N3AS Network for Neutrinos, Nuclear Astrophysics, and Symmetries
PHYSICS FRONTIER CENTER
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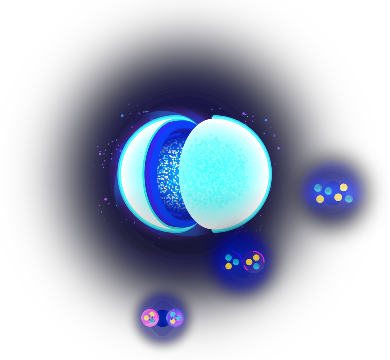
PennState

THE FATE OF TWIN STARS ON THE UNSTABLE BRANCH

November 16, 2021



Pedro Espino

Penn State University

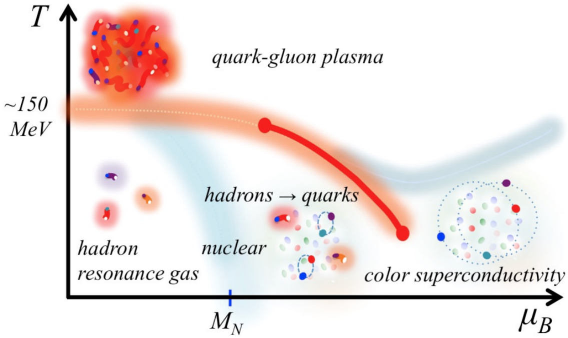
INTRODUCTION

INTRODUCTION

The **fate** of **twin stars** on the **unstable branch**

EQUATION OF STATE (EOS) OF DENSE MATTER

QUANTUM CHROMODYNAMICS (QCD) PHASE DIAGRAM



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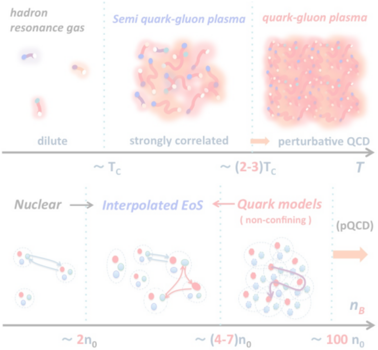

[1] Baym et al., Rept.Prog.Phys. 81 (2018) 5, 056902

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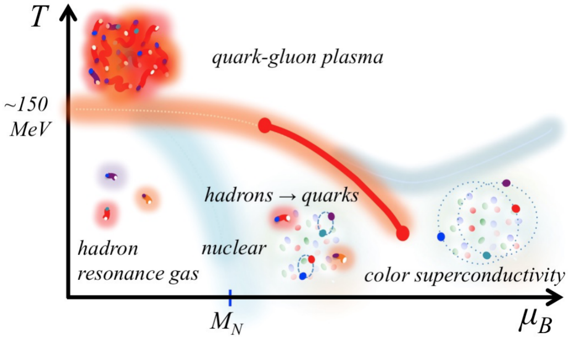

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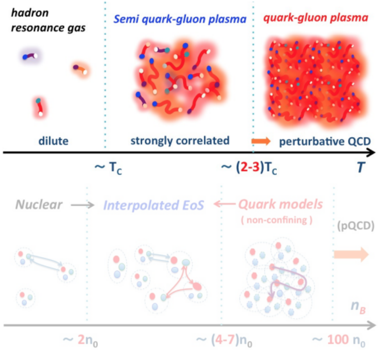

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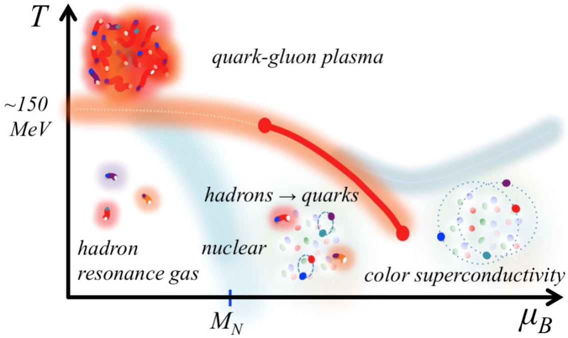

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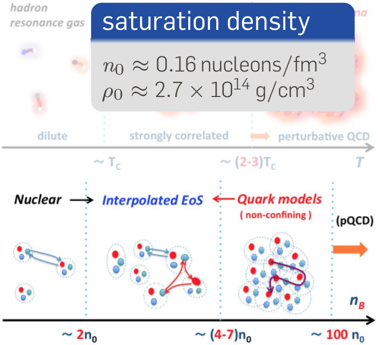


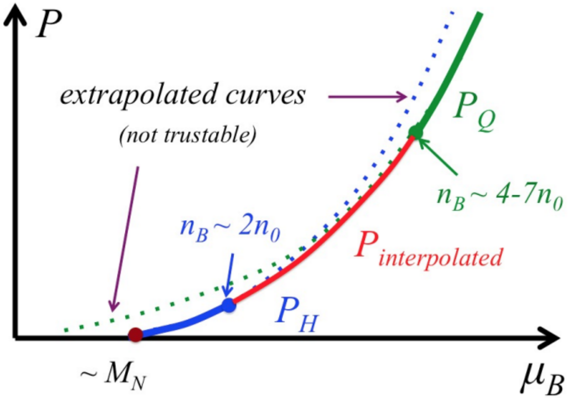
saturation density

$n_0 \approx 0.16\ \text{nucleons}/\text{fm}^3$
$\rho_0 \approx 2.7 \times 10^{14}\ \text{g}/\text{cm}^3$

[1] Baym et al., Rept.Prog.Phys. 81 (2018) 5, 056902

DENSE MATTER EOS

The problem is usually approached by treating the hadronic and quark phases under separate models

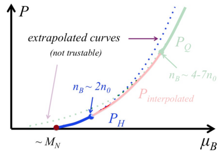


[2]

One then interpolates between the two phases and matches the EOSs under some construction

[2] Baym et al., Rept.Prog.Phys. 81 (2018) 5, 056902

HADRONIC EOS



Many-body approach

– Use understanding of symmetric nuclear/ pure neutron matter at n_0
– Determine nucleon-nucleon potentials from scattering data associated with symmetric nuclei
– Solve many-body Schrödinger equation in the presence of these potentials
– Expand in terms of proton fraction

$$E_B(x, n_B) \approx E(0, n_B) - 4(1-x) n_B S_B + \cdots , \tag{1}$$

Relativistic mean-field approach

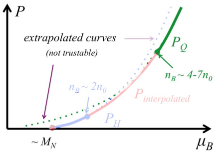
Write down EFT including relevant d.o.f (nucleons, electrons, muons, ...) interacting via exchange of mesons (scalar, vector, ...)

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu \partial_\mu - m_B)\psi + g_\sigma \sigma \bar{\psi}\psi - g_\omega \bar{\psi}\gamma^\mu \omega_\mu \psi + \cdots \tag{2}$$

Next, integrate out mediators. Assuming the mediator fields can be replaced with their mean field (static values) and extract an EOS

$$T^{\mu\nu} = \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi_\alpha)} \partial^\nu \phi_\alpha - g^{\mu\nu} \mathcal{L} \tag{3}$$

QUARK EOS



Bag models:

Treat entire quark phase as a bag of non-interacting quarks. Takes into accound the bare quark kinetic energy density ε_Q (non- interacting Fermi gas) and the bag constant B.

$$\varepsilon_Q = 2n_c n_f \int_0^{p_F} \frac{d^3p}{(2\pi^3)} |p| \tag{4}$$

Bag constant B: difference in energy density between non-perturbative (QCD ground state) and perturbative (devoid of all particles and condensates) vacua. Total energy density is then

$$\varepsilon = \varepsilon_Q + B \tag{5}$$

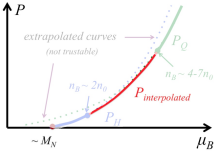
Relativistic mean-field approach (Nambu-Jona-Lasinio models)

Write down EFT including relevant d.o.f (quarks) interacting via exchange of particles (scalar, vector, ...)

$$\mathcal{L} = \bar{q}(i\gamma^\mu \partial_\mu - m_q + \mu_q \gamma^0) q + \mathcal{L}^{(4)} + \mathcal{L}^{(6)} \tag{6}$$

Extract EOS in mean-field approach

FROM HADRONS TO QUARKS



Maxwell construction:

Impose local charge neutrality within the system. This leads to strict separation of phases (sharp interface between quark and hadronic regions). Transition is isobaric and can happen over a finite range in densities, defined by

$$P^H(\mu_B^H, \mu_e^H) = P^Q(\mu^Q, \mu_e^Q) \tag{7}$$

$$\mu_B^H = 3\mu^Q \tag{8}$$

μ_e is discontinuous across interface

Gibbs construction

Impose global charge neutrality over entire system (quarks + hadrons). Allows for mixed phase regions. all quantities are continuous accross the transition

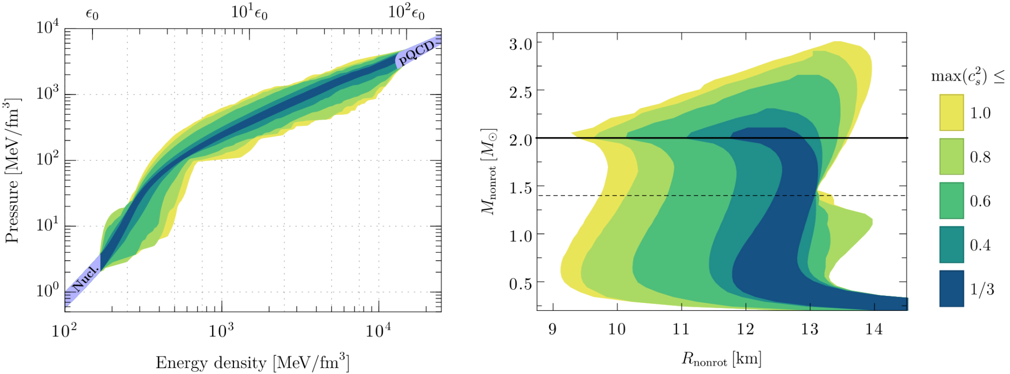
$$P^H(\mu_B^H, \mu_e^H) = P^Q(\mu^Q, \mu_e^Q) \tag{9}$$

$$\mu_B^H = 3\mu^Q \tag{10}$$

$$\mu_e^H = \mu_e^Q \tag{11}$$

Many interpolation techniques, e.g.: high-order polynomial interpolation, spectral expansions, ...

OBSERVATIONAL CONSTRAINTS



[3]

[3] Annala et al., arXiv:2105.05132v2 (2021)

TWIN STARS

HYBRID STARS -- THE THIRD FAMILY OF COMPACT STARS

PHYSICAL REVIEW VOLUME 172, NUMBER 5 25 AUGUST 1968

Equation of State at Supranuclear Densities and the Existence of a Third Family of Superdense Stars*†

ULRICH H. GERLACH‡§
Palmer Physical Laboratory, Princeton University, Princeton, New Jersey

(Received 14 December 1967; revised manuscript received 10 May 1968)

This paper presents a method for deducing the equation of state of "cold" matter at supranuclear densities from astronomical data. In particular, from the masses and the radii of a sequence of superdense stars composed of degenerate matter, one can determine the equation of state. The relationship between the equation of state and the mass-radius curve is used to construct an equation of state that allows a third family of superdense stars.

INTRODUCTION

First family: White dwarfs

$M : 0.3 - 1.0\,\mathrm{M}_\odot$

$R : 7000\,\mathrm{km}$

$C \sim 0.0001$

Second family: Neutron stars

$M : 1.0 - 2.0\,\mathrm{M}_\odot$

$R : 10 - 15\,\mathrm{km}$

$C \sim 0.2$

Third family: Hybrid stars

$M \gtrsim M_{\mathrm{NS}}$

$R \lesssim R_{\mathrm{NS}}$

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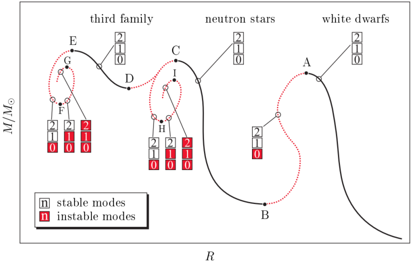

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4K. Schertler et al., Nucl. Phys. A677 (2000) 463, [astro-ph/0001467]

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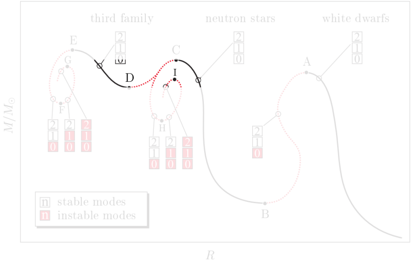

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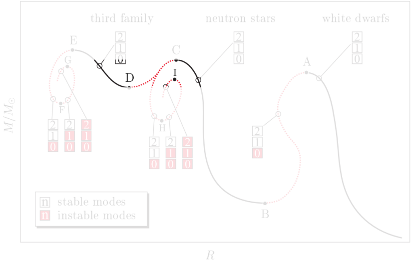

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Twin stars:

"Due to partially overlapping mass regions of the neutron star branch and the branch of the third family it is possible that non-identical stars of the same mass can exist. Such pairs are refered to as 'neutron star twins'"

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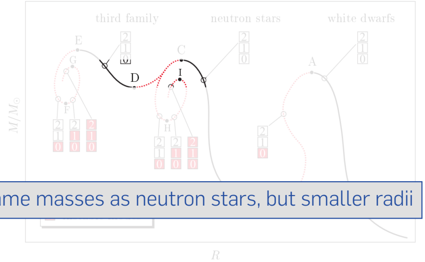

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Twin stars: Hybrid stars with the same masses as neutron stars, but smaller radii

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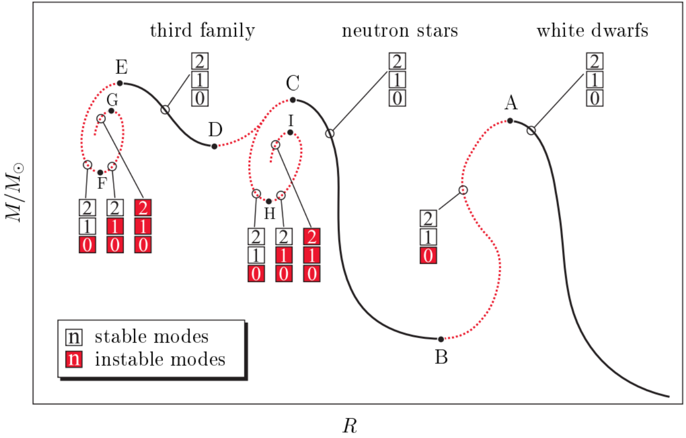
STABILITY: THE TURNING POINT THEOREM

TURNING POINT THEOREM



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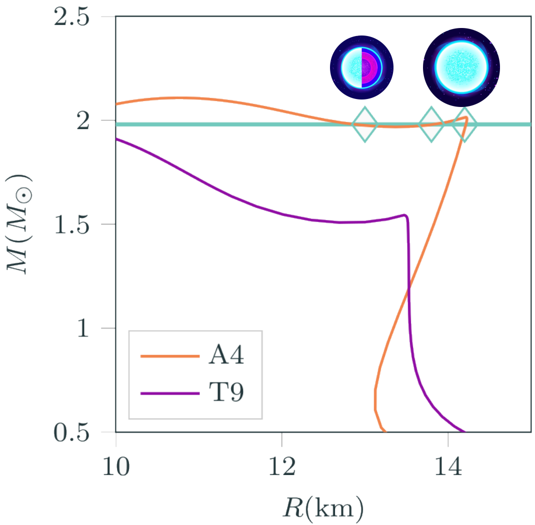
UNSTABLE TWIN STARS



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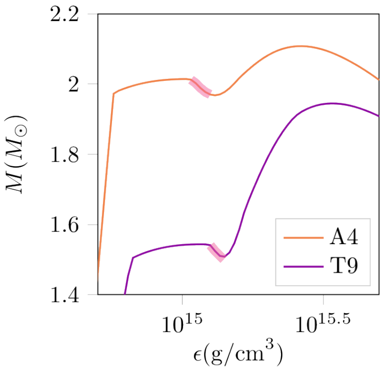


For a sequence of stars of constant entropy S and angular momentum J, instability to radial perturbations arises when

$$\left.\frac{\partial M}{\partial \epsilon_c}\right|_{S,J} \leq 0 \tag{12}$$

UNSTABLE TWIN STARS

The fate of twin stars on the unstable branch

Hybrid hadron-quark stars with the same mass as neutron stars, but with different radii.

The turning point-theorem indicates that stars which satisfy

$$\left.\frac{\partial M}{\partial \epsilon_c}\right|_{J,S} \leq 0,$$

are susceptible to radial instabilities

QUESTIONS

Some questions we are interested in answering:

- What dynamics can we expect from the evolution of unstable branch hybrid twin stars?
- Does the evolution preferably go toward the hadronic branch? hybrid branch?

- What gravitational wave signals can we expect from the evolution of unstable branch hybrid twin stars?

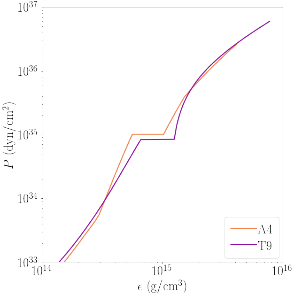
METHODS

METHODS: GENERAL

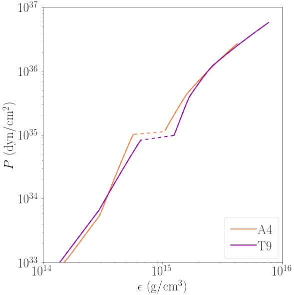
- We consider parametrizations of two hybrid hadron-quark descriptions of the dense matter EOS.

· we use a **piecewise polytropic** fit of the $P(\rho)$ functional for each of these EOSs, ensuring that stellar properties are largely unchanged

· Using these finite sound speed versions of the EOS alleviates problems associated with the numerical evolution of fluid with $c_s = 0$



$$\boxed{c_s^2 \equiv \partial P/\partial \epsilon}$$



- We construct stars with different amounts of rotation to cover as much of the solution space as possible.
- We evolve these initial data using 3D general relativistic hydrodynamics while inciting different radial perturbations.

METHODS: RADIAL PERTURBATIONS

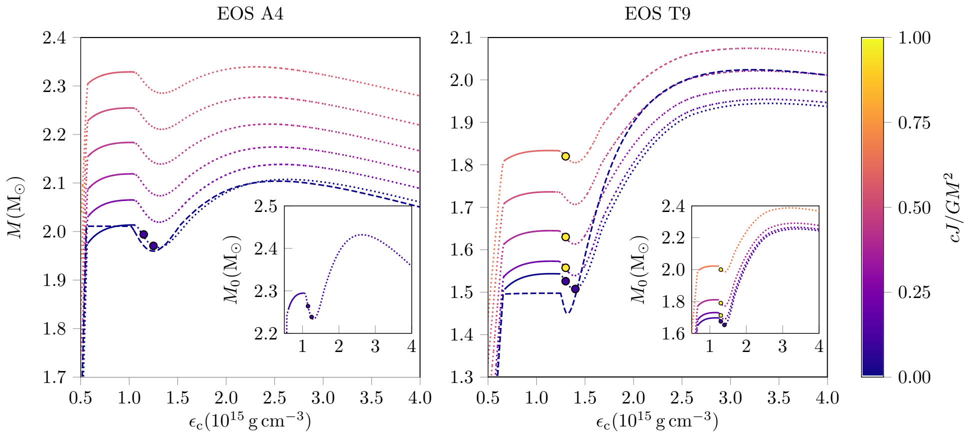
- We begin all evolutions in one of three ways:

· Equilibrium evolution: no radial perturbations are explicitly excited at the start of evolution
· Positive pressure perturbation: a radial perturbation is excited by increasing the pressure everywhere in the star by a small amount.
· Negative pressure perturbation: a radial perturbation is excited by decreasing the pressure everywhere in the star by some amount. The pressure perturbations take the following form

$$P(t=0,\mathbf{x}) \longrightarrow (1+\xi)P(t=0,\mathbf{x}), \tag{13}$$

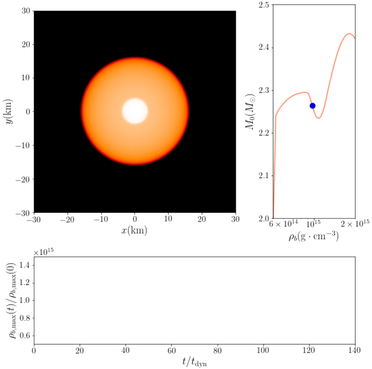
where $\mathbf{x}$ indicates the spatial coordinates, and ξ is a small number that can be either positive or negative in cases where we add or deplete pressure, respectively.

METHODS: INITIAL DATA



RESULTS: NON-ROTATING MODELS

NON-ROTATING MODELS: EQUILIBRIUM EVOLUTION AND POSITIVE PRESSURE PERTURBATION

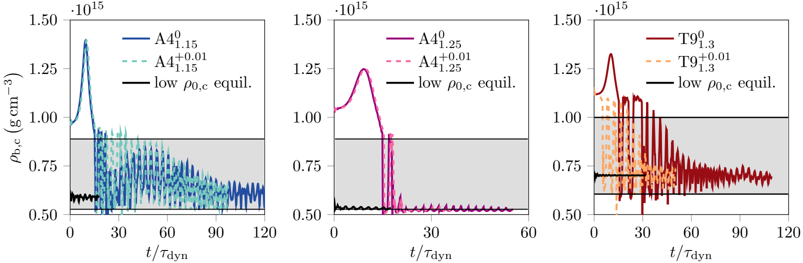


- The model initially contracts, leading to a **momentary collapse** and increasing in size of the quark core.
- As more matter enters the deconfined phase with relatively high pressure, the collapse is halted and the model **reverts into expansion**.
- Eventually the model settles close to the the twin star at lower densities.

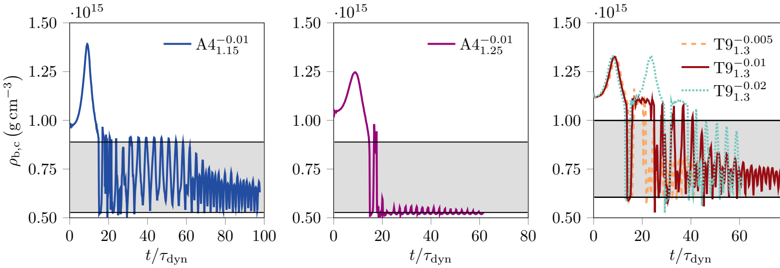
NON-ROTATING MODELS: EQUILIBRIUM EVOLUTIONS AND POSITIVE PRESSURE PERTURBATIONS

——— Equilibrium

– – – – – Positive pressure perturbation

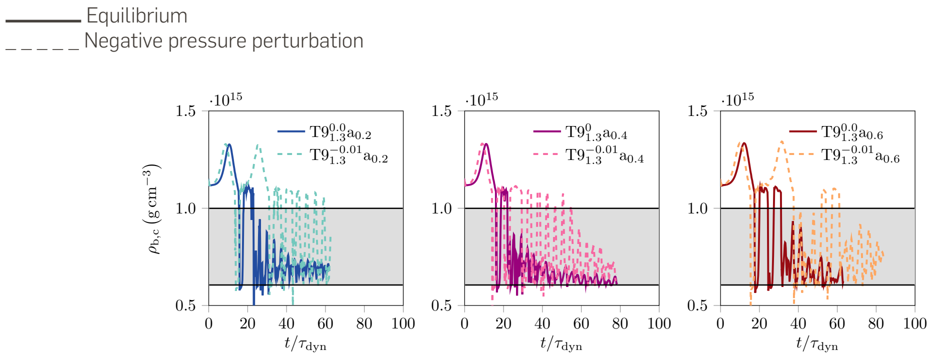


NON-ROTATING MODELS: PRESSURE DEPLETION

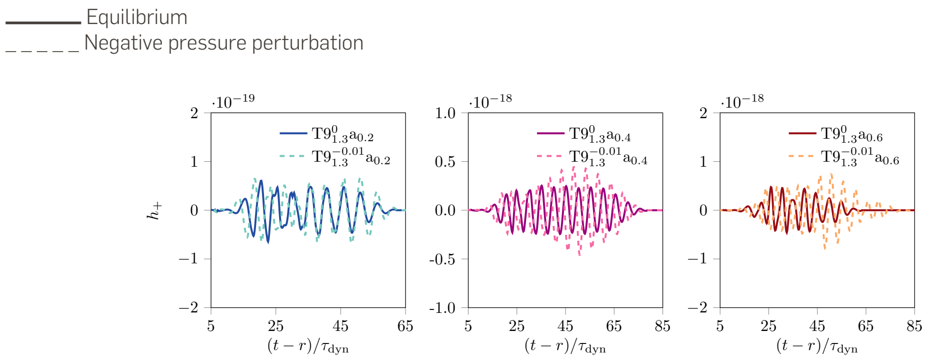


RESULTS: ROTATING MODELS

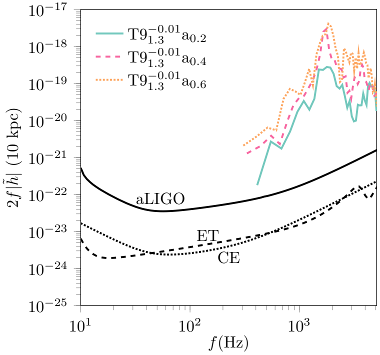
ROTATING MODELS: PRESSURE DEPLETION



ROTATING MODELS: GRAVITATIONAL RADIATION



ROTATING MODELS: DETECTABILITY OF GWS



- At $d_{\text{source}} = 10$ kpc, assuming an ideal detector orientation, the gravitational waves associated with the transition from an unstable hybrid star to a stable hadronic star are detectable.
- Likely progenitors of unstable branch hybrid stars are core-collapse supernovae (CCSN) or white dwarf- neutron star mergers. Each of these progenitor systems are expected to produce GWs which peak at lower frequencies:

$$f_{\text{peak}}^{\text{CCSN}} \sim 1 \text{ kHz} \qquad f_{\text{peak}}^{\text{WDNS}} \sim 0.01 - 0.1 \text{ Hz}$$

CONCLUSION

SUMMARY

We have explored the stability of hybrid hadron-quark stars on the unstable branch:

- → Unstable branch hybrid twin stars migrate toward the stable neutron star branch:
  - → we tested stars accross two EOSs, which sample the wide range of viable hybrid hadron quark stars
  - → we consider several perturbations of different sizes
- → In the time they transition to the stable neutron star branch, the stars undergo strong radial oscillations.
- → Rotating hybrid stars on the unstable branch may produce detectable GW bursts for close-by sources.
- → Future work:
  - → inducing strong radial oscillations in hybrid stars in dynamical capture binaries (e.g. consider eccentric hybrid star mergers)
  - → exciting strong radial oscillations/phase transitions in WDNS binaries. Considering GW signal at different frequencies.

BONUS: TURNING POINT THEOREM

A STABILITY CRITERION FOR MANY-PARAMETER EQUILIBRIUM FAMILIES

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Received 1981 November 6; accepted 1982 January 6

ABSTRACT

Theorems are established which let one detect instabilities without recourse to the usual perturbation analysis. The method applies to any system whose stable equilibria maximize a functional S at fixed values of *one or more parameters* E^{α}. It generalizes the "turning point method" by inferring instability from the behavior *in equilibrium* of the E^{α} and of their conjugate parameters $\partial S/\partial E^{\alpha}$. The "cusp catastrophe" and the black hole equilibrium family illustrate the approach. In connection with the latter, an Appendix proves that the Gibbs free energy is an analytic function of its natural arguments, as would be expected if all the equilibria belonged to a *single thermodynamic phase*.

Subject headings: hydrodynamics — instabilities

- Let $\mathcal{M}$ be a manifold of "all possible configurations and states" of a system
- S and E are C3 functions on $\mathcal{M}$
- Equilibria: X are points in $\mathcal{M}$ at which S is an extremum with respect to infinitesimal variations dX along which $dE = 0$
- Unstable equilibria: X is unstable if each of its neighborhoods contains a state of strictly greater S, at the same E
- this holds for systems which are characterized by an extremum and whose equilibria fall along a one-parameter sequence (first applied to NSs by Friedman (86))

BONUS: NEWMAN-PENROSE FORMALISM

An Approach to Gravitational Radiation by a Method of Spin Coefficients*

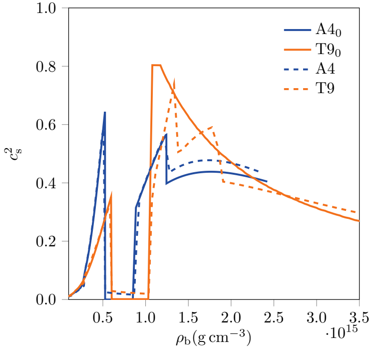
EZRA NEWMAN
University of Pittsburgh, Pittsburgh, Pennsylvania
AND
ROGER PENROSE†
Syracuse University,‡ Syracuse, New York
(Received September 29, 1961)

A new approach to general relativity by means of a tetrad or spinor formalism is presented. The essential feature of this approach is the consistent use of certain complex linear combinations of Ricci rotation coefficients which give, in effect, the spinor affine connection. It is applied to two problems in radiation theory; a concise proof of a theorem of Goldberg and Sachs and a description of the asymptotic behavior of the Riemann tensor and metric tensor, for outgoing gravitational radiation.

- Focus on $s = -2$ spin-weighted spherical harmonic decompositions of the Newman-Penrose scalar Ψ_4.
  - The coefficients of the spin-weighted decomposition are labeled $\Psi_4^{l,m}$
- Extract $\Psi_4^{l,m}$ from numerical simulations at fixed co-centric spheres increasing radii (typically ranging from $r \sim 50M$ to $r \sim 200M$.
- We compute the gravitational wave strain from

$$\Psi^4 = \ddot{h}_+ - i\ddot{h}_\times \,. \tag{14}$$

BONUS: EOS PROPERTIES



- → Both EOSs are modelled by RMF models in the hadronic phase
- → EOS A4: Originally a 4-segment piecewise polytrope above $n \sim n_0$
- → EOS T9: Quark phase based on constant sound speed ($c_s = 1$) parametrization

$$P(\epsilon) = \begin{cases} P_{\mathrm{tr}} & \epsilon_{\mathrm{tr}}^{\mathrm{T9_0}} \leq \epsilon \leq 1.9\epsilon_{\mathrm{tr}}^{\mathrm{T9_0}} \,, \\ P_{\mathrm{tr}} + c_{\mathrm{s}}^2(\epsilon - 1.9\epsilon_{\mathrm{tr}}^{\mathrm{T9_0}}) & \epsilon \geq 1.9\epsilon_{\mathrm{tr}}^{\mathrm{T9_0}} \,, \end{cases} \tag{15}$$