Mode-by-Mode Relative Binning

Nathaniel Leslie (Berkeley), Liang Dai (Berkeley),

and Geraint Pratten (Birmingham)

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Computational Challenges

Original Relative Binning

Precessing Models

“Twisting- up” Procedure

2 New Schemes

Bin Selection

Testing Methods

Target Accuracy

Results

Conclusions

Overview

Computational challenges of parameter estimation

likelihood

prior

- Goal is to map a posterior: $$P(\Phi|d) = \frac{P(d|\Phi)p(\Phi)}{P(d)}$$

parameter-independent normalization

- High dimensional parameter space:
  - 2 masses, 6 spin components, 2 sky localization angles, 3 orientation angles, distance, and time offset: 15 total parameters.
- Large number of required likelihood evaluations: $\sim 10^7 - 10^8$
- Likelihood evaluations can be expensive:
  - Runtime is dominated by model calls at each frequency: there are $T f_{\max} \sim 10^4$ frequencies for BBH merger signals (more for BNS).
  - At roughly 5 s per frequency, a single posterior can take a week or longer.

(Application/Extension: Finstad, Brown: arXiv:2009.13759)

Relative Binning

(Introduced by Zackay, Dai, Venumadhav: arXiv:1806.08792)

- Key idea: Ratio of any given waveform $h(f)$ over *fiducial waveform* $h^0(f)$ (usually the MLE) is smooth in frequency space; it can be approximated by a piecewise linear interpolant.

$$r(f) = \frac{h(f)}{h^0(f)} \approx r^0(h,b) + r^1(h,b)\ (f - f_c(b))$$

center frequency of bin b

- Compute likelihood using overlaps:

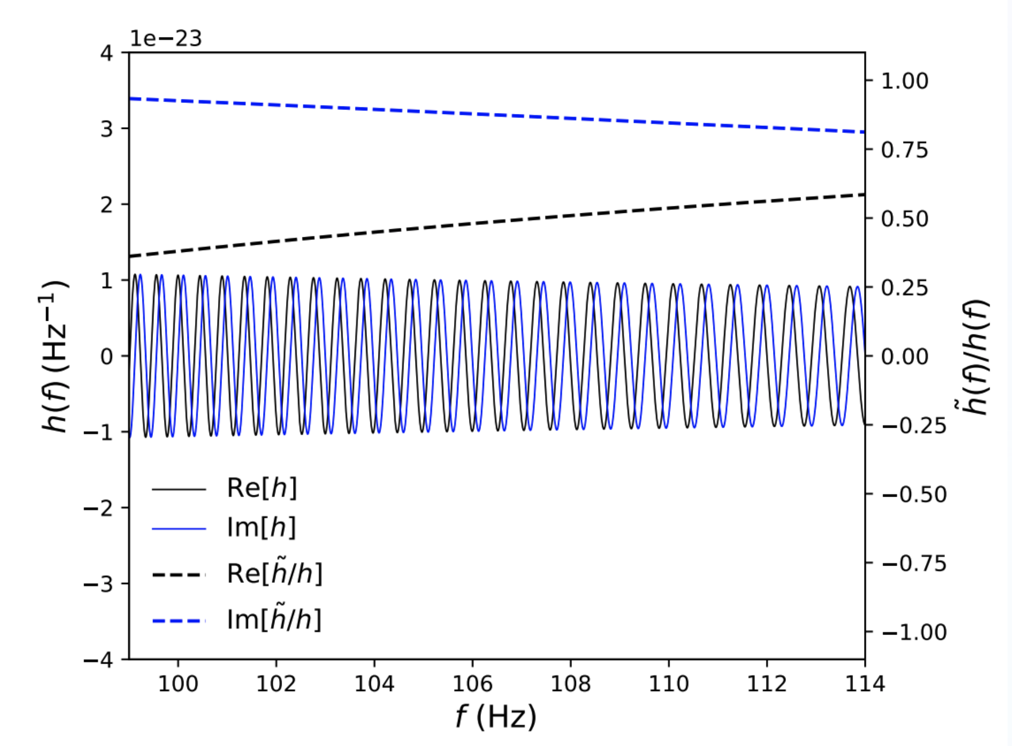
$$\ln \mathcal{L} = \mathfrak{Re}\left(Z[d(f),h(f)]\right) - \frac{1}{2} Z[h(f),h(f)]\,,$$

where $$Z[d(f),h(f)] = 4\sum_f \frac{d(f)\,h^*(f)}{S_n(f)/T}\,.$$

- Precompute *summary data*:

$$A^0(b) = 4\sum_{f\in b} \frac{d(f)\,h^{0*}(f)}{S_n(f)/T} \qquad A^1(b) = 4\sum_{f\in b} \frac{d(f)\,h^{0*}(f)}{S_n(f)/T}\,(f - f_c(b))$$

$$B^0(b) = 4\sum_{f\in b} \frac{h^0(f)h^{0*}(f)}{S_n(f)/T} \qquad B^1(b) = 4\sum_{f\in b} \frac{h^0(f)h^{0*}(f)}{S_n(f)/T}(f - f_c(b))$$



Compute overlaps from linear coefficients:

$$Z[d(f),h(f)] \approx \sum_b A^0(b) r^{0*}(h,b) + A^1(b) r^{1*}(h,b)$$

$$Z[h(f),h(f)] \approx \sum_b \left(B^0(b)|r^0(h,b)|^2 + 2B^1(b)\mathfrak{Re}[r^0(h,b) r^{1*}(h,b)]\right)$$

Precessing Models

- Since original RB, new models
  - incorporate higher spherical harmonic modes
  - and account for the effects of spin-orbit precession.
- Models that incorporate such effects
  - fit numerical simulations better,
  - fit LIGO/Virgo data better,
  - estimate parameters more accurately and precisely for injected signals,
  - and have resulted in the discovery of entirely new solutions.
- Here we use IMRPhenomXPHM

“Twisting-up” Procedure

Detailed in Pratten et. al.: arXiv:2004.06503

- The waveform is broken up into spherical harmonic modes:

$$h = h_+ - ih_\times = \sum_{\ell \geq 2} \sum_{m=-\ell}^{\ell} h^J_{\ell,m} \, {}_{-2}Y_{\ell,m}(\theta, \varphi)$$

- Modes in the J frame are related to modes in the L frame by a rotation of the coordinate system:

$$h^J_{\ell,m} = \sum_{m'=-\ell}^{\ell} e^{-im\alpha} e^{-im'\gamma} d^\ell_{mm'}(\beta) h^L_{\ell,m'}$$

Euler angles: α, β, γ

- As a result, the strain can be written as sum over L-frame modes in the following ways:
  - 1) Time-dependent augmentations of the L-frame mode with coefficients:

$$h(f) = \sum_{\ell,m'} C_{\ell,m'}(f) \, \hat{h}_{\ell,m'}(f) \, , \qquad \text{where } \hat{h}_{\ell,m'}(f) = h^L_{\ell,m'}(f) \, e^{-2\pi i f t_0} \, .$$

  - 2) The entire term for each L-frame mode:

$$h(f) = \sum_{\ell,m'} h_{\ell,m'}(f) \, .$$

Scheme 1

- For this scheme, we use the first decomposition:

$$h(f) = \sum_{\ell,m'} C_{\ell,m'}(f)\, \hat{h}_{\ell,m'}(f), \qquad \text{where } \hat{h}_{\ell,m'}(f) = h^L_{\ell,m'}(f)\, e^{-2\pi i f t_0}.$$

- We linearize the ratio of time-dependent L frame modes and we linearize the coefficient in each bin:

$$r_{\ell,m'}(f) = \frac{\hat{h}_{\ell,m'}(f)}{\hat{h}^0_{\ell,m'}(f)} \approx r^0_{\ell,m'}(h,b) + r^1_{\ell,m'}(h,b)\,(f - f_c(b)) \quad C_{\ell,m'}(f) \approx C^0_{\ell,m'}(h,b) + C^1_{\ell,m'}(h,b)\,(f - f_c(b))$$

- Similarly, we precompute summary data, and we need them for each mode or mode pair:

$$A^0_{\ell,m'}(b) = 4 \sum_{f\in b} \frac{d(f)\, \hat{h}^{0*}_{\ell,m'}(f)}{S_n(f)/T} \qquad A^1_{\ell,m'}(b) = 4 \sum_{f\in b} \frac{d(f)\, \hat{h}^{0*}_{\ell,m'}(f)}{S_n(f)/T}\,(f - f_c(b))$$

$$B^0_{\ell,m',\tilde{\ell},\tilde{m}'}(b) = 4 \sum_{f\in b} \frac{\hat{h}^0_{\ell,m'}(f)\, \hat{h}^{0*}_{\tilde{\ell},\tilde{m}'}(f)}{S_n(f)/T} \qquad B^1_{\ell,m',\tilde{\ell},\tilde{m}'}(b) = 4 \sum_{f\in b} \frac{\hat{h}^0_{\ell,m'}(f)\, \hat{h}^{0*}_{\tilde{\ell},\tilde{m}'}(f)}{S_n(f)/T}\,(f - f_c(b))$$

- Compute overlaps with $r^0_{\ell,m'}(h,b)$, $r^1_{\ell,m'}(h,b)$, $C^0_{\ell,m'}(h,b)$, $C^1_{\ell,m'}(h,b)$ and summary data.

Scheme 2

- For this scheme, we use the second simpler decomposition:

$$h(f) = \sum_{\ell,m'} h_{\ell,m'}(f)\,.$$

- We linearize the ratio of the entire L frame mode component:

$$r_{\ell,m'}(f) = \frac{h_{\ell,m'}(f)}{h^0_{\ell,m'}(f)} \approx r^0_{\ell,m'}(h,b) + r^1_{\ell,m'}(h,b)\,(f - f_c(b))$$

- Similarly, we precompute summary data, and we need them for each mode or mode pair:

$$A^0_{\ell,m'}(b) = 4\sum_{f\in b} \frac{d(f)\,h^{0*}_{\ell,m'}(f)}{S_n(f)/T} \qquad A^1_{\ell,m'}(b) = 4\sum_{f\in b} \frac{d(f)\,h^{0*}_{\ell,m'}(f)}{S_n(f)/T}\,(f - f_c(b))$$

$$B^0_{\ell,m',\tilde{\ell},\tilde{m}'}(b) = 4\sum_{f\in b} \frac{h^0_{\ell,m'}(f)\,h^{0*}_{\tilde{\ell},\tilde{m}'}(f)}{S_n(f)/T} \qquad B^1_{\ell,m',\tilde{\ell},\tilde{m}'}(b) = 4\sum_{f\in b} \frac{h^0_{\ell,m'}(f)\,h^{0*}_{\tilde{\ell},\tilde{m}'}(f)}{S_n(f)/T}\,(f - f_c(b))$$

- Compute overlaps with $r^0_{\ell,m'}(h,b)$, $r^1_{\ell,m'}(h,b)$, and summary data.

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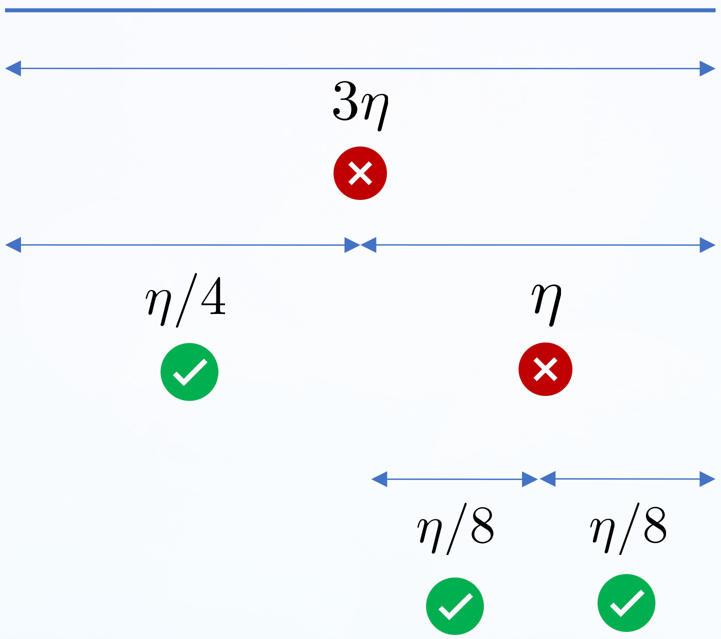
Bin Selection

- Because the sampling dominates PE runtime, bin selection algorithm can be more thorough.
- Key points:
  - Pick bins that achieve a target error η on a *test waveform* (near MLE).
  - In each iteration, bisect to obtain bins with error below $\eta/\mathrm{targetN}$.
  - Number of bins obtained is $\mathrm{targetN}$ for next iteration.
  - Convergence is empirically guaranteed, independent of initial $\mathrm{targetN}$.

Example:
Iteration 1: $\mathrm{targetN}$= 2, accept error $\eta/2$
Iteration 2: $\mathrm{targetN}$= 3, accept error $\eta/3$



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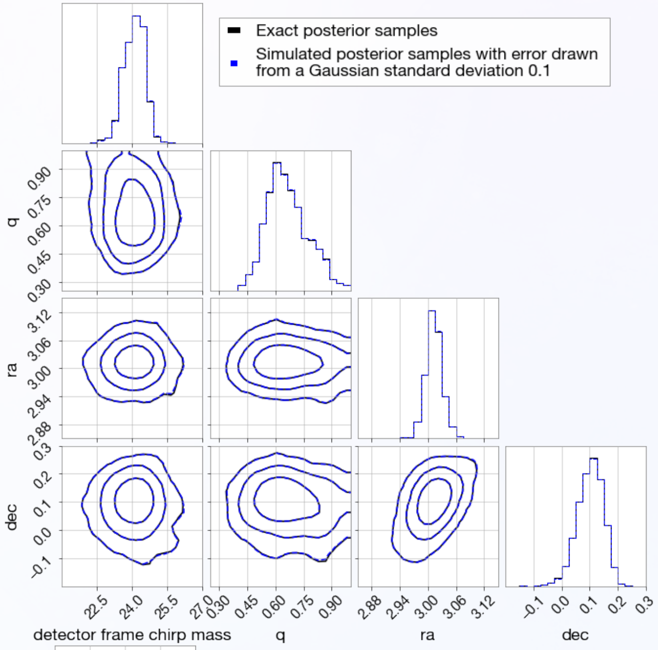
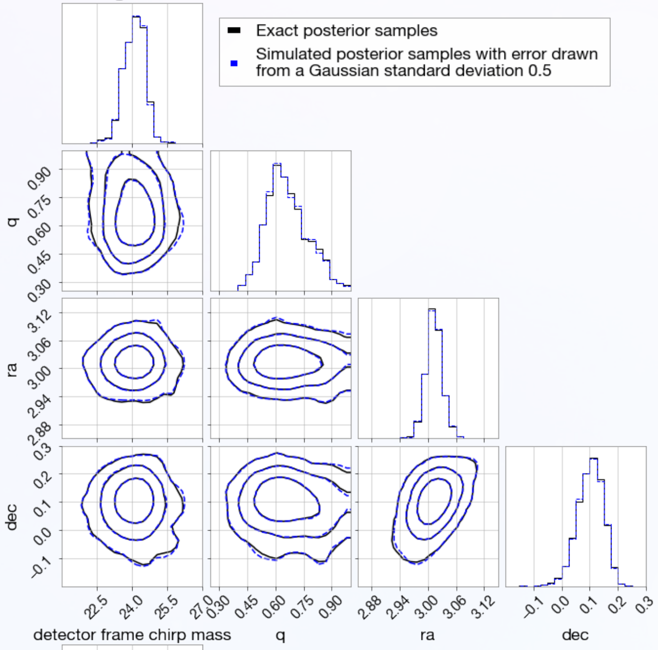
Testing Methods

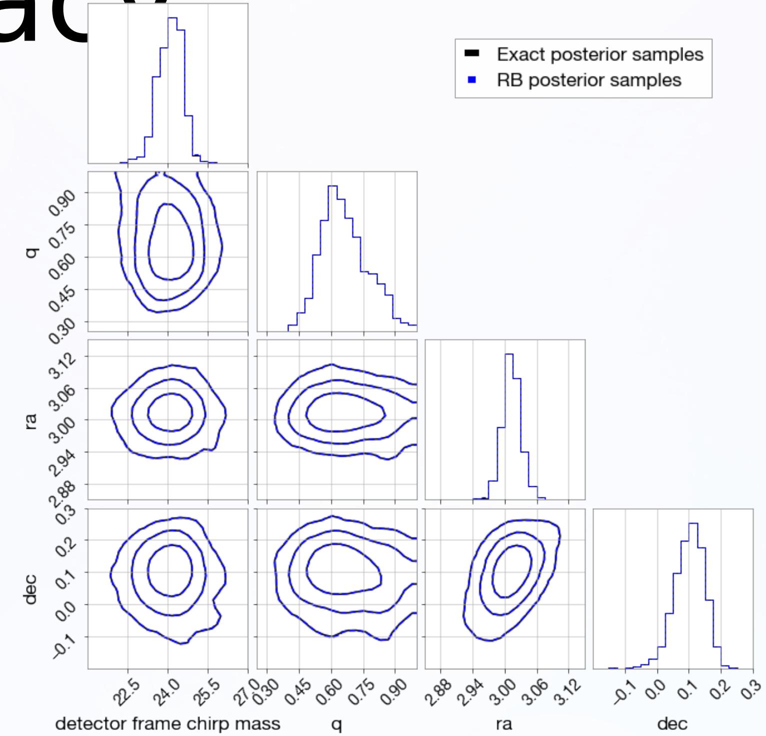
- Samples of IMRPhenomXPHM model gathered by running MultiNest using the original relative binning algorithm
- We target events with either strong precession effects or large contributions from subdominant modes:
  - Large in-plane spins
  - Large total mass
  - Small mass ratio $q = m_2/m_1 < 1$
- Five events (4 injected events, 1 real event)
  - Injection with all of the above, except q = 2/3 (High q)
  - Injection with all of the above and q = 1/10 (Low q)
  - Injection based on alternative solution of GW151226 (GW151226-like)
  - Injection based on alternative solution of GW190521 (GW190521-like)
  - Real event: GW190814

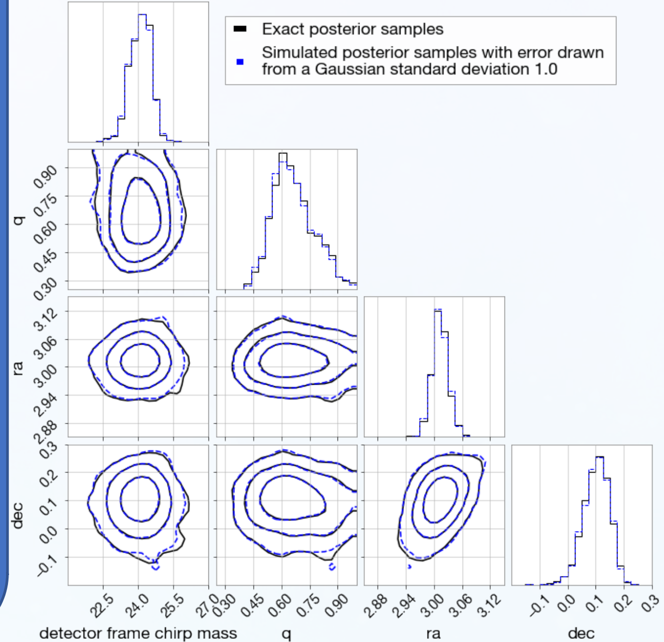
Target Accuracy

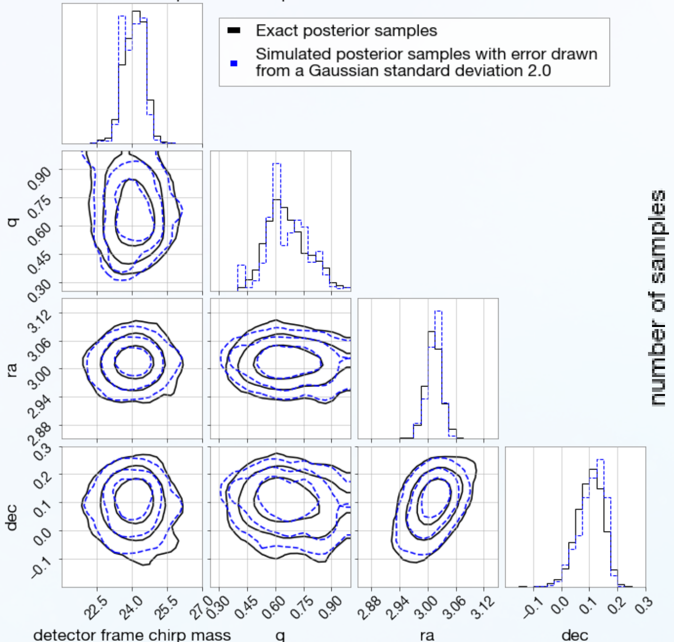
(high q injection)

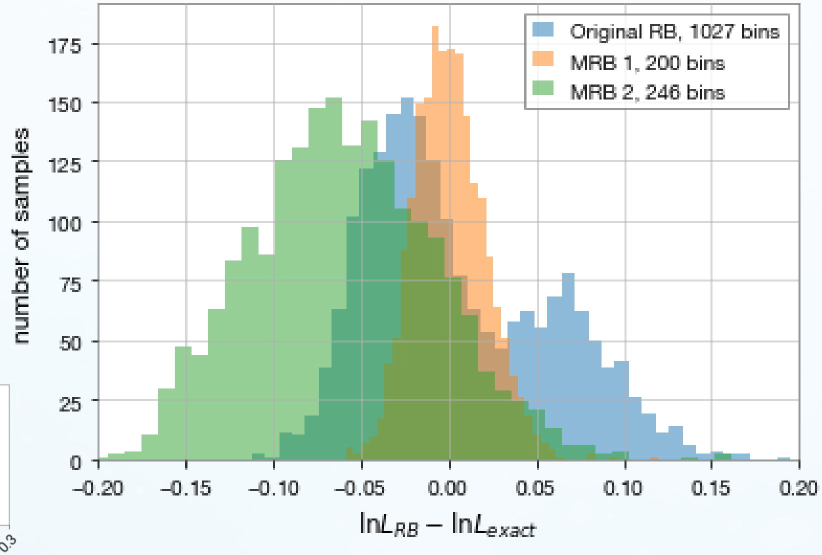












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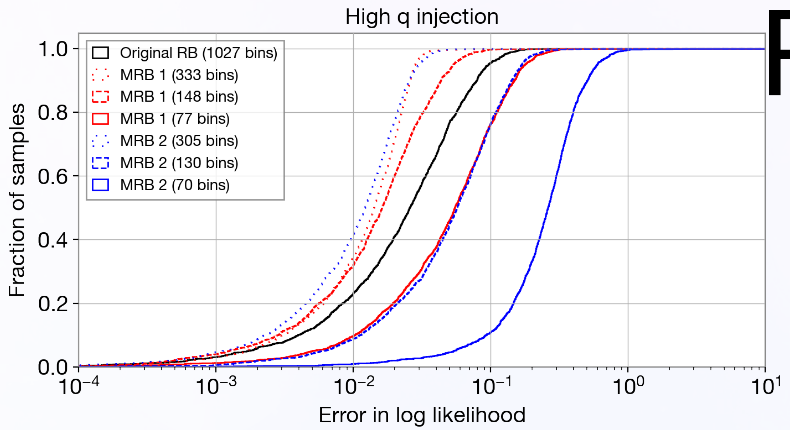

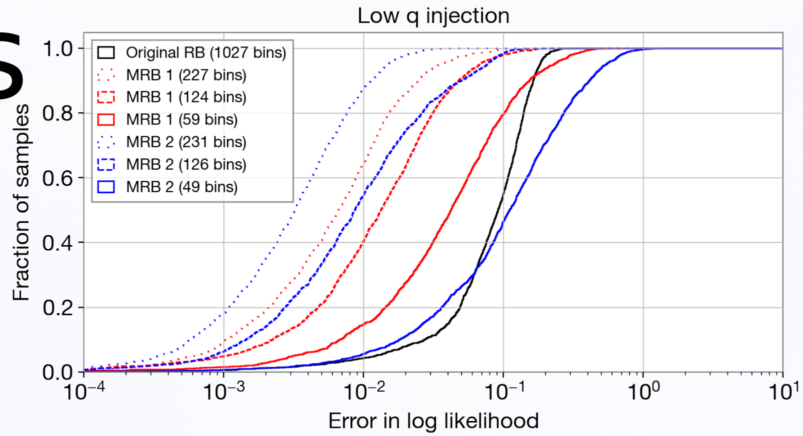

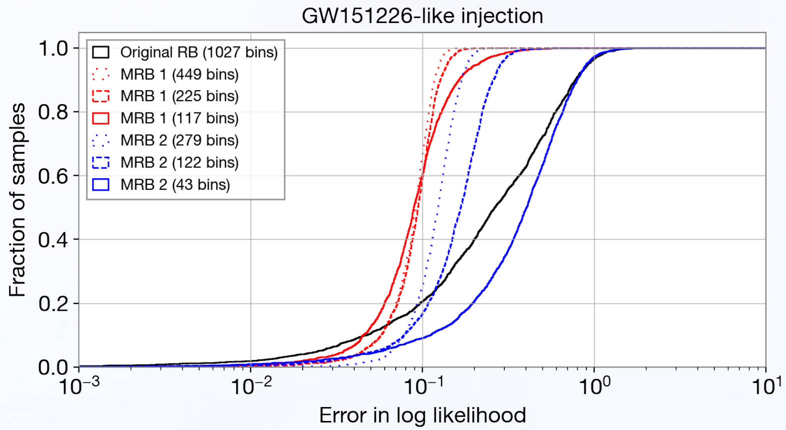

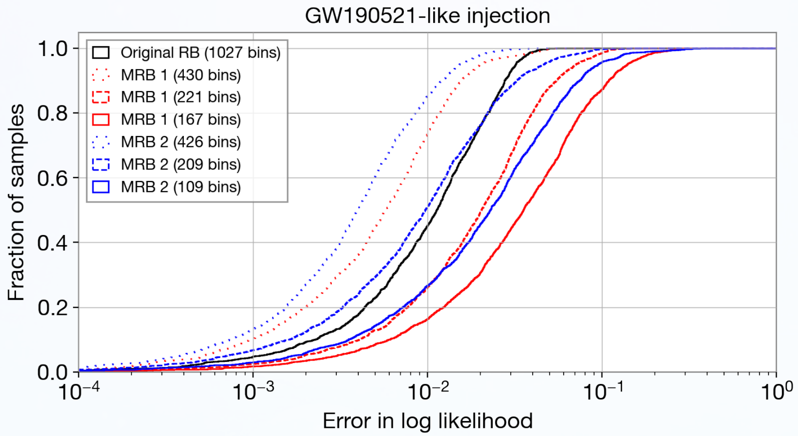

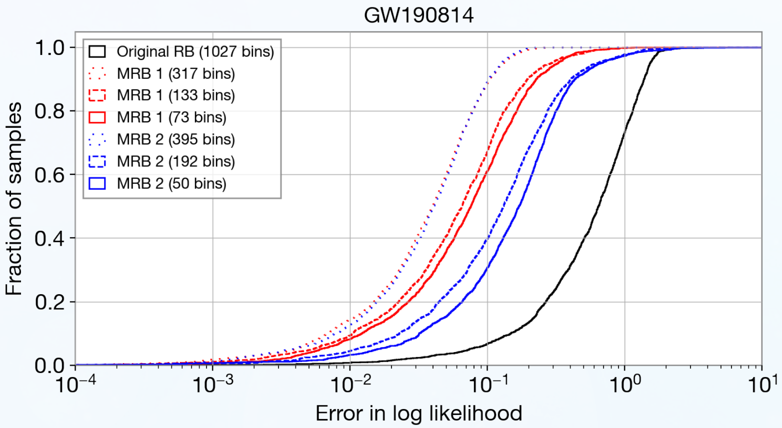

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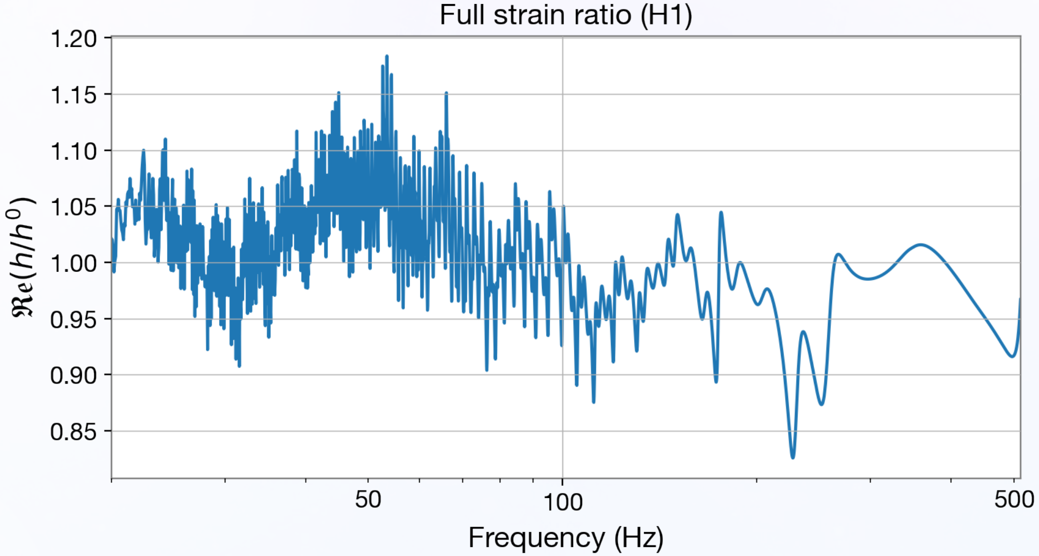
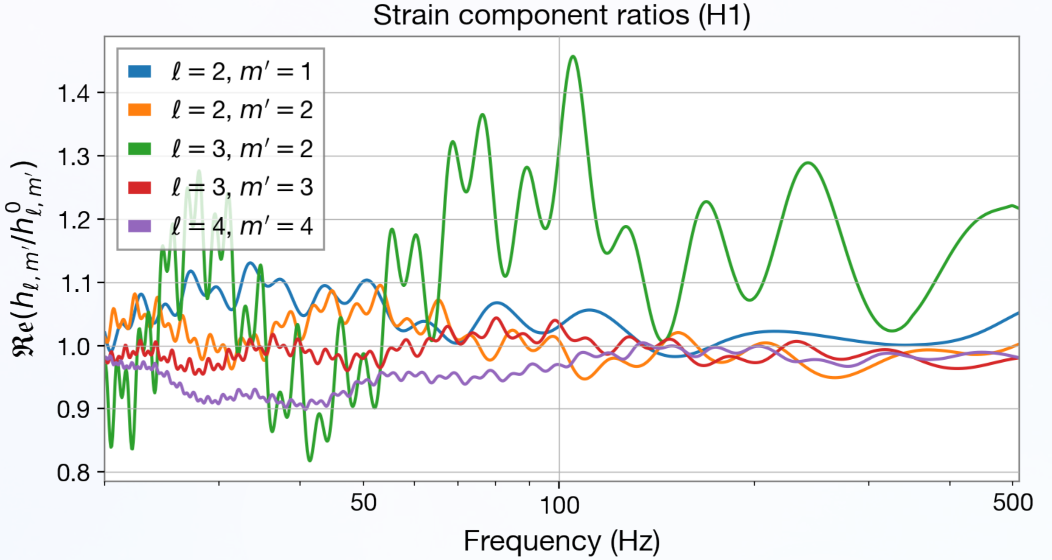
Smoothness Comparison (GW190814)

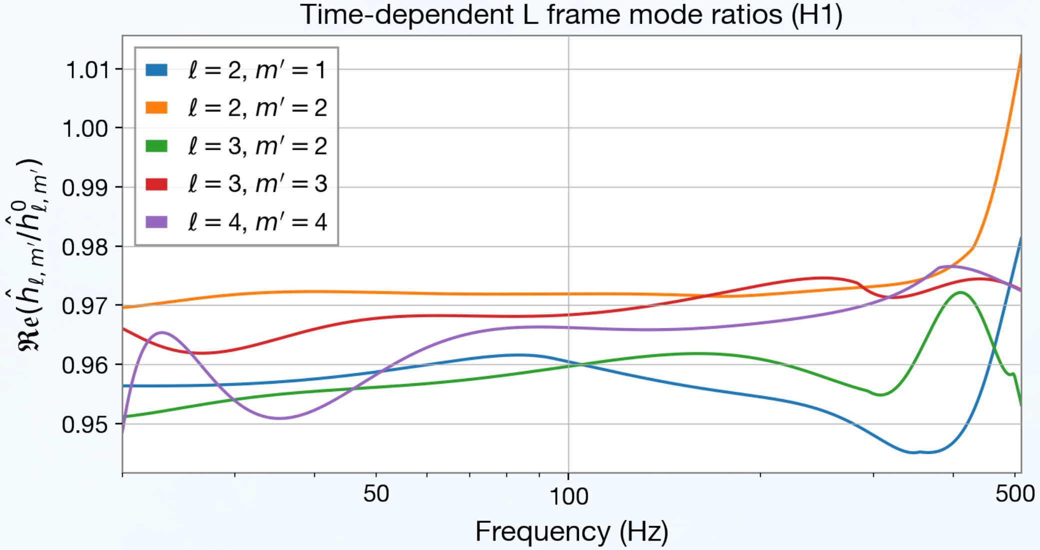
Linearized in original relative binning:



Linearized in Scheme 2:



Linearized in Scheme 1:



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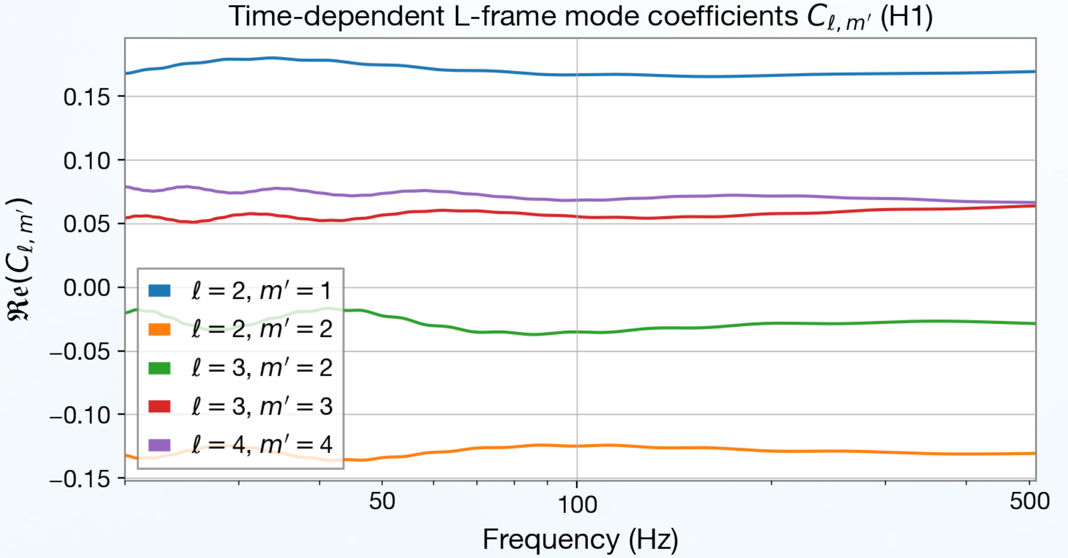

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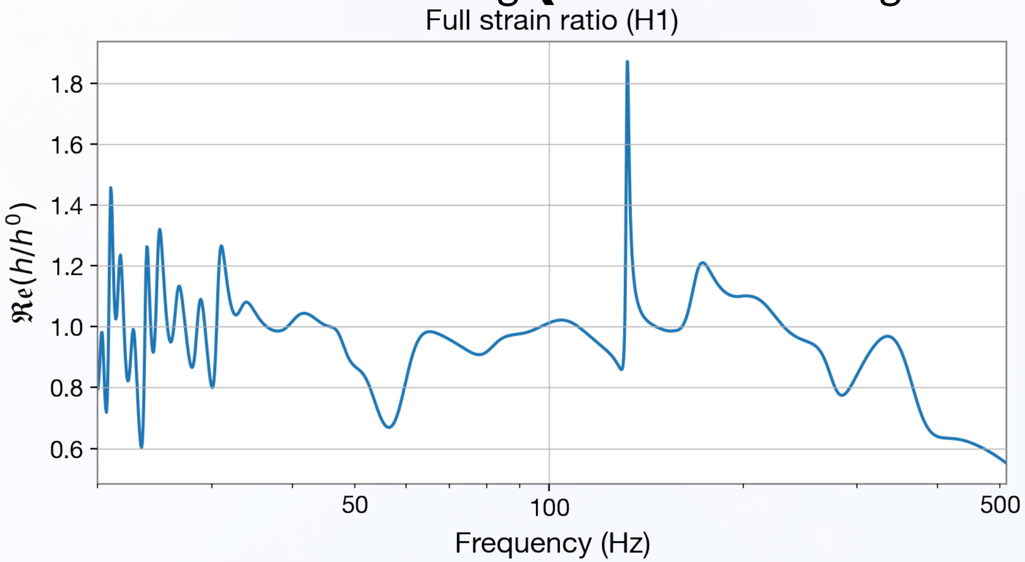
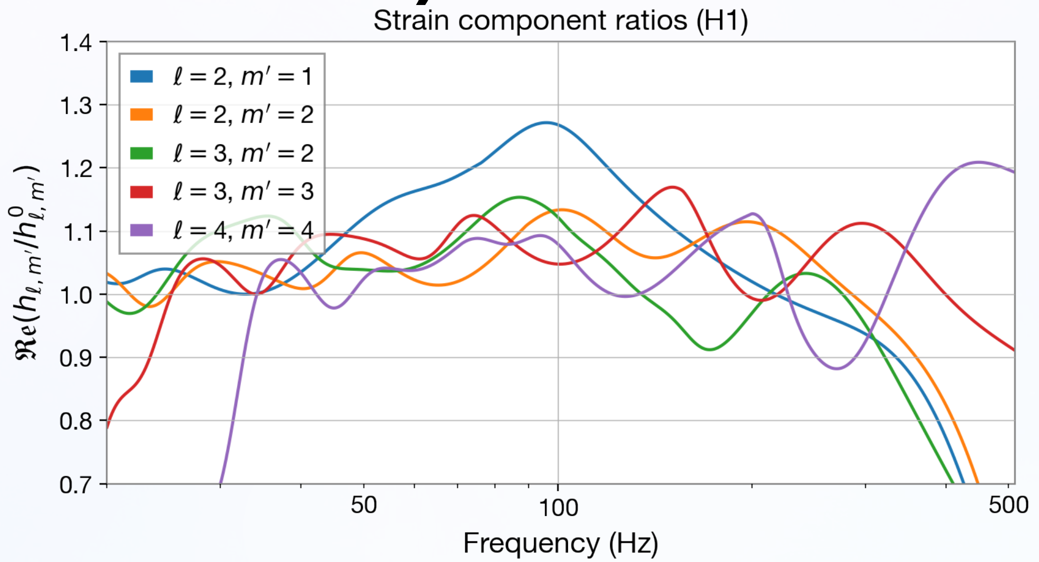
Smoothness Comparison (GW190521-like)

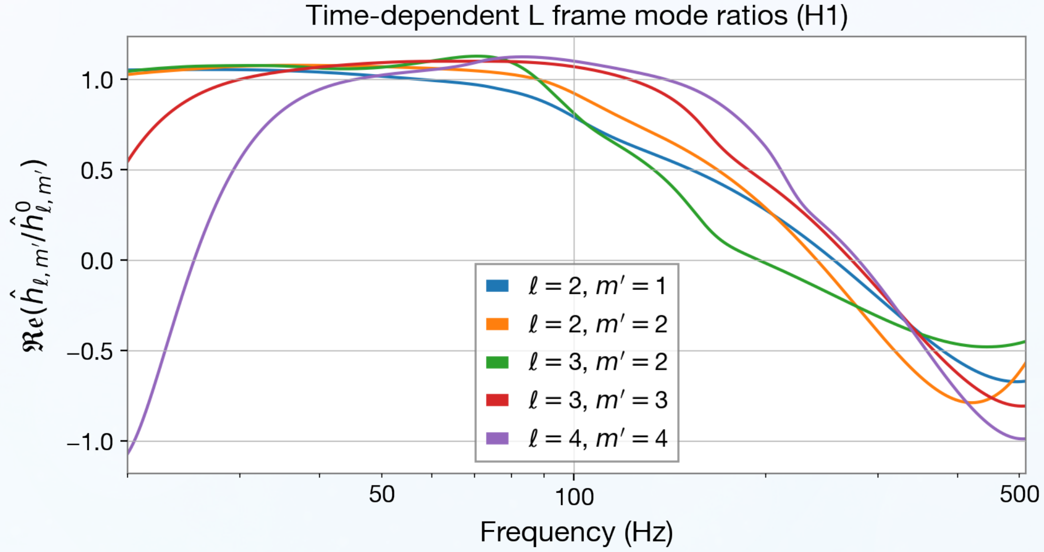
Linearized in original relative binning:



Linearized in Scheme 2:



Linearized in Scheme 1:



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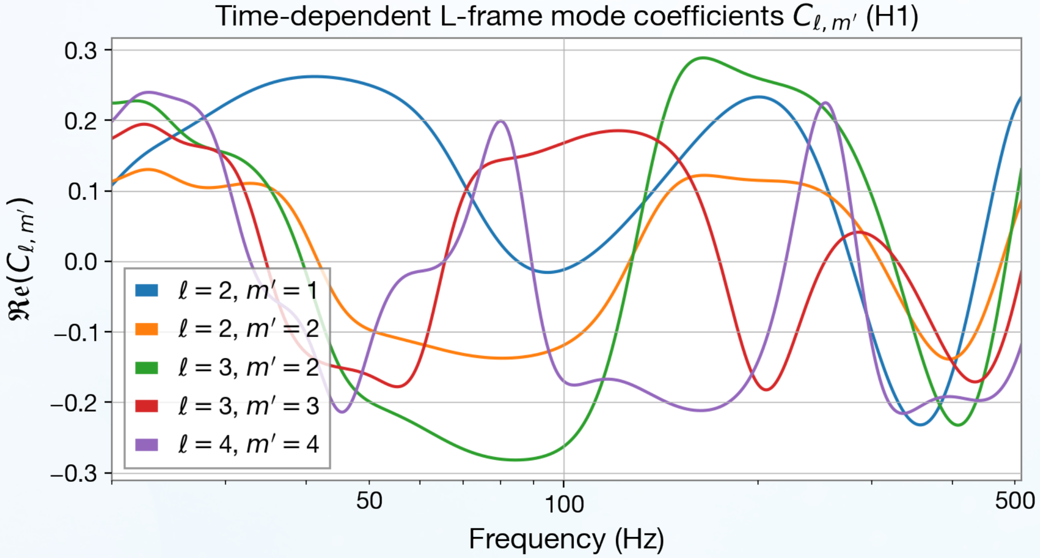

Conclusions

- Scheme 1 generally better, but Scheme 2 might be useful in some cases.
- Flexible bin selection allows speed and accuracy to be freely traded.
- Both schemes can achieve target error with roughly 100-200 bins or better.
- Optimal implementation could compute sub-1 ms likelihoods, full PE in a few hours or less.