Theory of Fast Flavor Conversion of Supernova neutrinos

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CONTENT OF THE TALK :

- **Background and challenges**

- **Our Works :**

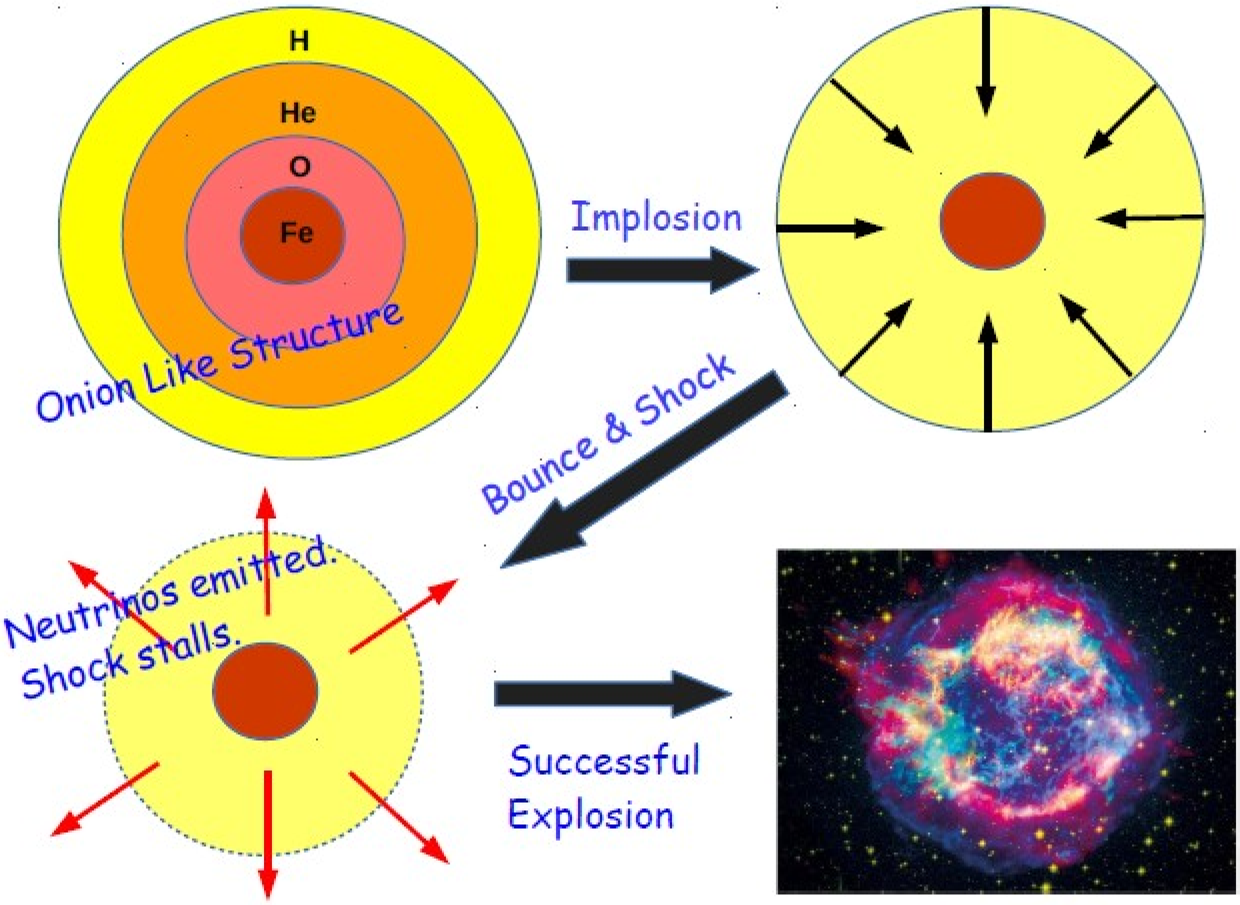
  a) Our own Numerical recipe
  b) Theoretical dvelopment in SN neutrino oscillations :

  1) Multipole diffusion (Why?)
  2) Transverse relaxation (When ?)
  3) Extent of Flavor Depolarization (How much ?)

  c) Extension towards multidimensional systems

- **Phenomenological Consequences and Conclusion**

SN Explosion & Neutrinos :



EQUATION OF MOTION :

- **Neutrino density matrix :** $\hat{\rho}[\vec{r}, E, \vec{p}, t] \equiv \hat{\rho}_{E,\vec{p}}$ $\quad \hat{\rho} = \begin{pmatrix} \langle \nu_e|\nu_e\rangle & \langle \nu_e|\nu_\mu\rangle & \langle \nu_e|\nu_\tau\rangle \\ \langle \nu_\mu|\nu_e\rangle & \langle \nu_\mu|\nu_\mu\rangle & \langle \nu_\mu|\nu_\tau\rangle \\ \langle \nu_\tau|\nu_e\rangle & \langle \nu_\tau|\nu_\mu\rangle & \langle \nu_\tau|\nu_\tau\rangle \end{pmatrix}$

$\langle \nu_i|\nu_i\rangle \longrightarrow$ Total flavor content

$\langle \nu_i|\nu_j\rangle \longrightarrow$ Amount of flavor conversion

- **Equations governing flavor evolution :**

$$\left(\partial_t + \vec{v}.\vec{\nabla}\right) \hat{\rho}_{E,\vec{p}} = [\hat{H}_{E,\vec{p}},\ \hat{\rho}_{E,\vec{p}}] \qquad \hat{H}_{E,\vec{p}} = H_E^{vac} + H^{mat} + H_{\vec{p}}^{self}$$

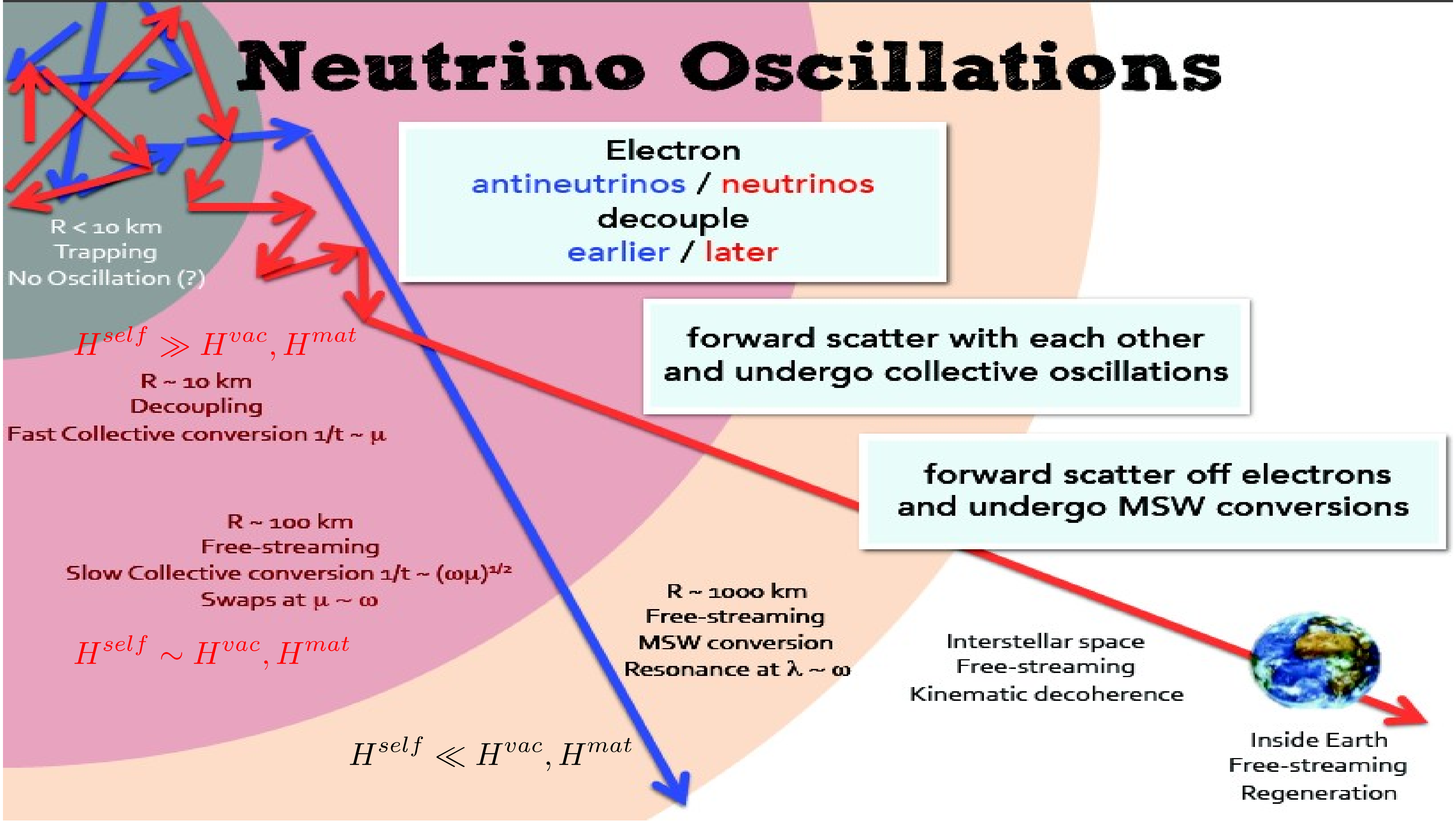
$$\underbrace{H_E^{vac} = \frac{\Delta m^2}{2E}}_{\text{Vacuum term}} \qquad \underbrace{H^{\rm mat} = \sqrt{2} G_F n_e}_{\text{Matter background (MSW)}} \qquad \underbrace{H_{\vec{p}}^{\rm self} = \int d^3\vec{q}/(2\pi)^3 \left(1 - \vec{v}_{\vec{q}} \cdot \vec{v}_{\vec{p}}\right)\left(\hat{\rho}_{E',\vec{q}} - \bar{\hat{\rho}}_{E',\vec{q}}\right)}_{\text{Neutrino self-interaction, Collective effects}}$$

Vacuum term

Matter background (MSW)

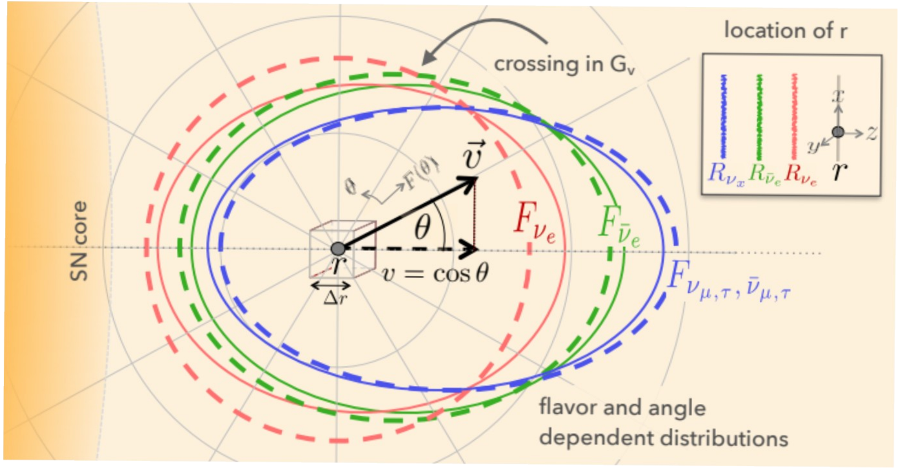
Neutrino self-interaction,
Collective effects

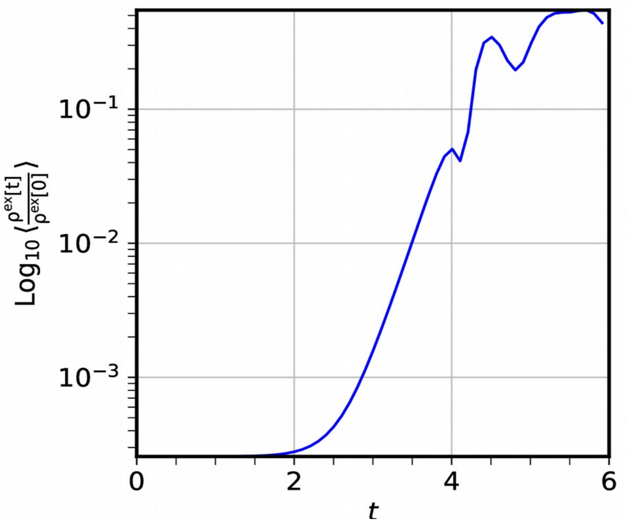
Neutrino Oscillations



Fast Flavor Conversions : A review

Chakraborty, Hansen, Izaguirre, Raffelt (2016)
Ray Sawyer (2005, 2015)





- Very close to the SN core (few km's) Rate $\propto$ Neutrino number density
- Rapid/faster compared to neutrino oscillation in vacuum/ordinary matter
- Requires a "zero" crossing in neutrino angular distribution and independent of energy (and mass hierarchy)

EQUATION OF MOTION (FFC) :

$g_{\vec{p}}$:
Neutrino angular distribution

We stick to two-flavor framework :

$$\hat{\rho}_{E,\vec{v}} = \begin{pmatrix} \langle \nu_e | \nu_e \rangle_{E,\vec{v}} & \langle \nu_e | \nu_x \rangle_{E,\vec{v}} \\ \langle \nu_x | \nu_e \rangle_{E,\vec{v}} & \langle \nu_x | \nu_x \rangle_{E,\vec{v}} \end{pmatrix} = \frac{Tr\left(\hat{\rho}_{E,\vec{v}}\right)}{2} \mathbb{I}_{2\times 2} + \frac{g_{E,\vec{v}}}{2} \vec{S}_{E,\vec{v}} \cdot \sigma$$

$$\hat{H}_{E,\vec{v}} = \frac{Tr\left(\hat{H}_{E,\vec{v}}\right)}{2} \mathbb{I}_{2\times 2} + \frac{1}{2} \vec{H}_{E,\vec{v}} \cdot \sigma$$

Chakraborty, Hansen, Izaguirre, Raffelt (2016)
Dasgupta, Mirizzi, Sen (2017)
Bhattacharyya, Dasgupta (2020)

$$\left(\partial_t + \vec{v}.\vec{\nabla}\right) \hat{\rho}_{E,\vec{v}} = \hat{H}_{E,\vec{v}} \hat{\rho}_{E,\vec{v}} - \hat{\rho}_{E,\vec{v}} \hat{H}_{E,\vec{v}} \longrightarrow \left(\partial_t + \vec{v}.\vec{\nabla}\right) \mathsf{S}_{\omega,\vec{v}} = \left(\mathsf{H}^{\text{vac}}_{\omega} + \mathsf{H}^{\text{mat}} + \mathsf{H}^{\text{self}}_{\vec{v}}\right) \times \mathsf{S}_{\omega,\vec{v}}$$

$$\mathsf{H}^{\text{mat}} = \sqrt{2} G_F (n_{e^-} - n_{e^+}) (0, 0, 1) \qquad \mathsf{H}^{\text{vac}}_{\omega} = \omega \left(\sin 2\vartheta, 0, \cos 2\vartheta \right)$$

$$\mathsf{H}^{\text{self}}_{\vec{v}} = \int d^3 \vec{p}\,'_{\omega', \vec{v}'} / (2\pi)^3 \, g_{\omega', \vec{v}'} \left(1 - \vec{v} \cdot \vec{v}'\right) \mathsf{S}_{\omega', \vec{v}'}$$

EQUATION OF MOTION (FFC) :

$g_{\vec{p}}$:
Neutrino angular distribution

We stick to two-flavor framework :

$$\hat{\rho}_{E,\vec{v}} = \begin{pmatrix} \langle \nu_e | \nu_e \rangle_{E,\vec{v}} & \langle \nu_e | \nu_x \rangle_{E,\vec{v}} \\ \langle \nu_x | \nu_e \rangle_{E,\vec{v}} & \langle \nu_x | \nu_x \rangle_{E,\vec{v}} \end{pmatrix} = \frac{Tr\left(\hat{\rho}_{E,\vec{v}}\right)}{2}\mathbb{I}_{2\times 2} + \frac{g_{E,\vec{v}}}{2}\vec{S}_{E,\vec{v}}\cdot\sigma$$

$$\hat{H}_{E,\vec{v}} = \frac{Tr\left(\hat{H}_{E,\vec{v}}\right)}{2}\mathbb{I}_{2\times 2} + \frac{1}{2}\vec{H}_{E,\vec{v}}\cdot\sigma$$

$$\left(\partial_t + \vec{v}.\vec{\nabla}\right)\hat{\rho}_{E,\vec{v}} = \hat{H}_{E,\vec{v}}\hat{\rho}_{E,\vec{v}} - \hat{\rho}_{E,\vec{v}}\hat{H}_{E,\vec{v}} \longrightarrow \left(\partial_t + \vec{v}.\vec{\nabla}\right)\mathsf{S}_{\omega,\vec{v}} = \left(\mathsf{H}^{\text{vac}}_{\omega} + \mathsf{H}^{\text{mat}} + \mathsf{H}^{\text{self}}_{\vec{v}}\right)\times\mathsf{S}_{\omega,\vec{v}}$$

$$\left(\partial_t + \vec{v}.\vec{\nabla}\right)\mathbf{S}_{\vec{v}} = \mu_0 \int d^3v' G_{\vec{v}'}\left(1 - \vec{v}\cdot\vec{v}'\right)\mathbf{S}_{\vec{v}'} \times \mathbf{S}_{\vec{v}}$$

$$\mathsf{H}^{\text{mat}} = \sqrt{2}G_F(n_{e^-} - n_{e^+})(0,0,1) \qquad \mathsf{H}^{\text{vac}}_{\omega} = \omega\left(\sin 2\vartheta, 0, \cos 2\vartheta\right)$$

$$\mathsf{H}^{\text{self}}_{\vec{v}} = \int d^3\vec{p}'_{\omega',\vec{v}'}/(2\pi)^3\, g_{\omega',\vec{v}'}\left(1 - \vec{v}\cdot\vec{v}'\right)\mathsf{S}_{\omega',\vec{v}'}$$

“Zero crossing” : FFC

$$\mathbf{S}_{\omega',\vec{v}'} \equiv \mathbf{S}_{\vec{v}'}$$

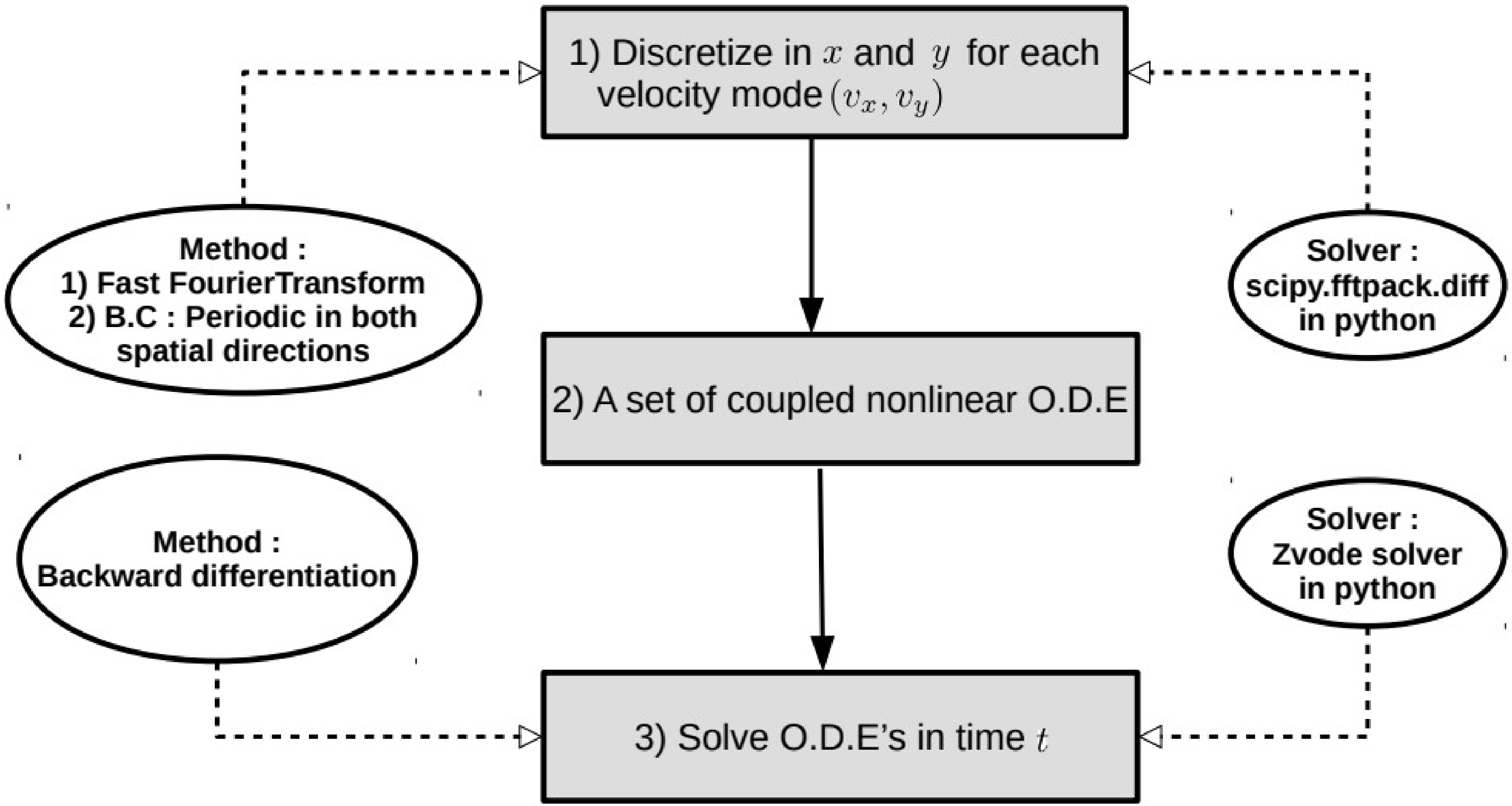
$$G_{\vec{v}} = \int d\omega\, g_{\omega,\vec{v}}$$

$\left|\mathsf{S}^{\perp}_{\vec{v}}\right|$ **: Flavor conversion** $\qquad \mathsf{S}^{\parallel}_{v}$ **: Total Flavor content**

Difficulties and Challenges :

What are the challenges ?	What did we do ?	
Huge Phase space dimensionality 3 sp. + 3 mom. + 1 time = 7 dim	Partially resolved 2 sp. + 2 mom. + 1 time = 5 dim.	✓
Large set of coupled nonlinear P.D.E's	Developed : a) Analytical techniques b) Numerical code	✓
Lack of numerical techniques to give accurate and precise result in the nonlinear regime	Developed our own code that gives accurate and precise answer even in the nonlinear regime	✓
Lack of analytical development / theory beyond the linear regime (Linear stability) which can predict the final outcome	Developed our own theory that can predict how, when and to what extent fast conversion can happen.	✓

OUR NUMERICAL RECIPE :



RESULTS : Irreversibility

Model :

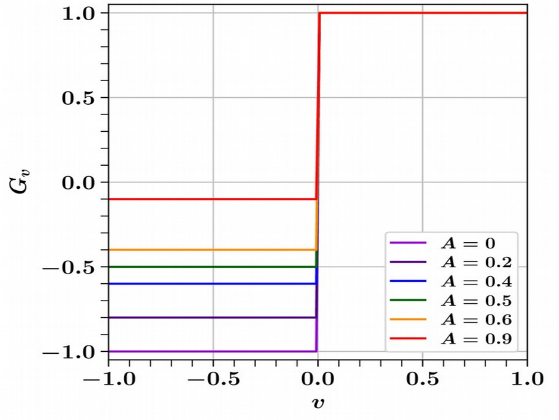
1Time + 1 Sp. + 1 Mom.

Initial Cond :

$$\mathsf{S}_v^{\|}\big|^{\mathrm{ini}} = +1$$
$$\left|\mathsf{S}_v^{\perp}\right|^{\mathrm{ini}} = 10^{-6}\delta\left(z\right)$$

Angular dist. :

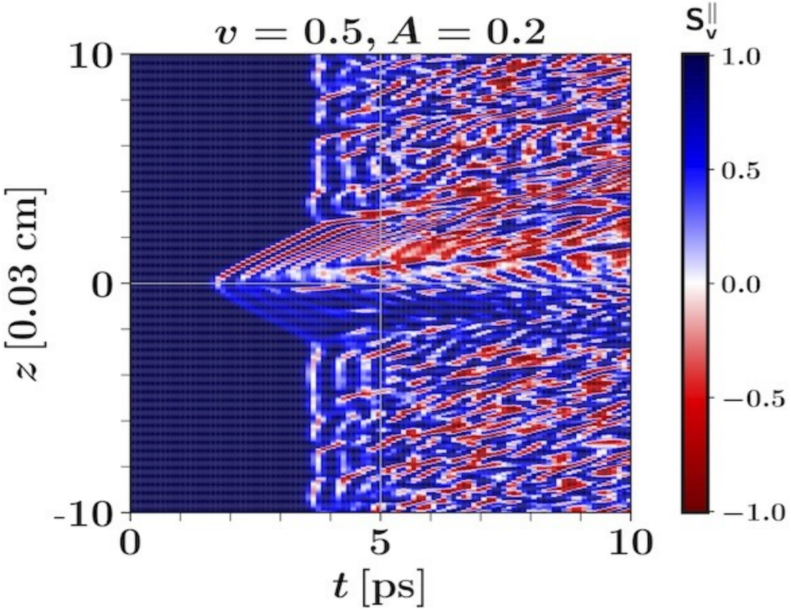
$$G_v = 1, v > 0; A - 1, v < 0$$

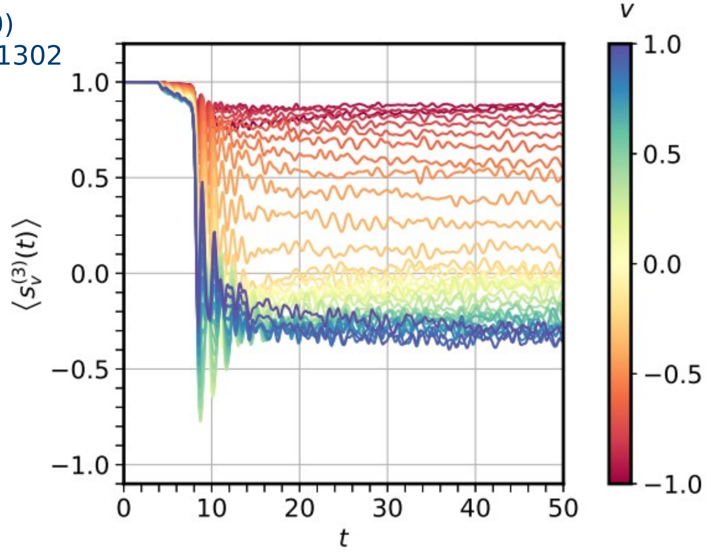


- **Irreversible and steady state behaviour in time**
- **length shrinks**

Bhattacharyya, Dasgupta (2020)
arXiv : 2005.00459

Bhattacharyya, Dasgupta (2020)
Phys.Rev.Lett. 126 (2021) 6, 061302





RESULTS : Multipole Diffusion

- In terms of multipole moments, $\mathrm{M}_n = \int_{-1}^{+1} dv\, G_v L_n \mathrm{S}_v$ and considering n as continuum we get :

$$\partial_t \mathrm{M}_n - \mathrm{M}_0 \times \mathrm{M}_n = \partial_z \left(\mathrm{M}_n + \partial_n \mathrm{M}_n/(2n+1) + \partial_n^2 \mathrm{M}_n/2\right) - \mathrm{M}_1 \times \left(\mathrm{M}_n + \partial_n \mathrm{M}_n/(2n+1) + \partial_n^2 \mathrm{M}_n/2\right)$$

- Further coarse graining over z , and using $2n+1 \approx 2n$

$$\partial_t \langle M_n \rangle = \frac{\langle M_1 \rangle}{2} \left(\partial_n^2 \langle M_n \rangle + \frac{1}{n} \partial_n \langle M_n \rangle \right)$$

Diffusion in multipole space

- The above equation remains same under $n \to an\,,\; t \to a^2 t \longrightarrow \langle M_n(t) \rangle = f\left(\frac{n^2}{t}\right) = f(\xi)$

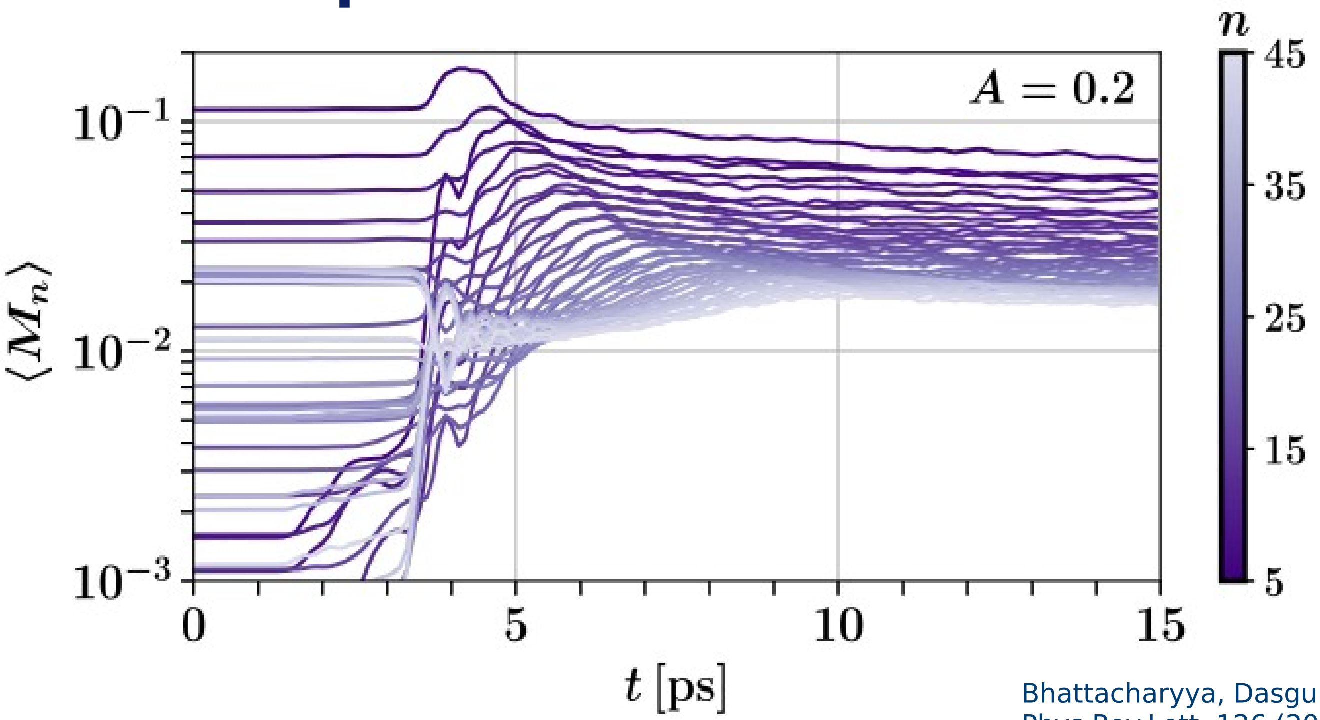
$$2\frac{d^2}{d\xi^2} f(\xi) + \left(1/\langle M_1 \rangle + 2/\xi\right) \frac{d}{d\xi} f(\xi) = 0$$

$$\langle M_n(t) \rangle = c_1\, \mathrm{Ei}\left[-n^2/\left(2\langle M_1 \rangle t\right)\right] + c_2$$

Georg G. Raffelt and Günter Sigl
Phys. Rev. D 75, 083002

Bhattacharyya, Dasgupta (2020)
Phys.Rev.Lett. 126 (2021) 6, 061302

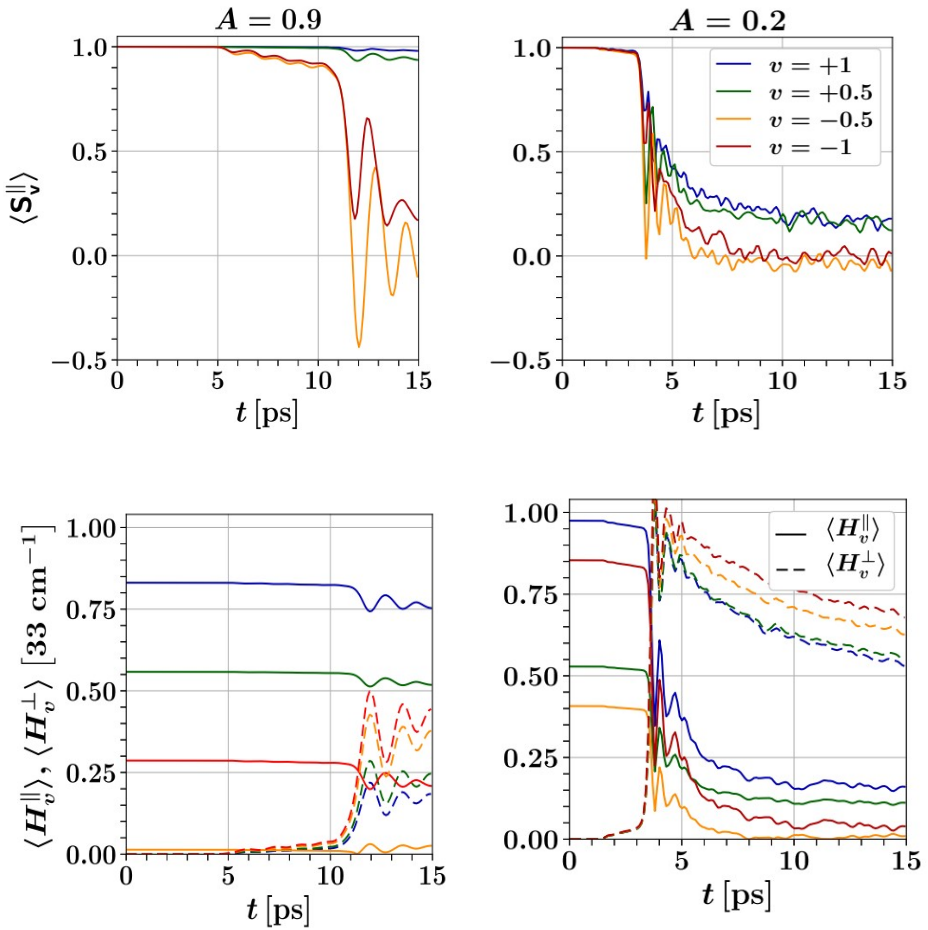
RESULTS : Multipole Diffusion

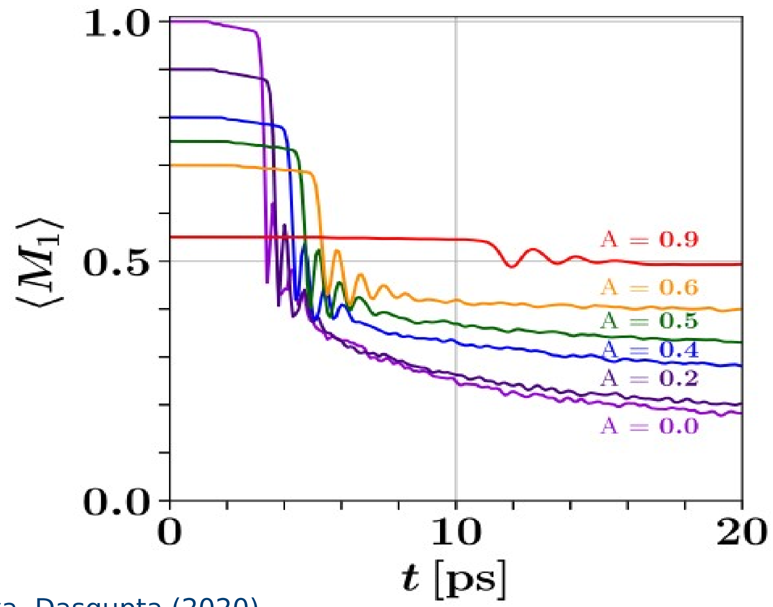


Bhattacharyya, Dasgupta (2020)
Phys.Rev.Lett. 126 (2021) 6, 061302

Power flow in multipole space from low to high n values and coarse-graining causes irreversibiltiy in time and also shrinking in the length for high n multipole moments

RESULTS : Transverse Relaxation





Bhattacharyya, Dasgupta (2020)
Phys.Rev.Lett. 126 (2021) 6, 061302

- **Modes for which $|H_v^{\perp}| \approx |H_v^{\|}|$, S_v crosses the transverse plane and gets depolarized**
- **Amount depends on lepton asymmetry and choice of v**

RESULTS : Flavor Depolarization

Depolarization factor :

$$f_v^{\rm D} = \tfrac{1}{2}\left(1 - \langle S_v\rangle^{\rm fin}/\langle S_v\rangle^{\rm ini}\right)$$

$f_v^{\rm D} = 0.5$ → Complete depolarization

$f_v^{\rm D} = 0$ → No depolarization

Multipole expansion upto linear order :

$$G_v \mathsf{S}_v^{\parallel}|_{fin} = \frac{\mathsf{M}_0^{fin}}{2} + \frac{3v\mathsf{M}_1^{fin}}{2} + O(v^2)$$

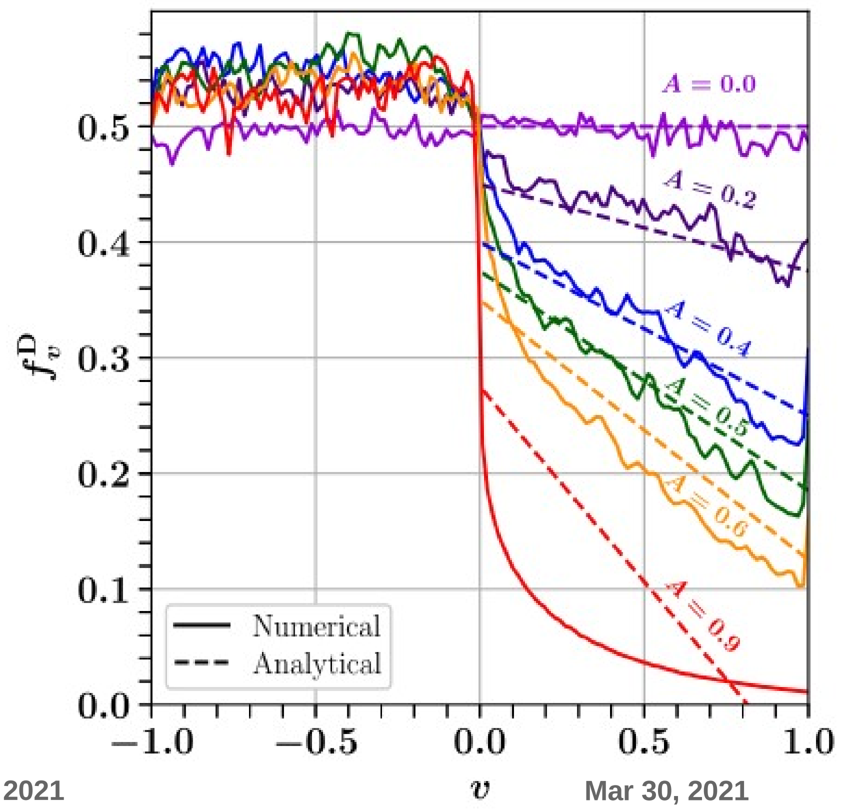
+

Lepton number conservation :

$$\left\langle \mathsf{M}_0^{ini}\right\rangle = \left\langle \mathsf{M}_0^{fin}\right\rangle = A \quad \left\langle \mathsf{M}_1^{fin}\right\rangle = \frac{A}{2}$$

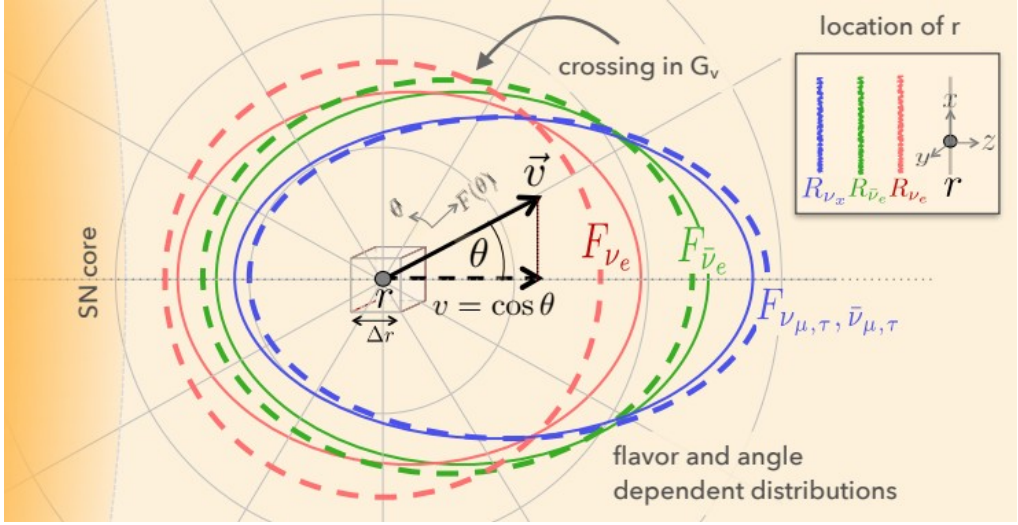
$$f_v^{\rm D} \approx 0.5, \text{if } v < 0$$

$$f_v^{\rm D} \approx \frac{1}{2} - \frac{A}{4} - \frac{3A}{8} v, \text{if } v > 0$$



CONCLUSION

- We have presented an ***analytical theory*** of fast neutrino flavor conversions in the ***nonlinear regime***.
- We showed fast conversions can bring different neutrino flavors ***close to each other*** (Flavor Depolarization) and ***irreversibility*** in the system.
- ***T2 relaxation*** and ***multipole diffusion*** governs such behaviour.
- We gave a strategy and a formula for computing the ***extent of flavor depolarization***



Phenomenological Consequences :

- Flavor depolarization can cause significant increse in ***neutrino heating rate*** and ***change the explosion scenario***.
- Including MSW conversions, propagation and eartheffects our formula will allow one to determine the ***final neutrino signal from a SN explosion***

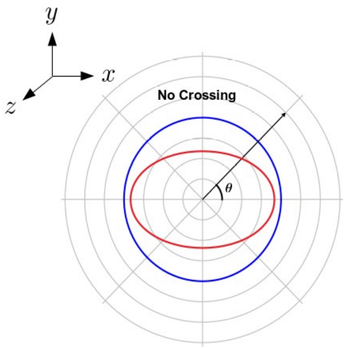
$$F^{\rm fin}_{\overset{(-)}{\nu}_e,\overset{(-)}{\nu}_\mu}[\vec{p}] = (1-f^{\rm D}_{\vec{p}})F^{\rm ini}_{\overset{(-)}{\nu}_e,\overset{(-)}{\nu}_\mu}[\vec{p}] + f^{\rm D}_{\vec{p}}F^{\rm ini}_{\overset{(-)}{\nu}_\mu,\overset{(-)}{\nu}_e}[\vec{p}]$$

- This signal can be the first ever direct probe of testing the ***neutrino-neutrino self-interaction***.
- The final output of fast conversions can have implications even in the ***nucleosynthesis of elements***, astrophysics of ***binary neutron star mergers***, ***diffuse SN background*** and can be detected in future experiments.

Extension towards multidimensional framework

2 (sp.) +2 (mom.)+1 (time) dim.

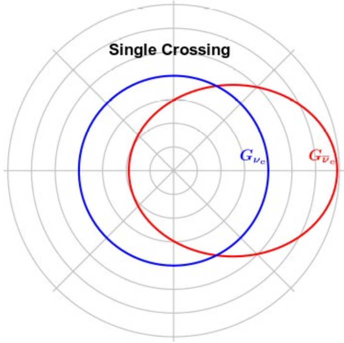
No excess



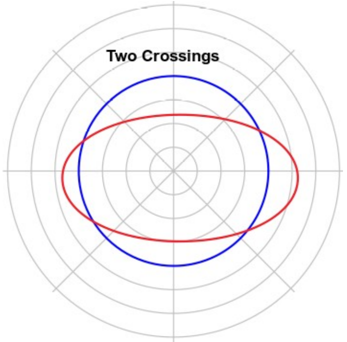
No Growth !!

Only Forward excess

$$G[\theta] = \begin{cases} \frac{A-1}{2\pi}, & \text{if } v_x = \cos\theta > 0 \\ \frac{1}{2\pi}, & \text{if } v_x = \cos\theta < 0\,. \end{cases}$$



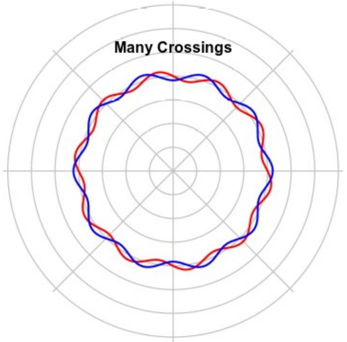
Forward and Backward excess



$$G[\theta] = \begin{cases} \frac{A-1}{2\pi}, & \text{if } v_x = \cos\theta > 0 \;\&\; v_y = \sin\theta > 0 \\ \frac{1}{2\pi}, & \text{if } v_x = \cos\theta < 0 \;\&\; v_y = \sin\theta > 0 \\ \frac{A-1}{2\pi}, & \text{if } v_x = \cos\theta < 0 \;\&\; v_y = \sin\theta < 0 \\ \frac{1}{2\pi}, & \text{if } v_x = \cos\theta > 0 \;\&\; v_y = \sin\theta < 0\,. \end{cases}$$

Turbulent density profile

$$G[\theta] = \frac{A}{2\pi} + c_1 \cos m\theta + c_2 \sin m\theta$$



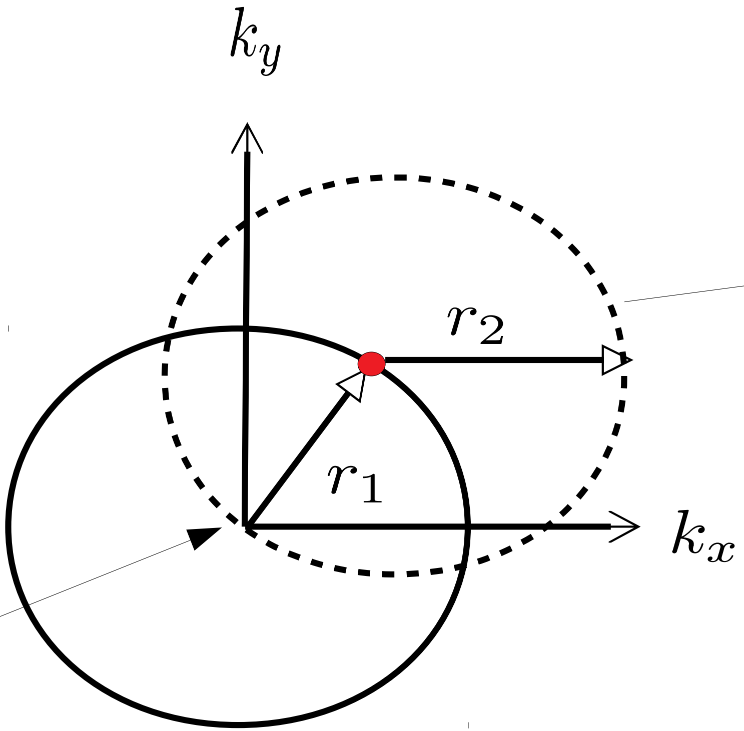
Numerical recipe : General Dispersion relation solver

$$\Pi^{\mu\nu}[\vec{k},\omega] = \eta^{\mu\nu} + \int d\Gamma\ v^{\mu}v^{\nu}\frac{G_{\vec{v}}}{k^l v_l}$$

Solver :
1) FSOLVE
2) QUAD

$$\mathbf{det}[\mathbf{\Pi}]_{\tilde{\mathbf{k}}\neq\mathbf{0}} = \mathbf{0}$$

Method :
1) Powell's Conjugate
2) Adaptive quadrature



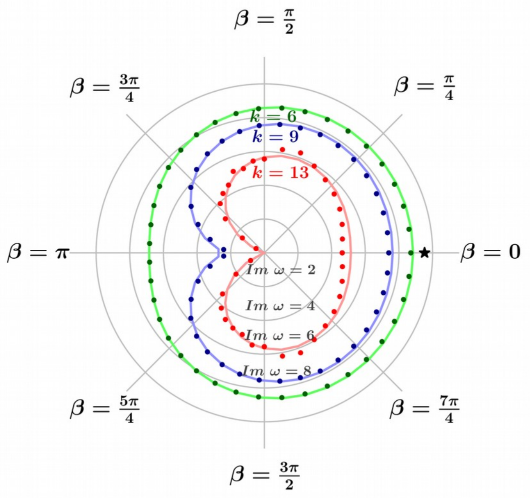
Bhattacharyya, Dasgupta (2021)
arXiv : 2101.01226

$$\mathbf{det}[\mathbf{\Pi}]_{\tilde{\mathbf{k}}=\mathbf{0}} = \mathbf{0}$$

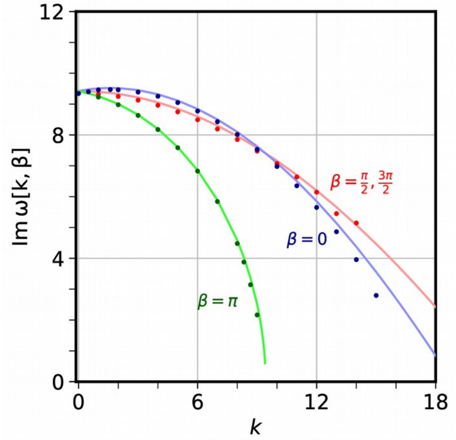
Simple algebraic equation

Results : Only forward excess / Single crossing

$$A = 0$$

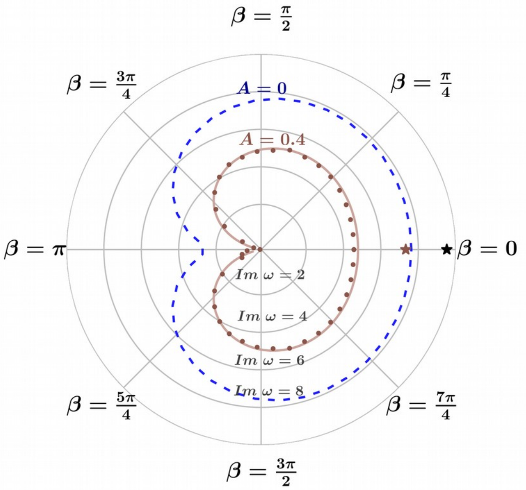


1) Linear growth rate has the same symmetry as ELN
2) k = 0 mode is unstable
3) Growth rate decreses as k increase.
4) Decrease is much slower along symmetry axis
5) Fourier mode with maximum growth rate located along non-symmetric axis.

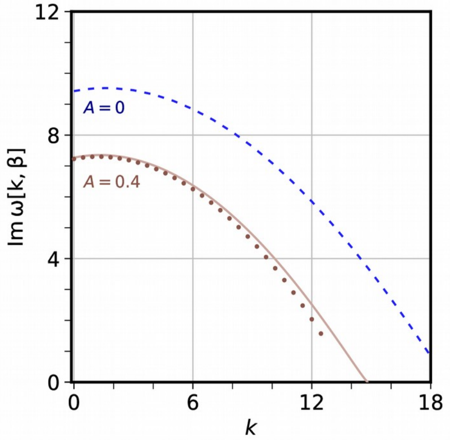


$$A \neq 0$$

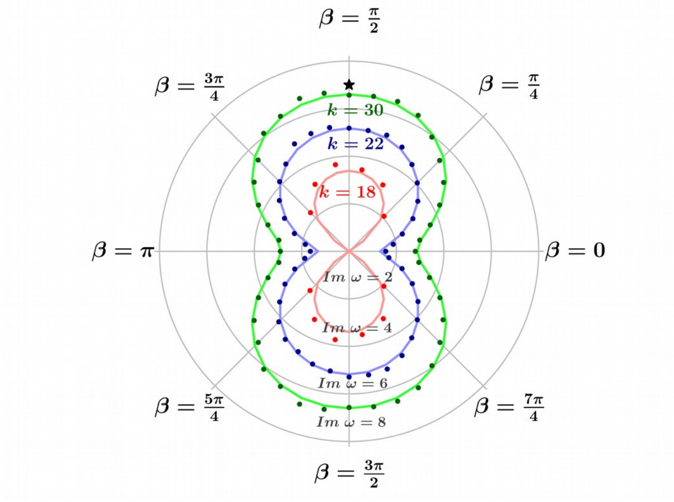
$$k_x = k \cos \beta$$
$$k_y = k \sin \beta$$



1) Above (1)-(5) points remain unchanged
2) Decrease as a function of k is much faster
3) Overall growth rate is suppressed.

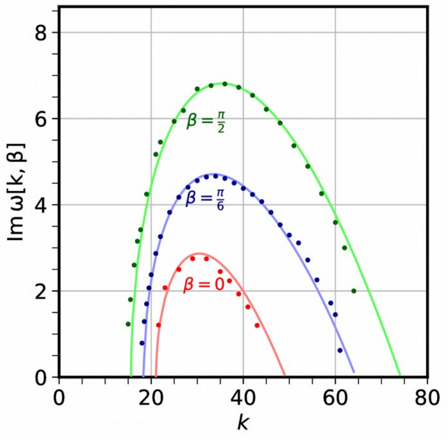


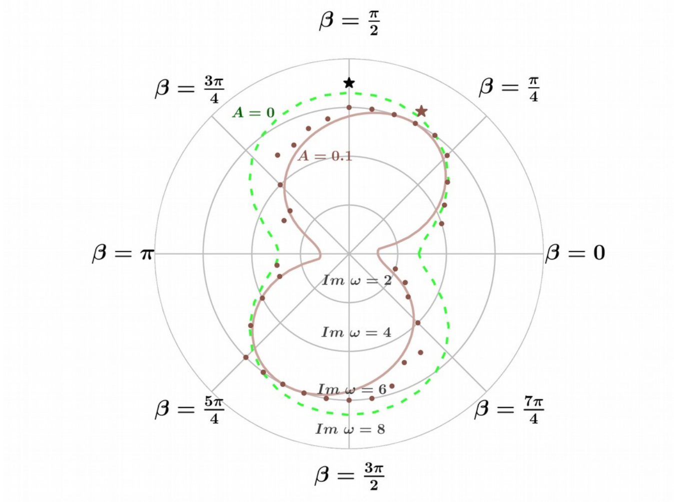
Results : Both forward and backward excess / Double crossing



$$A = 0$$

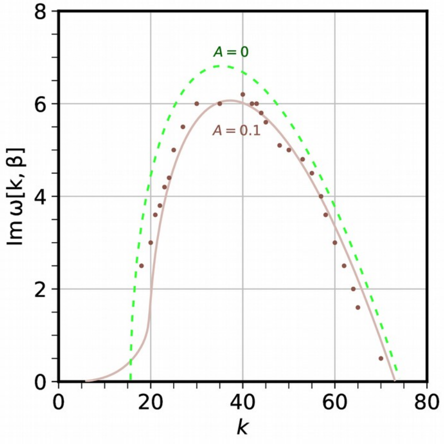
1) Linear growth rate has the same symmetry as ELN
2) k = 0 mode is stable
3) Growth rate shows a lorentzian nature w.r.t k
4) Wider along β = 90° than β = 0°
5) More growth along β = 90° than β = 0°
5) Fourier mode with maximum growth rate located along symmetric axis.



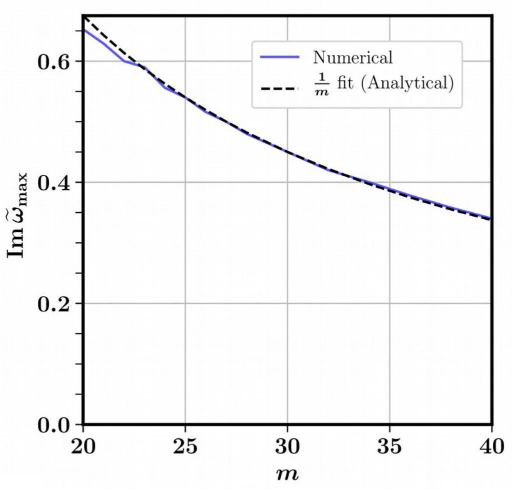


$$A \neq 0$$

1) Lepton asymmetry breaks symmetry between β = 90° than β = 0°
2) Shifts the symmetry towards diagonal
2) Previous (2), (3), (4) and (5) remains same.
3) Wideness is lesser compared to A = 0
4) Shifted more towards k = 0
5) Overall growth rate suppressed.
6) Fourier mode with maximum growth rate shifted along diagonals
7) All effects from (1)-(6) gets more pronounced with increse of A



Results : Turbulent profile / Multiple crossings



$$\mathbf{\Pi}[k,\beta,\omega] = \boldsymbol{\eta} + \boldsymbol{\psi}[\omega,k,\beta] = \boldsymbol{\eta} + \int_0^{2\pi} d\theta\ f[\theta,\omega,k,\beta]\, G[\theta]\, \mathbf{W}[\theta]$$

$$f[\theta,\omega,k,\beta] = \frac{1}{\omega - k\cos(\theta-\beta)} \qquad \mathbf{W}[\theta] = \begin{pmatrix} 1 & \cos\theta & \sin\theta \\ \cos\theta & \cos^2\theta & \sin\theta\cos\theta \\ \sin\theta & \sin\theta\cos\theta & \sin^2\theta \end{pmatrix}$$

$$\psi_{ij}[\omega_{\max},k_{\max},\beta_{\max}] = \int_0^{2\pi} d\theta\, f[\theta,\omega_{\max},k_{\max},\beta_{\max}]\, G[\theta] W_{ij}[\theta] = \int_0^{2\pi} d\theta\ \exp[F_{ij}[\theta]]$$

Bhattacharyya, Dasgupta (2021)
arXiv : 2101.01226

Saddle Point Approximation

$$F_{ij}[\theta] = F_{ij}[\theta_0] + (\theta-\theta_0)^2 \left.\frac{d^2 F_{ij}[\theta]}{d\theta^2}\right|_{\theta=\theta_0}$$

$$\psi_{ij}[\omega_{\max},k_{\max},\beta_{\max}] = \widetilde{G}[\theta_0]\frac{W_{ij}[\theta_0]}{m\,\widetilde{\omega}_{\max}} \longrightarrow Im\,\widetilde{\omega}_{\max} \propto \frac{1}{m}$$

Take away message :

1) The symmetries of $Im\,\omega$ in the k-β plane, its ***radial*** (i.e., vs. k) and ***angular*** (i.e., vs β) variation, the ***stability of the k=0 mode***, ***overall linear growth*** and ***the position of the Fourier mode with the highest growth rate***, etc., all have an ***intimate connection with the various symmetries*** of the neutrino angular distributions and one can analytically understand them in great detail

2) ELNs with ***large number of zero crossings*** lead to a relatively ***smaller growth rate***, essentially decreasing as 1/m where m is the number of crossings. We speculate that this may be important for many realistic environments where electron-neutrino and anti electron neutrino distributions are close to each other and crossings occur in the ELNs due to noise or fluctuations