Comparing Supernova Models Using Past and Future Neutrino Data

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In recent years, CCSN simulations have grown sophisticated enough to cover the neutrino signal for ~ 10 s in detail

Is it feasible to distinguish between these models, using either SN 1987A neutrino data or data from a future supernova?

Neutrino Emission Models

We'll consider three models, generously provided by the Garching group

- z9.6-SFHo
- s20-SFHo
- s27-LS220

Three progenitor models, two nuclear equations of state

Neutrino Emission Phases

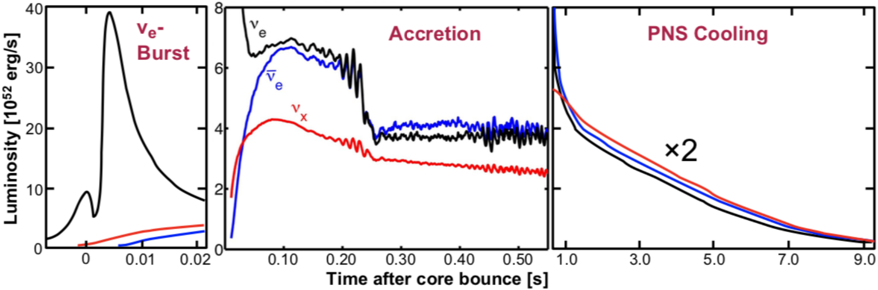


Figure from Janka (2017).

Neutrino emission occurs in three phases

- Electron neutrino breakout burst
- Accretion emission
- Proto-neutron star cooling

Predicted Neutrino Signal

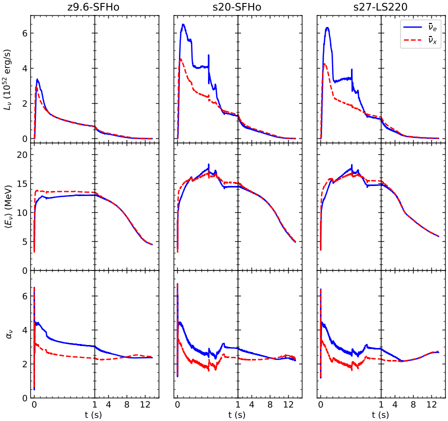
Neutrino spectrum:

$$f_{\nu_\beta}(E_\nu, t) = \frac{\left(\frac{E_\nu}{T_{\nu_\beta}}\right)^{\alpha_{\nu_\beta}} e^{-E_\nu/T_{\nu_\beta}}}{\Gamma(1+\alpha_{\nu_\beta})\, T_{\nu_\beta}}$$

with

$$\alpha_{\nu_\beta} = \frac{2\langle E_{\nu_\beta}\rangle^2 - \langle E^2_{\nu_\beta}\rangle}{\langle E^2_{\nu_\beta}\rangle - \langle E_{\nu_\beta}\rangle^2},$$

$$T_{\nu_\beta} = \frac{\langle E_{\nu_\beta}\rangle}{1+\alpha_{\nu_\beta}}$$



Past: SN 1987A

SN 1987A Data

Three detectors observed neutrinos from SN 1987A

- **Kamiokande II** (11 events)
- IMB (8 events)
- Baksan (5 events)

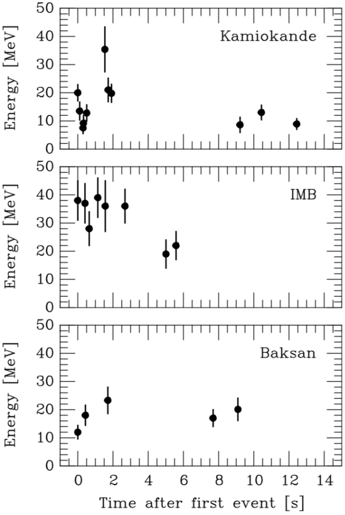


Figure from Janka (2017).

Kamiokande II

2.14 kton water Cherenkov detector

Best channel for SN neutrinos: inverse beta decay (IBD)
$\bar{\nu}_e + p \longrightarrow e^+ + n$

Only relevant species are $\bar{\nu}_e$ and $\bar{\nu}_x$

Only sensitive to accretion emission and PNS cooling

Time (s)	Energy (MeV)
0	20.0 ± 2.9
0.107	13.5 ± 3.2
0.303	7.5 ± 2.0
0.324	9.2 ± 2.7
0.507	12.8 ± 2.9
1.541	35.4 ± 8.0
1.728	21.0 ± 4.2
1.915	19.8 ± 3.2
9.219	8.6 ± 2.7
10.433	13.0 ± 2.6
12.439	8.9 ± 1.9

Predicted Event Rate

Predicted (unoscillated) flux at the detector:

$$F_{\nu_\beta}(E_\nu, t) = \frac{L_{\nu_\beta}}{4\pi d^2 \langle E_{\nu_\beta} \rangle} f_{\nu_\beta}(E_\nu, t)$$

Predicted event rate:

$$\frac{d^2 N}{dt dE} = B(E) + \int F_{\mathrm{det}}(E_e, t) \sigma(E_e) \frac{N_p \epsilon(E_e)}{\sigma_E \sqrt{2\pi}} \exp\left[-\frac{(E - E_e)^2}{2\sigma_E^2}\right] dE_e$$

where

- $B(E)$: Background rate
- N_p: Number of protons
- $\epsilon(E_e)$: Detector efficiency
- σ_E: Energy resolution

Oscillations

Incorporate oscillations with swapping fraction f:

$$F_{\mathrm{det}}(E_\nu, t) = f\, F_{\bar{\nu}_e}(E_\nu, t) + (1 - f) F_{\bar{\nu}_x}(E_\nu, t)$$

Three oscillations scenarios

- No oscillations (NO): $f = 1.0$
- MSW with normal mass hierarchy (NH): $f = 0.681$
- MSW with inverted mass hierarchy (IH): $f = 0.022$

Model Parameters

Time offset

- No way to determine when exactly the ν signal reaches the detector relative to the first event
- Therefore, syncing the model time with the time of the first detection is difficult

$$t = t_{\mathrm{det}} + t_{\mathrm{off}}$$

- Treat t_{off} as a model parameter

Supernova Distance

- Distance to SN 1987A has been measured to be $d = 51.4 \pm 1.2$ kpc
- We will also treat this as a parameter, with a restrictive prior

Bayesian Statistics

Bayesian Model Comparison

Bayes' Theorem:

$$P(M_\alpha|D) = \frac{P(D|M_\alpha)P(M_\alpha)}{P(D)}$$

- $P(M_\alpha)$ is the prior information about the models
- $P(D|M_\alpha)$ is the Bayesian evidence
- $P(D)$ is a normalization constant

Bayesian Model Comparison

Bayes' Theorem:

$$P(M_\alpha|D) = \frac{P(D|M_\alpha)P(M_\alpha)}{P(D)}$$

- $P(M_\alpha)$ is a uniform prior
- $P(D|M_\alpha) = \int d\vec{\theta}\,\mathcal{L}_\alpha(\vec{\theta})P(\vec{\theta}|M_\alpha)$ is the Bayesian evidence, $P(\vec{\theta}|M_\alpha) = P(t_{\rm off}|M_\alpha)P(d|M_\alpha)$ is the parameter prior
  - We take $P(t_{\rm off}|M_\alpha)$ to be uniform
  - $P(d|M_\alpha)$ is Gaussian, with $d = 51.4 \pm 1.2$ kpc
- $P(D) = \sum_\beta P(D|M_\beta)P(M_\beta)$ is the normalization constant

Bayes Factor

$B_{\alpha\beta}$	$\log B_{\alpha\beta}$	Strength of Evidence
1-3	0-1	Negligible
3-20	1-3	Positive
20-150	3-5	Strong
> 150	> 5	Very strong

To compare models, we use the Bayes factor

$$B_{\alpha\beta} = \frac{P(D|M_\alpha)}{P(D|M_\beta)} = \frac{P(M_\alpha|D)}{P(M_\beta|D)}\frac{P(M_\beta)}{P(M_\alpha)} = \frac{P(M_\alpha|D)}{P(M_\beta|D)}$$

Results of Bayesian Analysis

Model	d (kpc)	t_{off} (s)	$\langle N \rangle$	$P(M_\alpha \mid D)$
z9.6-SFHo (NO)	51.39	0.048	6.81	0.2807
z9.6-SFHo (NH)	51.39	0.036	7.17	0.2684
z9.6-SFHo (IH)	51.39	0.024	7.92	0.2037
s20-SFHo (NO)	51.45	0.054	19.5	0.0058
s20-SFHo (NH)	51.45	0.054	19.4	0.0060
s20-SFHo (IH)	51.45	0.026	19.3	0.0043
s27-LS220 (NO)	51.43	0.051	15.1	0.0913
s27-LS220 (NH)	51.43	0.051	15.1	0.0875
s27-LS220 (IH)	51.43	0.033	15.0	0.0523

$\frac{P(\mathrm{z9.6-SFHo(NO)}|D)}{P(\mathrm{s20-SFHo(NO)}|D)} \approx 48.4 \Rightarrow$ Strongly favored

$\frac{P(\mathrm{z9.6-SFHo(NO)}|D)}{P(\mathrm{s27-LS220(NO)}|D)} \approx 3.1 \Rightarrow$ Positively favored

$\frac{P(\mathrm{z9.6-SFHo(NO)}|D)}{P(\mathrm{z9.6-SFHo(IH)}|D)} \approx 1.4 \Rightarrow$ No preference among oscillation scenarios

SN 1987A Conclusions

We can draw limited conclusions about SN 1987A based on the data

Data seems to favor a model with a shorter accretion period

No insights about neutrino oscillations

Future Supernova

Prospects for Future SN Data

$\sim 1 - 3$ Galactic CCSN expected per century

Better detectors running now than in 1987, such as Super-K

We should see many more neutrinos from the next Galactic CCSN

Will it be enough to distinguish models?

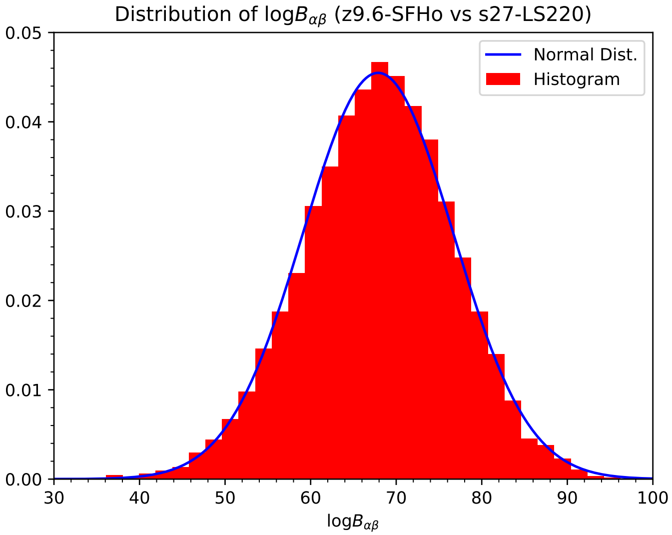
Example

Let's imagine we have an ideal water Cherenkov detector, $\mathcal{D}_I$

A SN occurs at 50 kpc, following z9.6-SFHo, with no oscillations

To simulate this, we can sample many possible datasets from the distribution for z9.6-SFHo
$\implies$ Compute the distribution of the Bayes factors

Distribution of log $B_{\alpha\beta}$



Example: Results

Comp. Model	$\log B_{\alpha\beta}$ ($t_{\rm off}$ unknown)	$\log B_{\alpha\beta}$ ($t_{\rm off}$ known)
s20-SFHo	113.7 ± 11.3	114.1 ± 11.3
s27-LS220	67.9 ± 8.8	68.2 ± 8.8

Take $\log B_{\alpha\beta} > 5.0$ to be our distinguishability threshold

We can distinguish models at a 95% confidence level with $\langle \log B_{\alpha\beta} \rangle - z\sigma[\log B_{\alpha\beta}] > 5.0$, with $z = 1.645$

$\Longrightarrow$ Both models can be distinguished from z9.6-SFHo at 50 kpc

We computed assuming both unknown and known time offsets, and find negligible difference

$\Longrightarrow$ For simplicity, take the time offset to be known

Super-Kamiokande

Large (32 kton) water Cherenkov detector

After spallation cut, background is negligible

Conservative efficiency: ~ 0.75

Imperfect energy resolution

Addition of Gd should allow for tagging of IBD events

Super-K Results

Model	z9.6-SFHo	s20-SFHo	s27-LS220
z9.6-SFHo	–	83.95 ± 9.66 75.8 ± 9.51 61.99 ± 9.0	50.34 ± 7.55 44.17 ± 7.39 34.68 ± 6.91
s20-SFHo	149.16 ± 22.93 124.56 ± 20.09 93.13 ± 16.63	–	22.9 ± 8.21 24.04 ± 8.49 26.88 ± 9.11
s27-LS220	88.6 ± 17.64 70.78 ± 15.0 50.02 ± 11.98	15.54 ± 4.6 15.98 ± 4.61 17.24 ± 4.69	–

Using Super-K rather than the ideal detector reduces $\log B_{\alpha\beta}$ values

All models are distinguishable at 50 kpc, regardless of the oscillation scenario

Conclusions

Applying Bayesian techniques, we find that SN 1987A ν data favors a short accretion period

These techniques could also be used in the event of a future Galactic CCSN to distinguish between state-of-the-art ν emission models

The three models could be distinguished up to 50 kpc, effectively anywhere in the Galaxy

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Bayesian Parameter Estimation

Bayes' Theorem:

$$P(\vec{\theta}|D, M_\alpha) = \frac{P(D|\vec{\theta}, M_\alpha)P(\vec{\theta}|M_\alpha)}{P(D|M_\alpha)}$$

- $\vec{\theta}$ are the model parameters
- $P(\vec{\theta}|M_\alpha)$ is the prior
- $P(D|\vec{\theta}, M_\alpha)$ is the likelihood function
- $P(D|M_\alpha)$ is a normalization constant

Bayesian Parameter Estimation

Bayes' Theorem:

$$P(\vec{\theta}|D, M_\alpha) = \frac{P(D|\vec{\theta}, M_\alpha)P(\vec{\theta}|M_\alpha)}{P(D|M_\alpha)}$$

- In our case, $\vec{\theta} = (t_{\text{off}}, d)$
- $P(D|\vec{\theta}, M_\alpha) = \frac{\langle N \rangle^N e^{-\langle N \rangle}}{N!} \prod_{i=1}^{N} p(E_i, t_i|\vec{\theta}, M_\alpha)$ is the likelihood
- $P(\vec{\theta}|M_\alpha) = P(t_{\text{off}}|M_\alpha)P(d|M_\alpha)$ is the prior
  - We take $P(t_{\text{off}}|M_\alpha)$ to be uniform
  - $P(d|M_\alpha)$ is Gaussian, with $d = 51.4 \pm 1.2$ kpc
- $P(D|M_\alpha) = \int d\vec{\theta}\, P(D|\vec{\theta}, M_\alpha)P(\vec{\theta}|M_\alpha)$

Bayesian Model Comparison

Bayes' Theorem:

$$P(M_\alpha|D) = \frac{P(D|M_\alpha)P(M_\alpha)}{P(D)}$$

- $P(D|M_\alpha) = \int d\vec{\theta}\, P(D|\vec{\theta}, M_\alpha)P(\vec{\theta}|M_\alpha)$ is the Bayesian evidence
- $P(M_\alpha)$ is a (uniform) prior
- $P(D) = \sum_\beta P(D|M_\beta)P(M_\beta)$ is a normalization constant

Goodness-Of-Fit Intuition

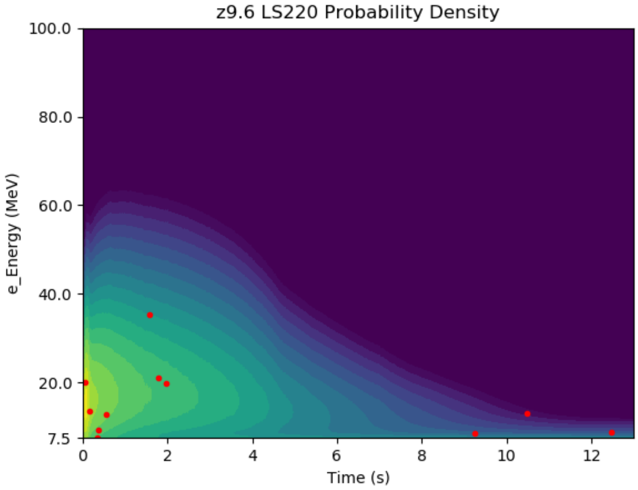

Figure 1: Plot of probability density and Kam II data.

Hypothesis Testing

- Specify null and alternative hypotheses H_0 and H_A
- Define a test statistic $\lambda \equiv \prod_i p(t_i, E_i)$
- Set threshold for and calculate p-value:

$$\text{p-value} \equiv \int_0^{\lambda^*} d\lambda \, \text{pdf}(\lambda)$$

- The p-value is the probability of observing a λ as likely or less likely than λ^*, assuming the null hypothesis is true
- If p-value $<$ threshold $= 0.050$, reject H_0 in favor of H_A

Normalized SN 1987A Comparison

Model	K	t_{off} (s)	$\langle N \rangle$	p-value	$p(M_i \| D)$
z9.6-SFHo (NO)	1.32	0.048	8.90	0.16	0.2638
z9.6-SFHo (NH)	1.26	0.036	8.96	0.20	0.2223
z9.6-SFHo (IH)	1.15	0.024	9.06	0.25	0.1359
s20-SFHo (NO)	0.46	0.054	9.14	0.19	0.0400
s20-SFHo (NH)	0.47	0.054	9.29	0.25	0.0385
s20-SFHo (IH)	0.47	0.026	9.21	0.39	0.0245
s27-LS220 (NO)	0.60	0.051	9.18	0.12	0.1108
s27-LS220 (NH)	0.60	0.051	9.16	0.17	0.1038
s27-LS220 (IH)	0.61	0.033	9.27	0.25	0.0603

Estimating the $\log B_{\alpha\beta}$ Distribution

Instead of Monte Carlo simulations, let's do some (mildly) clever mathematics

$$\langle \log B_{\alpha\beta} \rangle = \sum_N \mathrm{pois}(N; M_\alpha) \int d\vec{x}\; P(\vec{x}|M_\alpha) \log \frac{P(\vec{x}|M_\alpha)}{P(\vec{x}|M_\beta)}$$

After some tedium,

$$\langle \log B_{\alpha\beta} \rangle = \langle N \rangle_\alpha \log \tfrac{\langle N \rangle_\alpha}{\langle N \rangle_\beta} - \Delta_{\alpha\beta} + \langle N \rangle_\alpha \langle \log \tfrac{p(t,E|M_\alpha)}{p(t,E|M_\beta)} \rangle,$$

with $\Delta_{\alpha\beta} \equiv \langle N \rangle_\alpha - \langle N \rangle_\beta$

Can calculate $\sigma[\log B_{\alpha\beta}]$ similarly

Model Distributions

Previously, we included $\langle N \rangle$ predictions in our analysis

Let's now generalize our models to only the probability distributions $p(t, E|M_\alpha)$

Likelihood becomes $P(D|M_\alpha) = \prod_{i=1}^{N} p(E_i, t_i|M_\alpha)$

$\Longrightarrow$ All the model distributions are distinguishable at 10 kpc