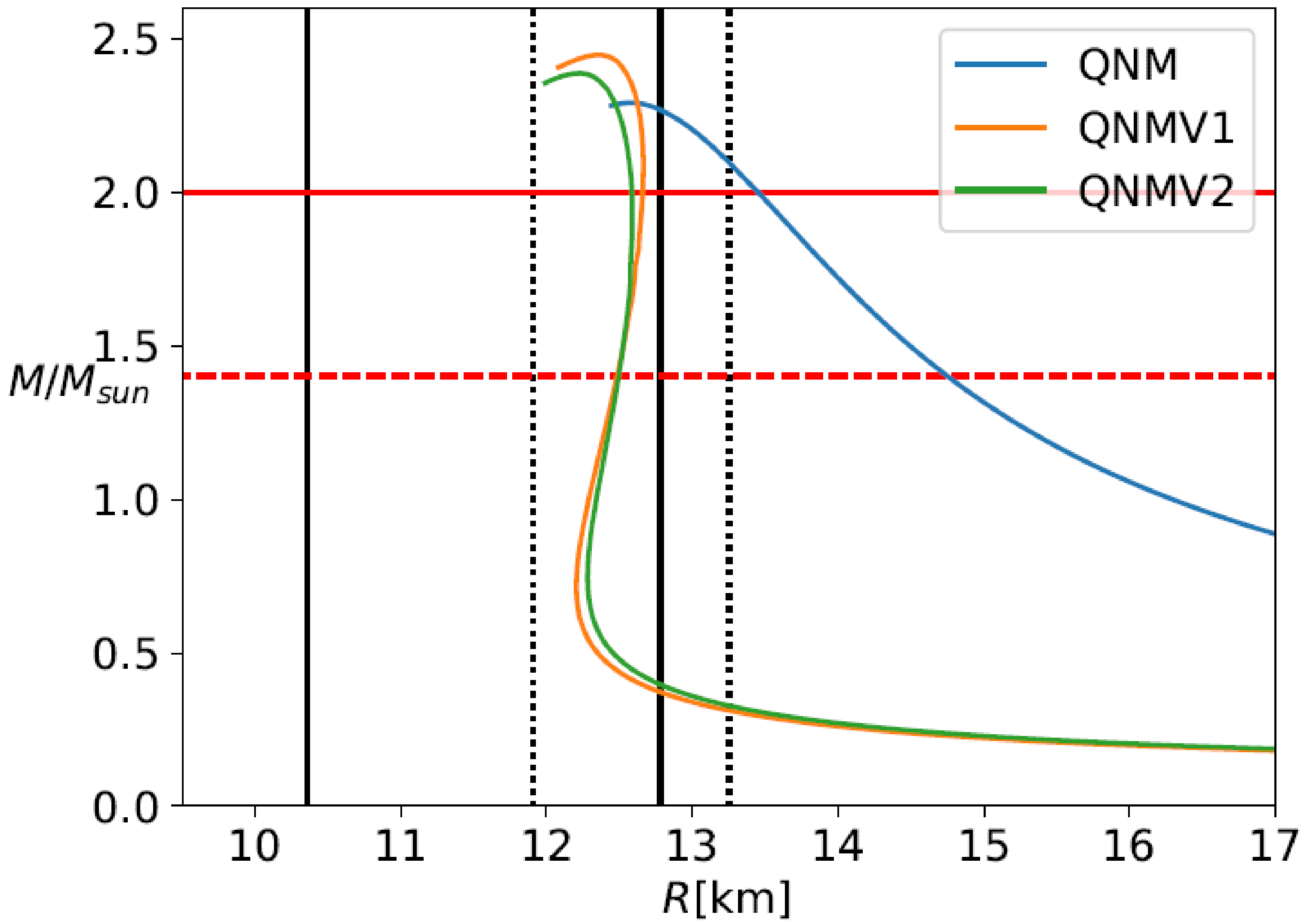
saturation density $=\rho_0 < n_0$

So, for low density we expand the hard-sphere contribution to energy density in $\frac{n_B^N}{n_0}$ and subtract off these terms from the energy functional.

$$2\left(1 - \frac{\mathrm{n}_B^N}{\mathrm{n}_0}\right) \int_{k_F}^{k_F+\Delta} \frac{d^3k}{(2\pi)^3} \sqrt{k^2 + M_N^2} - 2\int_{k_F}^{k_F+\Delta} \frac{d^3k}{(2\pi)^3} \sqrt{k^2 + M_N^2}$$

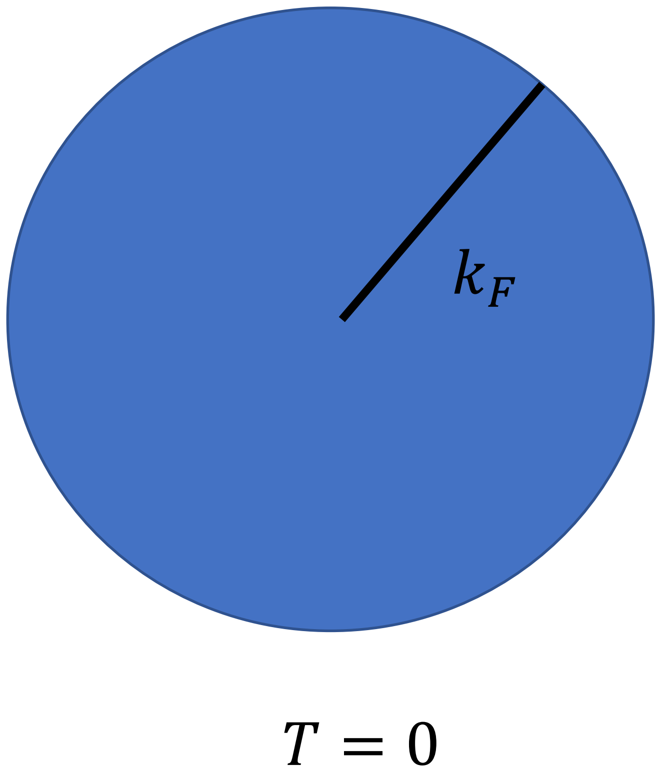
Keep form intact for singularity. But subtract off terms and add V_n to match low density nuclear physics.

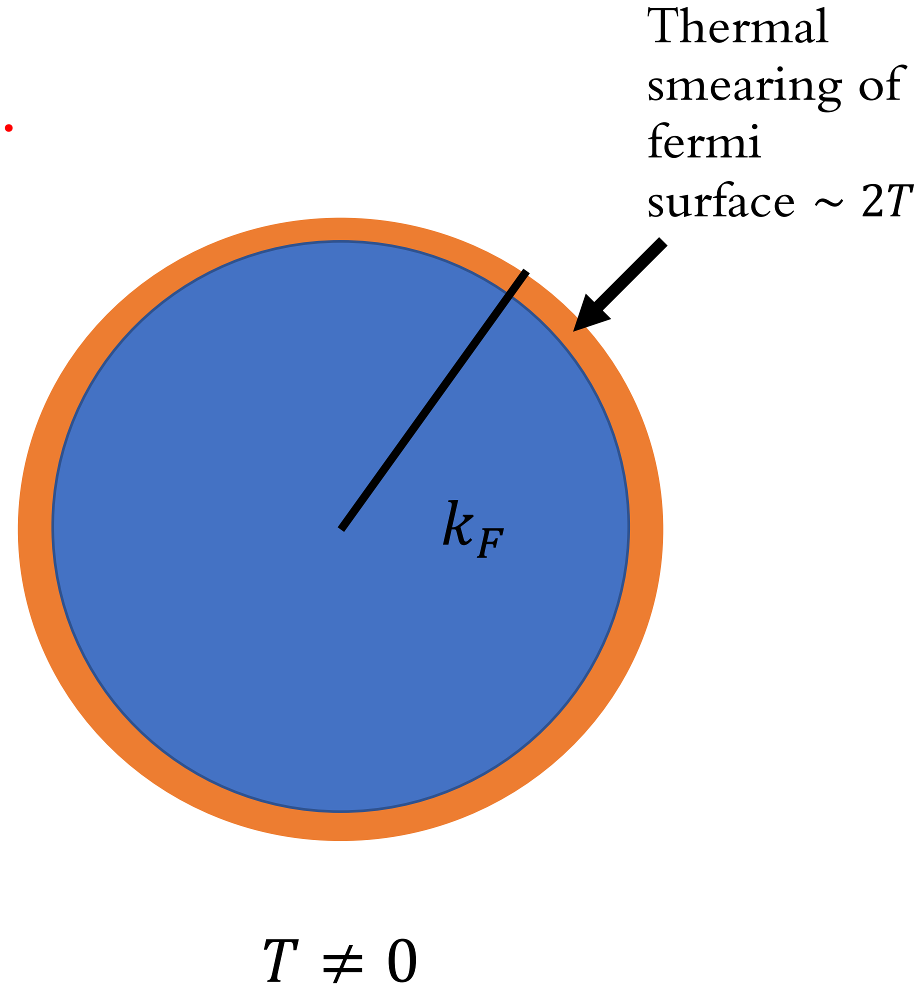
Mass radius for NS



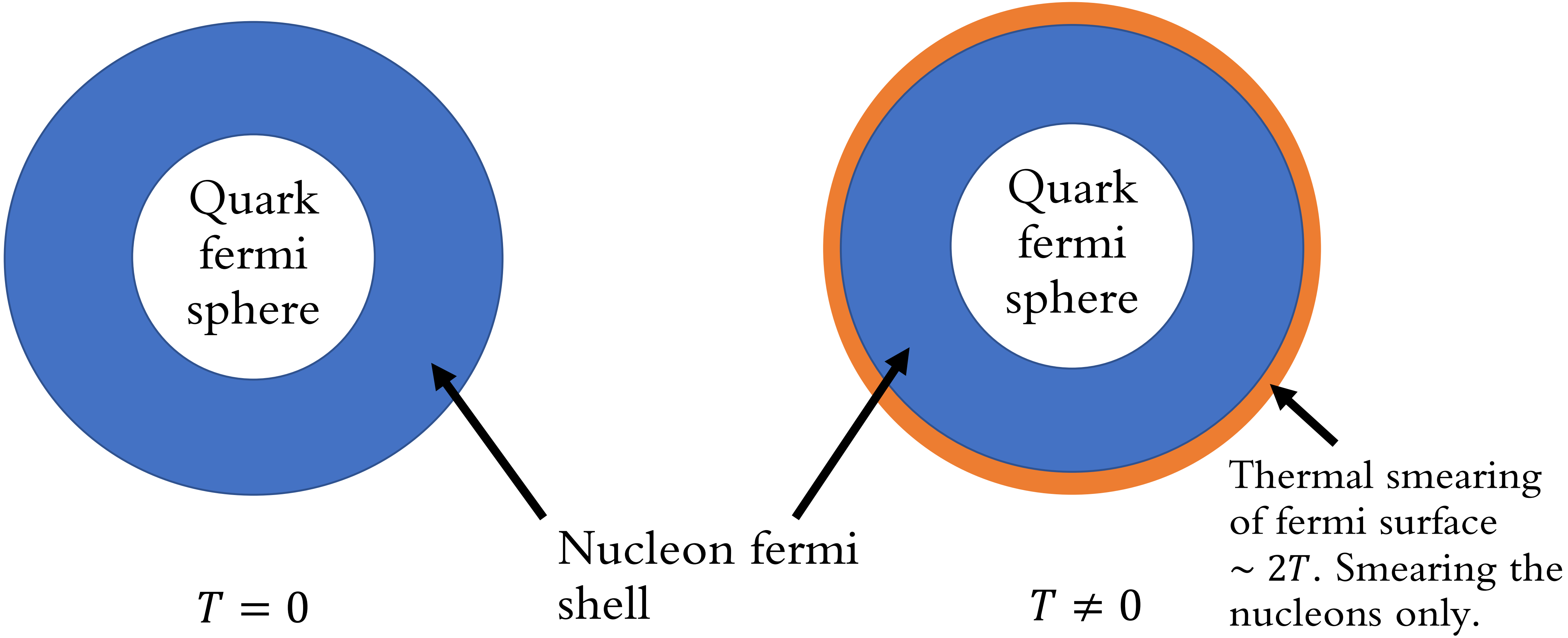
How do you introduce a finite temperature ?

Consider free fermion fermi sphere first.





Finite temperature quarkyonic matter



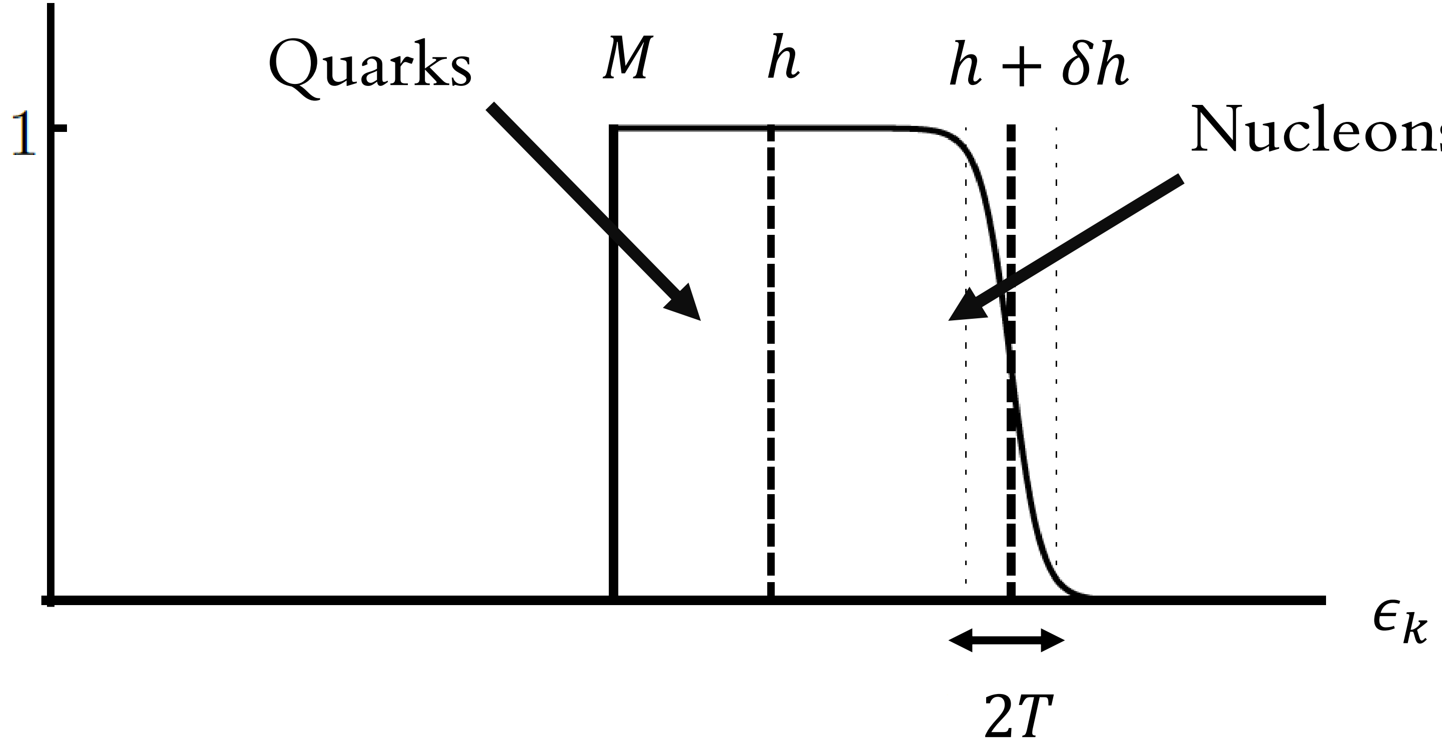
$T = 0$ $T \neq 0$

Sen, Warrington *Nucl.Phys.A* 1006 (2021) 122059

Distribution function

h = energy of the states at the bottom of the nucleon fermi shell

$h + \delta h$ = nucleon fermi energy or energy of the states at the top of the nucleon fermi shell



Energy density at finite temperature

$$\text{Energy density} = 4\left(1-\frac{\mathrm{n_N}}{\mathrm{n_0}}\right)\int\frac{d^3k}{(2\pi)^3}\sqrt{k^2+M_N^2}\,f_Q(k)$$

$$+4N_c\int\frac{d^3k}{(2\pi)^3}\sqrt{k^2+M_Q^2}f_N(k)$$

$$f_Q(k)=\theta\left(\frac{h}{N_c}-\epsilon_Q(k)\right)\frac{1}{\left(1+e^{\beta\left(M_Q+\frac{k^2}{2M_Q}-\frac{h+\delta h}{N_C}\right)}\right)}\;;\epsilon_Q(k)=M_Q+\frac{k^2}{2M_Q}$$

$$f_N(k)=\theta(\epsilon_N(k)-h)\frac{1}{\left(1+e^{\beta\left(M_N+\frac{k^2}{2M_n}-(h+\delta h)\right)}\right)}\;;\epsilon_N(k)=M_N+\frac{k^2}{2M_N}$$

Next ingredient: entropy

At finite temperature, minimizing the energy density E is not sufficient.

We need to minimize $E - Ts$ where T is the temperature and s is the entropy density.

So, we need an entropy functional (just like we needed the energy functional).

Entropy functional

μ = Chemical potential, $\beta = \frac{1}{T}$.

Free fermion entropy density:

$$s = \int \frac{d^3k}{(2\pi)^3}\left(\beta(\epsilon(k)-\mu)f(k) + \mathrm{Log}(1+\mathrm{e}^{-\beta(\epsilon(k)-\mu)})\right)$$

Entropy density = $s_Q + s_N$ with

$$s_Q = 2N_c \int \frac{d^3k}{(2\pi)^3}\theta\left(\frac{h}{N_c} - \epsilon_Q(k)\right)\left(\beta\xi_Q(k)f_Q(k) + \mathrm{Log}(1+e^{-\beta(\xi_Q(k))})\ \right)$$

$$\frac{s_N}{1-\frac{n_N}{n_0}} = 2\int \frac{d^3k}{(2\pi)^3}\theta(\epsilon_N(k)-h)\left(\beta\xi_N(k)f_N(k) + \mathrm{Log}(1+e^{-\beta(\xi_N(k))})\ \right)$$

$$\xi_Q(k) = \frac{k^2}{2M_Q} + M_Q - \frac{h+\delta h}{N_c} \qquad \xi_N(k) = \frac{k^2}{2M_N} + M_N - (h+\delta h)$$

What happens to quark onset at finite temperature ?

Employ a low temperature expansion, i.e. $\frac{T}{n^{2/3}/M} \ll 1.$

All nucleon configuration: $F_N(T) - F_N(0) = -\frac{1}{2}\left(\frac{2\pi}{3}\right)^{2/3}\left(1 - \frac{n_B}{n_0}\right)^{\frac{2}{3}} M n_N^{\frac{1}{3}} T^2$

All quark configuration: $F_Q(T) - F_N(0) = -\frac{1}{2}\left(\frac{2\pi}{3}\right)^{2/3} M n_B^{\frac{1}{3}} T^2$

As n_B approaches n_0 , all quark config becomes energetically even less costly at finite temperature!

Quark onset happens at lower density when temperature is increased. More quarks are produced after quark onset.

Follow up work

- Inclusion of strange quark. – (Duarte, Jeong, Hernandez-Ortiz 2003.02362).
- Beta equilibrium (Zhao, Lattimer 2004.08293).
- Finite temperature analysis of quarkyonic neutron matter?
- Beta equilibrium at finite temperature ? Transport properties ?